Coupling a Cosmic String to a TQFT

Sungwoo Hong

Enrico Fermi Institute
University of Chicago
Argonne National Laboratory

(Based on works in progress with
T. Daniel Brennan and Liantao Wang)

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- **Symmetry** plays the central role in theoretical physics!

- ✸ **Symmetry** plays the central role in theoretical physics!
- ✸ Recently, notion/concept of **Symmetry** has gone through explosive **generalizations**!

○ **Ordinary Symmetry** 0-form particle or local operator	→	**Higher-form Symmetry** 1-form, 2-form, … extended object: line, surface, …
○ **Group**	→	**Higher-Group**, **Non-Invertible Symmetries**
○ **Conserved in entire Spacetime**	→	**Subsystem Symmetries** (Fracton)

○ More…

(Q1) Are there **generalized symmetries** in **(3+1)d QFTs** that relevant for **particle physics**?

(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

(Q3) Can **generalized symmetry** provide **novel or meaningful solutions** to problems in **particle physics**?

Today's Talk

(Q1) Are there **generalized symmetries** in **(3+1)d QFTs** that relevant for **particle physics**?

(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

(Q3) Can **generalized symmetry** provide **novel or meaningful solutions** to problems in **particle physics**?

Outline

I. Axion-Maxwell Theory

II. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT

IR-Universal Observables from TQFT-Coupling

Anomaly-Inflow, Fermion Zero Modes

III. TQFT-Coupling 2: Gauging Discrete Subgroup of Axion Shift

Modification of Cosmic String Spectrum

IV. Conclusion

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I. Axion-Maxwell Theory

$$S = \int \frac{1}{2}\left(\partial_\mu a\right)^2 + \int \frac{1}{2g^2}\, F_{\mu\nu}F^{\mu\nu} - \int \frac{iK}{16\pi^2}\frac{a}{f}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

cf) $F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\, F_{\mu\nu}F_{\rho\sigma}$

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- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
  - 1-form magnetic
  - ✸ 3-group
  - ✸ Non-invertible symmetries (Cordova, Ohmori '22)

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$$\theta = \frac{a}{f} \;\rightarrow\; \theta + c$$

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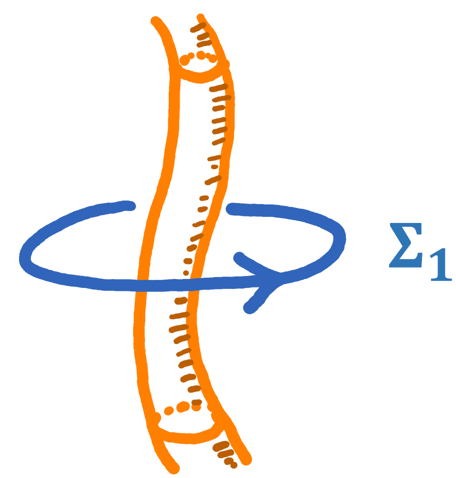
  - 0-form axion shift
  - 2-form axion winding
    - $Q(\Sigma_1) = \oint_{\Sigma_1} \frac{d\theta}{2\pi} = m$: Winding number
    - Charged object: cosmic string (M_2^{st})



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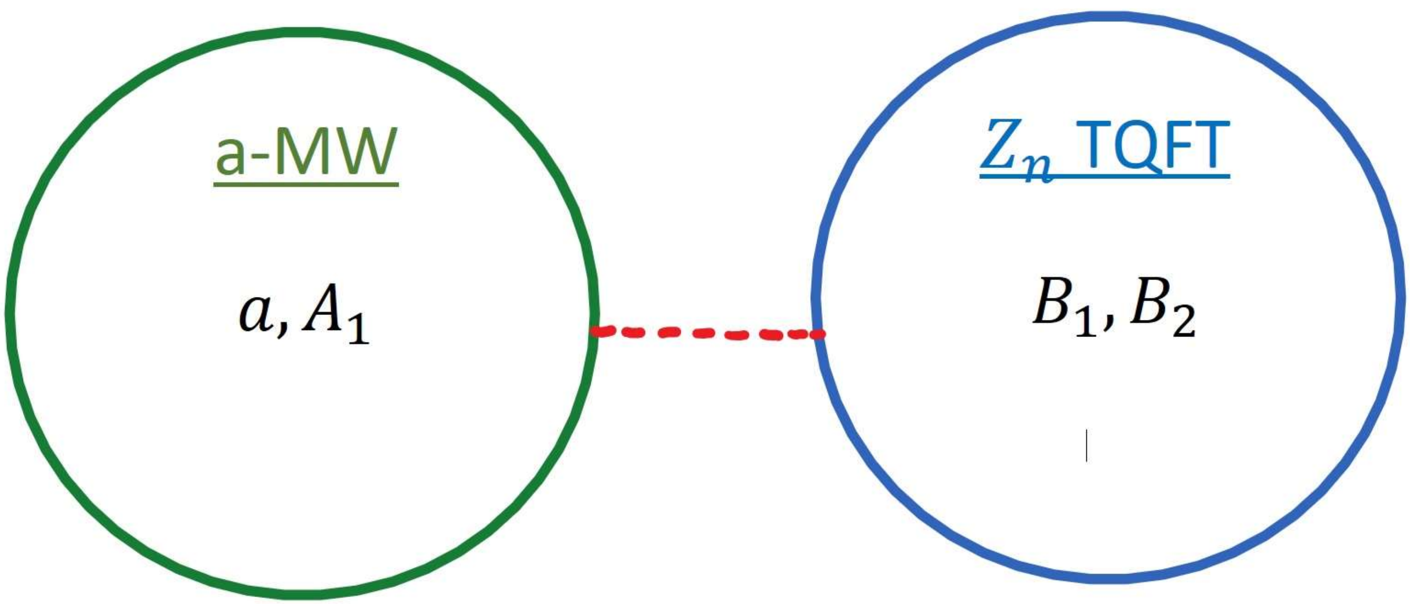

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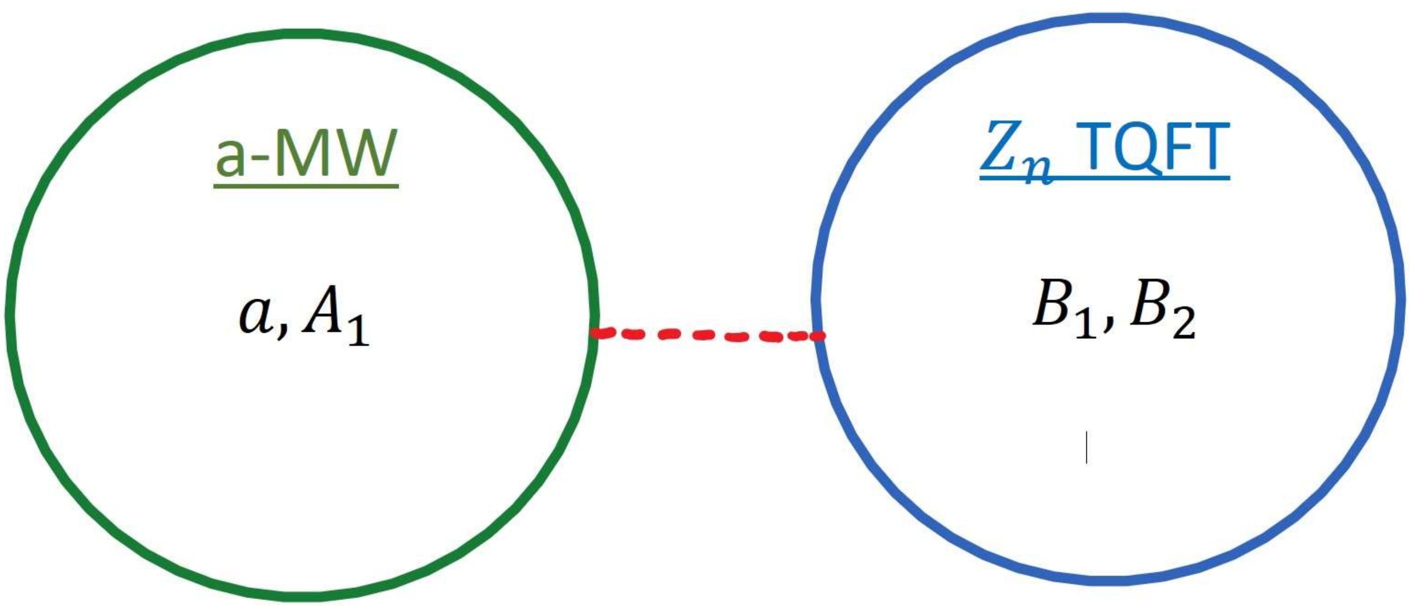

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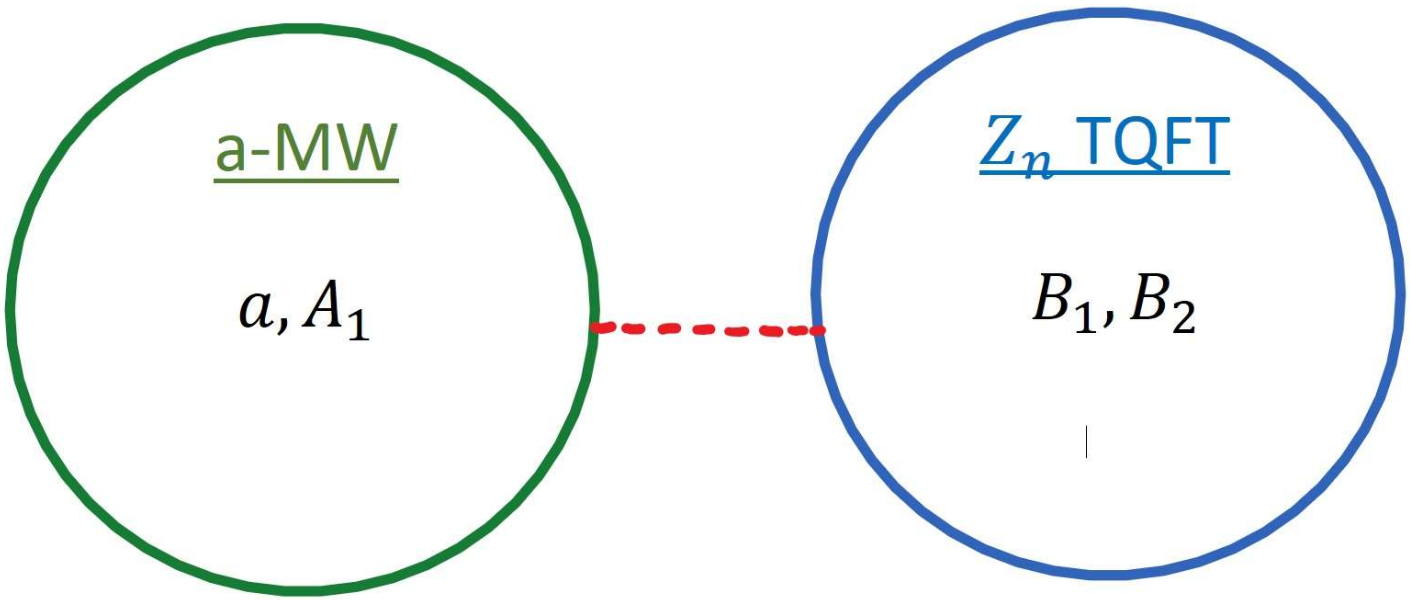
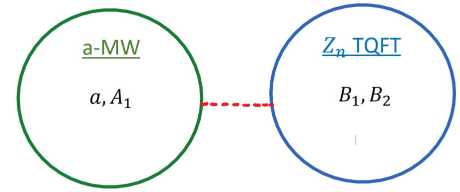


