

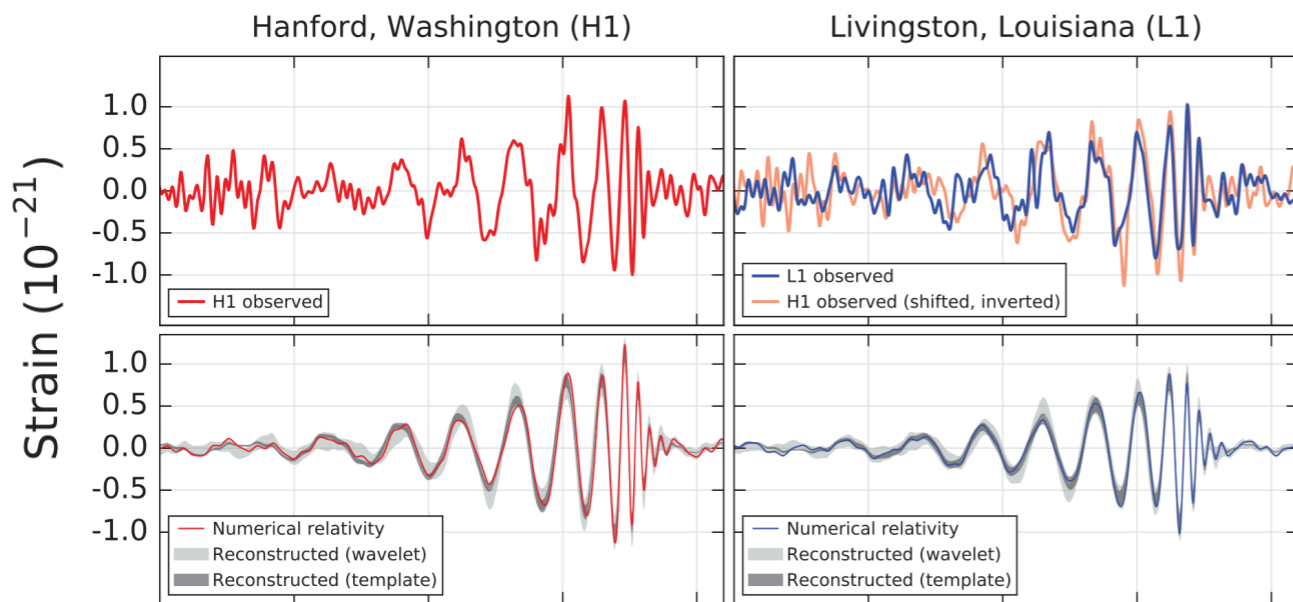
Probing primordial fluctuations through stochastic gravitational wave background anisotropies

Soubhik Kumar
UC Berkeley and LBL

w/ Raman Sundrum and Yuhsin Tsai, [2102.05665](#), *JHEP* 11 (2021) 107
w/ Yanou Cui, Raman Sundrum and Yuhsin Tsai, in progress



Gravitational waves are exciting!

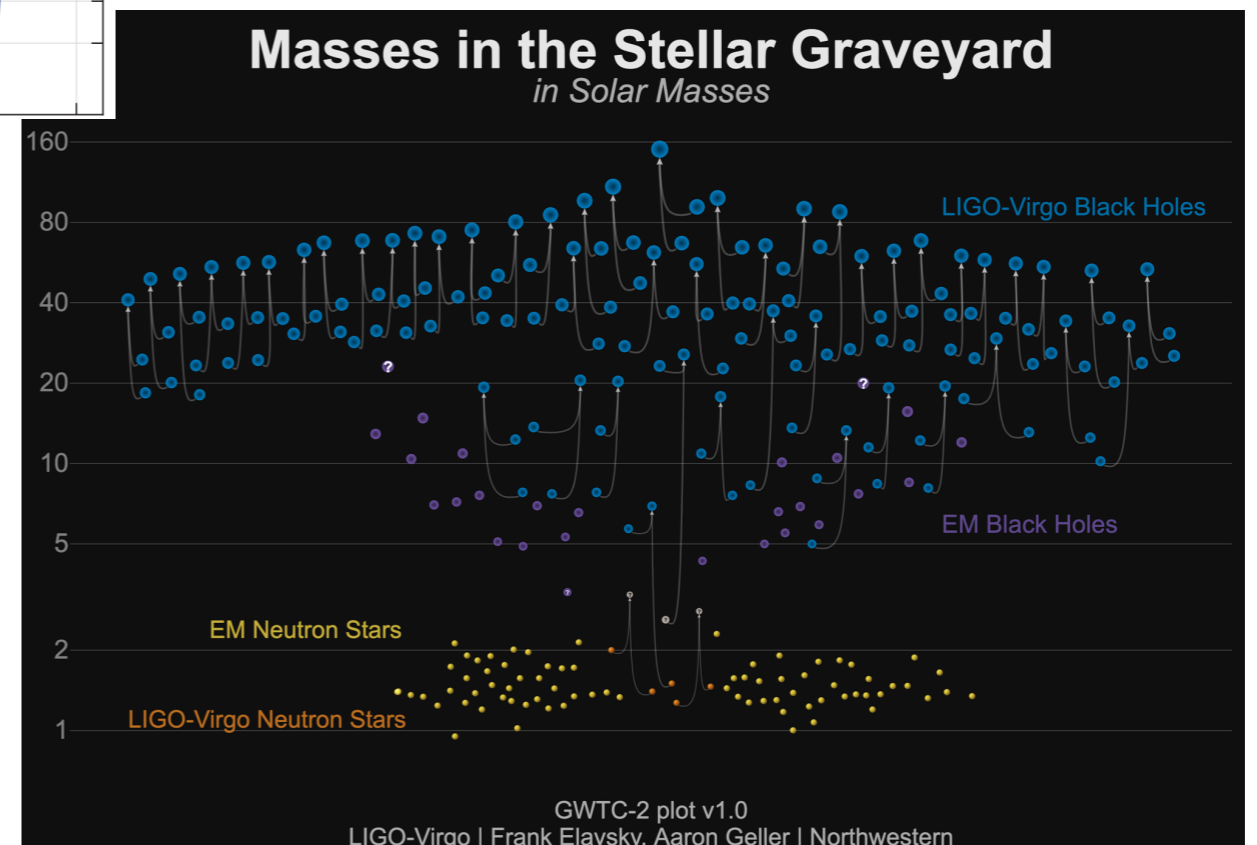


GW from Binary BH mergers first detected in 2015 by LIGO

Abbott et al., '16, LIGO-Virgo

Since then many more, including NS-BH, NS-NS
Multi-messenger astronomy

Abbott et al., '17, LIGO-Virgo



Stochastic GW Background (SGWB)

- However, **far-away** mergers not individually detected
- Combine to give a Stochastic GW Background (random temporal phase)

