Flavor diagonal matrix elements

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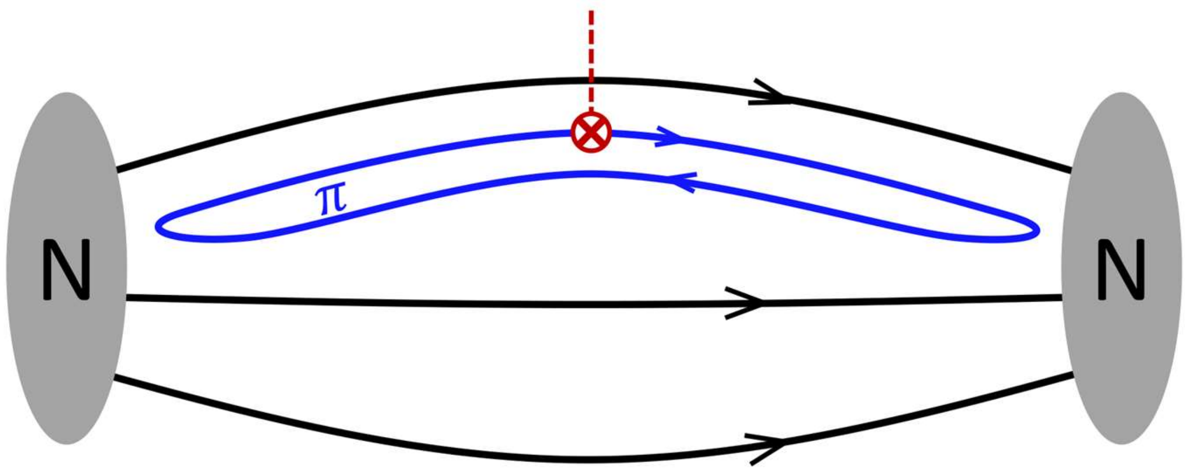

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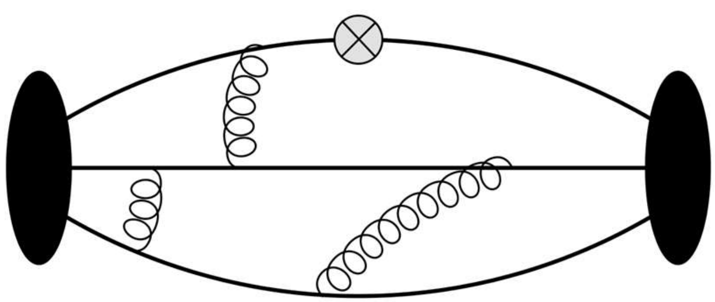
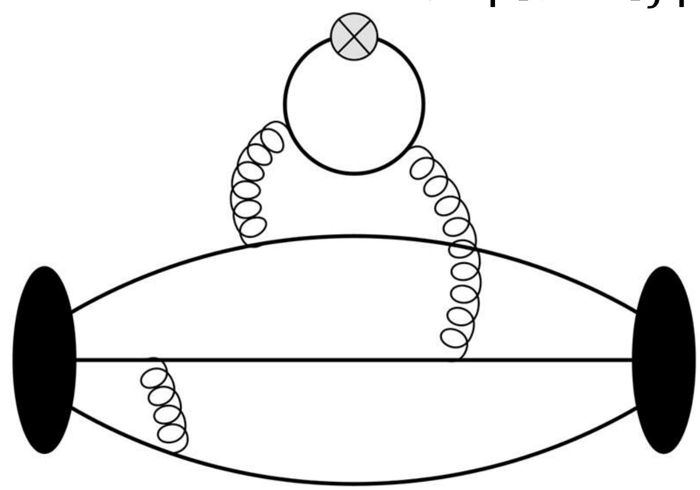
USQCD All Hands Meeting April 2021

Flavor diagonal matrix elements require high precision measurements of quark bilinear operators within the nucleon state for both "connected" and "disconnected" 3-point correlation functions,

Nucleon charges g_A, g_T, and g_S obtained from ME $\langle N | \bar{q}_i \, \Gamma \, q_j | N \rangle$