$F_B = dB_1$

II. $\boldsymbol{Z_n}$ Gauge Theory (BF theory)



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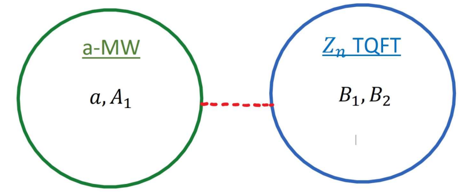

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(ii) Can be obtained from Abelian Higgs Model (AHM)

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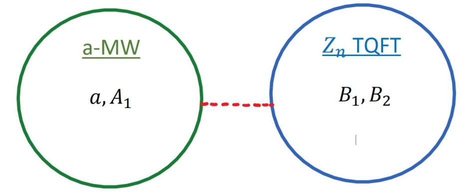

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$$\mathcal{L} \sim \Lambda^2 (nB_1) \wedge * \, (d\varphi) \sim \frac{in}{2\pi} B_1 \wedge d\, B_2 \qquad (dB_2 \sim * \, d\varphi)$$

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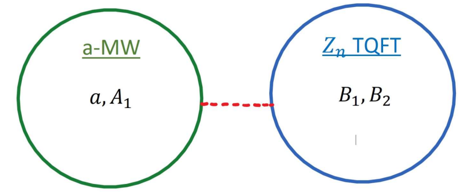

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(i) Describes discrete (Z_n) gauge theory

(ii) Can be obtained from Abelian Higgs Model (AHM)

(iii) Describes non-trivial vacuum structure and selection rules in terms of topological degrees of freedom B_1 , B_2

$$dB_1 = 0, \quad dB_2 = 0$$

$$\langle W_1(\Sigma_1, m)\, W_2(\Sigma_2, \ell)\rangle \sim e^{i\frac{2\pi}{n} m\ell\, \mathrm{Link}(\Sigma_1,\Sigma_2)}$$

$$W_1\,(\Sigma_1, m) = e^{im \oint_{\Sigma_1} B_1}\,, \quad W_2(\Sigma_2, \ell) = e^{i\ell \oint_{\Sigma_2} B_2}$$

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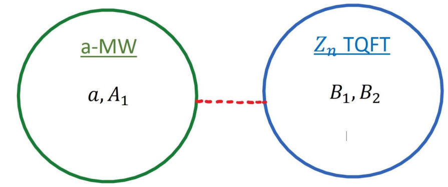

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(Q1) Can there be any **IR-Universal** (local) observable effect?

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Or can this arise as IR-EFT of some **standard UV QFT** relevant for particle physics?

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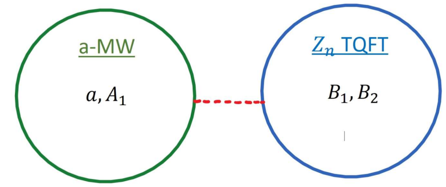

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✓ Illustrate importance of studying carefully the effects of remnant TQFT-couplings (GGS = essential tools)

II-1. IR-Universal Observables from TQFT-Coupling

- **Anomaly Inflow**

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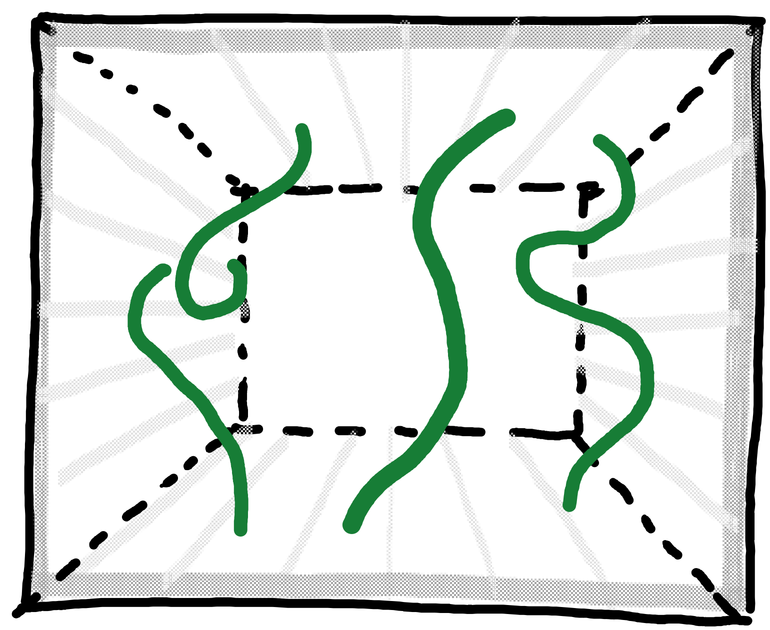

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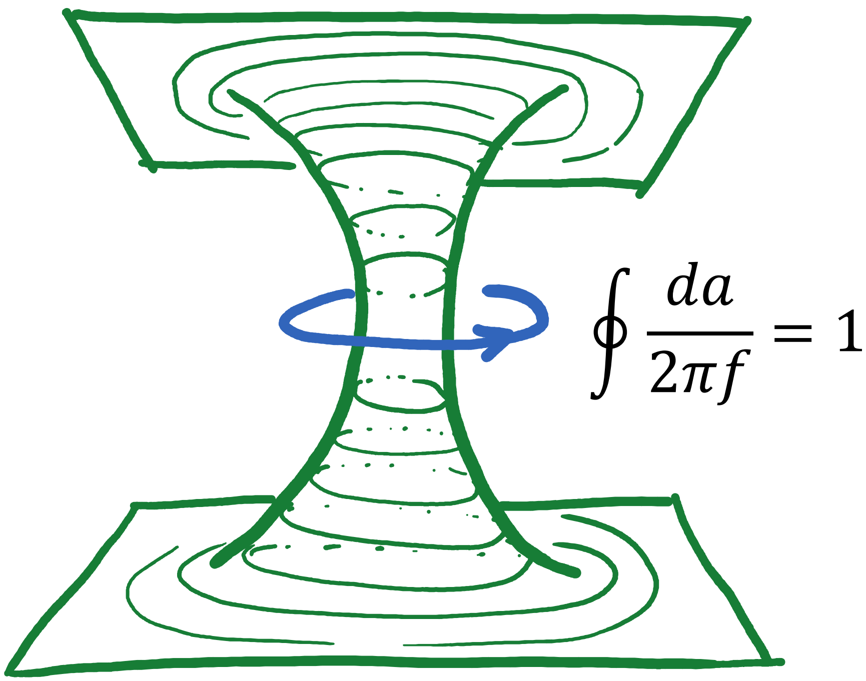

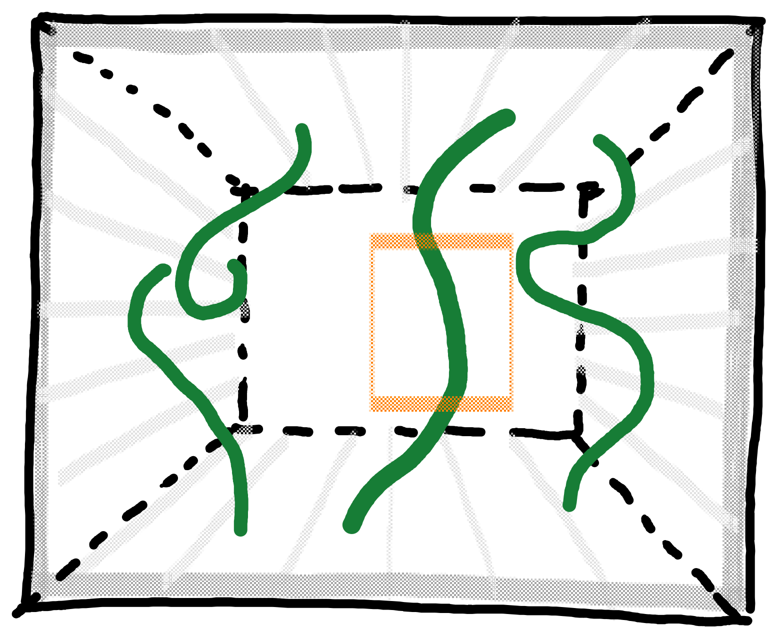

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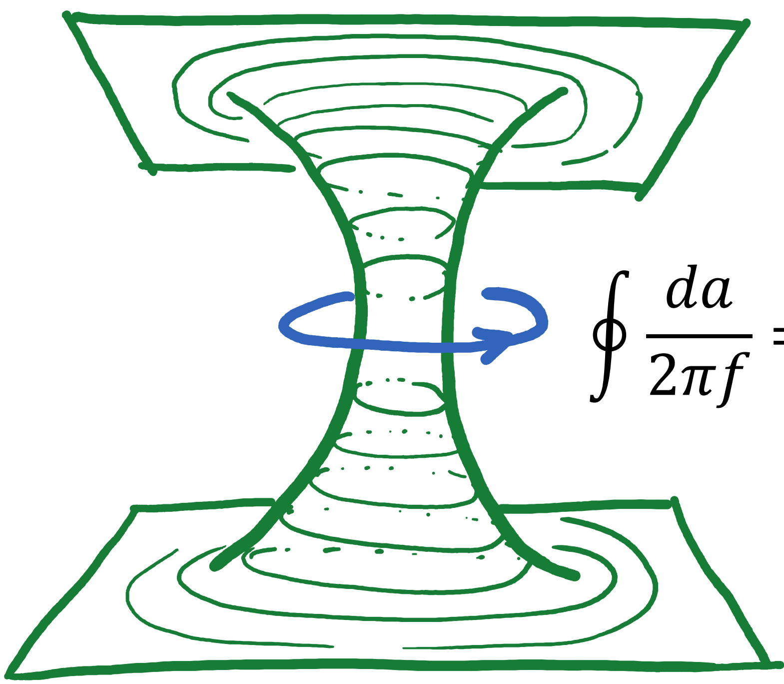

$$\oint \frac{da}{2\pi f} = 1$$

○ Consistency with $U(1)_A$ invariance :

$$A_1 \to A_1 + d\lambda$$

○ $S \supset \dfrac{iK_A}{8\pi^2} \displaystyle\int da \wedge A_1 \wedge F_A$

$$\downarrow$$

○ $\delta S = i \displaystyle\int \delta^{(2)}(M_2^{st}) \wedge \left(\lambda \frac{K_A}{4\pi} F_A\right)$

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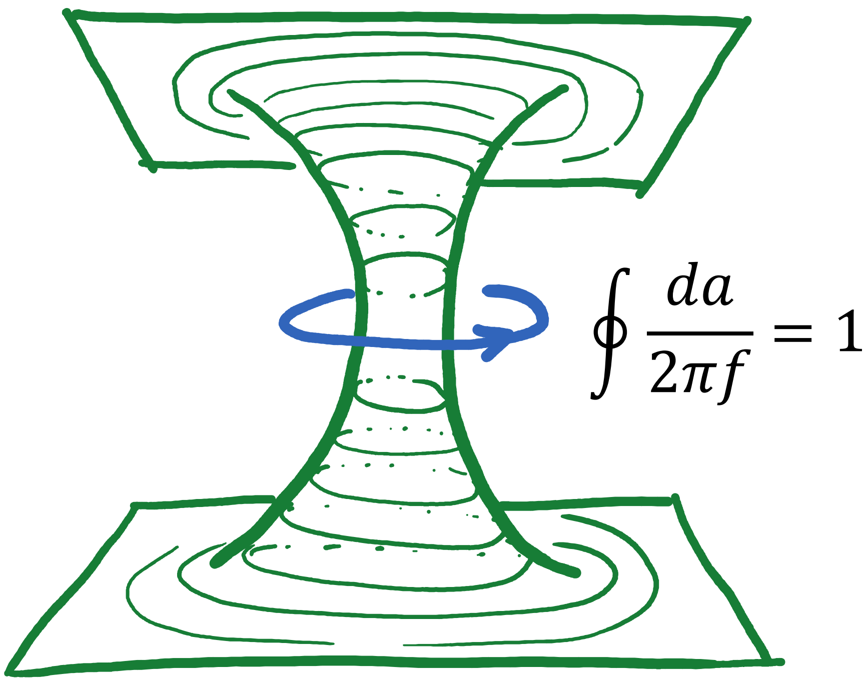

