

Gravitational Wave Gastronomy

CMSA | Phase Transitions and Topological Defects in the Early Universe

David Dunsky, Anish Ghoshal, Hitoshi Murayama, Yuki Sakakihara, and Graham White

Gravitational Wave Gastronomy

Definition:



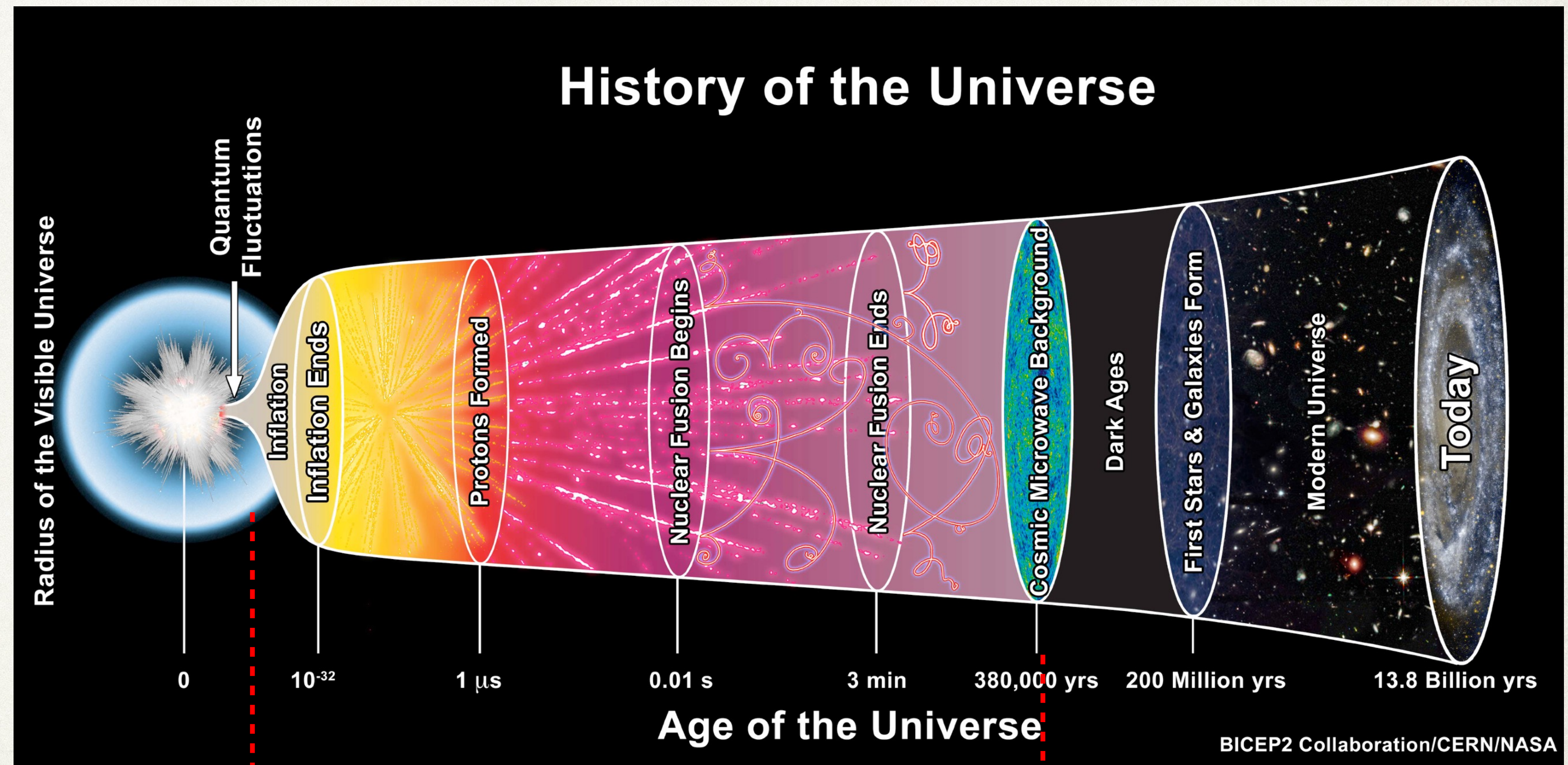
- *A savory variation of **Gravitational Wave Astronomy**.*
- *The search for gravitational wave signatures from the “eating” of one topological defect by another. Observed to occur in hybrid topological defects (i.e. domain walls bounded by strings, and strings bounded by monopoles.)*
- *Seasoned with equal parts of gravitational wave physics, early universe cosmology, and grand unification.*

Outline

- *Appetizers:* Gravitational Wave (GWs) as probes of the early universe, and relationship to Grand Unified Theories (GUTs)
- *Entree:* Dynamics and the GW signatures from hybrid topological defects
- *Dessert:* Outlook for distinguishing GUT symmetry breaking chains by such GW signatures

Gravity Waves as Probes of the Early Universe

- Early universe transparent to GWs
- Potentially remarkable probe of moments after Big Bang

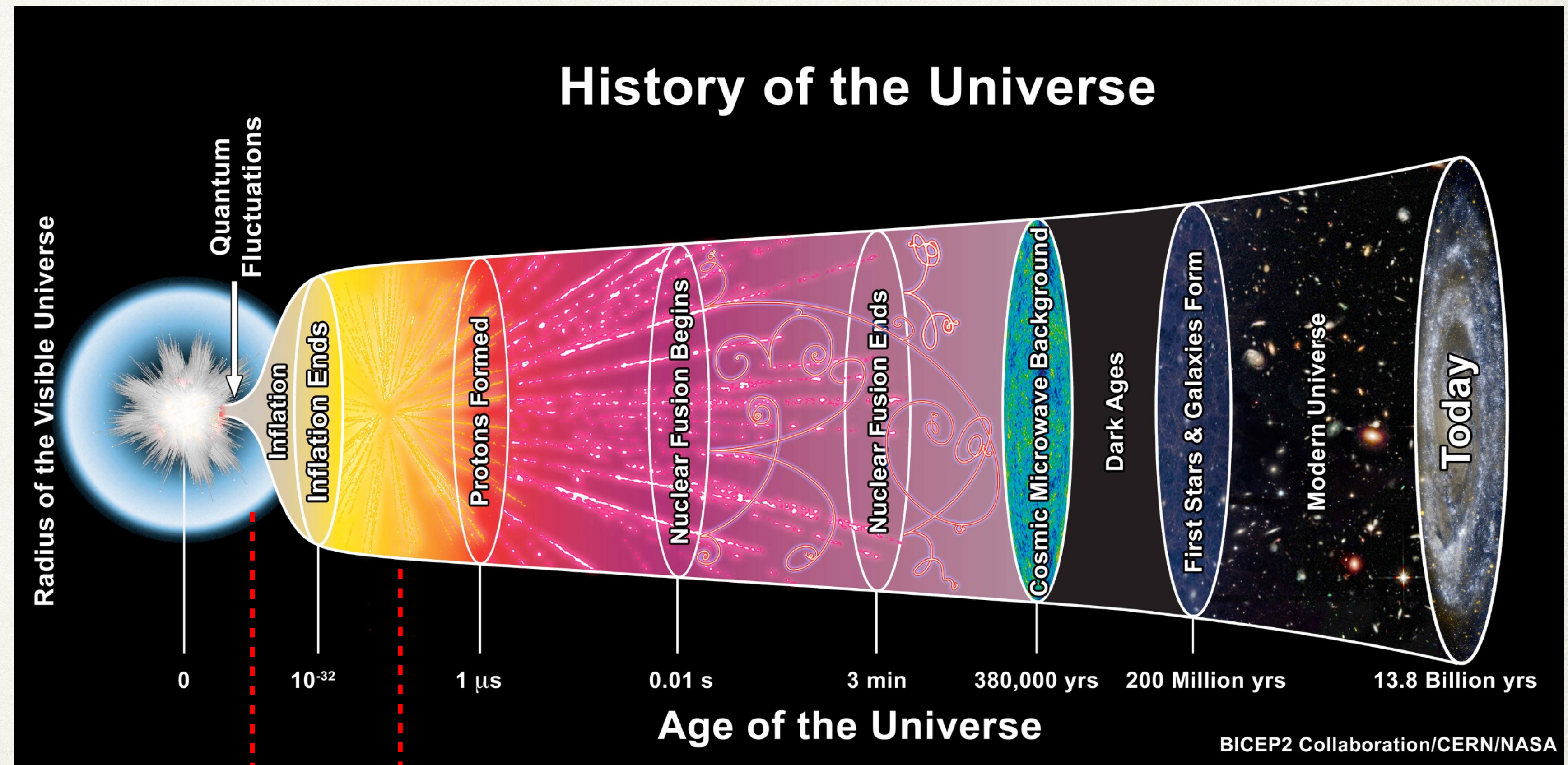


Electromagnetic waves (optical light) ←

Gravitational waves ←

Gravity Waves as Probes of the Early Universe

- Early universe transparent to GWs
- Potentially remarkable probe of moments after Big Bang



Gravitational waves ←

Colliders ←

Gravitational Waves and GUTs

- Grand Unified Theories (GUTs) intimately tied to gravitational waves:
 - True symmetries of nature are restored at high temperatures in early universe
 - As the universe expands and cools, gauge symmetries describing nature spontaneously break

$$G \rightarrow G_i \rightarrow G_{i+1} \rightarrow \cdots \rightarrow G_{\text{SM}}$$

← Colliders

Universe →

Gravitational Waves and GUTs

- Grand Unified Theories (GUTs) intimately tied to gravitational waves:
 - When breaking, many gauge groups leave behind topological relics if non-trivial vacuum

$$\pi_{2-D}(G_j/G_{j+1}) \neq 0$$

- $D = 0 \rightarrow$ Monopoles form



- $D = 1 \rightarrow$ Cosmic Strings form



- $D = 2 \rightarrow$ Domain Walls form



- Topological defects massive and can accelerate to relativistic speeds \rightarrow source of GW!

Hybrid Topological Defects

- For some chains, strings bounded by monopoles, or walls bounded by strings

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$$G \xrightarrow{\text{monopoles}} H \times U(1)$$



Hybrid Topological Defects

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$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$



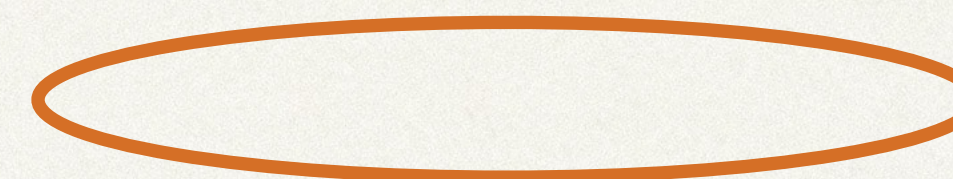
Hybrid Topological Defects

- For some chains, strings bounded by monopoles, or walls bounded by strings

$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$



$$G \xrightarrow{\text{strings}} H \times Z_2$$



$$(\pi_0(H \times Z_2) \rightarrow \pi_1(G/(H \times Z_2)))$$

Hybrid Topological Defects

- For some chains, strings bounded by monopoles, or walls bounded by strings

$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$



$$G \xrightarrow{\text{strings}} H \times Z_2 \xrightarrow{\text{walls}} H$$



Hybrid Topological Defects

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- Tension force of string or wall causes (relativistic) oscillations

Hybrid Topological Defects

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Gravitational radiation (Unique fingerprints of chains)

Hybrid Topological Defects

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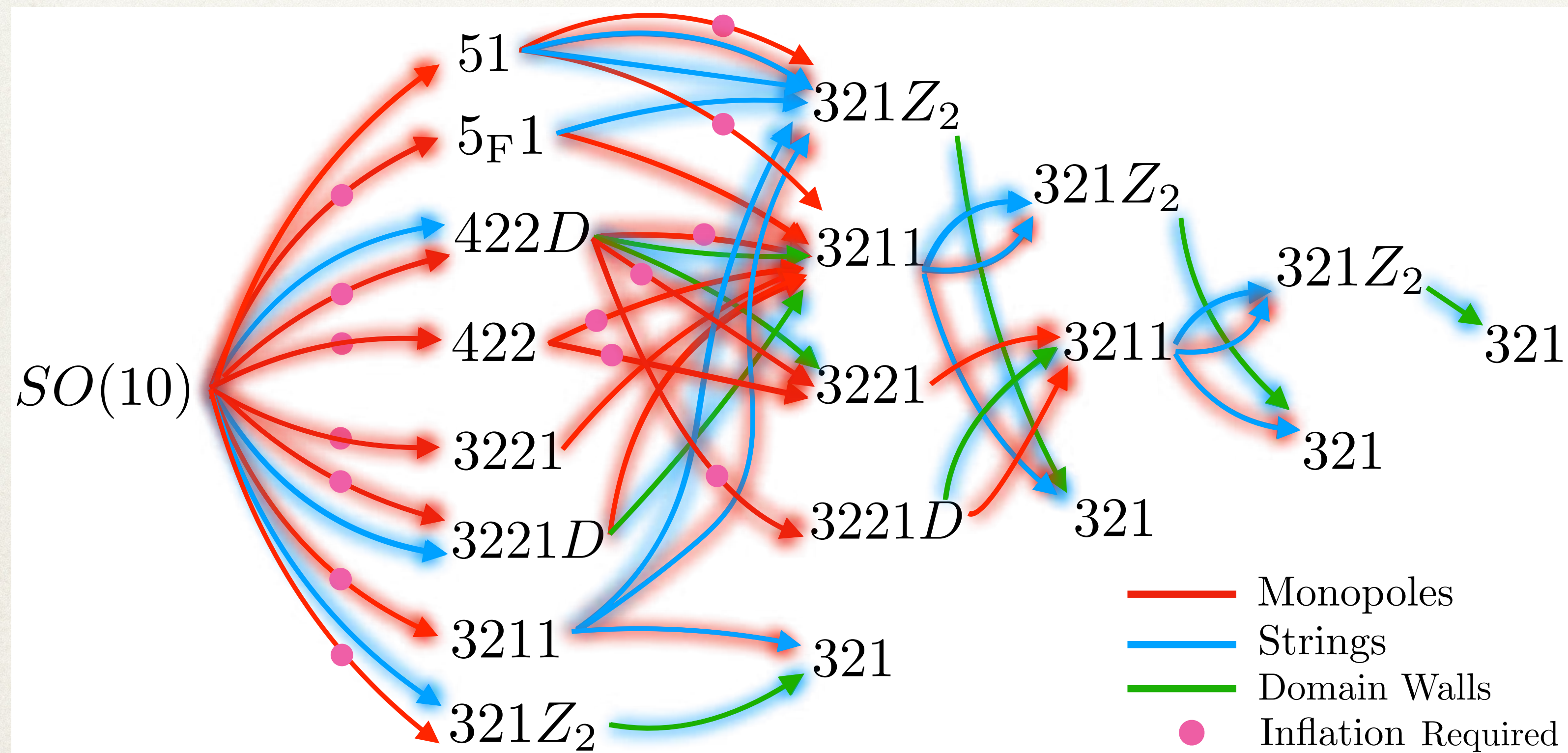
$$G \xrightarrow{\text{strings}} H \times Z_2 \xrightarrow{\text{walls}} H$$