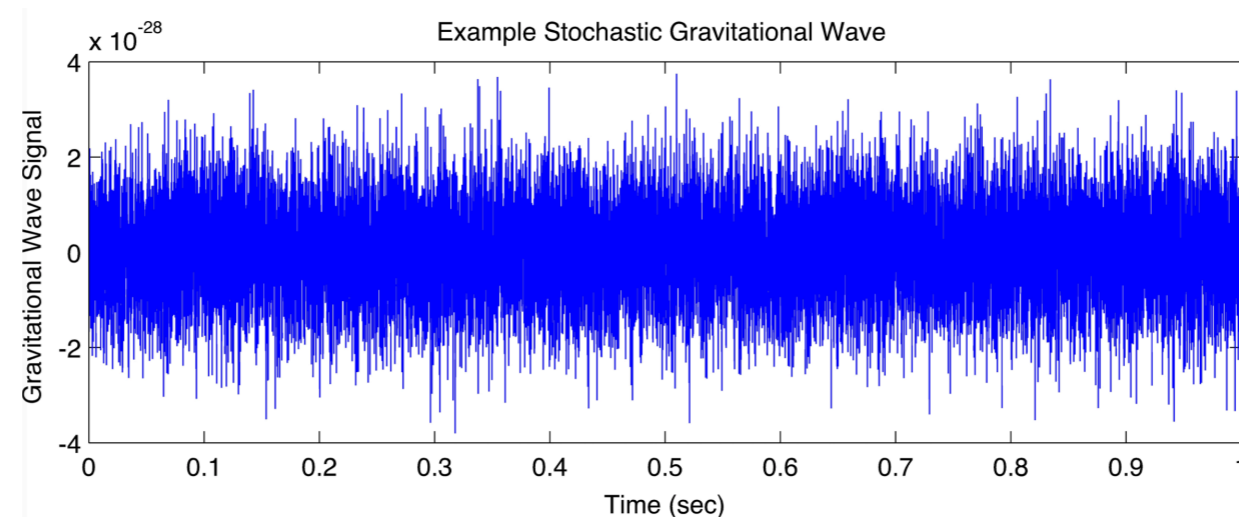


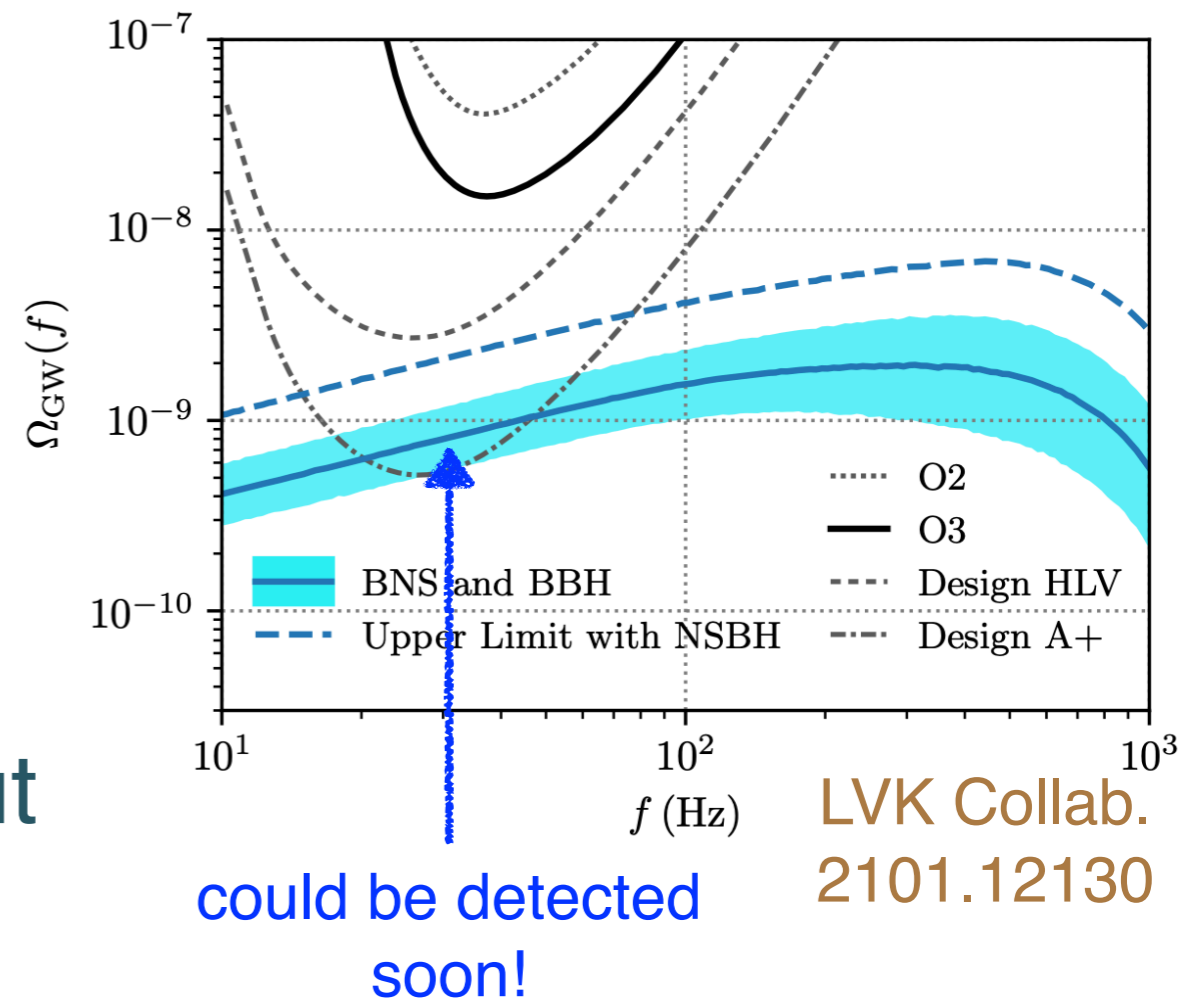
Image from LIGO

- Also a rich source of information and similar to CMB

Astrophysical SGWB

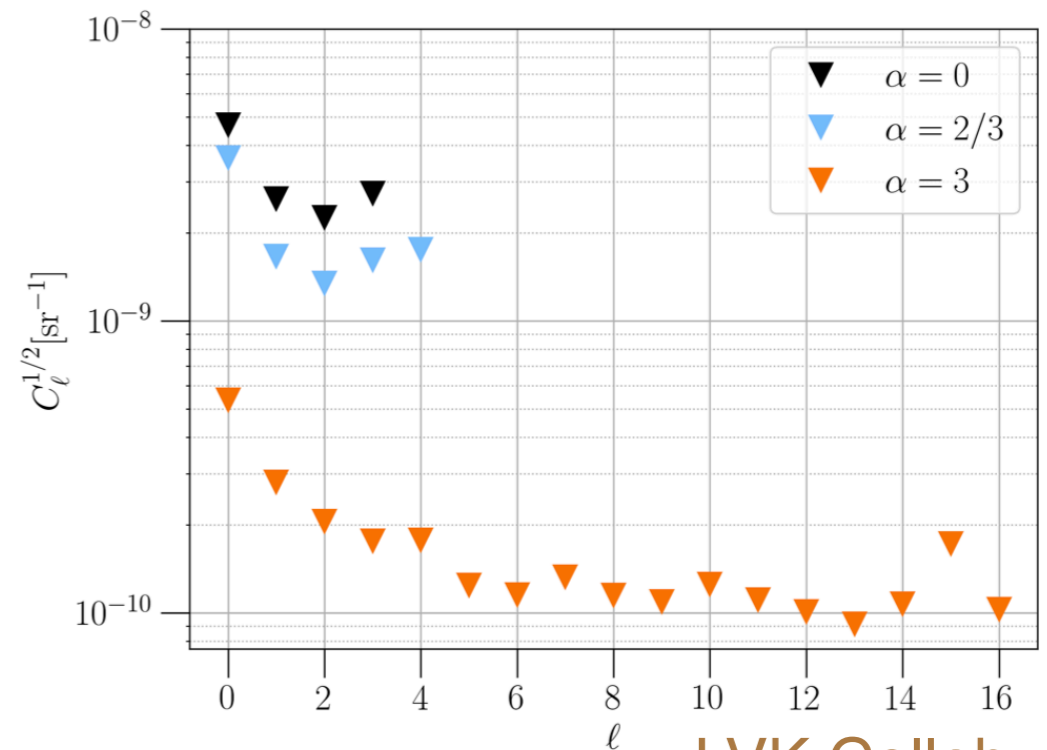
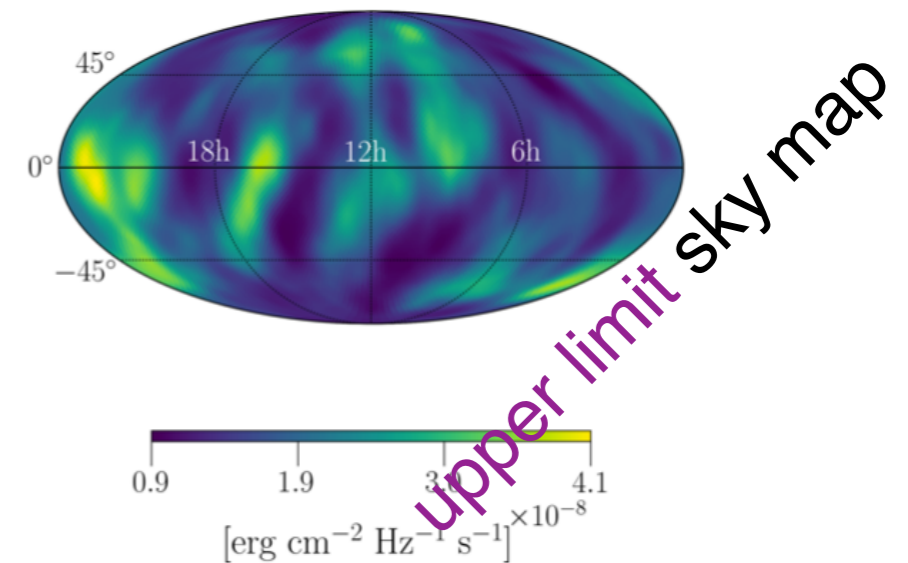
- Might be visible with LIGO upgrade
- ALIGO-AVirgo bound for $\alpha = 2/3$, $\Omega_{\text{GW}} \leq 3.4 \times 10^{-9}$ (at 25 Hz)
- Also relevant for LISA. Additionally **WD contributions** are also important. Learn about astrophysics.

