Chiral SM (light sector - nearly massless)

Mirror SM (dark sector - massive gapped)

How many Weyl fermions, per generation and in total, in the Standard Model (SM)?

Quantum Matter Adventure to Beyond the Standard Model Prediction

Juven Wang

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arXiv: 1910.14668 [JHEP], 2106.16248 [PRD], 2111.10369, 2202.13498
2012.15860 [PRD], 2008.06499, 2006.16996,
2112.14765, 2204.08393 [PRD], 1810.00844 [JMP], 1809.11171 [PRR],
2204.14271 [Sym], 2207.14813, ...

w/ **Yuta Hamada** (HU/KEK), **Zheyan Wan** (YMSC), **Yi-Zhuang You** (UCSD), ...

PhaseTrans-Topo-Defect Early Universe, Thur, Aug 4, 2022

Two crucial tools in our work:

- **What is Mass**? **How to give Mass**?

- Other than **Perturbative Local Anomaly**, use **Nonperturbative Global Anomaly** cancellation (or matching) to predict Beyond the Standard Model (BSM) physics.

Two crucial tools in our work:

• **What is Mass**? **How to give Mass**?

- Energy eigenvalues of quantum Hamiltonian $\hat{H}$ has a gap $\Delta E = E_{\text{1st}} - E_0 > 0$. (Dispersion $E_p = \sqrt{m^2 + p^2}$ above the energy gap, $m = \Delta E$.)
- matter field correlator decays $\exp(-x/\xi)$, with $\xi = 1/m$.

- mean-field vs non-mean-field mass terms in the Lagrangian.
- Symmetric Mass Generation Interaction $\mathcal{O}\psi\tilde{\psi}$: $\langle \mathcal{O} \rangle = 0$ and $\langle \psi\tilde{\psi} \rangle = 0$, but $\Delta E \neq 0$.

• Other than **Perturbative Local Anomaly**, use **Nonperturbative Global Anomaly** cancellation (or matching) to predict Beyond the Standard Model (BSM) physics.

Ideas developed from the quantum matter and quantum field theory frontier may guide us to explore BSM new physics. we propose a few such ideas. **Three messages**:

Part 1. Neutrinos: a right-handed neutrino (massless/massive) carries a $\mathbb{Z}_{16}$ class mixed gauge-gravitational global anomaly index, which could be replaced by interacting 4d or 5d gapped topological quantum field theory, or 4d gapless conformal field theory. These theories provide new neutrino mass mechanisms [arXiv:2012.15860].

Part 2. Deconfined quantum criticality between Grand Unified Theories: dictated by a $\mathbb{Z}_2$ class mixed gauge-gravitational global anomaly, a gapless quantum critical region can happen between Georgi-Glashow and Pati-Salam models as deformation of the Standard Model, where Beyond the Standard Model physics and Dark Gauge sector occur as neighbor phases [arXiv:2106.16248, arXiv:2112.14765, arXiv:2204.08393].

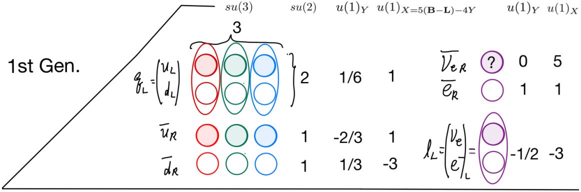
Part 3. Strong CP problem may be solved by a new solution involving Symmetric Mass Generation [arXiv:2204.14271, arXiv:2207.14813, ...].

Chiral SM (light sector - nearly massless)

Mirror SM (dark sector - massive gapped)

How many Weyl fermions, per generation and in total, in the Standard Model?

Standard Model (SM) with $(15+1)N_f$ Weyl fermions coupled to Yang-Mills gauge $su(3)_c \times su(2)_{\mathrm{L}} \times u(1)_Y$ in representation (rep):



$\times (N_f = 3)$.

Each Weyl fermion $\mathbf{2}_L$ of Spin(3,1)

$$\bar{d}_R \oplus l_L \oplus q_L \oplus \bar{u}_R \oplus \bar{e}_R \oplus ?\bar{\nu}_R$$
$$= (\overline{\mathbf{3}}, \mathbf{1})_{2,L} \oplus (\mathbf{1}, \mathbf{2})_{-3,L} \oplus (\mathbf{3}, \mathbf{2})_{1,L} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-4,L} \oplus (\mathbf{1}, \mathbf{1})_{6,L} \oplus ?(\mathbf{1}, \mathbf{1})_{0,L}.$$

SM gauge group $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$ with $q = 1, 2, 3, 6$.

If we drop some quarks or leptons in our theory, the God will yell: "the perturbative local anomalies do not cancel!" How about the cancellation of nonperturbative global anomalies?

Open Issues:

- Neutrino $\bar{\nu}_R$ exists or not? (15 vs 16 fermions) How many $\bar{\nu}_R$ for $N_f = 3$? How do ν_L and $\bar{\nu}_R$ get masses? Dirac vs Majorana?

Spacetime and internal symmetry of the Standard Model (SM)?

15 or 16 Weyl fermion multiplet $(\psi_L)_{\rm I} = (\bar{d}_R \oplus l_L \oplus q_L \oplus \bar{u}_R \oplus \bar{e}_R)_{\rm I} \oplus n_{\nu_{{\rm I},R}} \bar{\nu}_{{\rm I},R}$.
Here ${\rm I}, {\rm J} \in \{1, 2, \dots, N_f = 3\}$.

Lagrangian:

$$\mathcal{L}_{\rm SM} = \sum_{I=1,2,3} -\frac{1}{4} F^{\rm a}_{I,\mu\nu} F^{{\rm a}\mu\nu}_{I} - \frac{\theta_3}{64\pi^2} g_3^2 \epsilon^{\mu\nu\mu'\nu'} F^{\rm a}_{3,\mu\nu} F^{\rm a}_{3,\mu'\nu'} + \psi_L^\dagger ({\rm i}\bar{\sigma}^\mu D_{\mu,A})\psi_L$$

$$-(\psi_L^\dagger \phi \psi_R + {\rm h.c.}) + |D_{\mu,A}\phi|^2 - {\rm U}(\phi).$$

$\mathcal{L}_{\rm YH} = \psi_L^\dagger \phi \psi_R + {\rm h.c.}$ contains
$\mathcal{L}^d_{\rm YH} + \mathcal{L}^u_{\rm YH} + \mathcal{L}^e_{\rm YH} = \lambda^d_{\rm IJ} q_L^{{\rm I}\dagger} \phi d_R^{\rm J} + \lambda^u_{\rm IJ} \epsilon^{ab} q_{La}^{{\rm I}\dagger} \phi_b^* u_R^{\rm J} + \lambda^e_{\rm IJ} l_L^{{\rm I}\dagger} \phi e_R^{\rm J} + {\rm h.c.}$.

Classically, ${\rm U}(1)_{\mathbf{B}}$ and ${\rm U}(1)_{\mathbf{L}}$.

Quantum mechanically, ABJ anomaly with gauged $G_{{\rm SM}_q}$ break ${\rm U}(1)_{\mathbf{B}} \times {\rm U}(1)_{\mathbf{L}}$ down to ${\rm U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\rm F}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$. Many GUTs violate $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$, but the SM preserves $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$.

- A proton decays (without $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$) vs is stable (with $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$).

S.Koran 2204.01741, JW-Wan-You 2204.08393

Spacetime and internal symmetry of the Standard Model (SM)?

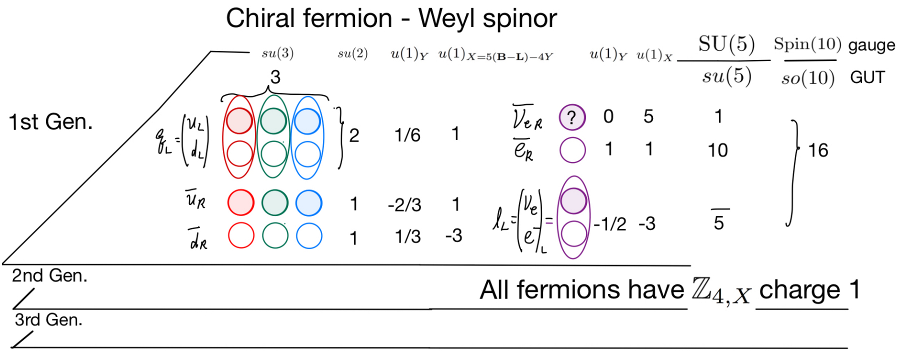
- Spacetime symmetry: Spin group. Diffeomorphism/grav. background. $\frac{\mathrm{Spin}}{\mathbb{Z}_2^{\mathrm{F}}} = \mathrm{SO}$.
- Internal symmetry: $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}} \times G_{\mathrm{SM}_q}$.
- Internal symmetry after gauging G_{SM_q}: $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}} \times \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}$.
  (n-form symmetry, see also Sungwoo Hong talk)

- We can replace the $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$ to a discrete $\mathbb{Z}_{4,X}$ (Wilczek-Zee '79) that is more robust and preserves the 4n fermion interactions (quarks and leptons with $\mathbb{Z}_{4,X}$ charges 1):

$$X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y.$$

For the SM with $\mathbb{Z}_{4,X}$, see Garcia-Etxebarria-Montero 1808.00009 and our work.

Standard Model and GUT anomaly cancellation



SM particle	SU(3)	SU(2)	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{\mathbf{B-L}}$	$\mathrm{U}(1)_X$	$\mathbb{Z}_{4,X}$	$\mathbb{Z}_2^F$
$\bar{d}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	−1/3	−3	1	1
l_L	$\mathbf{1}$	$\mathbf{2}$	−1/2	−1	−3	1	1
q_L	$\mathbf{3}$	$\mathbf{2}$	1/6	1/3	1	1	1
$\bar{u}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	−2/3	−1/3	1	1	1
$\bar{e}_R = e_L^+$	$\mathbf{1}$	$\mathbf{1}$	1	1	1	1	1
$\bar{\nu}_R = \nu_L$	$\mathbf{1}$	$\mathbf{1}$	0	1	5	1	1
ϕ_H	$\mathbf{1}$	$\mathbf{2}$	1/2	0	-2	2	0

Check: discrete $\mathbf{B} \pm \mathbf{L}$ symmetries, local vs global anomalies, cobordism.