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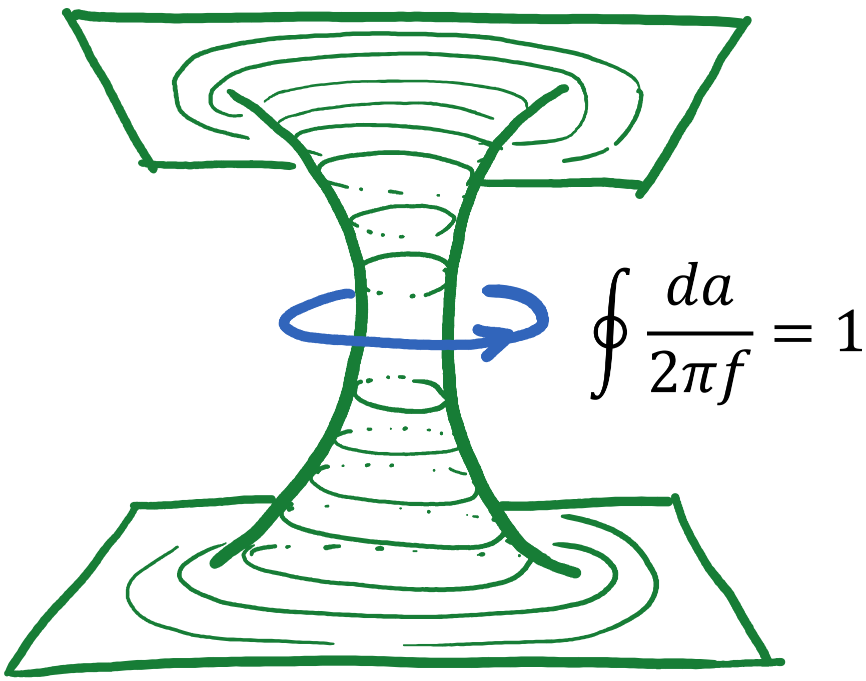

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- Violation of $U(1)_A$ - inv localized on the string core
- This 'gauge anomaly' must be cancelled by an anomalous 2d QFT on M_2^{st}
- Anomalous $TQFT_4$ in "bulk" + Anomalous QFT_2 on the "boundary" = 0

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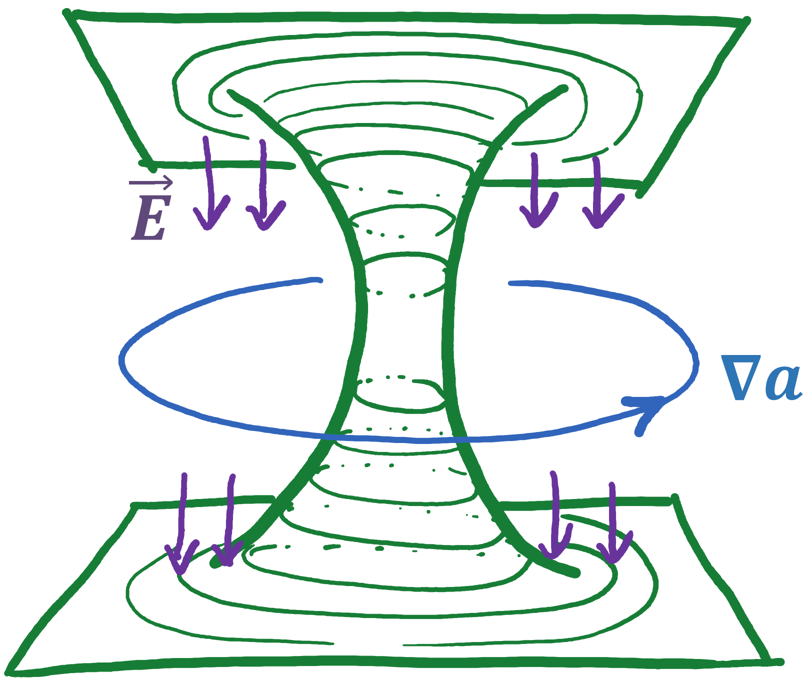
$$S \supset \frac{iK_A}{8\pi^2}\int da \wedge A_1 \wedge F_A = i\int A_1 \wedge * J_1$$



- $* J_1 = \frac{K_A}{4\pi}\, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi}\, F_A \wedge \delta^{(2)}\left(M_2^{st}\right)$
- $\vec{j}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)

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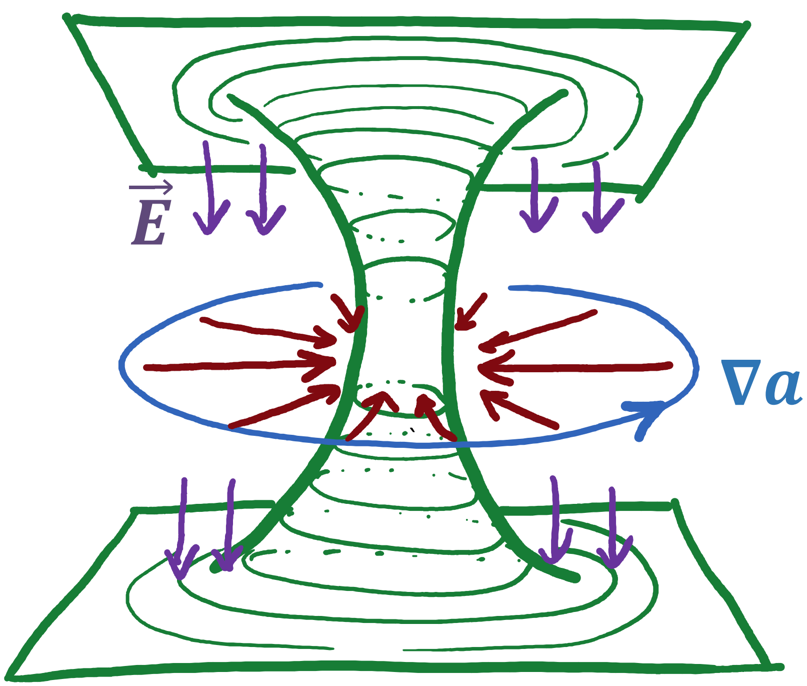

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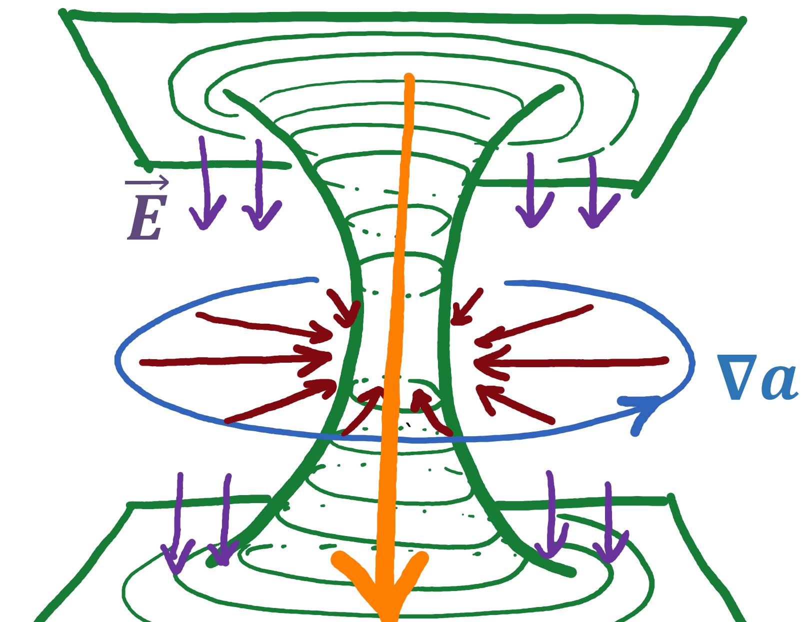

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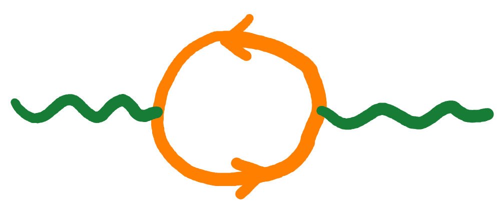
- $\vec{j}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)
- 2d chiral fermions $\{\alpha_i\,(z,t)\}$

$$d * J_1(2d) = -\frac{K_A}{4\pi}\, F_A$$



$$\sum_i Q_i^2 = K_A$$

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$$\delta_A S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B\right)$$

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2. $B_1 \to B_1 + d\lambda_B \,, \qquad \lambda_B = \frac{2\pi}{n}\kappa \,, \qquad \kappa = 0,1,\cdots,n-1$

$$\delta_B S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B\right)$$

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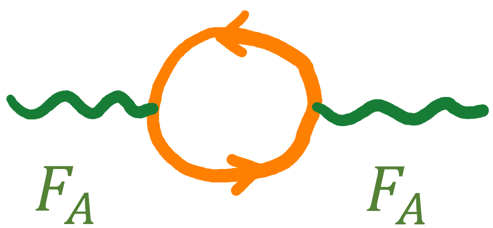

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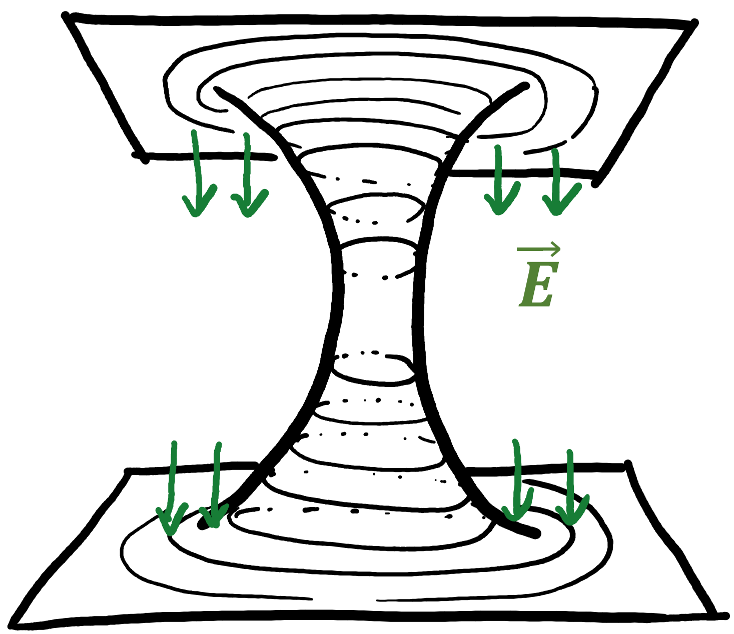