$$\Omega_{\text{GW}}(f) = \underbrace{\Omega_{\text{GW}}(f_{\text{ref}})}_{\text{uncertain}} \left(\frac{f}{f_{\text{ref}}} \right)^{2/3}$$



Anisotropic SGWB

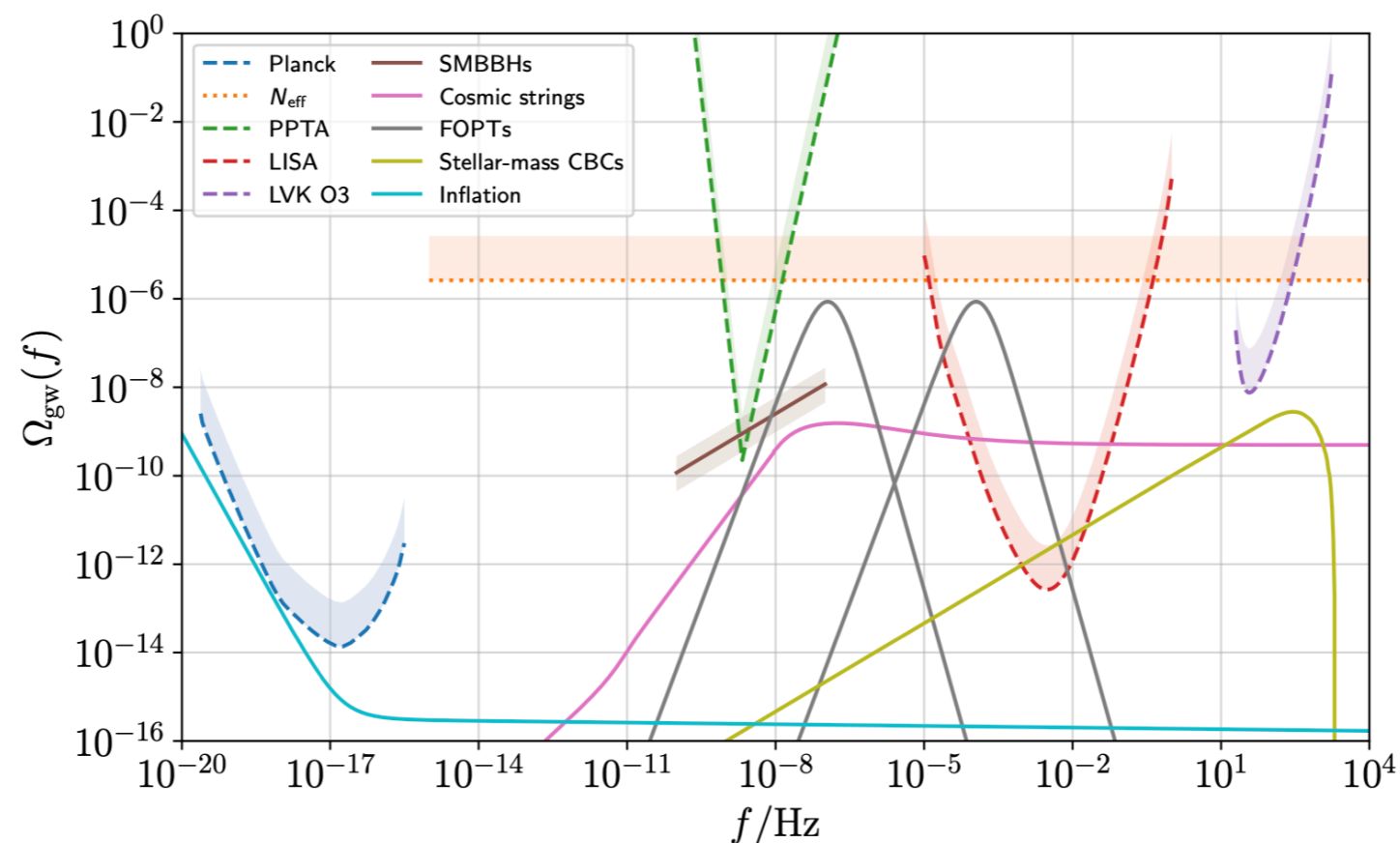
- **Anisotropies** in astrophysical SGWB are also expected.
- Astrophysical SGWB **biased** tracer of δ_m .
- Already constraint up to $\ell = 4$ by LIGO & VIRGO for the astrophysical component



LVK Collab.
2103.08520

Cosmological SGWB

- A **variety of cosmological sources** as well, e.g., phase transition, cosmic string, preheating, inflation etc.
- Strength is model dependent, but **can be above astrophysical SGWB**



Renzini et al.
2202.00178

Anisotropic Cosmological SGWB

- However, **anisotropies** will also generically be present in cosmological component.
- Very important from particle physics point of view, since GW “**free stream**” and can preserve pristine primordial info.
- Potentially **new map** independent of the CMB

