The Boring Monopole

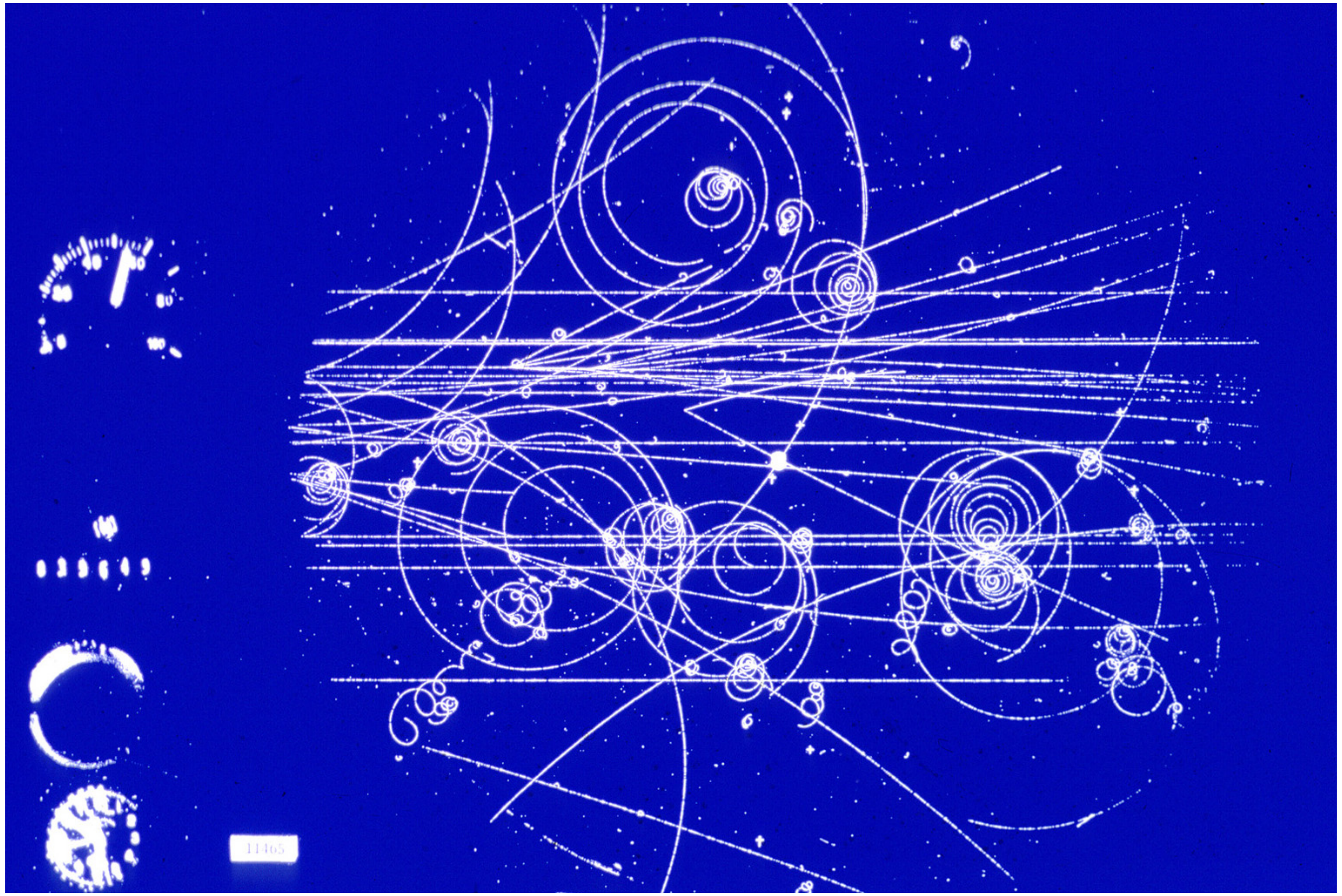
Michael Nee

Based on: hep-ph/2202.11102

w/ Prateek Agrawal

Impurity-driven phase transitions

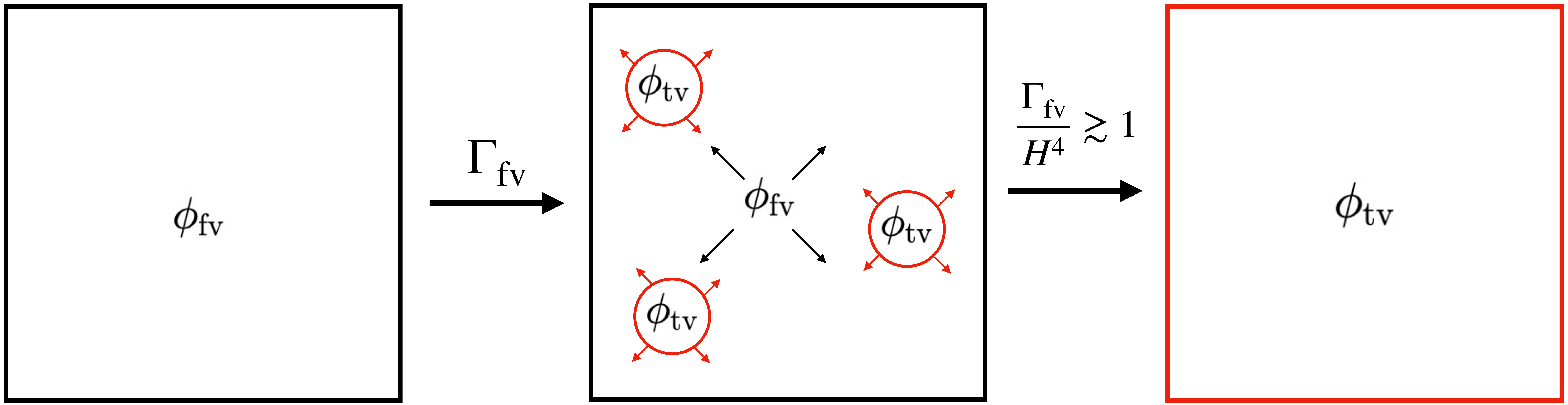
- Bubble chambers
- Condensation/rain
- Water bubbles
- …



https://cds.cern.ch/record/39474

Cosmological Phase transitions

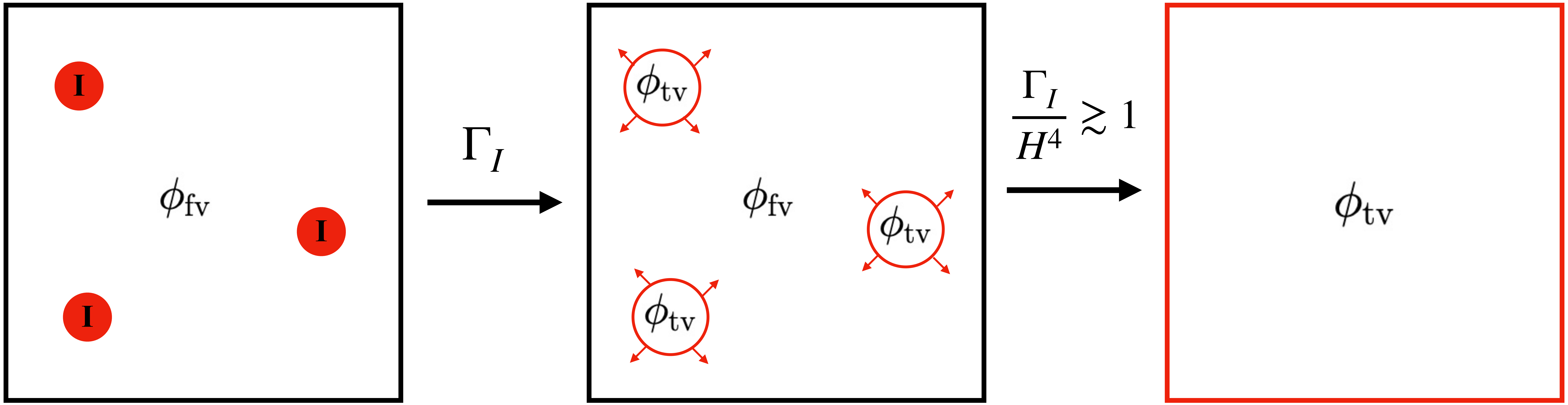
Guth & Weinberg: *Nucl. Phys. B.* B212, 1983



$$\Gamma_{\rm fv} = \frac{1}{\rm Vol.}\frac{dN_{\rm bubbles}}{dt} \sim \phi_{\rm fv}^4 e^{-B_{\rm fv}}$$

$$H = \left(\frac{8\pi G_N}{3}V\left(\phi_{\rm fv}\right)\right)^{1/2}$$

Impurity-driven phase transitions



$$\Gamma_I \sim n_I \phi_{fv} e^{-B_I}$$

$$\Gamma_{fv} \sim \phi_{fv}^4 e^{-B_{fv}}$$

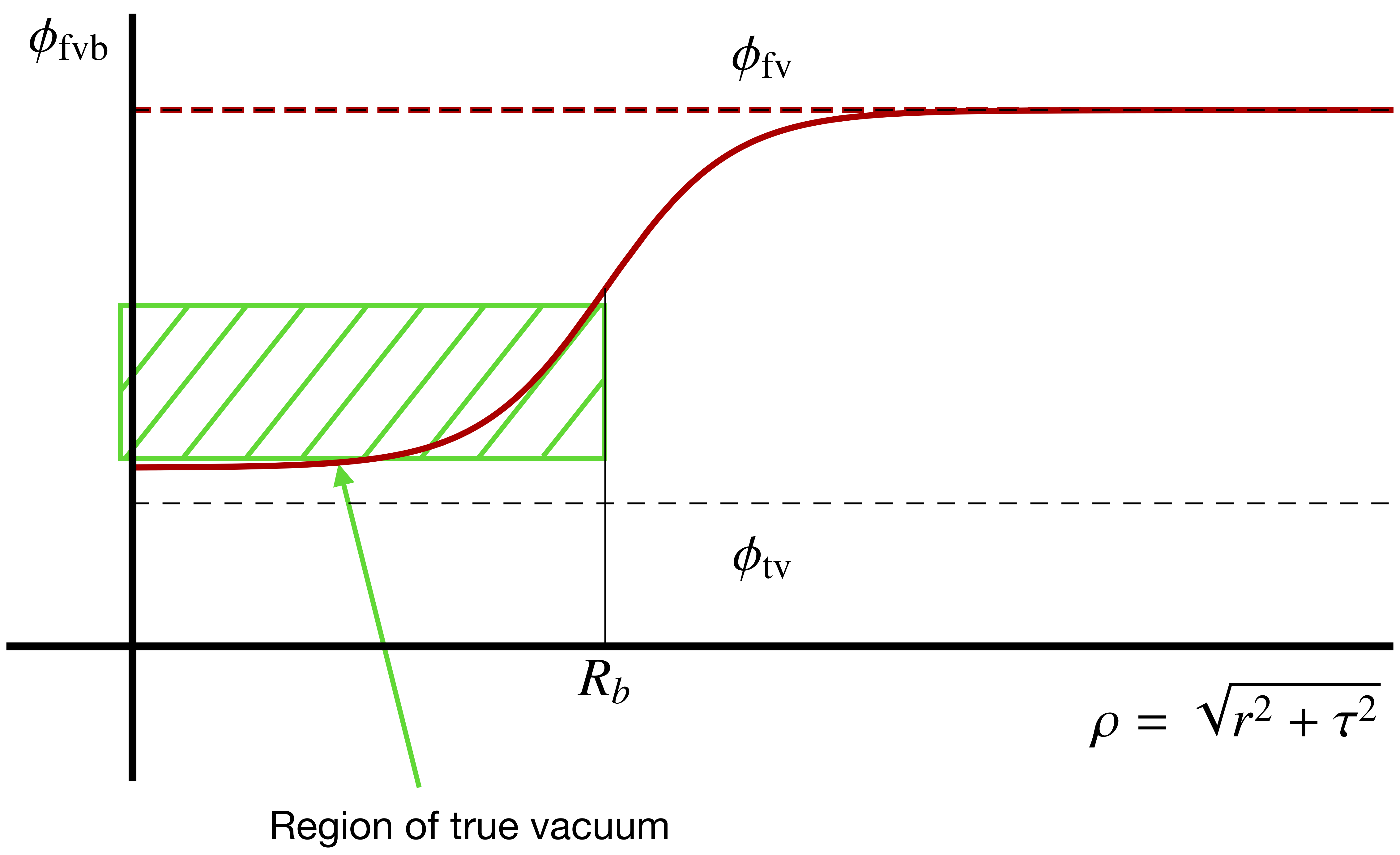
$$B_I < B_{fv} \implies \Gamma_I \ll \Gamma_{fv}$$

