# Pairwise States = Dressed States

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The Hebrew U. of Jerusalem & UC Berkeley

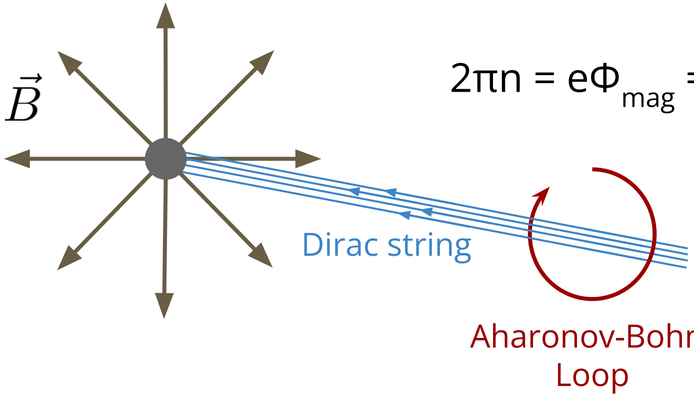
CSMA, Harvard and MIT Workshop
August 2022

2208.XXXX w/ C. Csáki, Z-Y. Dong, J. Terning, S. Yankielowicz

# Magnetic Monopoles

Sources of U(1) field with non-trivial winding number $\pi_1[\mathrm{U}(1)] = \mathbb{Z}$

$$\vec{B} = \frac{g}{4\pi r^2}\hat{r}$$




$2\pi n = e\Phi_{mag} = 4\pi q$ $q \equiv eg/4\pi$

$q = n/2$

At $r \gg m^{-1}$ effectively abelian

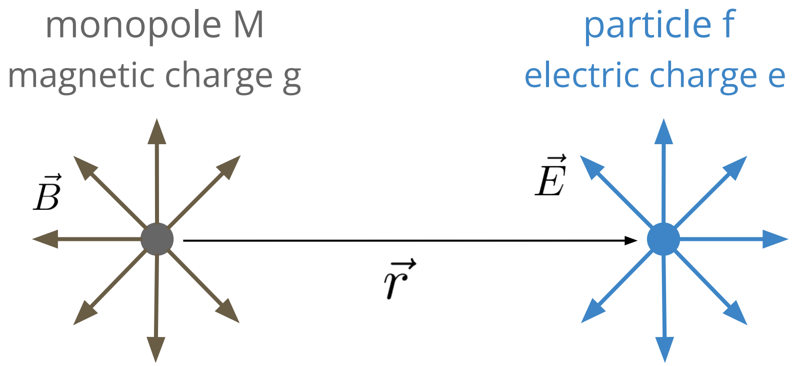
Maxwell's eqs → Need Dirac string* Dirac '31

String unobservable → Dirac quantization Dirac '31, Wu & Yang '76

* Or nontrivial gauge bundle

# Monopoles and Charges: Angular Momentum in EM Field

Thomson 1904




$$\vec{J}_{\text{field}} = \frac{1}{4\pi} \int \vec{r}\,' \times \left( \vec{E} \times \vec{B} \right) d^3r\,' = -\frac{g}{4\pi} \int \left( \vec{\nabla}\,' \cdot \vec{E} \right) \hat{r}\,' d^3r' = -eg\hat{r}$$

Distance independent!

# Some Overarching Themes and Questions

What is the action for electrodynamics + Monopoles ? How do we quantize it?

Weinberg '65; Schwinger '66; Zwanziger '71

What is the space of quantum multiparticle states with charges and monopoles?

Where does the extra angular momentum enter in the quantum (field) theory?

What is the role of Dirac quantization in the quantum field theory?

What is the role of the Dirac String in the quantum field theory?