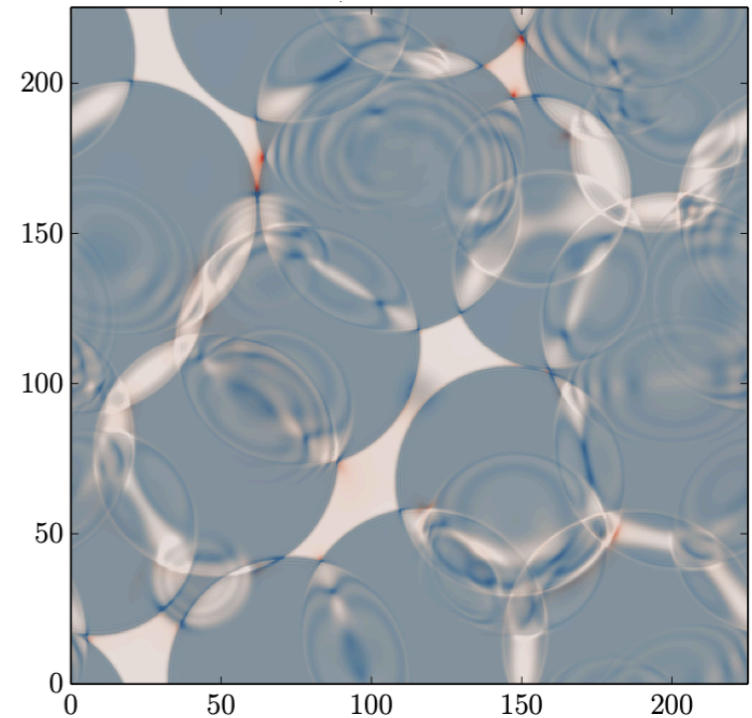
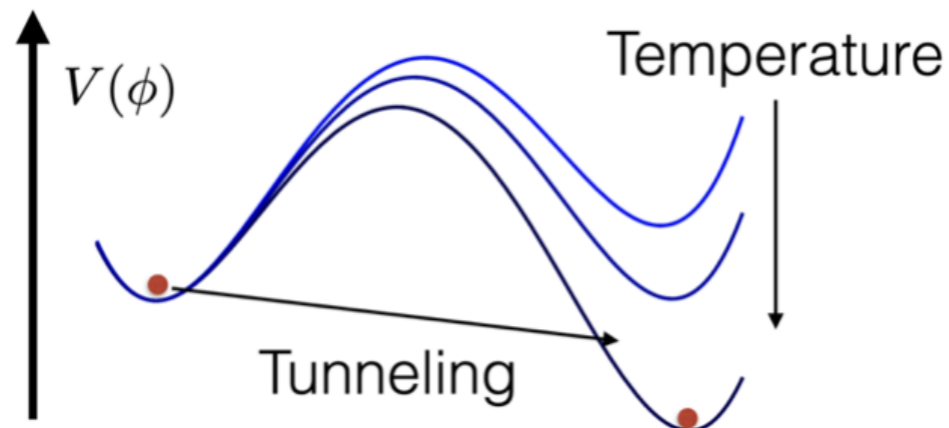
Broad question:

suppose we detect an anisotropic SGWB,
what can we learn from it?

Outline

- Phase transition and SGWB anisotropy
- Non-Gaussianity of SGWB
- Adiabatic vs. Isocurvature scenario
- Conclusion

GW from first order phase transitions



- **Bubble collisions**, sound waves, turbulence

Cutting et al.
1802.05712

$$\Omega_{\text{GW}}^{\text{peak}} h^2 = 1.3 \times 10^{-6} \left(\frac{H_{\text{PT}}}{\beta} \right)^2 \left(\frac{\alpha}{1 + \alpha} \right)^2$$

$$\alpha = \rho_{\text{vac}} / \rho_{\text{rad}} \quad \text{fractional energy density}$$

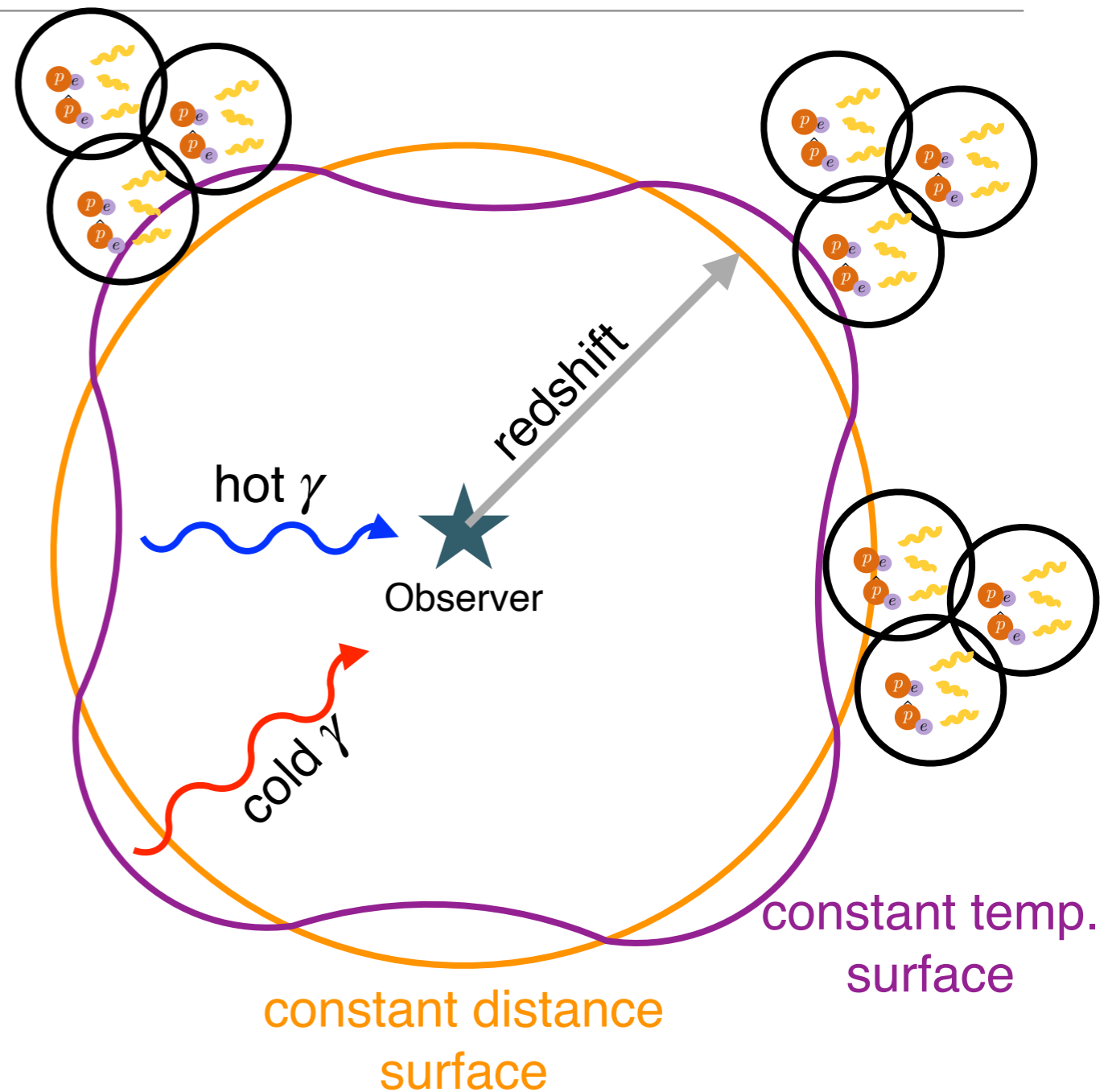
$$\omega_{\text{GW}}^{\text{peak}} = 0.04 \text{ mHz} \left(\frac{\beta}{H_{\text{PT}}} \right) \left(\frac{T_n}{\text{TeV}} \right)$$