Spacetime-internal symmetry ($\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B-L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B+L}} \times G_{\mathrm{SM}_q}$).

4d Anomaly (5d iTQFT) contained in the cobordism group ($N_f = 3$):

$$
\begin{aligned}
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B-L}} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{\mathrm{SM}_q}) &= (\mathbb{Z}^{11}) \times (\mathbb{Z}_9 \times \mathbb{Z}_3^7). \\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{\mathrm{SM}_q}) &= \begin{cases} (\mathbb{Z}^5 \times \mathbb{Z}_2 \times \mathbb{Z}_4^2 \times \mathbb{Z}_{16}) \times (\mathbb{Z}_9 \times \mathbb{Z}_3^4), & q = 1, 3. \\ (\mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}) \times (\mathbb{Z}_9 \times \mathbb{Z}_3^4), & q = 2, 6. \end{cases} \\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B-L}} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{[1]}) &= (\mathbb{Z}^2 \times \mathbb{Z}_{6/q}) \times (\mathbb{Z}_9 \times (\mathbb{Z}_3)^2 \times (\mathbb{Z}_3)^{3n_3}). \\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{[1]}) &= (\mathbb{Z}_{16} \times (\mathbb{Z}_4)^{n_2} \times \mathbb{Z}_{6/q}) \times (\mathbb{Z}_9 \times (\mathbb{Z}_3)^{2n_3}). \\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{Spin}(10)) &= \mathbb{Z}_2.
\end{aligned}
$$

4d Anomaly (5d iTQFT) of 15,16N_f-fermion $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$

Part 1:

JW-Wan-You arXiv:2112.14765

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L} \text{ or } X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}$-class local anomaly $\mathbf{B}-\mathbf{L}$ or $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y$: $\mathrm{U}(1)^3$ and $\mathrm{U}(1)$-grav^2:

$$\mathbf{Z}_5^{\mathrm{U}(1)} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} (A c_1^2 + \frac{1}{48} \mathrm{CS}_3^{T(\mathrm{PD}(c_1))})).$$

$-N_f + n_{\nu_R} \equiv -N_f + \sum_j n_{\nu_{j,R}} = -3 + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \dots.$

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• $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}_{16}$-class global anomaly of $\mathbb{Z}_{4,X}$-grav^2:

$$\boxed{\mathbf{Z}_5^{\mathbb{Z}_{4,X}} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} \frac{2\pi\,\mathrm{i}}{16} \eta_{4\mathrm{d}}(\mathrm{PD}(A_{\mathbb{Z}_{2,X}}))).}$$

4d Anomaly (5d iTQFT) of 15,16N_f-fermion $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$

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Part 2:

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{Spin}(10)$-symmetry.

$\mathbb{Z}_2$-class global anomaly. $p \in \mathbb{Z}_2$:

$$\mathbf{Z}_5 \equiv \exp(\mathrm{i}\pi\, \mathrm{p} \int_{M^5} w_2 w_3)|_{w_2 w_3(TM) = w_2 w_3(V_{\mathrm{SO}(n)})}$$

- We can also include $\mathbb{Z}_{2N_f, \mathbf{B}+\mathbf{L}}$ background field.

arXiv:2204.08393

Logic to Ultra Unification

- $\mathbb{Z}_{16}$ global anomaly cancellation application.

Assumptions:

1. Standard Model (SM) G_{internal}: Lie algebra $su(3) \times su(2) \times u(1)$.
   $G_{\text{SM}_q} \equiv \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)}{\mathbb{Z}_q}, \quad q = 1, 2, 3, 6.$
2. $15 \times (N_f = 3)$ Weyl fermions (spacetime Weyl spinors) observed, applicable to both SM and SU(5) GUT.
3. Discrete Baryon−Lepton number preserved (or not) at high energy: $\mathbb{Z}_{4, X \equiv 5(\mathbf{B}-\mathbf{L})-4Y} \supset \mathbb{Z}_2^{\mathrm{F}}$, so $X^2 = (-1)^F$, also dynamically gauged at higher energy due to no global symmetry in quantum gravity (if we embed the theory into quantum gravity).

Check: Perturbative local & nonperturbative global anomalies via cobordism.

Logic to Ultra Unification

Consequences: $\mathbb{Z}_{16}$ anomaly index as total $(N_f = 3) \cdot (15 = -1 \mod 16)$.

$$(-(N_f = 3) + \sum_{j=e,\mu,\tau,\ldots} n_{\nu_{j,R}} + \nu_{\text{new hidden sectors}}) = 0 \mod 16.$$

Anomaly-cancellation?

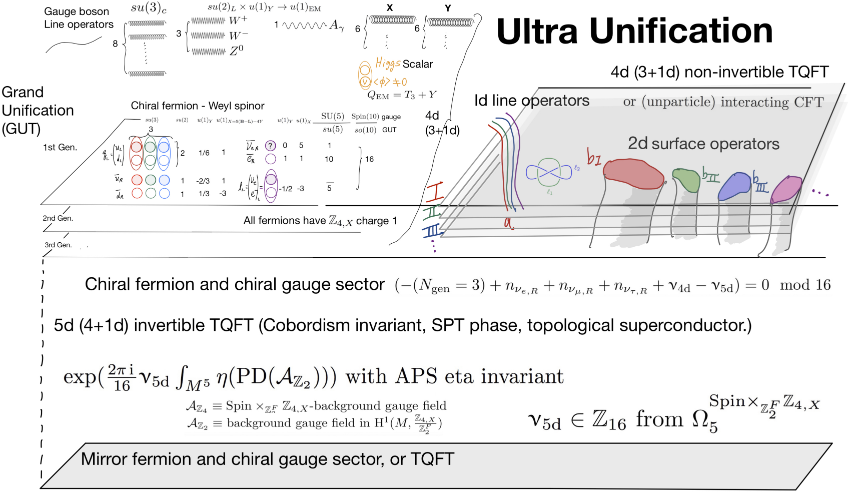
(1) **Standard Lore:** R-handed neutrino (16th Weyl) $n_{\nu_{j,R}} = 1$.
$\mathbb{Z}_{4,X}$ preserved (gapless fermion) vs broken (gap) by **Dirac** or **Majorana** mass.

(2) **My proposal:** New hidden sectors beyond SM (∼ Lieb-Schultz-Mattis thm) :

1. $\mathbb{Z}_{4,X}$-symmetry-preserving anomalous gapped 4d TQFT (**Topological** Mass). (Topo.Green-Schwarz mechanism. Boundary topological order [2+1d Vishwanath-Senthil'12].)
2. $\mathbb{Z}_{4,X}$-5d invertible TQFT (SPTs) by cobordism invariant $\eta(\text{PD}(\mathcal{A}_{\mathbb{Z}_2}))$.
3. $\mathbb{Z}_{4,X}$-gauged-5d-noninvertible TQFT (SETs) + gravity.
4. $\mathbb{Z}_{4,X}$-symmetry-breaking gapped phase (e.g. Landau phase or 4d TQFT).
5. $\mathbb{Z}_{4,X}$-symmetry-preserving gapless or breaking gapless (e.g., extra CFT).

HEP-PH **Gapped Extended Excitation/Objects beyond Particle Physics**.
HEP-PH **Gapless Unparticle CFT Physics**.

Part 1. Neutrinos: a right-handed neutrino (massless/massive) carries a $\mathbb{Z}_{16}$ class mixed gauge-gravitational global anomaly index, which could be replaced by interacting 4d or 5d gapped topological quantum field theory, or 4d gapless conformal field theory. These theories provide new interacting neutrino mass mechanisms.

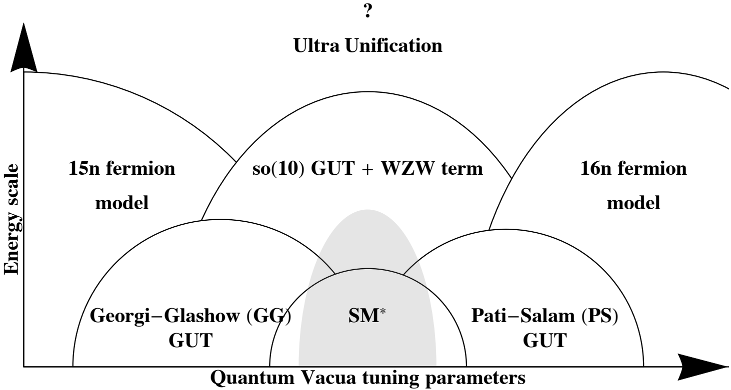


Chiral SM (light sector - nearly massless)

Mirror SM (dark sector - massive gapped)

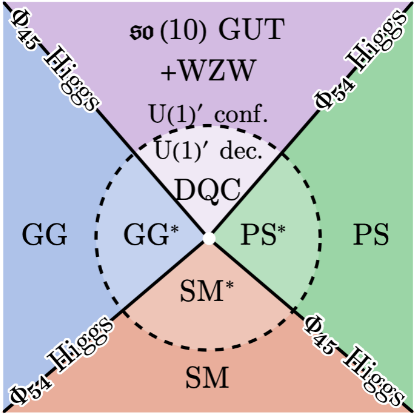
15 vs 16 Chessmen $\sim$ Weyl fermions in the Standard Model.