- Tension force of string or wall causes (relativistic) oscillations

↓
Gravitational radiation (Unique fingerprints of chains)
↓
Decay

Symmetry Breaking Chains



Group abbreviations

$$51 = SU(5) \times U(1)_X$$

$$5_F 1 = SU(5)_{\text{flipped}} \times U(1)_{\text{flipped}}$$

$$422 = SU(4)_c \times SU(2)_L \times SU(2)_R$$

$$3221 = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$3211 = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$$

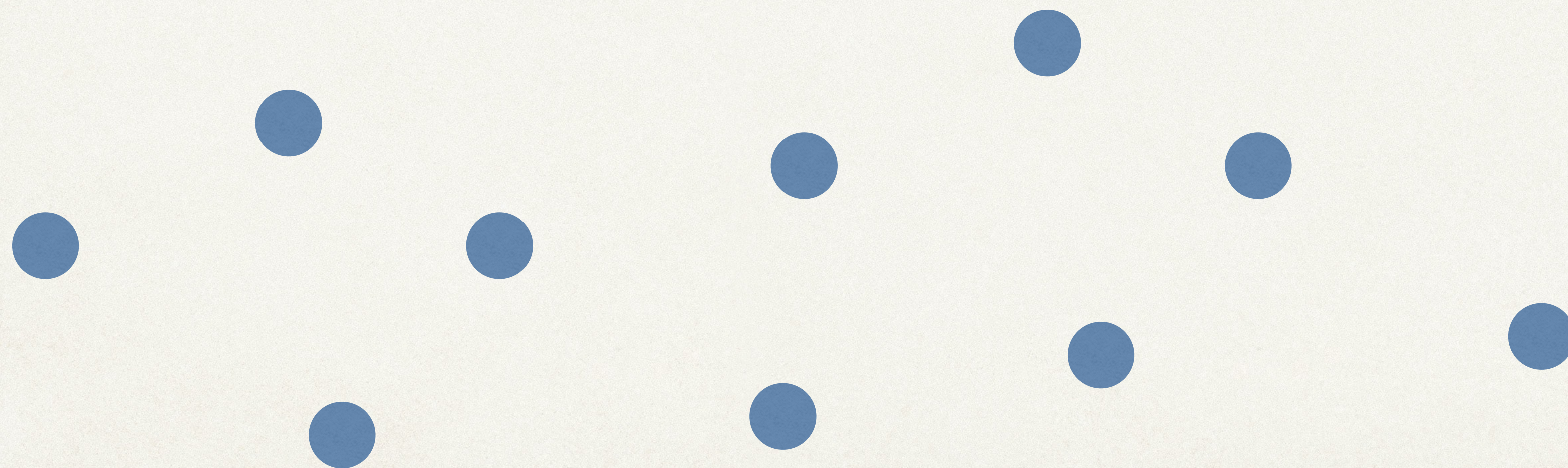
$$321 = SU(3)_c \times SU(2)_L \times U(1)_Y$$

D = D-Parity

Four Course Menu

- Monopoles eating strings (nucleation) *See also Dror '19, Buchmuller '21 (Kai's talk!)*

Monopole formation



Four Course Menu

- Monopoles eating strings (nucleation)

Monopole formation \longrightarrow Inflation

Four Course Menu

- Monopoles eating strings (nucleation)

Monopole formation \longrightarrow Inflation \longrightarrow String formation



Four Course Menu

- Monopoles eating strings (nucleation)

Monopole formation → Inflation → String formation → Wait...



Four Course Menu

- Monopoles eating strings (nucleation)

Monopole formation \longrightarrow Inflation \longrightarrow String formation \longrightarrow Nucleation

- String rest mass converted to monopole kinetic energy



Decay via gravitational radiation



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Decay via gravitational radiation



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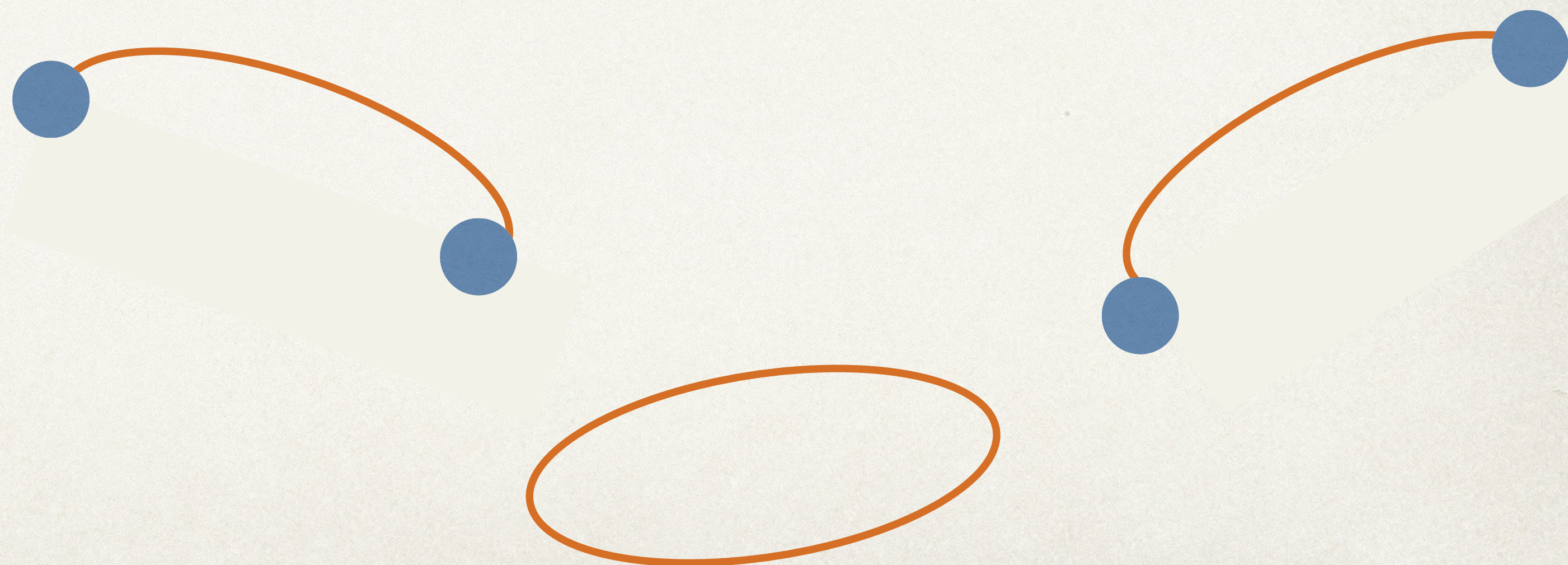
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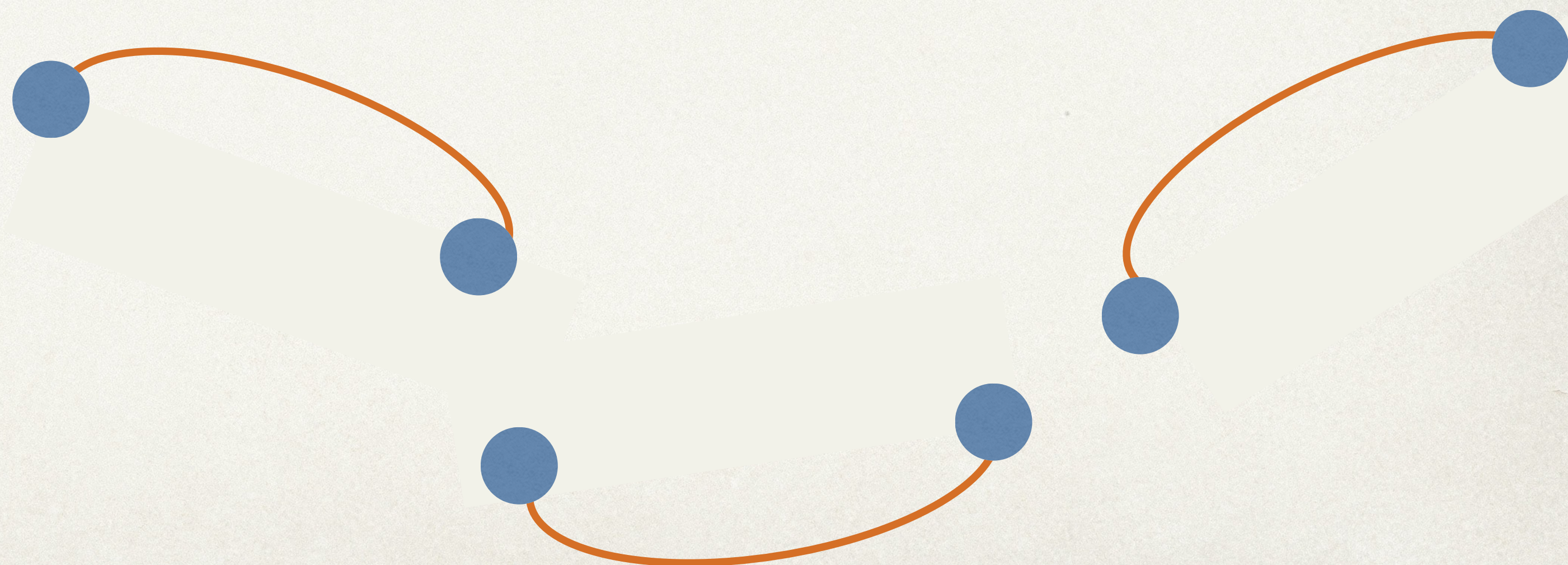
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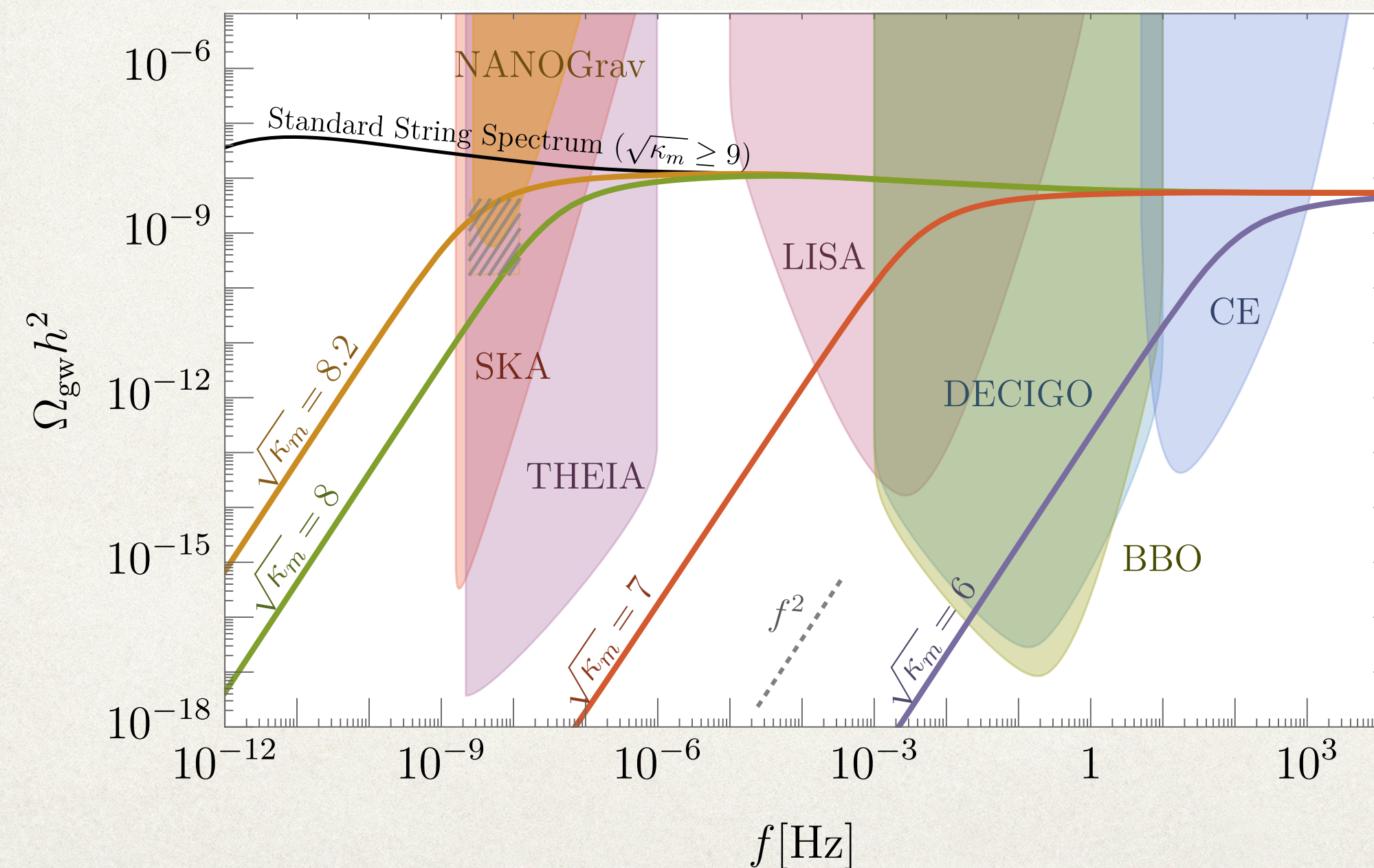
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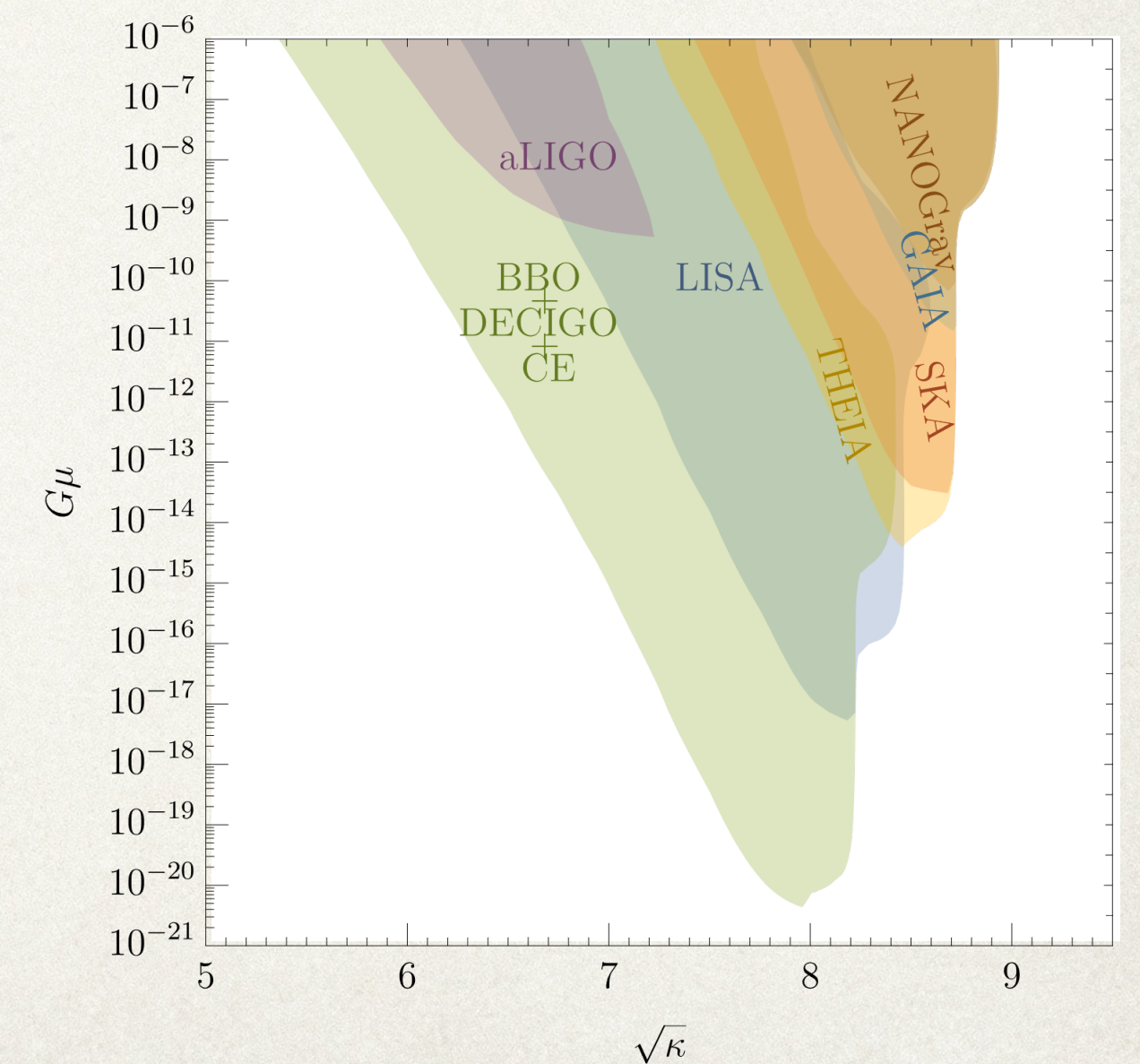
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Monopole formation \longrightarrow Inflation \longrightarrow String formation \longrightarrow Nucleation



