

# QCD spectroscopy



**RAÚL BRICEÑO**

✉ [rbriceno@odu.edu](mailto:rbriceno@odu.edu)

🌐 <http://bit.ly/rbricenoPhD>

🐦 @RaulBriceno12



# QCD spectroscopy

*Amplitude analysis*

QCD

*Experiments*



CLAS12



BESIII



# QCD spectroscopy

Amplitude analysis

GOAL:  
*Get insights to the governing patterns and rules of QCD from emergent phenomena*

Observables to test our understanding:

- Purely hadronic scattering,
- Production and decay,
- Exotic states,
- ...

Possible outcomes:

- Source of masses,
- Role of glue,
- Structure of excited states,
- ...

Experiments

QCD

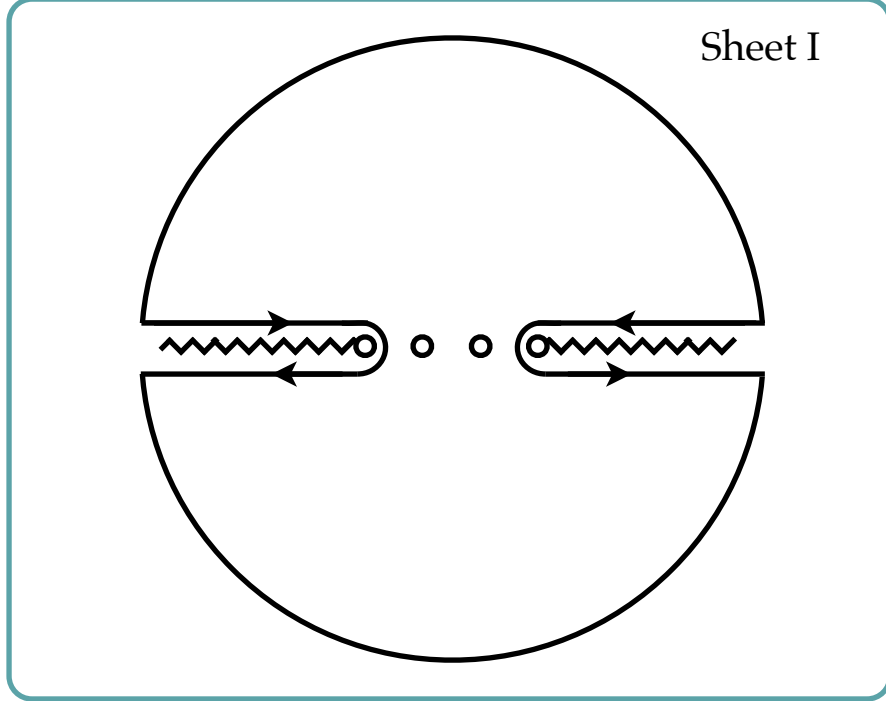
$$|n\rangle_{\text{QCD}} = c_0 \text{ (gluon ball) } + c_1 \text{ (quark-antiquark pair) } + c_2 \text{ (quark-gluon) } + c_3 \text{ (quark-antiquark-gluon) } + c_4 \text{ (quark-antiquark-gluon-gluon) } + \dots$$

... perhaps there is a hierarchy [.e.g.  $c_0 > c_1 > c_2 > c_3 > c_4$ ]

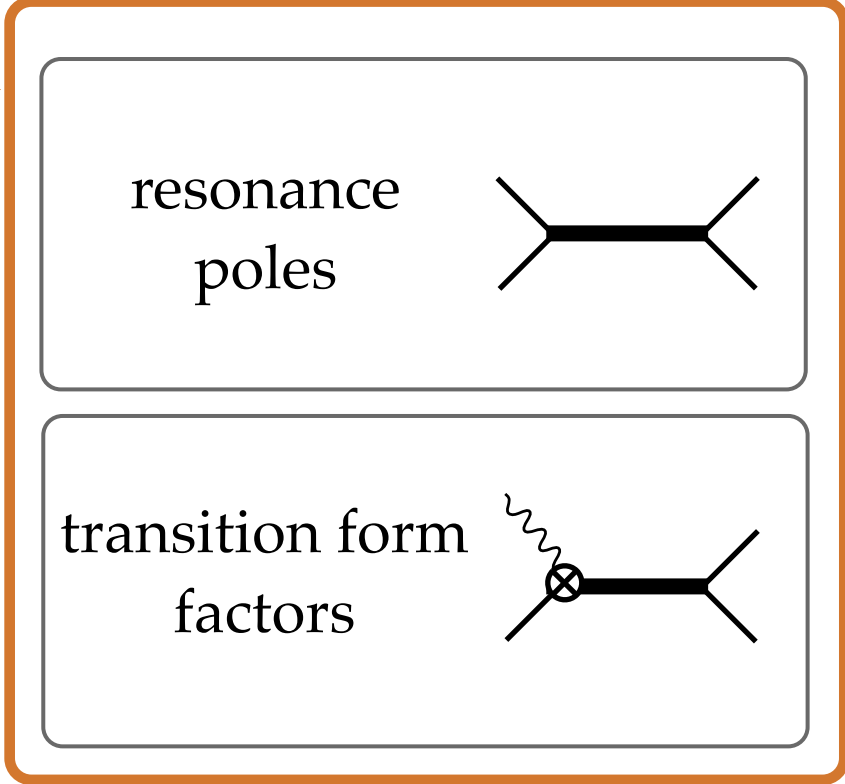
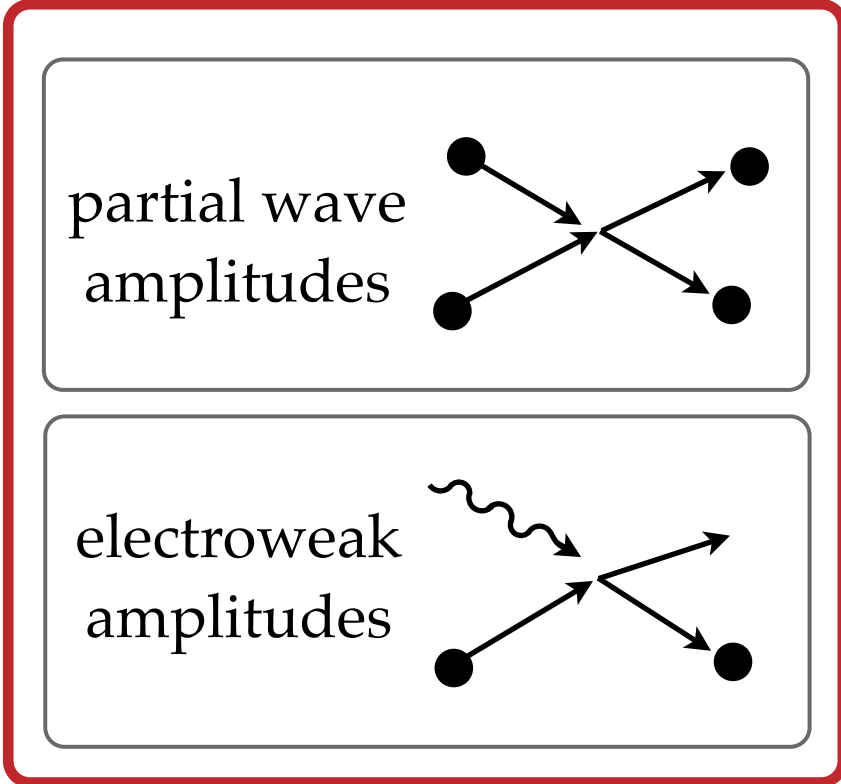
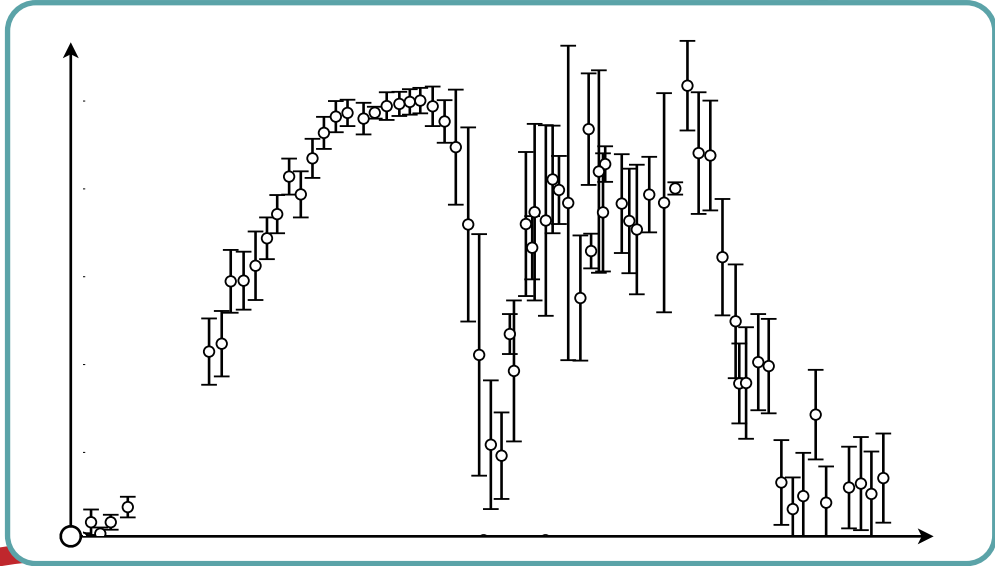
# QCD spectroscopy

QCD

Amplitude analysis

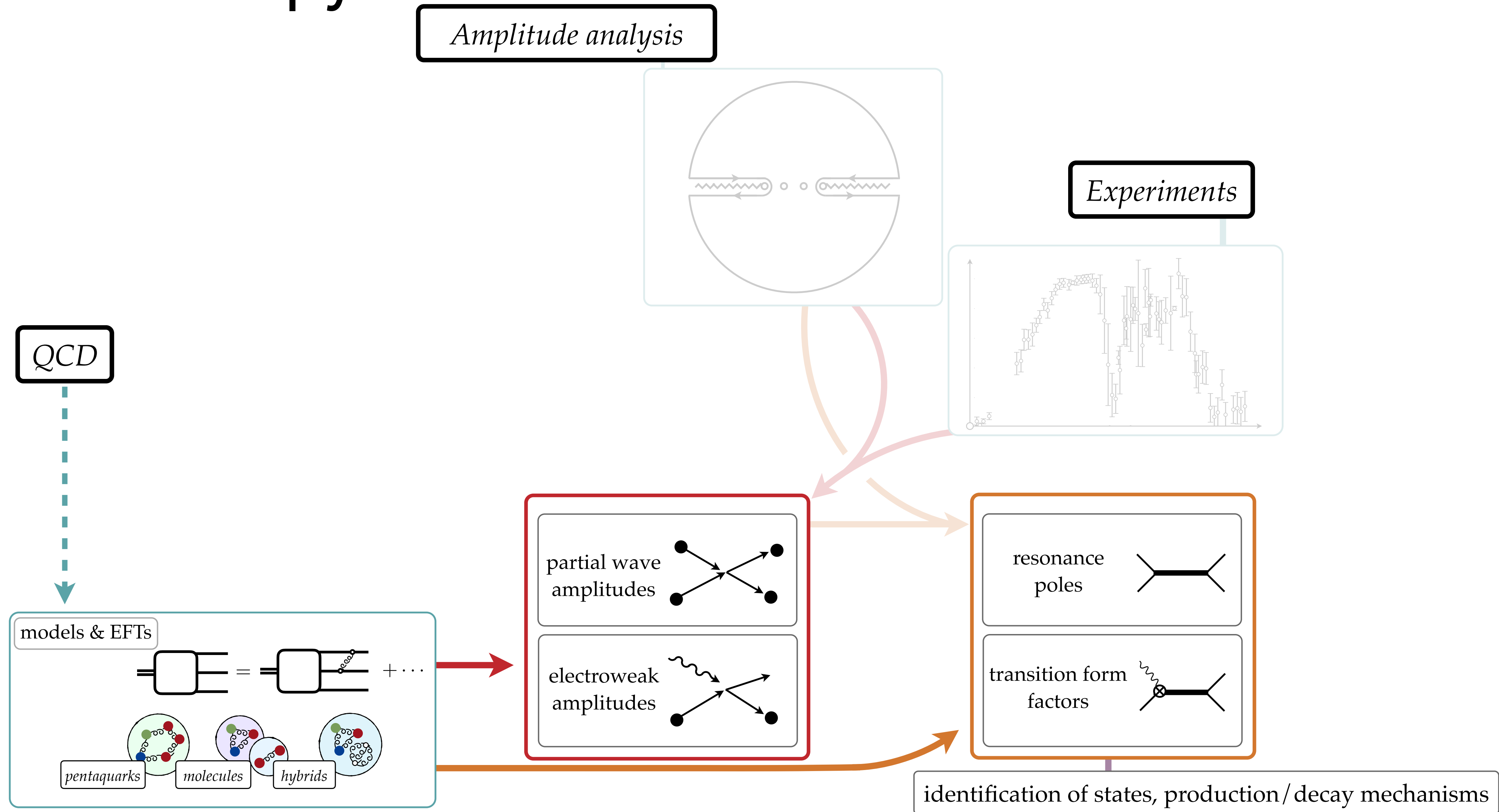


Experiments

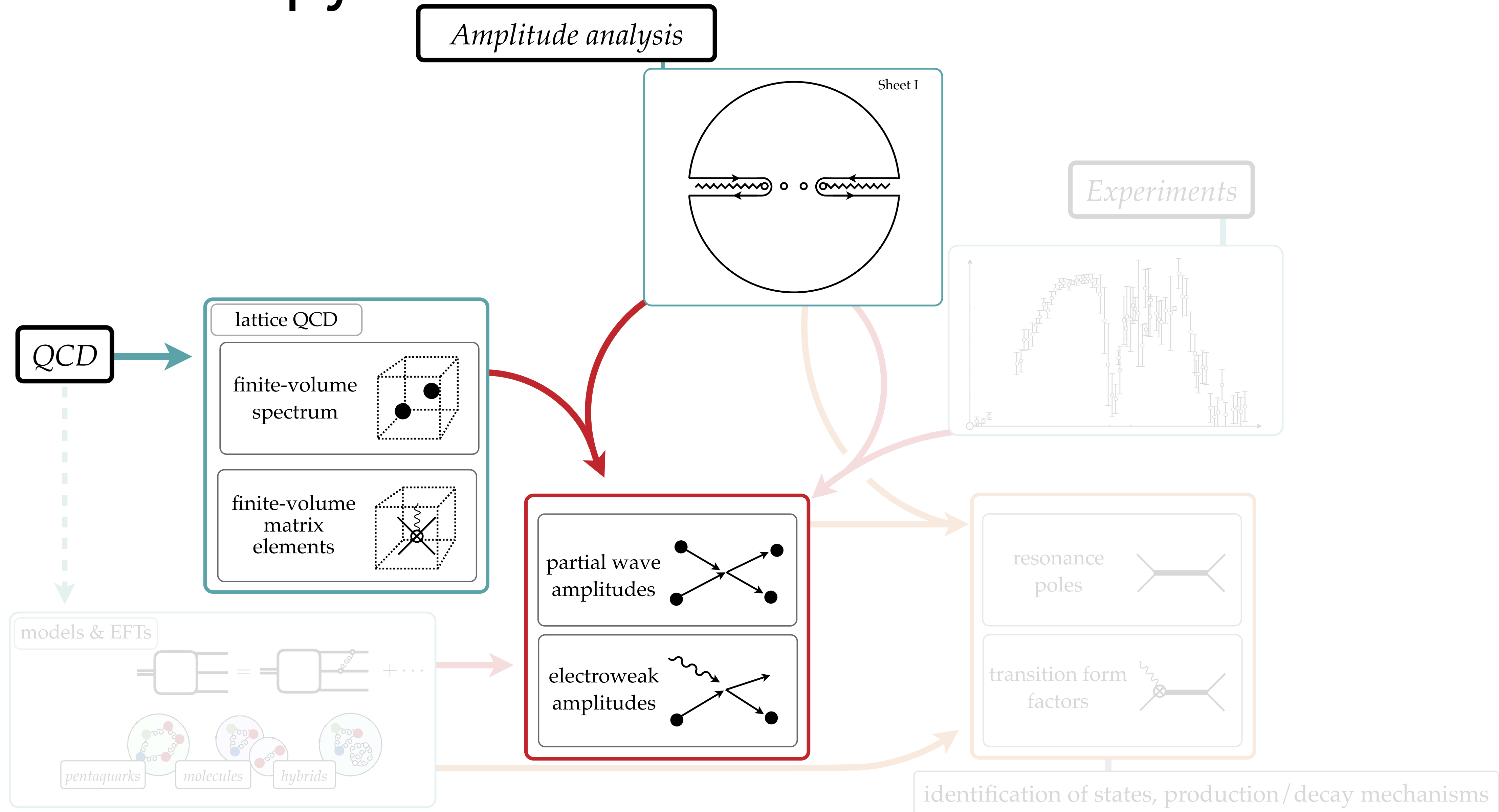


identification of states, production/decay mechanisms

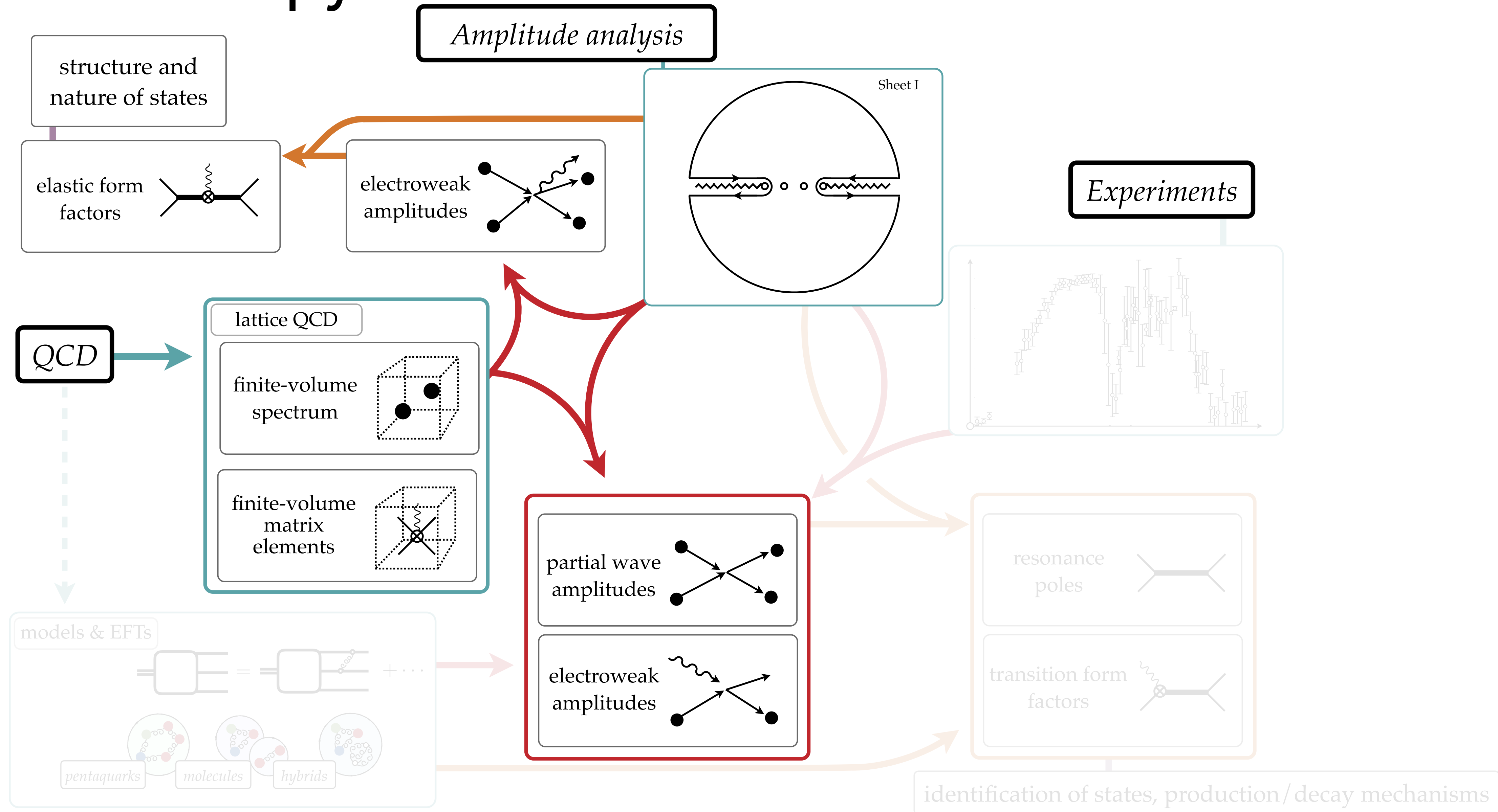
# QCD spectroscopy



# QCD spectroscopy



# QCD spectroscopy



# 2+1 minimum requirements

Two “musts” for few-body systems:

☑ Generalized eigenvalue problem (GEVP),

☑ large basis of ops,

$$\mathcal{O}_b \sim \bar{q} \Gamma_b q, \pi\pi, K\bar{K}, \dots, 3\pi, \dots$$

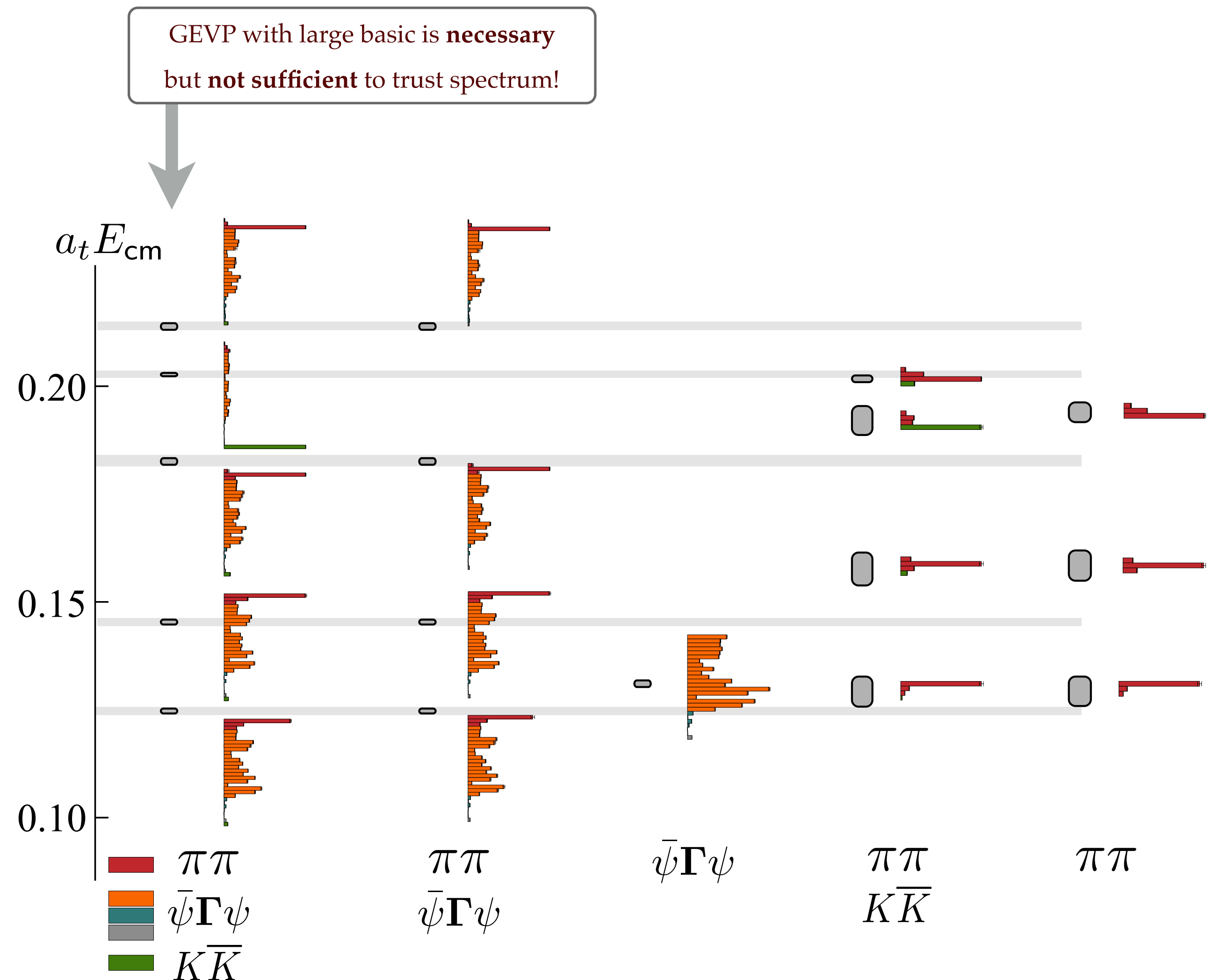
☑ diagonalization,

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$$

☑ Finite-volume formalisms.