Connected



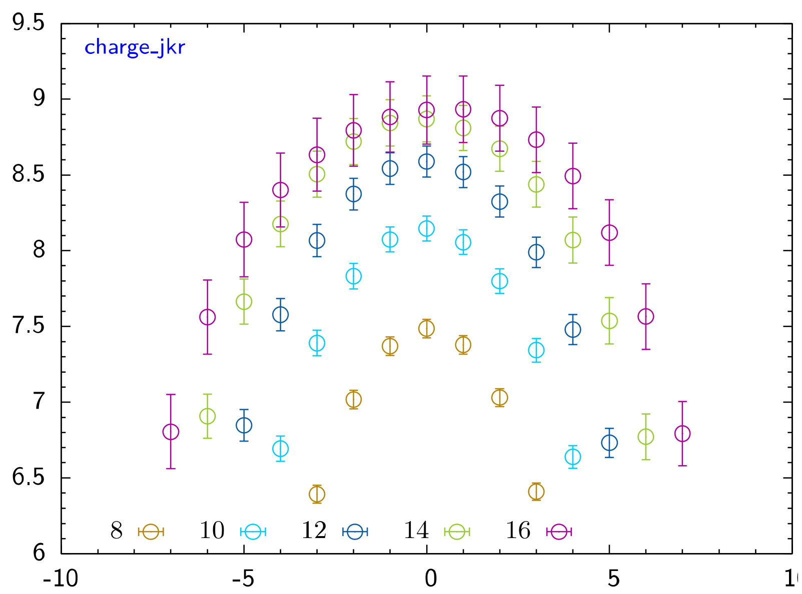
Disconnected

Outline of talk

- Methodology for calculation of 3-point functions very well established
  - Signal in connected versus disconnected
- Removing excited state contributions (ESC)
  - Fits using the spectral decomposition to connected plus disconnected terms
  - What states contribute? Do $N\pi$ states contribute?
- Renormalization
  - Constructing the full mixing matrix
- Chiral-Continuum-Finite-Volume fits

When are n-state ESC fits reliable?

- Spectral decomposition of 3-point function tells us
  - Data for a given τ should be symmetric about $\tau/2$
  - Convergence should be monotonic for large enough τ especially when only "one" excited state contribution is left
  - Only positive parity intermediate states contribute in the fits

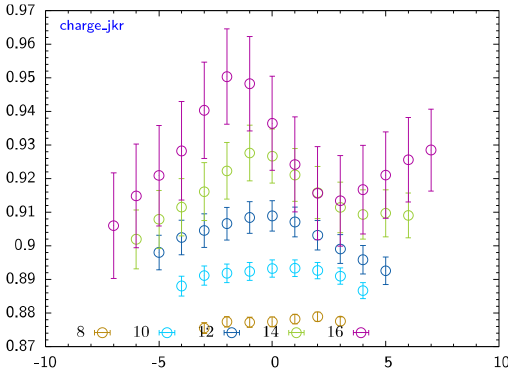


All discussion based on Clover-on-HISQ data

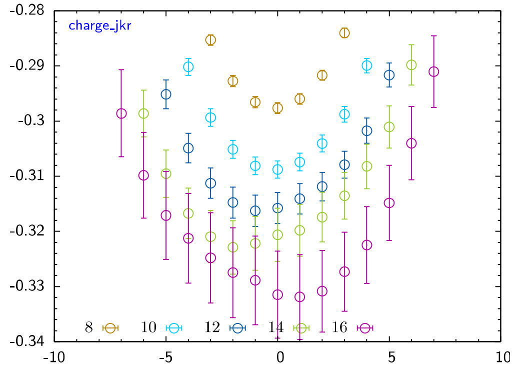
$a \approx 0.09fm,\ M_\pi = 135\ MeV,\ M_\pi L = 3.9$

- Source-sink time separation τ =8,10,12,14,16 (0.7~1.4fm)
- 1290/1270 configs
- 128 measurements of connected per config
- 10,000 sources for disconnected loop per config

