$K \to \pi\pi$ in RBC/UKQCD

Masaaki Tomii (UConn)

USQCD All Hands Meeting
April 30, 2021

K → ππ w/ periodic BCs

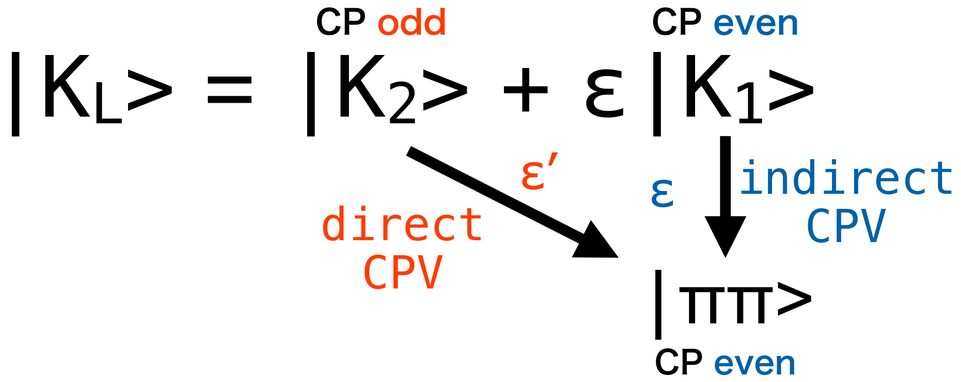
- Co-investigators

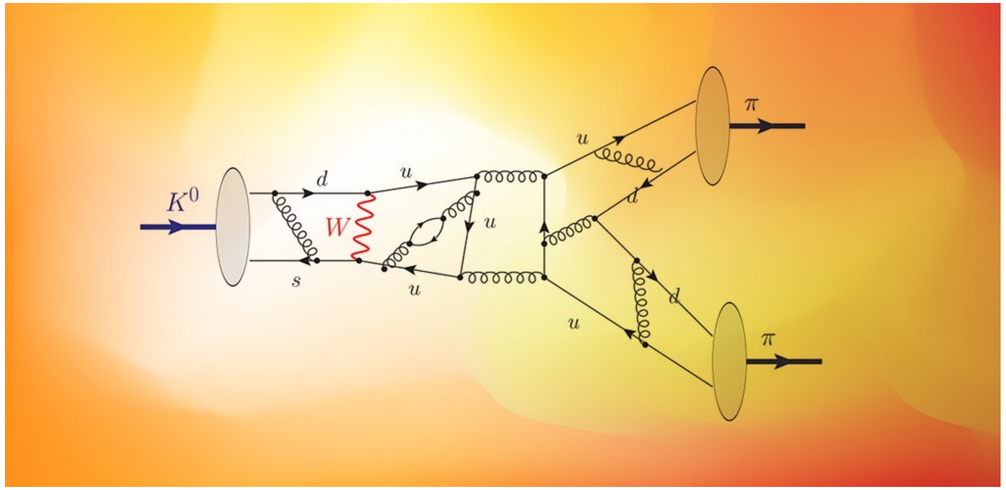
  T. Blum (PI, UConn/RBRC), D. Hoying (MSU), T. Izubuchi (BNL/RBRC), L. Jin (UConn/RBRC), C. Jung (BNL), A. Soni (BNL), MT (UConn)