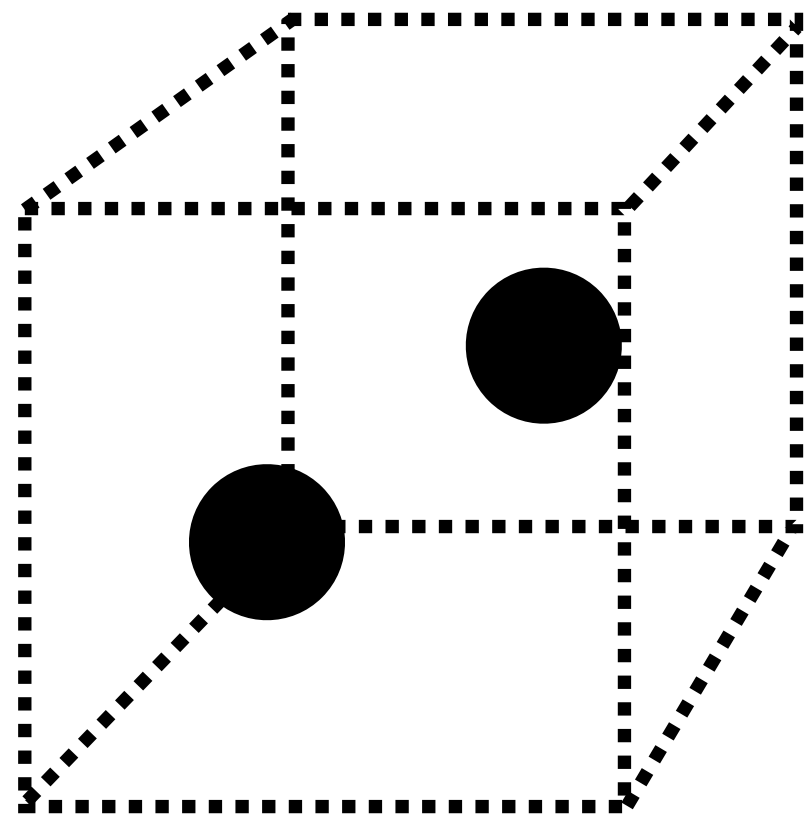
One powerful tool to make GEVP practical:

☑ Distillation [Peardon, *et al.* (HadSpec, 2009)].





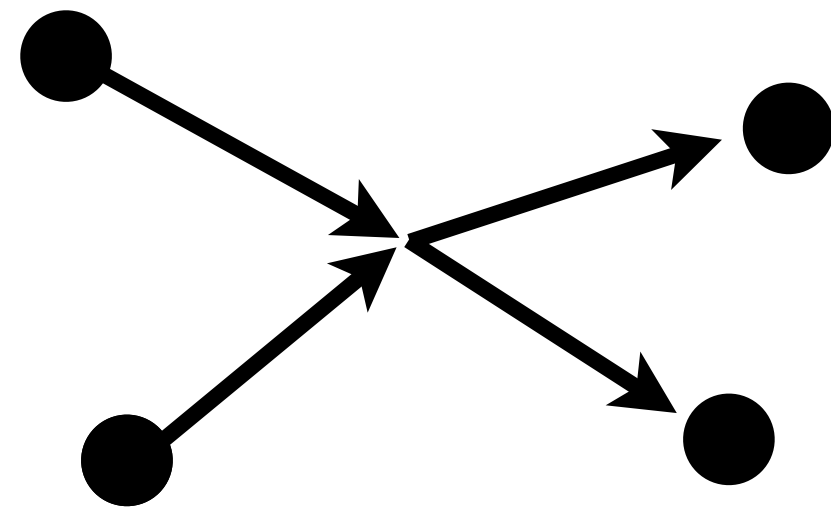
# Two-hadron systems



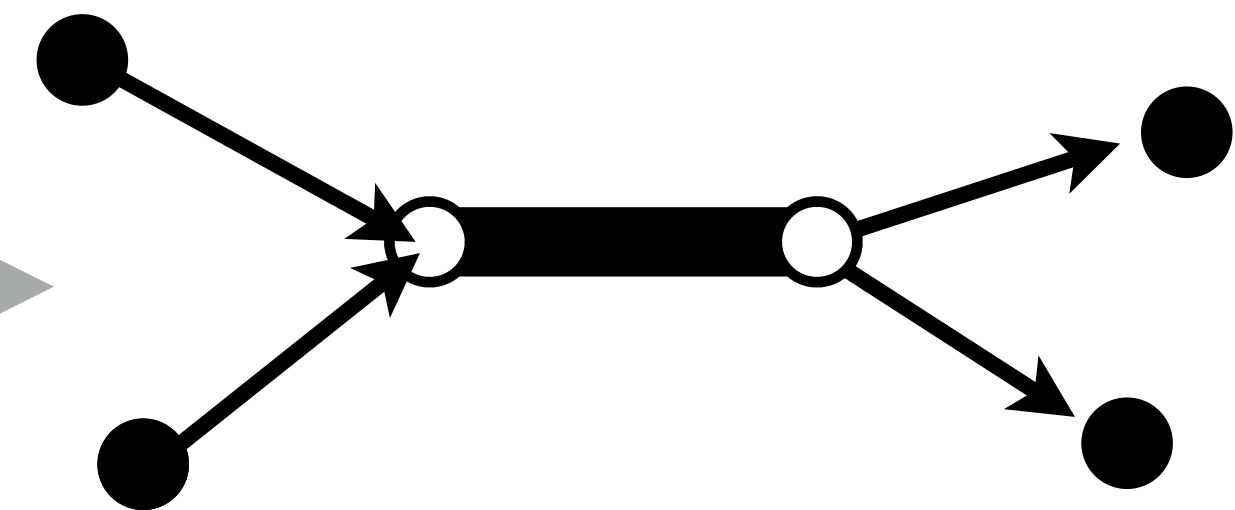
finite-volume  
spectroscopy



Lüscher (1986, 1991)  
Rummukainen & Gottlieb (1995)  
Kim, Sachrajda, & Sharpe (2005)  
Christ, Kim & Yamazaki (2005)  
Feng, Li, & Liu (2004)  
Hansen & Sharpe (2021)  
RB & Davoudi (2012)  
RB (2014)



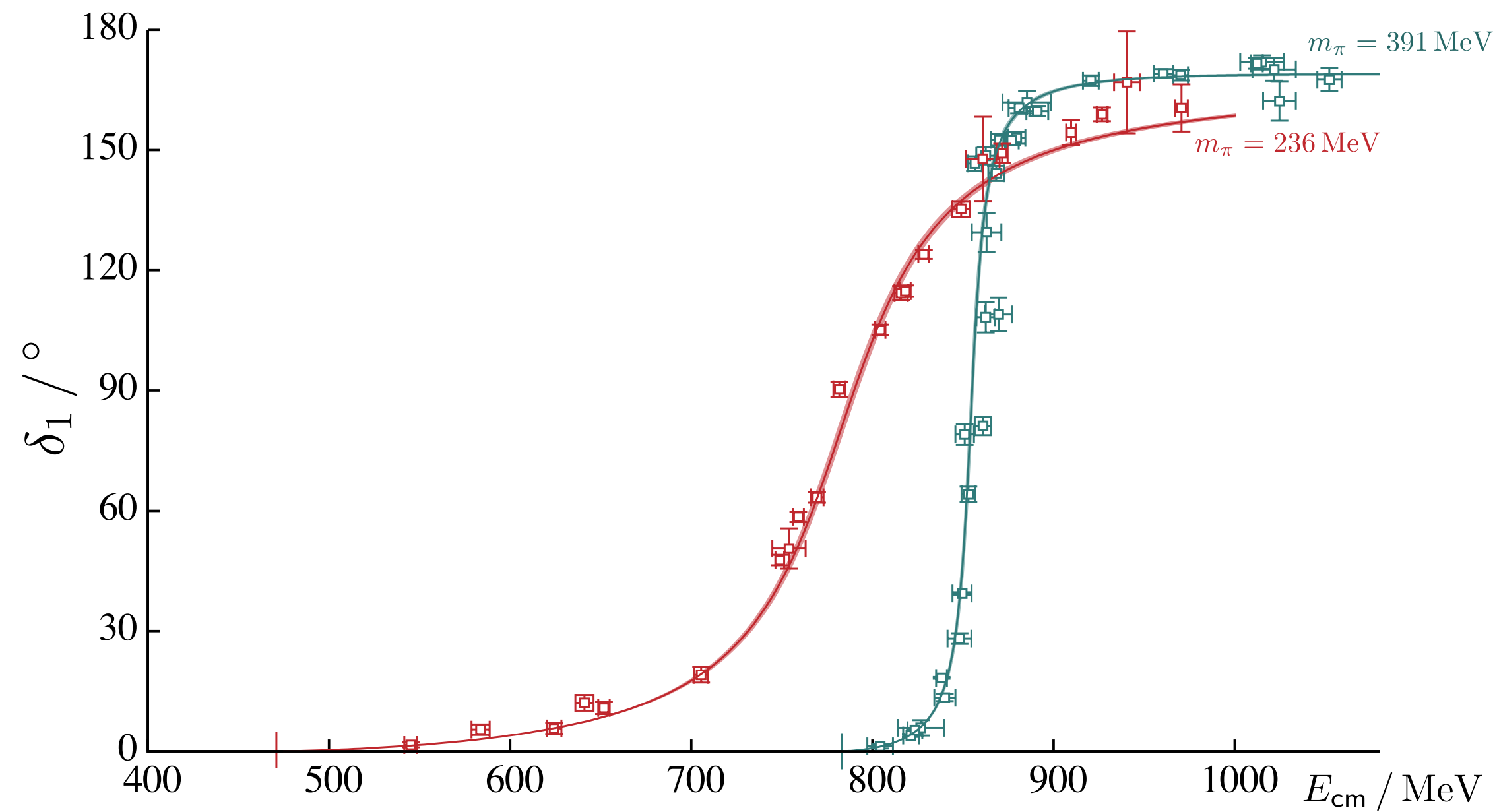
infinite-volume  
scattering amplitudes



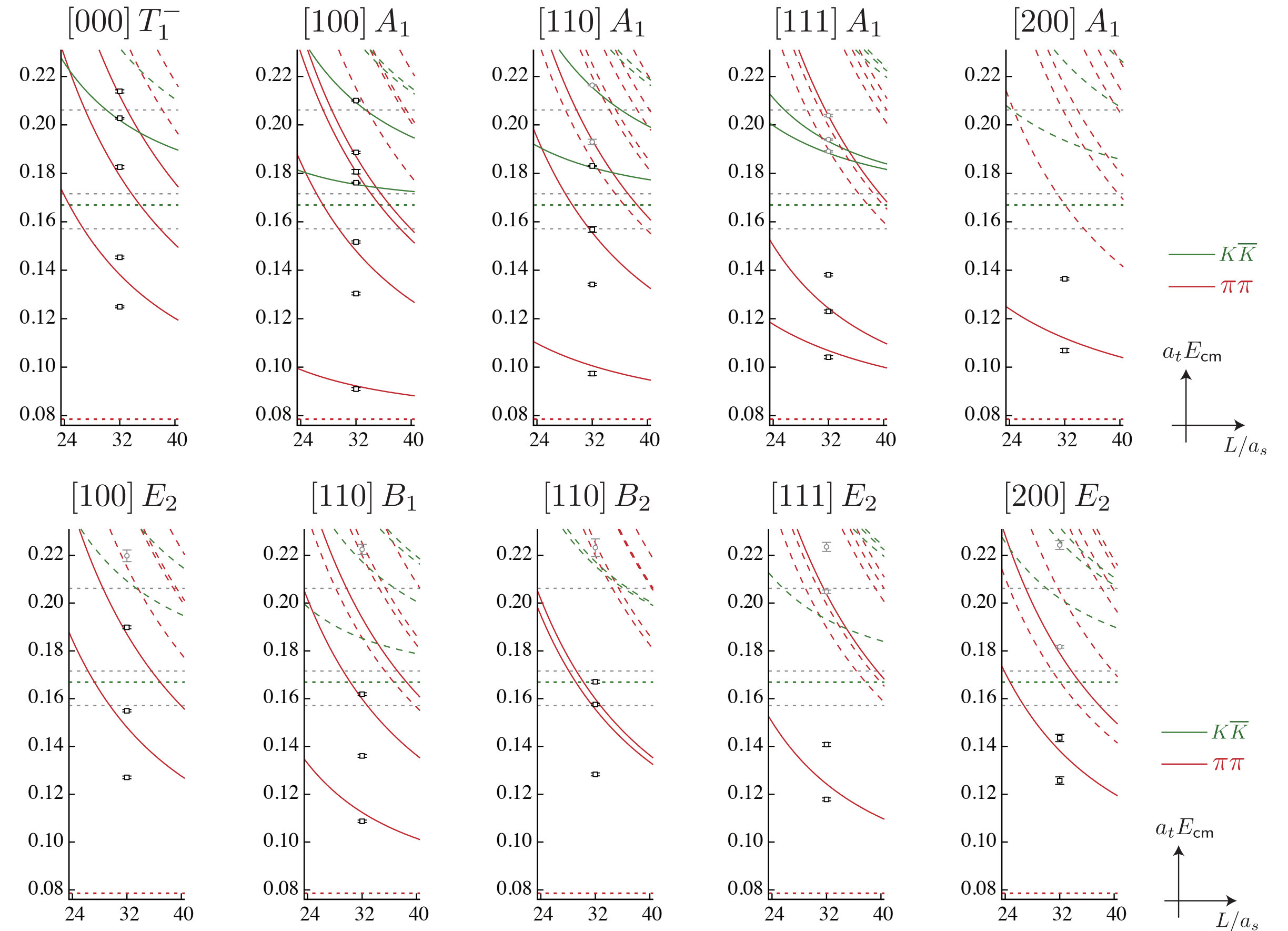
bound state and  
resonance poles

# $\pi\pi$ scattering

( $l=1$  channel)

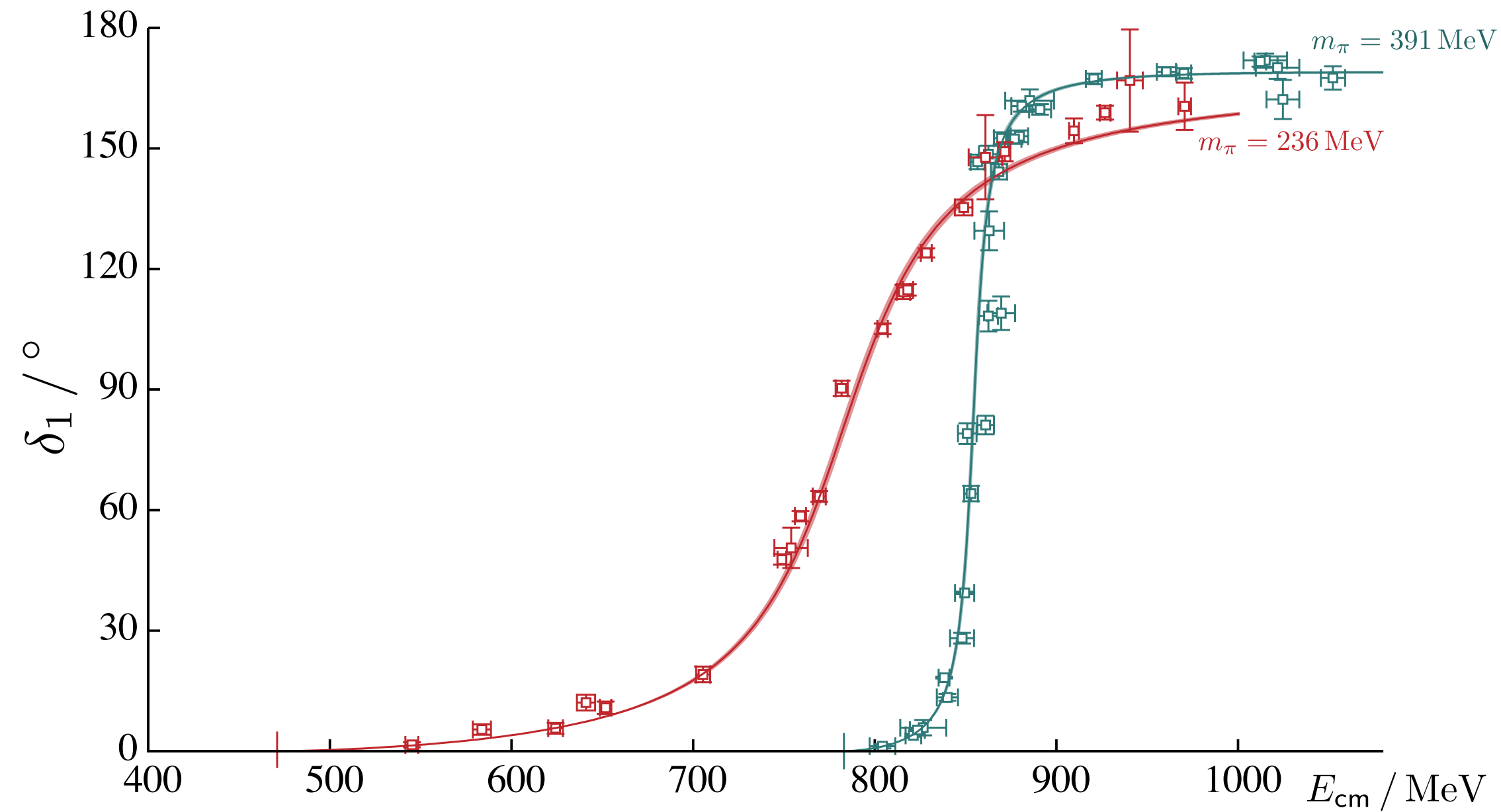


$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

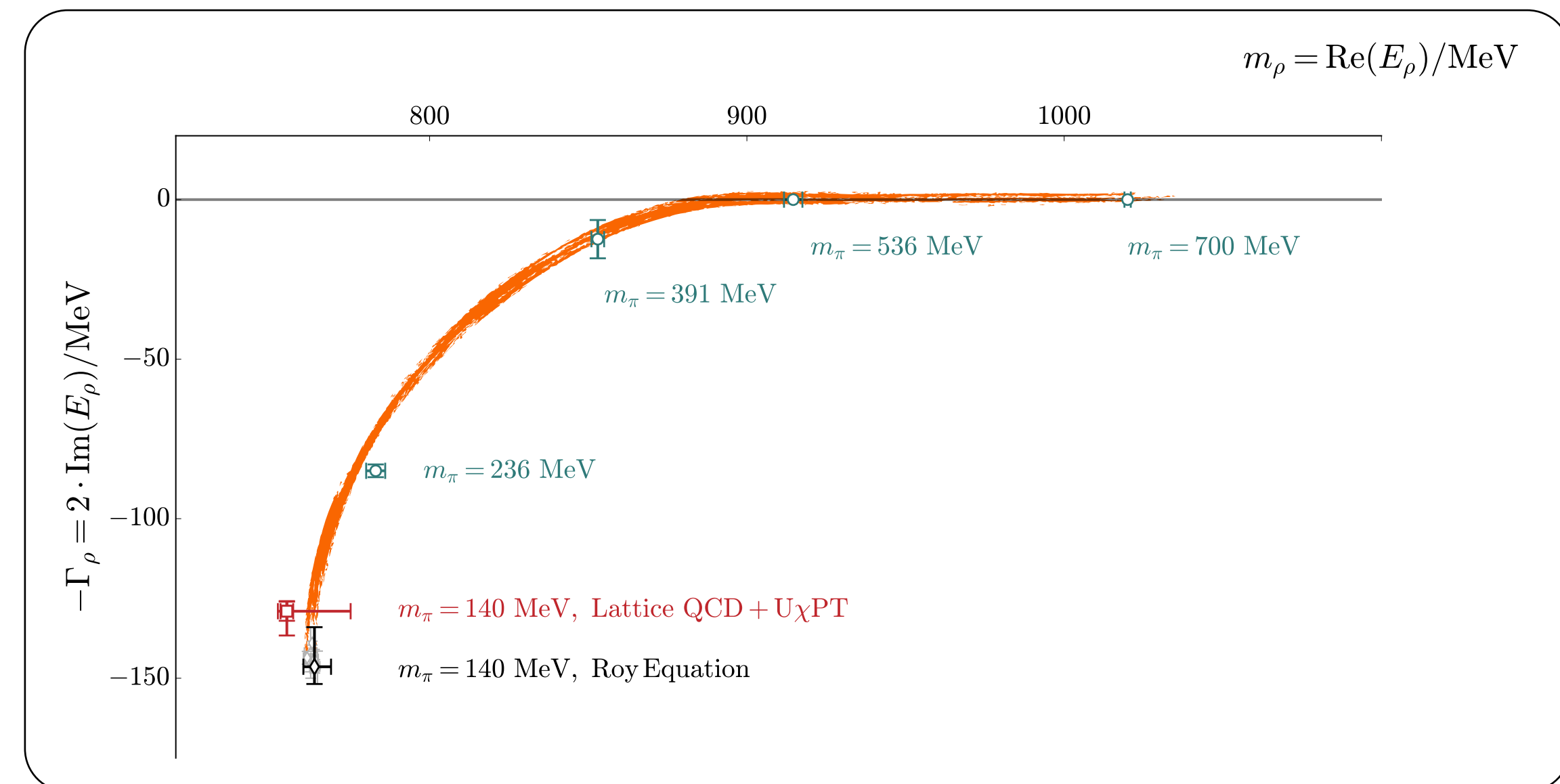
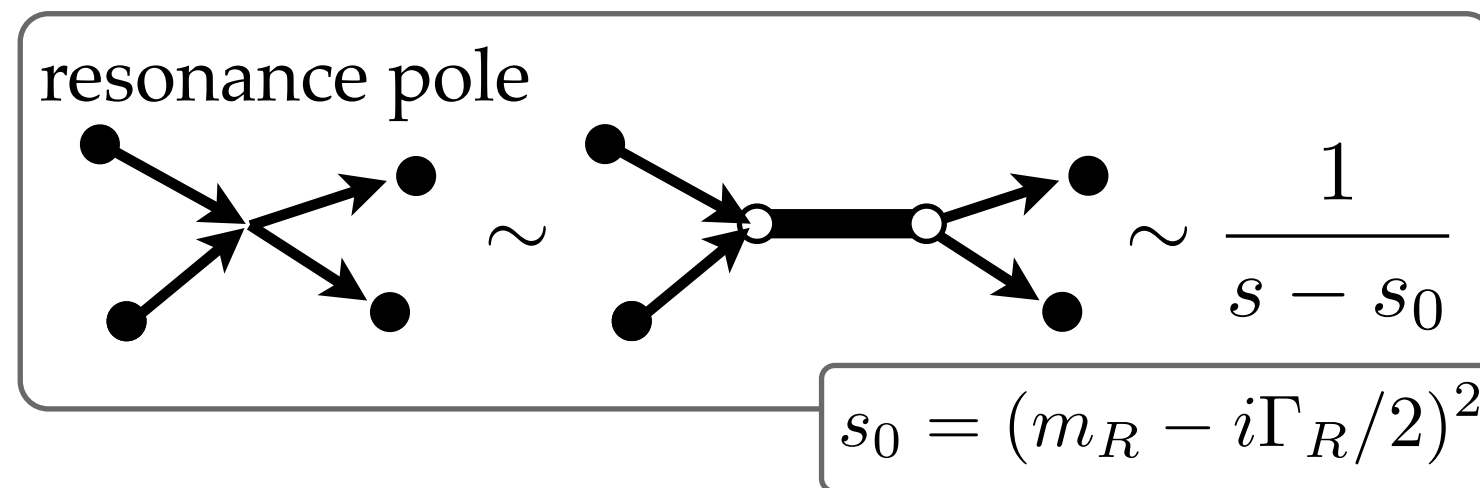


# $\pi\pi$ scattering

( $l=1$  channel)

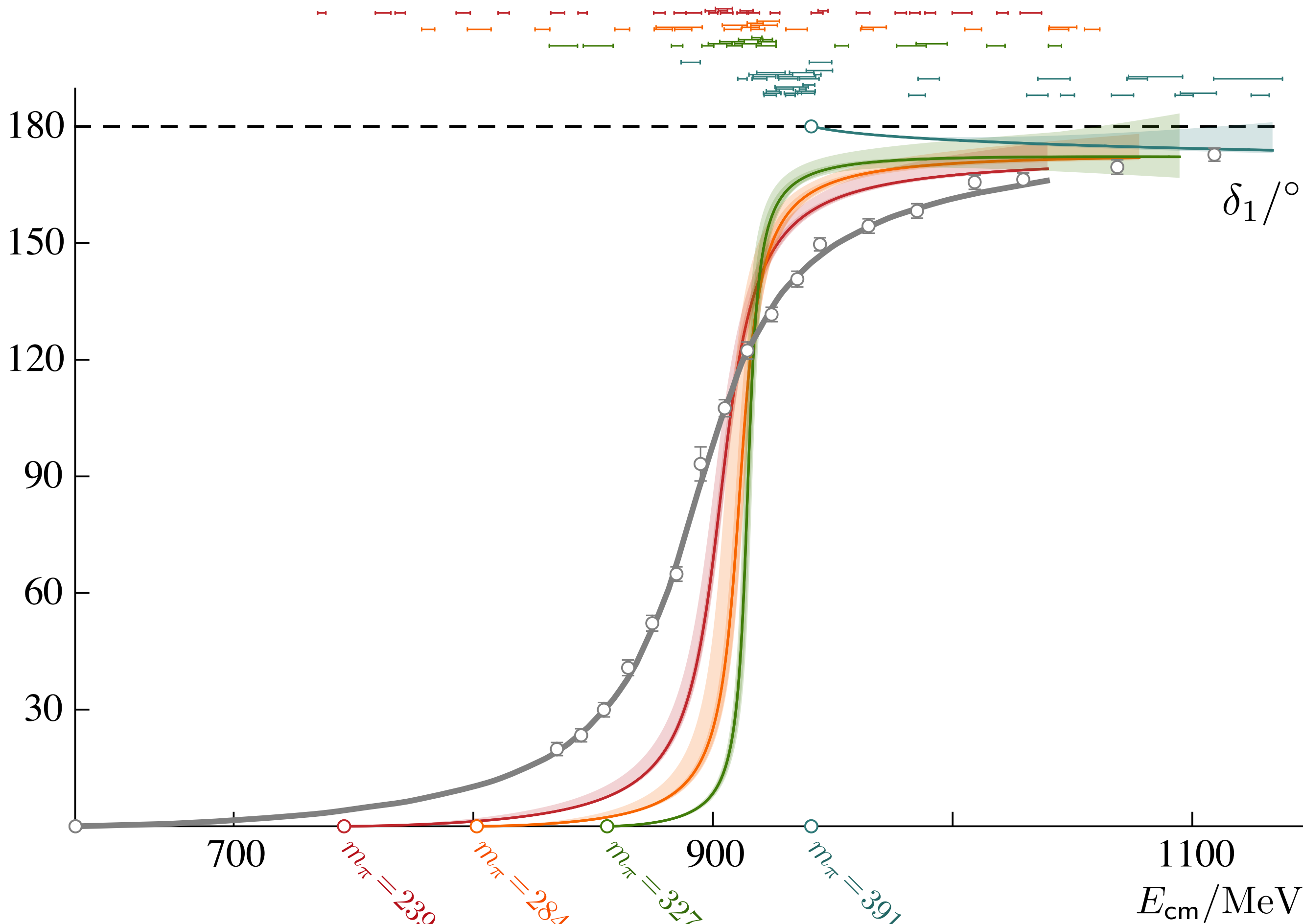
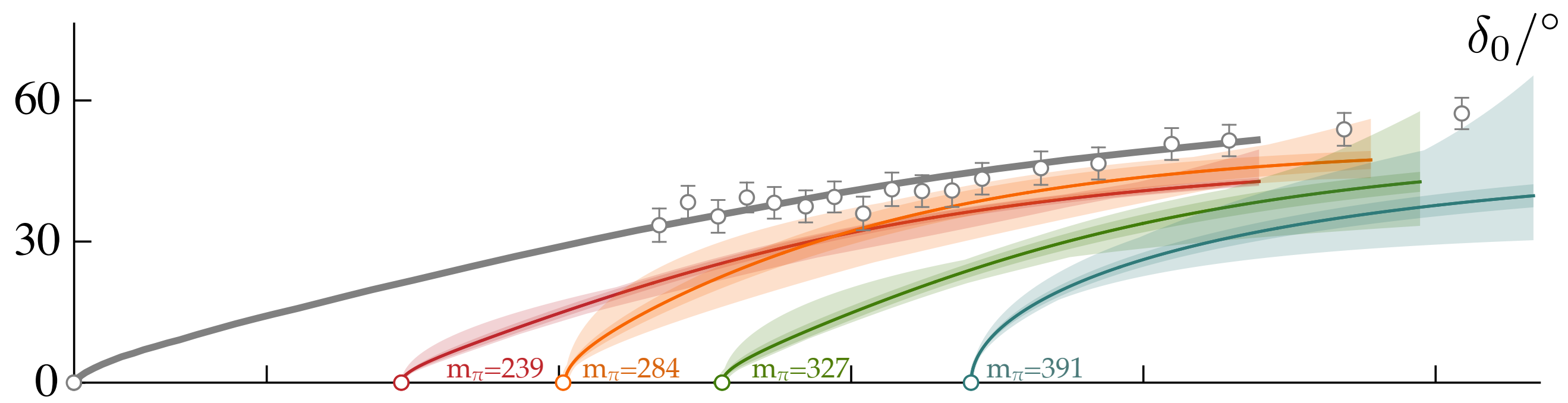