Part 2. Deconfined quantum criticality between Grand Unified Theories: dictated by a $\mathbb{Z}_2$ class global anomaly, a gapless quantum critical region can happen between Georgi-Glashow and Pati-Salam models as deformation of the Standard Model, where Beyond the Standard Model physics and Dark Gauge sector occur as neighbor phases [arXiv:2106.16248, arXiv:2112.14765, arXiv:2204.08393].



Quantum Phase Diagram

- Goal: understand phase transitions among SM/GUTs
  - Higgs condensation will break Spin(10) to its subgroups



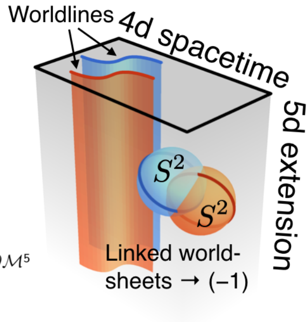
$\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2} \mathrm{Spin}(n \geq 7)) = \mathbb{Z}_2$,
by $w_2 w_3(TM) = w_2 w_3(V_{\mathrm{SO}(n)})$.

- **Model 1**: so(10) GUT + GUT-Higgs. No anomaly.
- **Model 2**: so(10) GUT + GUT-Higgs + WZW-like topological term from GUT-Higgs that matches the $w_2 w_3$ anomaly.

Deformation $\mathrm{U}(\Phi_{\mathbf{R}}) = \left(r_{45}(\Phi_{45})^2 + \lambda_{45}(\Phi_{45})^4\right) + \left(r_{54}(\Phi_{54})^2 + \lambda_{54}(\Phi_{54})^4\right) + \dots$.

Recall GUT breaking in David Dunsky talk

Mod-2 Class Wess-Zumino-Witten Term matches $w_2 w_3$ anomaly



- 1d monopole worldlines in 4d extended to 2d worldsheets in 5d.
- A WZW-like term, defined in the 5d extension, assigns (–1) sign to every linked pairs of monopole 2d worldsheets.

$$\exp(\mathrm{i} S^{\mathrm{WZW}}) = \exp(\mathrm{i}\pi \int_{\mathcal{M}^5} B(\tilde{\Phi}^{\mathrm{bi}}_{\mathbf{54}}) \smile \delta C(\hat{\Phi}^{\mathrm{bi}}_{\mathbf{45}}))\Big|_{\mathcal{M}^4 = \partial \mathcal{M}^5}$$

't Hooft-Polyakov monopoles:

GG $C(\hat{\Phi}^{\mathrm{bi}}_{\mathbf{45}}) \in \mathrm{H}^2(\frac{\mathrm{SO}(10)}{\mathrm{U}(5)}, \mathbb{Z}_2) = \mathbb{Z}_2$.

PS $B(\tilde{\Phi}^{\mathrm{bi}}_{\mathbf{54}}) \in \mathrm{H}^2(\frac{\mathrm{SO}(10)}{\mathrm{SO}(6)\times\mathrm{SO}(4)}, \mathbb{Z}_2)$ contains a $\mathbb{Z}_2$.

- Low energy of WZW can be matched by a critical CFT region: U(1) dark photon couples to new fermionic partons. Double spin DSpin structure.

		$\mathfrak{u}(1)^{\mathrm{dark}}_{\mathrm{gauge}}$	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)_{Y_1}$	$\mathfrak{u}(1)_{X_1}$	$\mathfrak{su}(5)$	$\mathfrak{so}(10)$
c		1	$\mathbf{3}$	$\mathbf{1}$	-2	-2	$\mathbf{5}$	$\mathbf{10}$
f		1	$\mathbf{1}$	$\mathbf{2}$	3	-2		
c'		1	$\bar{\mathbf{3}}$	$\mathbf{1}$	2	2	$\bar{\mathbf{5}}$	
f'		1	$\mathbf{1}$	$\mathbf{2}$	-3	2		

See Isabel Garcia Garcia talk on dark photon.

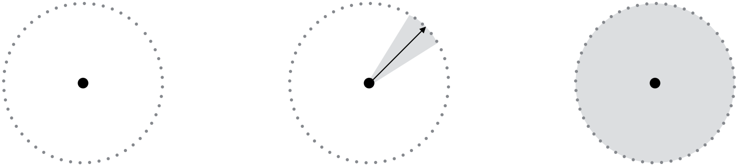
Part 3. Strong CP problem may be solved by a new solution involving Symmetric Mass Generation (SMG).

CP violating theta angle $|\bar{\theta}_3| < 10^{-10}$ for $\bar{\theta}_3 \equiv \theta_3 + \arg(\det(M_u M_d))$.
Path integral incldues $\exp(\mathrm{i}\theta_3 n^{(3)})$, $\quad n^{(3)} \equiv \frac{1}{8\pi^2}\int \mathrm{Tr}[F^{(j)} \wedge F^{(j)}]$. M_u and M_d are two rank-3 matrices specifying the Yukawa-Higgs coupling, for u, c, t-type and d, s, b-type quarks.

Challenge: Left-handed quarks participate weak interaction ($\not{P}$, $\not{C}$), CKM's δ_{CP} is order 1 ($\not{CP}$). Näive expect: $\bar{\theta}_3$ order 1.

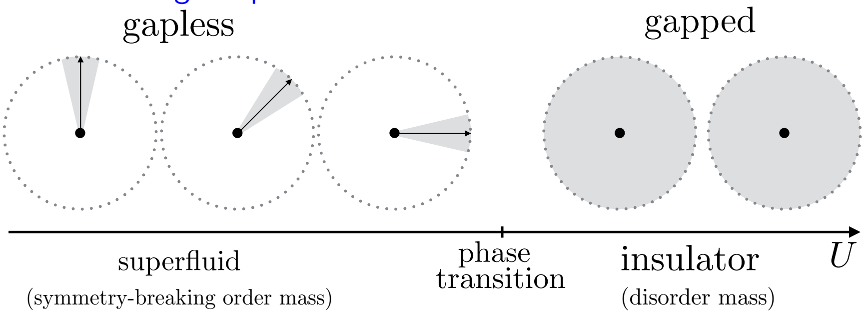
Under chiral transformations,

$\theta_3 \mapsto \theta_3 - \sum_{f=1,2,3}(\alpha_{u_{Lf}} + \alpha_{d_{Lf}} - \alpha_{u_{Rf}} - \alpha_{d_{Rf}})$.
$m_q \mapsto m_q \mathrm{e}^{\mathrm{i}(\alpha_{q_L} - \alpha_{q_R})}$.



(a) massless (gapless / no mass) (b) mean-field mass (symmetry-breaking order mass) (c) interacting SMG mass (disorder mass)

Part 3. Strong CP problem solution:



Hubbard model $\hat{H} = -\sum_{<i,j>} b_i^\dagger b_j + \text{h.c.} + U \sum_i (b_i^\dagger b_i)(b_i^\dagger b_i - 1)$.

- Gapless superfluid phase:

Peccei-Quinn symmetry-breaking, Goldstone mode, axion, etc.

- Gapped insulator phase:

One Strong CP solution: u quark obtains its mass fully from Symmetric Mass Generation (SMG), other quarks obtain portion of its mass from SMG, other than dominantly by Higgs mechanism.

Symmetric Mass Generation (SMG) examples:

- 0+1d Fidkowski-Kitaev interaction gaps 8 Majorana fermions preserving $\mathbb{Z}_2^{\mathrm{T}} \times \mathbb{Z}_2^{\mathrm{F}}$:

$\hat{H}_{\mathrm{int}} = -(c_1 c_2 c_3 c_4 + \text{h.c.}). \quad \langle c_i c_j \rangle = \langle c_i^\dagger c_j \rangle = 0.$

- 1+1d chiral $\mathrm{U}(1)$ 3-4-5-0 model - SMG interaction (JW-Wen):

$S_{\text{int}} \equiv \int \mathrm{d}^2 x \big(g_1 (\psi_{L,3})(\psi_{L,4}^\dagger)^2 (\psi_{R,5})(\psi_{R,0})^2 + g_2 (\psi_{L,3})^2 (\psi_{L,4})(\psi_{R,5}^\dagger)^2 (\psi_{R,0}) + \text{h.c.} \big)$

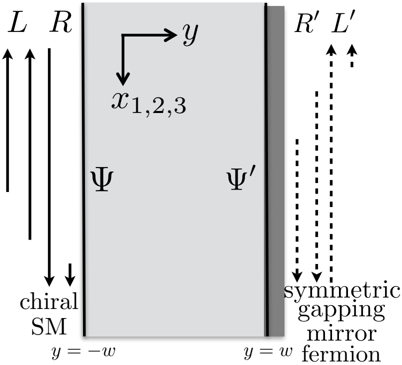
$S_{\text{disorder}} \equiv \int \mathrm{d}^2 x \Big(\phi_1^2 \psi_{L,3}\psi_{R,5} + \phi_1^\dagger \psi_{L,4}^\dagger \psi_{R,0} + \phi_2^\dagger \psi_{L,3} \psi_{R,5}^\dagger + \phi_2^2 \psi_{L,4} \psi_{R,0} + \text{h.c.} + \frac{1}{\sqrt{|g_1|}} \phi_1^\dagger \phi_1 + \frac{1}{\sqrt{|g_2|}} \phi_2^\dagger \phi_2 \Big)$ requires no-mean-field mass $\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_1^2 \rangle = \langle \phi_2^2 \rangle = 0.$

- 3+1d chiral SM:

- smooth s-confinement (Razamat-Tong) — difficult complex mass matrix.
- parton-Higgs

$S_{\text{int}} \equiv \int \mathrm{d}^4 x \big(\phi(\bar{d}_R q_L + \bar{\nu}_R \bar{l}'_R + \bar{e}_R l_L) + \phi^{\dagger 2} \bar{u}_R d'_L \big) + \mathrm{h.c.}$ requires no-mean-field mass $\langle \phi \rangle = \langle \phi^2 \rangle = 0.$

Part 3. Another Strong CP solution (with mirror fermions) by Symmetric Mass Generation. Fermion doubling.