False Vacuum Tunnelling

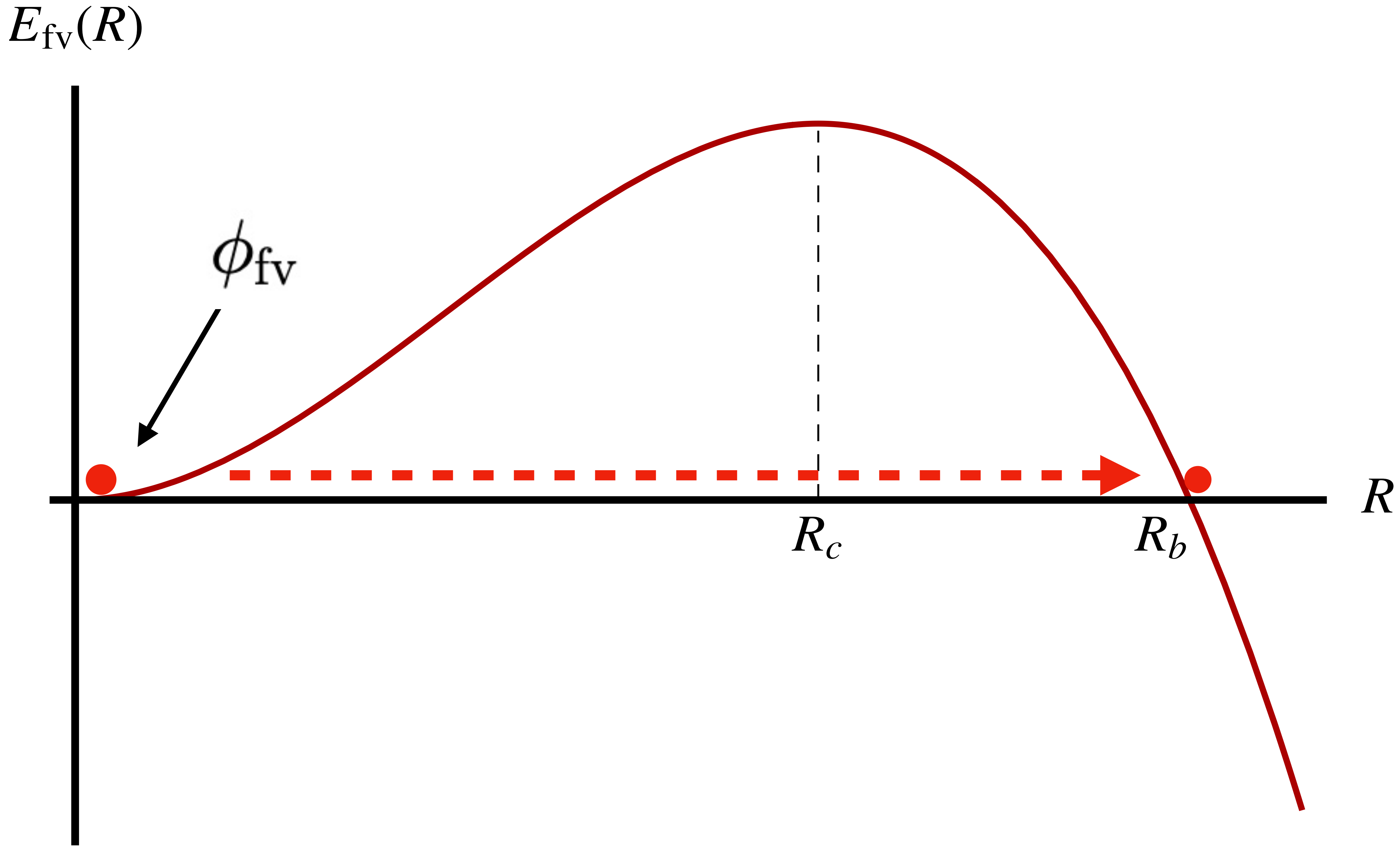
Coleman: *Phys.Rev.D* 15 (1977), Linde: *Nucl.Phys.B* 216 (1983)

Bounce Profile

$$B_{\rm fv} = S_E\left[\phi_{\rm fvb}\right] - S_E\left[\phi_{\rm fv}\right]$$



Thin Wall picture



Monopoles as Impurities

’t Hooft: Nucl.Phys.B 79 (1974), Polyakov: JETP Lett. 20 (1974)

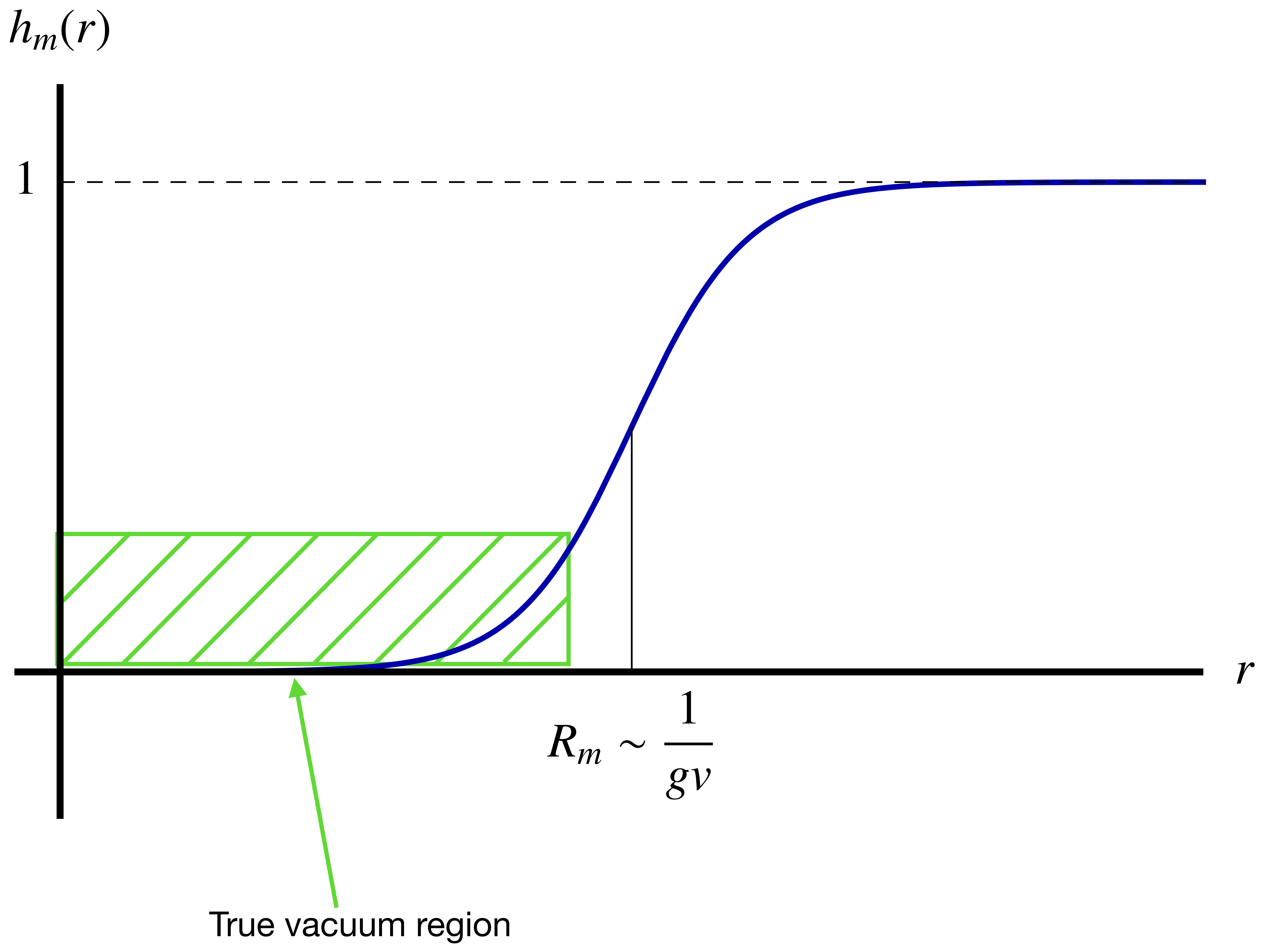
- Monopole solutions:

$$\phi^a = v\hat{r}^a h_{\rm m}(r)$$

$$A_i^a = \epsilon^{iam}\hat{r}^m \left(\frac{1 - u_{\rm m}(r)}{gr}\right)$$

- Continuity requires

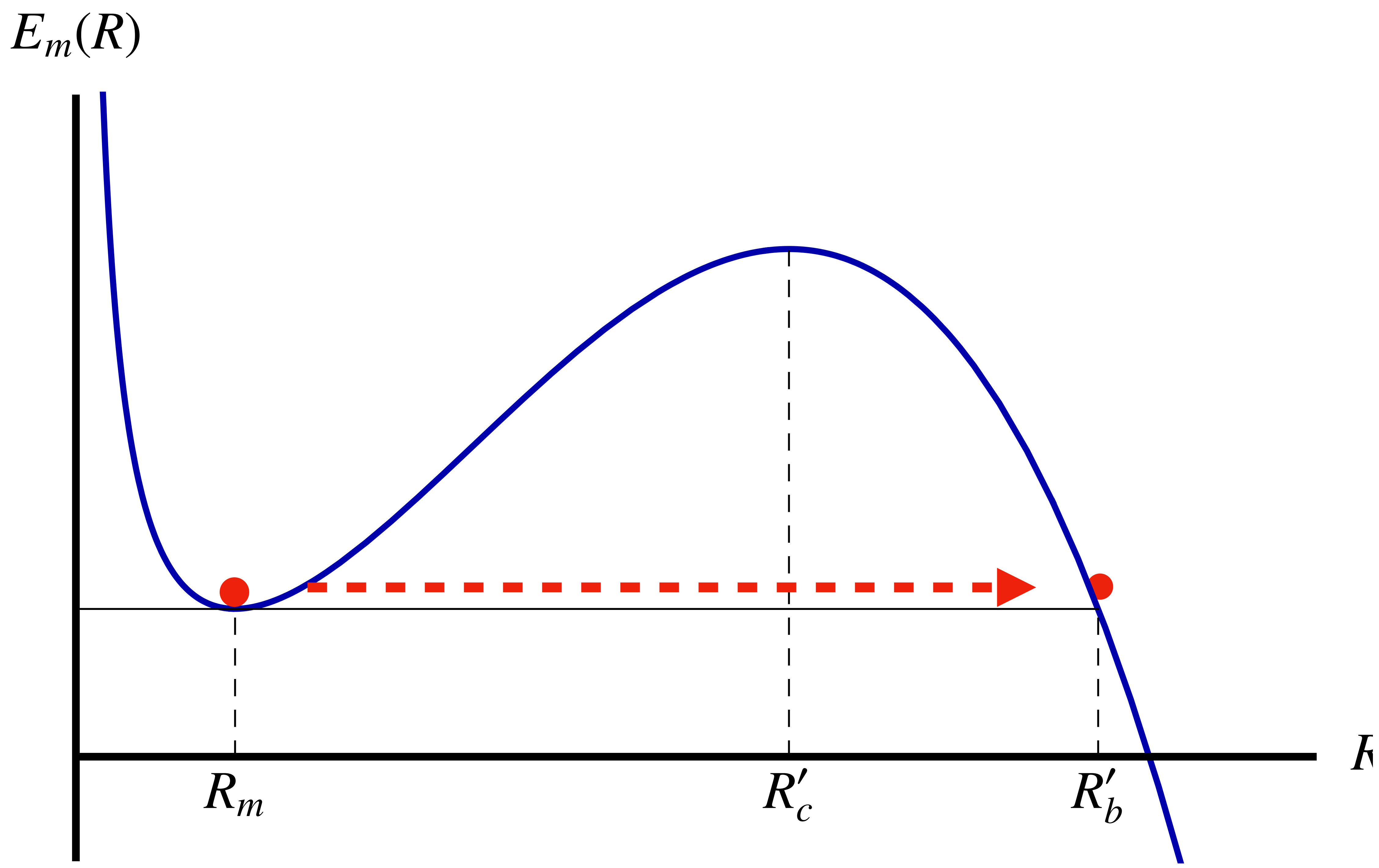
$$h_{\rm m}(0) = 0, u_{\rm m}(0) = 1$$



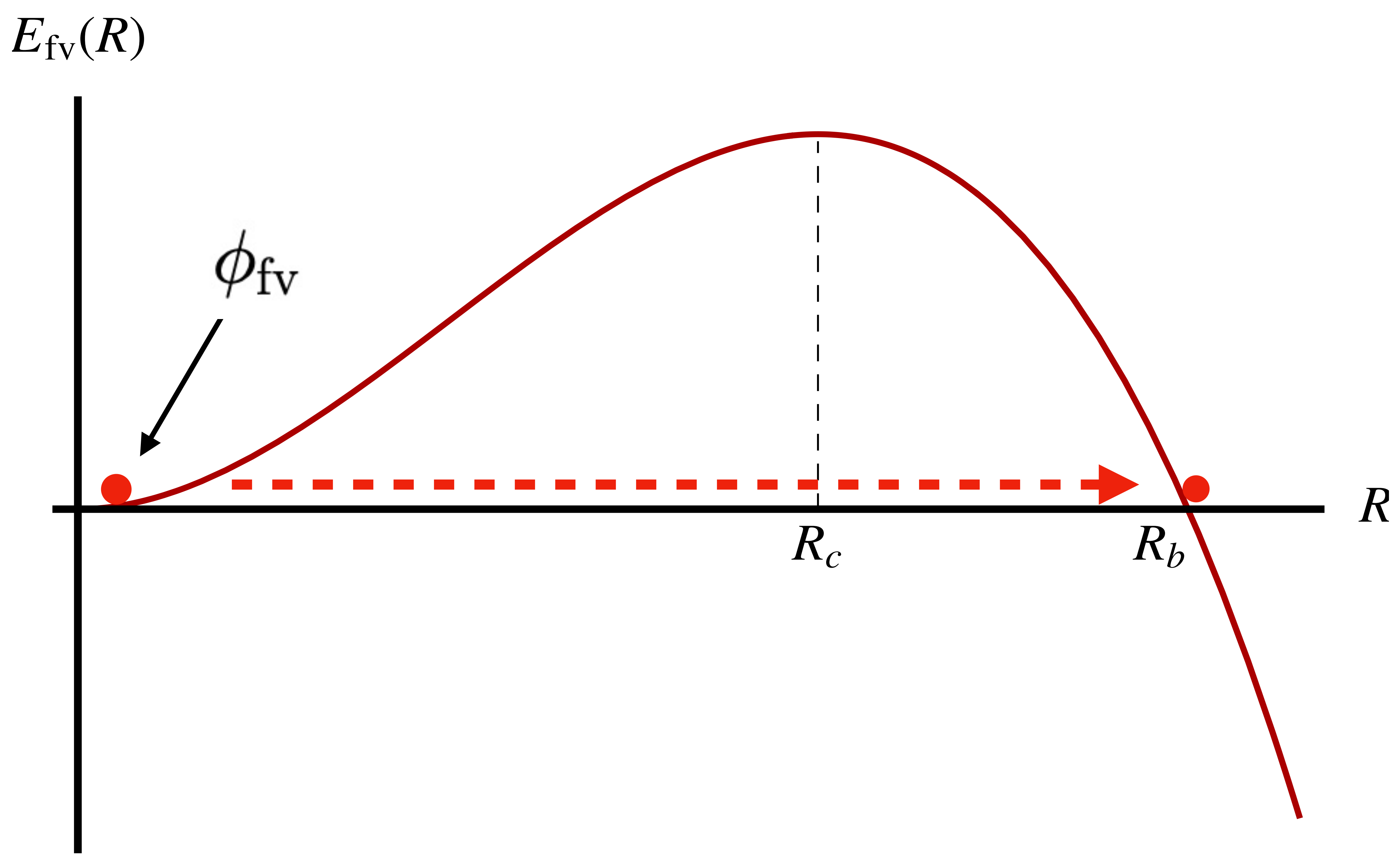
Monopoles as Impurities

Steinhardt: Phys. Rev. D 24, 842 (1981), Nucl.Phys.B 190 (1981); Kumar, Paranjape & Yajnik: hep-ph/0908.3949, hep-ph/1006.0693

Monopole Tunnelling

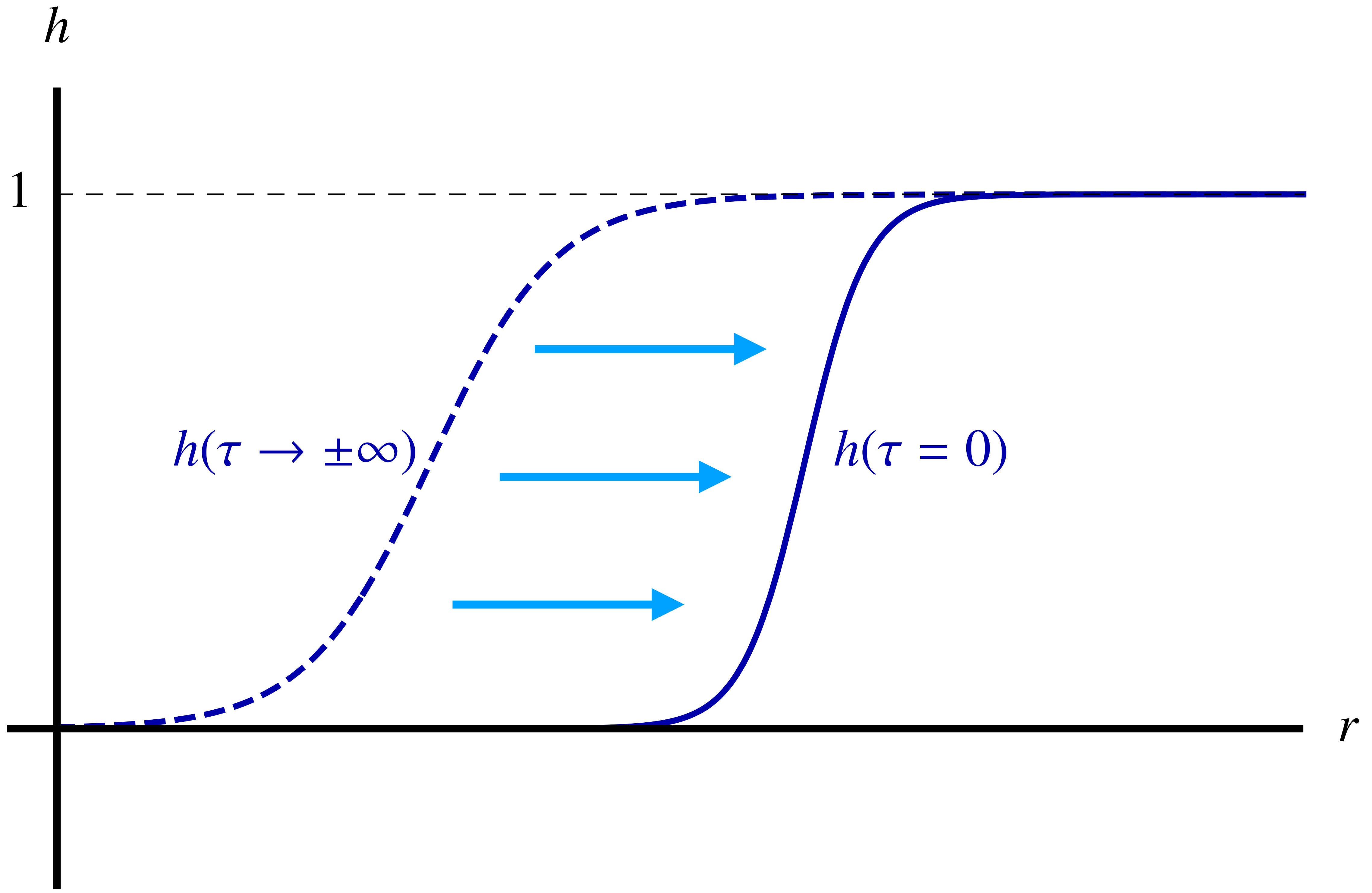


FV Tunnelling



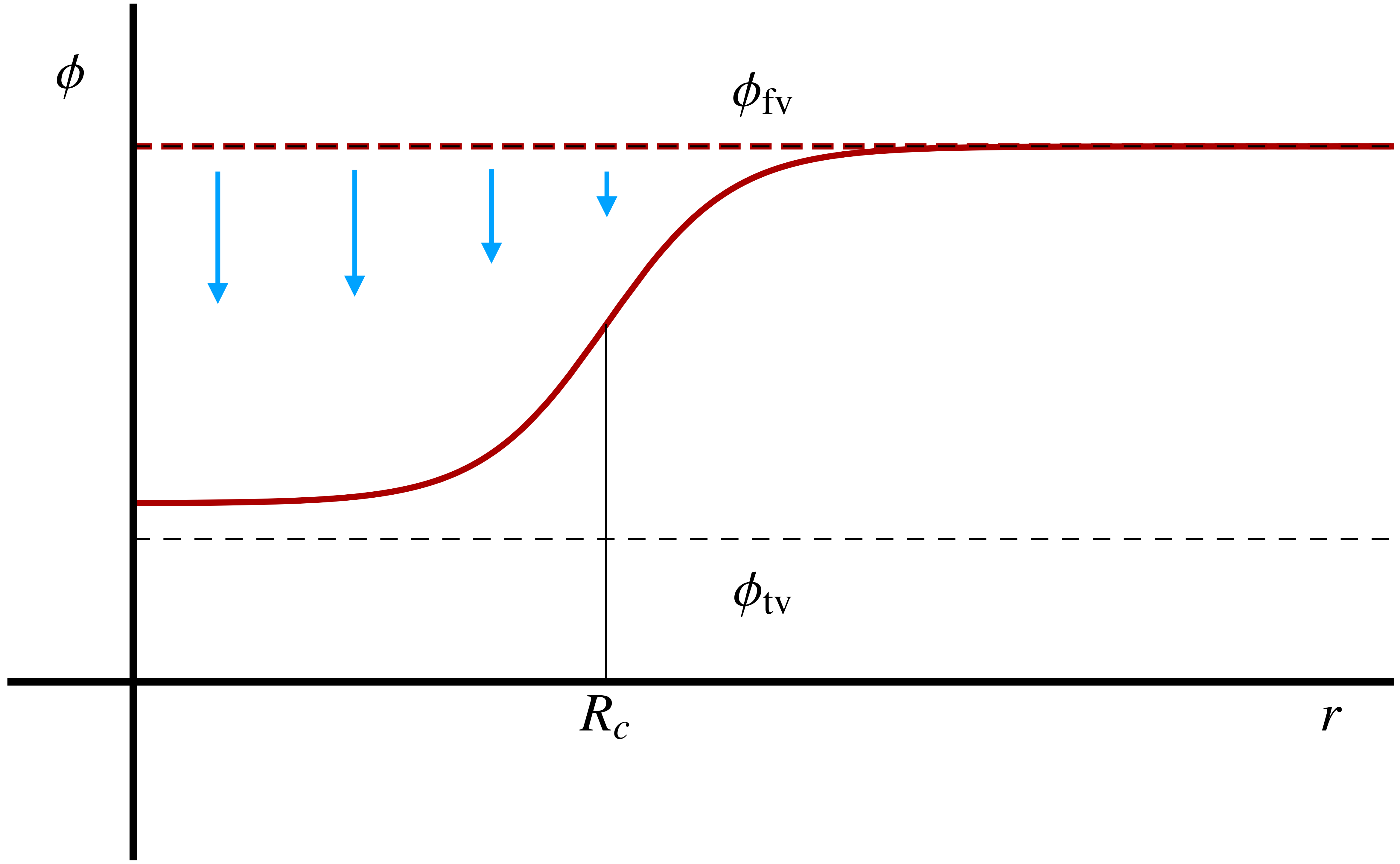
Monopole Bounce

Monopole Tunnelling



$$B_{\mathrm{m}} = S_E\left[h_{\mathrm{mb}}, u_{\mathrm{mb}}\right] - S_E\left[h_{\mathrm{m}}, u_{\mathrm{m}}\right]$$

FV Tunnelling

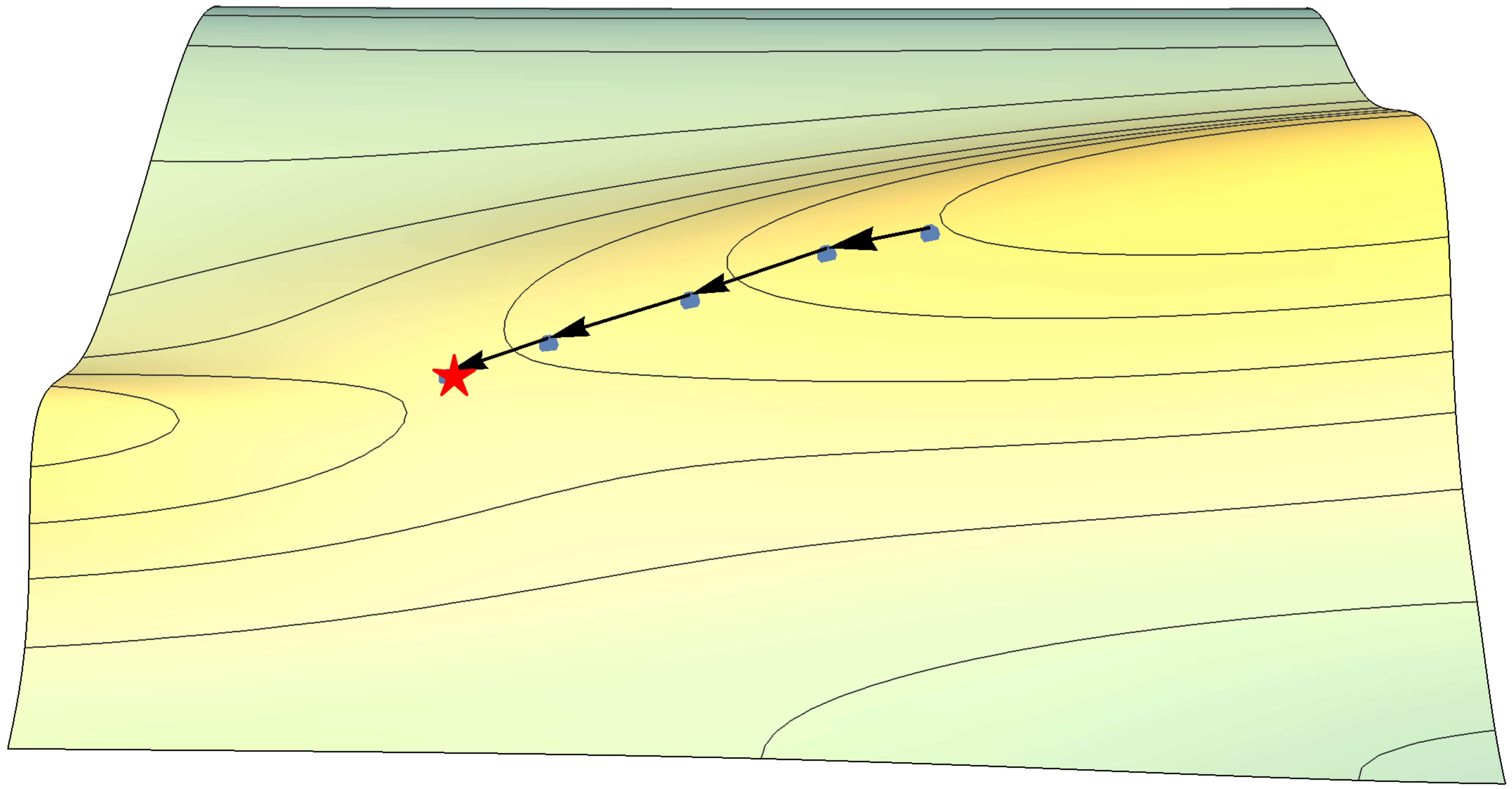


$$B_{\mathrm{fv}} = S_E\left[\phi_{\mathrm{fvb}}\right] - S_E\left[\phi_{\mathrm{fv}}\right]$$