$$\beta / H_{\text{PT}} \equiv d \ln \Gamma / dt \quad \text{duration}$$

Huber, Konstandin, 0806.1828

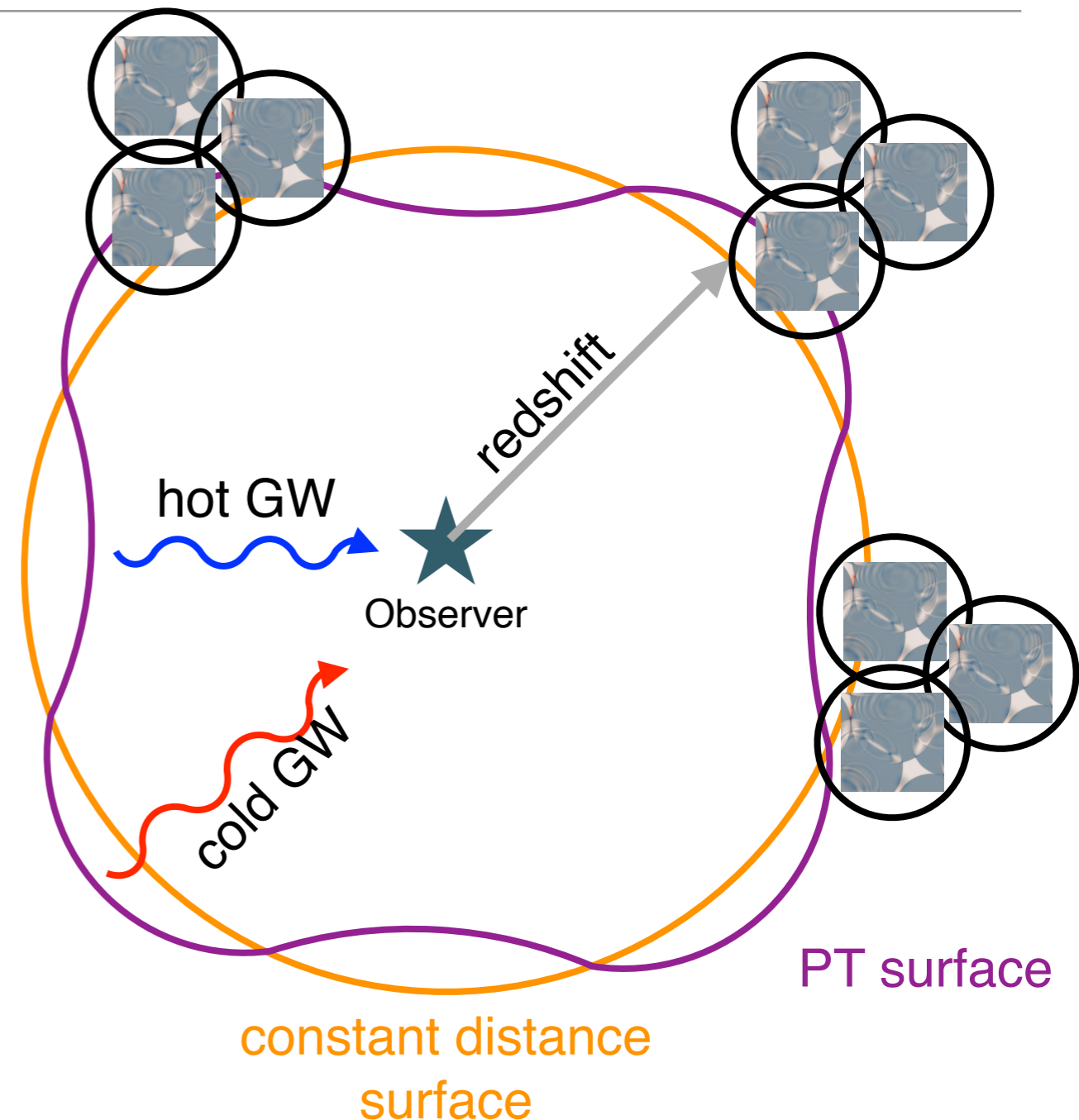
Anisotropy in CMB

- **Single field** inflation → each Hubble patch undergoes the **same history**, but with **time delay**
- Results into anisotropies in the CMB
- **Independent of microphysics** of decoupling



Anisotropic SGWB from PT

- Similar argument in the context of PT
- Surface CMB decoupling \leftrightarrow **surface of PT**
- Inflation generates **large scale fluctuations** in the fluid undergoing PT
- Imprinted in GW



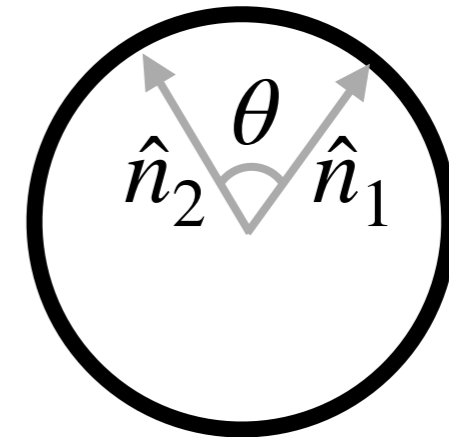
Hook et al.
1803.10780

Characterizing anisotropy

- Similar to CMB: $\delta_{\text{GW}}(f, \hat{n}) \equiv \frac{\delta\rho_{\text{GW}}}{\rho_{\text{GW}}} = \frac{\rho_{\text{GW}}(f, \hat{n}) - \bar{\rho}_{\text{GW}}(f)}{\bar{\rho}_{\text{GW}}(f)}$

$$C^{\text{GW}}(\theta) \equiv \langle \delta_{\text{GW}}(\hat{n}_1) \delta_{\text{GW}}(\hat{n}_2) \rangle.$$

$$C^{\text{GW}}(\theta) = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}^{\text{GW}} P_{\ell}(\cos \theta),$$



- For **scale invariant** spectrum,

$$C_{\ell}^{\text{GW}} \propto [\ell(\ell + 1)]^{-1} \quad \text{i.e.} \quad \delta_{\text{GW}}(\theta \sim 1/\ell) \propto 1/\ell$$

small-scale modes have less power,
important restriction on the number of visible modes

Angular sensitivity of future missions

$$\Omega_{\text{GW}}^{\text{peak}} h^2 = 1.3 \times 10^{-6} \left(\frac{H_{\text{PT}}}{\beta} \right)^2 \left(\frac{\alpha}{1 + \alpha} \right)^2$$

$$\delta_{\text{GW}}(\theta \sim 1/\ell) \propto 1/\ell \times 10^{-4}$$

small β
motivated
by approx.
conformal
theories

LISA benchmark

DECIGO/BBO benchmark

$$(\beta/H_{\text{PT}})^2 = 10$$

$$T_n \approx 10 - 100 \text{ TeV}$$

$$\alpha \sim 0.1$$

$$\ell \lesssim 10$$

$$(\beta/H_{\text{PT}})^2 = 100$$

$$T_n \approx 10^4 \text{ TeV}$$

$$\alpha \sim 0.1$$

$$\ell \lesssim 100$$

Konstandin, Servant
1104.4791
Agashe et al.
1910.06238

Future missions can be probe up to $\ell \lesssim 100$,
also within their angular resolution

Summarizing cosmo. anisotropies

- Probing power spectrum is already very interesting, but we can have **detectable anisotropies** in SGWB
- First order PT are quite **typical BSM phenomena** that can imprint such anisotropies
- GW “**free stream**” so primordial information preserved and can be independent from CMB
- What can we learn from such a map?