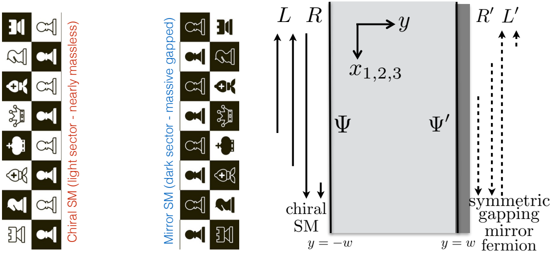


Domain wall fermions. Similar but different from Parity Solution.

$\mathbb{Z}_2^{\mathrm{PR}}$: $\psi_L(t,x) \mapsto \psi'_R(t,-x)$, $\psi_R(t,x) \mapsto \psi'_L(t,-x)$, $\quad F \wedge F \mapsto -F' \wedge F'(t,-x)$.

$\mathbb{Z}_2^{\mathrm{PR}}$ imposes $\theta_3 \mathrm{Tr}[F \wedge F] + \theta'_3 \mathrm{Tr}[F' \wedge F'] = 0$ by $\theta'_3 = -\theta_3$. F and F' are same field (SM's SU(3)) on two sides of domain wall.

Another Strong CP solution (with mirror fermions) by SMG



• Nielsen-Ninomiya (NN) fermion-doubling ⇒ Domain wall fermion remove doubling ⇒ regularize (lattice) nonperturb. chiral fermion.

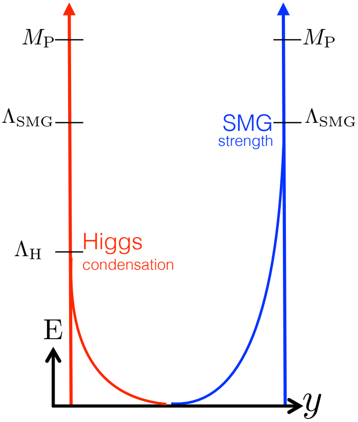
• $\not{P}$ ⇒ Left-right model (but symmetry/gauge group is not necessarily doubled). Parity-reflection symmetry $\mathbb{Z}_2^{\text{PR}}$.

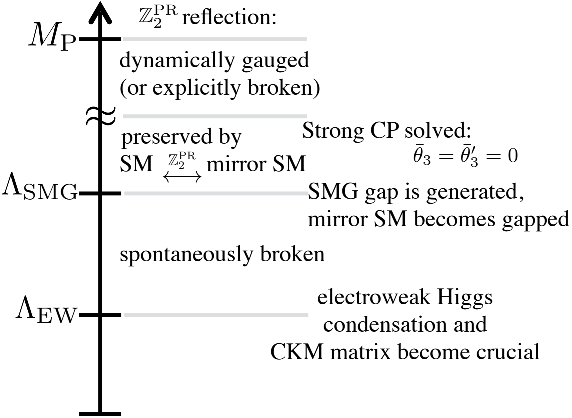
$$\bar{\theta}_3 + \bar{\theta}'_3 \equiv \theta_3 + \theta'_3 + \arg(\det(M_u M_d M'^{\dagger}_{\text{SMG}}))$$

• Higgs vs SMG $M'^{\dagger}_{\text{SMG}}$ (in contrast, twin Higgs $M'^{\dagger}_u M'^{\dagger}_d$). No mean-field mass for SMG $M'^{\dagger}_{\text{SMG}} = 0$. Energy scale $\Lambda_{\text{H}} < \Lambda_{\text{SMG}}$.

• Anomaly-free condition ⇔ Topological gapping condition (iff).

Part 3. Strong CP problem may be solved by a new solution involving Symmetric Mass Generation.





Ways to give mass

What is mass? correlation function (of the corresponding operators/excitations/states) decaying exponentially.

Mass mechanism	Symmetry Property	Topological Order with low energy TQFT	Description:
(1) Anderson-Higgs	Symmetry Breaking	✗	Mean-Field
(2) Confinement: Chiral SB	Symmetry Breaking	✗	Mean-Field
(3) Confinement: s confinement	Symmetry Preserving	✗	Many-Body or Interacting
(4) Symmetric Mass Generation (Anomaly-Free)	Symmetry Preserving **Part 3: solve Strong CP**	✗	Many-Body or Interacting
(5) Symmetric Gapped Topological Order (Anomalous)	Symmetry Preserving **Part 1: replace ν_R**	✓	Many-Body or Interacting
(6) Symmetry Extension Gapped/Gapless (Anomaly Trivialized)	Symmetry Extension $K \to \tilde{G} \xrightarrow{\iota} G$ **Part 2: dQC CFT**	✓: TO/TQFT if K is gauged, and if spacetime dim $d \geq 3$ ✗: no TO/TQFT if $\tilde{G}$ remains ungauged.	Many-Body or Interacting

Overview: JW-You 2204.14271.

Summary

Part 1. Neutrinos: a right-handed neutrino (massless/massive) carries a $\mathbb{Z}_{16}$ class mixed gauge-gravitational global anomaly index, which could be replaced by interacting 4d or 5d gapped topological quantum field theory, or 4d gapless conformal field theory. These theories provide possible new neutrino mass mechanisms [arXiv:2012.15860].

Part 2. Deconfined quantum criticality between Grand Unified Theories: dictated by a $\mathbb{Z}_2$ class mixed gauge-gravitational global anomaly, a gapless quantum critical region can happen between Georgi-Glashow and Pati-Salam models as deformation of the Standard Model, where Beyond the Standard Model physics and Dark Gauge sector occur as neighbor phases [arXiv:2106.16248, arXiv:2112.14765, arXiv:2204.08393].

Part 3. Strong CP problem may be solved by a new solution involving Symmetric Mass Generation [arXiv:2204.14271, arXiv:2207.14813, ...].

Back Up Slides:

Ways to give mass

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Overview: JW-You 2204.14271.

Neutrino ν_R vs TQFT sector

(Conventional) Quadratic neutrino mass term:

Dirac mass with some Higgs: $(\bar{\nu}_R \phi_H^\dagger \nu_L + \bar{\nu}_L \phi_H \nu_R)$.

Majorana mass: $\frac{\mathrm{i}\, m_{\text{Maj}}}{2}(\chi^{\mathrm{T}} \sigma^2 \chi + \chi^\dagger \sigma^2 \chi^*)$.

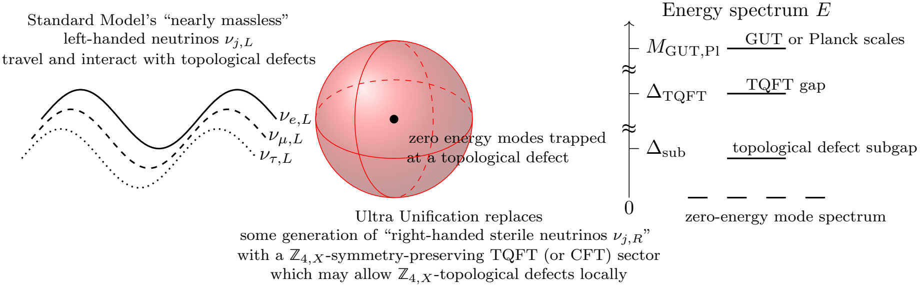
Both Dirac and Majorana masses:

$$\frac{1}{2}\Bigg(\Big(\Big(l_{L\nu_e}, l_{L\nu_\mu}, l_{L\nu_\tau}\Big) \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|}, \chi^\dagger_{\nu_e}, \chi^\dagger_{\nu_\mu}, \chi^\dagger_{\nu_\tau} \Big) \begin{matrix} & 3 & 3 \\ 3 & 0 & M_{\text{Dirac}} \\ 3 & M_{\text{Dirac}} & M_{\text{S}} \end{matrix} \begin{pmatrix} l_{L\nu_e} \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|} \\ l_{L\nu_\mu} \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|} \\ l_{L\nu_\tau} \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|} \\ \chi^\dagger_{\nu_e} \\ \chi^\dagger_{\nu_\mu} \\ \chi^\dagger_{\nu_\tau} \end{pmatrix} + h.c. \Bigg).$$

Seesaw mechanism:
3 mass eigenstates have small mass $\simeq \frac{|M_{\text{Dirac}}|^2}{|M_{\text{S}}|} \ll |M_{\text{Dirac}}|$ for ν_L-like.
3 mass eigenstates have large mass M_{S} for ν_R-like.

New Proposal on Neutrino Mass

In contrast to the traditional seesaw mechanism $\frac{M_{\text{Dirac}}^2}{M_S} \ll M_{\text{Dirac}}$.



The energy spectrum near the defect has energy subgap

$$\Delta_{\text{sub}} \lesssim \frac{\Delta_{\text{TQFT}}^2}{M_{\text{GUT,Pl}}}. \qquad \Delta_{\text{small}} \lesssim \frac{\Delta_{\text{sub}}^2}{\Delta_{\text{TQFT}}} \ll M_{\text{Dirac}}.$$

(In analogy with the vortex subgap $\Delta_{\text{sub}} \simeq \frac{\Delta_{\text{SC}}^2}{E_F}$ of superconductor gap Δ_{SC} and Fermi energy E_F.) Both can give the left-handed neutrinos small masses.

(JW, Harvard HEP-String lunch Dec 1, 2020 and arXiv v3: 2012.15860)

Neutrino ν_R vs TQFT sector

Conventional: Neutrino pairs up to get Dirac (left-handed ν_L with right-handed ν_R) or Majorana (right-handed ν_R with itself) mass.

My talk:
Some of right-handed neutrinos ν_R may be replaced by 4d TQFT/5d iTQFT. Left-handed neutrinos ν_L travel and interfere with the zero modes of topological defects of TQFT.