$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$



# $\pi K$ scattering

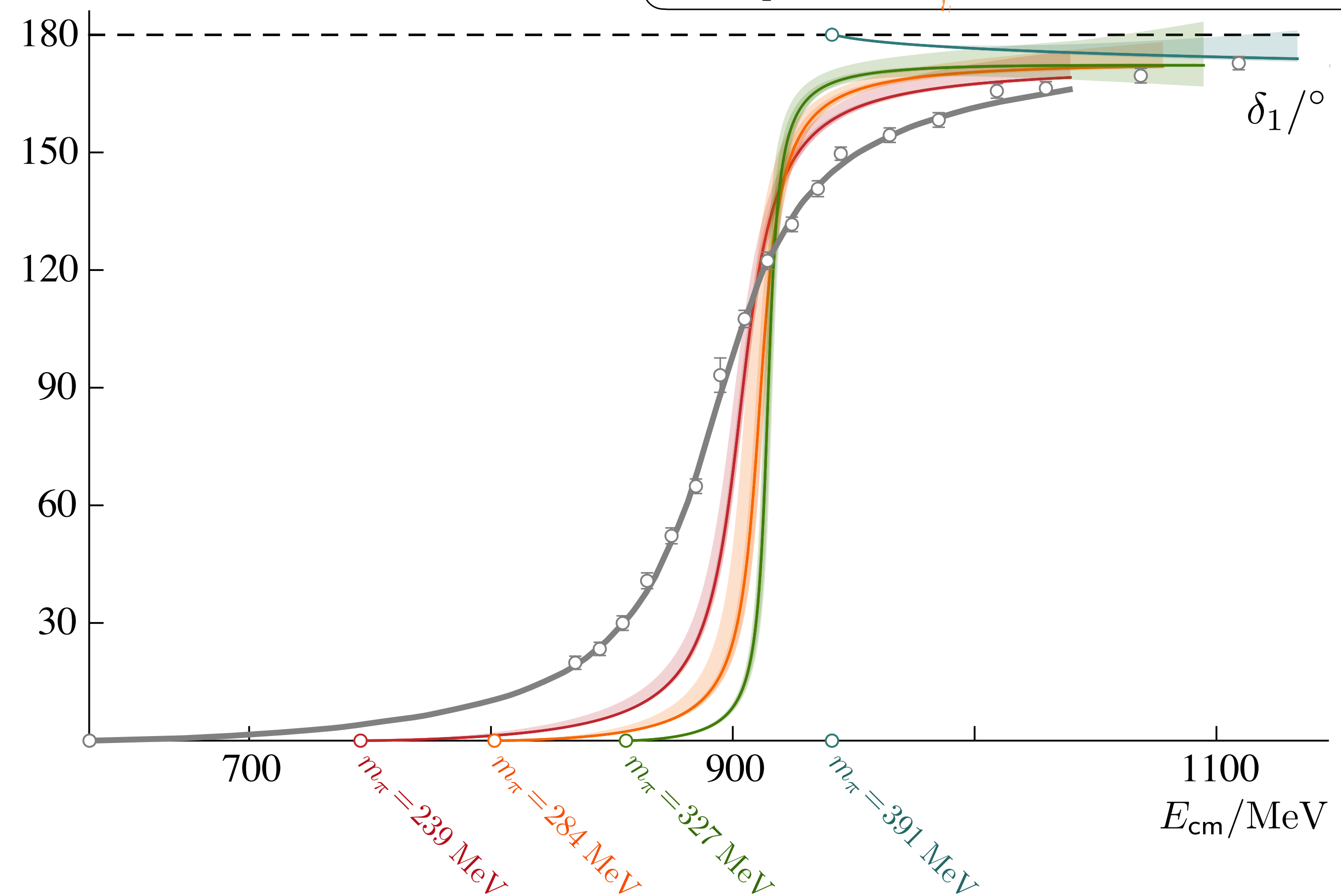
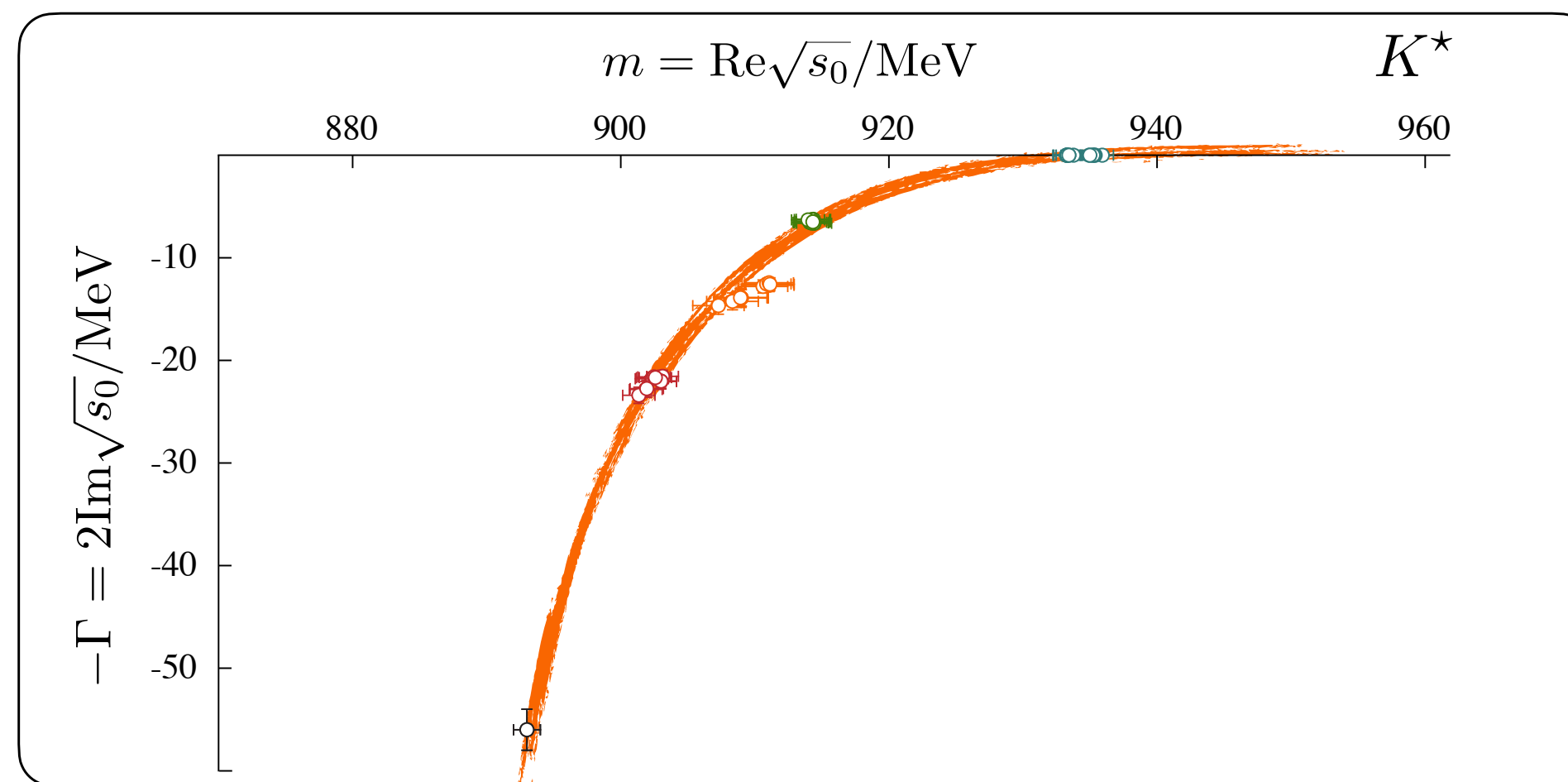
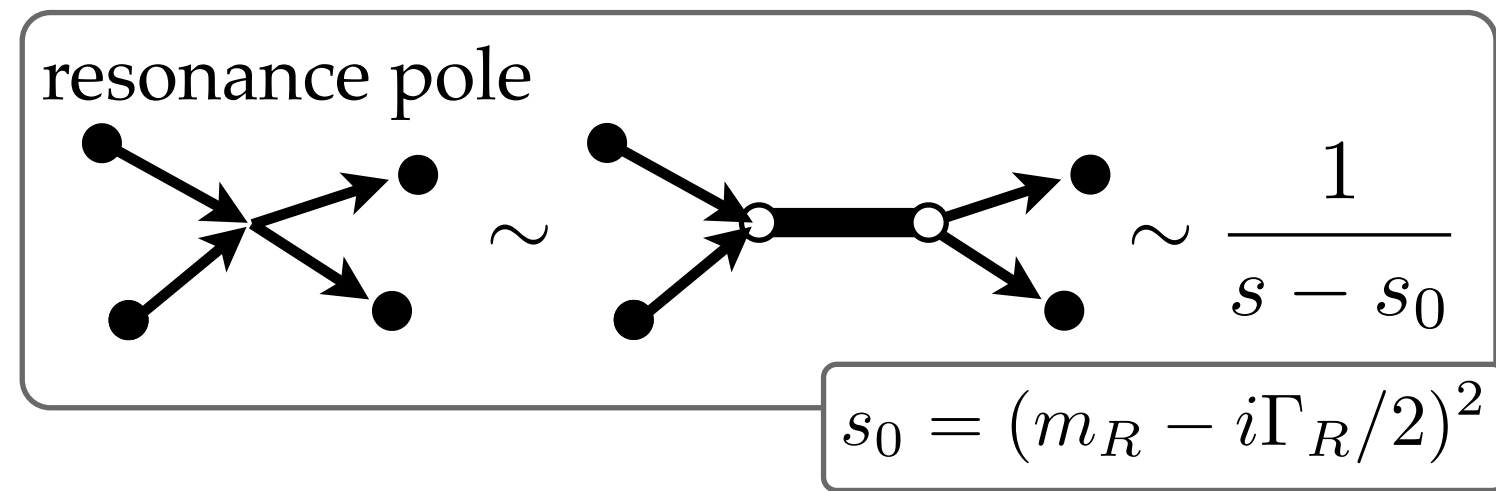
( $l=1/2$  channel)



$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

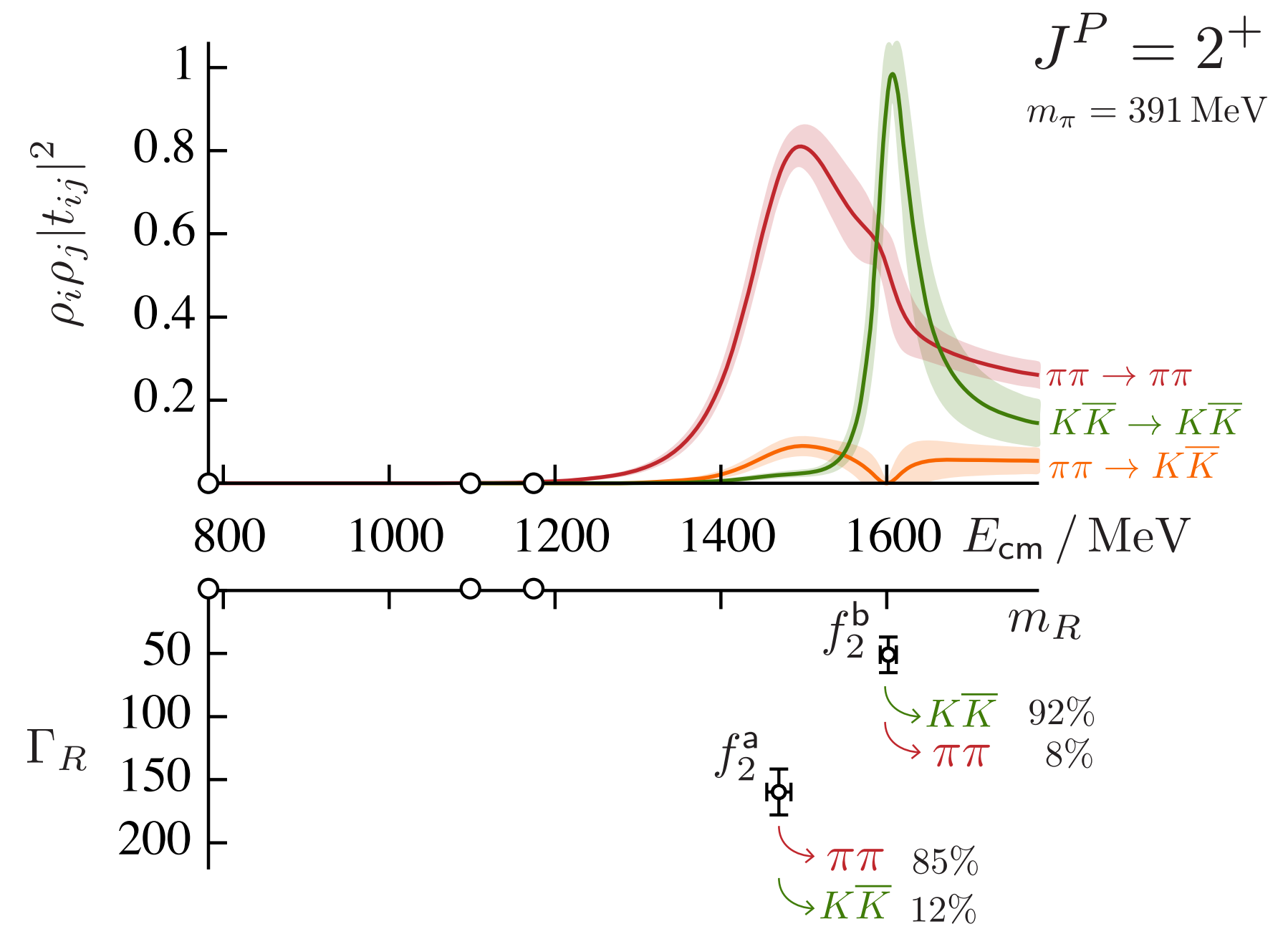
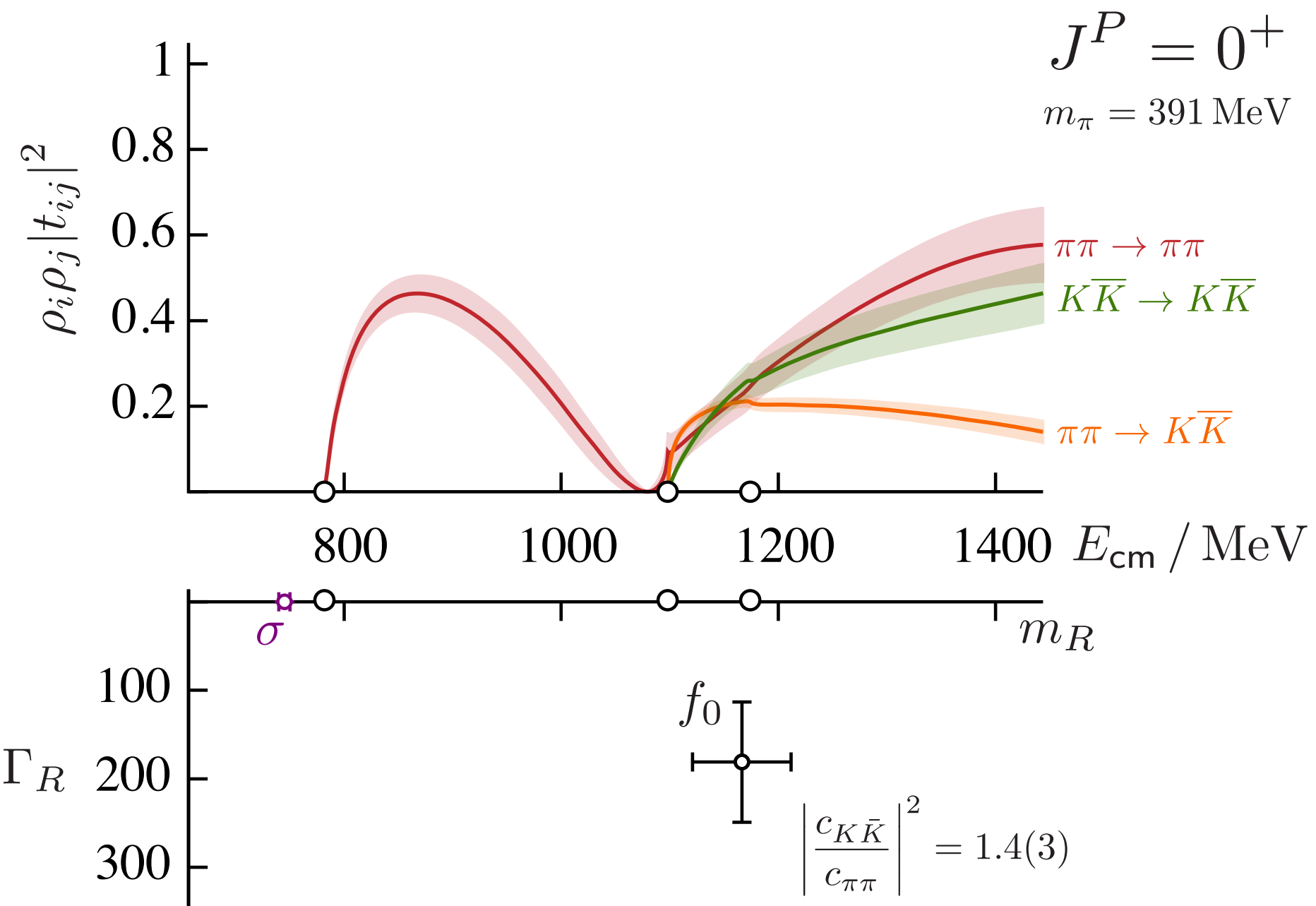
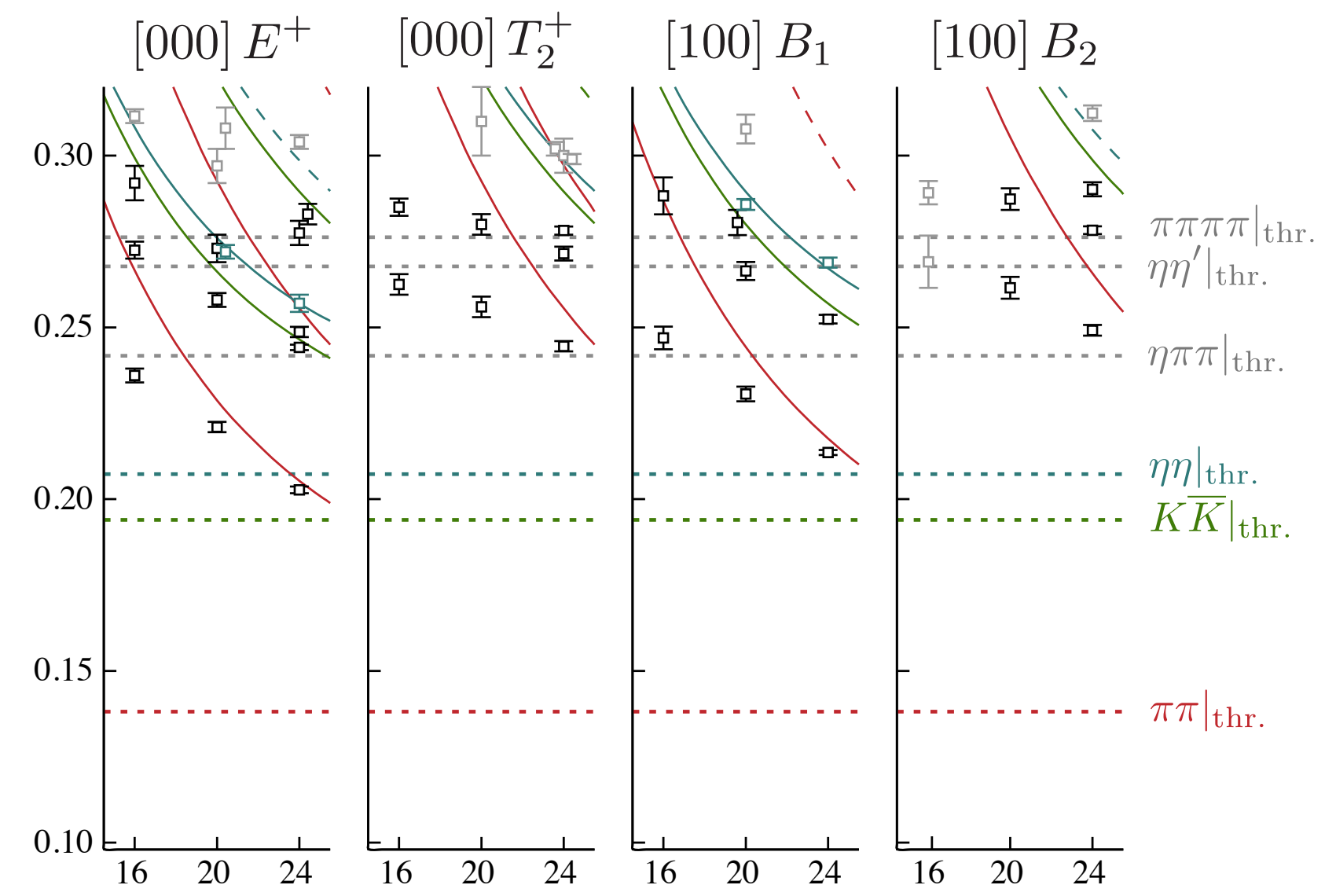
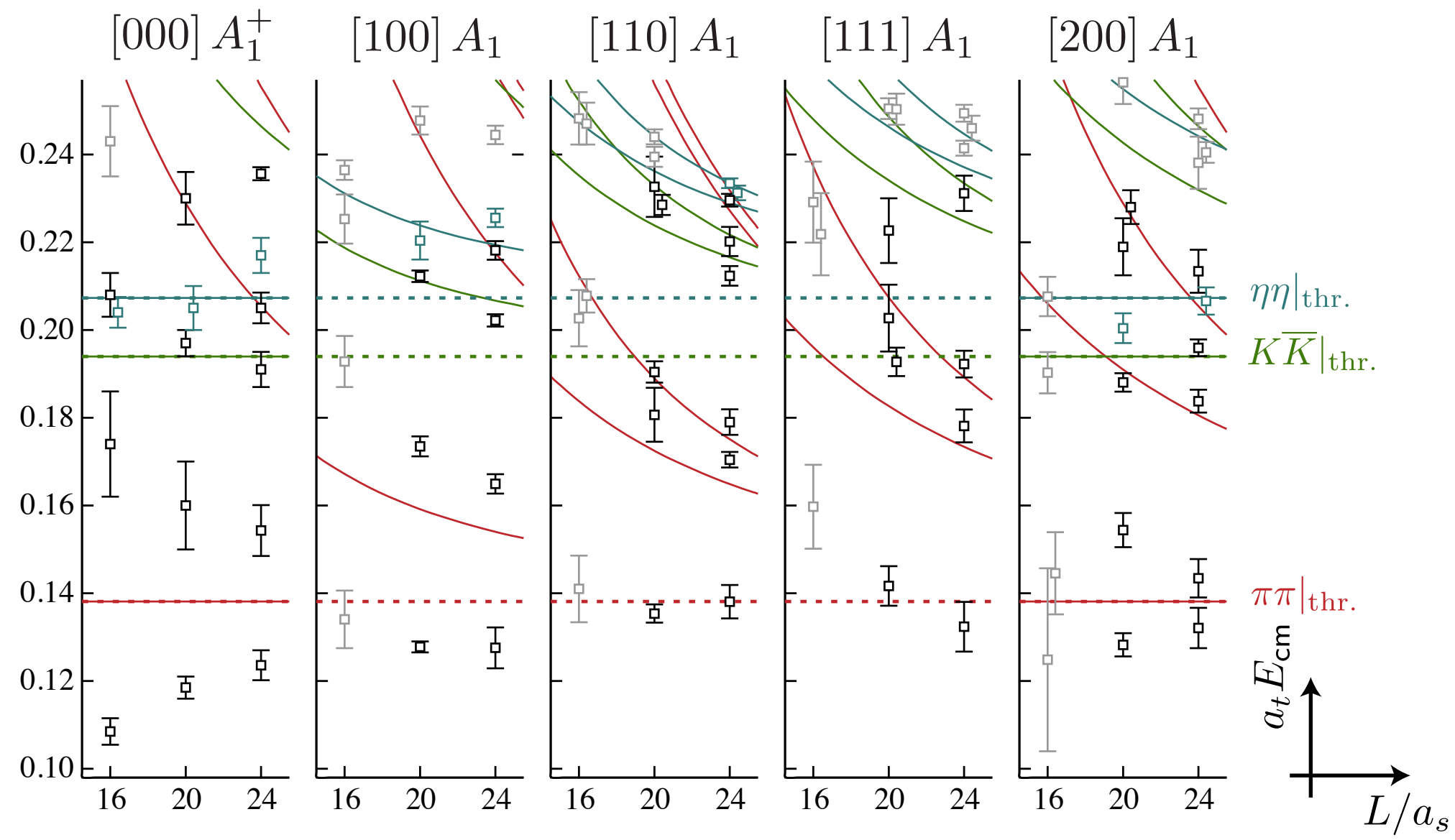
# $\pi K$ scattering