(See Kai's talk for more details)

Cosmic Course	IR	UV
Monopoles eating strings (Nucleation)	f^2	f^0



Four Course Menu

- Monopoles eating strings (nucleation)
- Strings eating walls (nucleation)

String formation



Four Course Menu

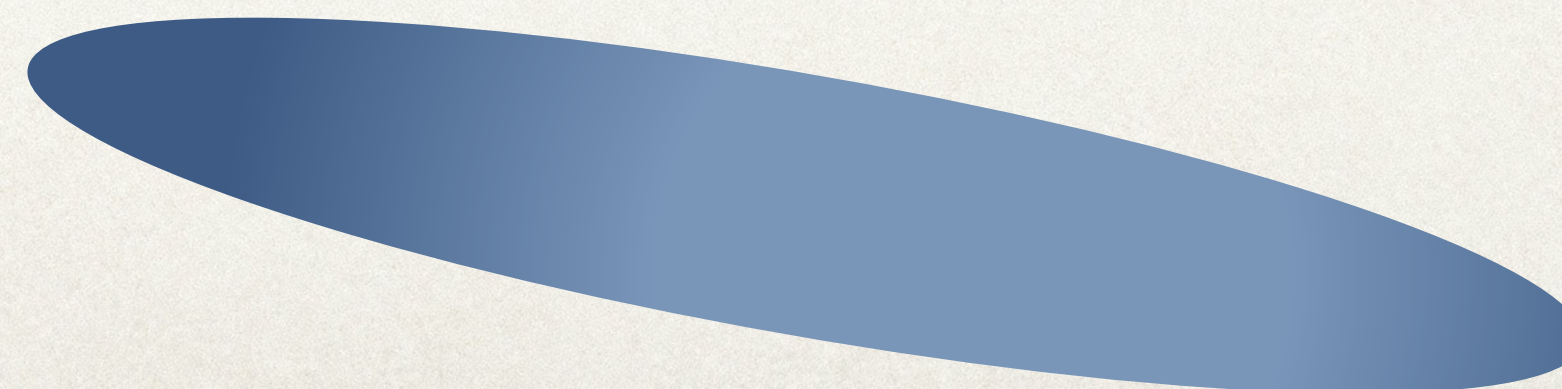
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String formation \longrightarrow Inflation

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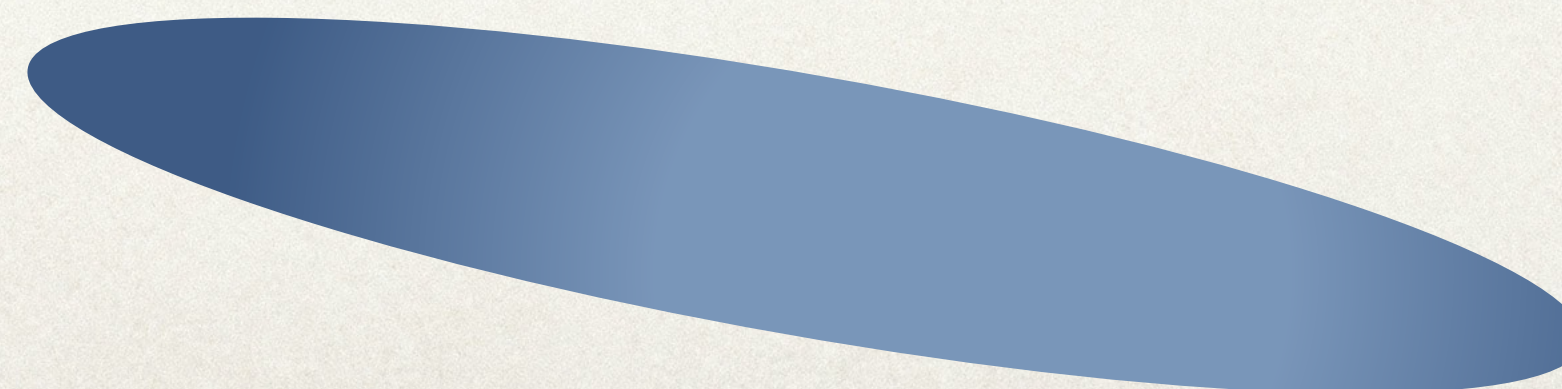
String formation → Inflation → Wall formation



Four Course Menu

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String formation → Inflation → Wall formation → Wait...



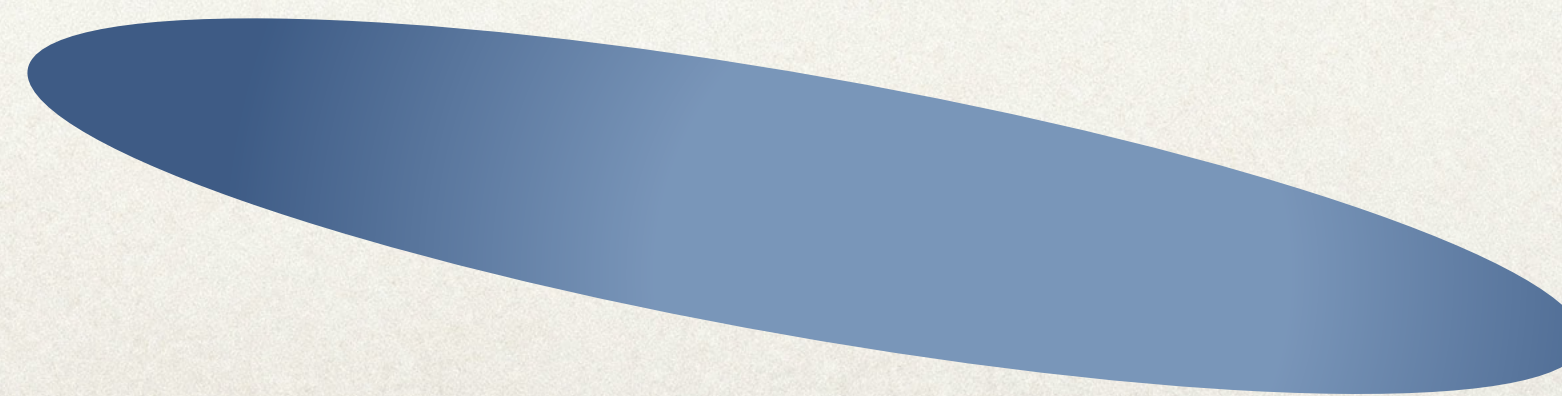
Four Course Menu

- Monopoles eating strings (nucleation)

- Strings eating walls (nucleation)

String formation → Inflation → Wall formation → Nucleation

- Wall rest mass converted to string kinetic energy (string expands and eats wall)