- Δ_{TQFT} gap replaces the right-handed ν_R mass M_{S}.
- Vortex subgaps give the left-handed ν_L mass.
- Mixed scenarios.

Classify dd Anomalies by $d+1$d iTQFT/SPTs via Cobordism

Kapustin'14, ..., Freed-Hopkins'16 (systematic)
Unitarity of Lorentz $\sim$ Reflection positivity of Euclidean.
$d+1$d invertible TQFT with reflect.pos in Euclidean signature
$\Rightarrow$ anomaly of dd reflect.pos Euclidean QFT.
$\Rightarrow$ anomaly of dd unitary Lorentz QFT. Take $d=4$ and $d+1=5$.

Here we only concern a cobordism group $\mathrm{TP}_{d+1}(G) \equiv \Omega_G^{d+1}$,
Note $(\mathrm{TP}_{d+1}(G))_{\text{tors}} = (\Omega_{d+1}^G)_{\text{tors}}$ and $(\mathrm{TP}_{d+1}(G))_{\text{free}} = (\Omega_{d+2}^G)_{\text{free}}$.
tor: a torsion group (only a finite group part).

Bordism group $\Omega_D^G = \pi_D(MTG) \equiv \text{colim}_{k\to\infty}\pi_{D+k}(MTG)_k$.

Tools: Pontryagin-Thom construction, Thom-Madsen-Tillmann spectra, Adams spectral sequence, and Freed-Hopkins's theorem

Given a G structure, we will later show co/bordism group (abelian group classification) and $d+1$d topological terms (invertible TQFT or Symmetry Protected Topological states [SPTs]) and the anomaly of a $(d-1,1)$d unitary Lorentz QFT.

Anomalies of SM and GUT via cobordism:

hep-th/0607134: Freed. generalized cohomology. SU(n) or Spin(n). Miss **B − L**.

1604.06527: Freed-Hopkins, Cobordism group $\Omega_G^d \equiv \mathrm{TP}_d(G)$ —

1808.00009: Inaki Garcia-Etxebarria, Miguel Montero.
$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4,\ \mathrm{Spin} \times \mathrm{SU}(n),\ \mathrm{Spin} \times \mathrm{Spin}(n)$.

1809.11171: JW-Wen. $G = \frac{\mathrm{Spin} \times \mathrm{Spin}(10)}{\mathbb{Z}_2^{\mathrm{F}}},\ \mathrm{Spin} \times \mathrm{SU}(5)$.

1910.11277: Joe Davighi, Ben Gripaios, Nakarin Lohitsiri.
$G = \mathrm{Spin} \times G_{\mathrm{SM}_q},\ \mathrm{Spin} \times \mathrm{Spin}(n)$, other GUTs

1910.14668: Wan-JW. (Subtle twisted cases.)
$G = \mathrm{Spin} \times G_{\mathrm{SM}_q},\ \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\mathrm{SM}_q},\ \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \mathrm{SU}(5)$,
$\mathrm{Spin} \times \mathrm{Spin}(n),\quad \frac{\mathrm{Spin} \times \mathrm{Spin}(n)}{\mathbb{Z}_2^{\mathrm{F}}}$ e.g., $n = 10, 18$, other GUTs

Conservative view vs Optimistic view (New Arena Beyond the SM).

Spacetime and internal symmetry of the Standard Model (SM)?

- Spacetime symmetry: Spin group. Diffeomorphism/grav. background. $\frac{\mathrm{Spin}}{\mathbb{Z}_2^{\mathrm{F}}} = \mathrm{SO}$.

- Internal symmetry: $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}} \times G_{\mathrm{SM}_q}$.
- Internal symmetry after gauging G_{SM_q}: $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}} \times \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}$.
- Determine the 1-form symmetry of $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_{\tilde{Y}}}{\mathbb{Z}_q}$ gauge theory:

	$Z(G_{\mathrm{SM}_q})$	$\pi_1(G_{\mathrm{SM}_q})^\vee$	1-form e sym $G^e_{[1]}$	1-form m sym $G^m_{[1]}$
G_{SM_q}	$\mathbb{Z}_{6/q} \times \mathrm{U}(1)$	$\mathrm{U}(1)$	$\mathbb{Z}^e_{6/q,[1]}$	$\mathrm{U}(1)^m_{[1]}$

Denote 1-form symmetry $G_{[1]} = G^e_{[1]} \times G^m_{[1]} = \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}$.

(See also Sungwoo Hong talk)

- No C, P, T discrete symmetry within SM.
- We can replace the $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$ to a discrete $\mathbb{Z}_{4,X}$ (Wilczek-Zee '79) that is more robust and preserves the 4n fermion interactions (quarks and leptons with $\mathbb{Z}_{4,X}$ charges 1):

$$X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y.$$

For the SM with $\mathbb{Z}_{4,X}$, see Garcia-Etxebarria-Montero 1808.00009 and our work.

Check 4d anomaly via 5d cobordism group with G_{SM_q}, $\mathbf{B}-\mathbf{L}$, discrete $\mathbf{B}+\mathbf{L}$ together form $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}} = \mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times \mathbb{Z}_{3,\mathbf{B}+\mathbf{L}}$, and gravitational backgrounds,

$$\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times \mathbb{Z}_{3,\mathbf{B}+\mathbf{L}} \times G_{\mathrm{SM}_q}) = \left(\mathbb{Z}^{11}\right) \times \left(\mathbb{Z}_9 \times \mathbb{Z}_3^7\right).$$

$$\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathbb{Z}_{3,\mathbf{B}+\mathbf{L}} \times G_{\mathrm{SM}_q}) = \left\{ \begin{array}{l} \left(\mathbb{Z}^5 \times \mathbb{Z}_2 \times \mathbb{Z}_4^2 \times \mathbb{Z}_{16}\right) \\ \left(\mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}\right) \end{array} \right.$$

Include one-form symmetries of the SM with gauged G_{SM_q},

$$\begin{aligned} &\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times \mathbb{Z}_{3,\mathbf{B}+\mathbf{L}} \times \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}) \\ &= \left(\mathbb{Z}^2 \times \mathbb{Z}_{6/q}\right) \times \left(\mathbb{Z}_9 \times (\mathbb{Z}_3)^2 \times (\mathbb{Z}_3)^{3n_3}\right). \\ &\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathbb{Z}_{3,\mathbf{B}+\mathbf{L}} \times \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}) \\ &= \left(\mathbb{Z}_{16} \times (\mathbb{Z}_4)^{n_2} \times \mathbb{Z}_{6/q}\right) \times \left(\mathbb{Z}_9 \times (\mathbb{Z}_3)^{2n_3}\right). \end{aligned}$$

For $q = 1, 2, 3, 6$ and we define $q' \equiv 6/q \equiv 2^{n_2} \cdot 3^{n_3} = 6, 3, 2, 1$ as $(n_2, n_3) = (1, 1), (0, 1), (1, 0), (0, 0)$ respectively.

$\mathrm{U}(1)_{\mathbf{B}}$ and $\mathrm{U}(1)_{\mathbf{L}}$ under dynamically gauged $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$

$\mathrm{U}(1)_{\mathbf{B}}$-$\mathrm{U}(1)_Y^2$, $\mathrm{U}(1)_{\mathbf{L}}$-$\mathrm{U}(1)_Y^2$ $\mathrm{U}(1)_{\mathbf{B}}$-$\mathrm{SU}(2)^2$ and $\mathrm{U}(1)_{\mathbf{L}}$-$\mathrm{SU}(2)^2$ local anomalies. Under $\mathrm{U}(1)_{\mathbf{B}}$ and $\mathrm{U}(1)_{\mathbf{L}}$ transformations on $(\psi_L)_{\mathrm{I}}$:

$$(\bar{d}_R \oplus l_L \oplus q_L \oplus \bar{u}_R \oplus \bar{e}_R)_{\mathrm{I}} \oplus n_{\nu_{\mathrm{I},R}} \bar{\nu}_{\mathrm{I},R} \sim \left((\overline{\mathbf{3}},\mathbf{1})_2 \oplus (\mathbf{1},\mathbf{2})_{-3} \oplus (\mathbf{3},\mathbf{2})_1 \oplus (\overline{\mathbf{3}},\mathbf{1})_{-4} \oplus (\mathbf{1},\mathbf{1})_6\right)_{\mathrm{I}} \oplus n_{\nu_{\mathrm{I},R}} (\mathbf{1},\mathbf{1})_0$$

ABJ anomalies show:

$$\int [\mathcal{D}\psi_L][\mathcal{D}\psi_L^\dagger] \mathrm{e}^{\mathrm{i}\left(\int \mathrm{d}^4x \left(\mathcal{L}_{\mathrm{SM}} + \alpha_{\mathbf{B}}(\partial_\mu J^\mu_{\mathbf{B}}) + \alpha_{\mathbf{L}}(\partial_\mu J^\mu_{\mathbf{L}})\right) - 18(\alpha_{\mathbf{B}}+\alpha_{\mathbf{L}}) N_f n^{(1)} - (\alpha_{\mathbf{B}}+\alpha_{\mathbf{L}}) N_f n^{(2)}\right)}.$$

Instanton number $n^{(N)} \equiv \int \mathrm{d}^4x \frac{g_1^2}{32\pi^2} \epsilon^{\mu\nu\mu'\nu'} F_{N,\mu\nu} F_{N,\mu'\nu'} \in \mathbb{Z}$ on spin manifolds.

- $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$ symmetry: $\alpha_{\mathbf{B}-\mathbf{L}} \equiv \alpha_{\mathbf{B}} = -\alpha_{\mathbf{L}}$, its Ward identity $\langle \partial_\mu (J^\mu_{\mathbf{B}} - J^\mu_{\mathbf{L}}) \rangle = 0$.
- $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$ symmetry: $\alpha_{\mathbf{B}+\mathbf{L}} \equiv \alpha_{\mathbf{B}} = \alpha_{\mathbf{L}}$, under SU(2) instanton, $\alpha_{\mathbf{B}+\mathbf{L}} \in \frac{2\pi}{2N_f}\mathbb{Z}$.