( $l=1/2$  channel)



$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

# $\pi\pi-K\bar{K}$ ( $l=0$ channel)



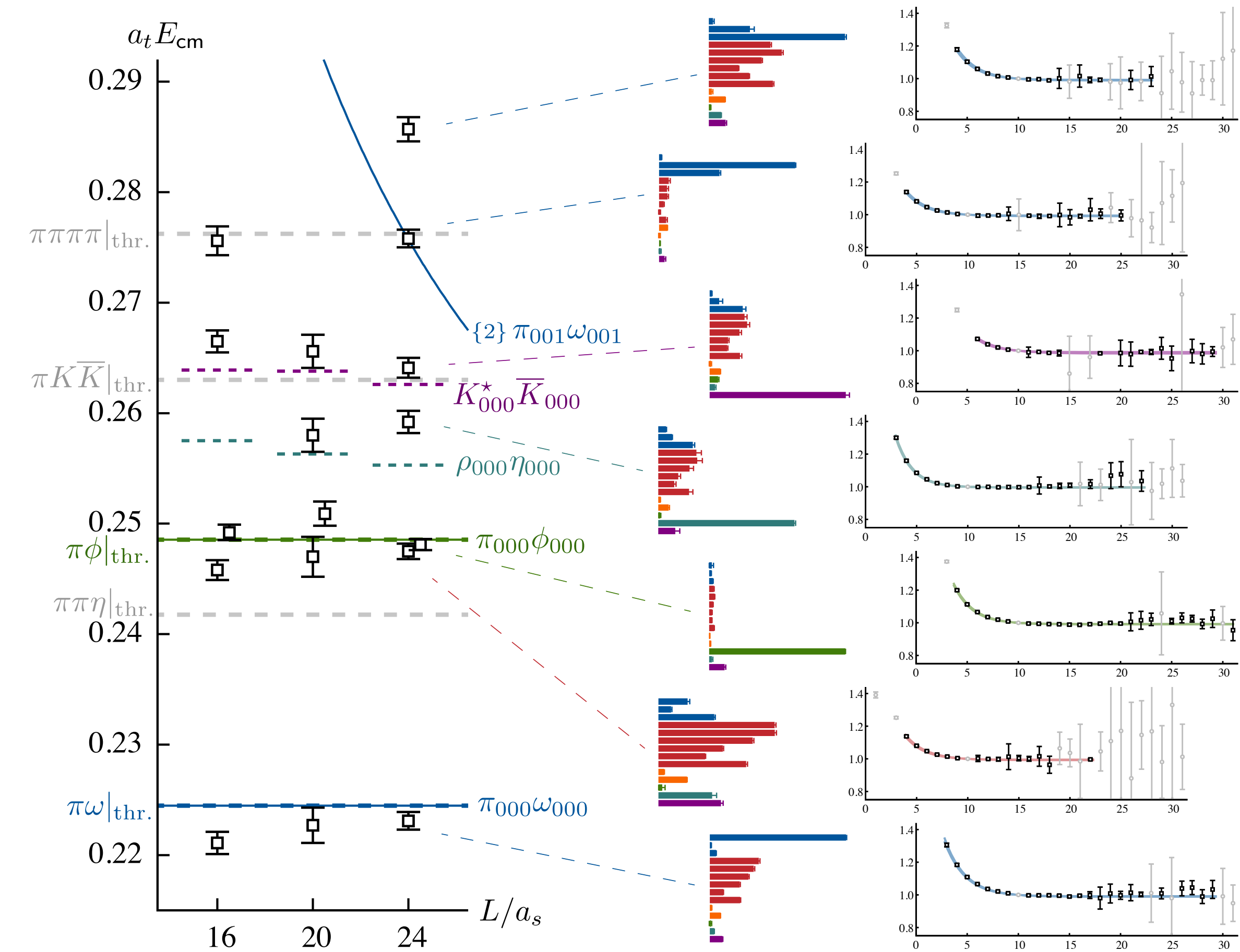
$m_\pi \sim 390 \text{ MeV}$

RB, Dudek, Edwards, & Wilson (2016)

RB, Dudek, Edwards, & Wilson (2017)

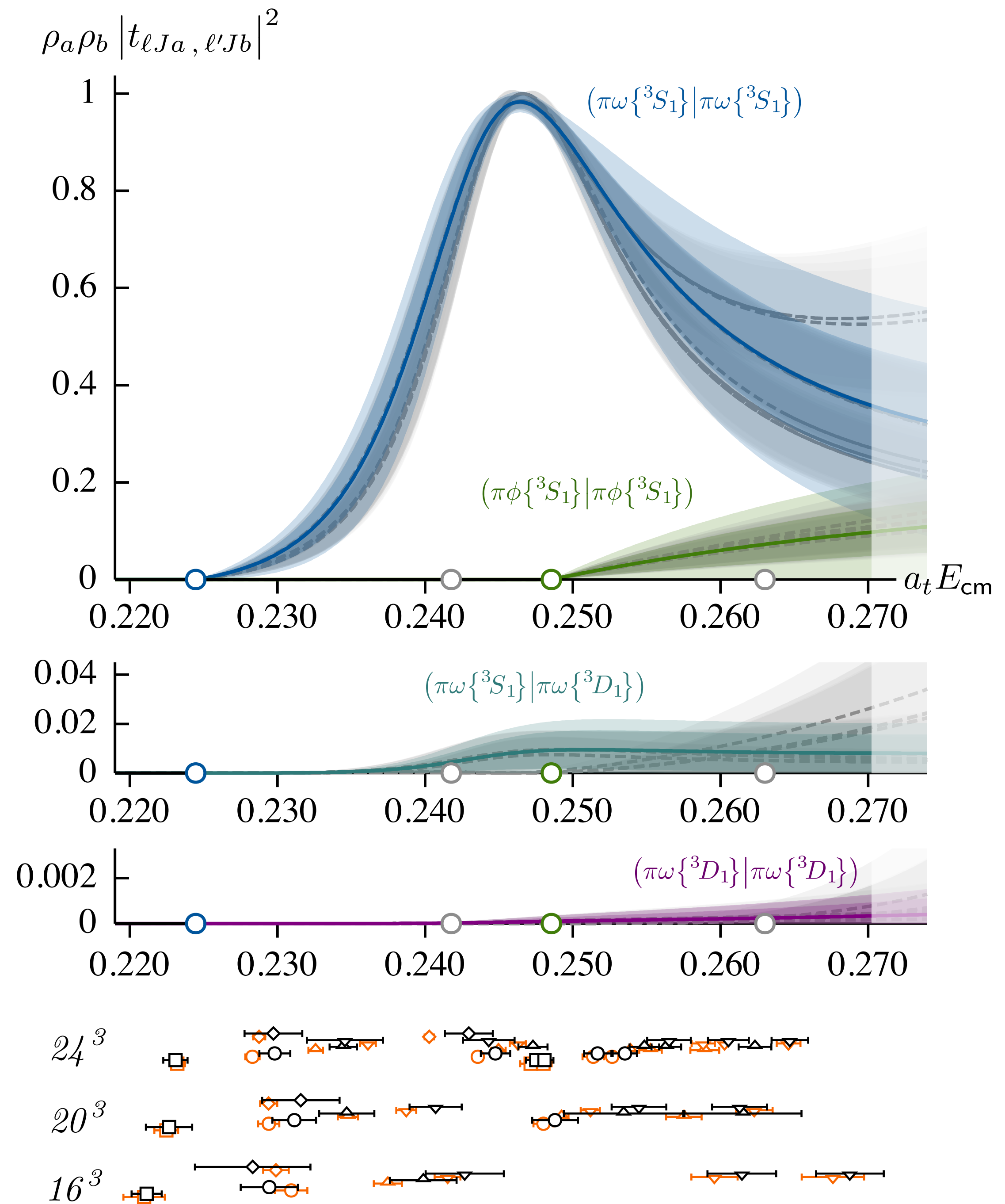
# $\pi\omega, \pi\phi, \dots \Leftrightarrow b_1$

$(I^G = 1^+, J^{PC} = 1^{+-})$



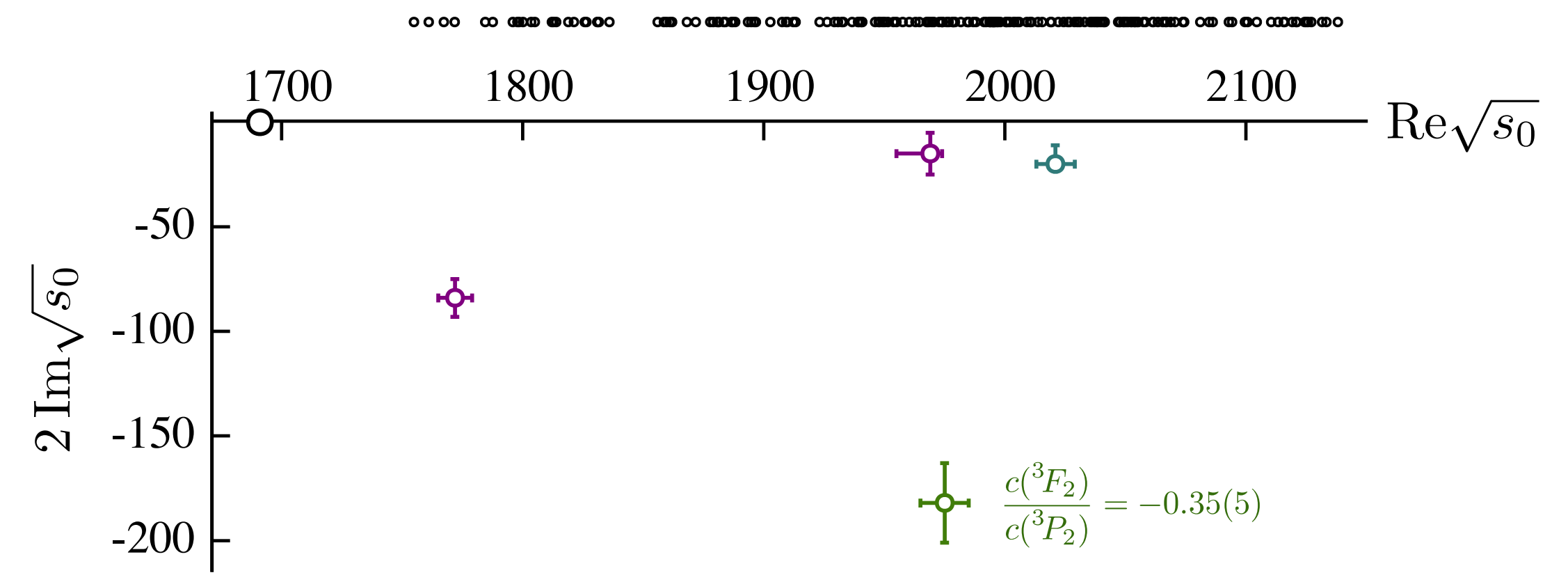
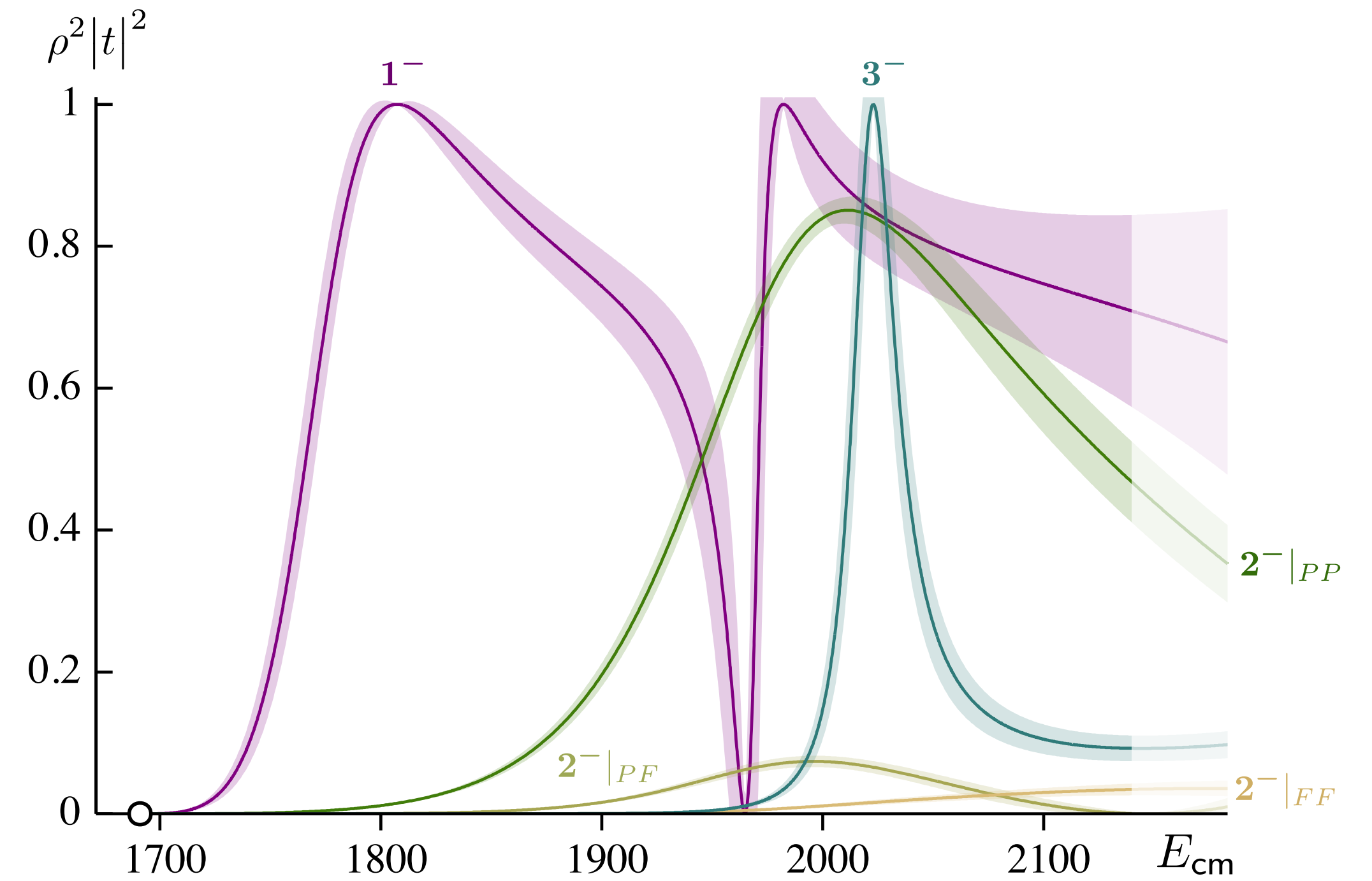
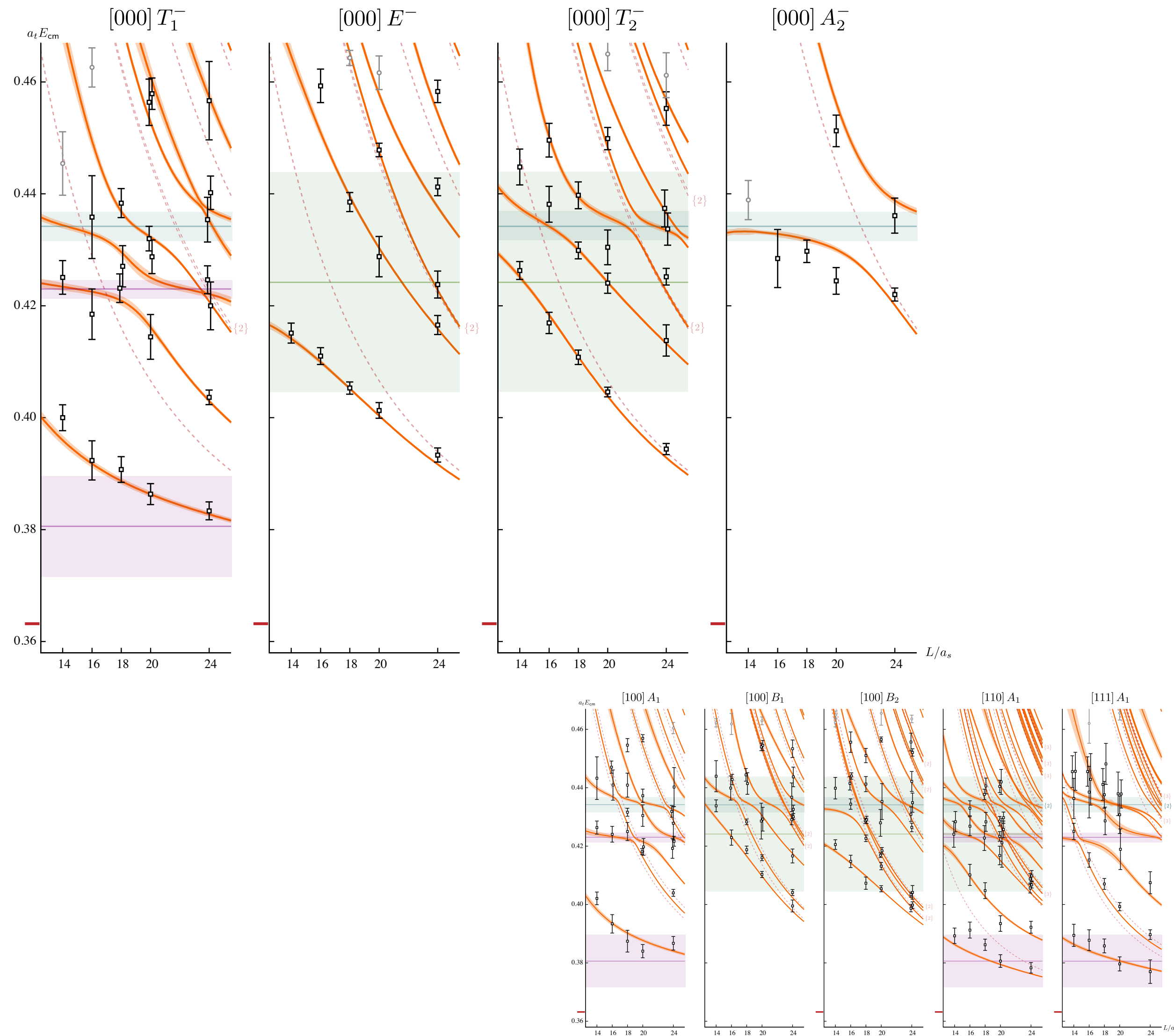
$m_\pi \sim 390 \text{ MeV}$

Woss, Dudek, Edwards, Thomas, & Wilson (2020)



# $\pi\rho, KK^*, \eta\omega, \dots \Leftrightarrow \omega^*, \rho^*$

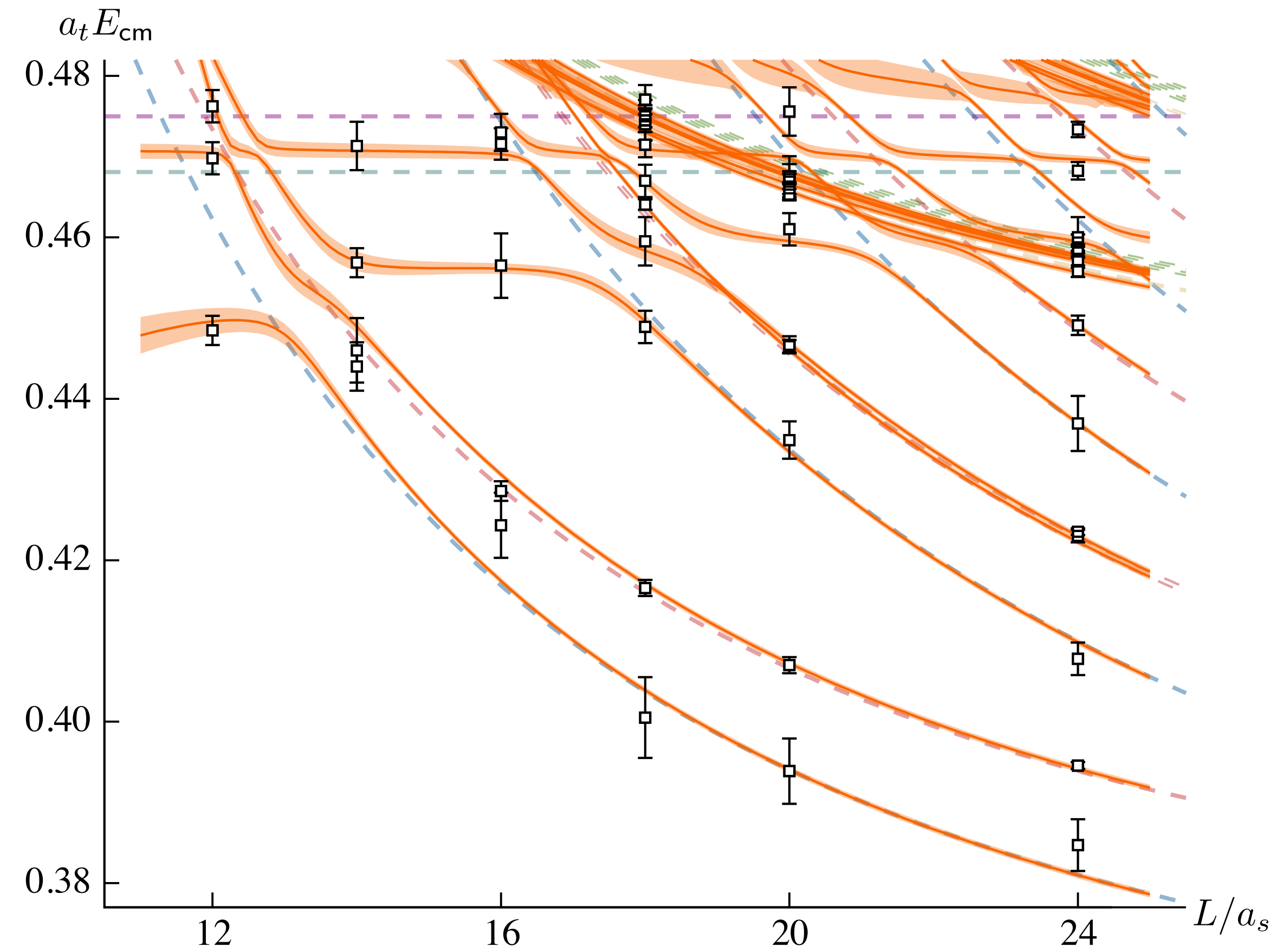
$(J^{PC} = 1^{--}, 2^{--}, 3^{--})$  192 energy levels...world record!