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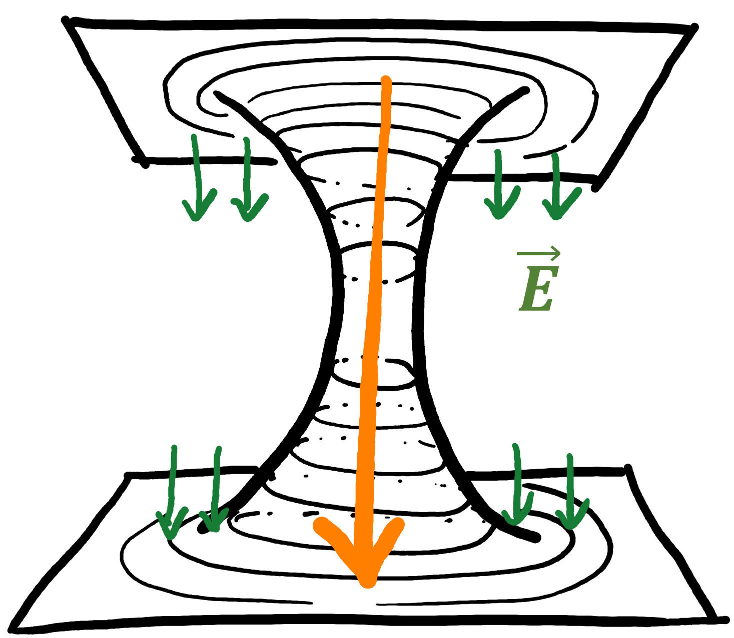

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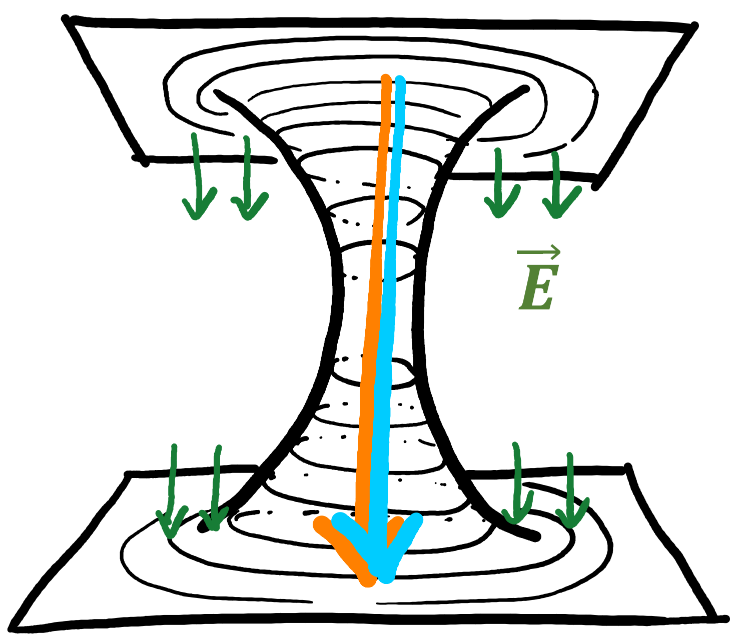

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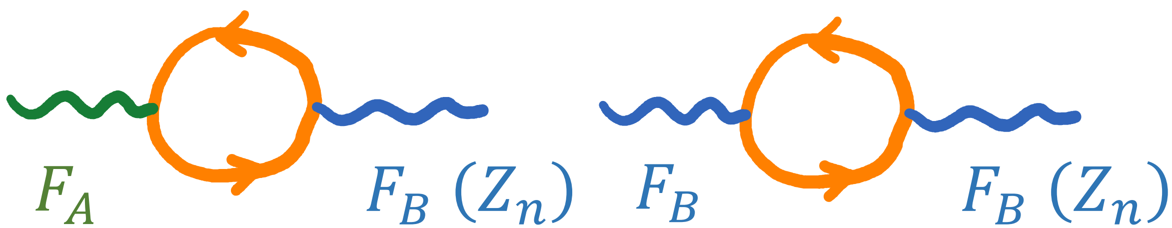

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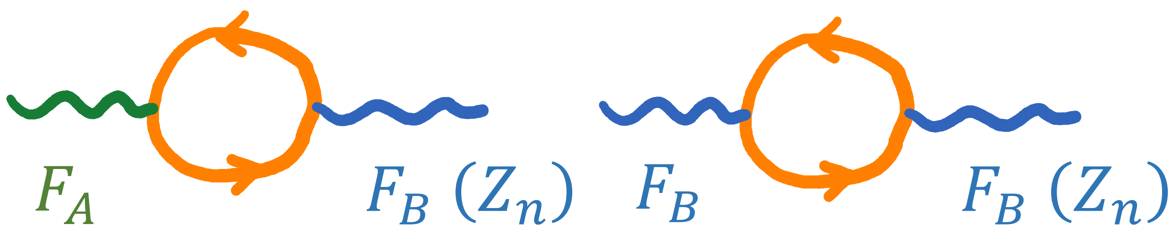

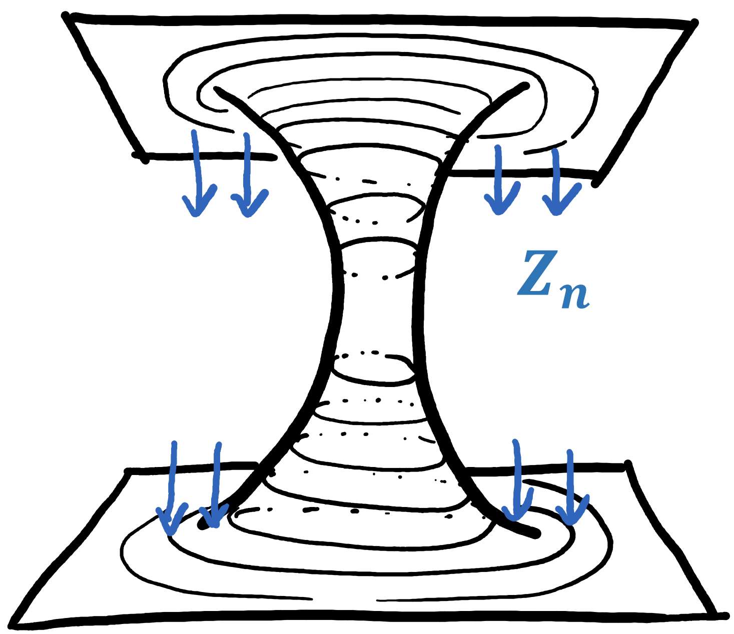

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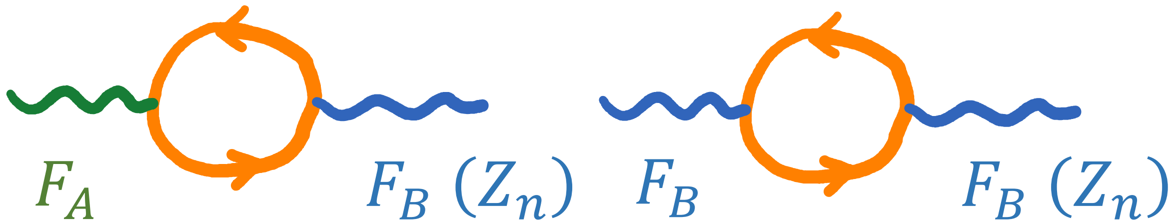

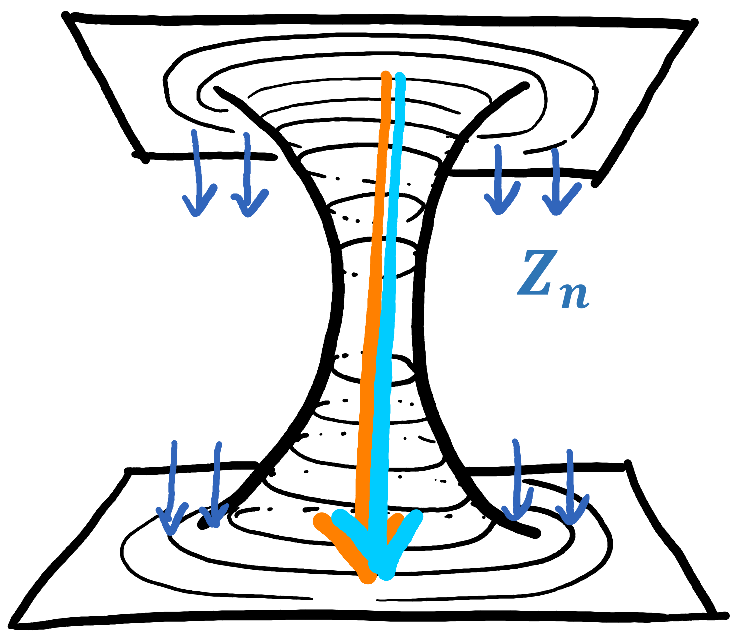

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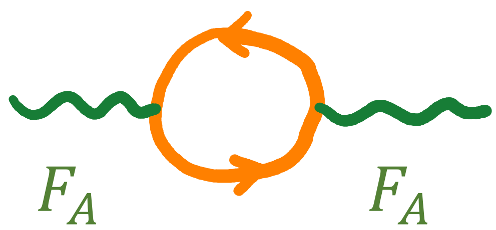

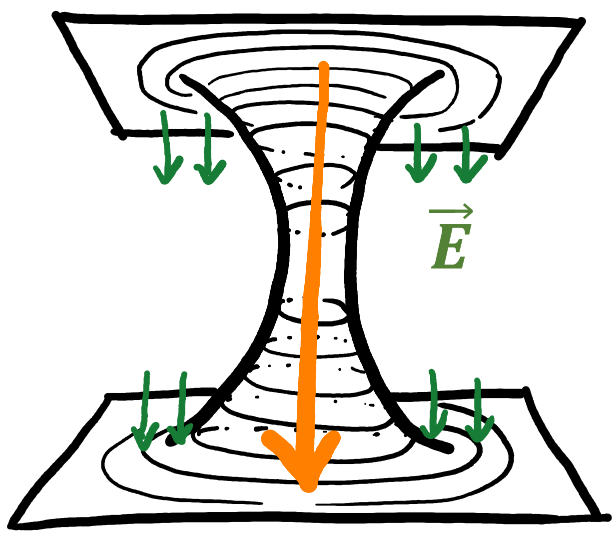

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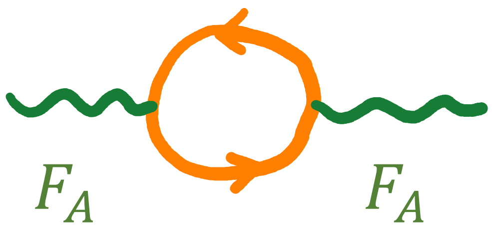