	SM	LR	PS	$su(5)$	flipped $u(5)$	$so(10)$	UU
$\mathbf{B}-\mathbf{L}$	$\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$	$\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$	$\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$	$\mathrm{U}(1)_X$	$\mathrm{U}(1)_{X_2}$	$\mathbb{Z}_{4,X}$	discrete
$\mathbf{B}+\mathbf{L}$	$\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$	$\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$	$\mathbb{Z}_2^{\mathrm{F}}$	$\mathbb{Z}_2^{\mathrm{F}}$	$\mathbb{Z}_2^{\mathrm{F}}$	$\mathbb{Z}_2^{\mathrm{F}}$	$\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$, $\mathbb{Z}_2^{\mathrm{F}}$

Many processes, like $p^+ \to e^+\pi^0$, have $\Delta(\mathbf{B}-\mathbf{L}) = 0$ and $\Delta(\mathbf{B}+\mathbf{L}) = -2$

Koran 2204.01741, JW-Wan-You 2204.08393 (check gauge-gravity anomalies w/ 1-symmetry, including $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$, or $\mathbb{Z}_{4,X} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$)

Classify 4d Anomalies by 5d iTQFT/SPTs via Cobordism

$\mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\mathrm{SM}_{q=6}}$ and $G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \mathrm{SU}(5)$:

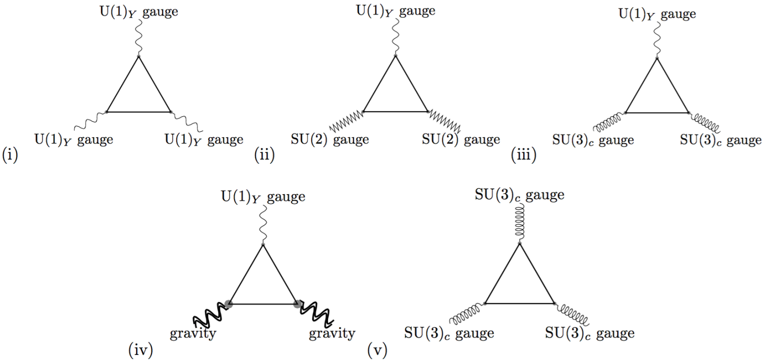
Cobordism group $\mathrm{TP}_d(G)$ with $G_{\mathrm{SM}_q} \equiv (\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_q$ with $q = 1, 2, 3, 6$		
	classes	cobordism invariants
	$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\mathrm{SM}_6}$	
5d	$\mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}$	$\mu(\mathrm{PD}(c_1(\mathrm{U}(3))))$, $c_1(\mathrm{U}(3))^2 \mathrm{CS}_1^{\mathrm{U}(3)}$, $\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(2)} + \mathrm{CS}_1^{\mathrm{U}(3)} c_2(\mathrm{U}(2))}{2} \sim \frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(2)} + c_1(\mathrm{U}(3)) \mathrm{CS}_3^{\mathrm{U}(2)}}{2}$ $\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(3)} + \mathrm{CS}_1^{\mathrm{U}(3)} c_2(\mathrm{U}(3)) + \mathrm{CS}_5^{\mathrm{U}(3)}}{2} \sim \frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(3)} + c_1(\mathrm{U}(3)) \mathrm{CS}_3^{\mathrm{U}(3)} + \mathrm{CS}_5^{\mathrm{U}(3)}}{2}$, $\mathrm{CS}_5^{\mathrm{U}(3)}$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{U}(3))$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{U}(2))$, $c_1(\mathrm{U}(3))^2 \eta'$, $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$
	$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \mathrm{SU}(5)$	
5d	$\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_{16}$	$\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{SU}(5)} + \mathrm{CS}_5^{\mathrm{SU}(5)}}{2}$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{SU}(5))$, $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$

$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathrm{Spin}(10)$:

$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathrm{Spin}(N)$ for $N \geq 7$, e.g. $\mathrm{Spin}(N) = \mathrm{Spin}(10)$ or $\mathrm{Spin}(18)$ for SO(10) or SO(18) GUT		
5d	$\mathbb{Z}_2$	$w_2(TM) w_3(TM) = w_2(V_{\mathrm{SO}(N)}) w_3(V_{\mathrm{SO}(N)})$

JW-Wen '18 1809.11171, Wan-JW'19 1910.14668

I. (Local) Anomalies of $\mathrm{Spin}(d) \times G_{\mathrm{SM}_q}|_{(\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1))/\mathbb{Z}_q}$



1. $\mathrm{U}(1)_Y^3$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_1(\mathrm{U}(1))^2$ and 6d $c_1(\mathrm{U}(1))^3$
2. $\mathrm{U}(1)_Y$-$\mathrm{SU}(2)^2$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_2(\mathrm{SU}(2))$, 6d $c_1(\mathrm{U}(1))c_2(\mathrm{SU}(2))$
3. $\mathrm{U}(1)_Y$-$\mathrm{SU}(3)_c^2$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_2(\mathrm{SU}(3))$, 6d $c_1(\mathrm{U}(1))c_2(\mathrm{SU}(3))$
4. $\mathrm{U}(1)_Y$-(gravity)^2: 4d anomaly from 5d $\mu(\mathrm{PD}(c_1(\mathrm{U}(1))))$, 6d $\frac{c_1(\mathrm{U}(1))(\sigma - \mathrm{F}\cdot\mathrm{F})}{8}$
5. $\mathrm{SU}(3)_c^3$: 4d anomaly from 5d $\frac{1}{2}\mathrm{CS}_5^{\mathrm{SU}(3)}$, 6d $\frac{1}{2} c_3(\mathrm{SU}(3))$
6. 4d global Witten SU(2) anomaly from 5d $c_2(\mathrm{SU}(2))\tilde{\eta}$, 6d $c_2(\mathrm{SU}(2))\mathrm{Arf}$.
   It becomes part of local anomaly in $\mathbb{Z}$ when $q = 2, 6$.

Davighi-Gripaios-Lohitsiri1910.11277, Wan-JW 1910.14668, Davighi-Lohitsiri 2001.07731 JW 2006.16996.

II. (Local+Global) Anomalies: $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\text{internal/gauge}}$

Focus on $\mathbb{Z}_{4,X} = Z(\mathrm{Spin}(10)) \subset \mathrm{U}(1)_X$ where $X = 5(\mathbf{B}-\mathbf{L}) - 4Y$.

$G = \mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$ and $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathrm{SU}(5)$:

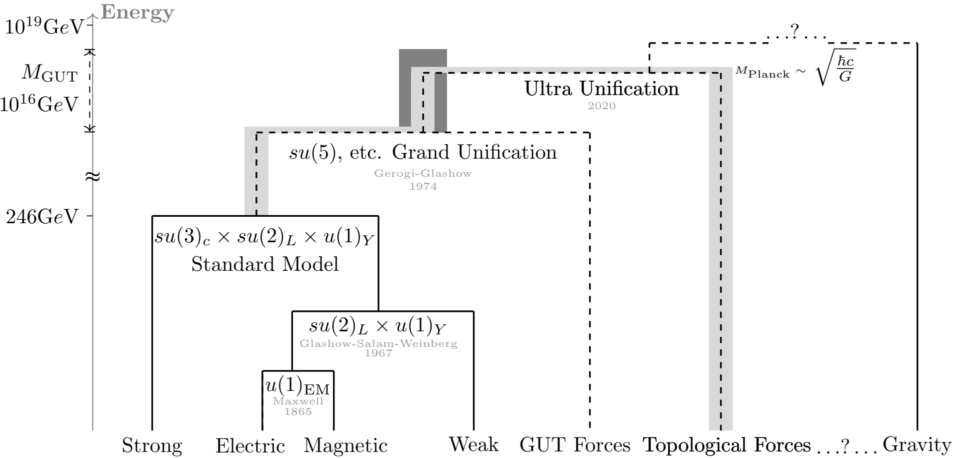
$$\mathrm{TP}_{d=5}(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}) = \begin{cases} \mathbb{Z}^5 \times \mathbb{Z}_2 \times \mathbb{Z}_4^2 \times \mathbb{Z}_{16}, & q = 1,3. \\ \mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}, & q = 2,6. \end{cases}$$

$$\mathrm{TP}_{d=5}(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathrm{SU}(5)) = \mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_{16}.$$

$\mathcal{A}_{\mathbb{Z}_2} \in \mathrm{H}^1(M, \mathbb{Z}_{4,X}/\mathbb{Z}_2^{\mathrm{F}})$ is a $\mathbb{Z}_2$-gauge field of $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X}$-manifold.

1. Mutated Witten SU(2) anomaly $c_2(\mathrm{SU}(2))\tilde{\eta}$:
   4d $\mathbb{Z}_2$ to $\mathbb{Z}_4$ global anomaly free ($q = 1,3$): $c_2(\mathrm{SU}(2))\eta'$.
   4d $\mathbb{Z}_2$ to $\mathbb{Z}$ local anomaly free ($q = 2,6$): $\frac{1}{2}\mathrm{CS}_1^{\mathrm{U}(2)} c_2(\mathrm{U}(2)) \sim \frac{1}{2} c_1(\mathrm{U}(2))\mathrm{CS}_3^{\mathrm{U}(2)}$.
2. $(\mathcal{A}_{\mathbb{Z}_2})c_2(\mathrm{SU}(2))$: 4d $\mathbb{Z}_2$ global anomaly free ($q = 2,6$)
3. $(\mathcal{A}_{\mathbb{Z}_2})c_2(\mathrm{SU}(3))$: 4d $\mathbb{Z}_2$ global anomaly free
4. $c_1(\mathrm{U}(1))^2\eta'$: 4d $\mathbb{Z}_4$ global anomaly free
5. $(\mathcal{A}_{\mathbb{Z}_2})c_2(\mathrm{SU}(5))$: 4d $\mathbb{Z}_2$ global anomaly free
6. $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$: $\Omega_5^{\mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4} \cong \Omega_4^{\mathrm{Pin}^+} = \mathbb{Z}_{16}$.
   4d $\mathbb{Z}_{16}$ global anomaly not canceled for $15N_f$ Weyl fermions. Alternative stories?