Numerical Procedure

Aim:

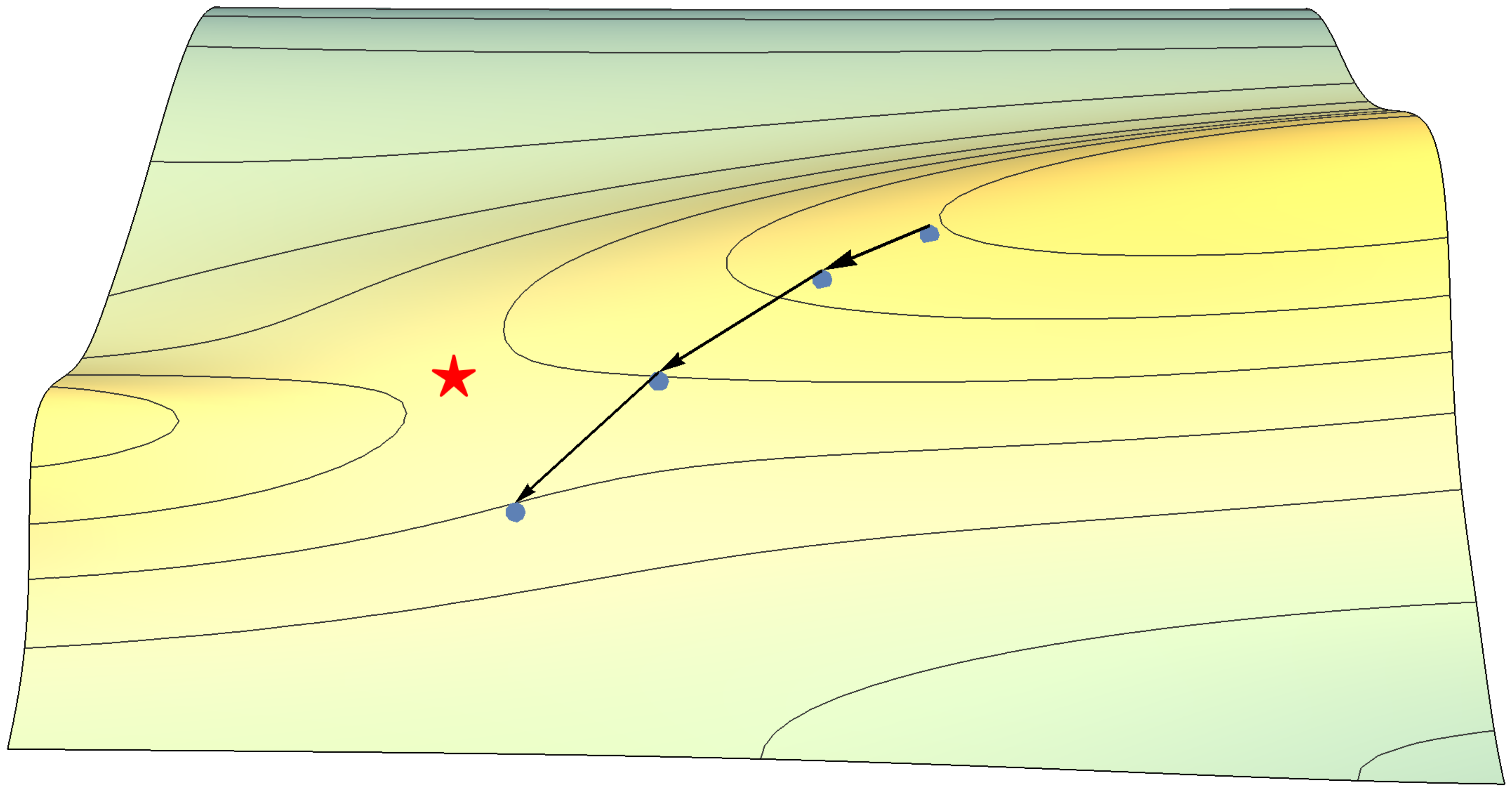
- Start with initial guess and improve by minimising action



Numerical Procedure

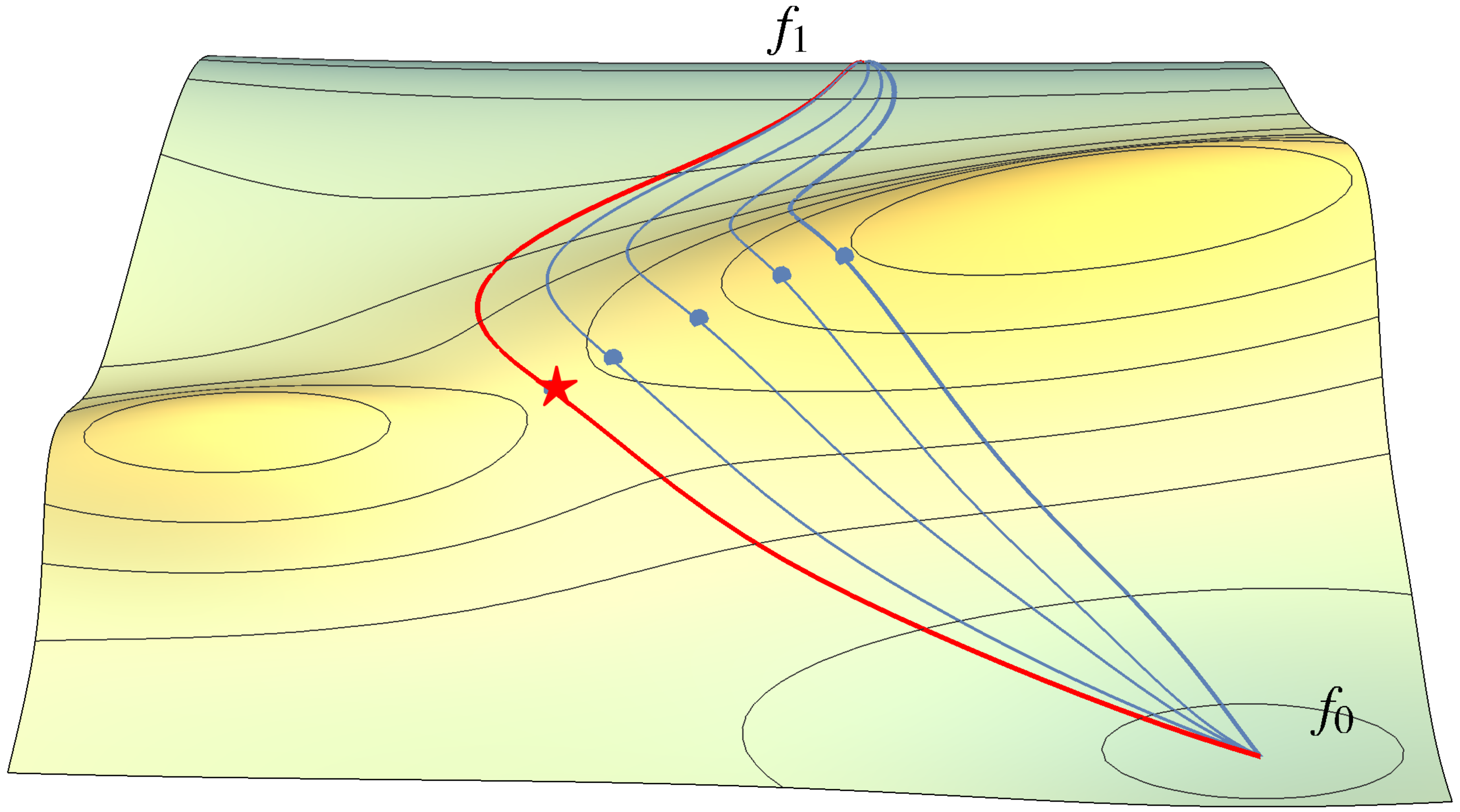
Problem:

- We are searching for saddle, not a minimum
- Runaway directions impossible to avoid



Mountain Pass Theorem

- $I[f]$ a functional
- f_0 a local minimum, f_1 such that $I[f_1] < I[f_0] = 0$
- $\gamma \in$ continuous paths from f_0 to f_1
- $\min_{\gamma} \max_{\alpha \in [0,1]} I[\gamma(\alpha)]$ is a saddle point

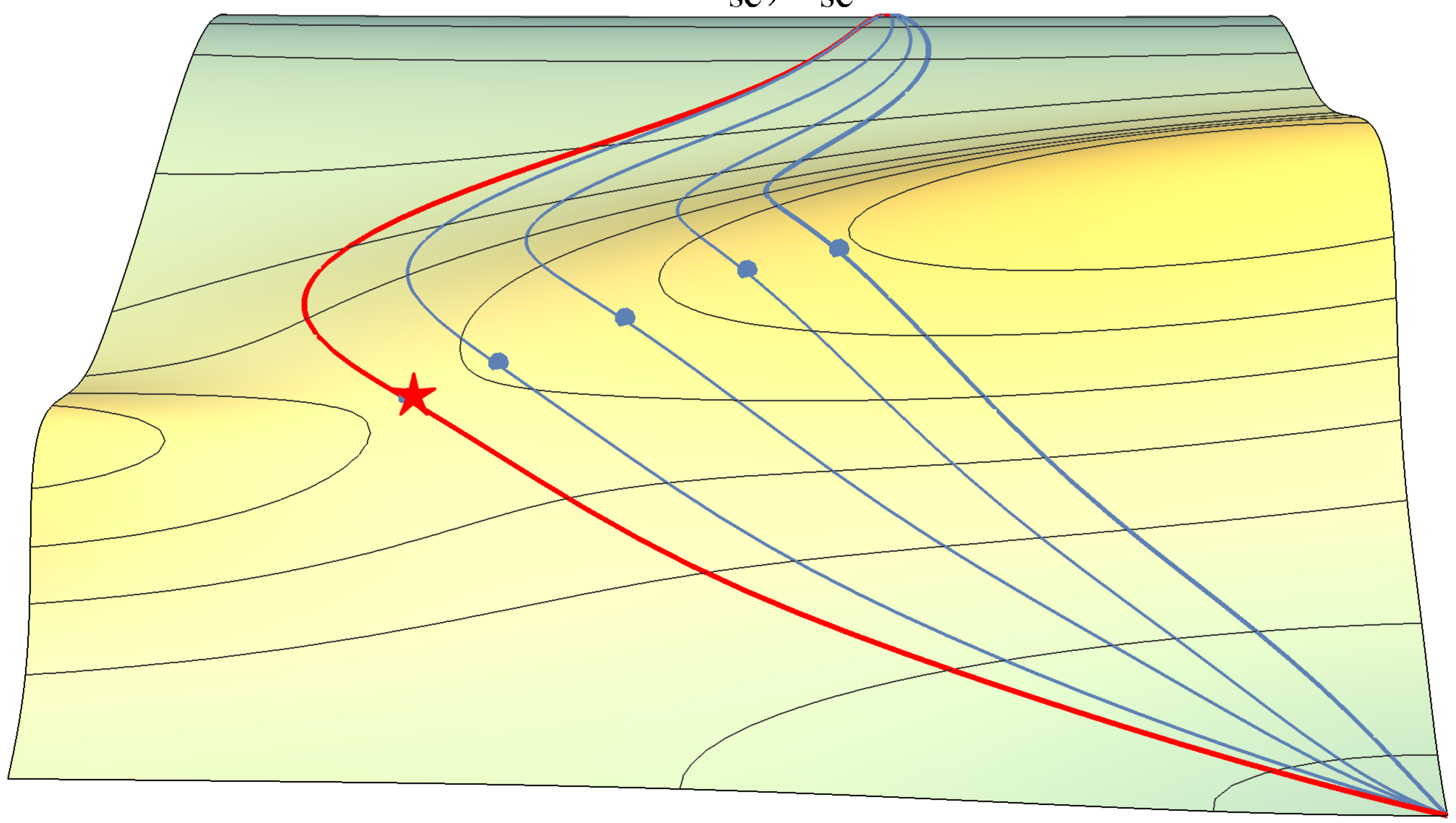


Mountain Pass Theorem

$$I[h, u] = S_E[h, u] - S_E[h_{\rm m}, u_{\rm m}]$$

satisfies all of the above conditions:

- $h_{\rm m}, u_{\rm m}$ is minimum
- need to find supercritical bubble where $I[h_{\rm sc}, u_{\rm sc}] < 0$
- Saddle point is the monopole bounce