Zwanziger '72; Csaki, Hong, Shirman, Terning, OT, Waterbury '20

Csaki, Hong, Shirman, Terning, OT '21 (PRL); Csaki, Shirman, Terning, OT '21; This work

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} But also

Zwanziger '72; Csaki, Hong, Shirman, Terning, OT, Waterbury '20

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# What Are the Quantum Multiparticle States for Charges and Monopoles?

Extra angular momentum in EM field, sourced by all *pairs* of charges and monopoles

$$\longrightarrow \quad |\text{charge}, \text{monopole}\rangle \neq |\text{charge}\rangle \otimes |\text{monopole}\rangle$$

Charges-monopole multiparticle states don't live in a *Fock Space*

How do we define charge-monopole multiparticle states?

The definition should reflect extra EM angular momentum in the EM field

# Electric-Magnetic "Pairwise" States

Zwanziger '72

Csaki, Hong, Shirman, Terning, OT, Waterbury '20

Csaki, Hong, Shirman, Terning, OT, PRL '21

*Electric-magnetic* multiparticle states $\left|\underbrace{p_1, \ldots, p_n}_{\text{momenta}};\ \underbrace{\sigma_1, \ldots, \sigma_n}_{\text{spins / helicities}};\ \underbrace{q_{12},\ q_{13}, \ldots, q_{n-1,n}}_{\text{pairwise helicities}}\right\rangle$

The pairwise helicities are the Dirac-quantized $q_{ij} = \dfrac{e_i g_j - e_j g_i}{4\pi}$

They are the S-matrix manifestation of the extra angular momentum

$$\Delta \vec{J}_{ij} = -q_{ij} \hat{r}_{ij}$$

carried by the electromagnetic field that's sourced by the dyon (or charge-monopole) pair (i,j)

# Electric-Magnetic "Pairwise" States

Zwanziger '72

Csaki, Hong, Shirman, Terning, OT, Waterbury '20

Csaki, Hong, Shirman, Terning, OT, PRL '21

Under a Lorentz transformation, transform with an extra pairwise Little Group phase:

$$U(\Lambda)\ |p_1,\ldots,p_n\,;\,\sigma_1,\ldots,\sigma_n\,;\,q_{12},q_{13},\ldots q_{n-1,n}\rangle =$$

$$\underbrace{e^{i\sum_{i<j} q_{ij}\varphi(p_i,p_j,\Lambda)}}_{\text{pairwise phase}} \prod_{i=1}^{n} \mathcal{D}^{i}_{\sigma_i'\sigma_i}\ |\Lambda p_1,\ldots,\Lambda p_n;\sigma_1',\ldots,\sigma_n';q_{12},q_{13},\ldots,q_{n-1,n}\rangle$$

The pairwise phase φ leads to modified angular momentum selection rules

in the scattering of charges, monopoles and dyons

# The Pairwise Little Group; A Short Dive-In

Consider the charge monopole state $|p_1, p_2; q_{12}\rangle$

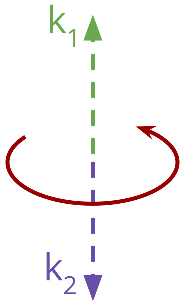
How does it transform under Lorentz?

1. Define the COM momenta $k^{\mu}_{1,2} = \left(E^c_{1,2}, 0, 0, \pm p_c\right)$

2. The pairwise Little Group (LG) is the subgroup of Lorentz which keeps $k_1$, $k_2$ invariant, i.e. U(1) rotations around the z-axis

3. The pairwise helicities $q_{ij}$ label representations of each $U(1)_{ij}$

$$U\left[R_z(\varphi)\right]|k_1, k_2; q_{12}\rangle = e^{iq_{12}\varphi}\,|k_1, k_2; q_{12}\rangle$$




Pairwise Little Group = z-rotations

$$p_c = \sqrt{\frac{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}{s}}$$

$$E^c_i = \sqrt{m_i^2 + p_c^2}$$

# The Pairwise Little Group; A Short Dive-In

Zwanziger '72

Consider the charge monopole state $|p_1, p_2; q_{12}\rangle$

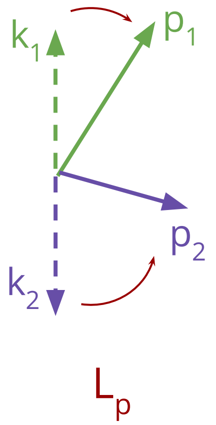
How does it transform under Lorentz?