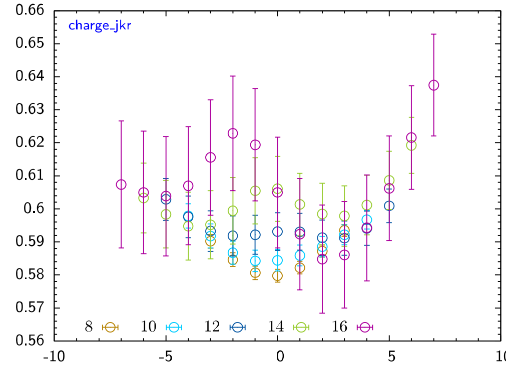
$g_A^{u,conn}$



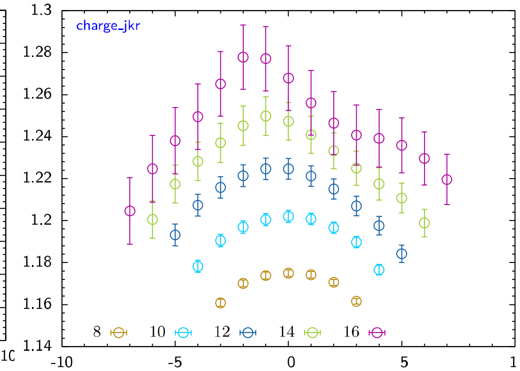
$g_A^{d,conn}$

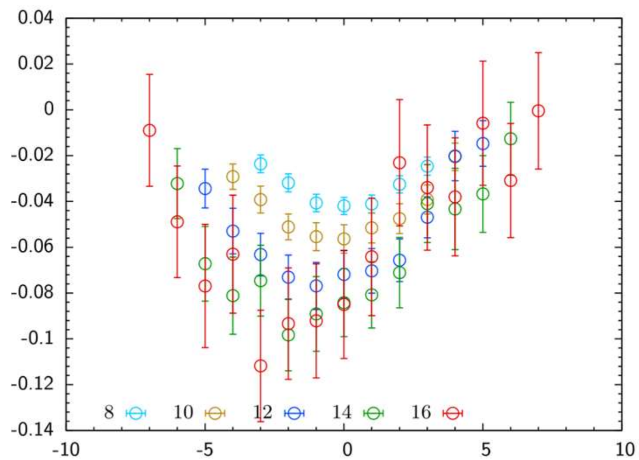


$g_A^{u+d,conn}$

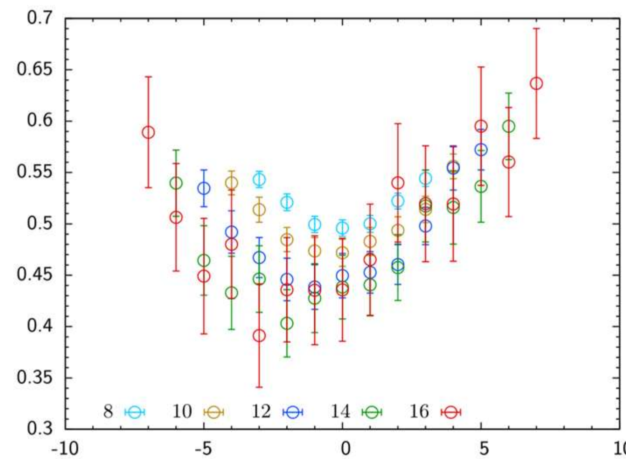


g_A^{u-d}





$g_A^{l,disc}$



g_A^{u+d}

Also see “anatomy of ESC” in appendix D in arXiv:2103.05599

g_A

- Signal in $g_A^{u,conn}$, $g_A^{d,conn}$, $g_A^{l,disc}$ versus τ improves together
- ESC in g_A^{u-d} adds

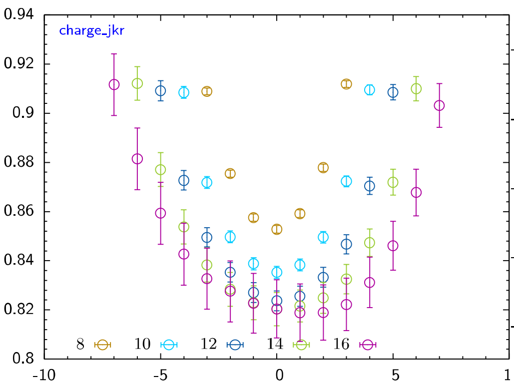
  ⇒ Excited-state fits are reasonable

  ⇒ Open issue of contribution of $N\pi$ state. (arXiv:2103.05599)
- ESC in $g_A^{u+d,conn}$ subtracts

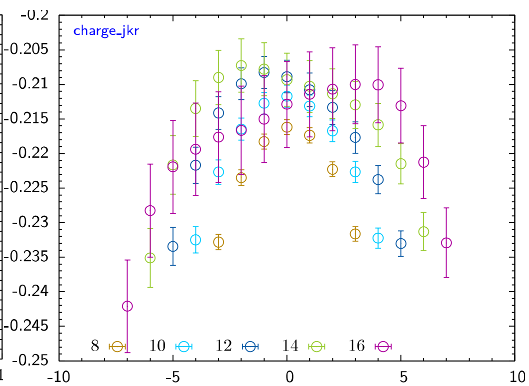
  ⇒ τ dependence hard to resolve
- $g_A^{l,disc}$ converges to a more negative value ≈ -0.1