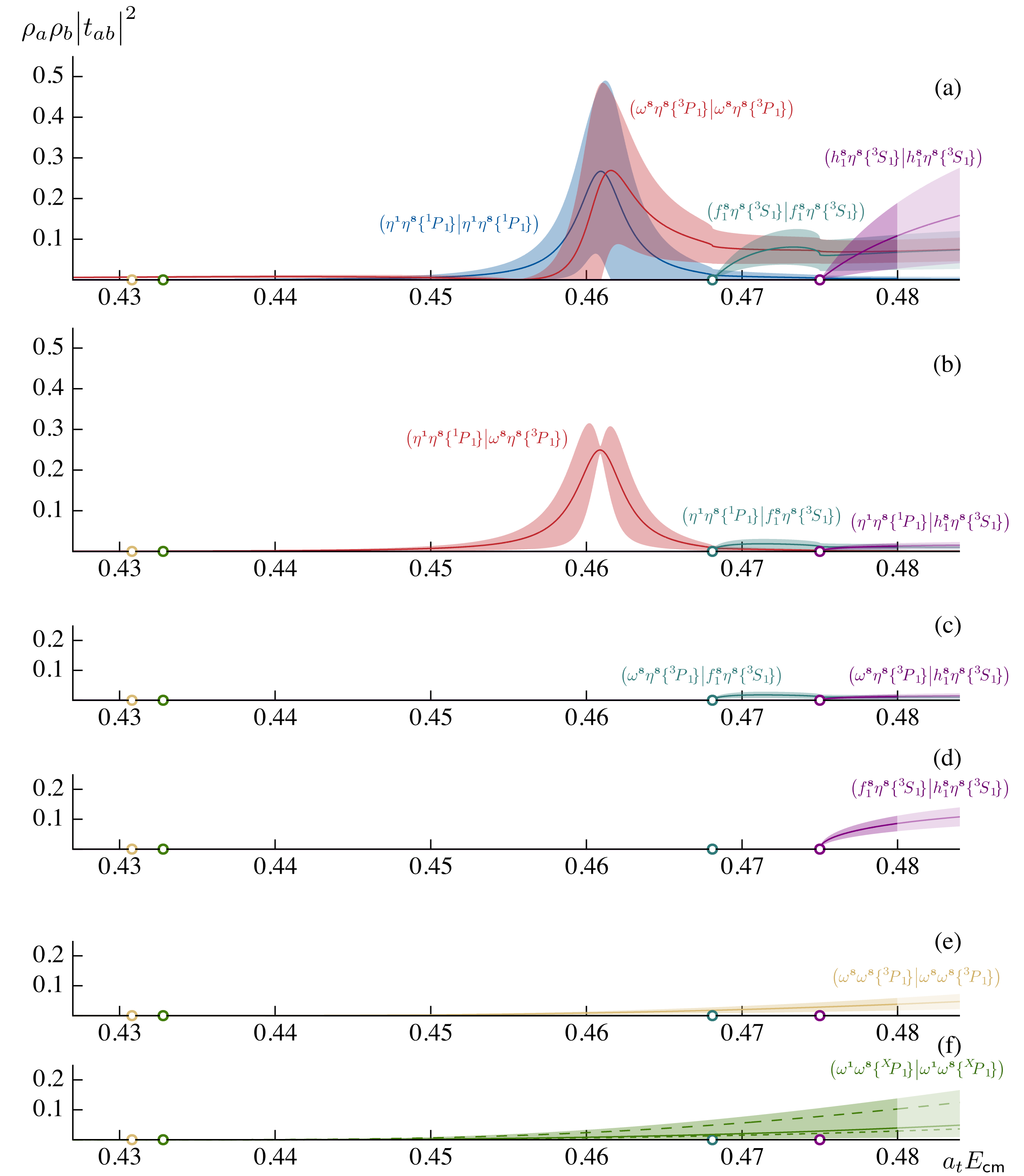


# $\pi\eta, \pi\rho, \dots \leftrightarrow \pi_1$

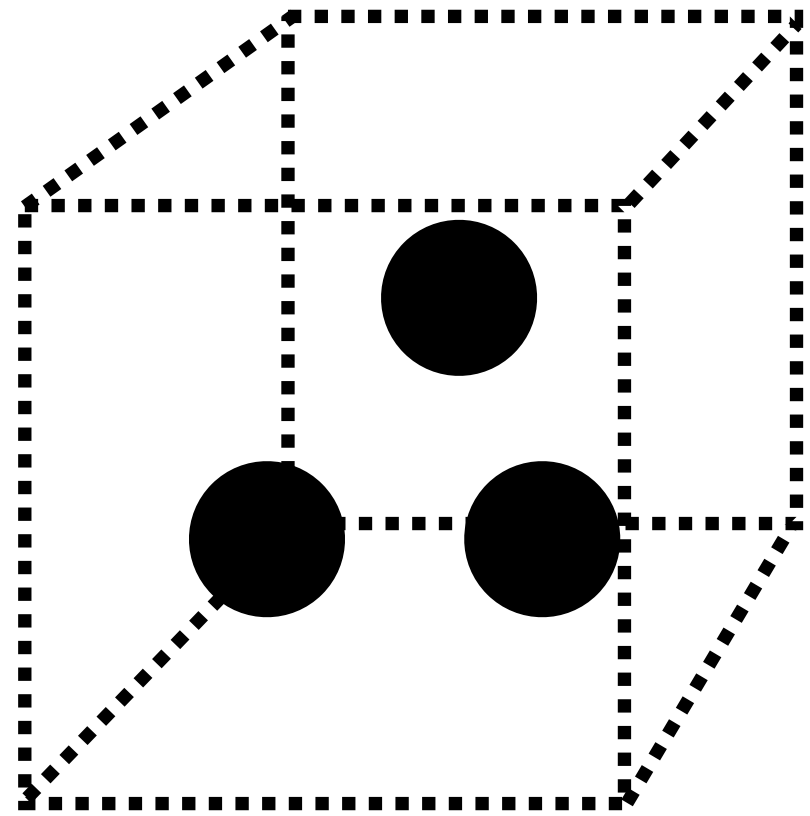
$(J^{PC} = 1^{-+}, \text{first exotic resonant amplitude // 8 channels!})$



$m_\pi \sim 700 \text{ MeV}$   
 Woss, Dudek, Edwards, Thomas, & Wilson (2020)



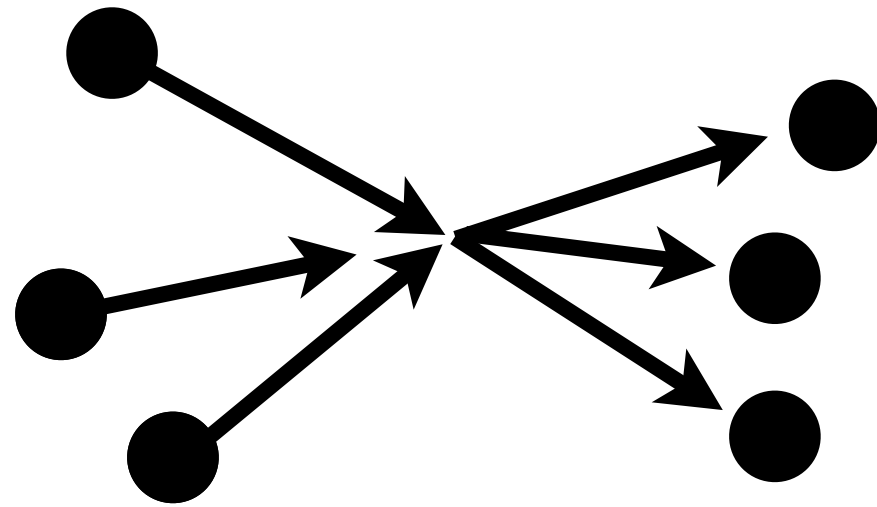
# Three-hadron systems



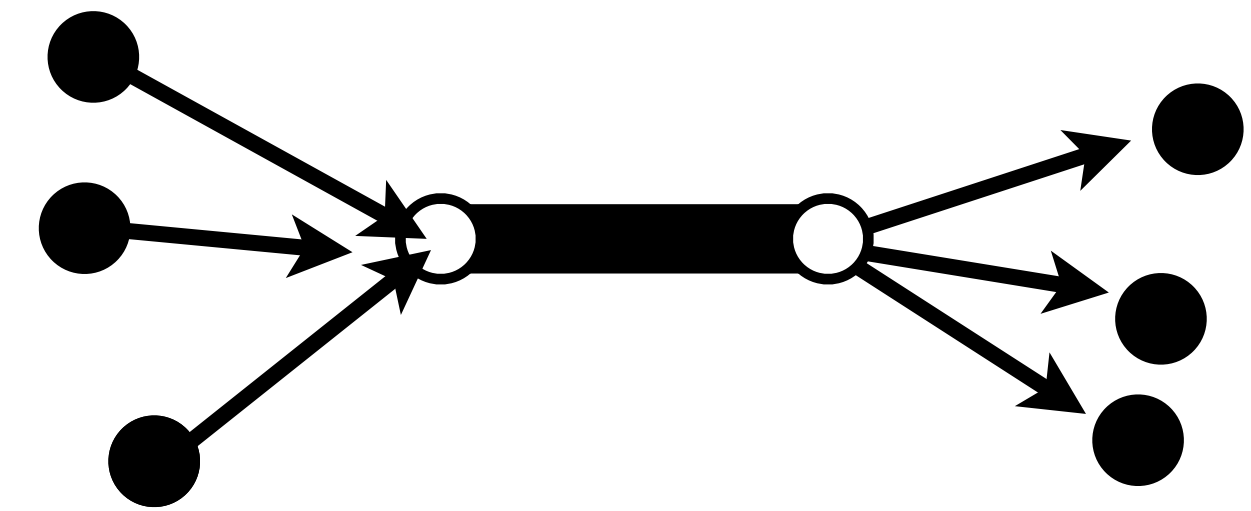
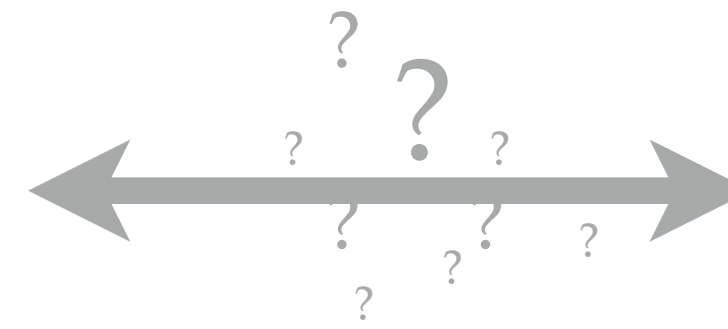
finite-volume  
spectroscopy



Hansen & Sharpe ('14, '15)  
Mai & Döring ('17)  
RB, Hansen & Sharpe ('18)  
Hansen, Romero-Lopez & Sharpe ('20)  
Blanton & Sharpe ('20)  
Jackura et al. ('20)



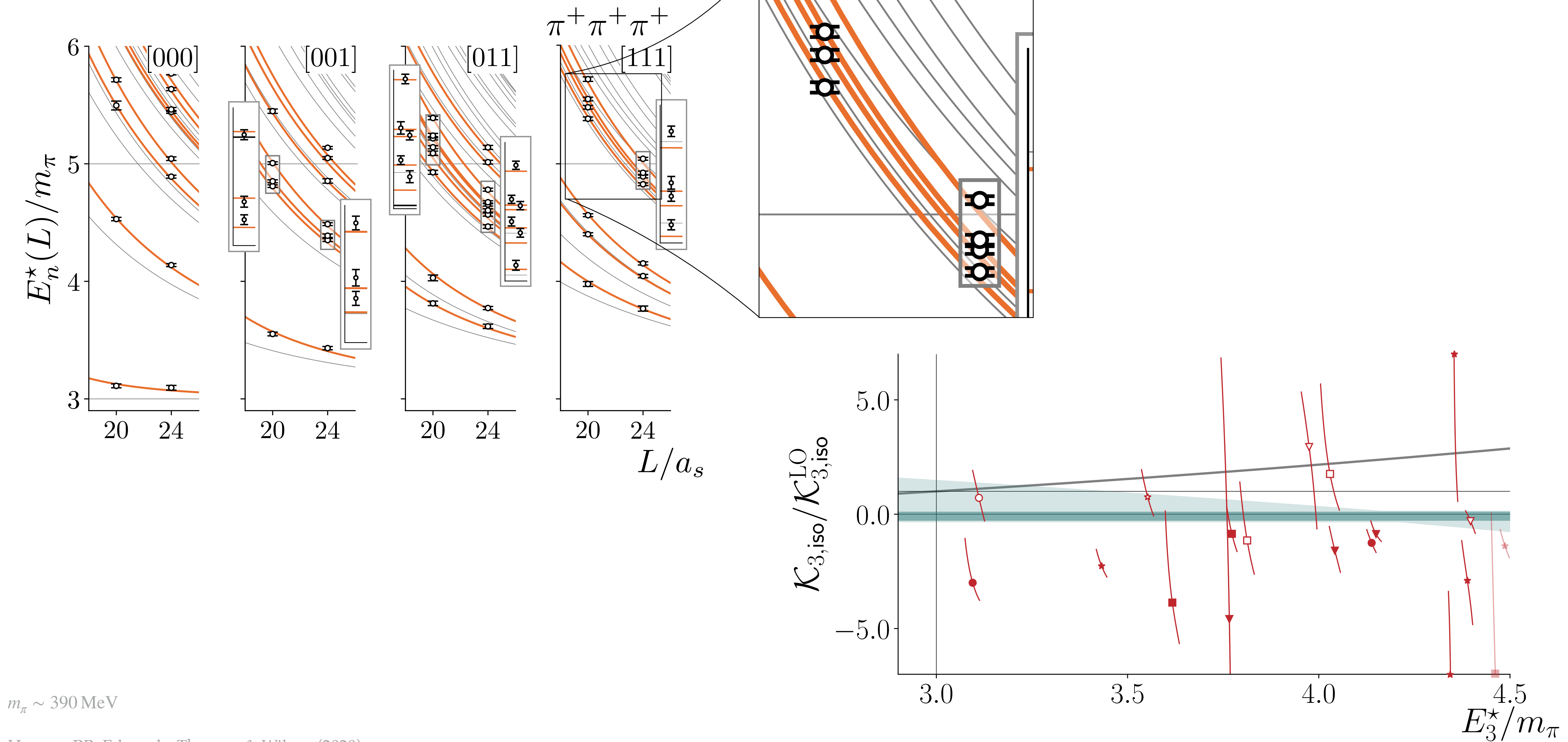
infinite-volume  
scattering amplitudes



bound state and  
resonance poles

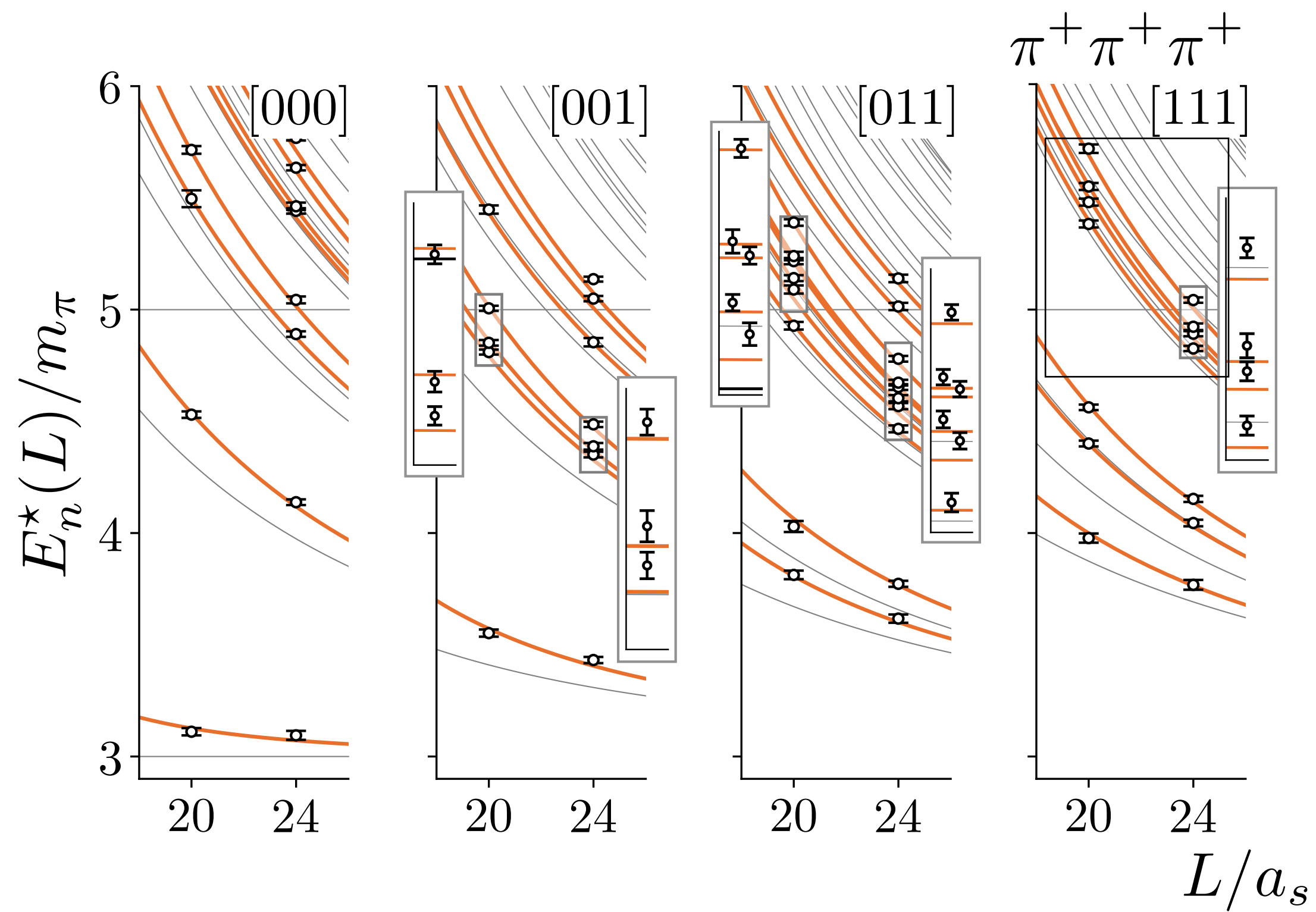
# $\pi\pi\pi$

( $l=3$  channel, first three-body scattering amplitude)

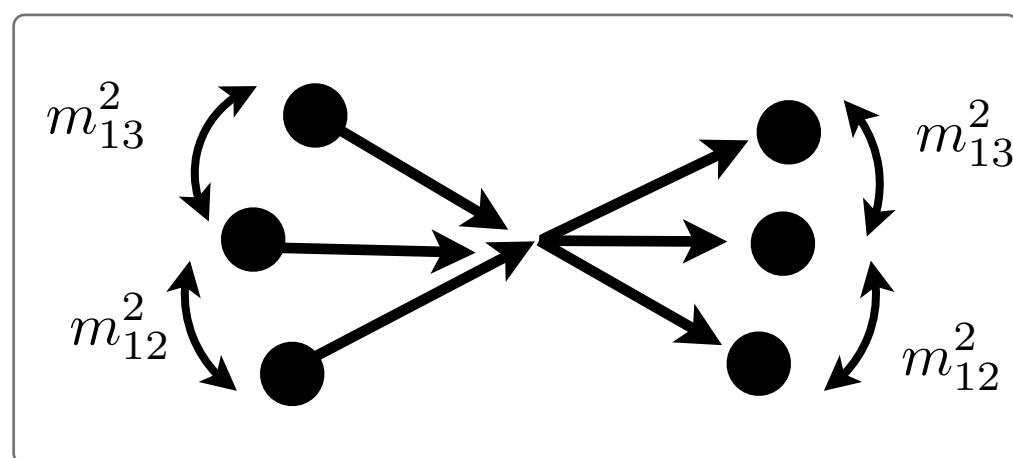
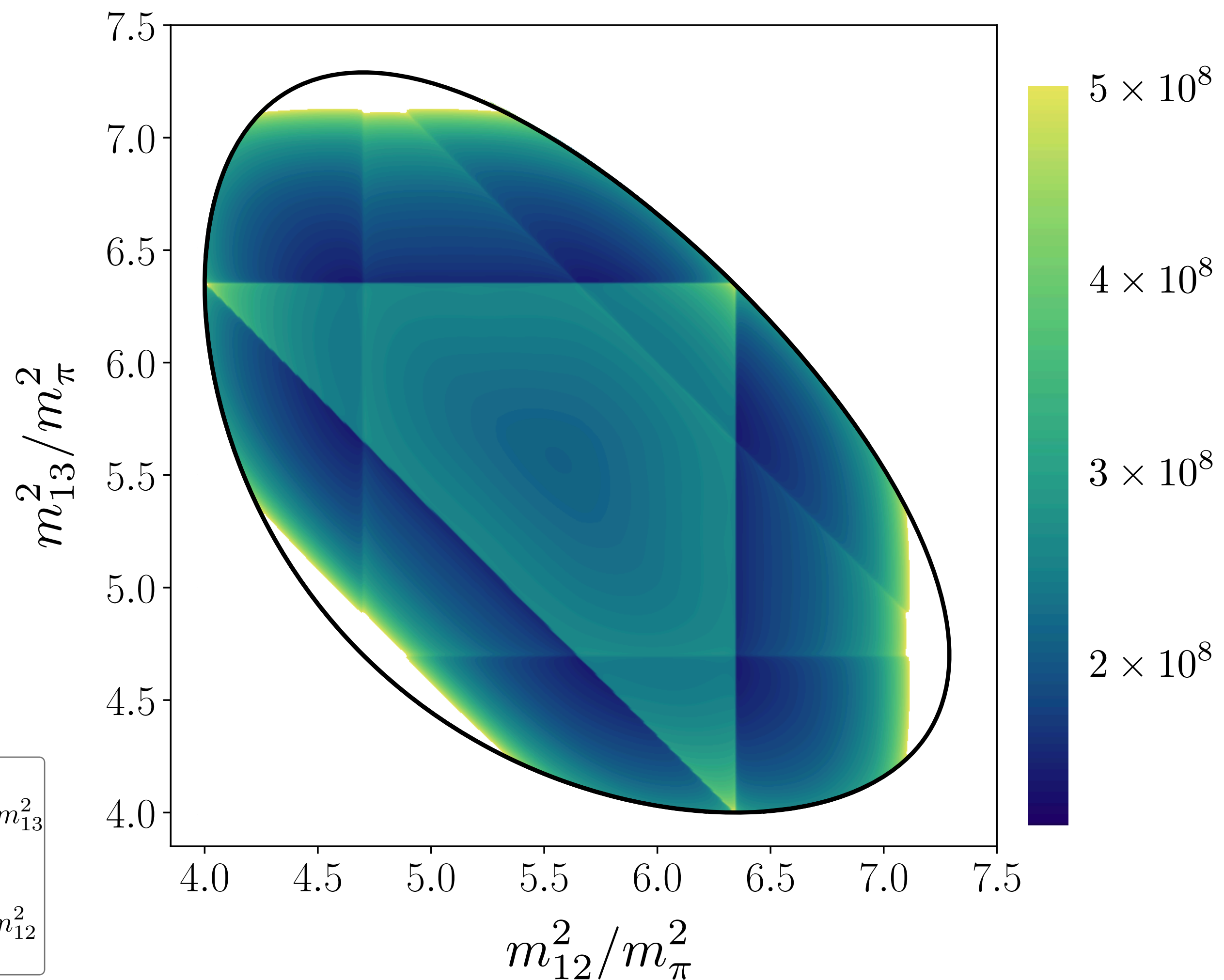


# $\pi\pi\pi$

( $l=3$  channel, first three-body scattering amplitude)



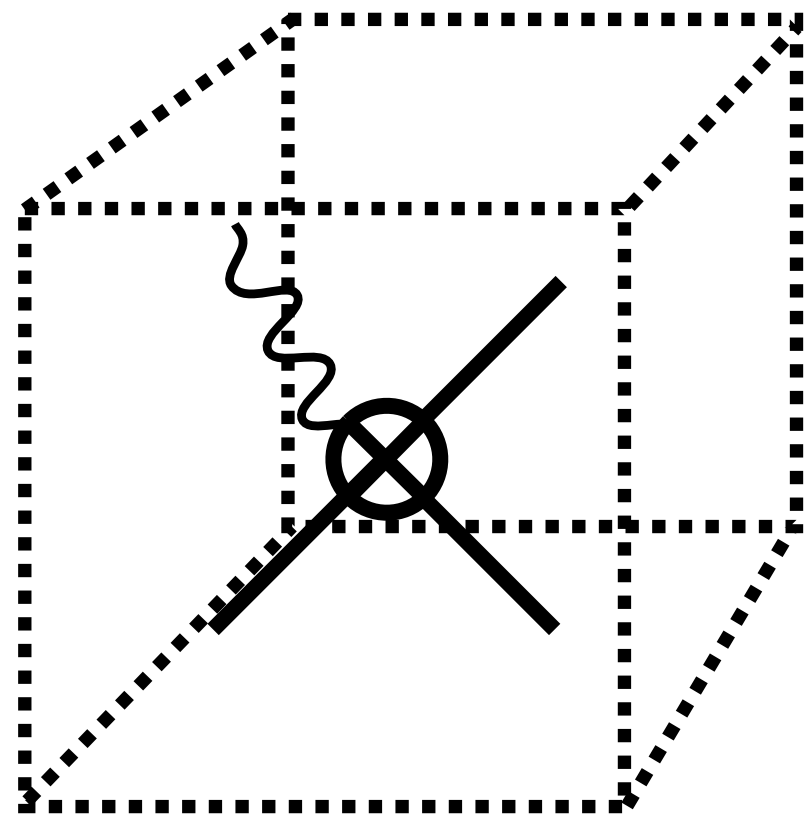
$\pi^+ \pi^+ \pi^+$



$m_\pi \sim 390$  MeV

Hansen, RB, Edwards, Thomas, & Wilson (2020)