4. Define $L_p$ so that $p^\mu_{1,2} = [L_p]^\mu{}_\nu \, k^\nu_{1,2}$

5. Under any Lorentz transformation $\Lambda$

$$U[\Lambda]\,|p_1, p_2; q_{12}\rangle = e^{iq_{12}\varphi(p_1,p_2,\Lambda)}\,|\Lambda p_1, \Lambda p_2; q_{12}\rangle$$

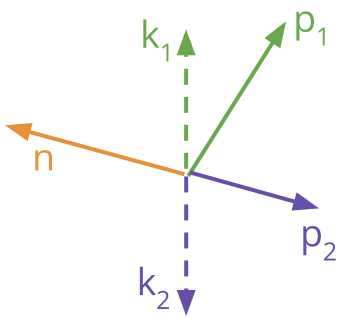
where $R_z[\phi(p_i, p_j, \Lambda)] = L^{-1}_{\Lambda p}\Lambda L_p$



# The Phase $\varphi(p_1, p_2, \Lambda)$, Explicitly

Zwanziger '72

We fix the freedom in $L_p$ by choosing arbitrary vector $n^\mu$ and let




$$[L_p]^\mu{}_2 = \hat{\epsilon}^\mu(p_1, p_2, n)$$

The pairwise phase is then

$$\cos\left[\varphi(p_1, p_2, \Lambda)\right] = \hat{\epsilon}(p_1, p_2, \Lambda^{-1} n) \cdot \hat{\epsilon}(p_1, p_2, n)$$

$n^\mu$ is the pairwise LG analog of the Dirac string for the monopole

$$\hat{\epsilon}^\mu(abc) \equiv \frac{\epsilon^{\mu\nu\rho\sigma} a_\nu b_\rho c_\sigma}{|\epsilon^{\mu\nu\rho\sigma} a_\nu b_\rho c_\sigma|}$$

# Pairwise States: Results

- Derived all 3pt amplitudes involving charges, monopoles, and dyons
- Reproduced the forced chirality flip in the lowest PW for fermion-monopole scattering
- Reconstructed monopole spherical harmonics from pairwise spinor-helicity variables

Csaki, Hong, Shirman, Terning, OT, Waterbury ‘20

- Derived geodesics in Taub-NUT background as the classical limit of “pairwise” amplitudes

Kol, O’Connell, OT, ‘21

- Defined pairwise states for mutually-non-local branes — Csaki, Fan, Terning, OT, upcoming

- Proposed an on-shell derivation for monopole catalysis of nucleon decay (the Rubakov-Callan effect)

Csaki, Shirman, Terning, OT, ‘21

# A Lingering Question and Its Straightforward Answer

What is the actual origin of the "pairwise" charge-monopole states?

How does QED+monopoles "know" to create such complicated multiparticle states?

Today - a straightforward answer

$$|p_1, p_2; q_{12}\rangle \quad = \quad |p_1, p_2\rangle\!\rangle$$

pairwise
states

IR-dressed
charge-monopole states
of monopole-QED (QEMD)

# Dressed States in QED

The QED S-matrix is IR-finite when taken between asymptotic states "dressed" by soft photons

$$S_{\text{QED,finite}} = \langle\langle p_1^{out}, \ldots, p_m^{in} | p_1^{in}, \ldots, p_n^{in} \rangle\rangle$$