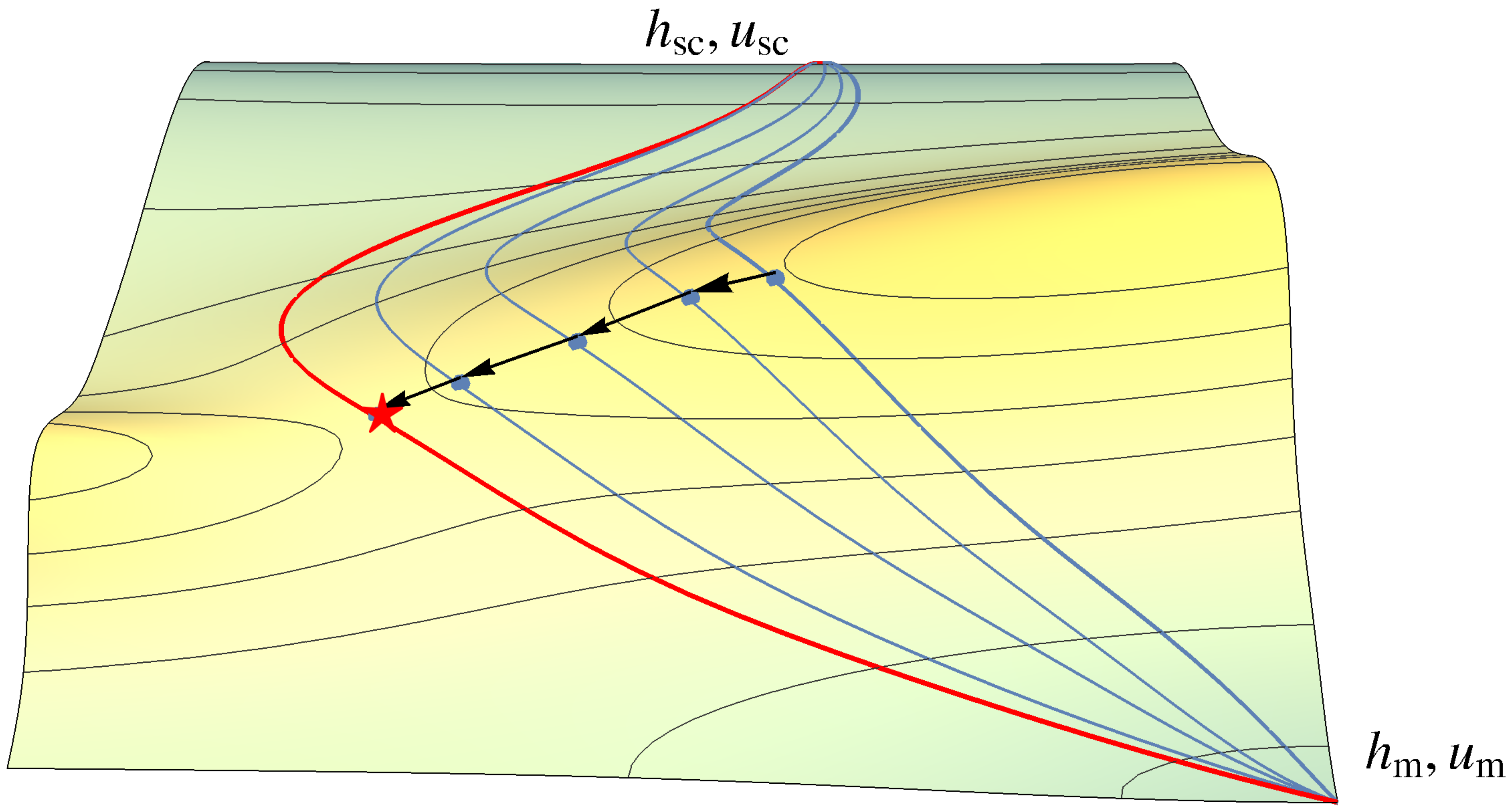


Mountain Pass Algorithm

- Start with $h_\alpha(r,\tau), u_\alpha(r,\tau)$
- Pick $\bar{\alpha}$ such that $I[h_\alpha, u_\alpha]$ is maximised and update:

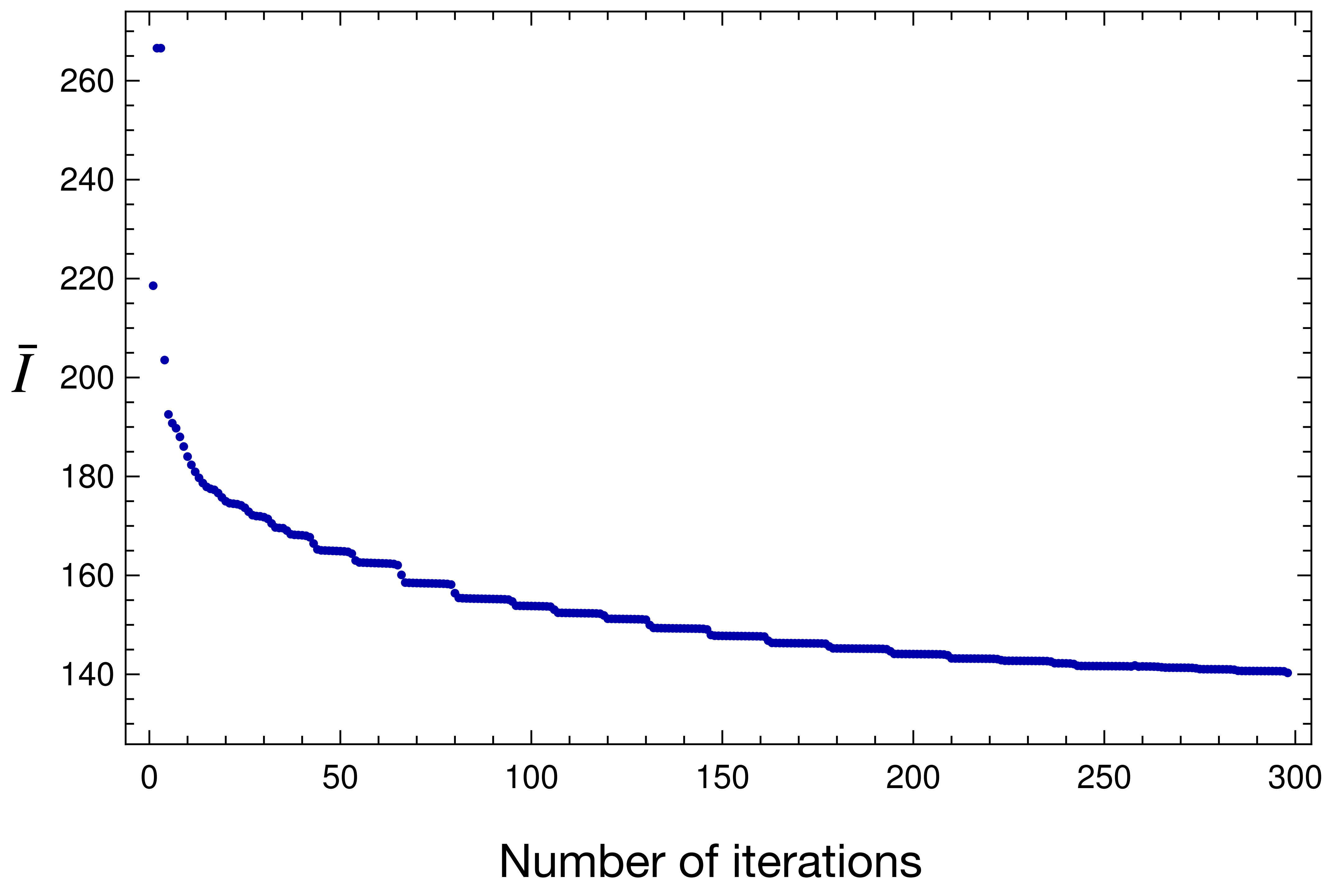
$$h_{\bar{\alpha}}^{n+1} = h_{\bar{\alpha}}^{n} - \beta_n \frac{\delta I}{\delta h_{\bar{\alpha}}^{n}},$$

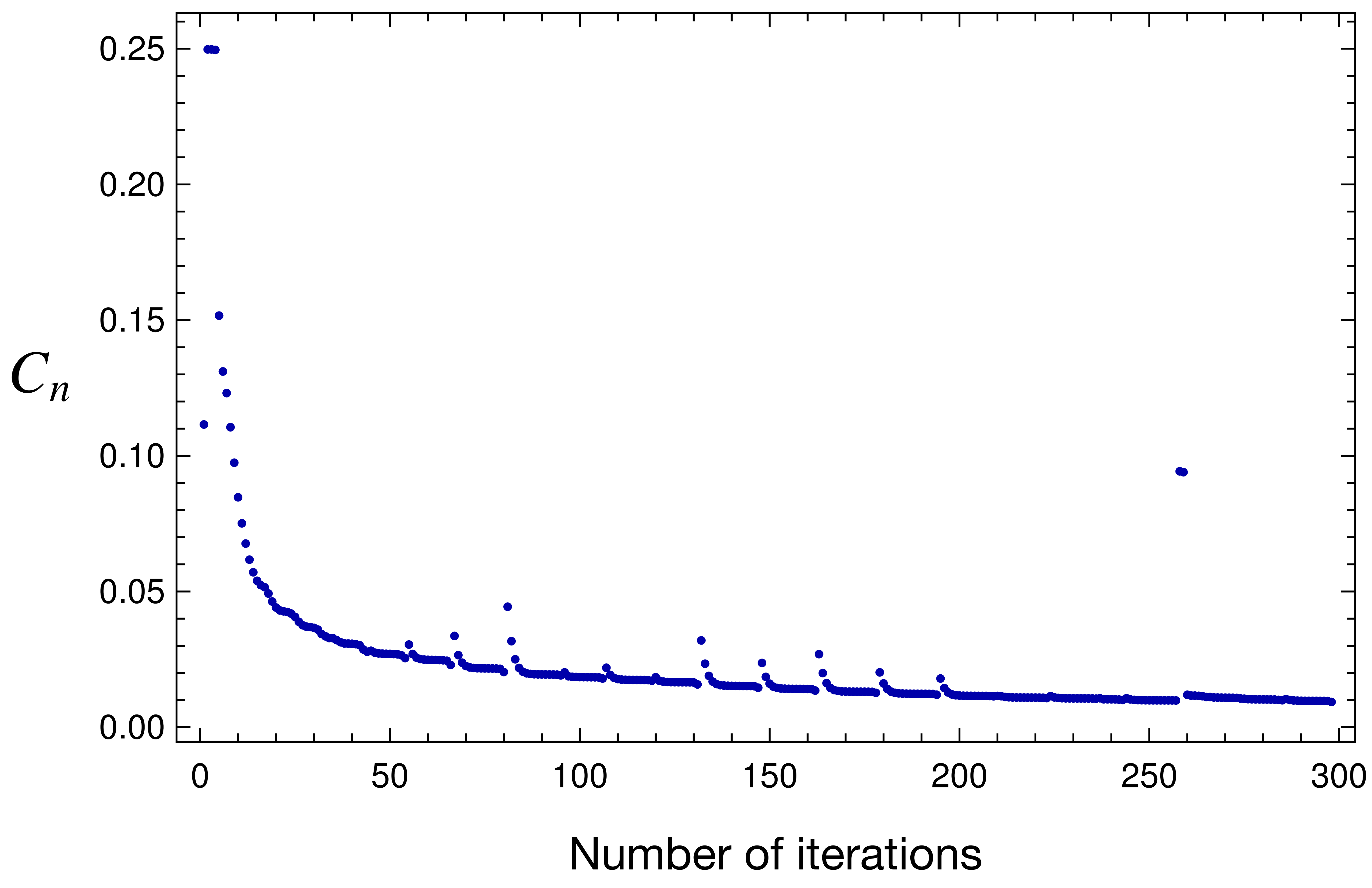
- Update for all α to preserve continuity and fix endpoints