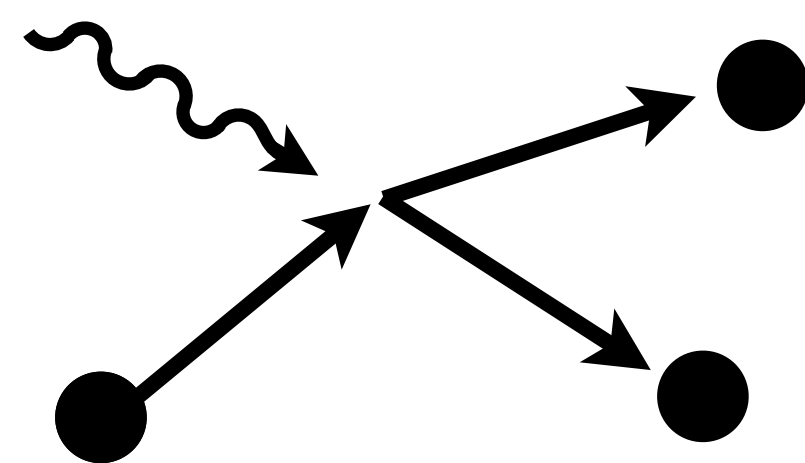
# Transition amplitudes



finite-volume  
spectroscopy



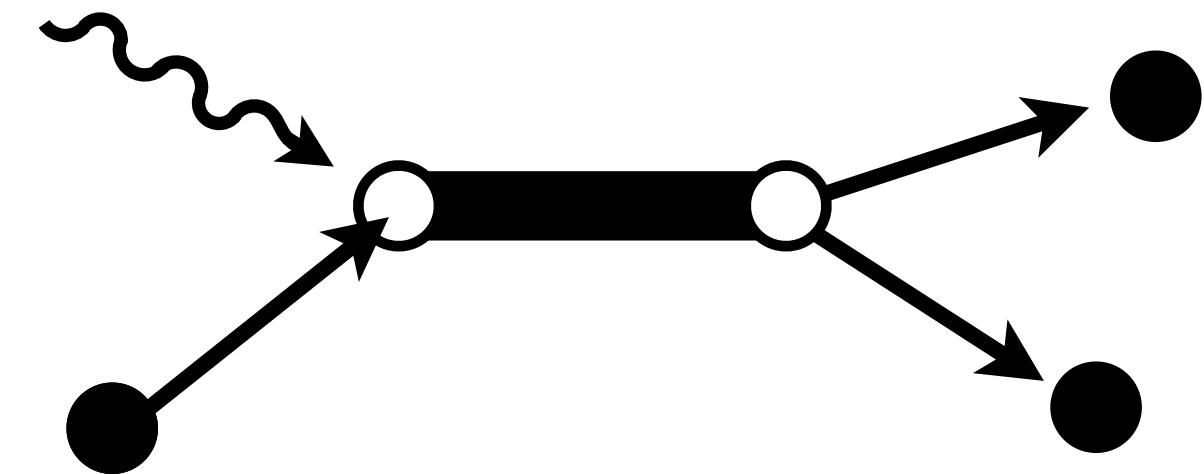
Lellouch & Lüscher (2000)  
Kim, Sachrajda, & Sharpe (2005)  
Christ, Kim & Yamazaki (2005)  
Hansen & Sharpe (2012)  
RB, Hansen Walker-Loud (2014)  
RB & Hansen (2015)



infinite-volume  
scattering amplitudes

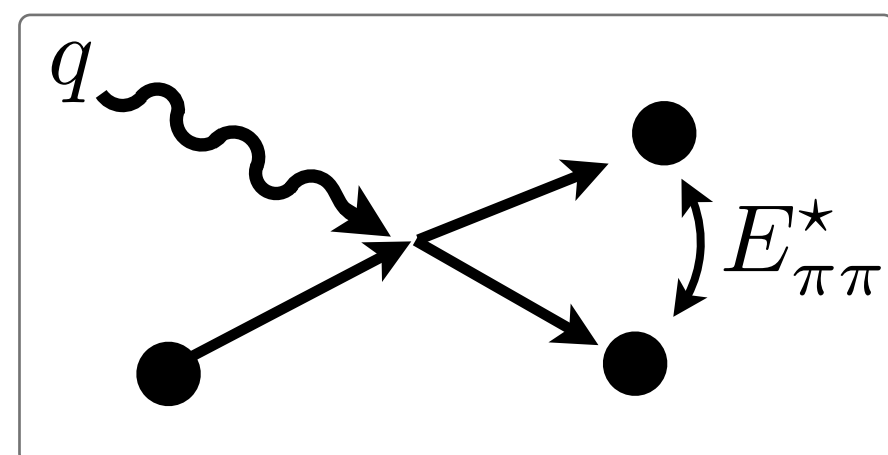
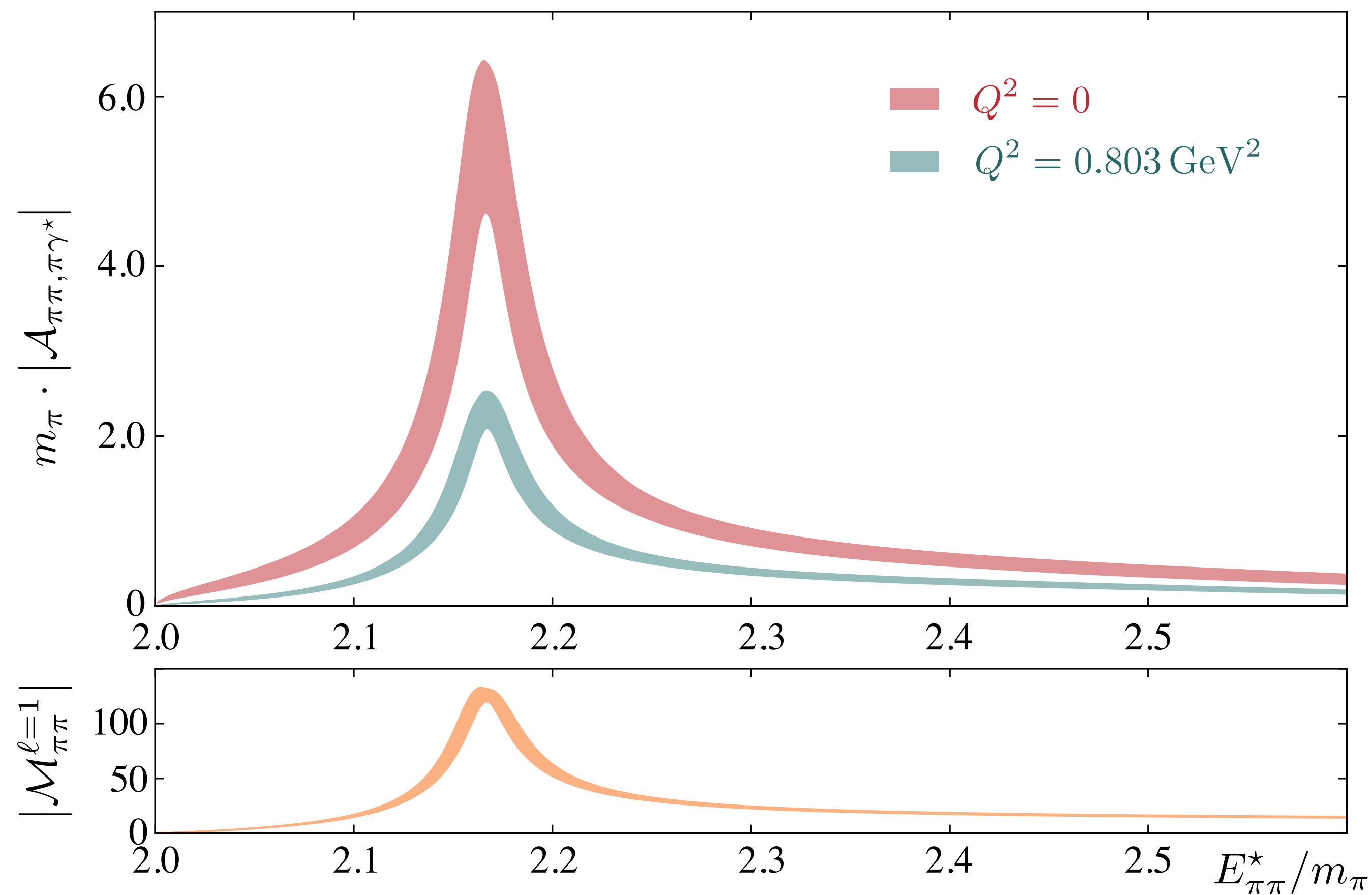
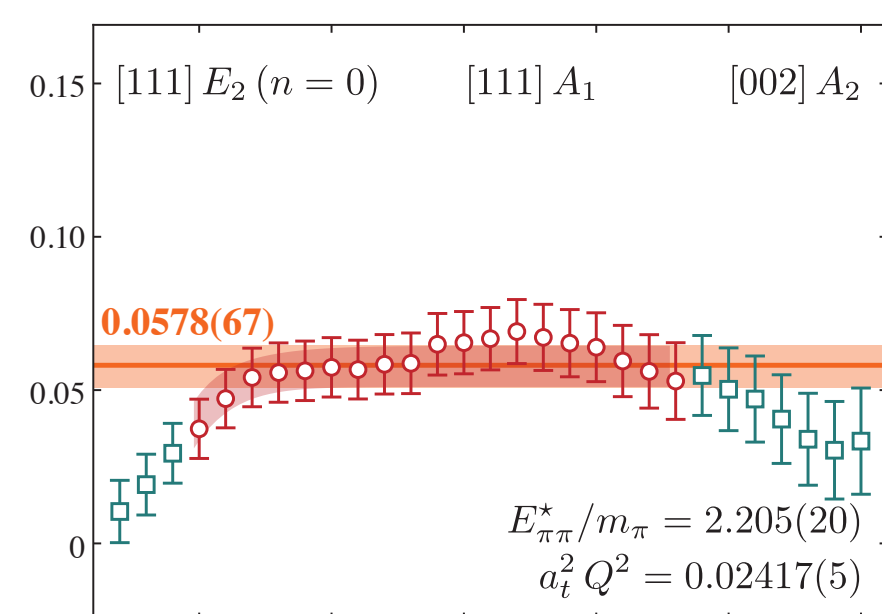
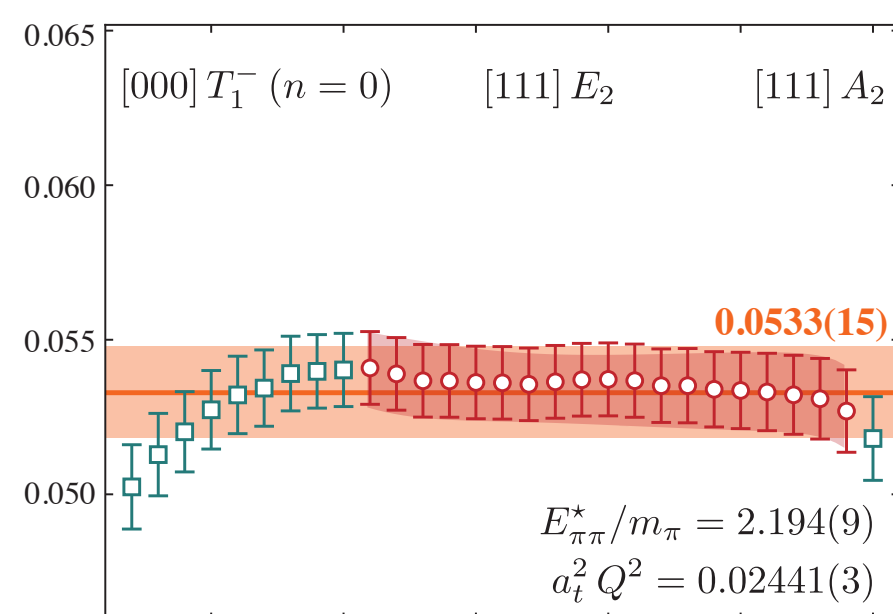
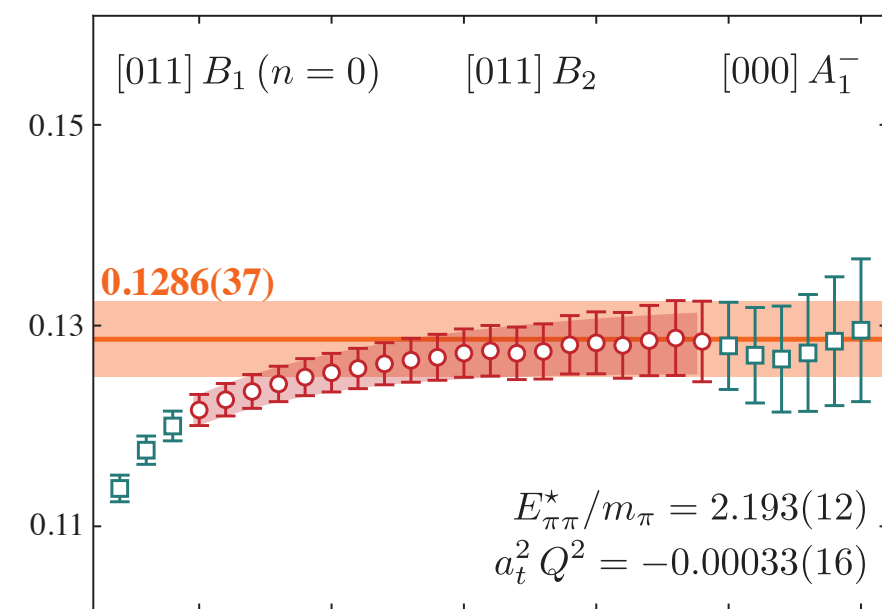
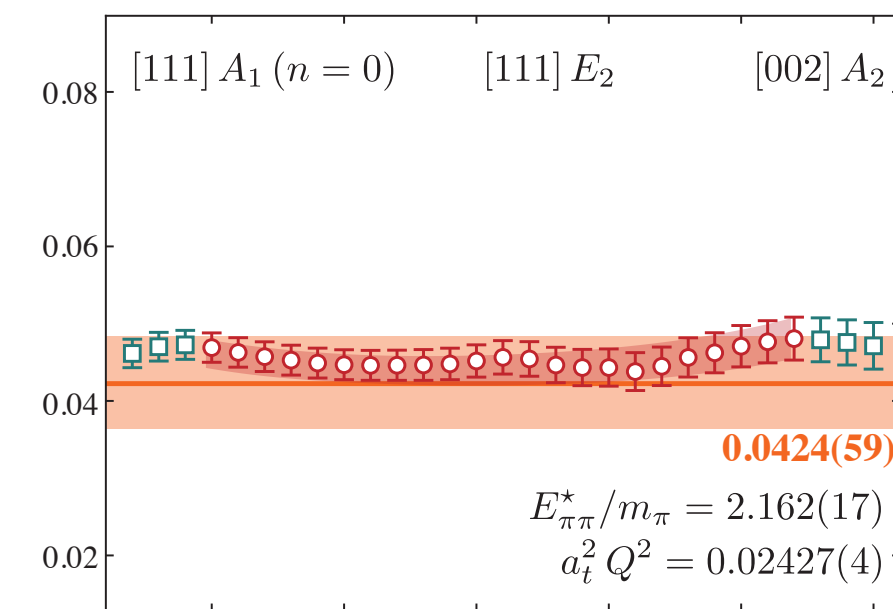


RB, Jackura, Ortega-Gama,  
Sherman ('21)



bound state and  
resonance poles

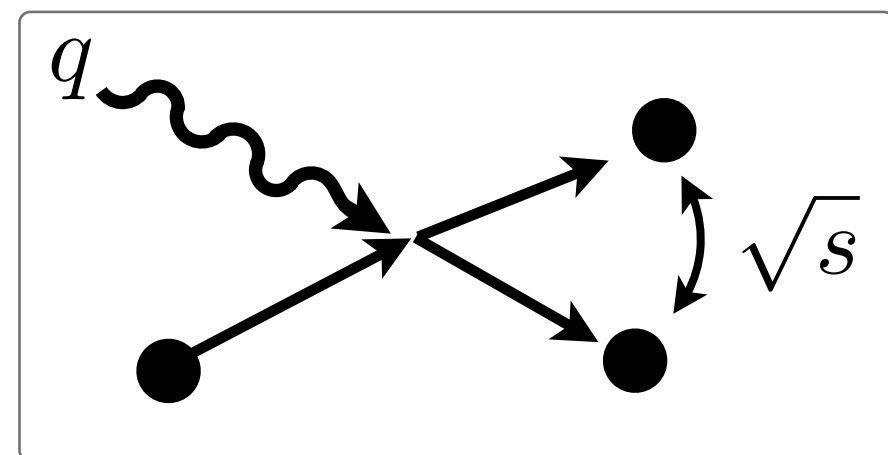
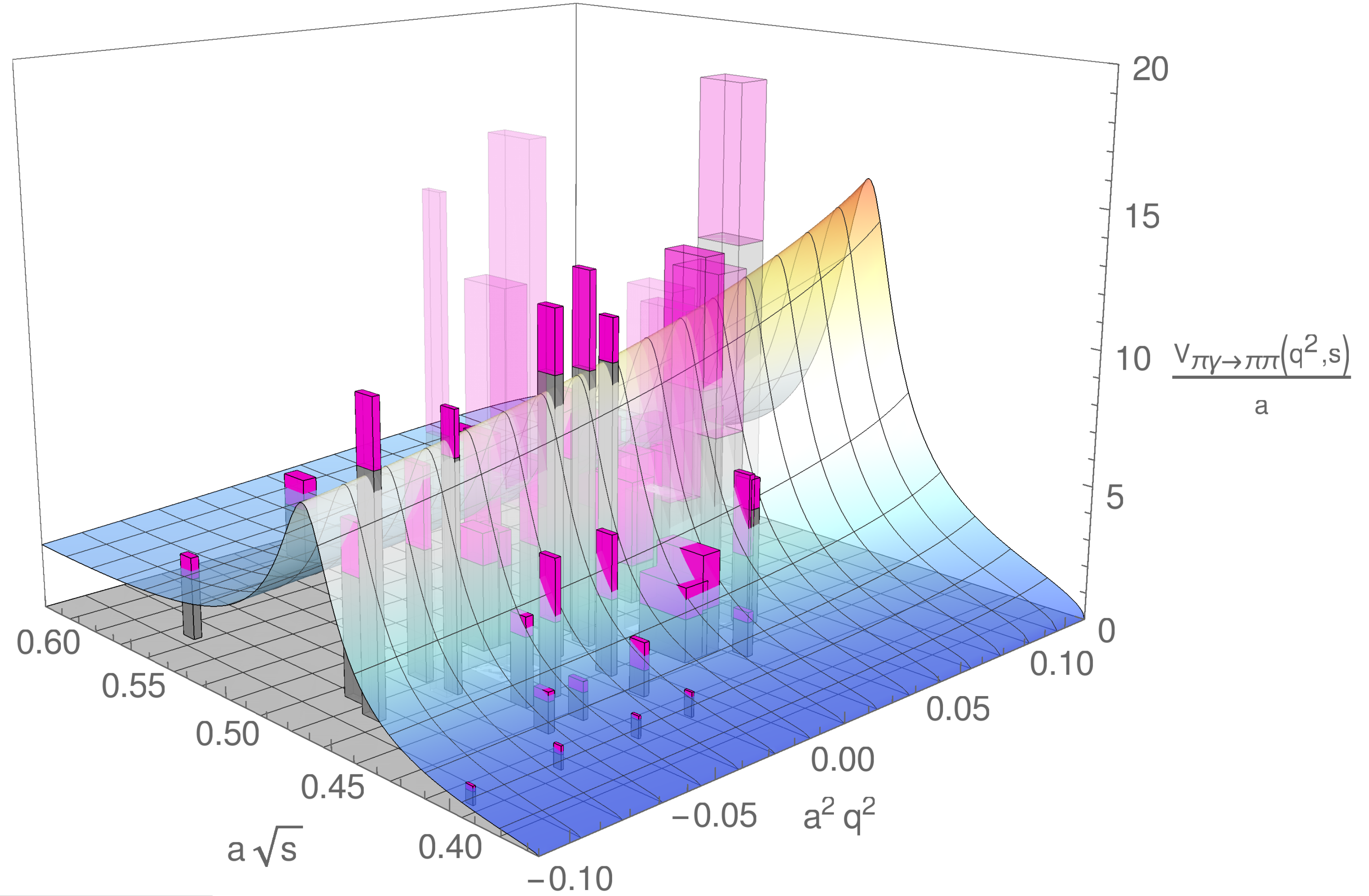
# $\pi\gamma \Rightarrow \pi\pi$



$m_\pi \sim 390 \text{ MeV}$

RB, Dudek, Edwards, Shultz, & Thomas (2015)

$\pi\gamma \Rightarrow \pi\pi$

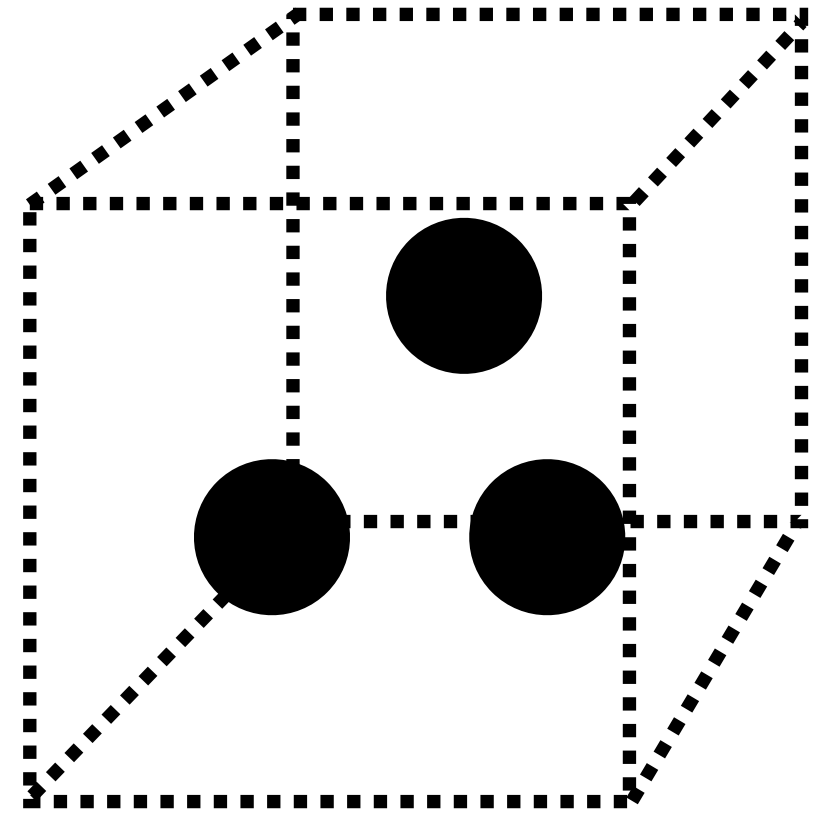


$m_\pi \sim 320 \text{ MeV}$

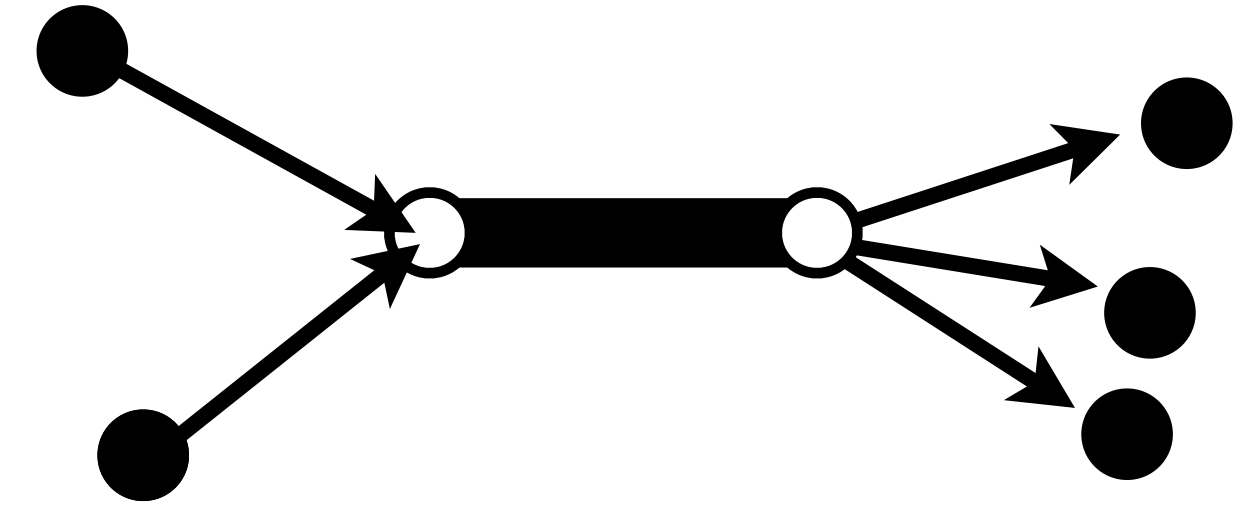
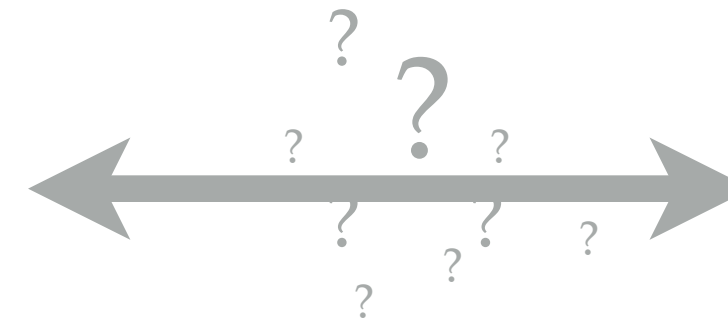
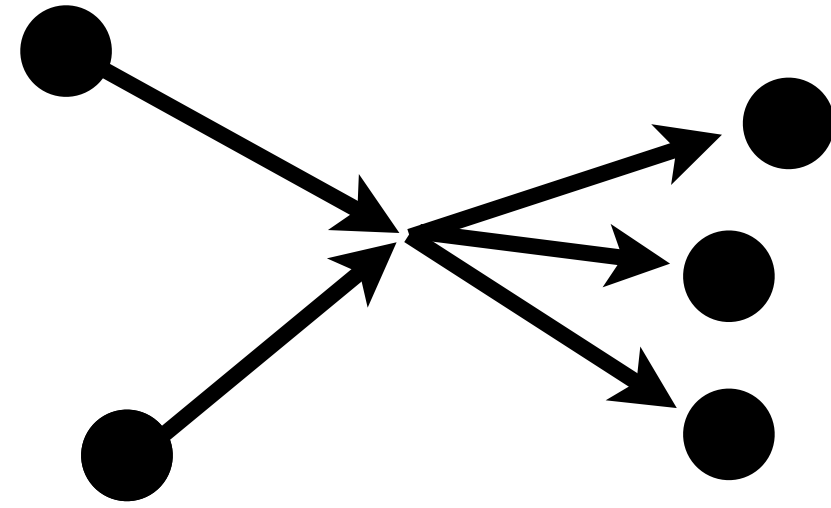
Alexandrou, Leskovec, Meinel, Negele,

Paul, Petschlies, Pochinsky, Rendon, Syritsyn(2018)