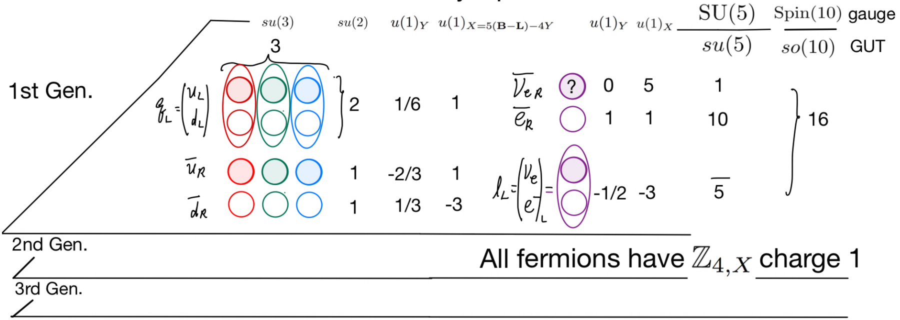
Fundamental Physics embodies Ultra Quantum Matter



HEP-phenomenology: beyond 0d particle physics (to gapped extended TQFT objects or gapless unparticle CFT). **Quantum Matter in Math/Physics**.

Standard Model and GUT anomaly cancellation

Chiral fermion - Weyl spinor



SM particle	SU(3)	SU(2)	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$	$\mathrm{U}(1)_X$	$\mathbb{Z}_{4,X}$	$\mathbb{Z}_2^F$
$\bar{d}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	−1/3	−3	1	1
l_L	$\mathbf{1}$	$\mathbf{2}$	−1/2	−1	−3	1	1
q_L	$\mathbf{3}$	$\mathbf{2}$	1/6	1/3	1	1	1
$\bar{u}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	−2/3	−1/3	1	1	1
$\bar{e}_R = e_L^+$	$\mathbf{1}$	$\mathbf{1}$	1	1	1	1	1
$\bar{\nu}_R = \nu_L$	$\mathbf{1}$	$\mathbf{1}$	0	1	5	1	1
ϕ_H	$\mathbf{1}$	$\mathbf{2}$	1/2	0	-2	2	0

Check: discrete $\mathbf{B} \pm \mathbf{L}$ symmetries, global anomalies, cobordism.

Deformation Class of $15N_f$,$16N_f$-fermion $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$

Part 1:

JW-Wan-You arXiv:2112.14765

- $\mathrm{Spin} \times_{\mathbb{Z}_2^\mathrm{F}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L} \text{ or } X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}$-class local anomaly $\mathbf{B}-\mathbf{L}$ or $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y$: $\mathrm{U}(1)$-grav^2 and $\mathrm{U}(1)^3$:

$$\mathbf{Z}_5^{\mathrm{U}(1)} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} (A c_1^2 + \frac{1}{48} \mathrm{CS}_3^{T(\mathrm{PD}(c_1))})).$$

$-N_f + n_{\nu_R} \equiv -N_f + \sum_j n_{\nu_{j,R}} = -3 + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \dots.$

- $\mathrm{Spin} \times_{\mathbb{Z}_2^\mathrm{F}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}_{16}$-class global anomaly of $\mathbb{Z}_{4,X}$-grav^2:

$$\mathbf{Z}_5^{\mathbb{Z}_{4,X}} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} \frac{2\pi \mathrm{i}}{16} \eta_{4\mathrm{d}}(\mathrm{PD}(A_{\mathbb{Z}_{2,X}}))).$$

Part 2:

- $\mathrm{Spin} \times_{\mathbb{Z}_2^\mathrm{F}} \mathrm{Spin}(10)$-symmetry.

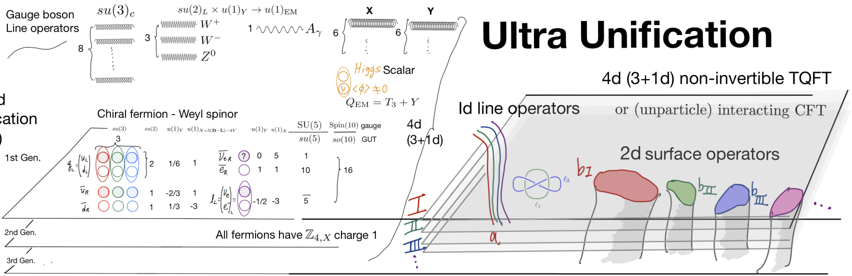
$\mathbb{Z}_2$-class global anomaly:

$$\mathbf{Z}_5 \equiv \exp(\mathrm{i}\pi \, \mathrm{p} \int_{M^5} w_2 w_3)|_{w_2 w_3(TM) = w_2 w_3(V_{\mathrm{SO}(n)})}$$

- We can also include $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$ background field.

arXiv:2204.08393

Ultra Unification 4d and 5d coupled quantum system



Chiral fermion and chiral gauge sector $(-(N_{\rm gen}=3) + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \nu_{\rm 4d} - \nu_{\rm 5d}) = 0 \mod 16$

5d (4+1d) invertible TQFT (Cobordism invariant, SPT phase, topological superconductor.)

$\exp(\frac{2\pi \mathrm{i}}{16} \nu_{\rm 5d} \int_{M^5} \eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2})))$ with APS eta invariant

$\mathcal{A}_{\mathbb{Z}_4} \equiv \mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathbb{Z}_{4,X}$-background gauge field

$\mathcal{A}_{\mathbb{Z}_2} \equiv$ background gauge field in $\mathrm{H}^1(M, \frac{\mathbb{Z}_{4,X}}{\mathbb{Z}_2^F})$

$\nu_{\rm 5d} \in \mathbb{Z}_{16}$ from $\Omega_5^{\mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathbb{Z}_{4,X}}$

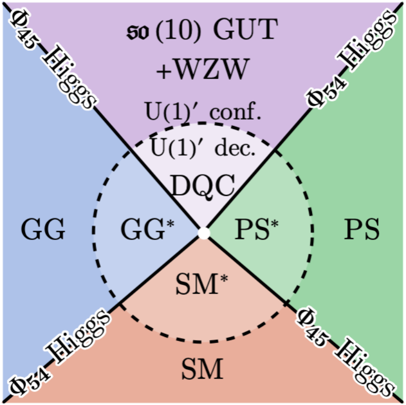
Mirror fermion and chiral gauge sector, or TQFT

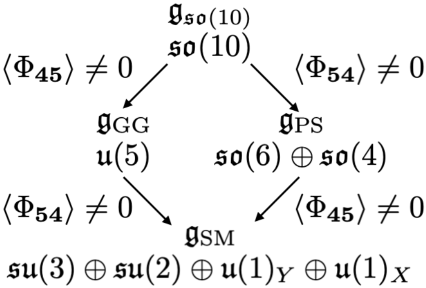
Application to Beyond SM: Neutrino Physics and Dark Matter.

Quantum Phase Diagram

- Goal: understand phase transitions among SM/GUTs
  - Higgs condensation will break Spin(10) to its subgroups
  - Each irrep. in 10x10 can be condensed separately

$$\boldsymbol{\Phi = \Phi_1 \oplus \Phi_{45} \oplus \Phi_{54}}$$

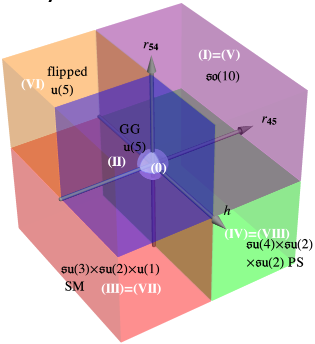


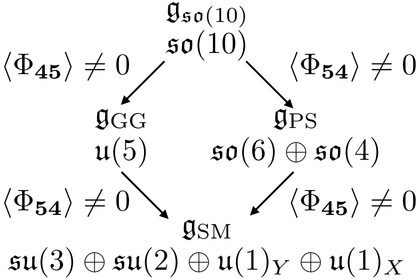


- Note: $\mathbf{1 + 45 + 54 = 10 \times 10 \leftarrow 16 \times 16 \times 16 \times 16}$
  Both Higgs fields can be written as four-fermion operators

Quantum Phase Diagram (Moduli space or Landscape)

- Goal: understand phase transitions among SM/GUTs
  - Introduce two scalar fields (GUT-Higgs fields)
    Φ_{45} - anti-symmetric rank-2 tensor rep. of so(10)
    Φ_{54} - symmetric traceless rank-2 tensor rep. of so(10)





- Note: $\mathbf{1} + \mathbf{45} + \mathbf{54} = \mathbf{10} \times \mathbf{10} \leftarrow \mathbf{16} \times \mathbf{16} \times \mathbf{16} \times \mathbf{16}$
  Both Higgs fields can be written as four-fermion operators

$$\mathrm{U}(\Phi_{\mathbf{R}}) = \Big(r_{45}(\Phi_{45})^2 + \lambda_{45}(\Phi_{45})^4\Big) + \Big(r_{54}(\Phi_{54})^2 + \lambda_{54}(\Phi_{54})^4\Big) + h\ \Phi_{45} \cdot (\langle \Phi_{45}^{\text{1st}} \rangle - \langle \Phi_{45}^{\text{2nd}} \rangle) + \dots.$$

Yang-Mills G gauge group theory with Weyl spinor. $so(10)$, GG, flipped, PS, SM. New Parton.