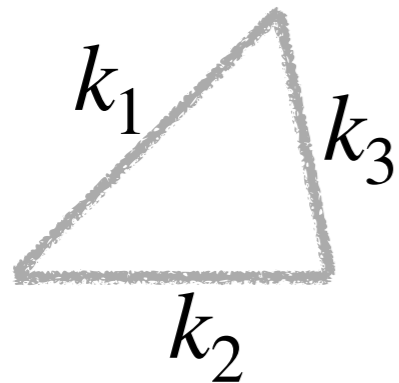
Outline

- Phase transition and SGWB anisotropy
- Non-Gaussianity of SGWB
- Adiabatic vs. Isocurvature scenario
- Conclusion

Non-Gaussianity of SGWB

- Primordial non-Gaussianity (NG) characterizes **interactions** of the inflaton (or metric) fluctuations ζ

$$F(k_1, k_2, k_3) = \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)}$$



$$P(k) = \langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle'$$

rotational
invariance

$$B(k_1, k_2, k_3) = \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle'$$

Simplified definition of CMB NG

- On very **large scales**, CMB anisotropies approximated by Sachs-Wolfe effect

$$\delta_\gamma \equiv \frac{\delta\rho_\gamma}{\rho_\gamma} = 4 \frac{\delta T}{T} \Big|_{\text{CMB}} \approx -\frac{4}{5}\zeta$$

- Motivates a simplified definition,

$$F_{\text{CMB}}(k_1, k_2, k_3) = \frac{2}{3} \frac{\langle \delta_\gamma(\vec{k}_1) \delta_\gamma(\vec{k}_2) \delta_\gamma(\vec{k}_3) \rangle}{P_\gamma(k_1)P_\gamma(k_2) + P_\gamma(k_1)P_\gamma(k_3) + P_\gamma(k_2)P_\gamma(k_3)}$$

Not using the standard $\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle$ notation

Bartolo et al.
1912.09433

Simplified definition of SGWB NG

- Compute the **large scale** anisotropy for GW (including only **Sachs-Wolfe** contribution)

$$\delta_{\text{GW}} \equiv \frac{\delta\rho_{\text{GW}}}{\rho_{\text{GW}}} \approx -\frac{4}{3}\zeta$$

- Analogously

$$F_{\text{GW}}(k_1, k_2, k_3) = \frac{10}{9} \frac{\langle \delta_{\text{GW}}(\vec{k}_1) \delta_{\text{GW}}(\vec{k}_2) \delta_{\text{GW}}(\vec{k}_3) \rangle}{P_{\text{GW}}(k_1)P_{\text{GW}}(k_2) + P_{\text{GW}}(k_1)P_{\text{GW}}(k_3) + P_{\text{GW}}(k_2)P_{\text{GW}}(k_3)}$$

- **Cosmic variance** limited sensitivity on ΔF_{GW}

$$\delta_{\text{GW}} \Delta F_{\text{GW}} \sim \frac{1}{\sqrt{\ell_{\text{max}}(\ell_{\text{max}} + 1)}} \sim \ell_{\text{max}}^{-1} \Rightarrow \Delta F_{\text{GW}} \sim \frac{10^4}{\ell_{\text{max}}}$$

Outline

- Phase transition and SGWB anisotropy
- Non-Gaussianity of SGWB
- **Adiabatic vs. Isocurvature scenario**
- Conclusion

Adiabatic perturbations

- Consider **single-field inflation** → single source of fluctuations → **correlated** large scale fluctuations in GW and CMB

$$\delta_{\text{GW}} \approx \frac{5}{3} \delta_{\gamma}$$

- However, primordial non-Gaussianities are **strongly constrained by existing data**, e.g., Planck, $F_{\text{CMB}} \lesssim 10$

$$\Delta F_{\text{GW}} \sim \frac{1}{\delta_{\text{GW}}} \times \frac{1}{\sqrt{\ell_{\text{max}}(\ell_{\text{max}} + 1)}} \sim 10^4 \ell_{\text{max}}^{-1}$$

- This along with $\ell_{\text{max}} \lesssim 100$ implies **CMB much better probe of NG**

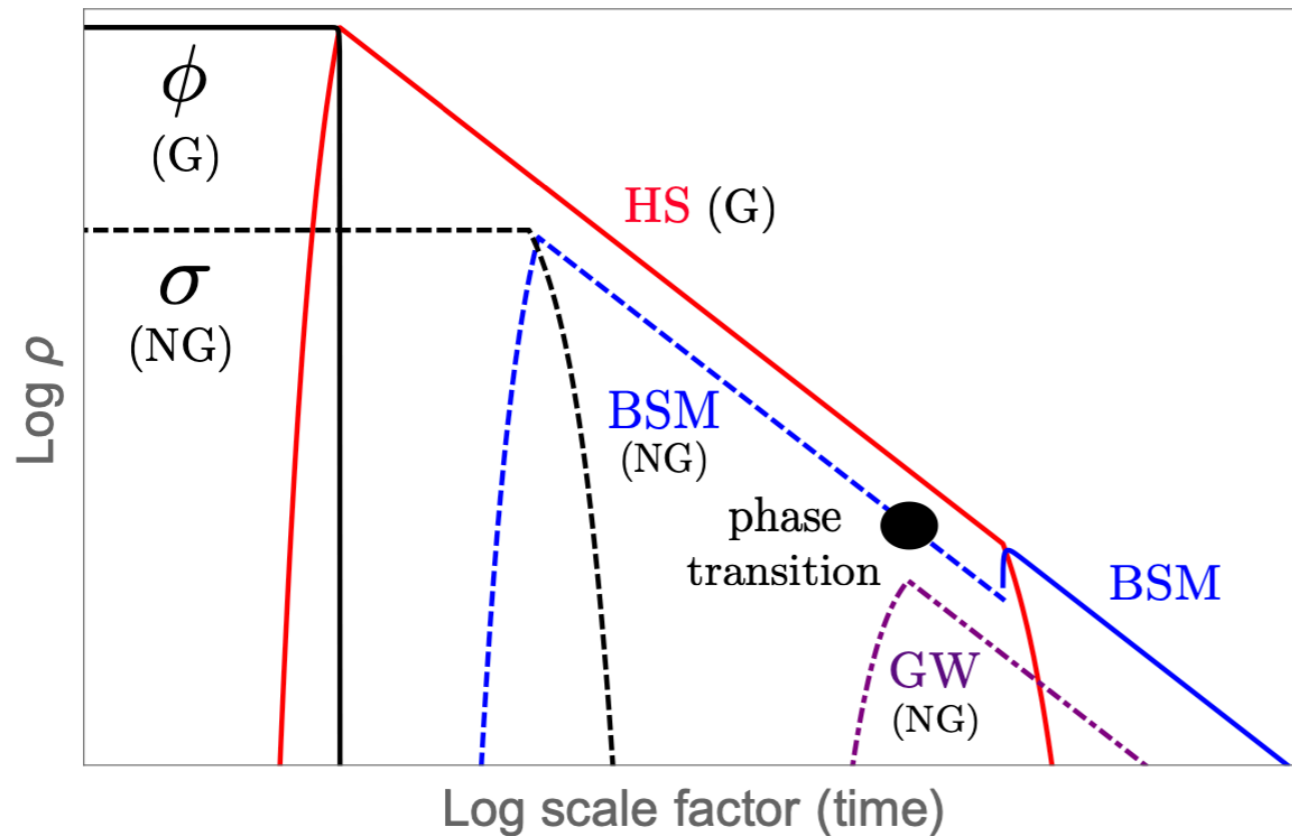
Isocurvature perturbations

- But a **very different conclusion** arises in the presence of **isocurvature perturbations**
- The PT sector can get **reheated differently** than the other sector

$$\delta_{\text{GW}} \not\approx \frac{5}{3} \delta_\gamma$$

- Consequently, large and observable NG in SGWB need **no longer be in conflict** with CMB constraints

A multi-sector reheating scenario



Inflaton decay products are highly Gaussian

Curvaton decay products are highly *non-Gaussian*

$$\delta_{\text{GW}} = -\frac{4}{3}\zeta_{\phi} + \frac{4}{3}S_{\sigma} \left(1 - \frac{4}{3}f_{\text{BSM}} \right)$$

$$f_{\text{BSM}} = \frac{\rho_{\text{BSM}}}{\rho_{\text{BSM}} + \rho_{\text{HS}}} \ll 1$$

$$\delta_{\gamma} = -\frac{4}{5}\zeta_{\phi} - \frac{4}{15}f_{\text{BSM}}S_{\sigma}$$

suppression factor

NG in CMB and GW

$$\delta_{\text{GW}} = -\frac{4}{3}\zeta_\phi + \frac{4}{3}S_\sigma \left(1 - \frac{4}{3}f_{\text{BSM}}\right) \quad \delta_\gamma = -\frac{4}{5}\zeta_\phi - \frac{4}{15}f_{\text{BSM}}S_\sigma$$

suppression factor

$$F_{\text{CMB}} \approx -\frac{5}{6} \left(\frac{f_{\text{BSM}}}{3}\right)^3 \left(\frac{P_S(k_1)P_S(k_2) + \text{perms.}}{P_\phi(k_1)P_\phi(k_2) + \text{perms.}}\right) F_{S_\sigma}$$