- RBC & UKQCD Collaborations

- Requests
  - 13 M KNL core-hours at JLab (or BNL)
  - 235 TB new tape storage

K → ππ & Direct CPV

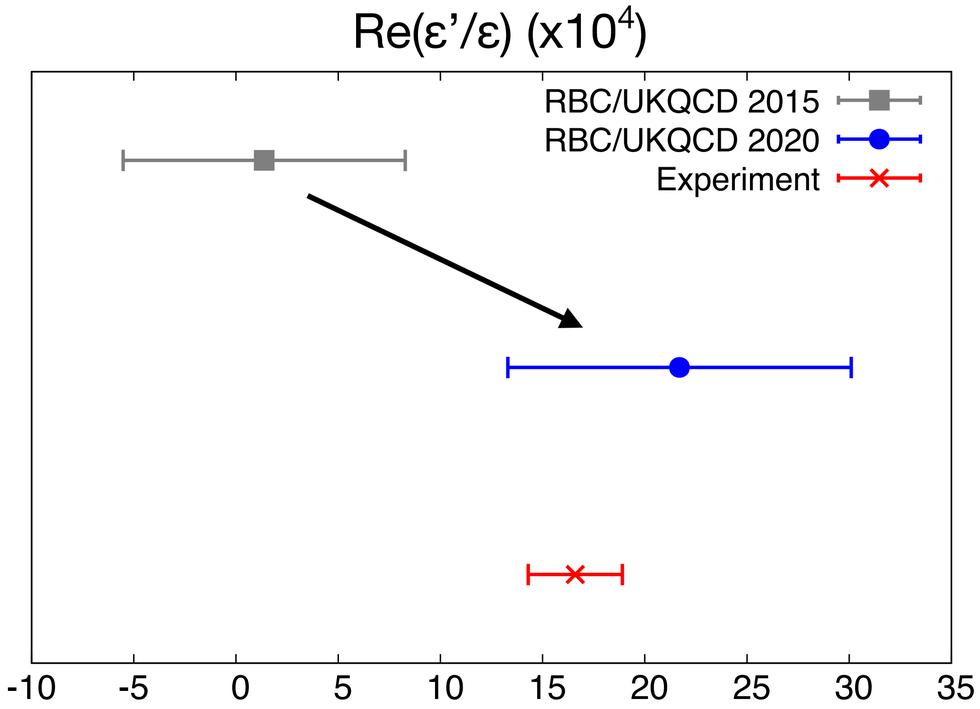




- ε' vs ε
  - Re $(\varepsilon'/\varepsilon)_{exp}$ = 16.6(2.3) x 10^{-4} (circa 2000)
  - Explained by SM?
- Key to understanding the nature of matter/anti-matter asymmetry

G-parity BC calculation done

- $E_{\pi\pi} = 2m_\pi \approx 280$ MeV state in Euclidean correlators prohibited
- Useful to extract $E_{\pi\pi} = m_K$ state at large time separations



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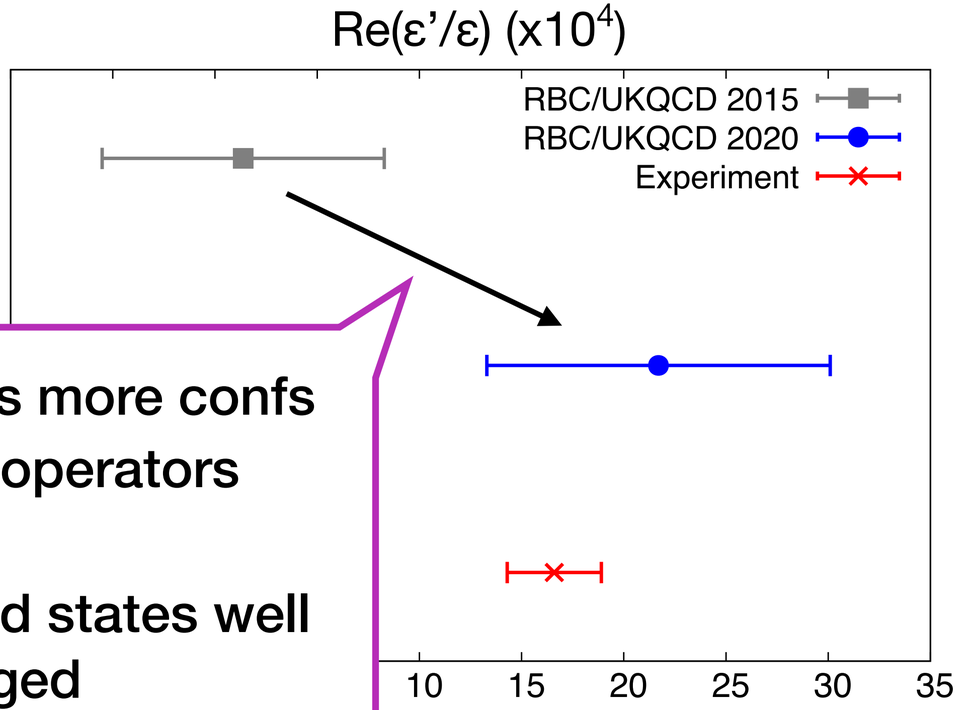