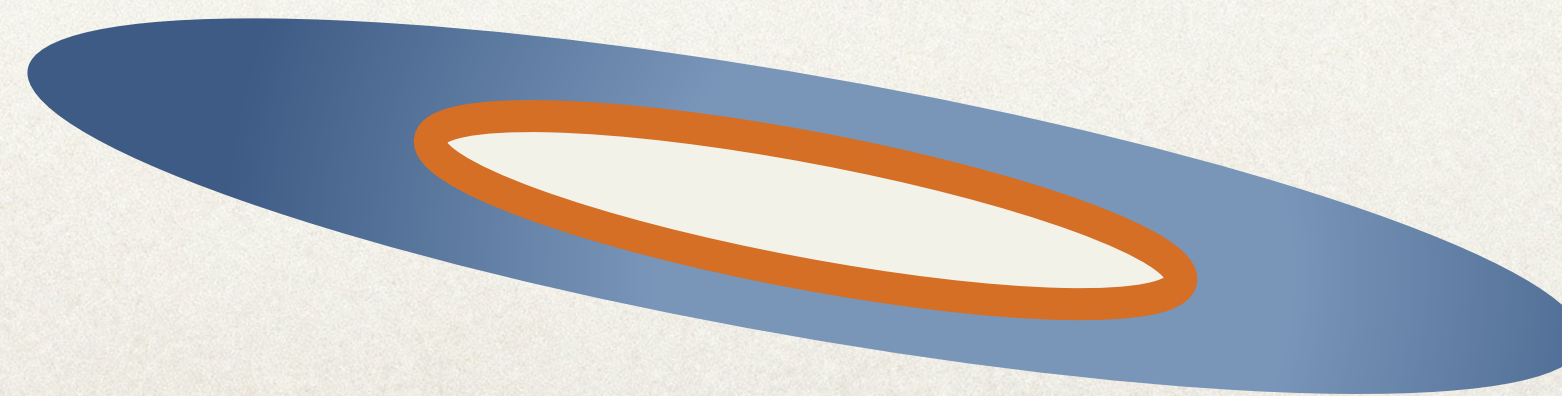
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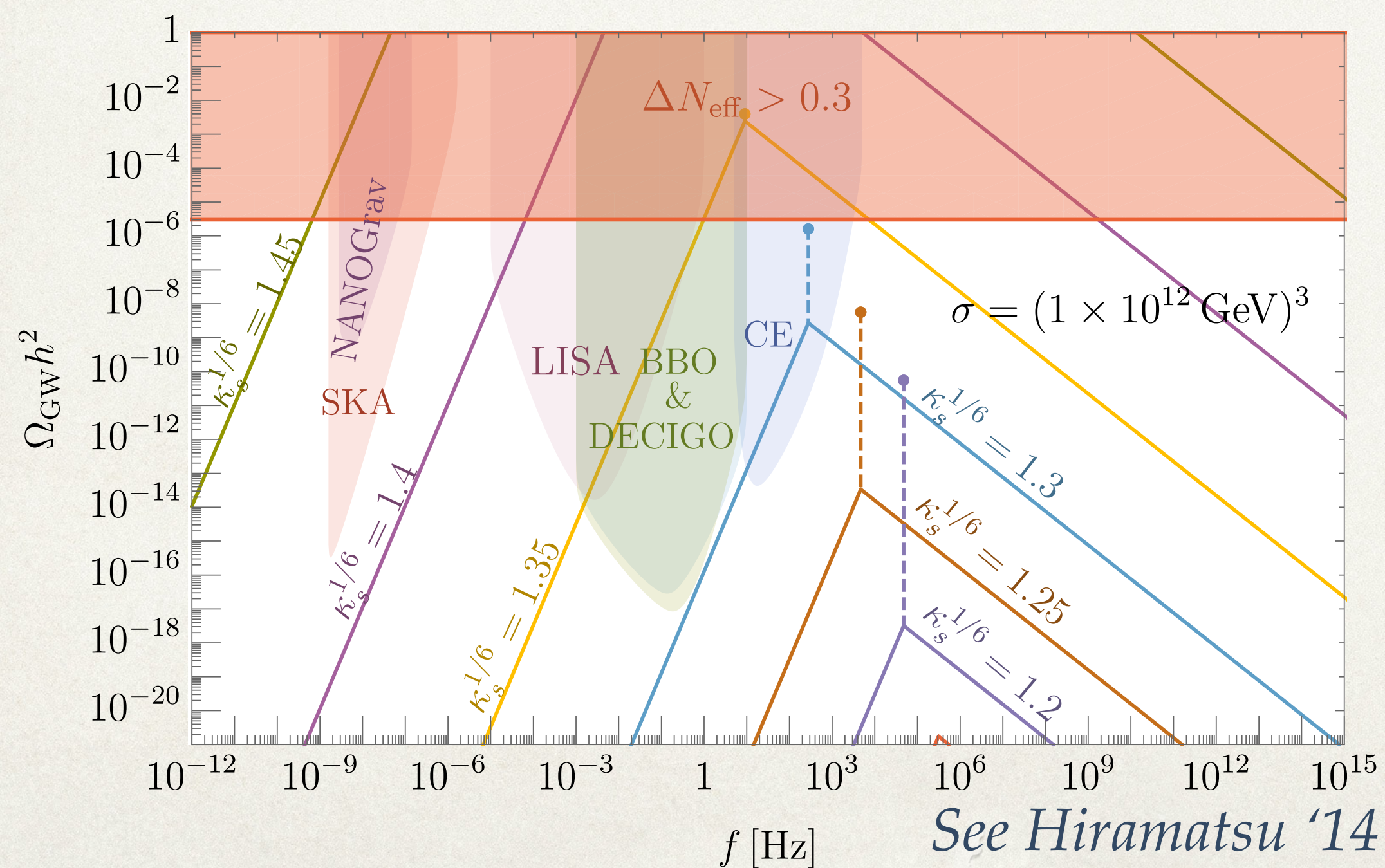
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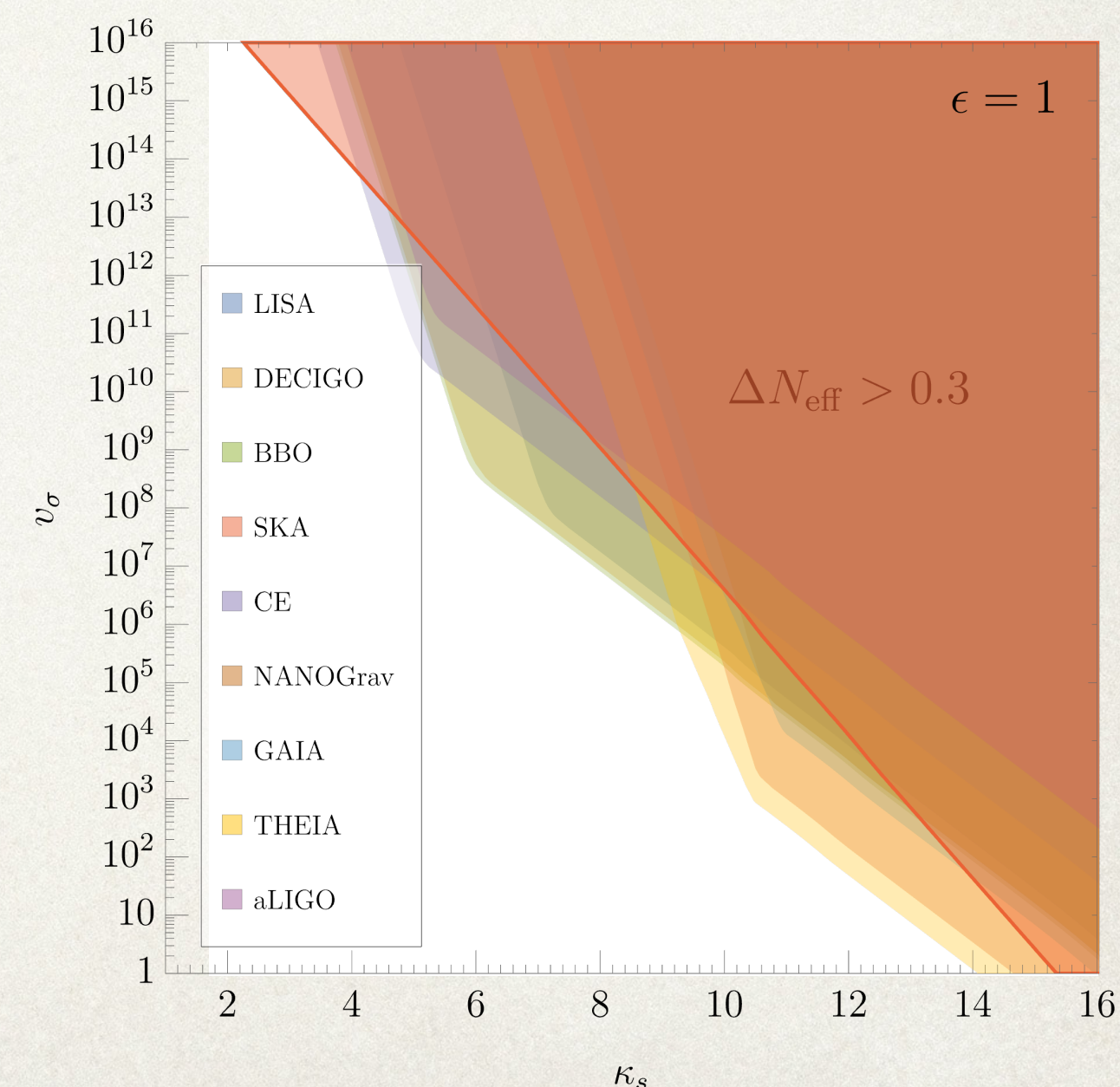
Four Course Menu

- Strings eating walls (nucleation)

String formation \longrightarrow Inflation \longrightarrow Wall formation \longrightarrow Nucleation



Cosmic Course	IR	UV
Monopoles eating strings (Nucleation)	f^2	f^0
Strings eating walls (Nucleation)	f^3	f^{-1}



Four Course Menu

- Monopoles eating strings (nucleation)
- Strings eating walls (nucleation)

← See [2111.08750](#) for more details

- Strings eating monopoles (collapse)
- Walls eating strings (collapse)

← Focus more on this talk.
Defects formed *after* inflation

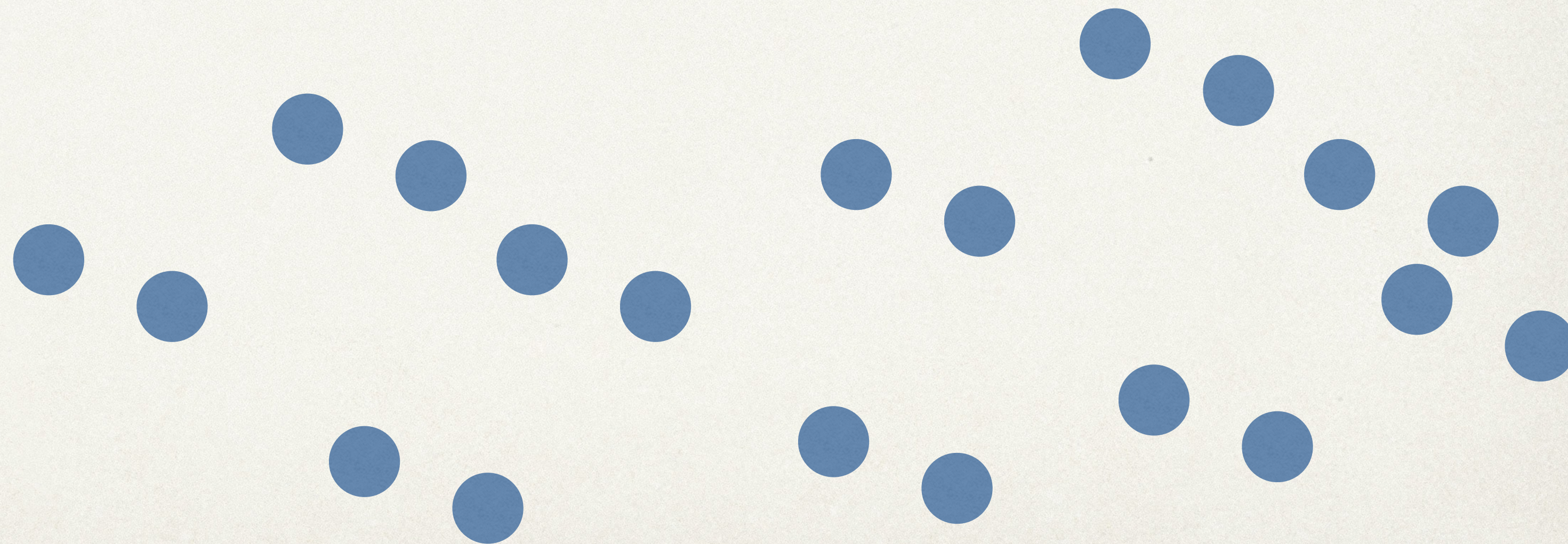
Strings Eating Monopoles (Collapse)

- Occurs in following scenario
Inflation \longrightarrow Monopole formation

$$n_m \sim H^3 \times \left(\frac{M_{\text{Pl}}}{v_m} \right)^2$$

Originally estimate
by Kibble

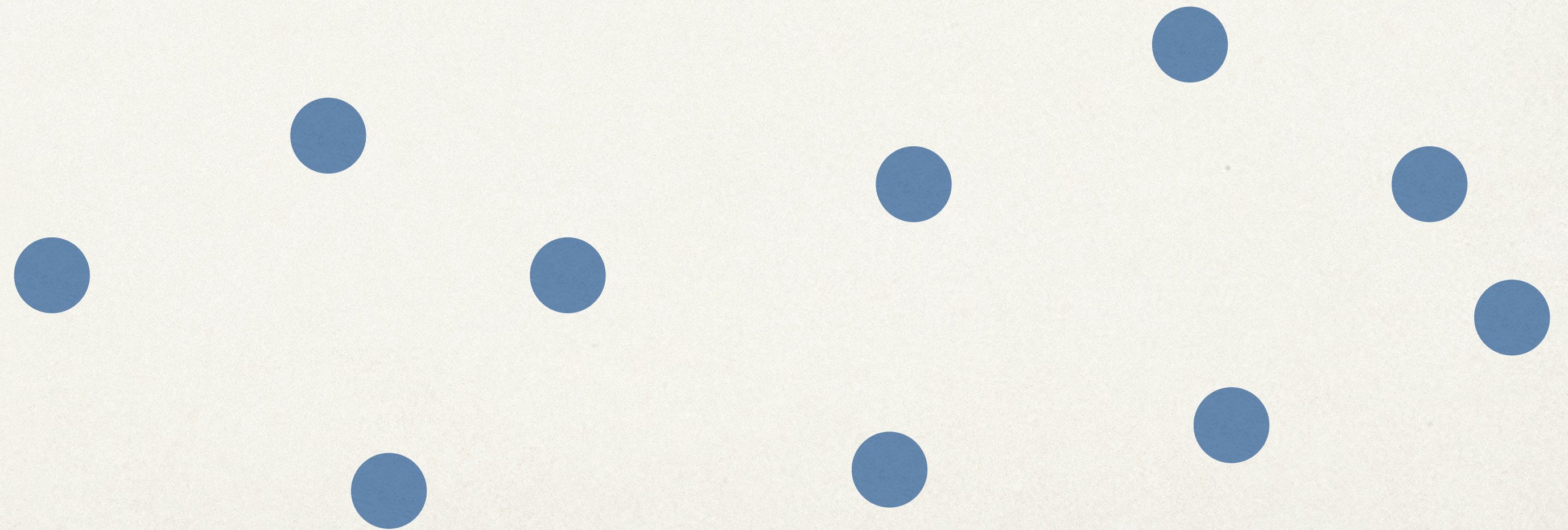
Zurek's
Enhancement



Strings Eating Monopoles (Collapse)

- Occurs in following scenario

Inflation → Monopole formation
& freeze-out



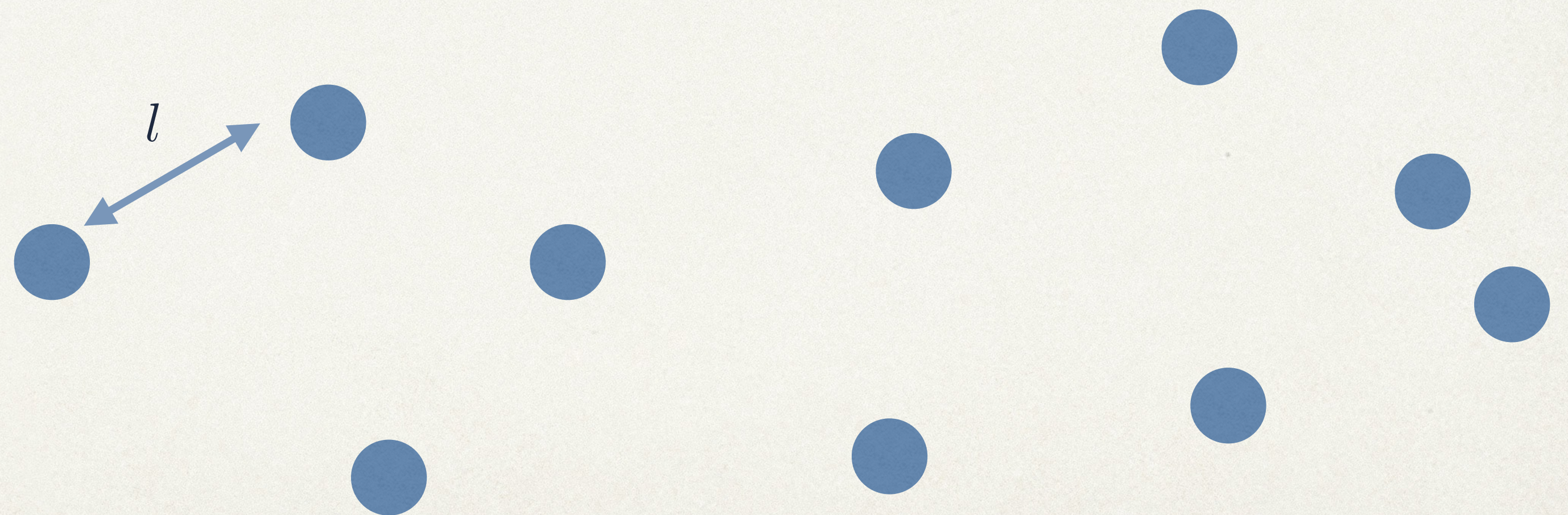
Strings Eating Monopoles (Collapse)