  ⇒ Makes the quark contribution to the proton spin smaller

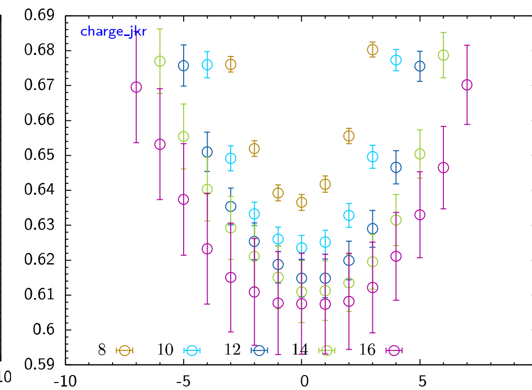
$g_T^{u,conn}$



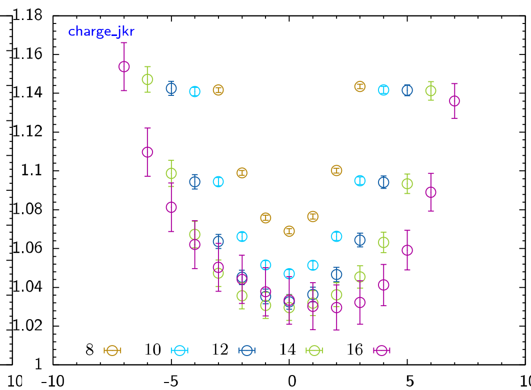
$g_T^{d,conn}$

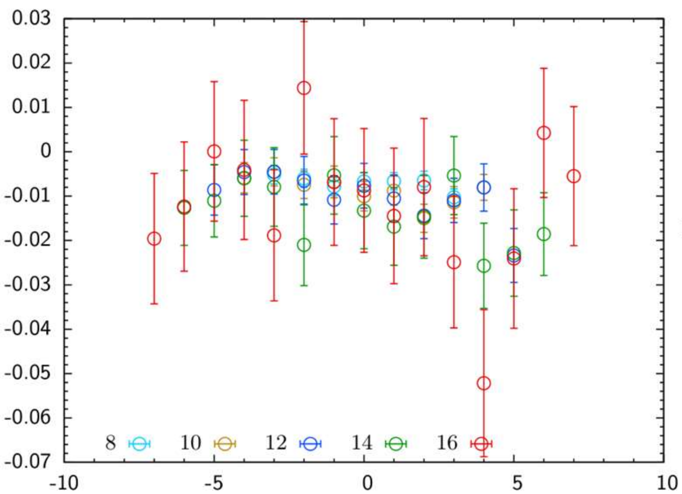


$g_T^{u+d,conn}$

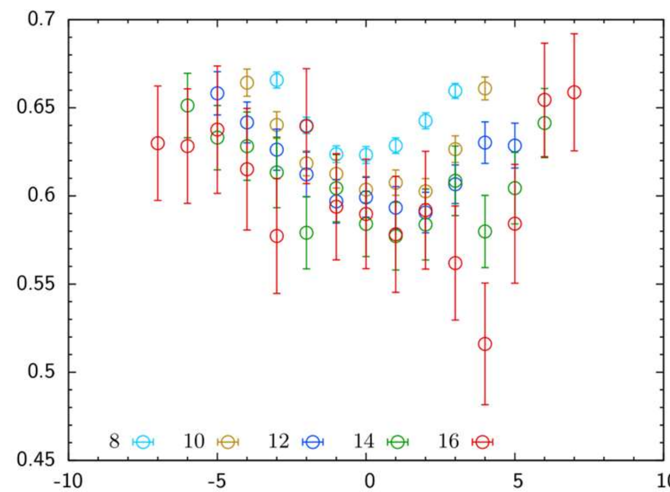


g_T^{u-d}



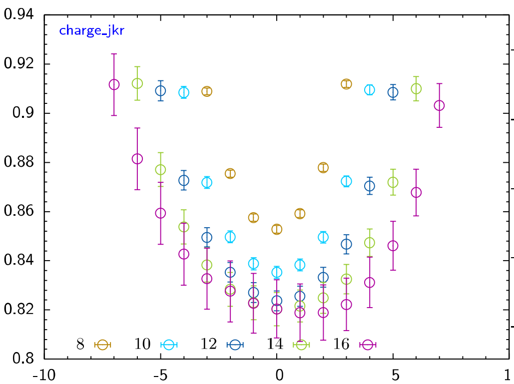


$g_T^{l,disc}$

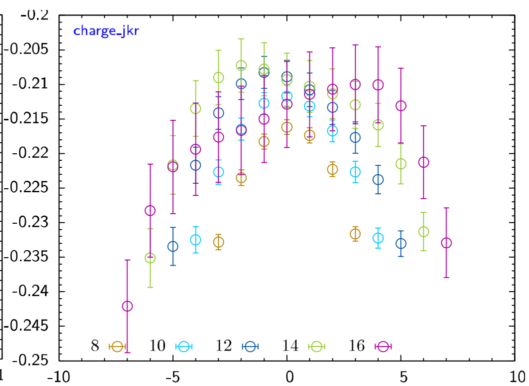


g_T^{u+d}