- 3+ times more confs
- # of ππ operators
  - 1 → 3
  - excited states well managed
- Step scaling in NPR

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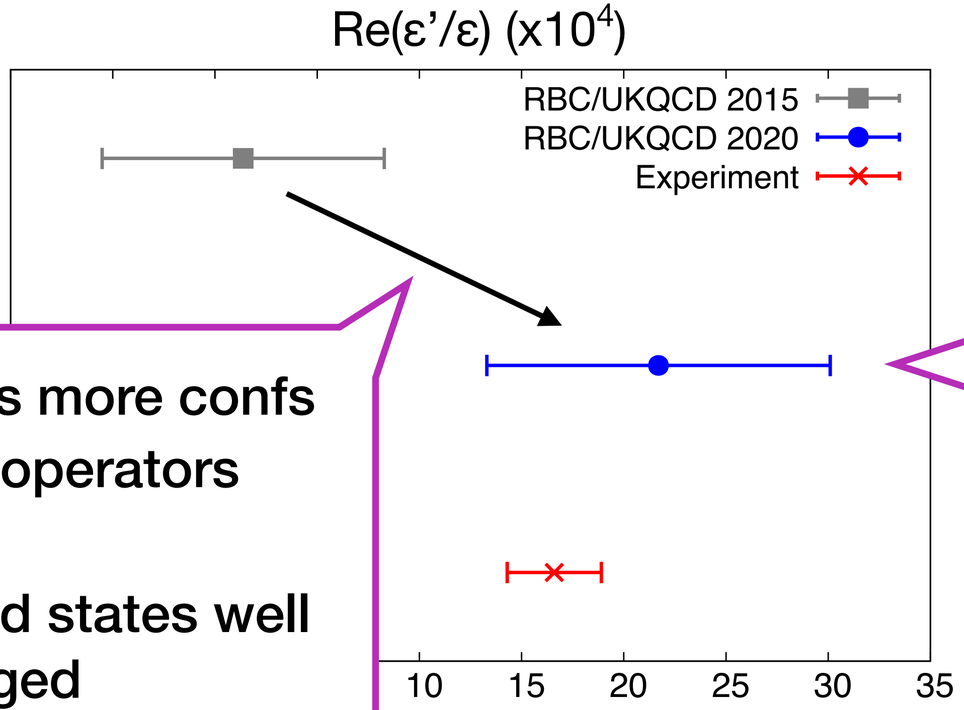

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$$21.7(2.6)_{\text{stat}}(6.2)_{\text{sys}}(5.0)_{\text{EM/IB}} \times 10^{-4}$$

- More independent calculations desired
- Systematic error
  - Isospin breaking effects
  - Finite lattice cutoff

Why periodic BCs?