- Occurs in following scenario

Inflation \longrightarrow Monopole formation & freeze-out \longrightarrow String formation \longrightarrow Annihilation

$$l \approx \frac{1}{n_m (T = v_\mu)^{1/3}}$$

- String rest mass converted to monopole kinetic energy



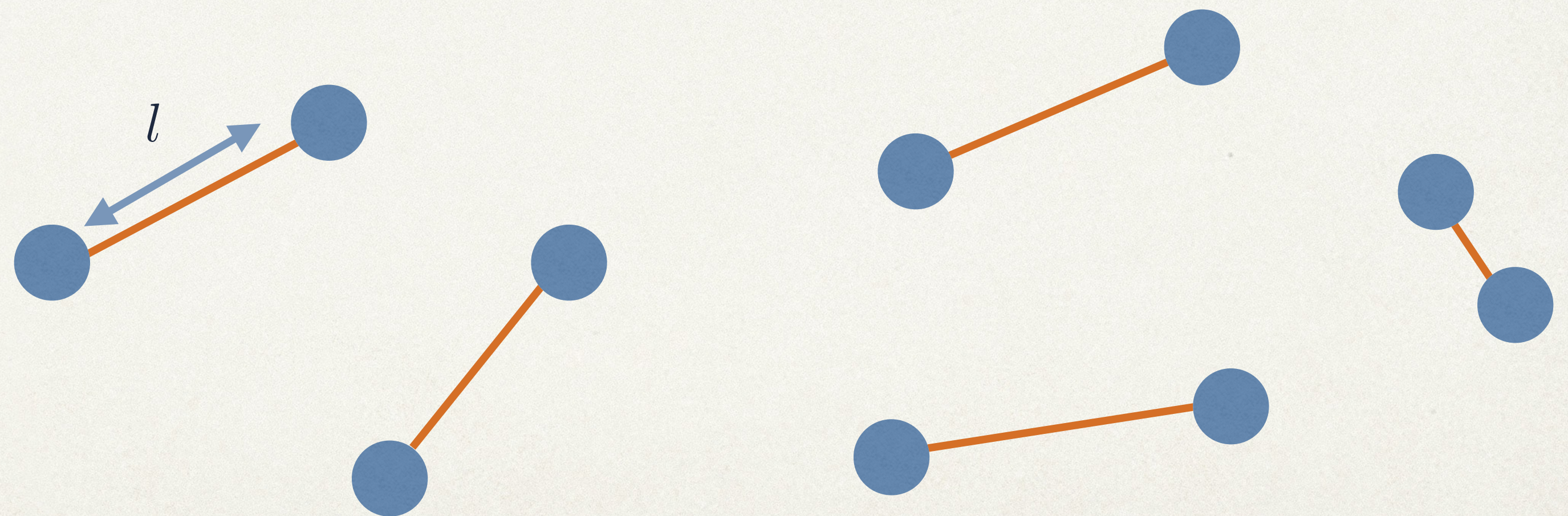
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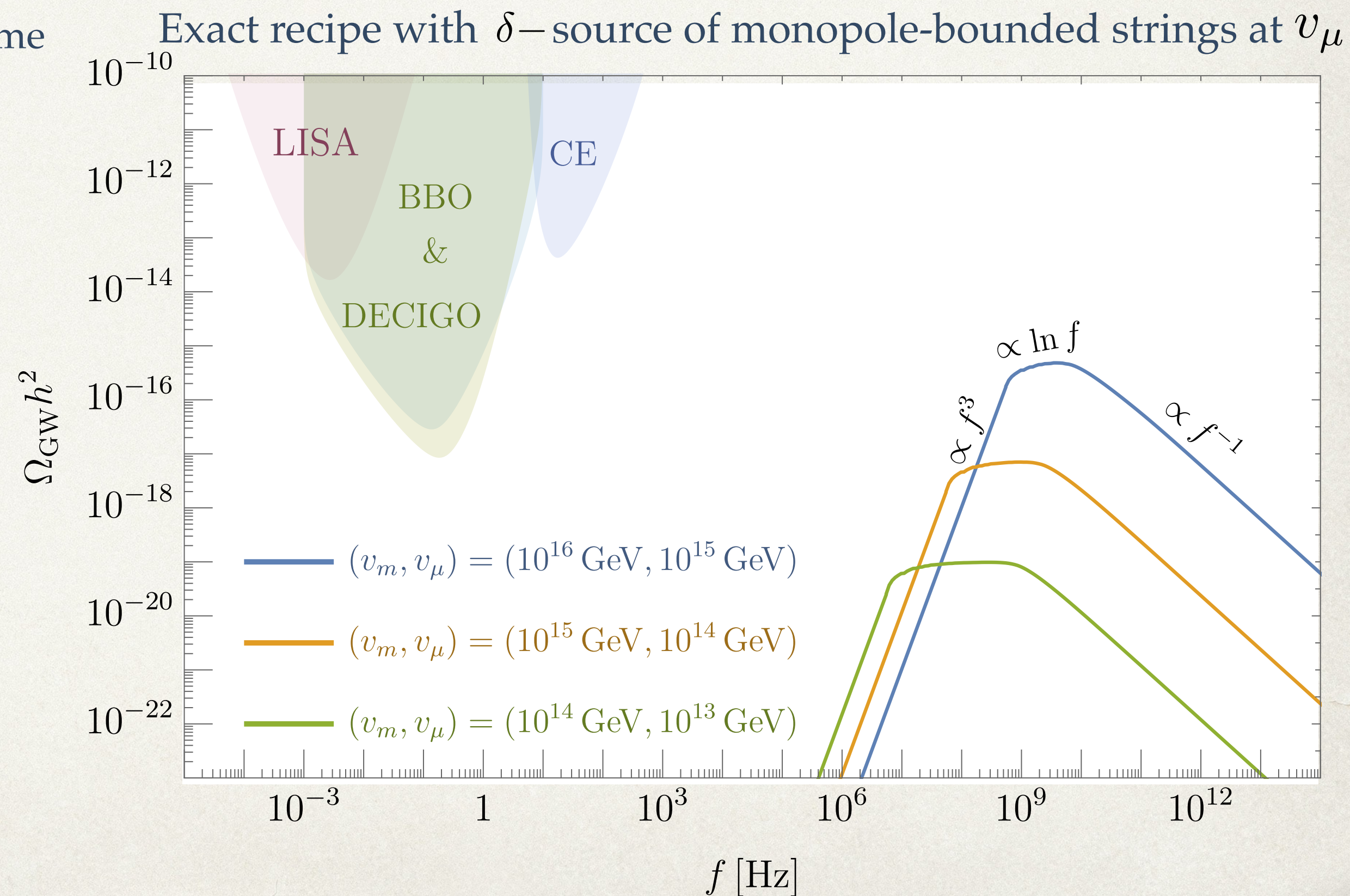
Monopole Burst Spectrum

- $\rho_{\text{GW,burst}} \approx n_m(v_\mu) \times P_{\text{GW}} \times \tau$
 $\sim G\mu^2$ (Roughly same as pure strings)
Leblond '09

- $\Omega_{\text{GW,burst}} \approx \frac{\rho_{\text{GW,burst}}}{\rho_{\text{BG}}(v_\mu)} \times \Omega_{\text{rad}} \sim 10^{-4} (G\mu)^{4/3}$

$$f_{\text{burst}} \sim \frac{1}{l} \frac{a(v_\mu)}{a(t_0)} \sim 10^8 \text{ Hz} \left(\frac{v_m}{10^{15} \text{ GeV}} \right)^{1/3}$$

- $\Omega_{\text{GW}} \propto f^{-1}$ and $\ln f$ at high frequencies,
 f^3 at lower frequencies



Walls Eating Strings (Collapse)

- Occurs in following scenario
Inflation → String formation



Walls Eating Strings (Collapse)

- Occurs in following scenario

Inflation \longrightarrow String formation \longrightarrow Wall formation \longrightarrow Eventual collapse

- Wall rest mass converted to string kinetic energy



Wall-bounded strings oscillate and decay via gravitational radiation

$$f_{\text{string}} \approx \mu \longrightarrow R_c \equiv \frac{\mu}{\sigma}$$
$$f_{\text{DW}} \approx \sigma R$$



Walls Eating Strings (Collapse)

- Occurs in following scenario

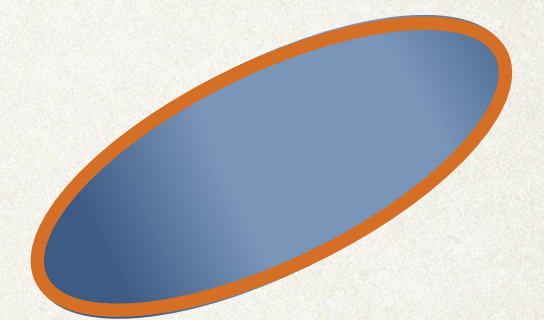
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Dynamics of Domain Wall Bounded Strings

- Combine domain wall and string Nambu-Goto actions

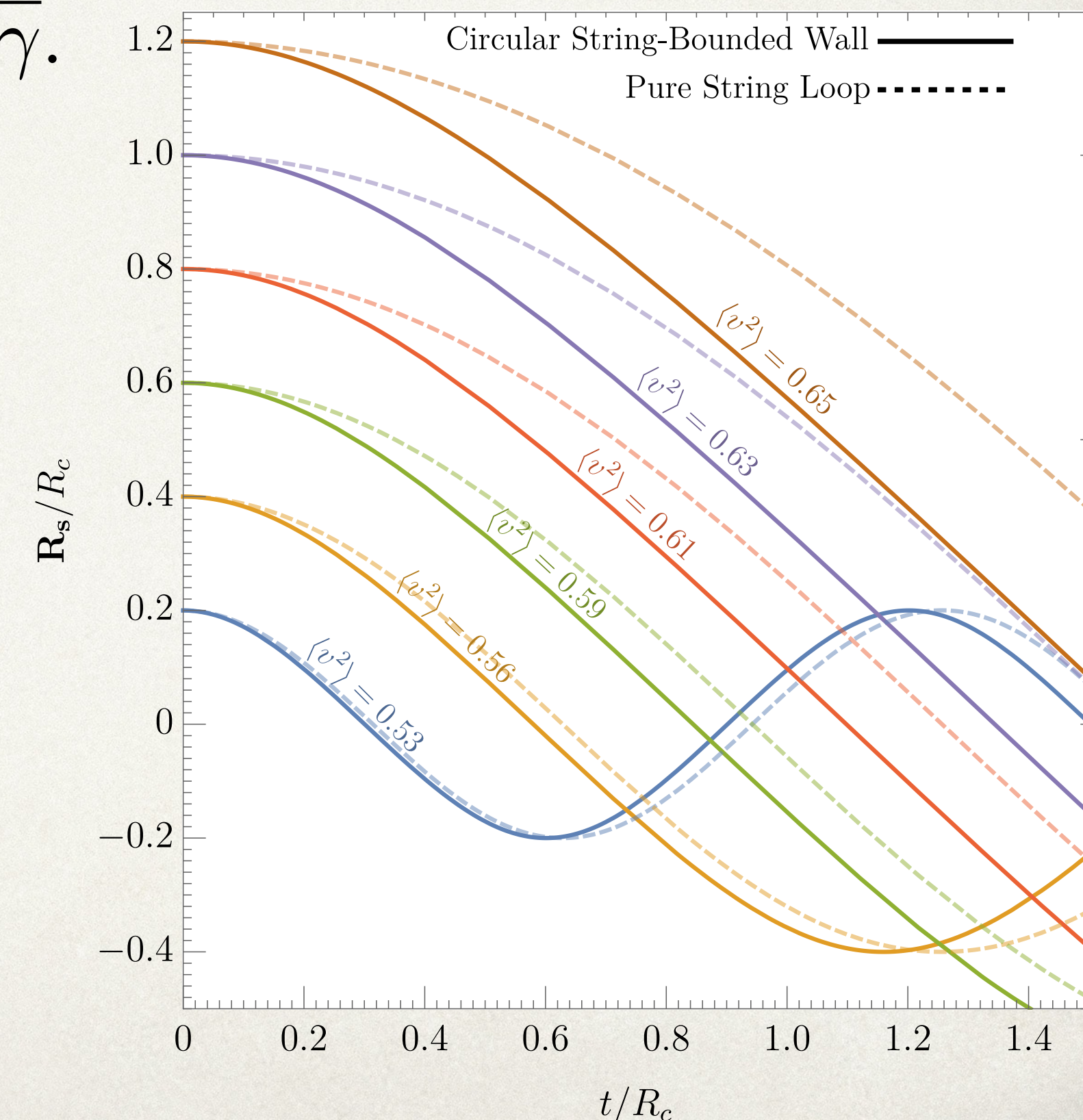
$$S = -\mu \int d^2\xi \sqrt{-\gamma} - \sigma \int d^3\xi \sqrt{-\gamma}.$$

- Identify boundary of domain wall with string radius

↓
E.O.M. of circular string boundary

String dominates dynamics for $|\mathbf{R}_s| < R_c$

Domain wall dominates dynamics for $|\mathbf{R}_s| > R_c$



Evolution of String-Wall Network

- Effect of wall tension on the infinite string network?