Faddeev-Kulish:
$$|p_1, \ldots, p_n\rangle\rangle = \exp\left[\mathcal{T} \int_{-\infty}^{\infty} dt\, V_I^{as}(t)\right] |p_1, \ldots, p_n\rangle$$

IR-dressed state

"bare" state

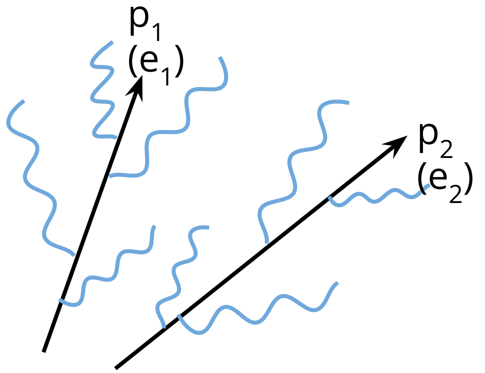
$$V_I^{as}(t) = -\lim_{t\to\pm\infty} \int d^3x\, [j^\mu A_\mu]$$

asymptotic potential:

generates retarded/advanced EM field associated with the charges moving with momenta $p_1,\ldots,p_n$

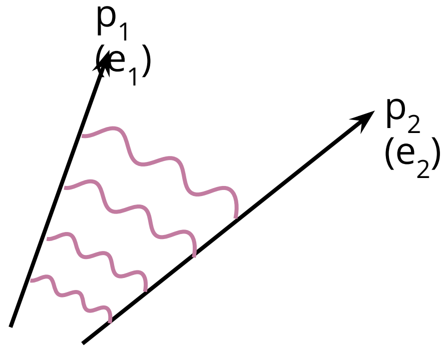
# Dressed States in QED

$$\exp\left[\mathcal{T}\int_{-\infty}^{\infty} dt\, V_I^{as}(t)\right] = e^{R_{FK}}\, e^{i\Phi_{FK}}$$




$$R_{FK} = -i\int_{-\infty}^{\infty} dt\ V_I^{as}(t)$$

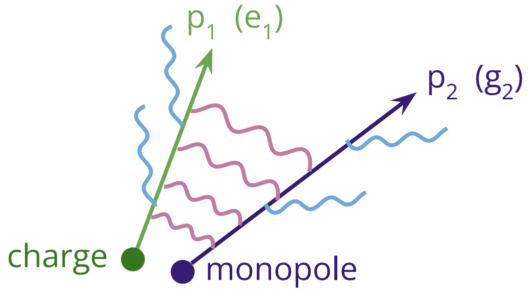
Real part of dressing (~soft photon creation operators)




$$\Phi_{FK} = \frac{i}{2}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2\ \left[V_I^{as}\left(t_1\right), V_I^{as}\left(t_2\right)\right]$$

Imaginary part of dressing (virtual photon exchange)

# Dressed States in Monopole-QED (QEMD)




Add dual interaction: $V_I^{as}(t) = -\lim_{t\to\infty}\int d^3x\left[j^\mu A_\mu + \tilde{j}^\mu \tilde{A}_\mu\right]$

$j^\mu$
electric current density

$\tilde{j}^\mu$
magnetic current density

Gauge field $A_\mu(x) = \sum_{a=\pm}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[\varepsilon_\mu^{*a}(\vec{k})a_a(\vec{k})e^{ik\cdot x} + \varepsilon_\mu^{a}(\vec{k})a_a^\dagger(\vec{k})e^{-ik\cdot x}\right]$

Dual gauge field $\tilde{A}_\mu(x) = \sum_{a=\pm}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[\tilde{\varepsilon}_\mu^{*a}(\vec{k})a_a(\vec{k})e^{ik\cdot x} + \tilde{\varepsilon}_\mu^{a}(\vec{k})a_a^\dagger(\vec{k})e^{-ik\cdot x}\right]$

same creation/annihilation ops.