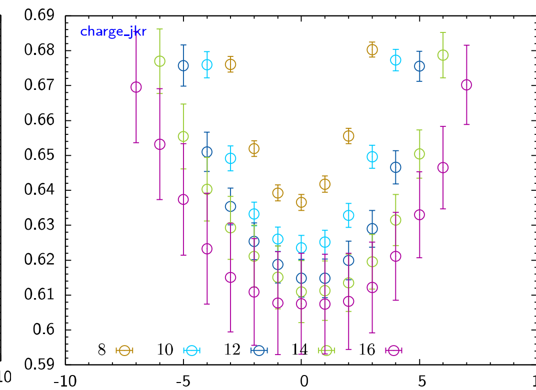
$g_T^{u,conn}$



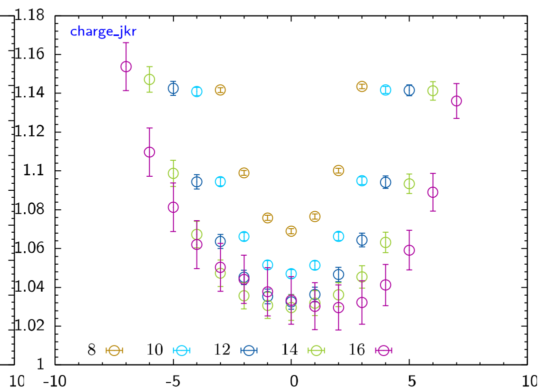
$g_T^{d,conn}$

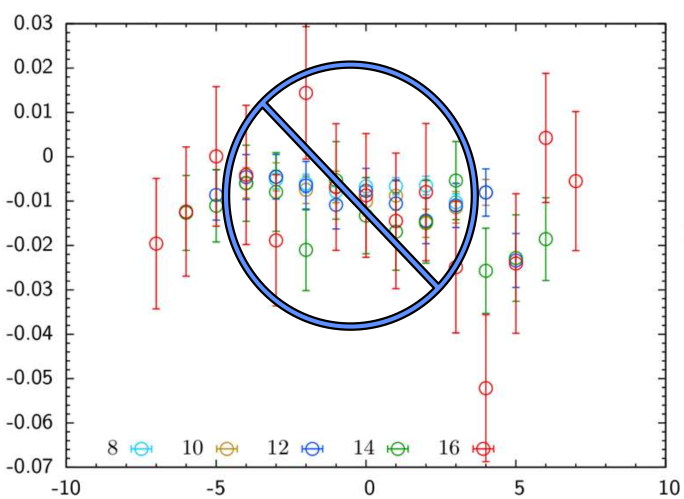


$g_T^{u+d,conn}$

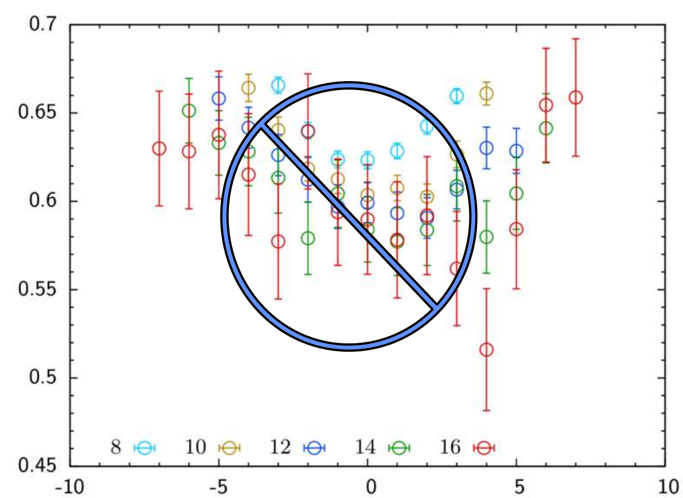


g_T^{u-d}





$g_T^{l,disc}$



g_T^{u+d}

g_T

- ESC in $g_T^{u,conn}$ larger than in $g_T^{d,conn}$

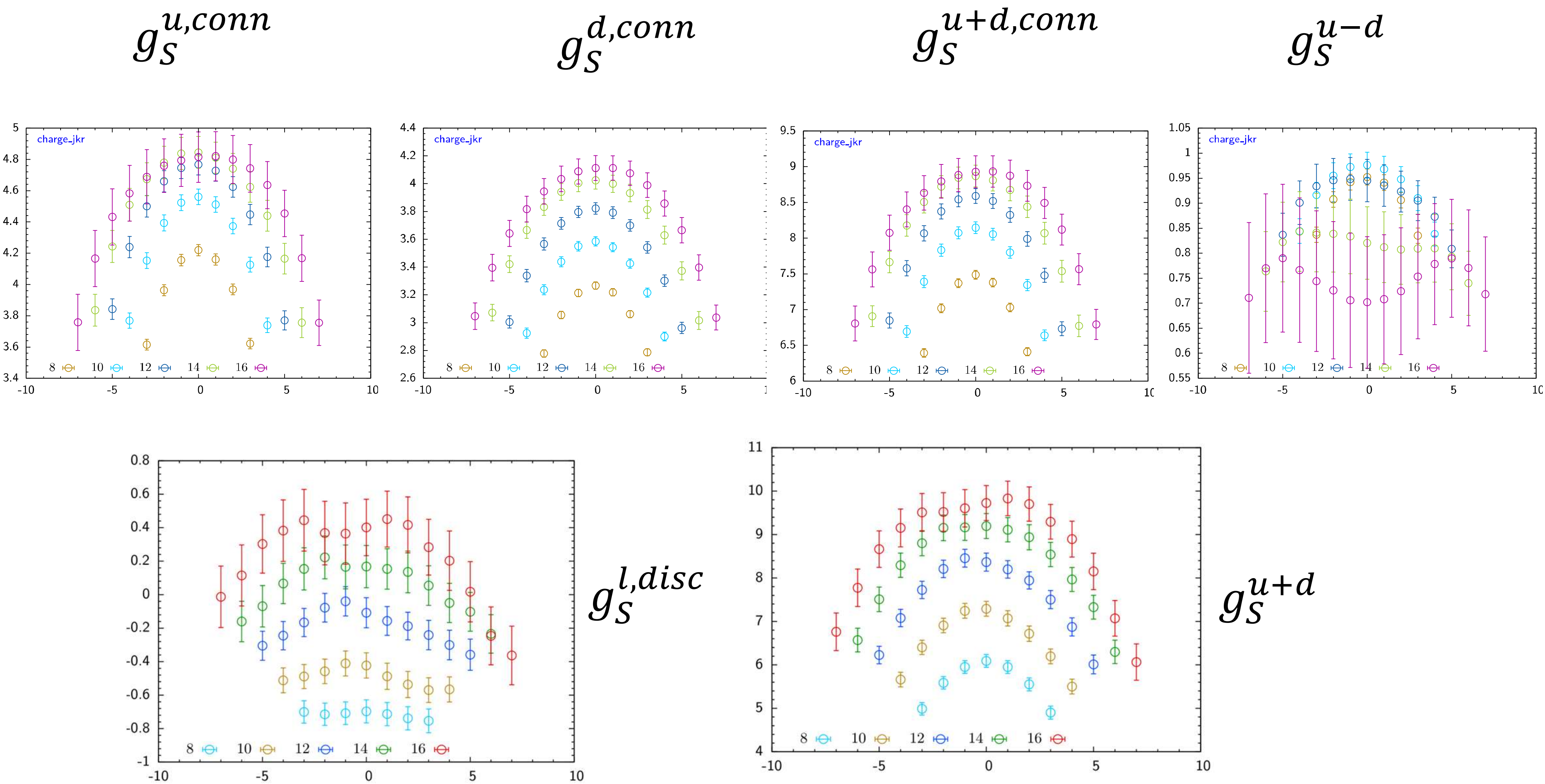
  ⇒ ESC $g_T^{u,conn}$ dominates all combinations