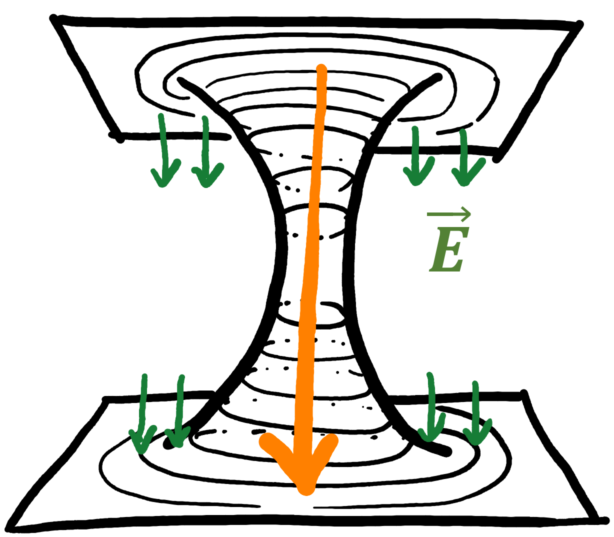

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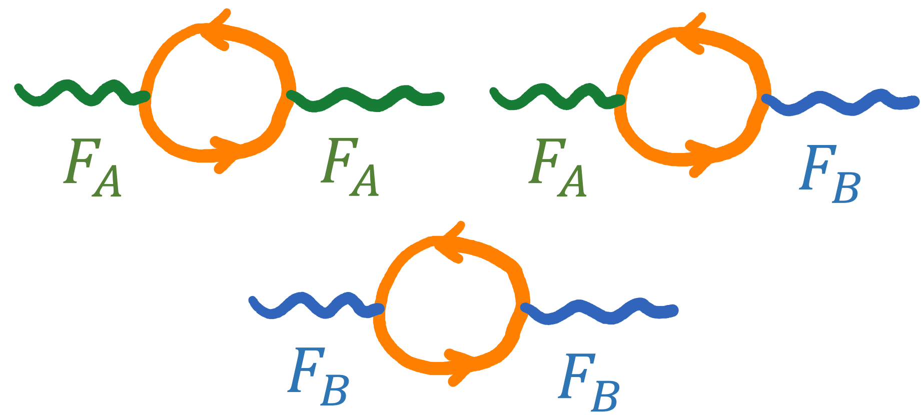

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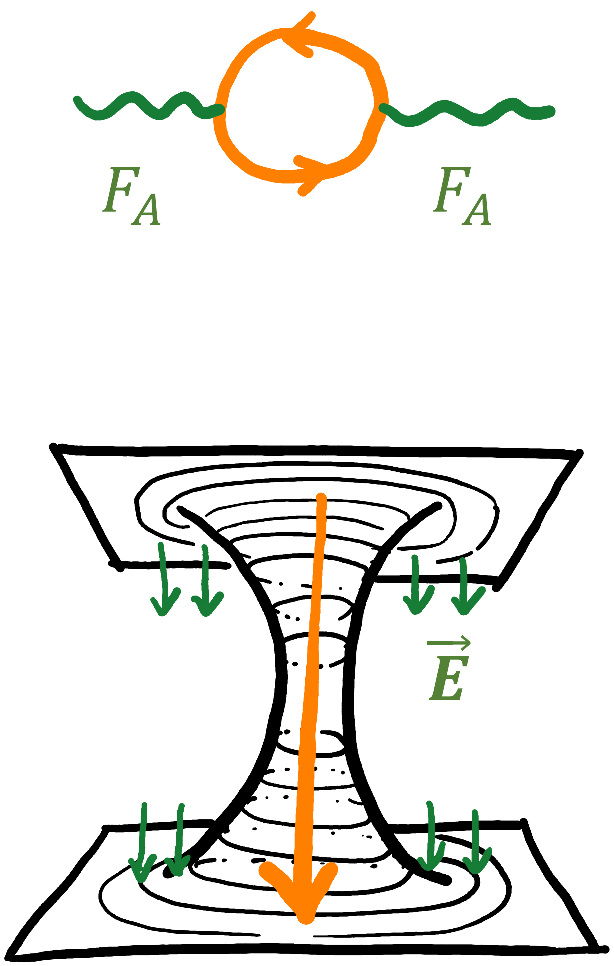

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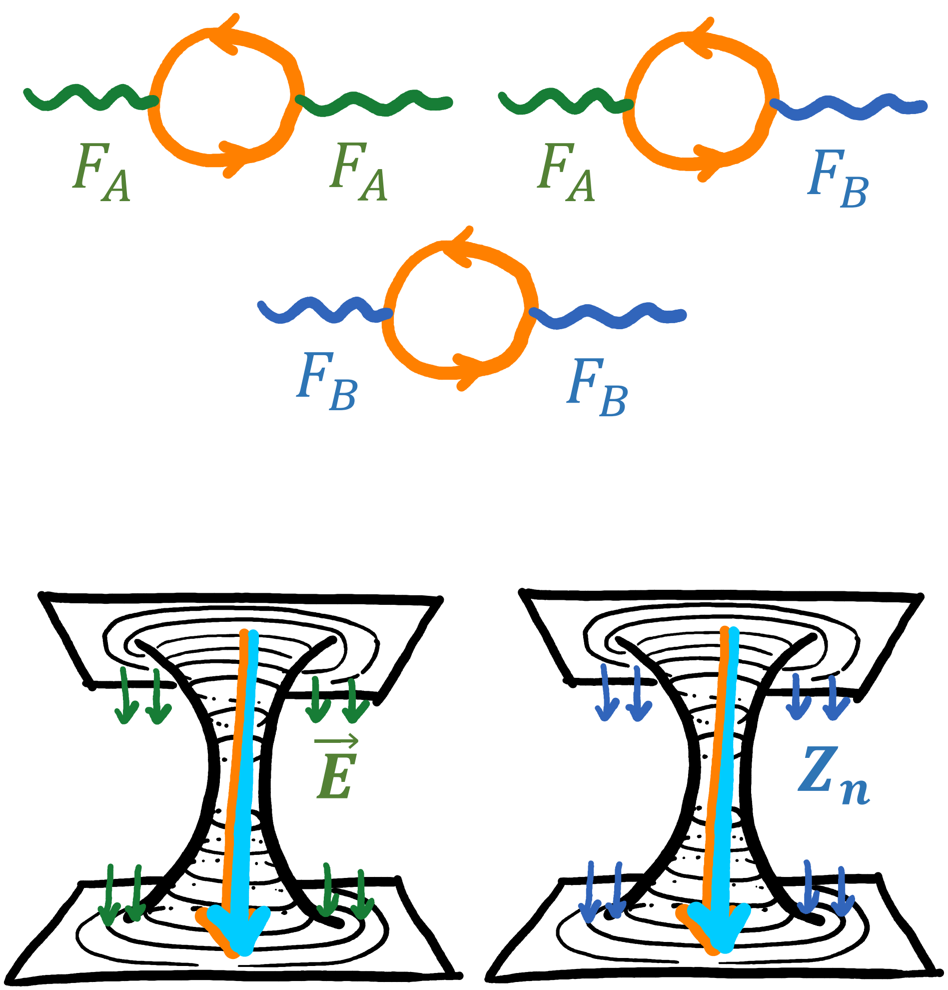

Outline

I. Axion-Maxwell Theory

II. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT
IR-Universal Observables from TQFT-Coupling
Anomaly-Inflow, Fermion Zero Modes

III. TQFT-Coupling 2: Gauging Discrete Subgroup of Axion Shift
Modification of Cosmic String Spectrum

IV. Conclusion

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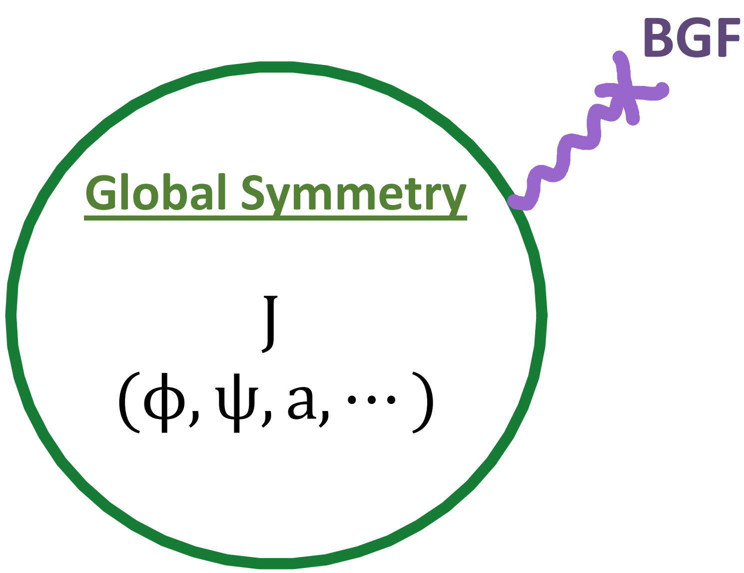
$$J^\mu \rightarrow S \supset -i \int A_\mu J^\mu$$



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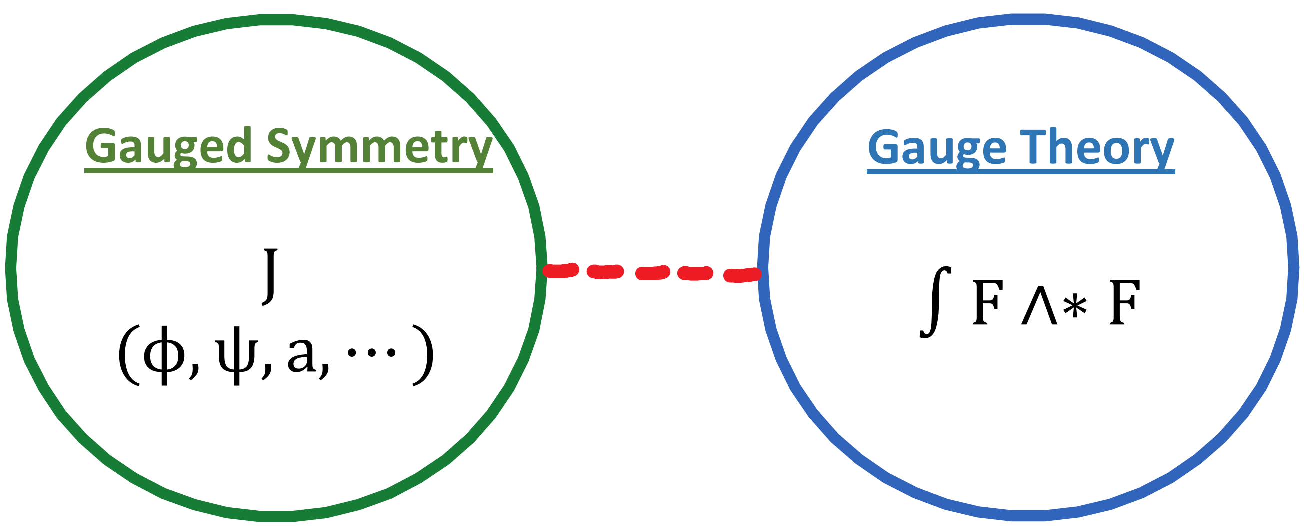

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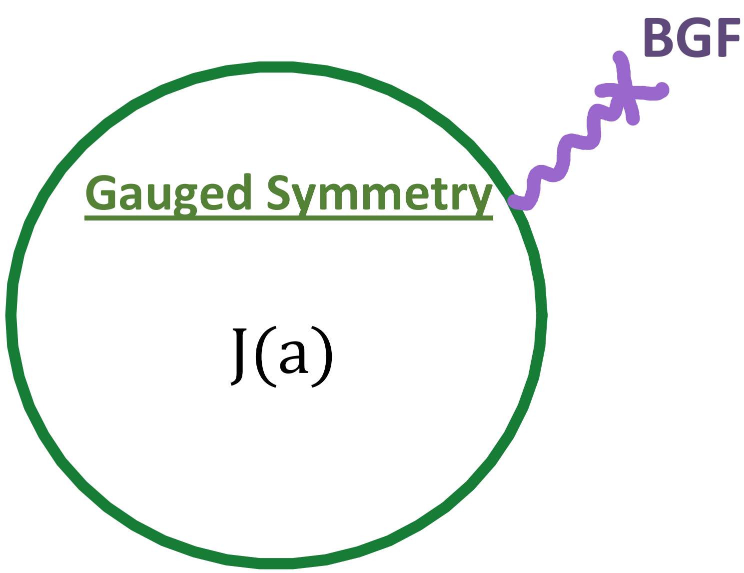