- Recipe:

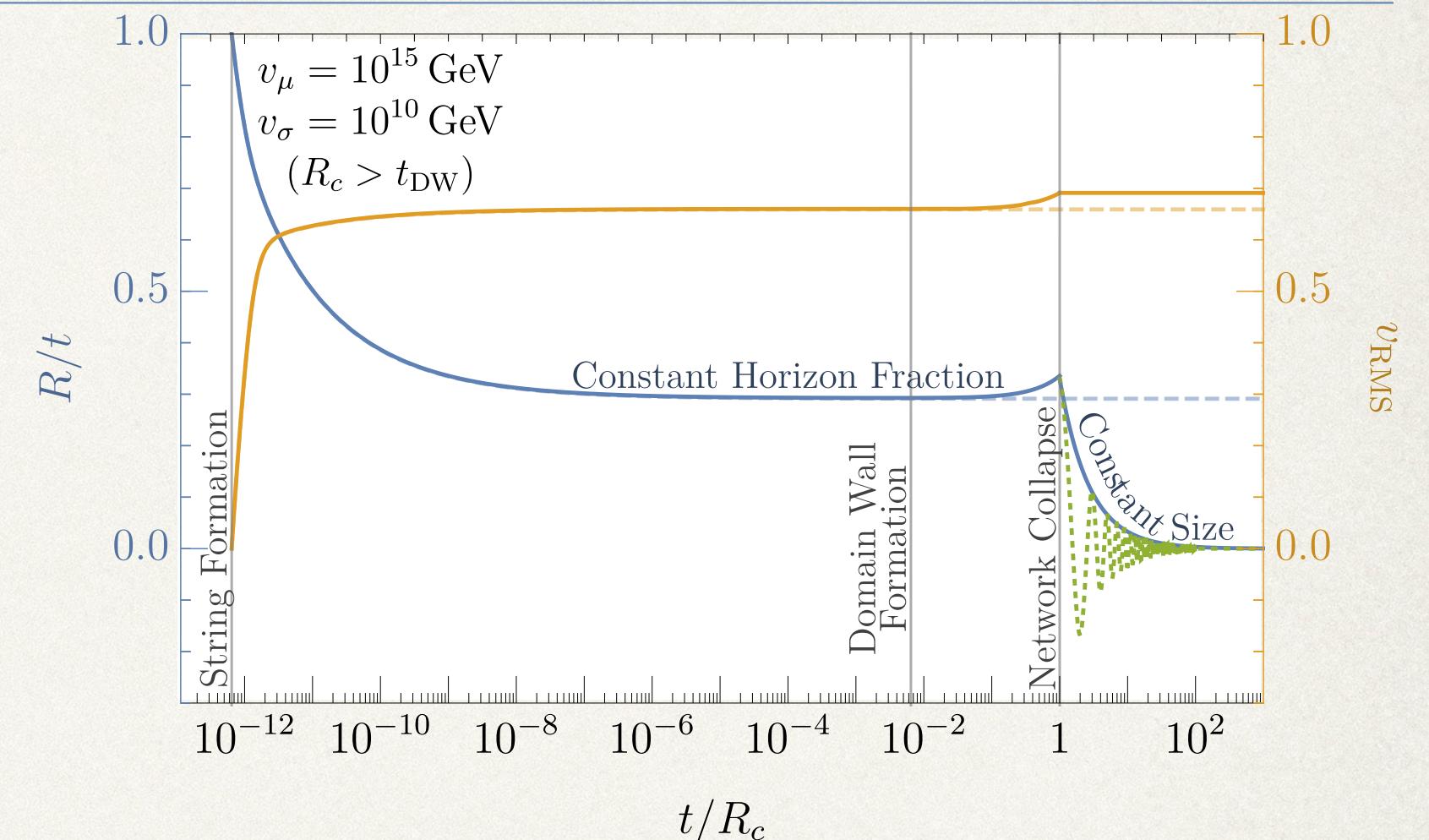
1a) Solve velocity averaged Nambu-Goto
E.O.M one-scale model ($\rho_\infty = \frac{\mu R}{R^3} = \frac{\mu}{R^2}$)

Martins, Shellard '96

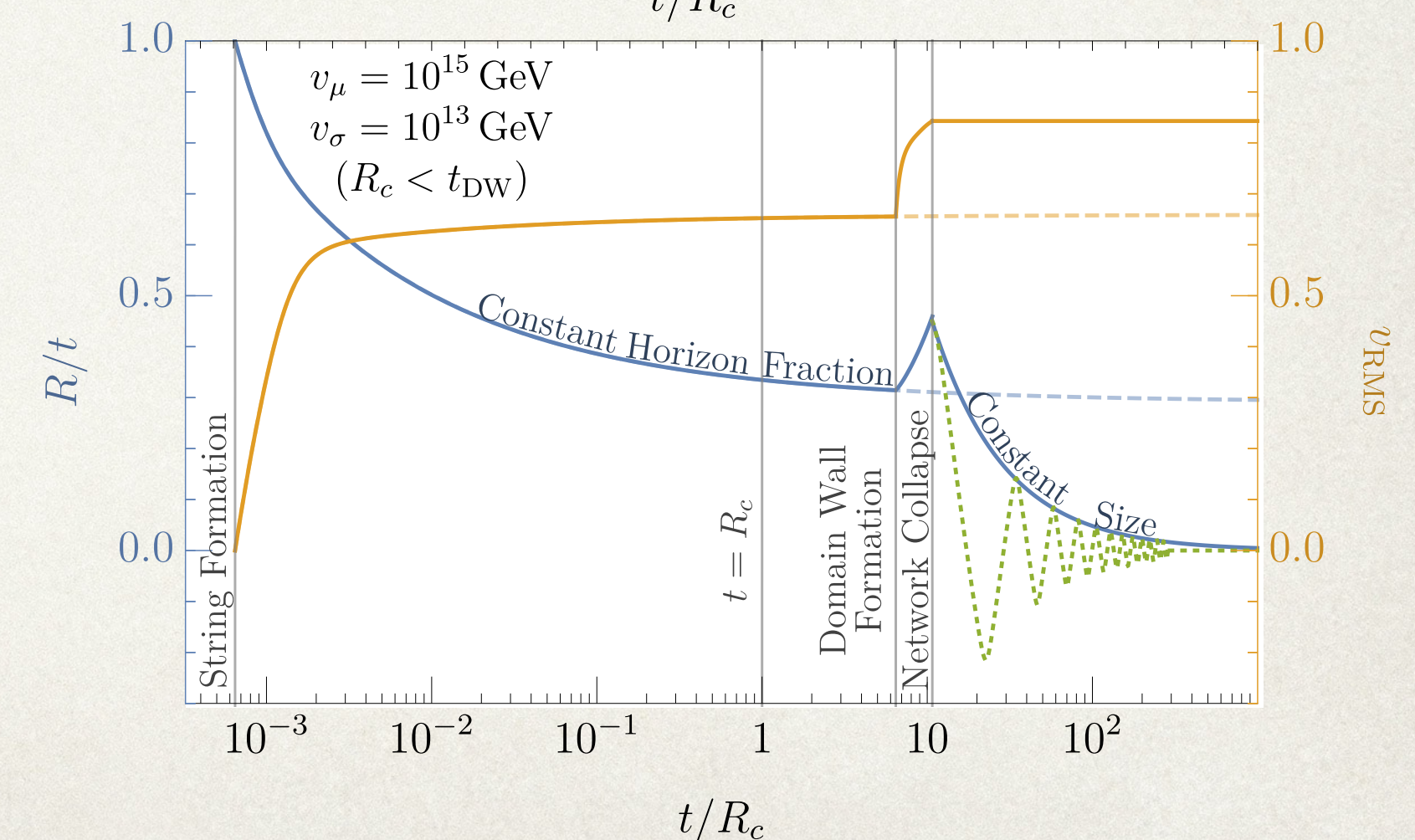
1b) Once wall tension begins affecting
evolution, infinite strings of curvature radius R
behave like wall bounded strings of radius R

2) Piecewise connect with solution of
circular string-bounded wall

Case 1:
 $t_{\text{DW}} < R_c$



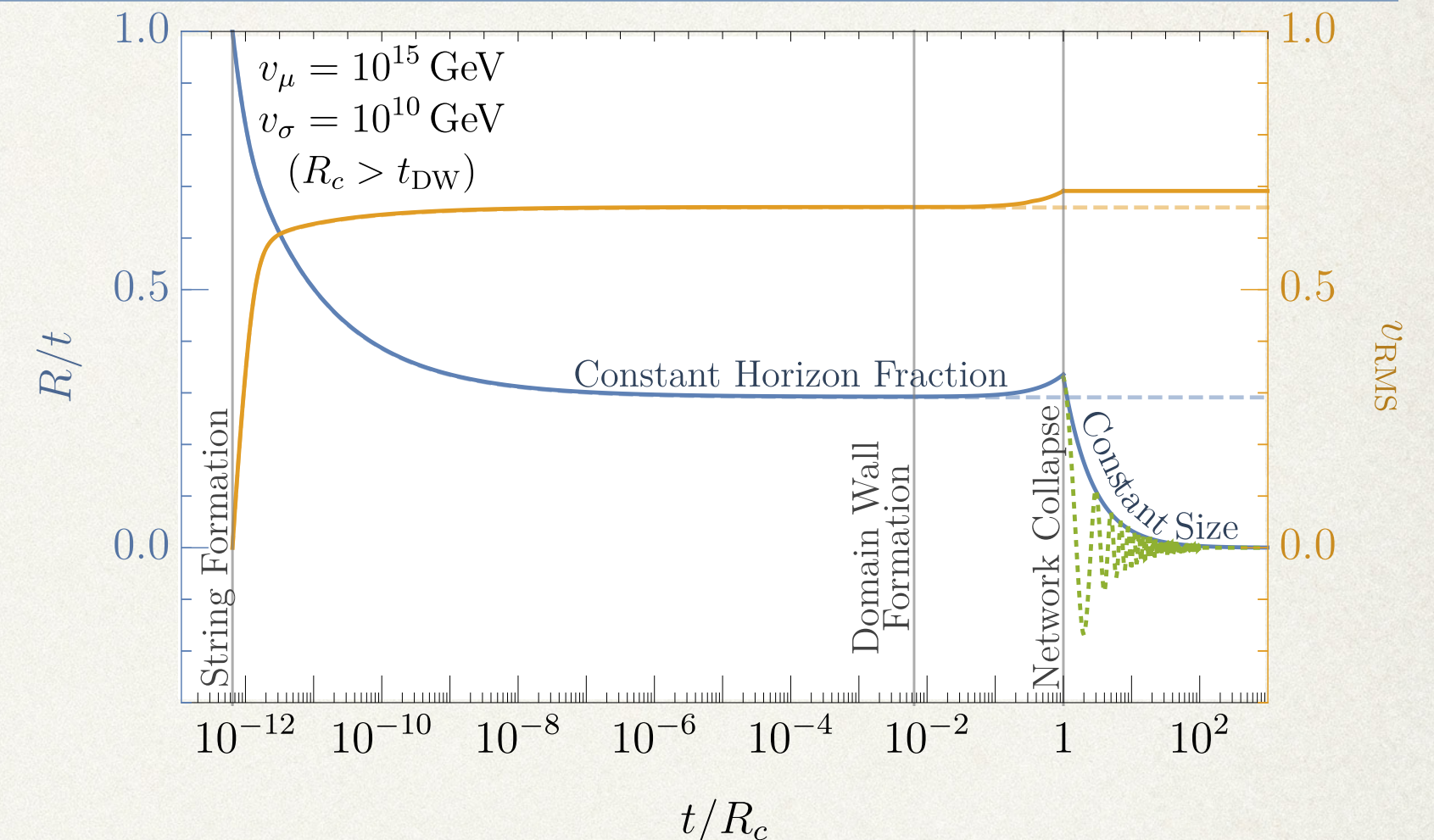
Case 2:
 $t_{\text{DW}} > R_c$



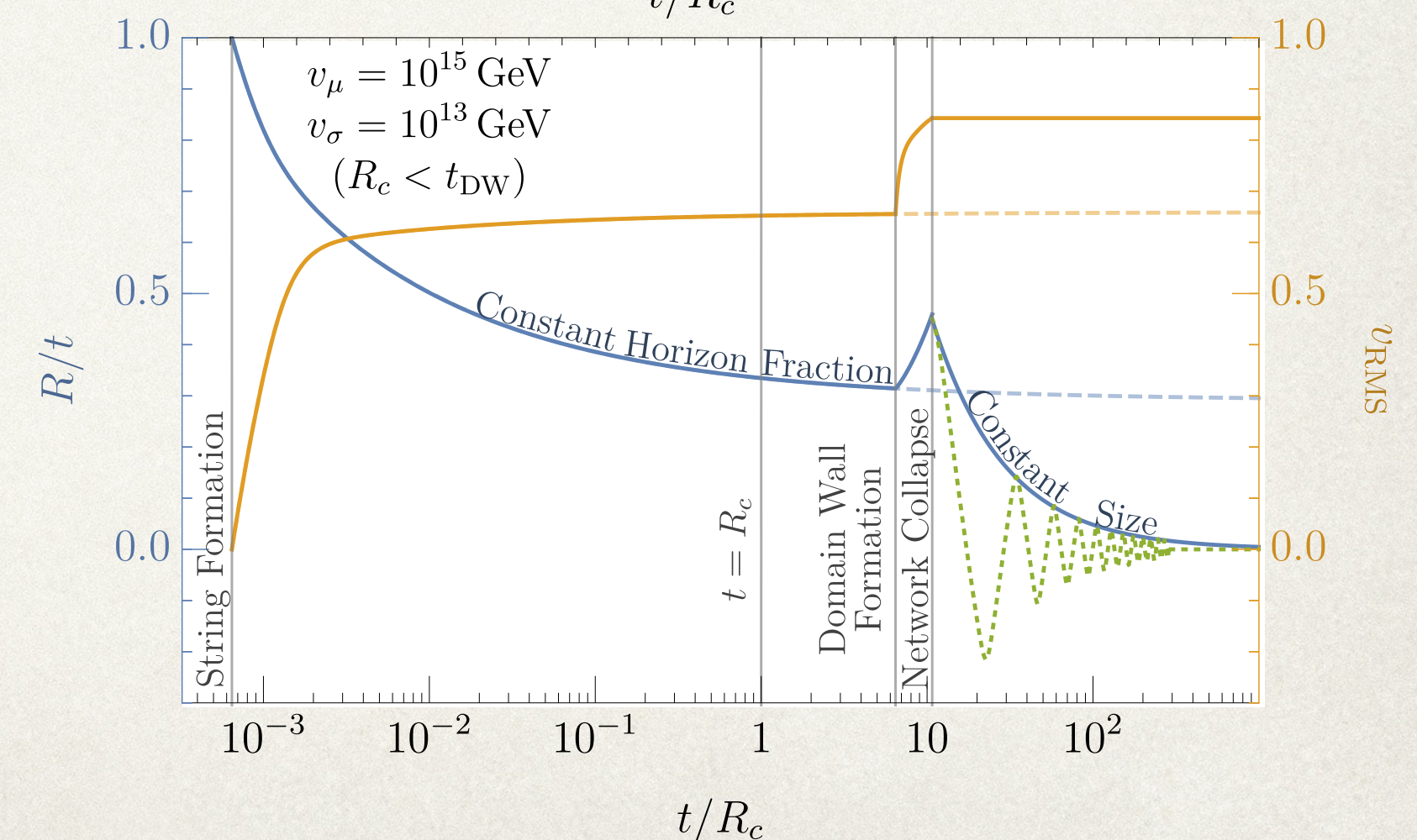
Evolution of String-Wall Network

- After attaching to strings, network evolves like usual string network if $R < R_c$
- After $R > R_c$, domain wall tension dominates, collapses (eating begins)
- Network collapse at $t_* \approx \text{Max}(R_c, t_{\text{DW}})$, as first proposed by *Everett, Vilenkin '82* and *Martin, Vilenkin '96*
- Domain wall bounded strings oscillate with constant mass before decaying via GW