“One photon, two descriptions”

# QEMD Dressed States = Pairwise States?

To O(eg) $\Phi_{FK}\,|p_1, p_2\rangle = q_{12}\,\varphi_{FK}(p_1, p_2)\,|p_1, p_2\rangle$

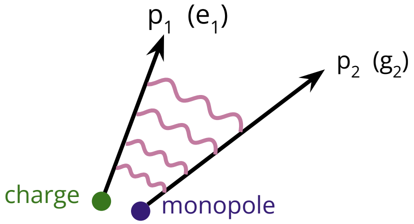
We want to show that the dressed state $e^{R_{FK}}\,e^{iq_{12}\varphi_{FK}}\,|p_1, p_2\rangle$

Transforms the same as the pairwise state $|p_1, p_2; q_{12}\rangle$

In other words, we want to show that

$$\Delta\varphi_{FK} \equiv \underbrace{\varphi_{FK}(\Lambda p_1, \Lambda p_2) - \varphi_{FK}(p_1, p_2)}_{\text{The shift in the soft photon phase}} = \underbrace{\varphi(p_1, p_2, \Lambda)}_{\text{The pairwise phase}}$$

# Calculating $\varphi_{FK}$ in QEMD




$$\Phi_{FK} = \frac{i}{2}\int_0^\infty dt_1 \int_0^{t_1} dt_2 \ [V_I^{as}(t_1), V_I^{as}(t_2)] - (t_1, t_2 \leftrightarrow -t_1, -t_2)$$

this has

$$e\left[p_1 \cdot \varepsilon^{*a}(\vec{k})\right] a_a(\vec{k}) + \dots$$

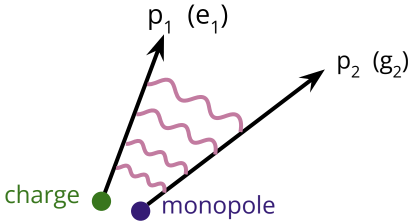
this has

$$g\left[p_2 \cdot \widetilde{\varepsilon}^{a}(\vec{k})\right] a_a^\dagger(\vec{k}) + \dots$$

$$\varphi_{FK} \sim \int d^3k \left[p_1 \varepsilon^a(\vec{k})\right]\left[p_2 \cdot \widetilde{\varepsilon}^a(\vec{k})\right] \sim \int d^3k \frac{\epsilon(p_1, p_2, n, k)}{n \cdot k + i\epsilon} + cc.$$

Integral over all soft photon exchanges between charge & monopole

# Calculating $\varphi_{FK}$ in QEMD



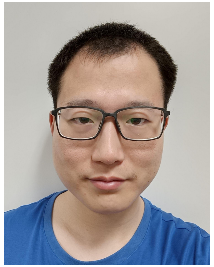

$$\varphi_{FK}\left(p_1, p_2, n\right) = \mathrm{Im}\left[\mathcal{I} + (n \leftrightarrow -n)\right] - (p_1 \leftrightarrow -p_1) - (p_2 \leftrightarrow -p_2)$$

$$\mathcal{I} = 2\pi i \int \frac{d^4k}{(2\pi)^4} \frac{\epsilon\left(p_1, p_2, n, k\right)}{\left(k^2 + i\epsilon\right)\left(p_1 \cdot k - i\epsilon\right)\left(p_2 \cdot k + i\epsilon\right)\left(n \cdot k + i\epsilon\right)}$$