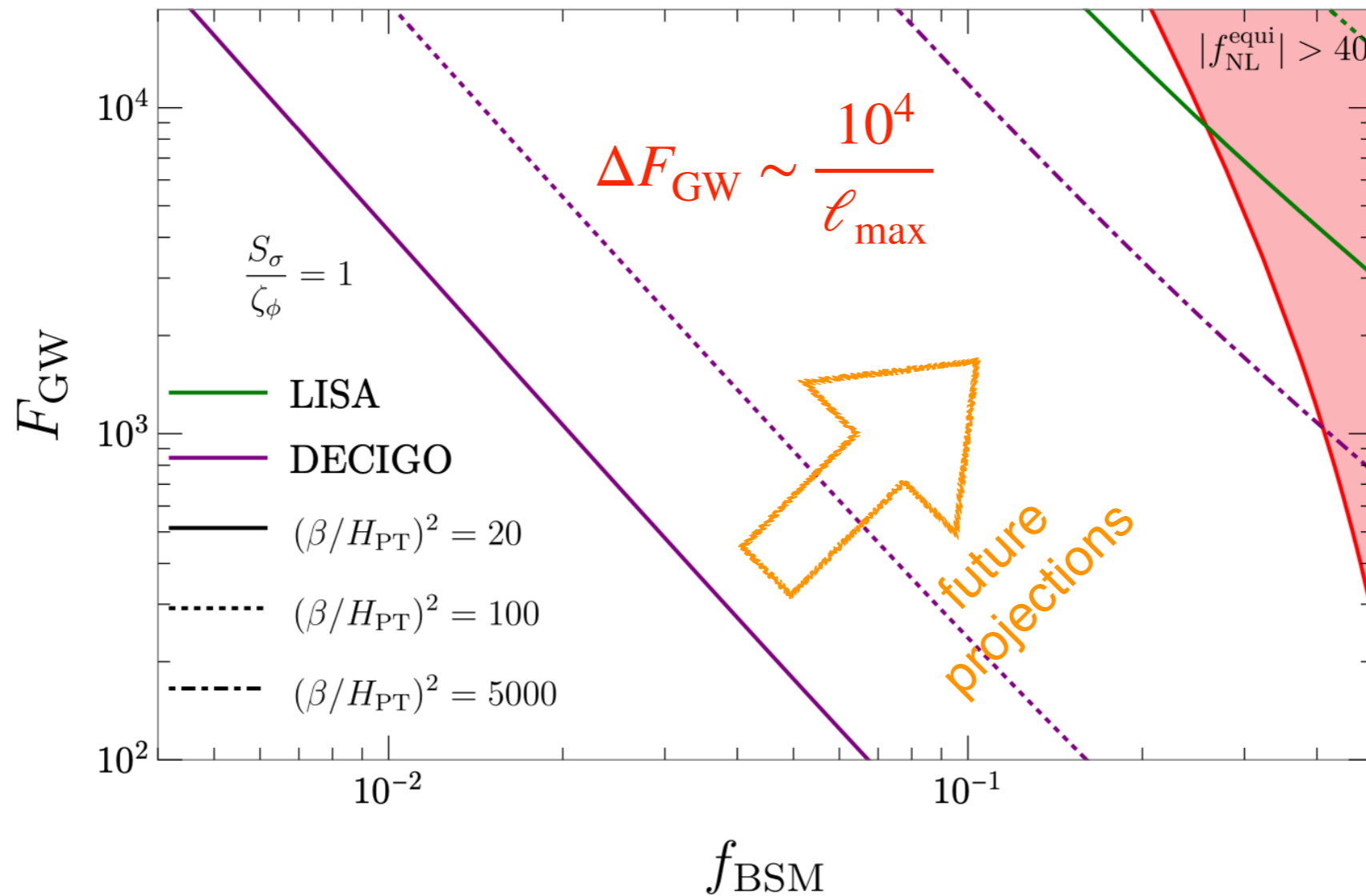
$$F_{\text{GW}} = \frac{5(P_S(k_1)P_S(k_2) + \text{perms.}) F_{S_\sigma}}{6(P_\phi(k_1) + P_S(k_1))(P_\phi(k_2) + P_S(k_2)) + \text{perms.}}$$

common

$$F_{\text{CMB}} \sim f_{\text{BSM}}^3 F_{\text{GW}}$$

NG in CMB can be easily hidden for $f_{\text{BSM}} \ll 1$

Reach of future missions



- SGWB more powerful than CMB or LSS in this scenario!
- No non-linear “clustering” unlike LSS

Details of non-Gaussian HS

- Fluctuations of σ are given by $S_\sigma = \frac{2\delta\sigma}{\sigma_0}$
- Light field, protected by **shift symmetry**

$$\mathcal{L}_\sigma = -\frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{\Lambda_\sigma^4}(\partial_\mu\sigma)^2(\partial_\nu\sigma)^2 + \dots$$

- Non-trivial **interactions** $\mathcal{L}_\sigma \supset 4\frac{\dot{\sigma}_0}{\Lambda_\sigma^4}\delta\sigma(\partial\delta\sigma)^2$

$$\langle\delta\sigma(\vec{k}_1)\delta\sigma(\vec{k}_2)\delta\sigma(\vec{k}_3)\rangle' = -\frac{7}{3}\frac{\dot{\sigma}_0}{\Lambda_\sigma^4}\frac{H_{\text{inf}}^5}{k^6} \quad |F_{S_\sigma}| = \frac{14}{9}\frac{H_{\text{inf}}\sigma_0\dot{\sigma}_0}{\Lambda_\sigma^4}$$

- **Benchmark** $\dot{\sigma}_0^2/\Lambda_\sigma^4 \lesssim 0.1$, $\dot{\sigma}_0 \sim H_{\text{inf}}^2$, $\sigma_0/H_{\text{inf}} \sim 10^4$ **within EFT control**
 $F_{\text{GW}} \sim 10^3$

Conclusions

- SGWB expected from **unresolved binary mergers**, but also arise in various **cosmological scenarios**.
- Anisotropies in SGWB are especially interesting as **probe of the early Universe**.
- In particular, anisotropic SGWB can **probe primordial NG** in ways complementary to CMB or LSS.
- Disentangling from the **astrophysical SGWB and its anisotropy**: interesting and important challenge!