Case 1:
 $t_{\text{DW}} < R_c$



Case 2:
 $t_{\text{DW}} > R_c$



Gravitational Power of String-Bounded Wall

- Quadrupole formula, $P_{\text{GW}} \approx \frac{G}{45} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle$, for string loop $P_{\text{GW}}^{(\text{string})} \sim G\mu^2$ and wall $P_{\text{GW}}^{(\text{wall})} \sim G\sigma M_{\text{DW}}$

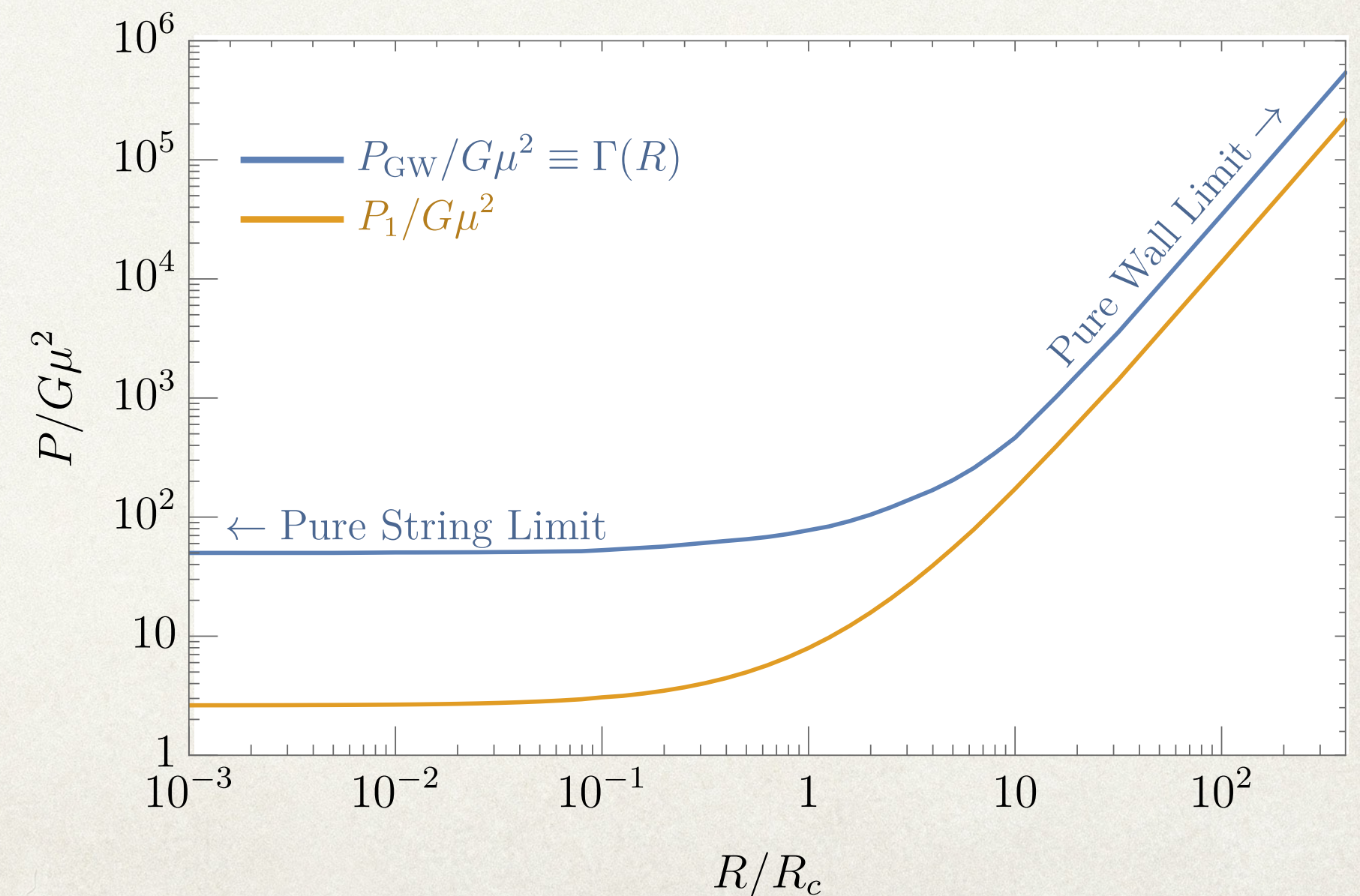


$$P_{\text{GW}}^{(\text{wall})} \gtrsim P_{\text{GW}}^{(\text{string})} \quad \text{when} \quad R > R_c$$

- In reality, expect highly relativistic string to dominate GW emission

- Numerically compute $P_n = \frac{G\omega_n^2}{\pi} \int d\Omega |T_{\mu\nu}^T(\mathbf{k}, \omega_n)|^2$
for circular string-bounded wall solution

Weinberg '72



Gravitational Wave Amplitude

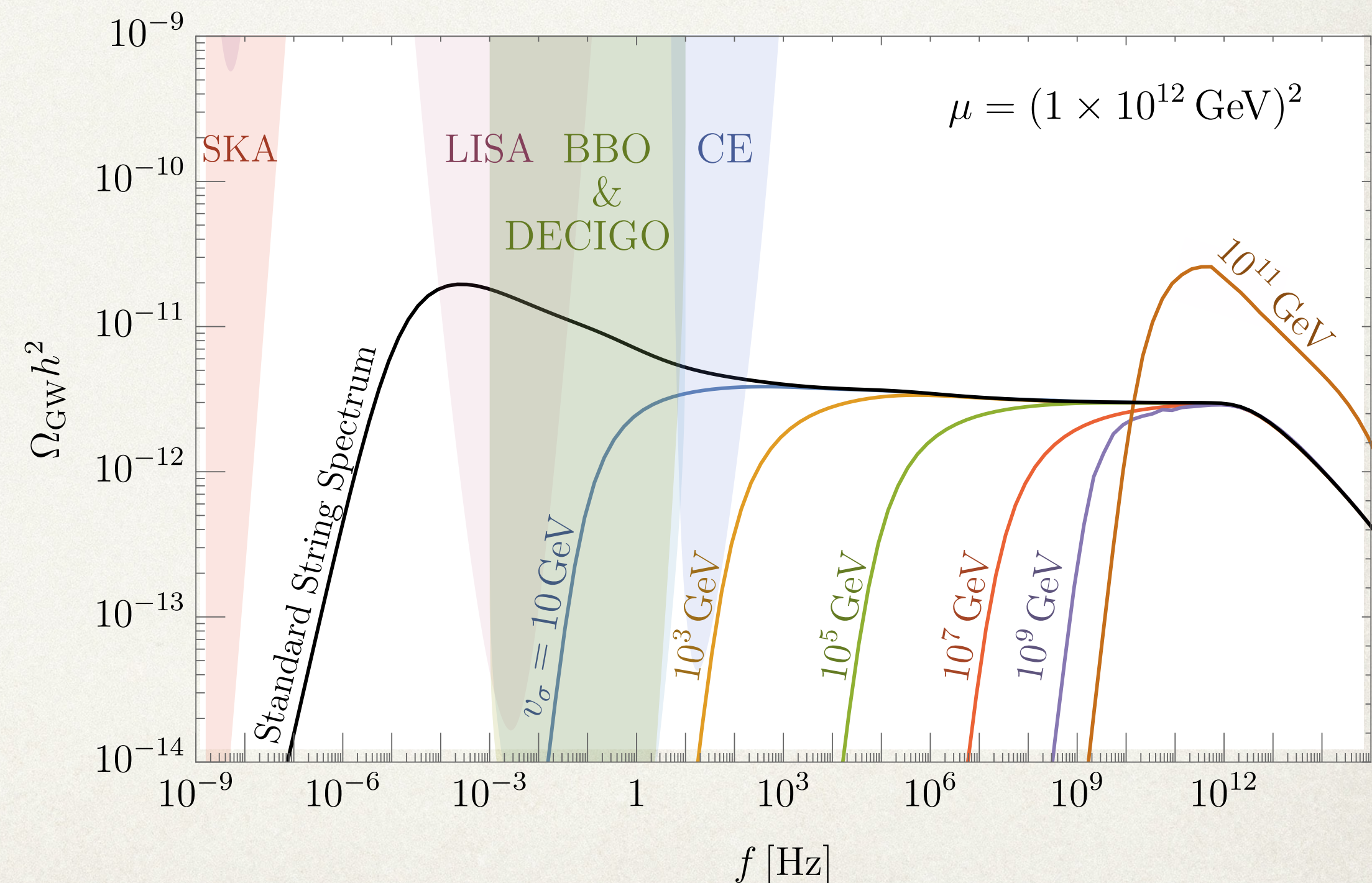
- Can confirm simple estimates more precisely

$$\Omega_{\text{GW}} h^2 = f \frac{d\rho_{\text{GW}}}{df} \frac{h^2}{\rho_c}$$

$$\frac{d\rho_{\text{GW}}(t)}{df} = \int_{t_{\text{sc}}}^t dt' \frac{a(t')^4}{a(t)^4} \int dl \frac{dn(l, t')}{dl} \frac{dP_l(l, t')}{df'} \frac{df'}{df}$$

\uparrow Expansion \uparrow One-scale model \uparrow Power \uparrow Redshift

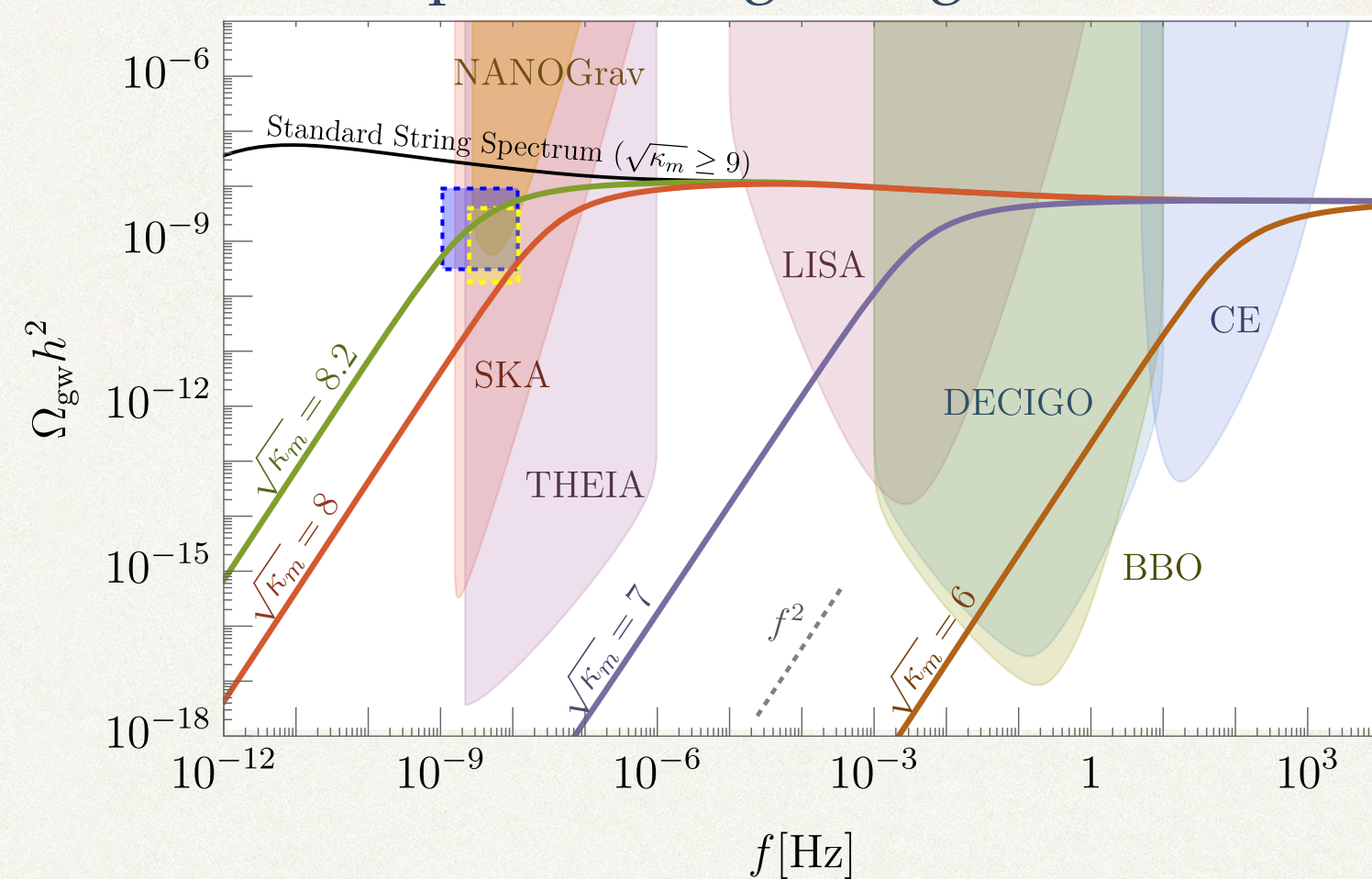
- Standard, flat, string spectrum at high frequencies, f^3 decaying spectrum at lower frequencies
- Domain wall “bump” before decay if $t_{\text{DW}} \gg R_c$