- Already have lattice ensembles with physical pion mass
  - 1 GeV, 24^3 x 64, 1.4 GeV, 32^3x64 and …
  - Continuum limit possible

- Hope to introduce QED/IB effects near future
  - Difficult with G-parity boundary conditions
  - Periodic BC study valuable

- Presence of $E_{\pi\pi} = 2m_\pi$ state challenging
  - S/N ratio of $E_{\pi\pi} = m_K$ state should be the same as G-parity BC

Ensembles

- RBC/UKQCD's 2+1-flavor ensembles with Möbius domain-wall fermions at physical pion & kaon masses
  - 24^3 x 64, a^{-1} = 1.0 GeV, 250 confs
    (will be done within this current allocation year)
  - 32^3 x 64, a^{-1} = 1.4 GeV, 200 confs
    (main target for next allocation year)

What to calculate

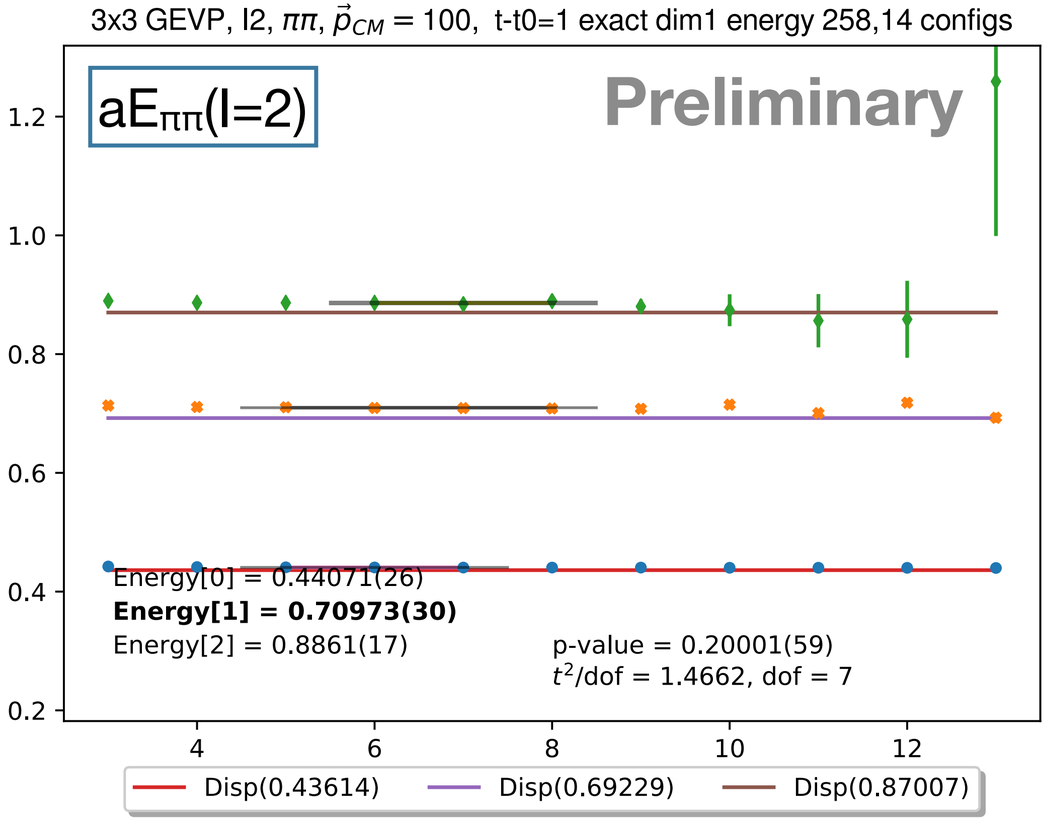
$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\}$$

- δ_I : ππ phase shifts (measurements done)
  - $\langle O_{\pi\pi}(t)\, O_{\pi\pi}(0)^\dagger \rangle$ & GEVP → ππ-state energies
  - Lüscher's formalism → ππ phase shifts
- $A_I = \langle (\pi\pi)_I \,|\, H_W \,|\, K \rangle$ (running & plan for 21-22 allocation year)
  - 3pt correlation functions
- I = 0 challenging — disconnected diagrams, power divergences

ππ scattering

Led by D. Hoying

- ππ energies from 2pt functions
  - GEVP w/ multiple operators
- I = 2 precisely calculated
- I = 0 analysis on going