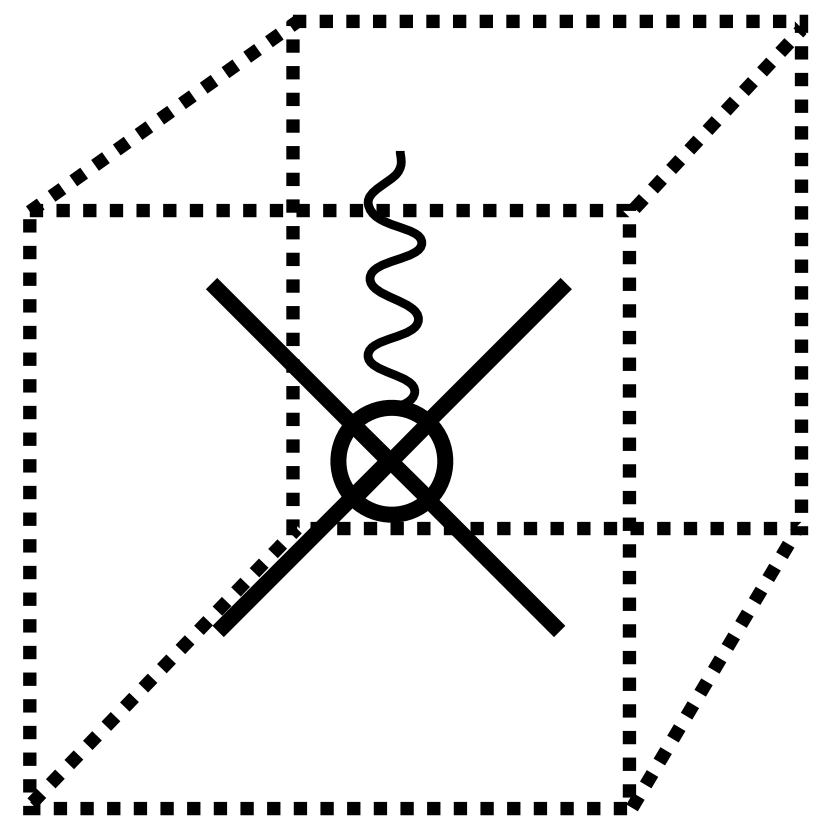
# Future of spectroscopy



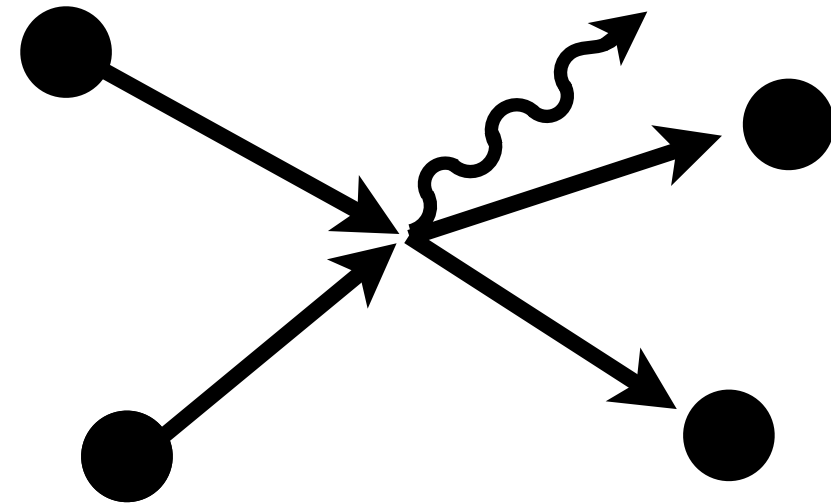
Hansen & Sharpe ('14, '15)  
Mai & Döring ('17)  
RB, Hansen & Sharpe ('18)  
Hansen, Romero-Lopez & Sharpe ('20)  
Blanton & Sharpe ('20)  
Jackura et al. ('20)



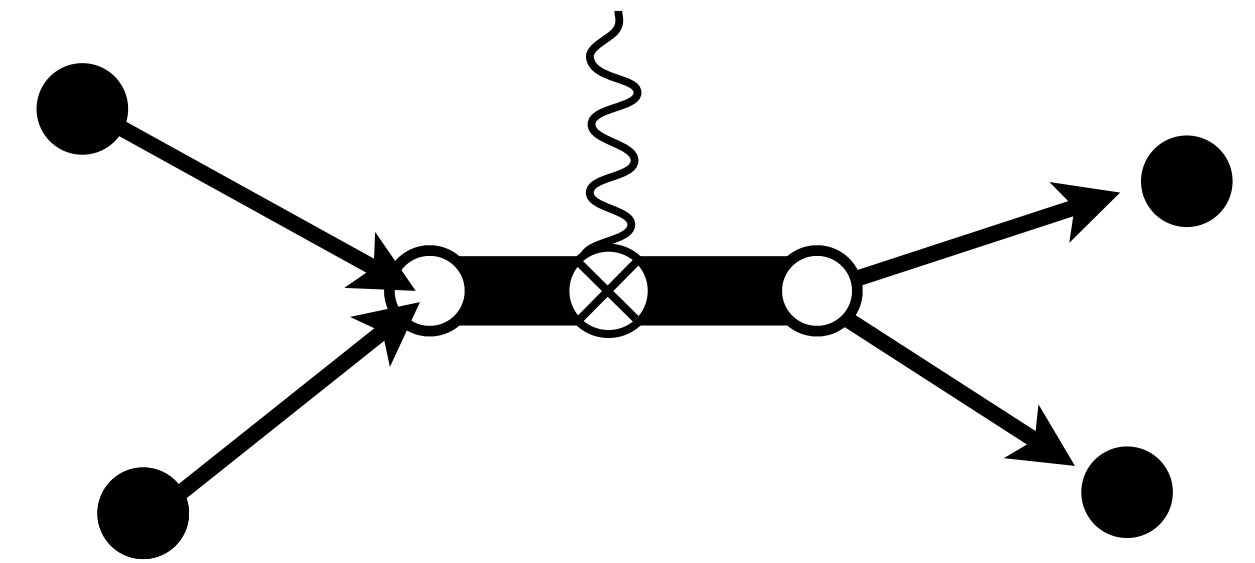
# Future of structure



RB & Hansen ('15)  
Baroni, RB, Jackura, Hansen  
& Ortega-Gama ('18)



RB, Jackura, Ortega-Gama,  
Sherman ('21)





# HadSpec

## Jefferson Lab



Edwards

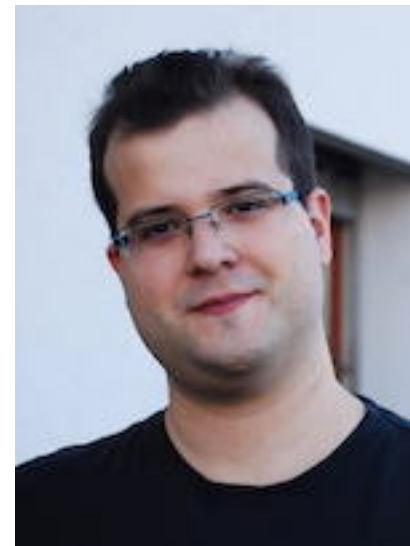


Chen



Winter

## Old Dominion University / Jefferson Lab



Leskovec



Jackura

## DAMTP, University of Cambridge



Wilson

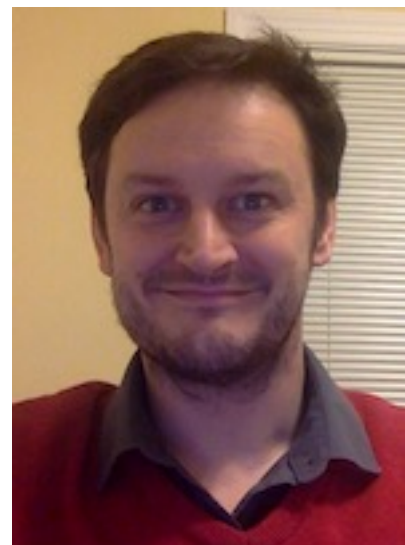


Thomas



Chakraborty

## William and Mary / Jefferson Lab



Dudek



Rodas

## Oak Ridge National Lab



Joó

## Edinburgh



Hansen

## Trinity College, Dublin



Ryan



Peardon

### Meson Spectrum

JHEP05 021 (2013)  
PRD88 094505 (2013)  
JHEP07 126 (2011)  
PRD83 111502 (2011)  
PRD82 034508 (2010)  
PRL103 262001 (2009)

### Baryon Spectrum

PRD91 094502 (2015)  
PRD90 074504 (2014)  
PRD87 054506 (2013)  
PRD85 054016 (2012)  
PRD84 074508 (2011)

### Scattering

arXiv: 2102.04973  
PRL 126 (2021)  
JHEP 02 (2021)  
JHEP 02 (2021)  
PRD 103 (2021)  
PRD 103 (2021)  
PRD 100 (2019)  
PRL 123 (2019)  
JHEP 07 (2018)  
JHEP 11 (2017)  
PRD 97 (2018)

PRL118 022002 (2017)  
JHEP011 1610 (2016)  
PRD93 094506 (2016)  
PRD92 094502 (2015)  
PRD91 054008 (2015)  
PRL113 182001 (2014)  
PRD87 034505 (2013)  
PRD86 034031 (2012)  
PRD83 071504 (2011)

### Electroweak

PRD93 114508 (2016)  
PRL115 242001 (2015)  
PRD91 114501 (2015)  
PRD90 014511 (2014)

### Techniques

PRD85 014507 (2012)  
PRD80 054506 (2009)  
PRD79 034502 (2009)  
PRD 101 (2020)

### Formalism

JHEP 04 (2021)  
JHEP 07 (2020)  
PRD 101 (2020)  
PRD 101 (2020)  
PRD 100 (2019)  
JHEP 10 (2019)  
PRD 100 (2019)  
PRD 100 (2019)  
PRD 99 (2019)  
PRD 98 (2018)

PRD95 074510 (2017)  
PRD94 013008 (2016)  
PRD92 074509 (2015)  
PRD91 034501 (2015)  
PRD89 074507 (2014)

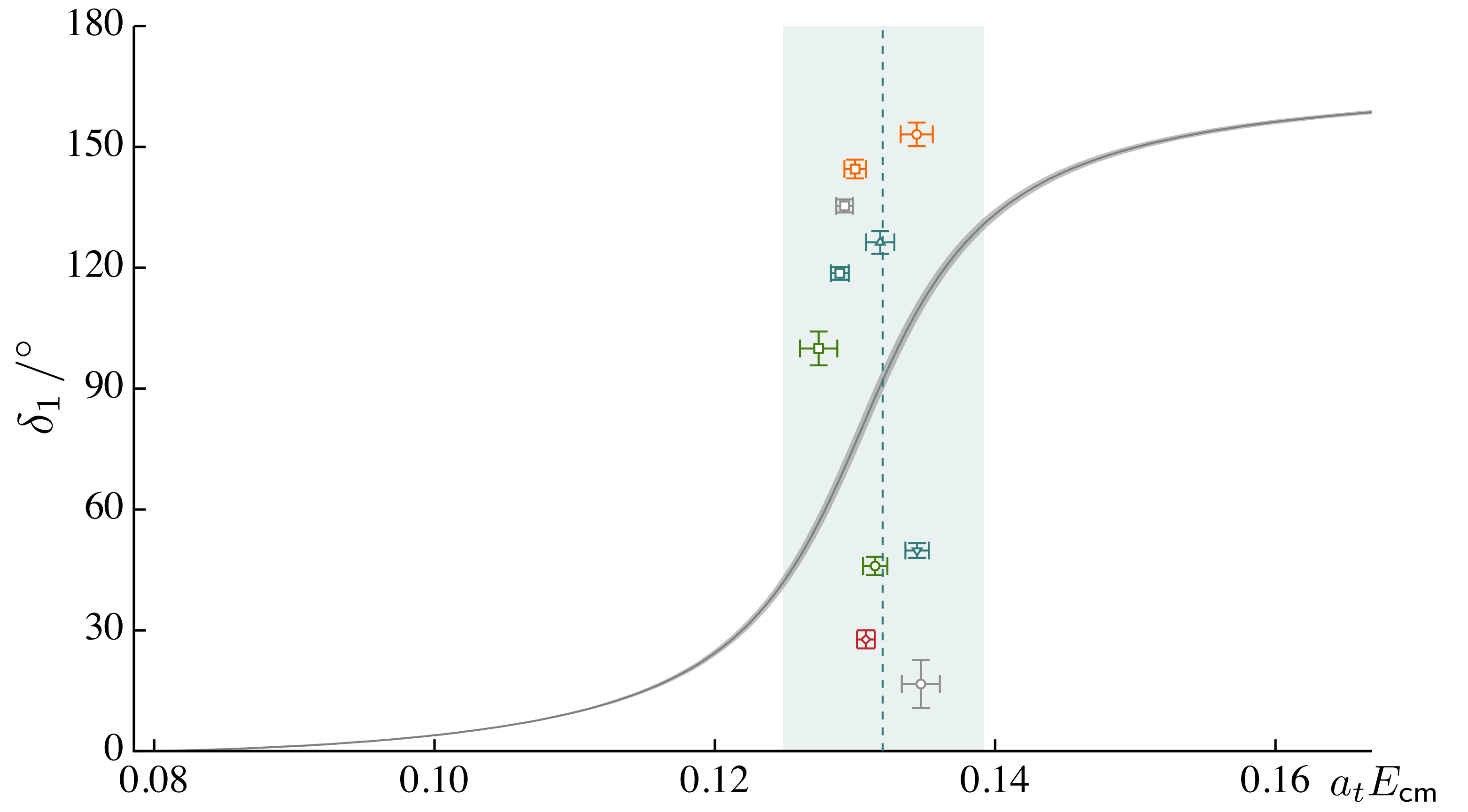
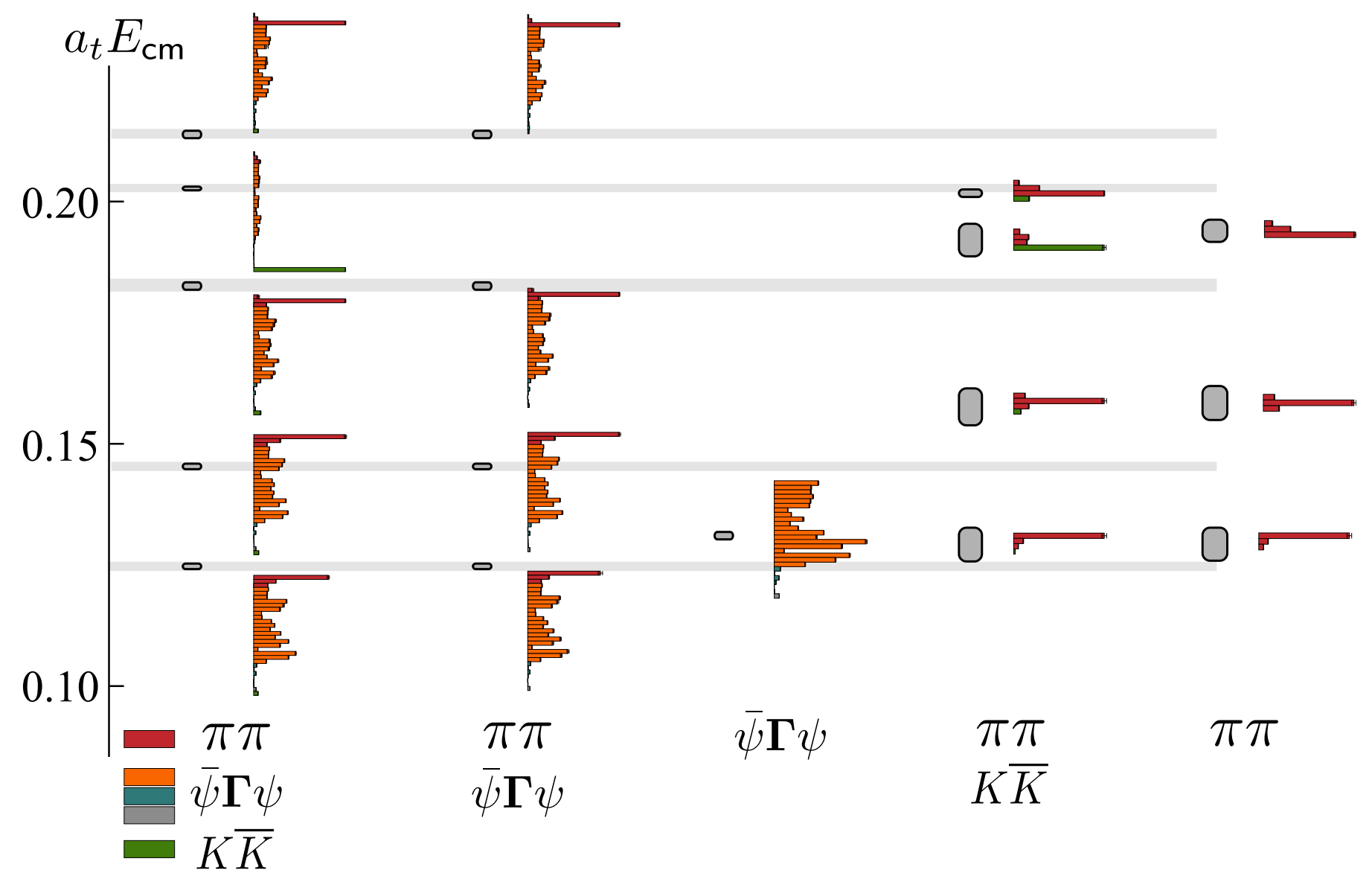
## Tata Institute, Mumbai



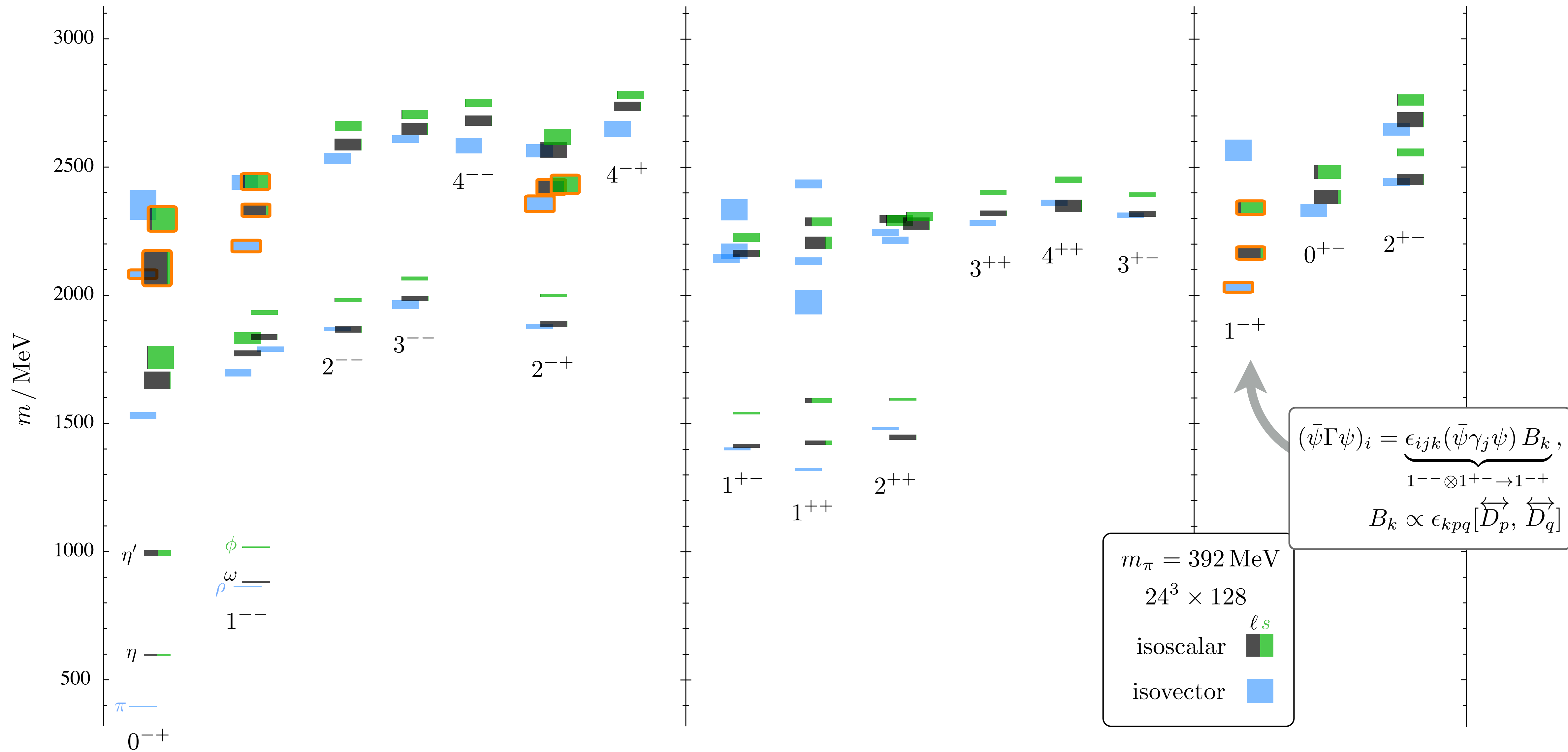
Mathur

Backup slides

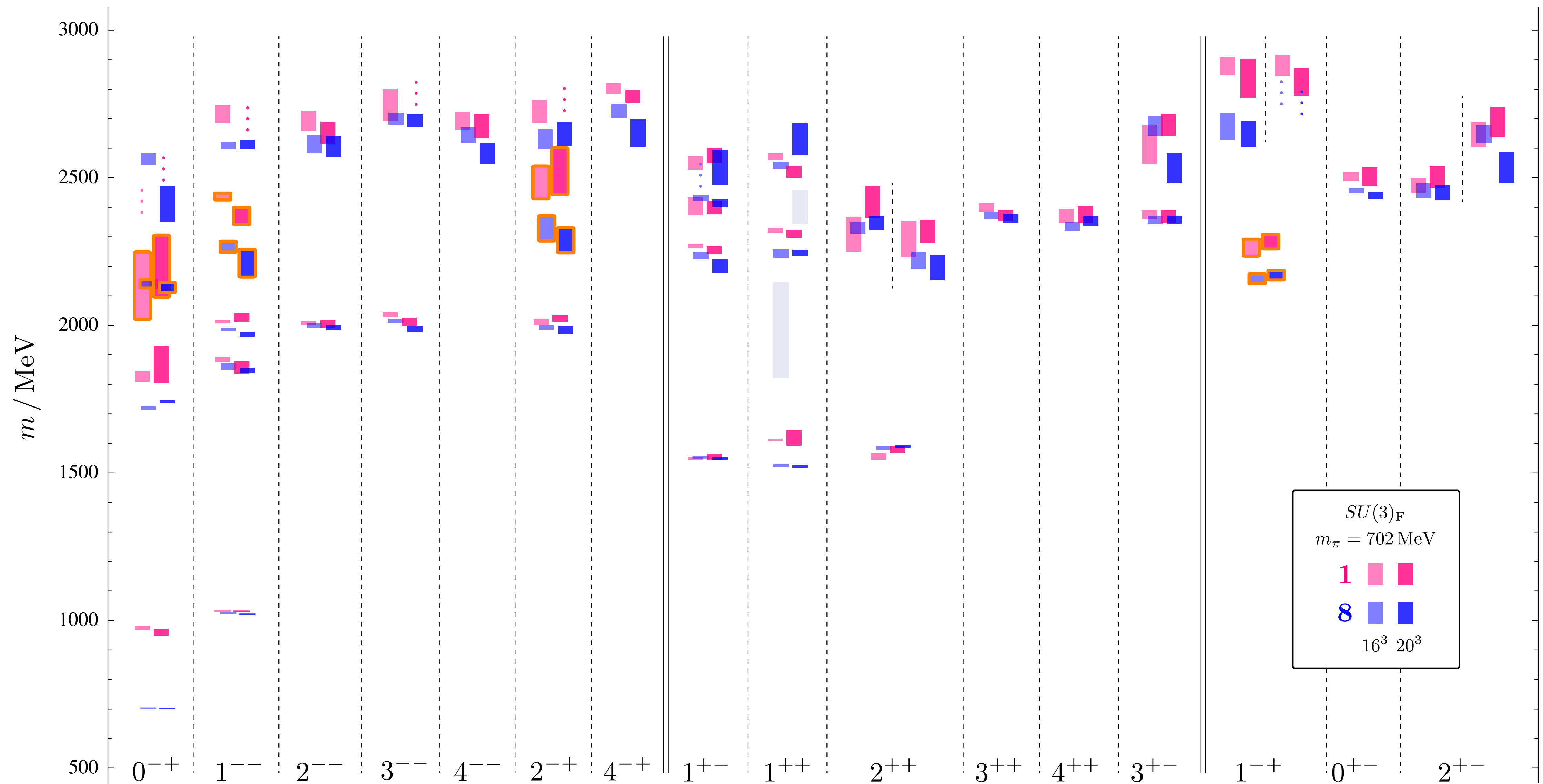
# GEVP necessary but not sufficient



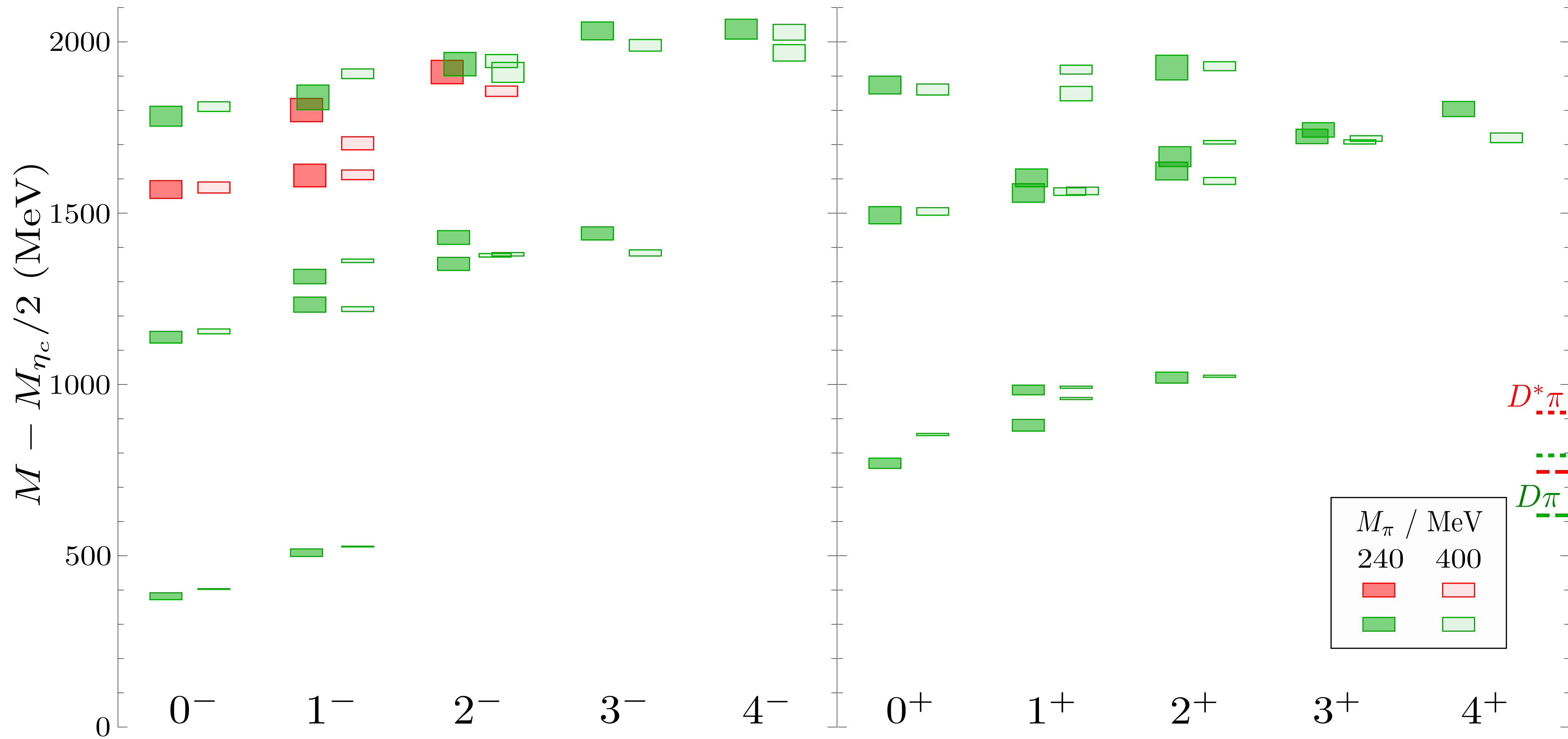
# Light mesons without multi-particle ops