$\mathfrak{so}(10)$:

	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$		$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$	$\mathfrak{so}(10)$
u_L, d_L	**3**	**2**	1	1	ν_{eL}, e_L	**2**	−3	−3	**16**
$\bar{u}_R$		**1**	−4	1	$\bar{\nu}_{eR}$?	**1**	0	5	
$\bar{d}_R$		**1**	2	−3	$\bar{e}_R$	**1**	6	1	

$\mathfrak{su}(5)$:

	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$		$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$	$\mathfrak{su}(5)$
u_L, d_L	**3**	**2**	1	1	$\bar{\nu}_{eR}$?	**1**	0	5	**1**
$\bar{u}_R$		**1**	−4	1	$\bar{e}_R$	**1**	6	1	**10**
$\bar{d}_R$		**1**	2	−3	ν_{eL}, e_L	**2**	−3	−3	$\bar{\mathbf{5}}$

$\mathfrak{su}(5)'$:

	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$		$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$	$\mathfrak{su}(5)'$
u_L, d_L	**3**	**2**	1	1	$\bar{e}_R$	**1**	6	1	**1**
$\bar{d}_R$		**1**	2	−3	$\bar{\nu}_{eR}$?	**1**	0	5	**10**
$\bar{u}_R$		**1**	−4	1	ν_{eL}, e_L	**2**	−3	−3	$\bar{\mathbf{5}}$

$\mathfrak{su}(4)\times\mathfrak{su}(2)_L\times\mathfrak{su}(2)_R$:

	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$		$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$	$\mathfrak{su}(4)\times\mathfrak{su}(2)_L\times\mathfrak{su}(2)_R$
u_L, d_L	**3**	**2**	1	1	ν_{eL}, e_L	**2**	−3	−3	**(4, 2, 1)**
$\bar{u}_R$		**1**	−4	1	$\bar{\nu}_{eR}$?	**1**	0	5	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$
$\bar{d}_R$		**1**	2	−3	$\bar{e}_R$	**1**	6	1	

SM:

	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$		$\mathfrak{su}(2)$	$\mathfrak{u}(1)^{Y_1}$	$\mathfrak{u}(1)^{X_1}$
u_L, d_L	**3**	**2**	1	1	ν_{eL}, e_L	**2**	−3	−3
$\bar{u}_R$		**1**	−4	1	$\bar{\nu}_{eR}$?	**1**	0	5
$\bar{d}_R$		**1**	2	−3	$\bar{e}_R$	**1**	6	1

New Parton:

	$\mathfrak{u}(1)^{\text{dark}}_{\text{gauge}}$	$\mathfrak{su}(3)$	$\mathfrak{su}(2)$	$\mathfrak{u}(1)_{Y_1}$	$\mathfrak{u}(1)_{X_1}$	$\mathfrak{su}(5)$	$\mathfrak{so}(10)$
c	1	**3**	**1**	−2	−2	**5**	**10**
f	1	**1**	**2**	3	−2		
c'	1	$\bar{\mathbf{3}}$	**1**	2	2	$\bar{\mathbf{5}}$	
f'	1	**1**	**2**	−3	2		

Field contents of the modified $so(10)$ GUT + WZW term

Field content	Spin $\equiv$ Spin(1,3)	Spin(10)	$\mathbb{Z}_2^F$	$\mathbb{Z}_2^{F'}$	$\mathrm{U}(1)'^{\text{dark}}_{\text{gauge}}$
Model I					
ψ	$\mathbf{2}_L$	$\mathbf{16}$	1	0	0
A	$\mathbf{4}$	$45_{\text{adj.}}$	0	0	0
$\Phi^{\text{bi}} = \Phi_{\mathbf{1}} \oplus \hat{\Phi}^{\text{bi}} \oplus \tilde{\Phi}^{\text{bi}}$	$\mathbf{1}$	$\mathbf{10} \otimes \mathbf{10} = \mathbf{100} = \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{54}$	0	0	0
ϕ	$\mathbf{1}$	$\mathbf{10}$	0	0	0
Model II (include Model I's above + extra below)					
ξ	$\mathbf{2}_L \oplus \mathbf{2}_R$	$\mathbf{10}$	0	1	1
a	$\mathbf{4}$	$\mathbf{1}$	0	0	$1_{\text{adj.}}$

$$S_{\text{YM-Weyl}} = \int_{M^4} \mathrm{Tr}(F \wedge \star F) + \mathrm{d}^4x\big(\psi_L^\dagger(\mathrm{i}\,\bar{\sigma}^\mu D_{\mu,A})\psi_L\big),$$

$$S_{\text{Higgs}} = \int_{M^4} \mathrm{d}^4x\big(|D_{\mu,A}\Phi_{\mathbf{R}}|^2 - \mathrm{U}(\Phi_{\mathbf{R}})\big),$$

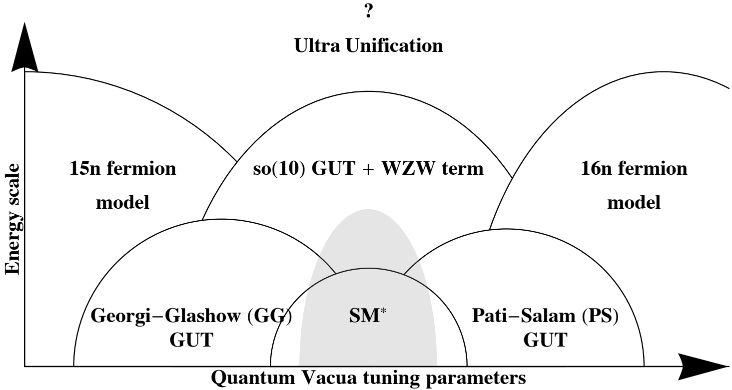
$$S_{\text{Yukawa}} = \int_{M^4} \mathrm{d}^4x\Big(\frac{1}{2}\phi^{\mathsf{T}}\Phi^{\text{bi}}\phi + \frac{1}{2}\sum_{a=1}^{5}\big(\psi_L^{\mathsf{T}}\,\mathrm{i}\,\sigma^2(\phi_{2a-1}\Gamma_{2a-1} - \mathrm{i}\,\phi_{2a}\Gamma_{2a})\psi_L + \text{h.c.}\big)\Big),$$

$$S^{\text{WZW}} = \frac{1}{\pi}\int_{M^5}\mathcal{B}(\Phi_{\mathbf{54}}) \wedge \mathrm{d}\,\mathcal{C}(\Phi_{\mathbf{45}})\Big|_{M^4=\partial M^5} = \pi\int_{M^5} B(\tilde{\Phi}^{\text{bi}}_{\mathbf{54}}) \smile \delta C(\hat{\Phi}^{\text{bi}}_{\mathbf{45}}))\Big|_{M^4=\partial M^5}.$$

$$S^{\text{WZW}}_{\text{QED}'_4} = \int_{M^4} \mathrm{d}^4x\; \bar{\xi}(\mathrm{i}\,\gamma^\mu D'_\mu - \tilde{\Phi}^{\text{bi}}_{\mathbf{54}} - \mathrm{i}\,\gamma^{\text{FIVE}}\hat{\Phi}^{\text{bi}}_{\mathbf{45}})\xi.$$

$$S^{\text{WZW}}_{\text{QED}'_5} = \int_{M^5} \mathrm{d}^5x\; \bar{\xi}(\mathrm{i}\,\tilde{\gamma}^\mu D'_\mu - m - \tilde{\gamma}^5\tilde{\Phi}^{\text{bi}}_{\mathbf{54}} - \tilde{\gamma}^6\,\mathrm{i}\,\hat{\Phi}^{\text{bi}}_{\mathbf{45}} - \mathrm{i}\,\tilde{\gamma}^5\tilde{\gamma}^\mu\tilde{\gamma}^\nu\mathcal{B}_{\mu\nu} - \mathrm{i}\,\tilde{\gamma}^6\tilde{\gamma}^\mu\tilde{\gamma}^\nu\mathcal{C}_{\mu\nu})\xi.$$

Ultra Unification (UU): SM or SM* as 4d EFT lives on the boundary of 5d gapped or gapless topological theory. Topological force and matter are involved in the unification. Many possible phases of our universe. Neighbor phases contain various $15N_f$ or $16N_f$ Weyl fermion SM or GUTs also as EFTs. The parent UU controls the tuning of these phase transitions.



UU replaces some of sterile neutrinos with new exotic gapped/gapless sectors (e.g., topological or conformal field theory, TQFT or CFT) or gravitational sectors with topological origins via global anomalies/cobordism constraints.

Summary

1. Cobordism class of the SM or GUT: 4d anomaly of the spacetime Spin, internal G_{SM_q}, $\mathbf{B}-\mathbf{L}$ (continuous $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$ and discrete $\mathbb{Z}_{4,X}$), and $\mathbb{Z}_{2N_f,\mathbf{B}+\mathbf{L}}$. Deformation class controls the quantum phase transitions between SM to neighbor GUT or beyond the SM phases.

2. UU replaces ν_R with 4d TQFT/5d iTQFT/4d CFT, etc, global anomaly/cobordism constraints similar to Lieb-Schultz-Mattis thm.

3. $\mathbb{Z}_{16}$-anomaly cancellation. **UU criticality**: $15N_f$ to $16N_f$ fermion ψ, topological quantum phase transition.

4. $\mathbb{Z}_2$-anomaly cancellation. **Deconfined quantum criticality**: Georgi-Glashow and Pati-Salam GUT transition, with SM + dark gauge force, and a modified $so(10)$-GUT +WZW term. Gapless CFT region.

- Neutrino mass from TQFT, vortex subgaps, zero modes, and mixed scenario with Dirac/Majorana mass.

- Proton stable in SM and some UU, but decay in many GUTs.

Thank you. Email: jw@cmsa.fas.harvard.edu.