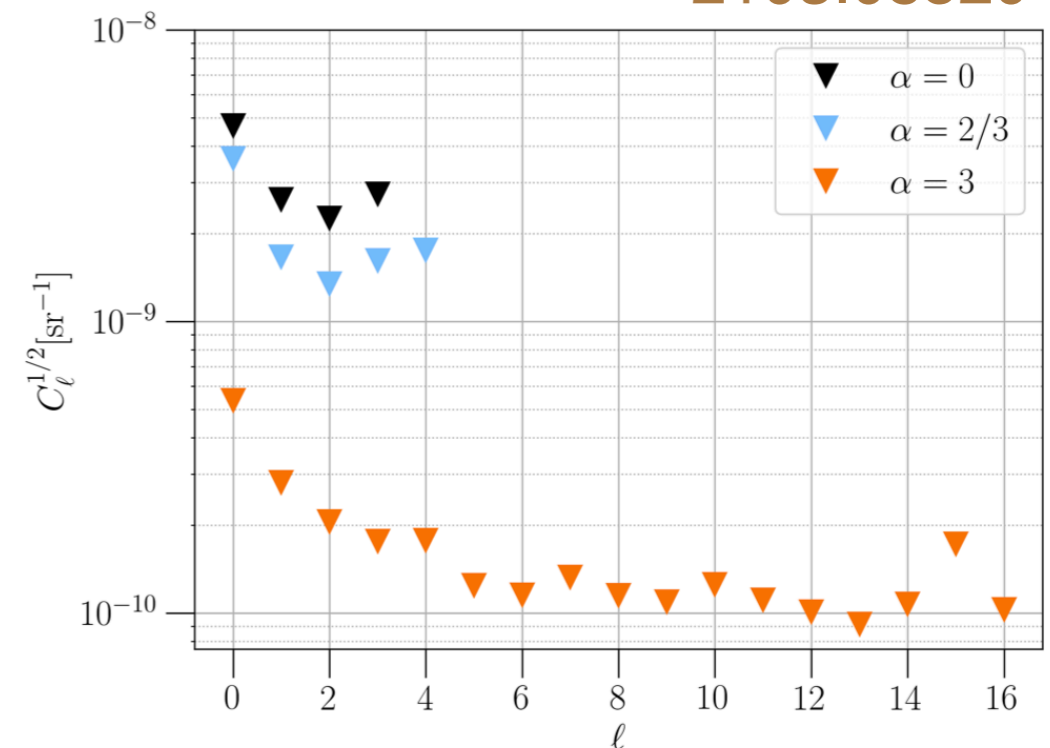
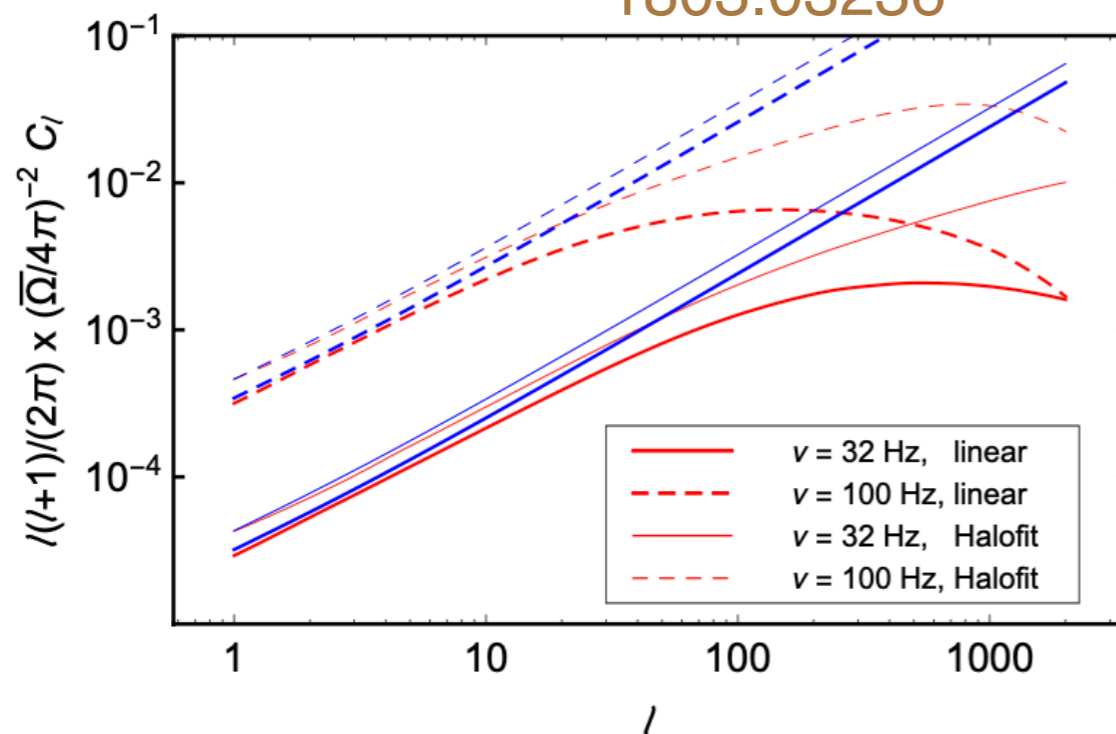
Thanks for your attention!

Anisotropy in astrophysical SGWB

- Astrophysical SGWB is a biased tracer of matter distribution

Cusin et al.
1803.03236

LVK Collab.
2103.08520



- For this talk, assume these are separable/subdominant

Derivation of SW Effect

- Newtonian gauge $ds^2 = a^2(\eta) (-(1 + 2\Phi)d\eta^2 + (1 - 2\Psi)d\vec{x}^2)$

- Curvature and isocurvature perturbations

$$\zeta = -\Psi - H \frac{\delta\rho}{\dot{\rho}} \qquad \zeta_i = -\Psi - H \frac{\delta\rho_i}{\dot{\rho}_i} \qquad S_{\text{GW}} \equiv 3(\zeta_{\text{GW}} - \zeta_\gamma)$$

$$\zeta = -\Psi - \frac{2}{3(1+w)H} (H\Phi + \dot{\Psi})$$

- CMB and GW anisotropy

$$\left. \frac{\Delta T}{T} \right|_{\text{CMB}} = \frac{1}{4} \delta_\gamma^{\text{prim}} + \Phi_{\text{MD}} = \zeta_\gamma + 2\Phi_{\text{MD}} \qquad \left. \frac{\Delta T}{T} \right|_{\text{GW}} = \frac{1}{4} \delta_{\text{GW}}^{\text{prim}} + \Phi_{\text{RD}} = \zeta_{\text{GW}} - \frac{4}{3} \zeta_{\text{RD}}$$

$$= \zeta_\gamma - \frac{6}{5} \zeta_{\text{MD}}$$

Derivation of SW Effect

$$\begin{aligned}\zeta_{\text{RD}} &= (1 - f_\nu - f_{\text{GW}})\zeta_\gamma + f_\nu\zeta_\nu + f_{\text{GW}}\zeta_{\text{GW}} \\ &= \zeta_\gamma + \frac{1}{3}f_{\text{GW}}S_{\text{GW}},\end{aligned}$$

$$\left.\frac{\Delta T}{T}\right|_{\text{CMB}} = -\frac{1}{5}\zeta_{\text{RD}} + \frac{1}{15}f_{\text{GW}}S_{\text{GW}}.$$

$$\left.\frac{\Delta T}{T}\right|_{\text{GW}} = -\frac{1}{3}\zeta_{\text{RD}} + \frac{1}{3}(1 - f_{\text{GW}})S_{\text{GW}}$$

$$\begin{aligned}\delta_\gamma &\equiv 4 \left.\frac{\Delta T}{T}\right|_{\text{CMB}} = -\frac{4}{5}(\zeta_{\gamma\text{HS}} + f_{\text{BSM}}(\zeta_{\gamma\text{BSM}} - \zeta_{\gamma\text{HS}})) \\ &= -\frac{4}{5}\zeta_\phi - \frac{4}{15}f_{\text{BSM}}S_\sigma,\end{aligned}$$

$$\begin{aligned}\delta_{\text{GW}} &\equiv 4 \left.\frac{\Delta T}{T}\right|_{\text{GW}} = -\frac{4}{3}\zeta_{\text{RD}} + \frac{4}{3}(1 - f_{\text{GW}})S_{\text{GW}} \\ &\approx -\frac{4}{3}\zeta_\phi + \frac{4}{3}S_\sigma \left(1 - \frac{4}{3}f_{\text{BSM}}\right)\end{aligned}$$