- ESC in g_T^{u-d} and $g_T^{u+d,conn}$ mainly driven by $g_T^{u,conn}$

  ⇒ Excited-state fits are reasonable

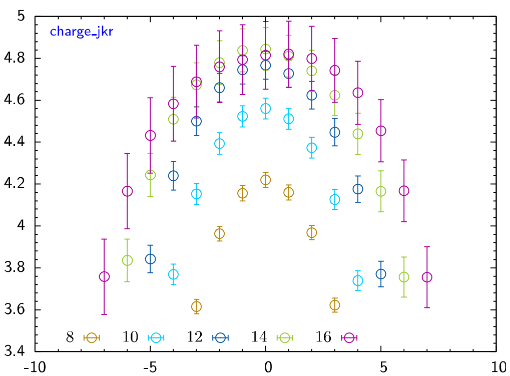
- $g_T^{l,disc}$ ~ -0.01 and noisy

  ⇒ A small contribution but makes g_T^{u+d} noisy

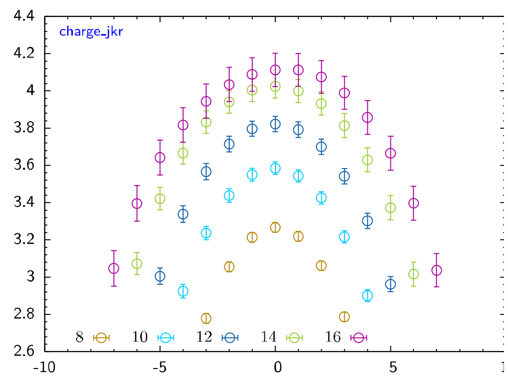


$g_S^{u,conn}$
$g_S^{d,conn}$
$g_S^{u+d,conn}$
g_S^{u-d}
charge_jkr
8 10 12 14 16
$g_S^{l,disc}$
g_S^{u+d}