Performance

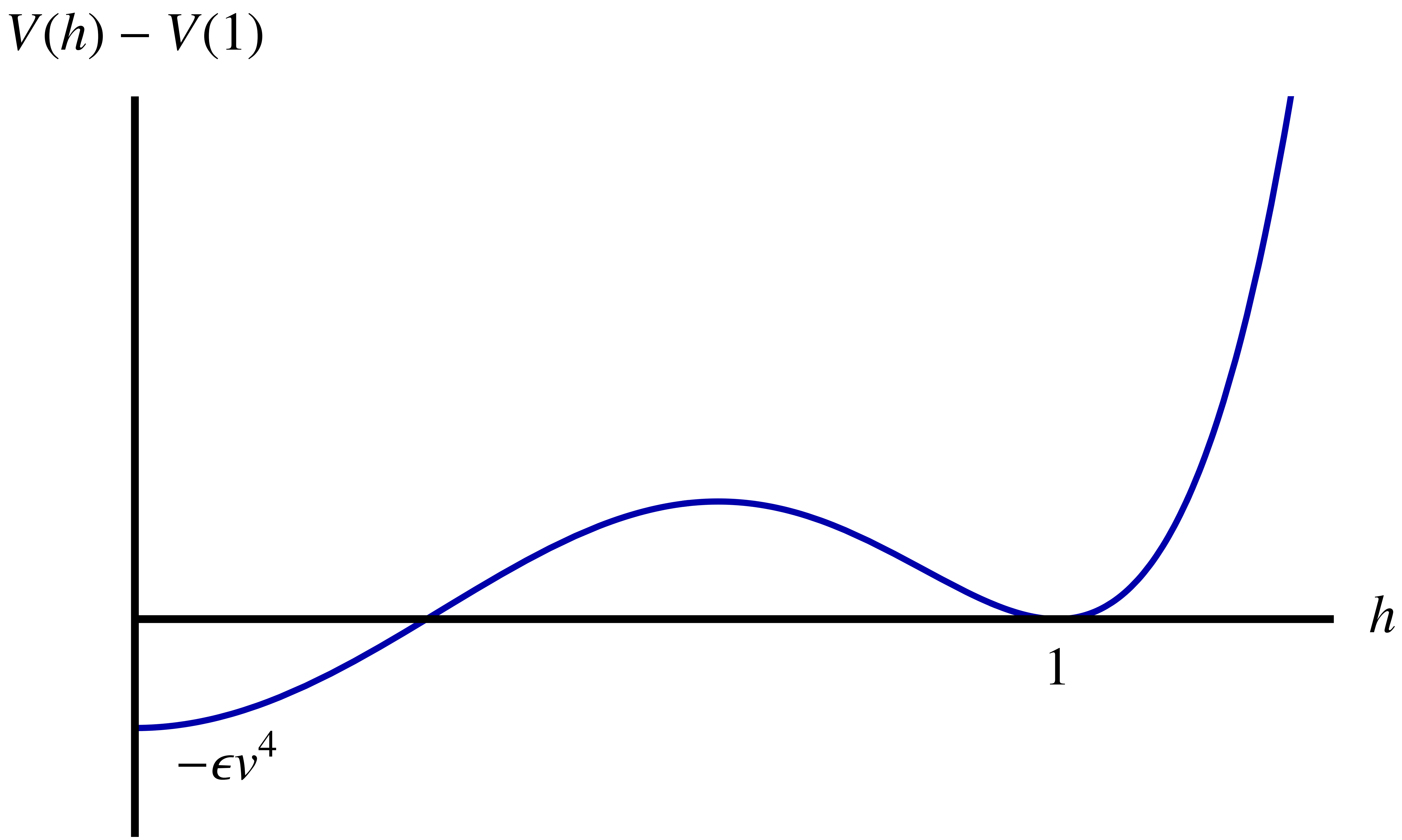
$$C_n = \frac{1}{N_{points}}\left[\sum_{points}\left(\left(\frac{\delta I}{\delta h_{\bar{\alpha}}}\right)^2 + \left(\frac{\delta I}{\delta u_{\bar{\alpha}}}\right)^2\right)\right]^{1/2}$$



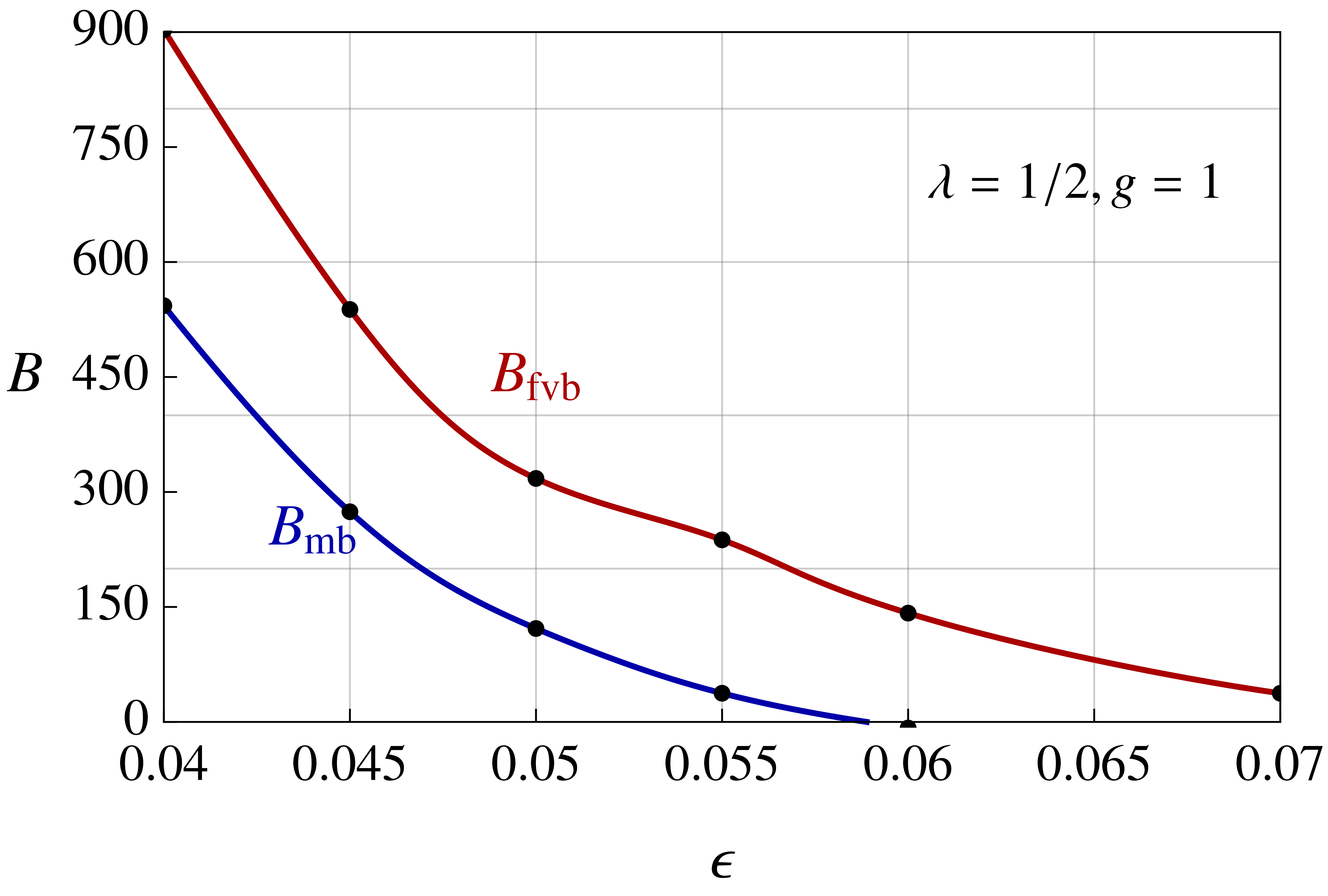


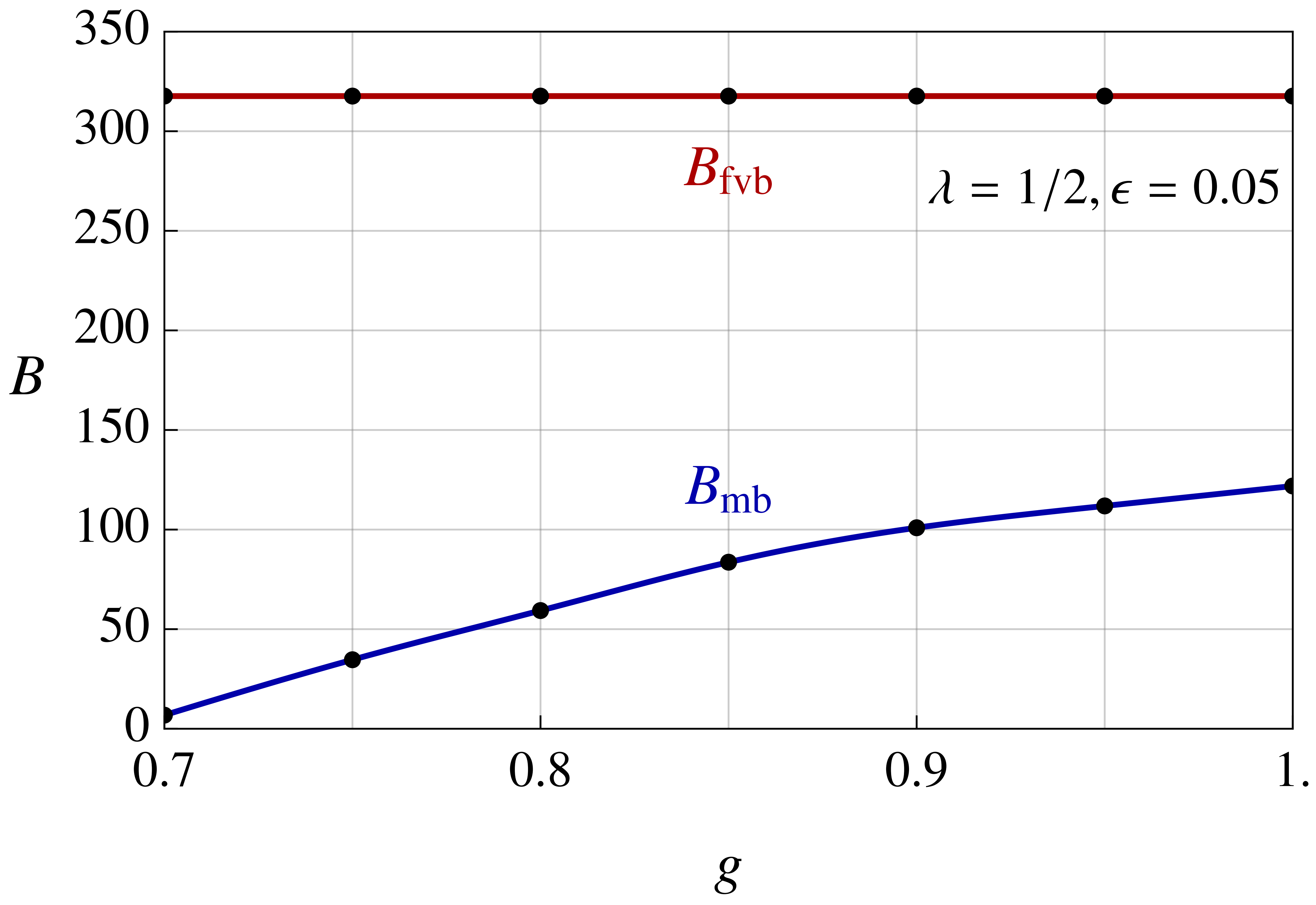
Scalar Potential

$$V(h) = \frac{\lambda v^4}{2}\left(h^2 - \left(2 - \frac{6\epsilon}{\lambda}\right)h^4 + \left(1 - \frac{4\epsilon}{\lambda}\right)h^6\right)$$



Results





$$4 \times 10^{-157} < e^{B_{\mathrm{mb}} - B_{\mathrm{fvb}}} < 8 \times 10^{-66}$$

Applications and future directions

Domain Walls catalysing the electroweak PT

Blasi & Mariotti: hep-ph/2203.16450

- SM + singlet scalar (S) odd under $\mathbb{Z}_2$ symmetry
- Two step phase transition: $(h, S) = (0,0) \rightarrow (0, v_s) \rightarrow (v_{\rm ew}, 0)$