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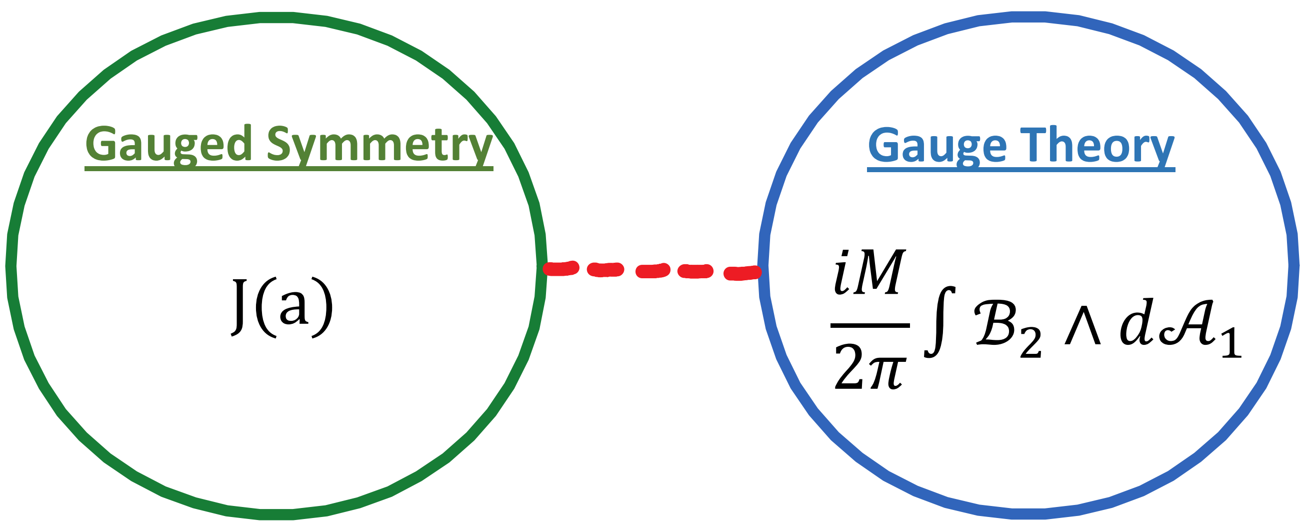

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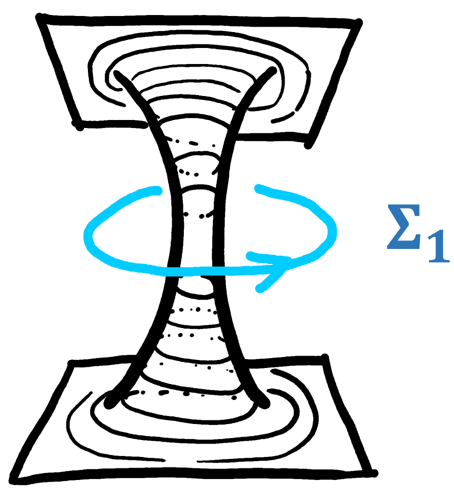

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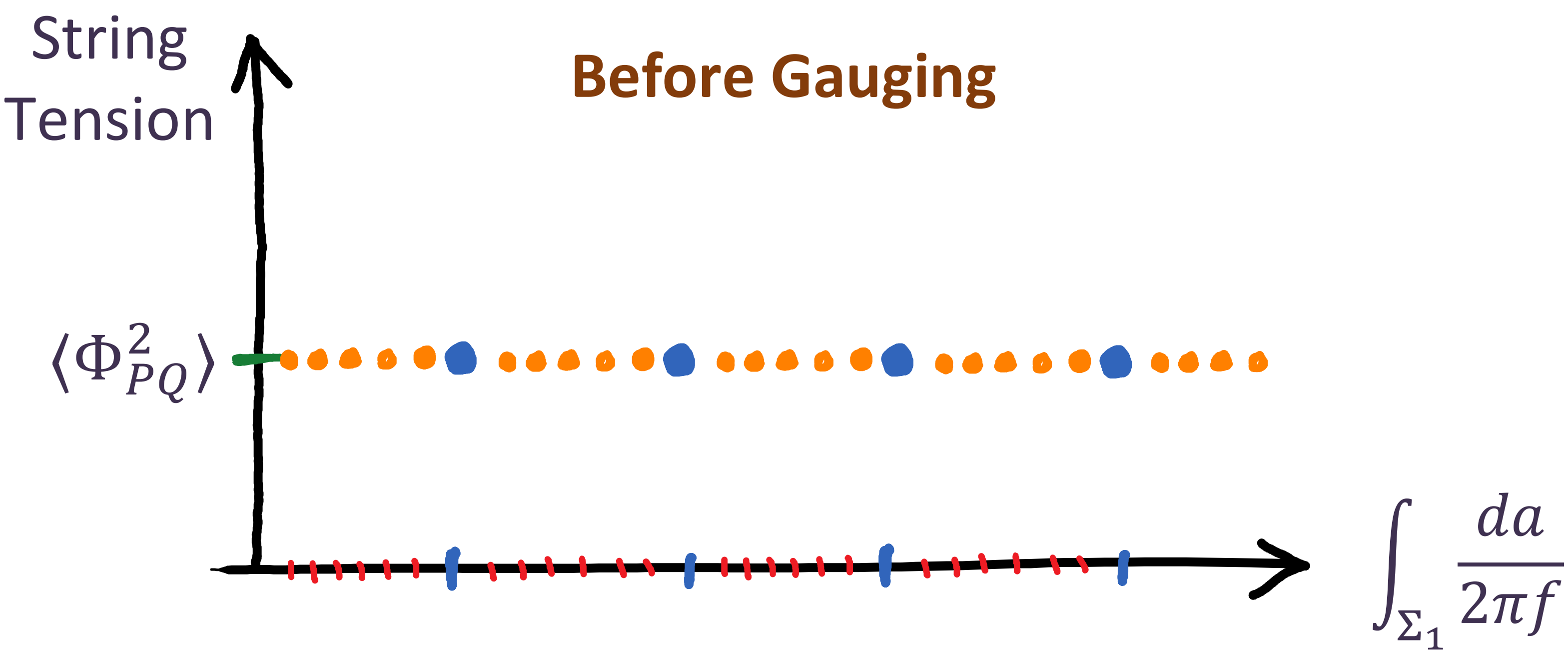

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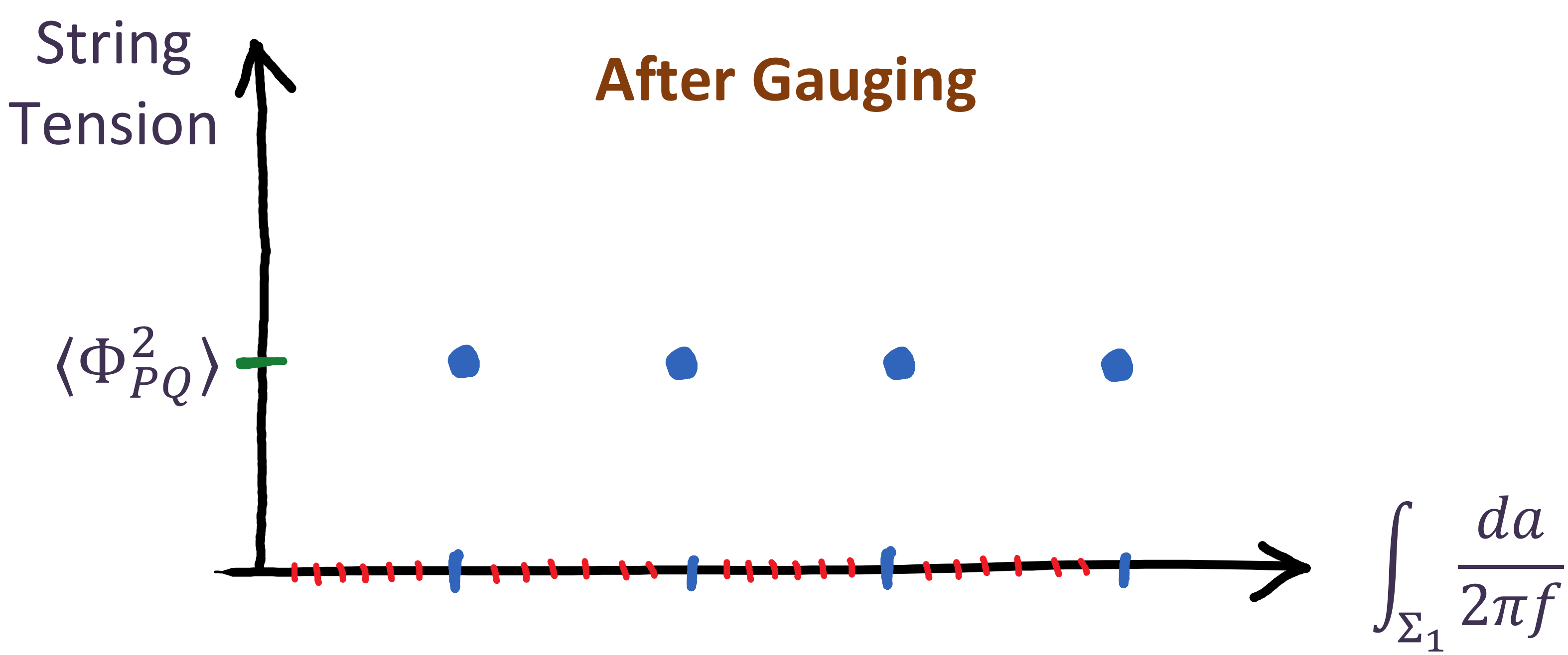