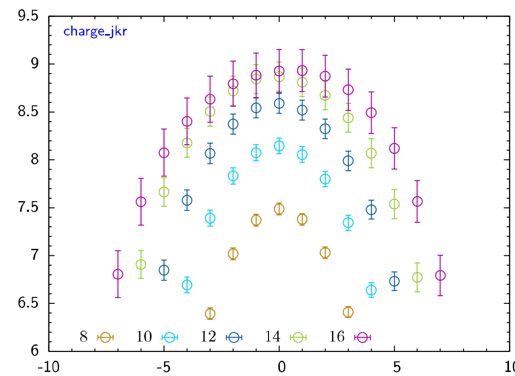
$g_S^{u,conn}$



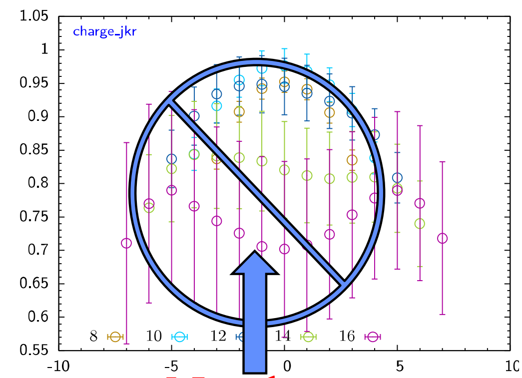
$g_S^{d,conn}$



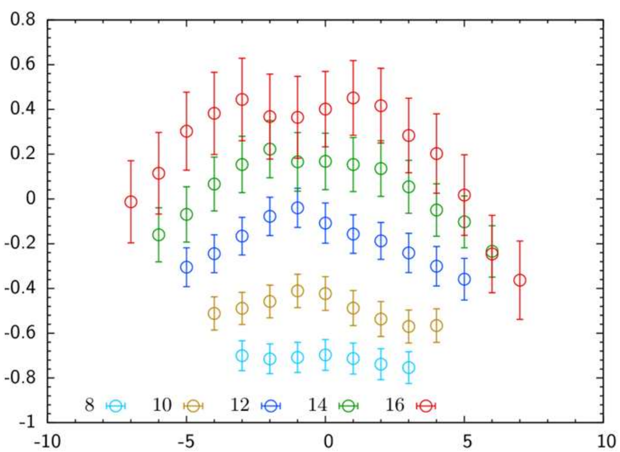
$g_S^{u+d,conn}$



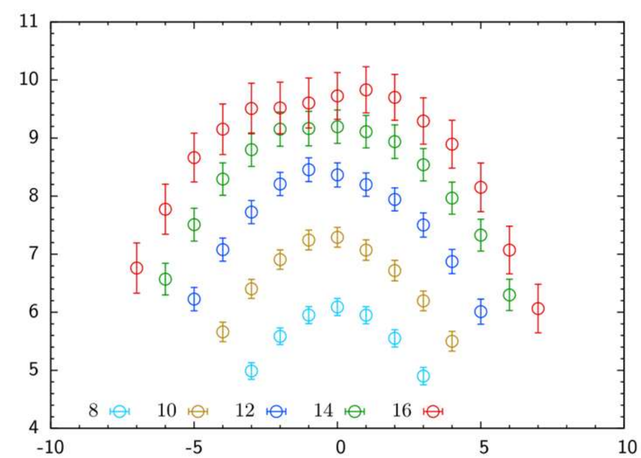
g_S^{u-d}



Needs very high statistics



$g_S^{l,disc}$



g_S^{u+d}

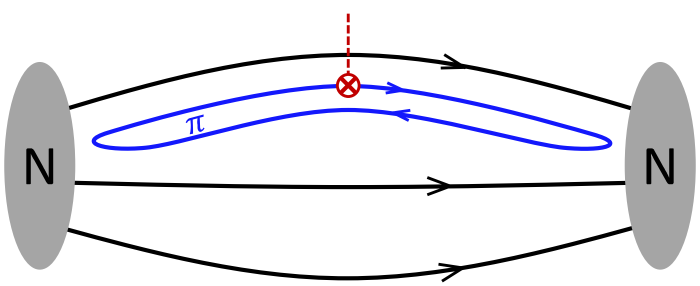
g_S

- ESC in $g_S^{u,conn}$ and $g_S^{d,conn}$ are similar
- ESC in g_S^{u-d} cancels