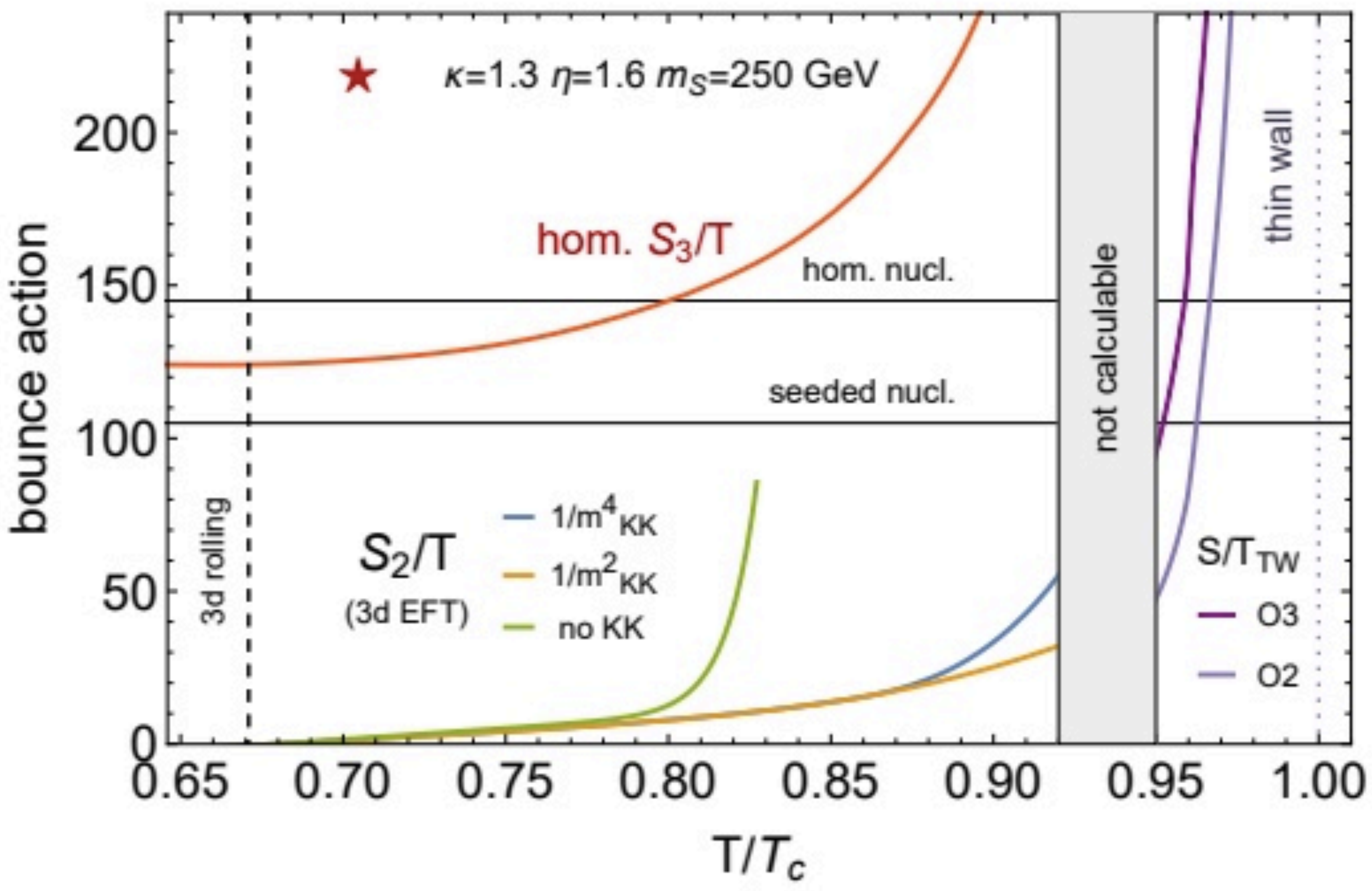
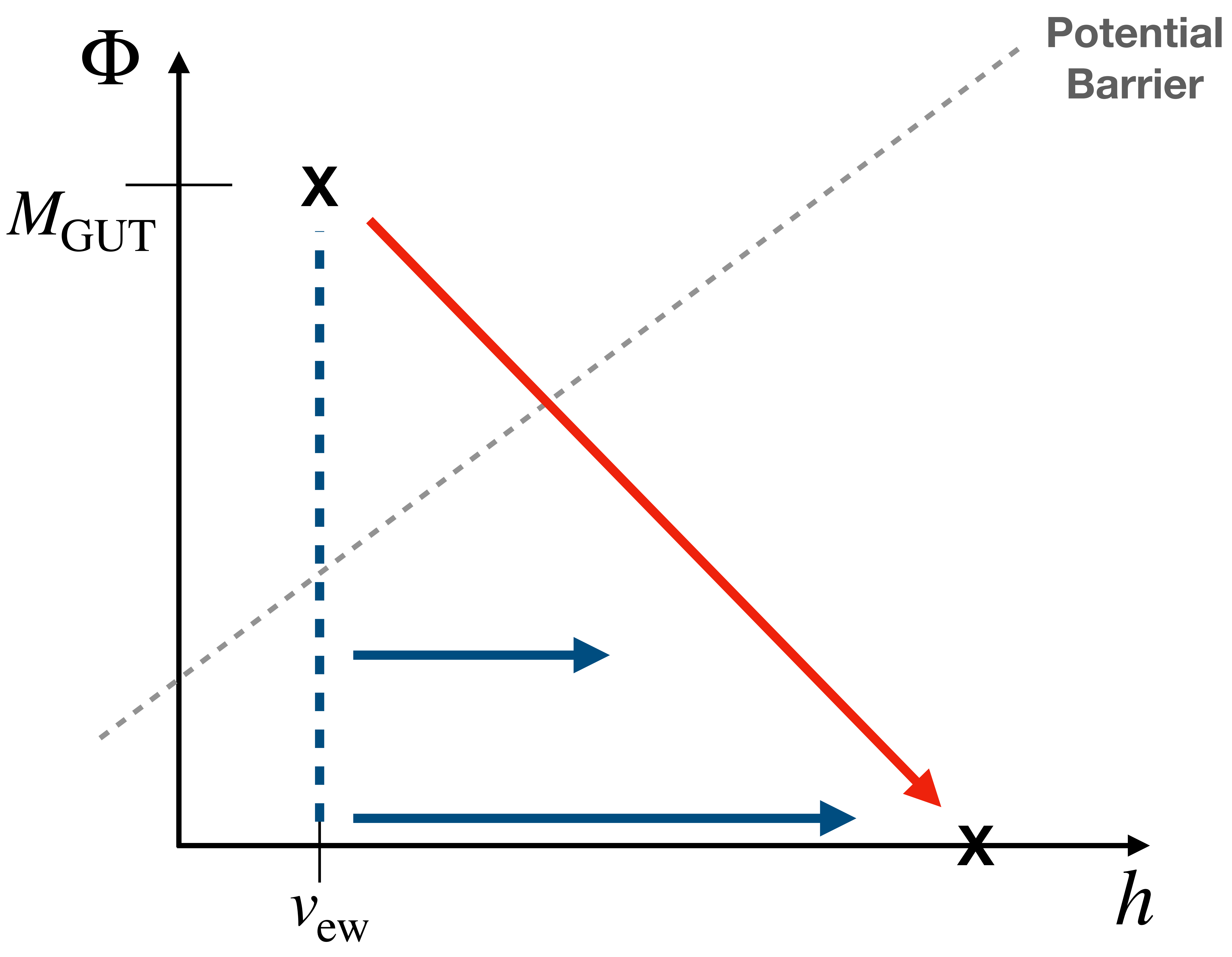


Figure from hep-ph/2203.16450

Applications and future directions

GUT monopoles and electroweak vacuum decay



Conclusions

- Catalysed phase transitions common in nature but seemingly not in cosmology
- Monopoles and other topological defects can act to catalyse cosmological phase transitions
- Results show effects can be dramatic due to exponential dependence on B
- Can modify electroweak phase transition and possibly electroweak vacuum stability in theories with monopoles

Backup Slides

Cosmological Impurities

- Monopoles:

  Steinhardt: Phys. Rev. D **24**, 842 (1981), *Nucl.Phys.B* 190 (1981)
  Kumar, Paranjape & Yajnik: hep-ph/0908.3949, hep-ph/1006.0693

- Other topological defects:

  Jensen & Steinhardt: *Phys.Rev.B* 27 (1983)
  Yajnik: *Phys.Rev.D* 34 (1986)
  Hosotani: *Phys.Rev.D* 27 (1983)
  Preskill & Vilenkin: *Phys.Rev.D* 47 (1993)
  Kusenko: *Phys.Lett.B* 406 (1997)
  Blasi & Mariotti: hep-ph/2203.16450

- Black holes:

  Hiscock: Phys. Rev. D 35, 1161 (1987)
  Berezin, Kuzmin, Tkachev: Phys.Lett.B 207 (1988), Phys.Rev.D 43 (1991)
  Gregory, Moss & Withers: hep-th/1401.0017,
  Burda, Gregory & Moss: hep-th/1503.07331, hep-th/1601.02152
  Shkerin & Sibiryakov: hep-th/2105.09331, hep-th/2111.08017

Monopoles as Impurities

- Monopole solutions:

$$\phi^a = v\hat{r}^a h_{\mathrm{m}}(r)$$

$$A_i^a = \epsilon^{iam}\hat{r}^m \left(\frac{1 - u_{\mathrm{m}}(r)}{gr}\right)$$

- Continuity requires

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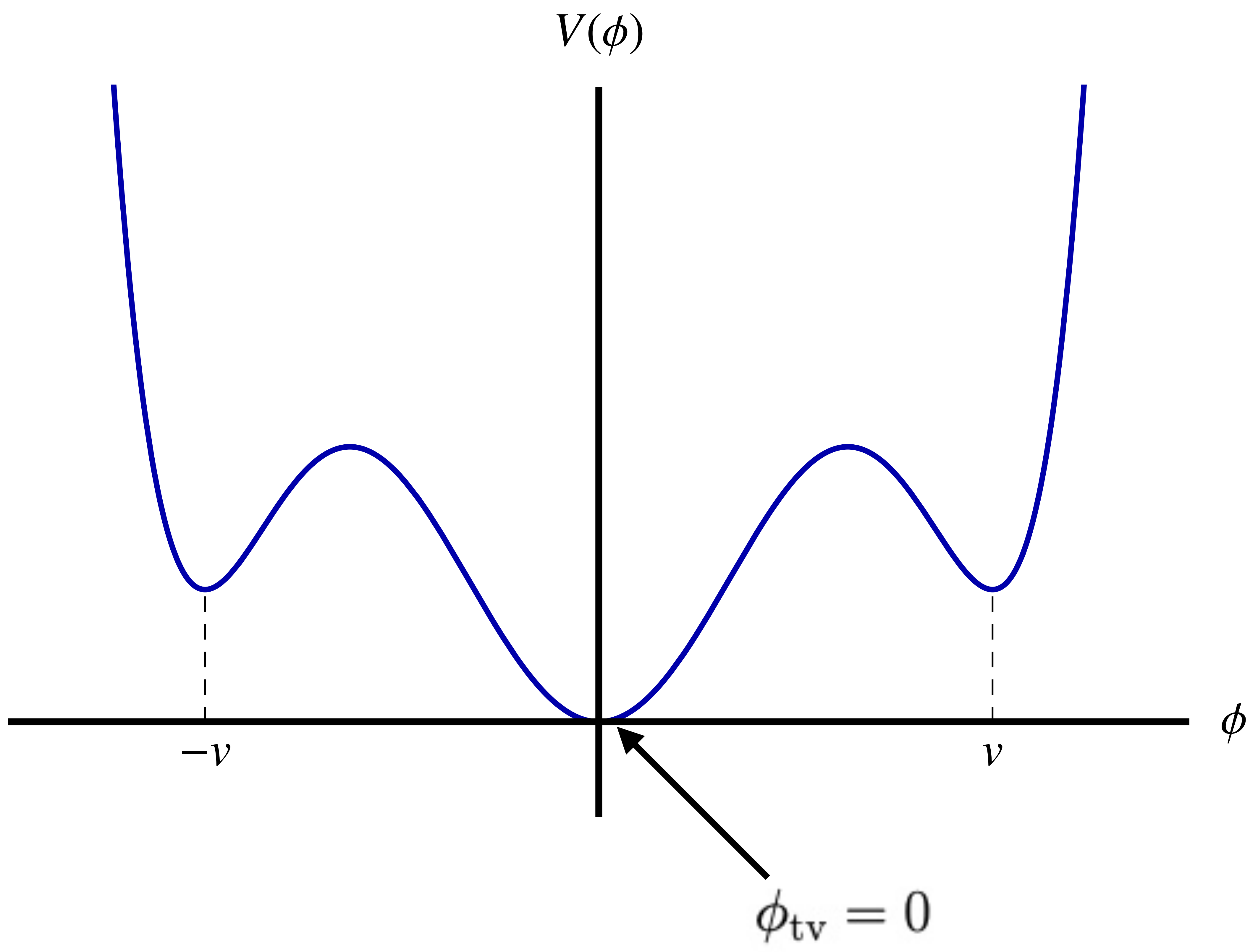


Magnetic Monopoles

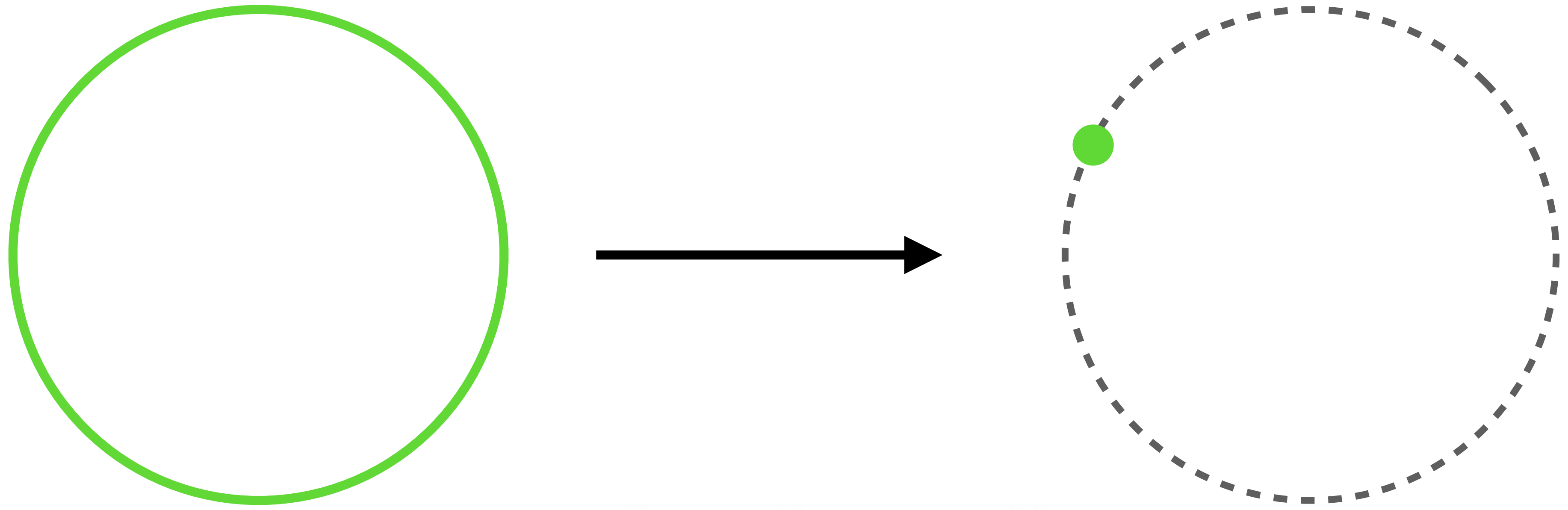
’t Hooft: Nucl.Phys.B 79 (1974), Polyakov: JETP Lett. 20 (1974)