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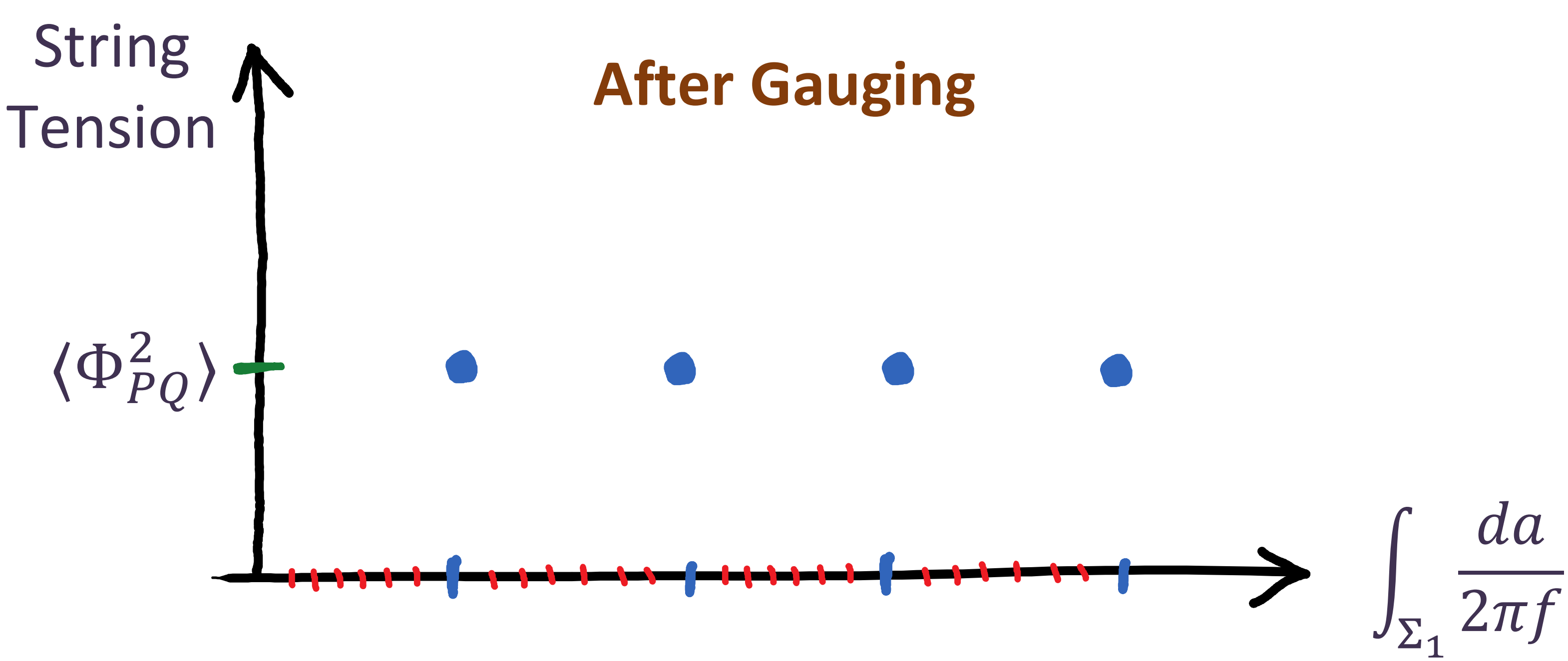

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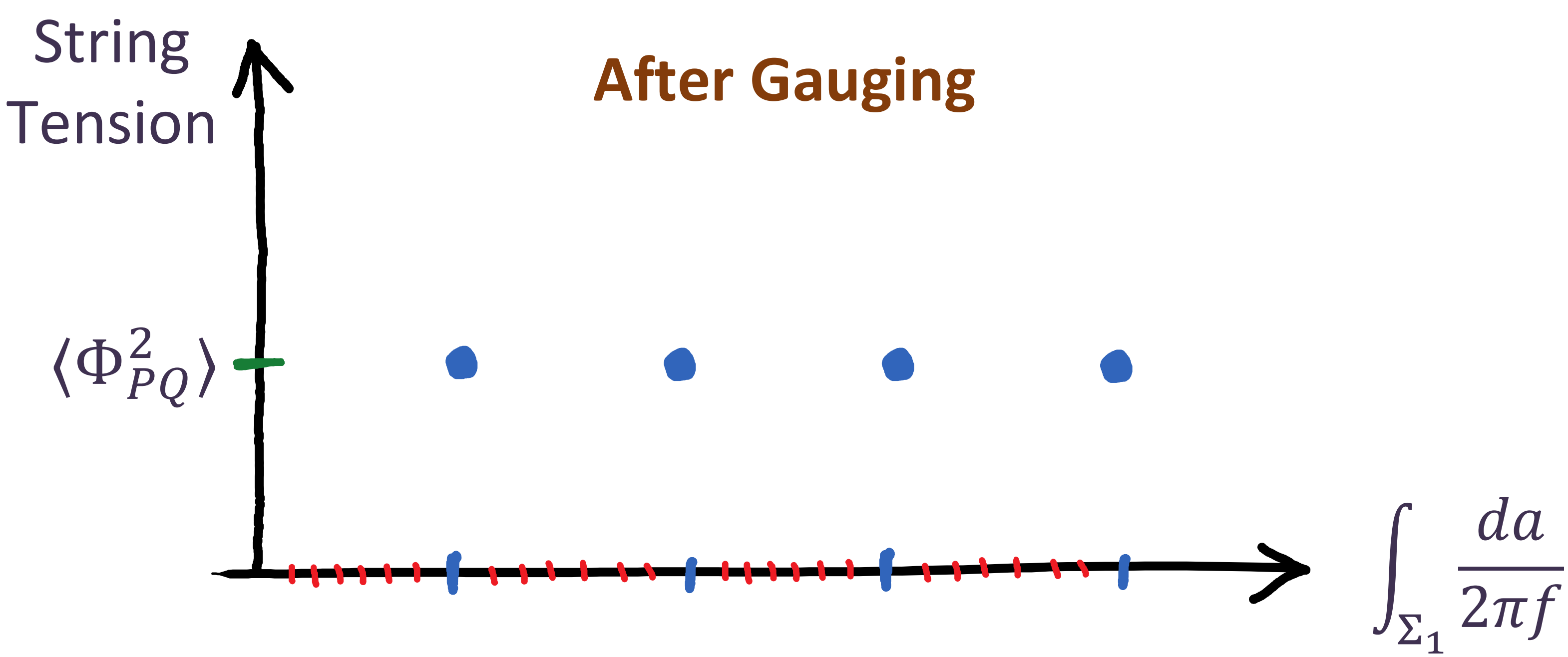

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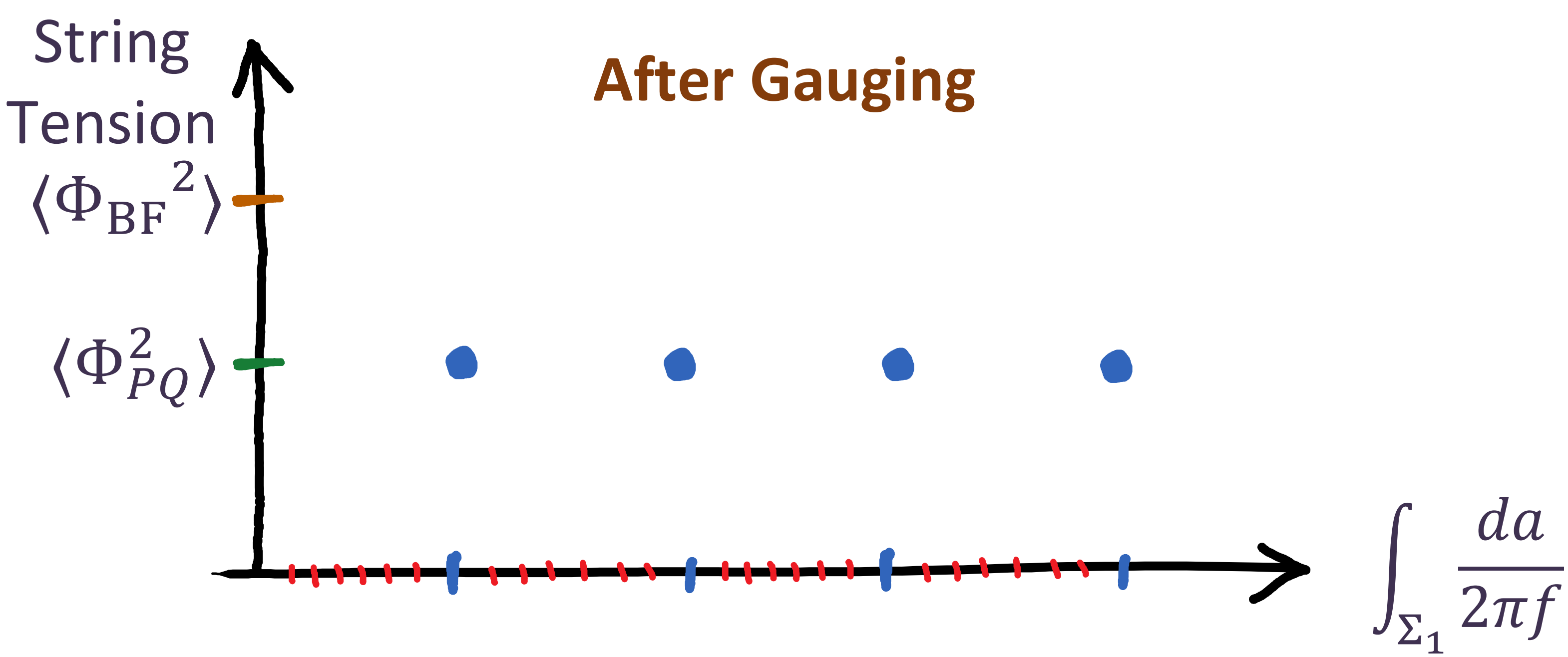

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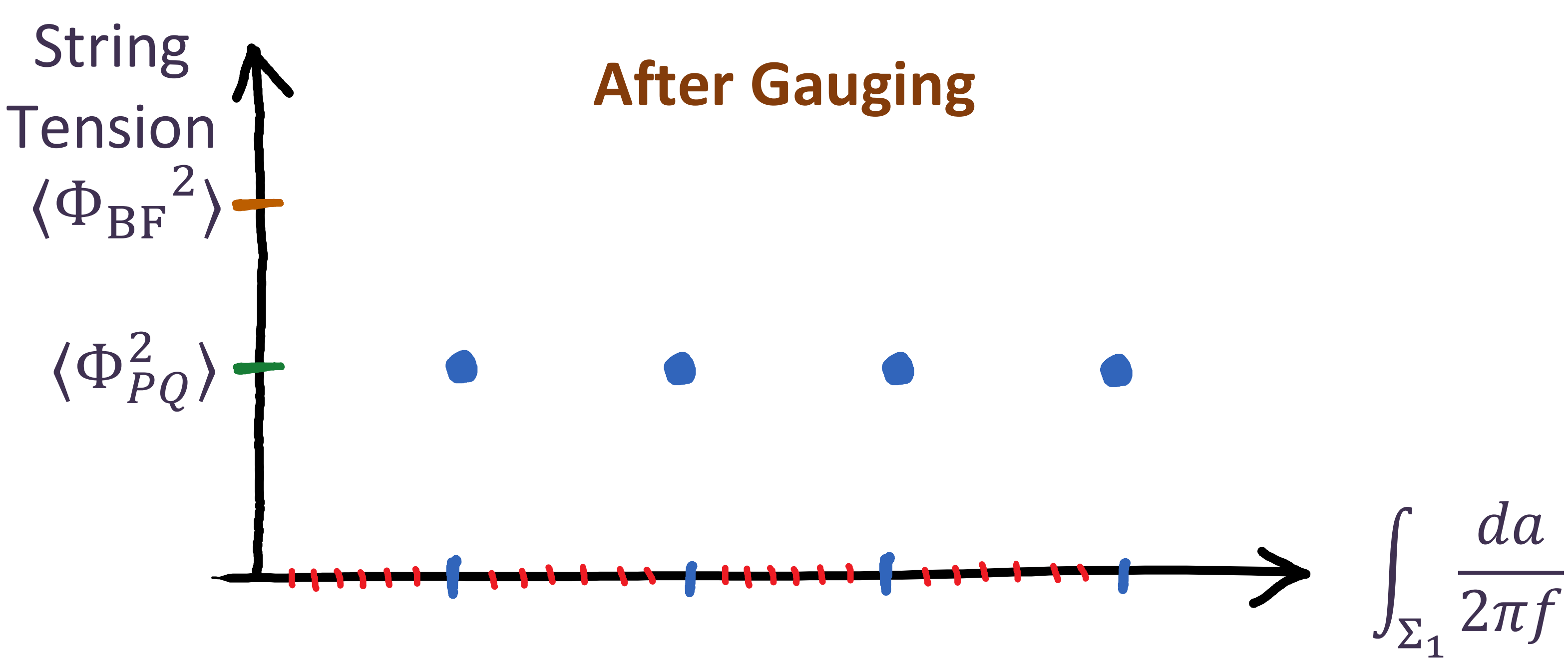
Gauging : Global Cosmic String $\rightarrow$ Local Cosmic String