  ⇒ Excited-state fits need high statistics
- ESC in $g_A^{u+d,conn}$ add

  ⇒ τ dependence well-resolved
- $g_S^{l,disc}$ has a good signal

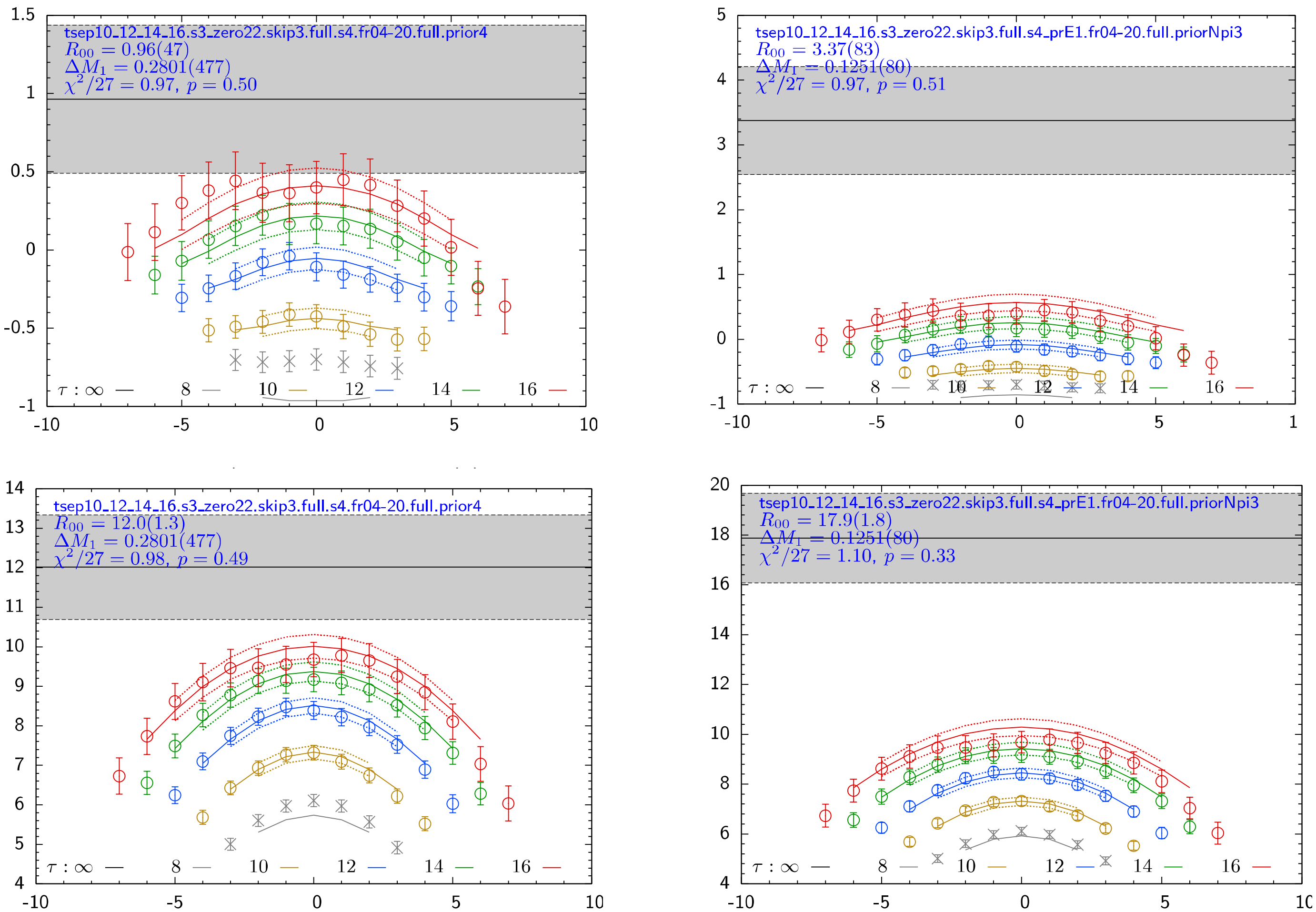
  ⇒ Makes a big contribution



without $N\pi$ | with $N\pi$

$g_S^{l,disc}$

g_S^{u+d}



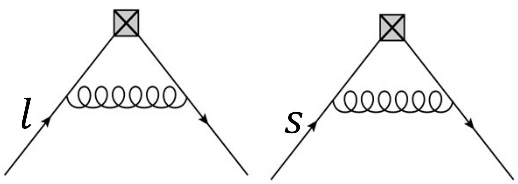
σ-term ~ 40 MeV | σ-term ~ 60 MeV

g_S and the nucleon sigma-term

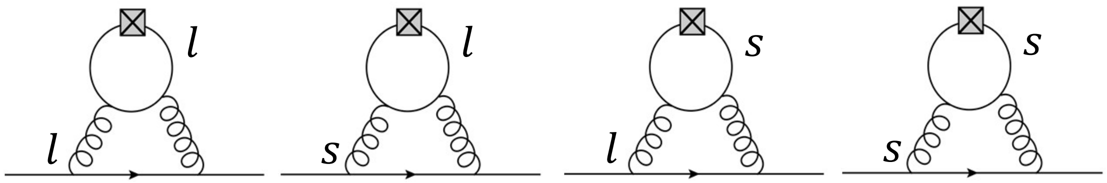
- Signal in $g_S^{u+d,conn}$ and $g_S^{l,disc}$ is good
  - $\Rightarrow \tau$ dependence well-resolved
- ESC in $g_S^{u+d,total}$ from u, d, *disc* terms adds
- n-state fits to get the ground state ME are good
- ME (Nucleon σ-term) depends strongly on whether $N\pi$ is included in excited-state fits
  - χ^2 are equally good
- χPT analysis points to significant contribution of $N\pi$ state

Paper in preparation

Z factors in RI-sMOM



2 connected diagrams



4 disconnected diagrams

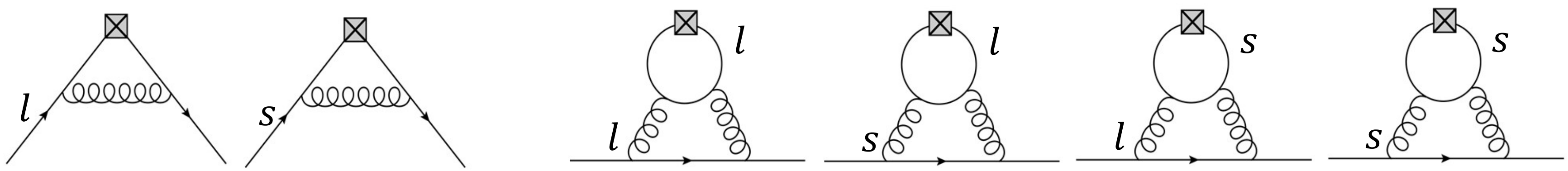
Work in the (- + s) basis

$$Z_\Gamma = \begin{pmatrix} Z_\Gamma^{u-d} & 0 & 0 \\ 0 & Z_\Gamma^{u+d,u+d} & Z_\Gamma^{u+d,s} \\ 0 & Z_\Gamma^{s,u+d} & Z_\Gamma^{s,s} \end{pmatrix} = \begin{pmatrix} c_l & 0 & 0 \\ 0 & c_l - 2d_{ll} & -2d_{sl} \\ 0 & -d_{ls} & c_s - d_{ss} \end{pmatrix}^{-1}$$

- $c_f = \frac{1}{Z_\psi^f} \mathrm{Tr}\left[\hat{P}_\Gamma \langle f | O_\Gamma^f | f \rangle_{\mathrm{conn}} \right]$

- $d_{ff'} = \frac{-1}{Z_\psi^f} \mathrm{Tr}\left[\hat{P}_\Gamma \langle f | O_\Gamma^{f'} | f \rangle_{\mathrm{disc}} \right]$

Z factors in RI-sMOM



Diagonal elements are roughly equal

Axial: $Z_A^{u-d,u-d} \sim Z_A^{u+d,u+d} \sim Z_A^{s,s}$

Tensor: $Z_T^{u-d,u-d} \sim Z_T^{u+d,u+d} \sim Z_T^{s,s}$

Scalar: $Z_S^{u-d,u-d} \sim Z_S^{u+d,u+d} \sim Z_S^{s,s}$

$$Z_S = \begin{pmatrix} Z_S^{u-d} & 0 & 0 \\ 0 & Z_S^{u+d,u+d} & Z_S^{u+d,s} \\ 0 & Z_S^{s,u+d} & Z_S^{s,s} \end{pmatrix} = \begin{pmatrix} 0.92338(5) & 0 & 0 \\ 0 & 0.910(4) & -0.012(3) \\ 0 & -0.007(2) & 0.914(1) \end{pmatrix}$$

Summary

- Higher statistics needed to resolve τ dependence in
  - $g_A^{l,disc}$, $g_A^{u+d,conn}$
  - $g_T^{l,disc}$, $g_T^{u+d,conn}$
  - $g_S^{u-d,conn}$
- Evidence of large contribution of $N\pi$ in ES fits to g_S^{u+d}
- Nucleon σ-term changes from ~40 MeV to ~60 MeV on including the $N\pi$ excited state