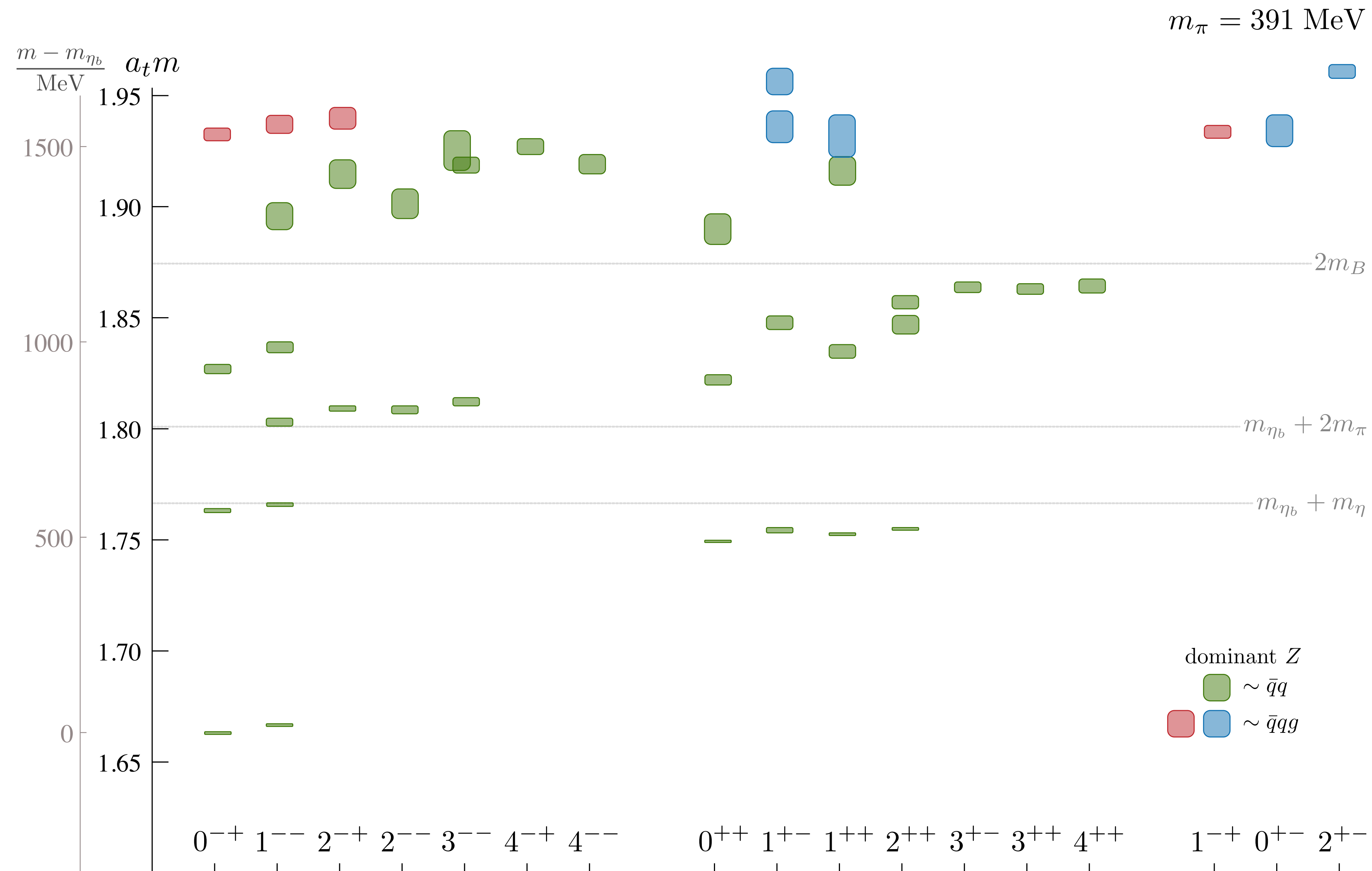
# Light mesons without multi-particle ops



# Charmonium without multi-particle ops

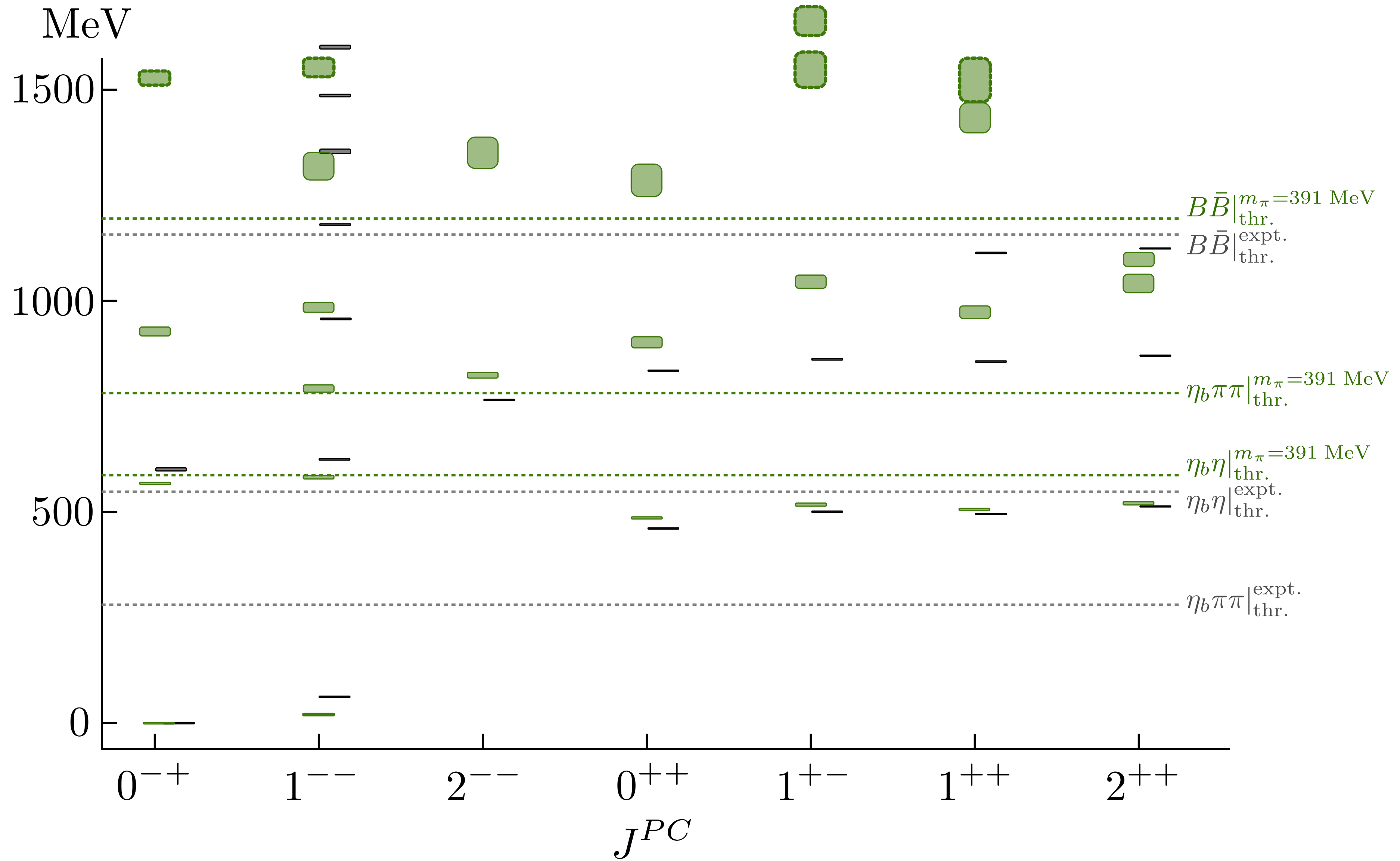


# Bottomonium without multi-particle ops



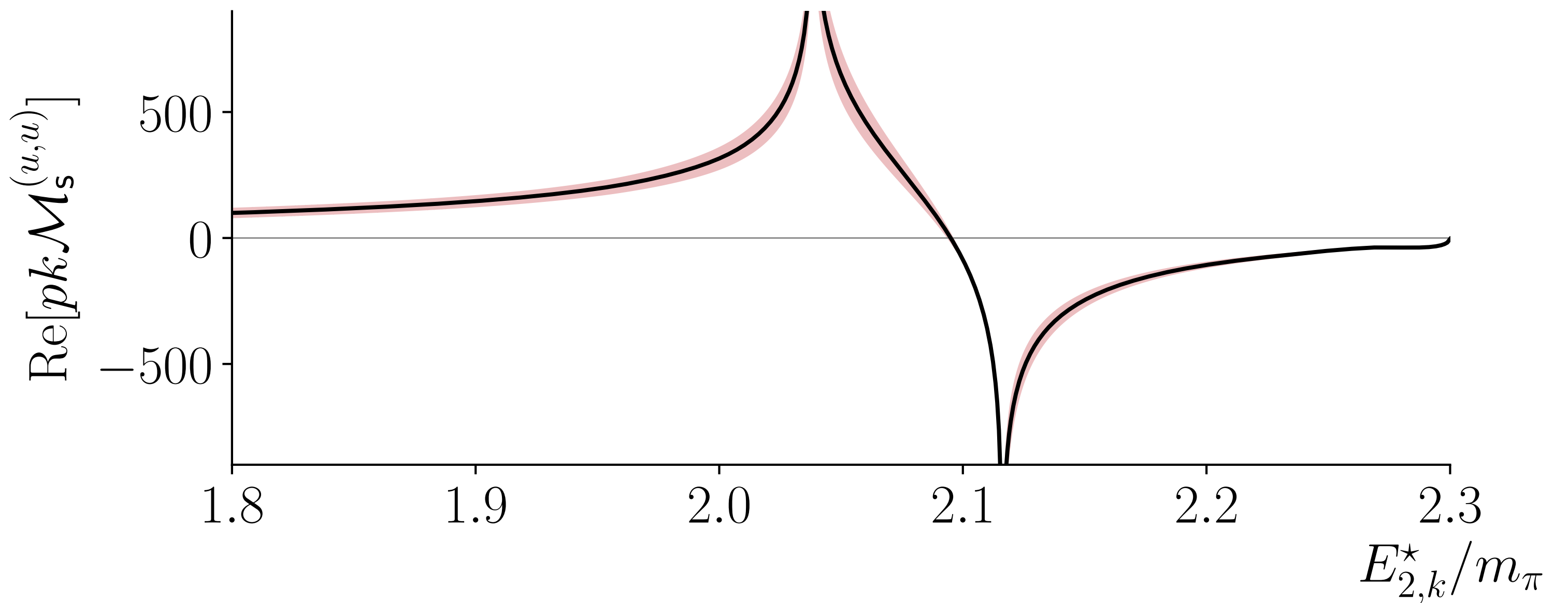
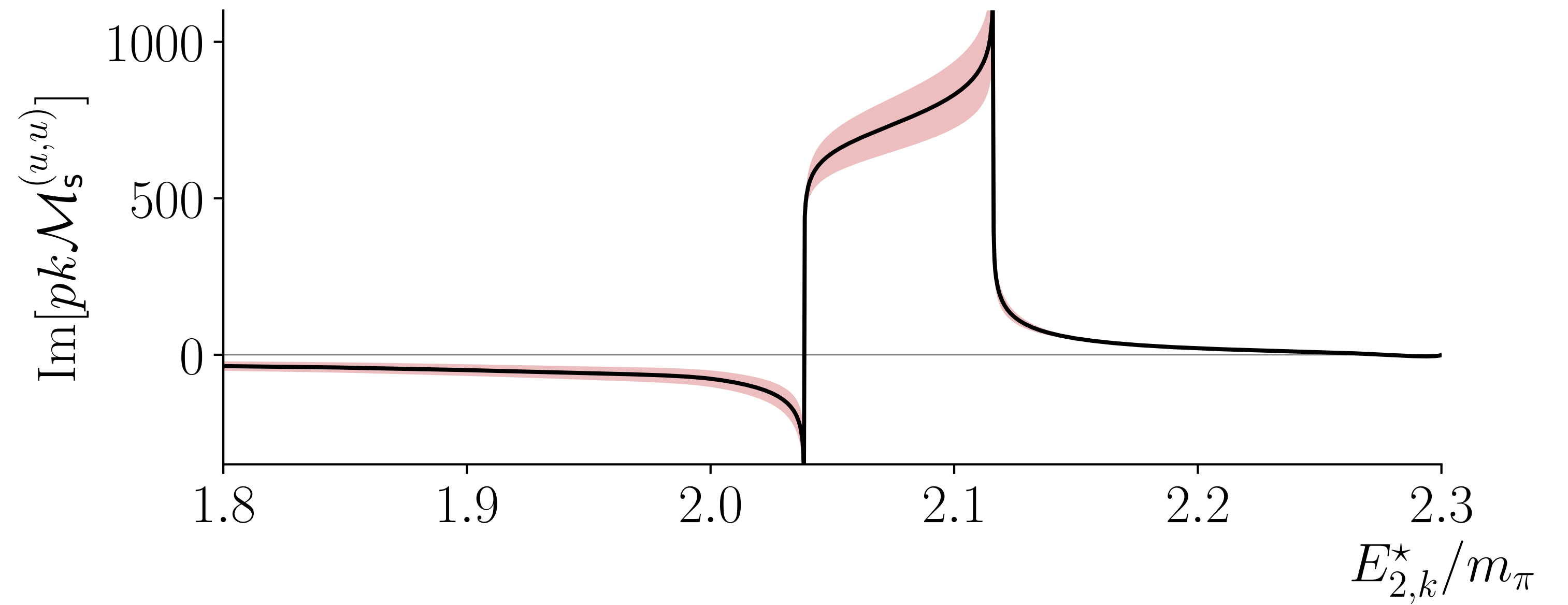
# Bottomonium without multi-particle ops

$$\frac{m - m_{\eta_b}}{\text{MeV}}$$





# three-body scattering



$m_\pi \sim 390 \text{ MeV}$

Hansen, RB, Edwards, Thomas, & Wilson (2020)

$$\mathcal{D}_s^{(u,u)}(p, k) = -\mathcal{M}_2(E_{2,p}^*)G_s(p, k, \epsilon)\mathcal{M}_2(E_{2,k}^*) - \mathcal{M}_2(E_{2,p}^*) \int_0^{k_{\max}} \frac{k'^2 dk'}{(2\pi)^2 \omega_{k'}} G_s(p, k', \epsilon) \mathcal{D}_s^{(u,u)}(k', k),$$

# Pheno $\pi_1$

Slide by Jozef Dudek

## experimental situation

5

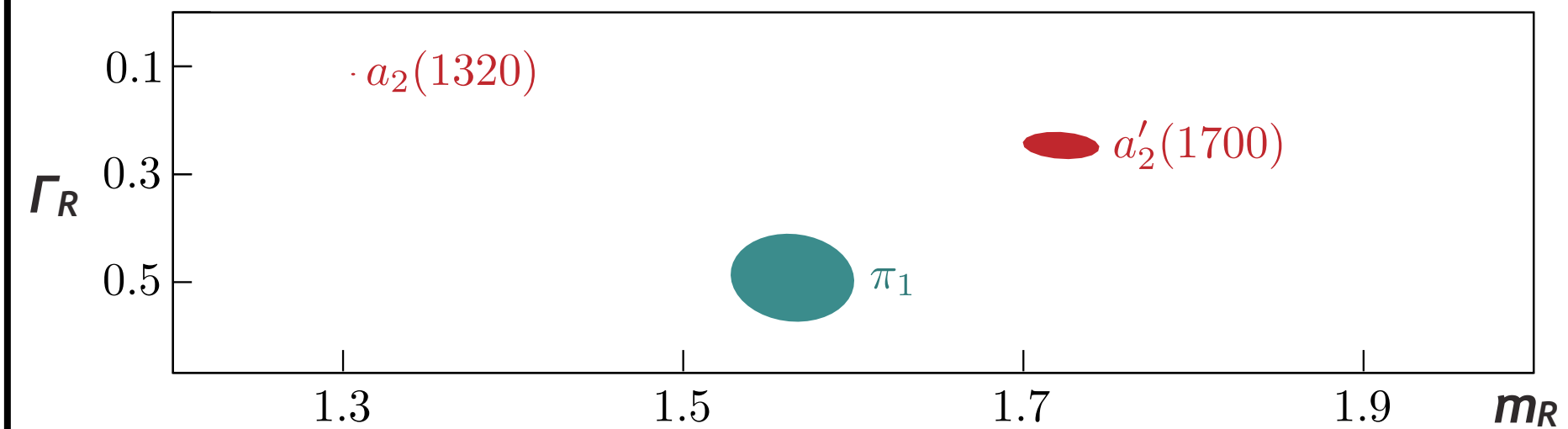
a recent JPAC analysis of COMPASS data on  $\pi\rho \rightarrow \pi\eta\rho$ ,  $\pi\rho \rightarrow \pi\eta'\rho$

### Determination of the Pole Position of the Lightest Hybrid Meson Candidate

A. Rodas,<sup>1,\*</sup> A. Pilloni,<sup>2,3,†</sup> M. Albaladejo,<sup>2,4</sup> C. Fernández-Ramírez,<sup>5</sup> A. Jackura,<sup>6,7</sup> V. Mathieu,<sup>2</sup>  
M. Mikhasenko,<sup>8</sup> J. Nys,<sup>9</sup> V. Pauk,<sup>10</sup> B. Ketzer,<sup>8</sup> and A. P. Szczepaniak<sup>2,6,7</sup>

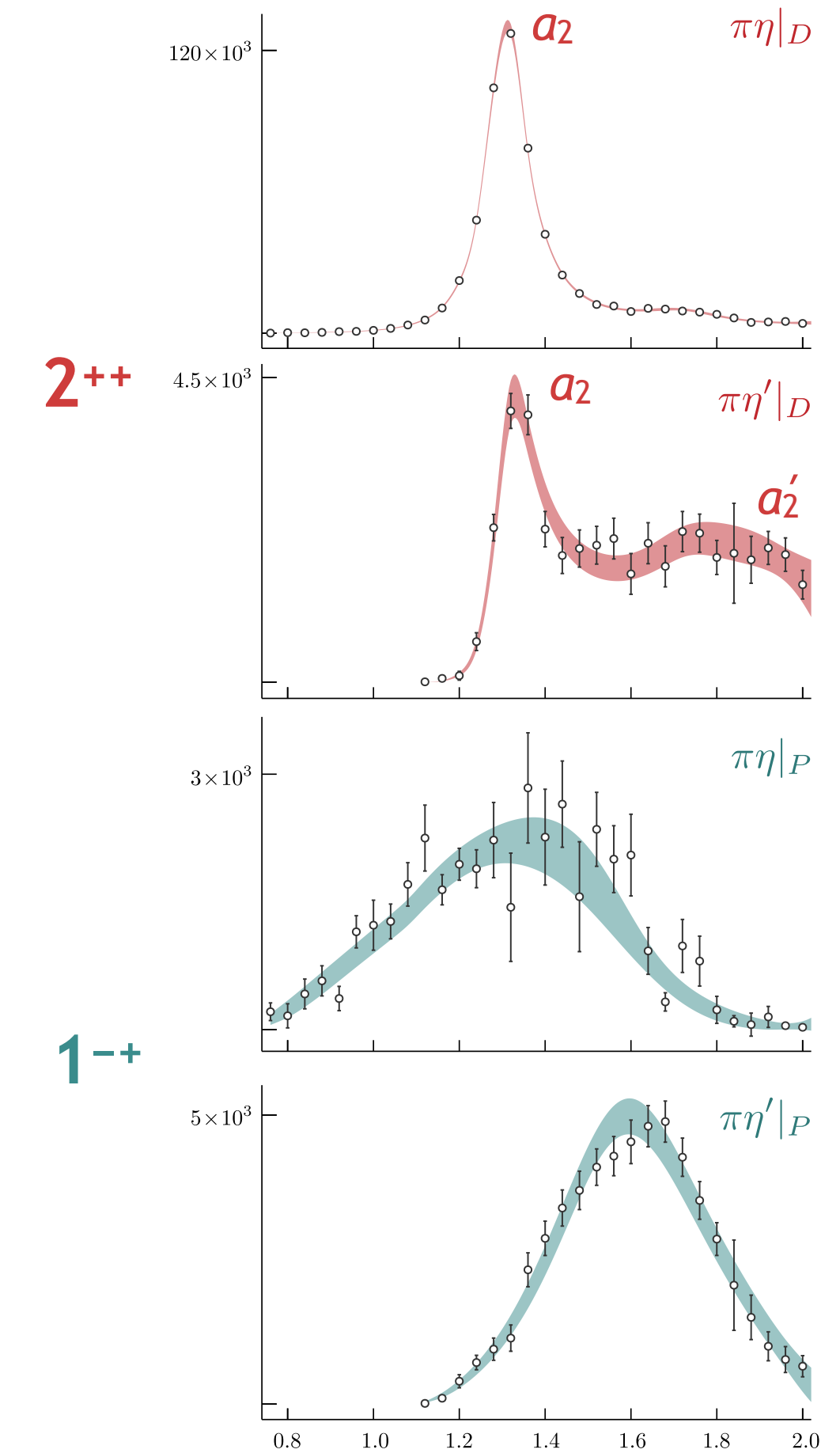
(Joint Physics Analysis Center)

pole singularity of a  $\pi_1$  resonance



$m_R=1564(89)$  MeV,  $\Gamma_R=492(115)$  MeV

a rather broad resonance



# Pheno $\pi_1$

Slide by Jozef Dudek

## crude extrapolation to physical point

17

core assumption: couplings scale only with the relevant barrier factor  $k^\ell$

use PDG masses & COMPASS/JPAC  $\pi_1$  mass

generates for a  $\pi_1$  at 1564 MeV:

$$\Gamma_{TOT} \sim 140\text{-}600 \text{ MeV}$$

$$\Gamma(\pi\eta) \approx 1 \text{ MeV}$$

$$\Gamma(\pi\eta') \approx 20 \text{ MeV}$$

$$\Gamma(\pi\rho) \approx 12 \text{ MeV}$$

$$\Gamma(\pi b_1) \sim 140\text{-}530 \text{ MeV}$$

JPAC/COMPASS candidate:

$$\Gamma_{TOT} \sim 492(115) \text{ MeV}$$

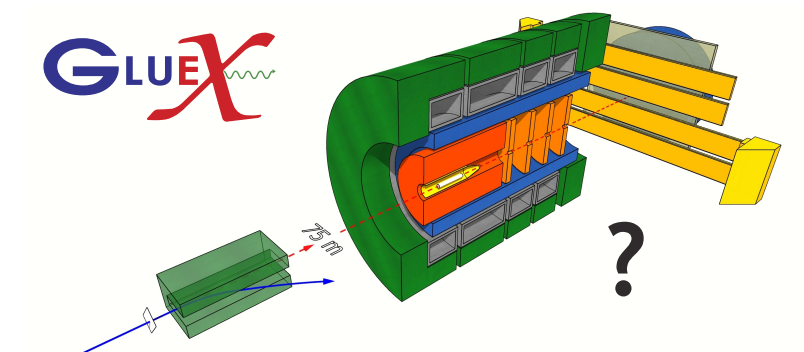
Kopf et al analysis:

$$\Gamma_{TOT} \sim 388(10) \text{ MeV}$$

$$\Gamma(\pi\eta') / \Gamma(\pi\eta) \sim 6.5(1)$$

if correct, suggests prior observations in  $\pi\eta$ ,  $\pi\eta'$ ,  $\pi\rho$   
are in heavily suppressed decay channels

$\pi b_1 \rightarrow \pi\pi\omega \rightarrow \pi\pi\pi\pi\pi$



# Lattice $\pi_1$ poles and couplings