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Tension $\sim \langle \Phi_{PQ}^2 \rangle$

$\Rightarrow$

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Winding $= 0, \cdots, M-1 \ \ (Z_M)$
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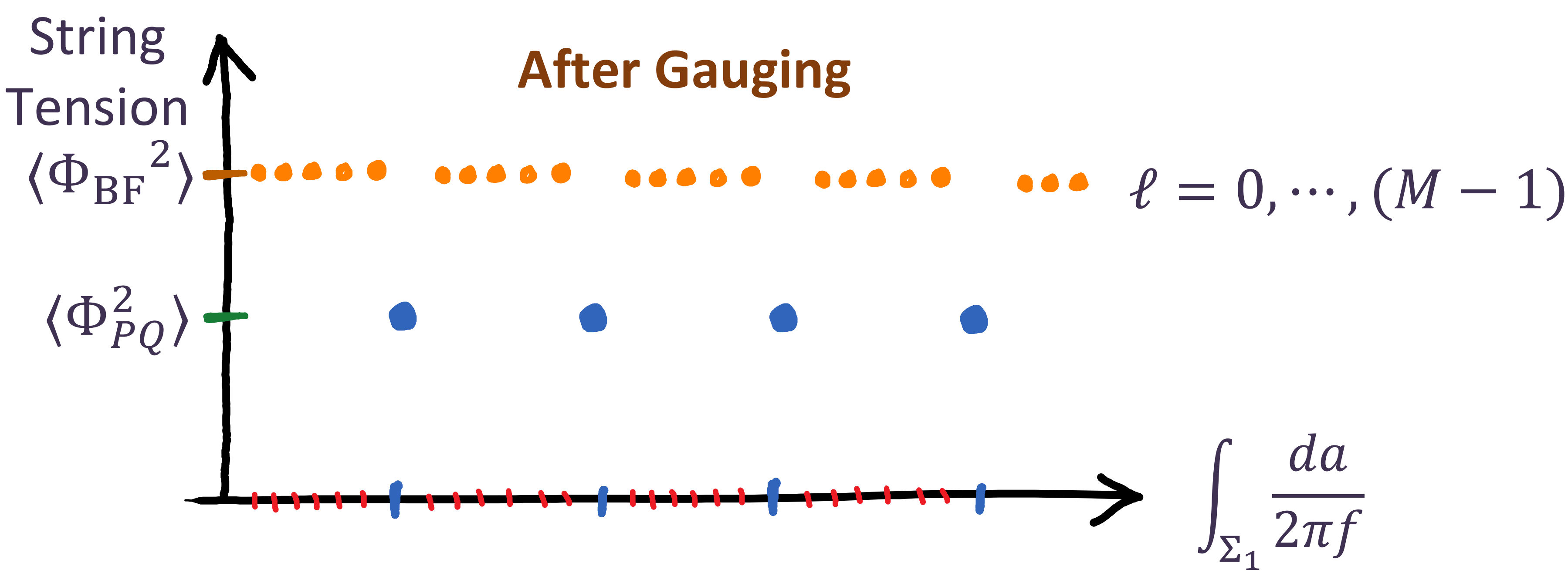

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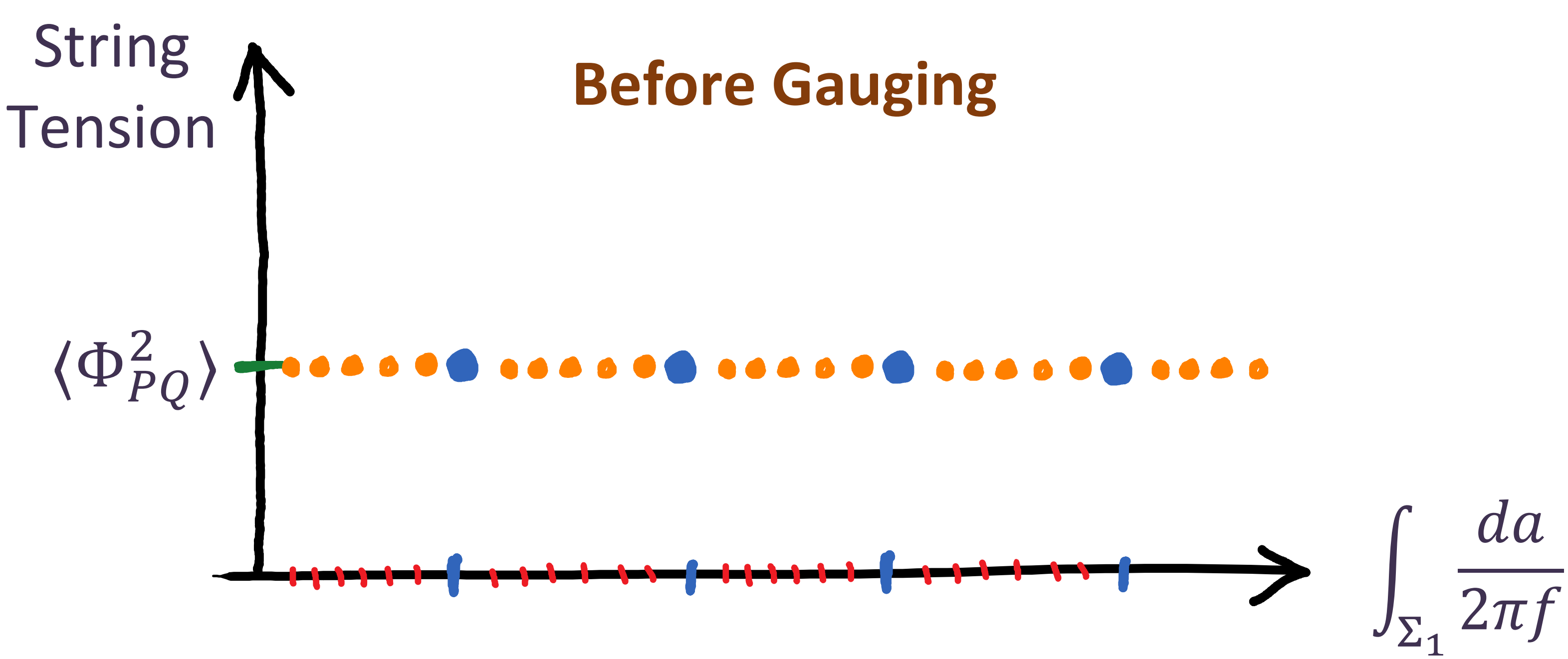

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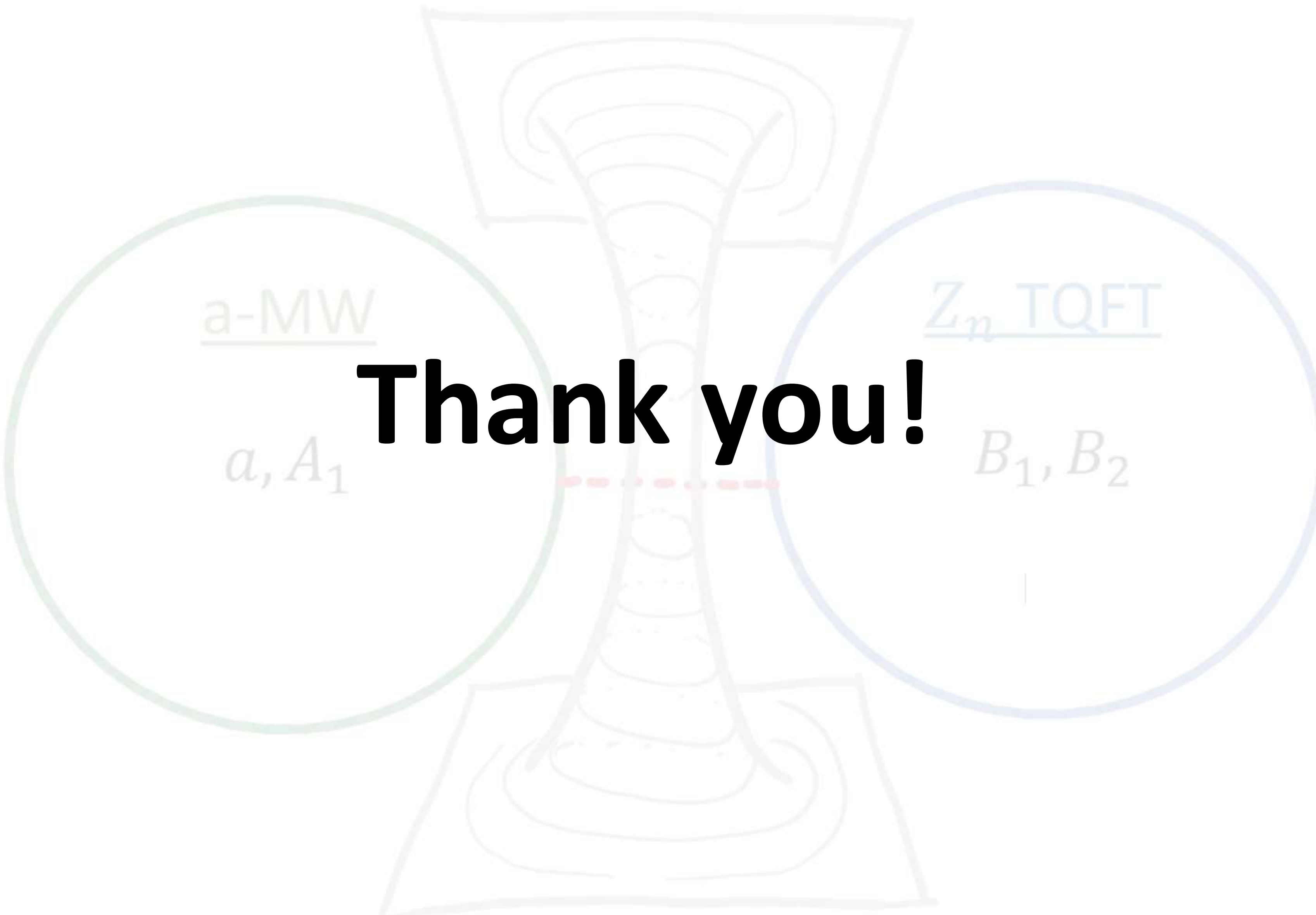
- ✓ Generalized Global Symmetry and their anomalies play the central role!
- ✓ This is just the beginning! (GGS in Particle Physics?)
  - ➢ Applications: e.g. **DM** charged under topological Z_n force
  - ➢ Wealth of **cosmic string physics**
    - > Axion-string + BF-string + Composite-string
    - > Non-Kibble mechanism for composite string production?
    - > Gravitational Waves
  - ➢ Axion-YM (QCD) coupled to a TQFT (on-going) : very rich!
  - ➢ Non-invertible symmetries

Thank you!



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- $Z_M{}^{(0)} \subset Z_K{}^{(0)}$ gauging
- This is gauging of 0-form (acts on local $a(x)$) and non-linearly realized discrete symmetry
  - $(\mathrm{d}\Phi - \mathrm{A}\Phi)^2$ vs $(\mathrm{d}\varphi - \mathrm{A})^2$
- Still good notion of axion-local-fluctuations (vs AHM)