All that remains is to calculate this Feynman integral

Alas! This integral is 0/0 and needs regularization…

# A Slick Trick



Zi-Yu Dong

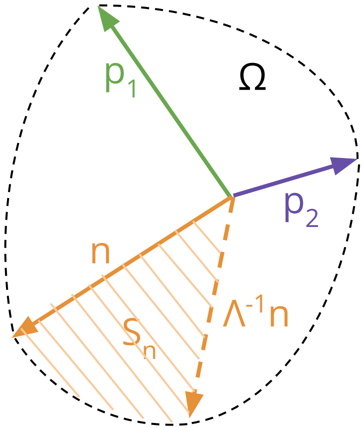
While $\mathcal{I}$ is ill-defined, what we need is actually $\Delta\varphi_{FK} \sim \Delta\mathcal{I}$

$$\Delta\mathcal{I} = \mathcal{I}\left(\Lambda p_1, \Lambda p_2, n\right) - \mathcal{I}\left(p_1, p_2, n\right)$$
$$= \mathcal{I}\left(p_1, p_2, \Lambda^{-1} n\right) - \mathcal{I}\left(p_1, p_2, n\right)$$

which *is* well defined as can be shown in position space via Stokes' theorem

In momentum space, we showed using Schwinger parameters that

$$\Delta\mathcal{I} = -\frac{i}{2\pi}\Omega\left(-p_1, p_2, n, \Lambda^{-1} n\right)$$




So the integral really computes the 4D solid angle between -$p_1$, $p_2$, n, and $\Lambda^{-1}$n

# Dressed = Pairwise!

Substituting in $\Delta\varphi_{FK} = \mathrm{Im}\,\Delta\mathcal{I} + (p_3 \leftrightarrow -p_3) - (p_1 \leftrightarrow -p_1) - (p_2 \leftrightarrow -p_2)$

The 4D solid angle degenerates to a dihedral angle

$$\cos\left[\Delta\varphi_{FK}\right] = \cos\left[\phi_{dih}\left(V_{123}, V_{124}\right)\right]$$

$$= \hat{\epsilon}(p_1, p_2, \Lambda^{-1}n) \cdot \hat{\epsilon}(p_1, p_2, n) = \cos\left[\varphi_{\text{pairwise}}(p_1, p_2, \Lambda)\right]$$

Shift in soft photon phase = pairwise Little Group phase

Dressed states of QEMD = pairwise multiparticle states ✓

# Bonus: Topology!

Terning, Verhaaren '18

The soft photon phase $\varphi_{FK}$ depends on the unphysical Dirac string

Can we measure it in an interference experiment?

For a closed path, we can apply Stokes' theorem

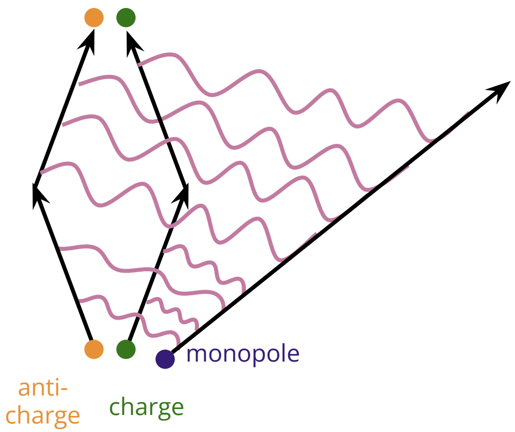
directly for $\varphi_{FK}$ and not just $\Delta\varphi_{FK}$

However, the result is an unobservable $2\pi q_{12}$ x integer !

In fact, the $\varphi_{FK}$ integral computes the topological linking number

between the charge worldline and the Dirac string worldsheet in 4D

This is the QFT generalization of the original Dirac quantization argument



# Conclusions

The multiparticle quantum states for charges and monopoles are not a Fock space;

They have pairwise helicities $q_{ij}$ and transform with pairwise LG phases $\varphi(p_i, p_j, \Lambda)$

The pairwise LG provides modified angular momentum selection rules constraining the scattering amplitudes of monopoles and charges

The pairwise states are equivalent to the soft-photon dressed states of monopole-QED

The pairwise LG phase $\varphi_{LG}$ is the shift of the soft photon phase $\varphi_{FK}$ under a Lorentz transformation

The soft photon phase $\varphi_{FK}$ is Dirac-string dependent but is unobservable due to topology