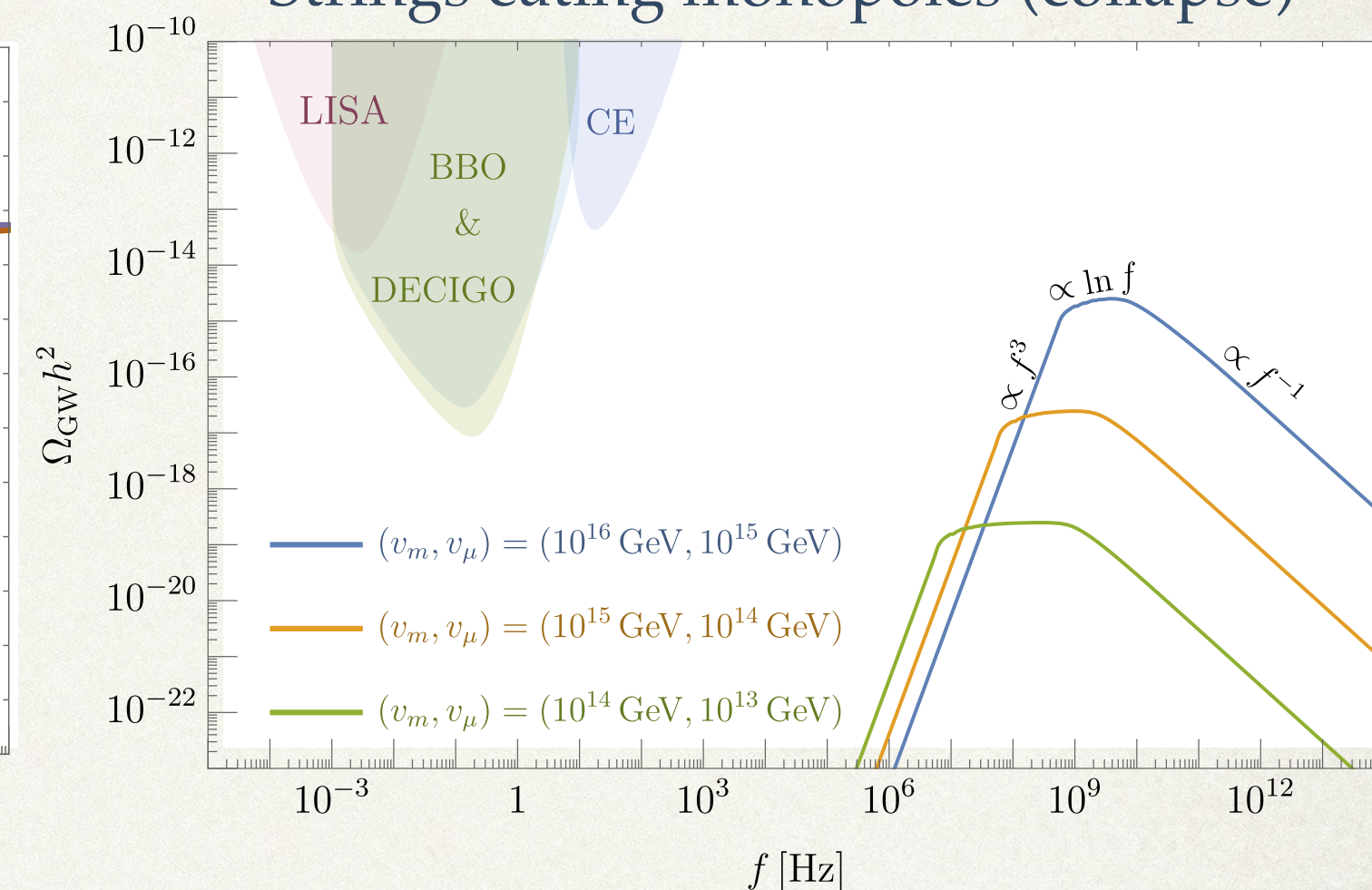
Overview of Spectra

	IR	UV	Generally high frequency signal?
Monopoles eating strings (Nucleation)	f^2	f^0	
Strings eating monopoles (Collapse)	f^3	$\ln f \rightarrow f^{-1}$	
Strings eating domain walls (Nucleation)	f^3	f^{-1}	
Domain walls eating strings (Collapse)	f^3	f^0	

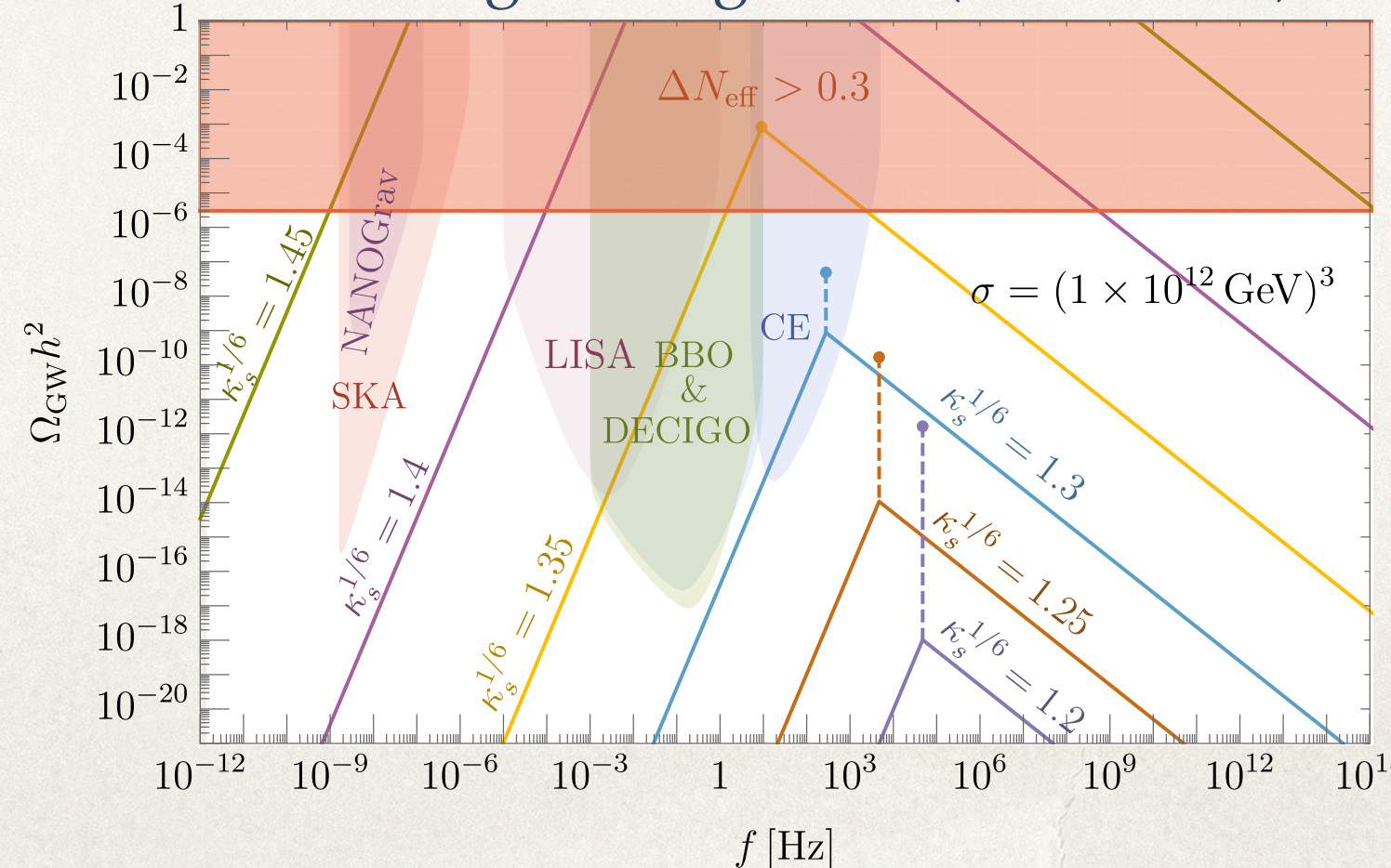
Monopoles eating strings (nucleation)



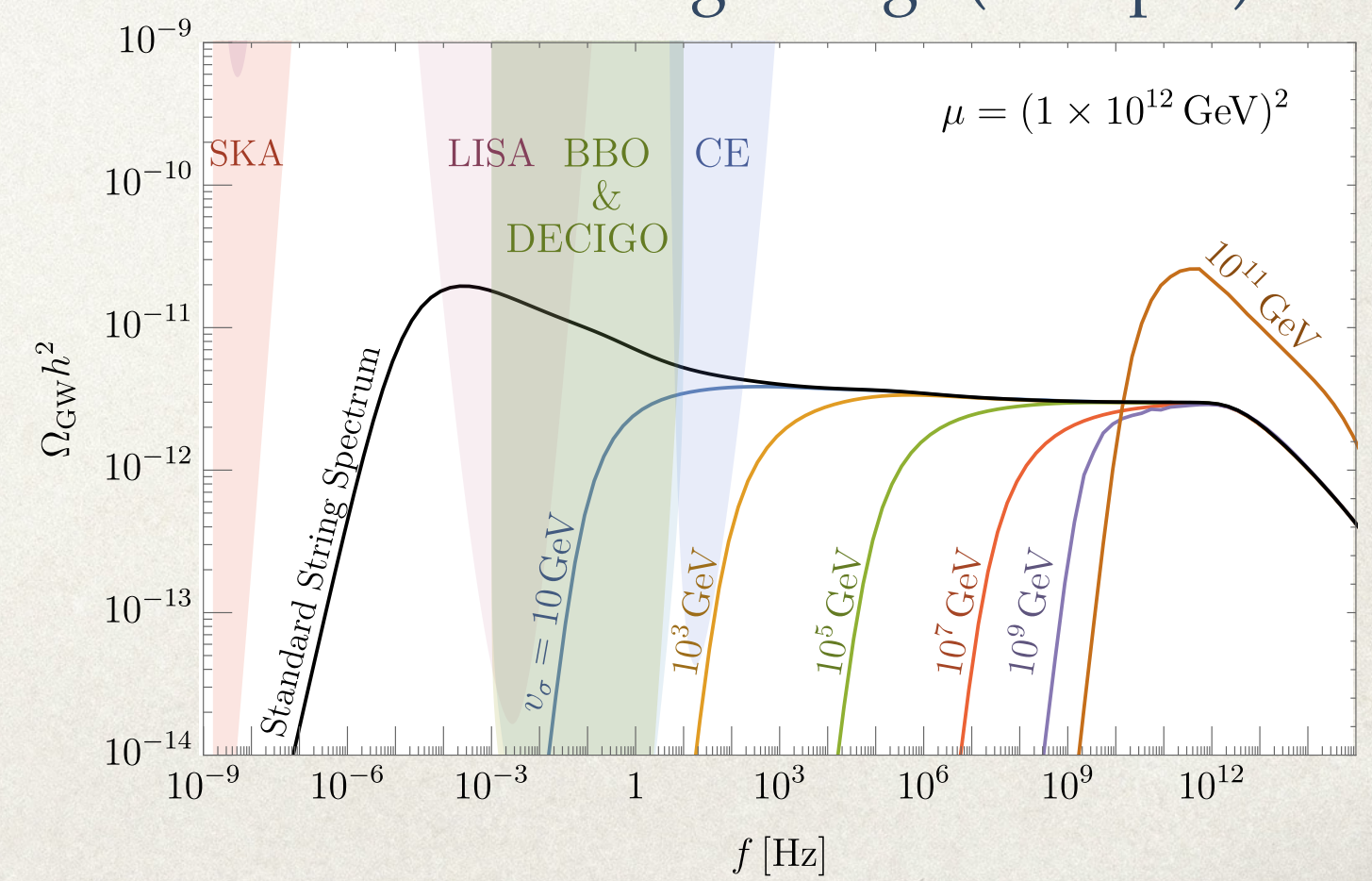
Strings eating monopoles (collapse)



Strings eating walls (nucleation)

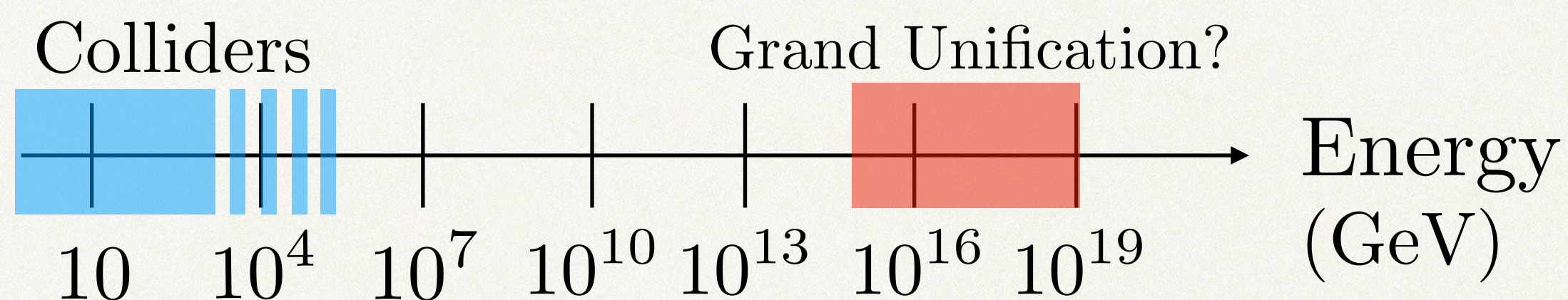


Walls eating strings (collapse)



Conclusions

- Gravitational waves provide amazing view into early universe and high energy physics



- GUT symmetry chains with hybrid topological defects produce unique gravitational wave fingerprints upon being “eaten”
- These signatures may allow us to understand our universe at a fundamental level

Gravitational Wave Basics: Computing Signal

- Measure strain through $\Omega_{\text{GW}} \equiv \frac{d\rho_{\text{GW}}}{d \ln f} \frac{1}{\rho_{\text{crit}}} \propto \text{Strain}^2$

- Recipe:

Estimate: $P_{\text{GW}} \sim G \langle \ddot{Q}^2 \rangle \longrightarrow \rho_{\text{GW}} \sim P_{\text{GW}} \times \Delta t \times n_{\text{sources}} \longrightarrow \Omega_{\text{GW}} \sim \left. \frac{\rho_{\text{GW}}}{\rho_{\text{BG}}} \right|_{\text{Emission}} \times \Omega_{\text{rad}}$

$f \sim \text{scale size of system} \Big|_{\text{Emission}}^{-1} \times \text{Redshift}$



Gravitational Wave Basics: Computing Signal

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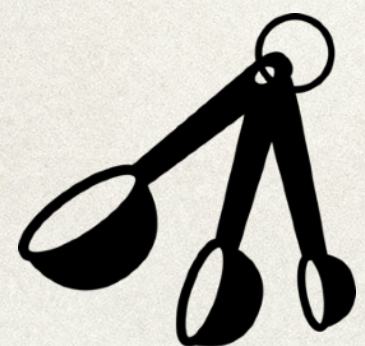
$f \sim \text{scale size of system} \bigg|_{\text{Emission}}^{-1} \times \text{Redshift}$



Exact:

$$P_{\text{GW}}(f) \sim G |T_{\mu\nu}^T(f)|^2$$

Weinberg '72



$$\frac{d\rho_{\text{GW}}(t)}{df} = \int dt' \frac{a(t')^4}{a(t)^4} \int dl \frac{dn(l, t')}{dl} \frac{dP(l, t')}{df'} \frac{df'}{df}$$

↑
↑
↑
↑
 Expansion Source Density Power Redshift