- Consider SU(2) gauge group broken to U(1) subgroup by triplet scalar

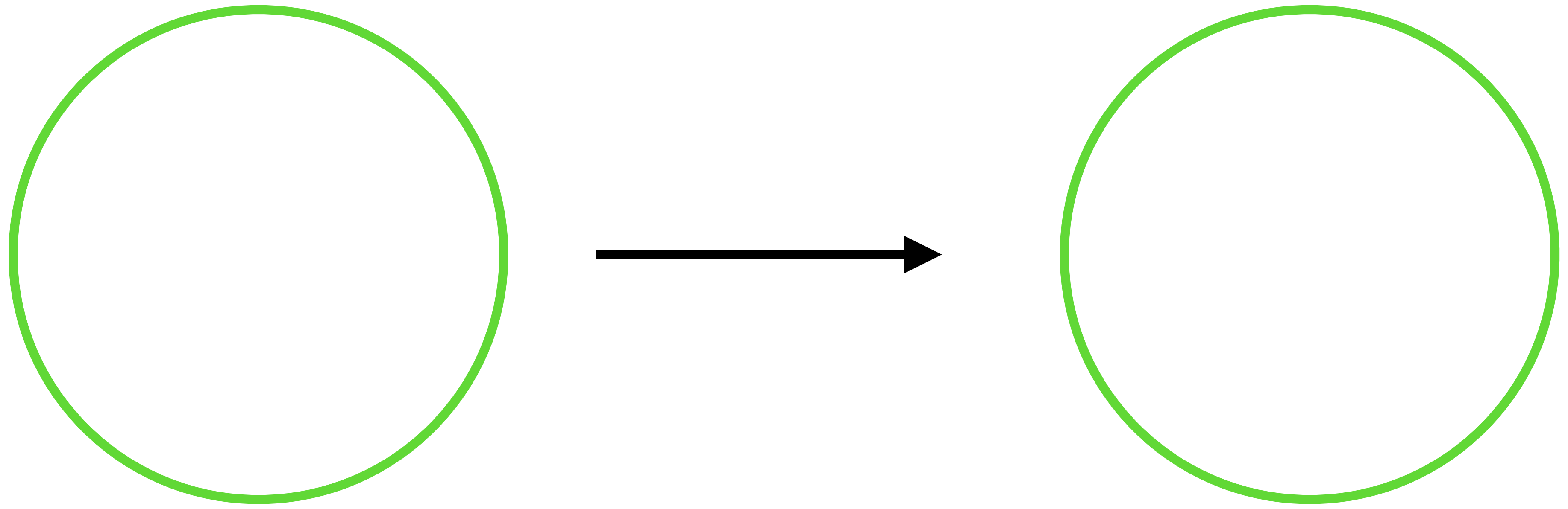
- Vacuum is a two-sphere:

$$\sum_{i=1}^{3} \phi^a \phi^a = v^2$$

- Have topologically distinct maps from spatial two-sphere to vacuum

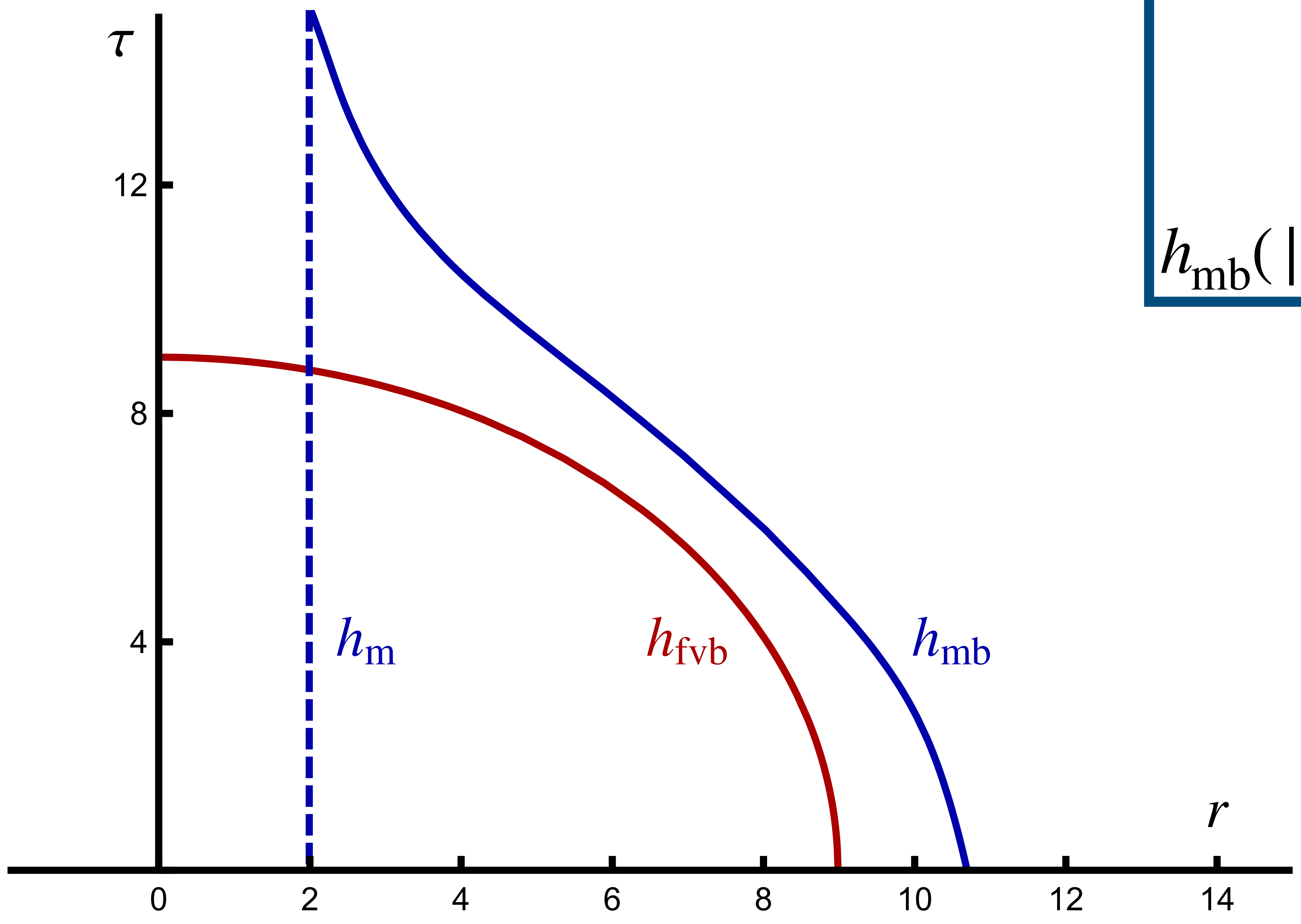


$$\phi^a_\infty(\theta, \varphi) = v\delta^{a3}$$



$$\phi^a_\infty(\theta, \varphi) = v\hat{r}^a$$

$O(4)$ symmetry



Monopole Tunnelling

$$\ddot{h}_{\mathrm{mb}} + h''_{\mathrm{mb}} + \frac{2}{r}h'_{\mathrm{mb}} = \frac{2h_{\mathrm{mb}}u^2_{\mathrm{mb}}}{r^2} + \frac{\partial V(vh_{\mathrm{mb}})}{\partial h}$$

$$\ddot{u}_{\mathrm{mb}} + u''_{\mathrm{mb}} = \frac{u_{\mathrm{mb}}(u^2_{\mathrm{mb}} - 1)}{r^2} + h^2_{\mathrm{mb}}u_{\mathrm{mb}}$$

$$h_{\mathrm{mb}}(|\tau| \to \infty, r) = h_m(r), \quad u_{\mathrm{mb}}(|\tau| \to \infty, r) = u_m(r)$$

FV Tunnelling

$$\phi''_{\mathrm{fvb}} + \frac{3}{\rho}\phi'_{\mathrm{fvb}} = \frac{\partial V(\phi_{\mathrm{fvb}})}{\partial \phi}$$

$$\rho = \sqrt{r^2 + \tau^2}$$

Gravitational Waves

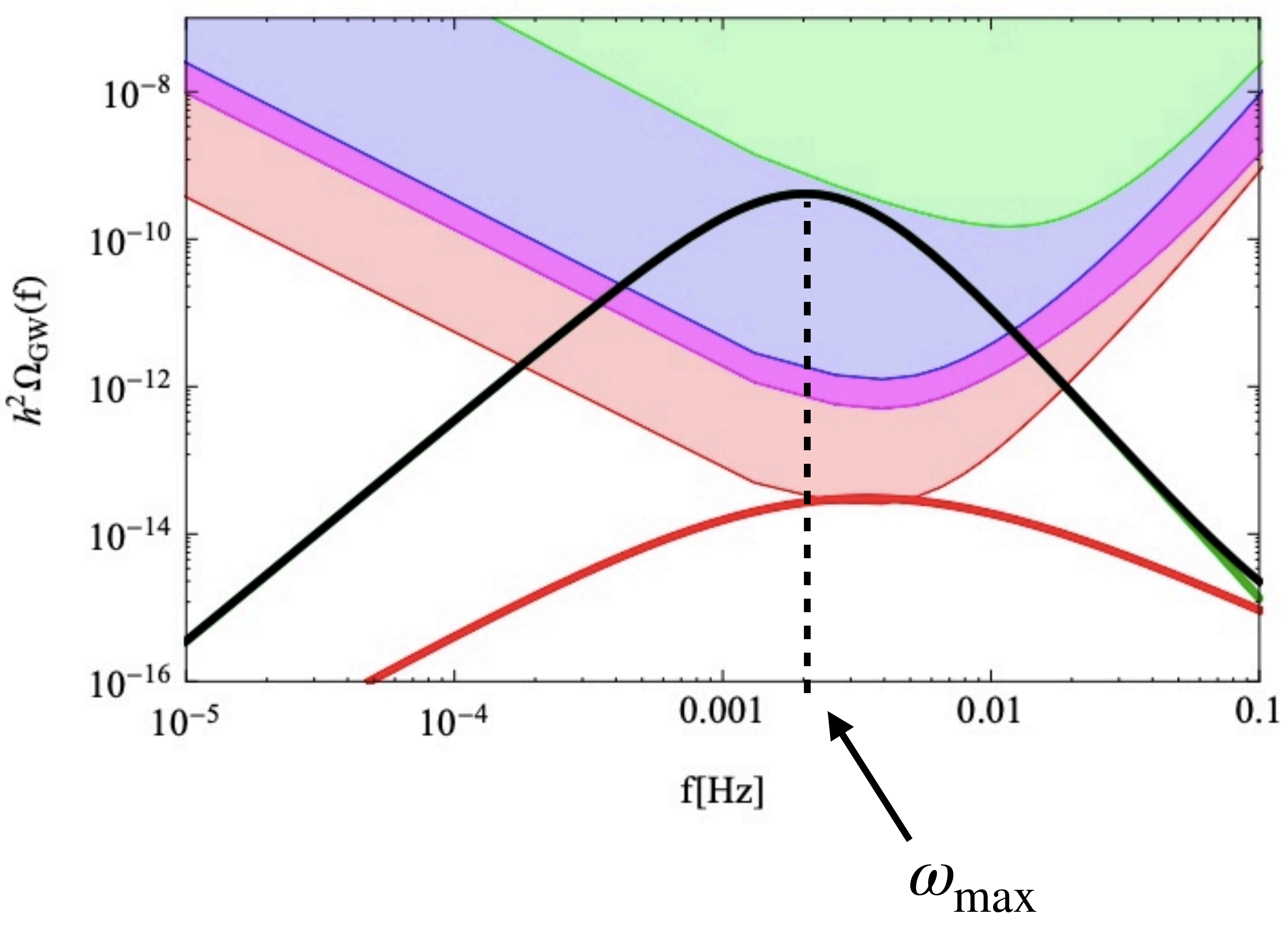
- General case:

$$\frac{E_{\mathrm{GW}}}{E_{\mathrm{vac}}} \propto \left(\frac{H}{\beta}\right)^2$$

$$\omega_{\mathrm{max}} \propto \beta$$

- β^{-1} = average separation of bubbles

Figure from hep-ph/1512.06239



Gravitational Wave Anisotropies

Bartolo et. al.: hep-ph/2201.08782; Geller et. al.: hep-ph/1803.10780

- LISA potentially sensitive to anisotropies in GW signal
- Monopole PT has signal which behaves like

$$\frac{\delta\rho_{GW}(\theta,\phi)}{\rho_{GW}} \propto \frac{\delta n_m(\theta,\phi)}{n_m}$$

- If $\delta n_m(\theta,\phi)/n_m$ independent of $\delta T(\theta,\phi)/T$ anisotropy may be observable

Results - early universe PTs

- Condition for monopoles to dominate:

$$n_m \gtrsim v^3 e^{B_{\rm mb}-B_{\rm fv}}\,.$$

- Kibble mechanism produces monopoles

$$n_{m,{\rm kibble}}(T) \simeq H^3 \left(\frac{v}{T}\right)^3$$

- Kibble produced monopoles dominate for:

$$\frac{T}{M_P} \gtrsim 1.7 \times 10^{-23} \left(\frac{e^{B_{\rm mb}-B_{\rm fv}}}{10^{-66}}\right)^{1/3}$$

Results - present day PTs

- Condition for monopoles to dominate:

$$\Omega_m > 1.3 \times 10^{-23} g^3 \left(\frac{M_m}{\mathrm{GeV}} \right)^4 \left(\frac{e^{B_{\mathrm{mb}} - B_{\mathrm{fv}}}}{10^{-66}} \right)$$

- Phase transition can be seeded by $O(1)$ monopole per Hubble today