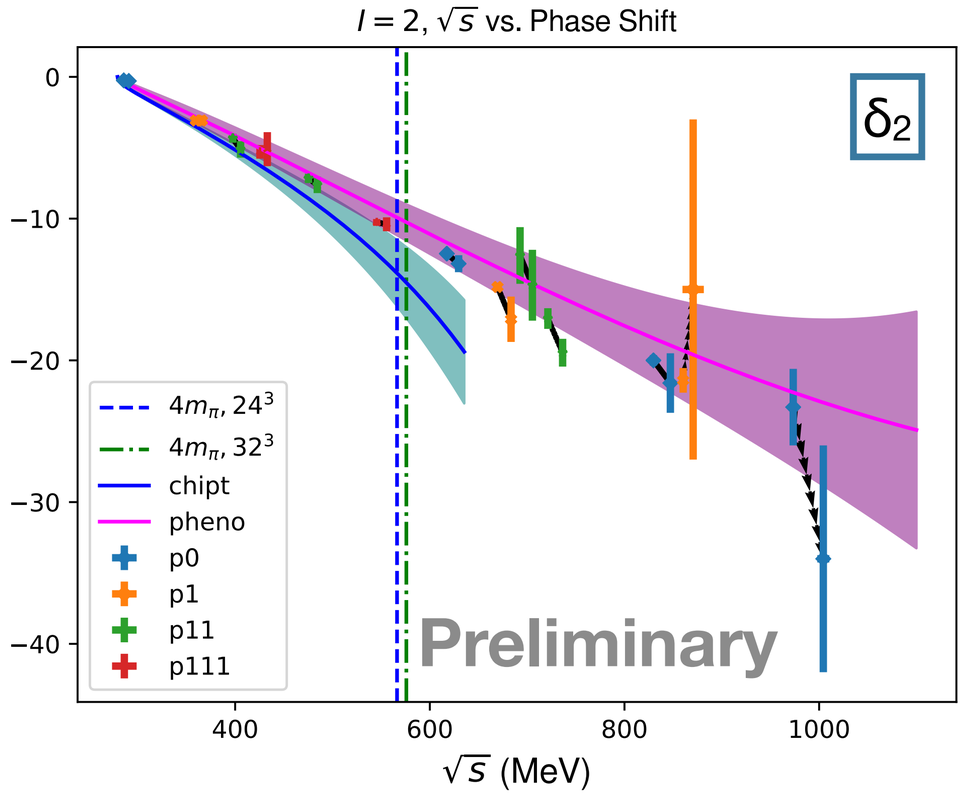


ππ scattering

Led by D. Hoying

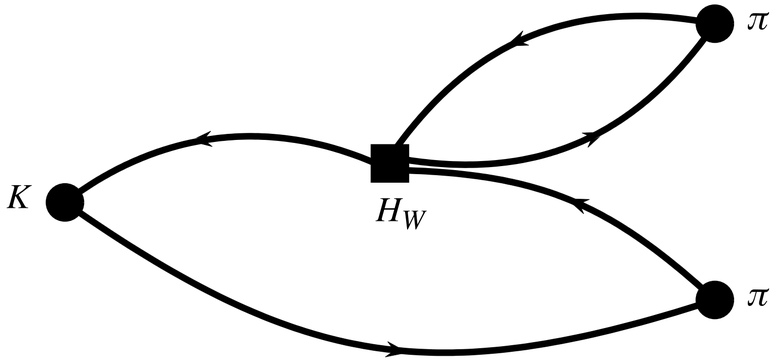
- ππ phase shifts
  - Lüscher's formula:

$$\tan\delta = -\frac{\pi^{3/2}\mathrm{k}}{\mathrm{Z}_{00}(1;\mathrm{k})}$$

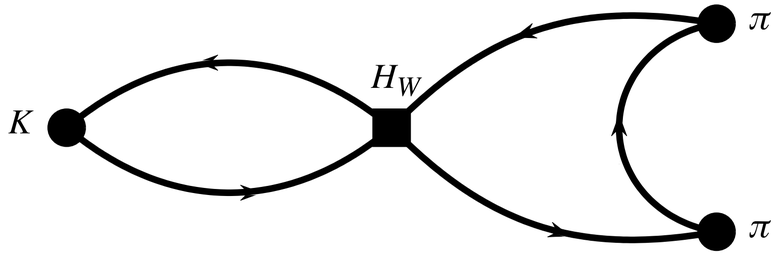


K → ππ calculation

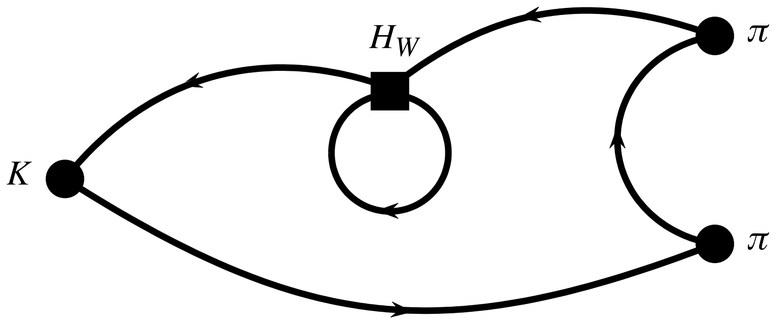
- 4 types of diagrams



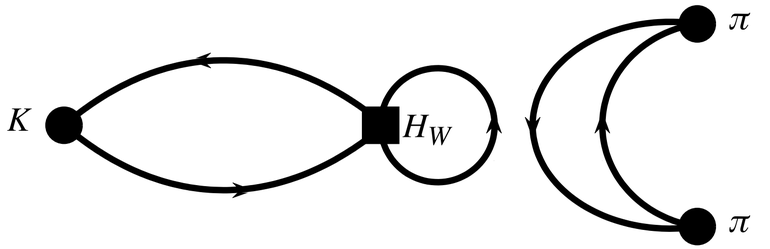
type 1



type 2



type 3



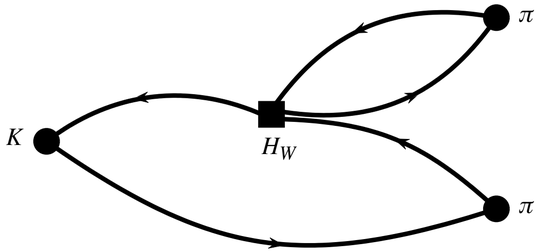
type 4

type 4 dominates stats. error

- Previous G-parity calculation
  - types 1,2: averaged over every 8 time translations
  - types 3,4: averaged over every time translation

- types 1,2 still expensive but no need of such precision

  $\rightarrow$ cost reduction?



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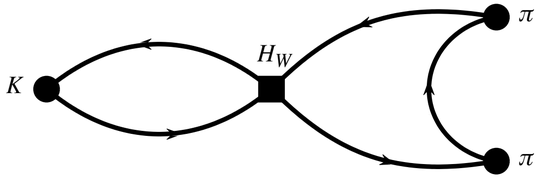

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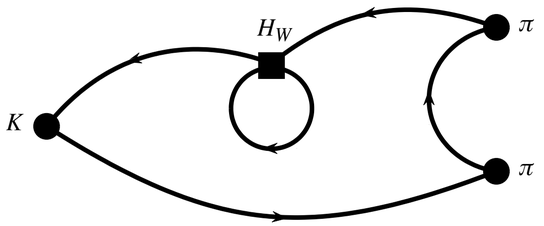

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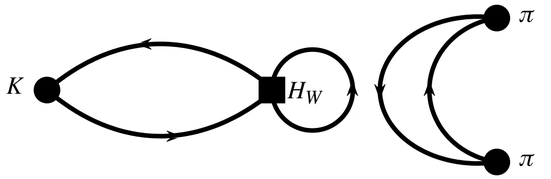
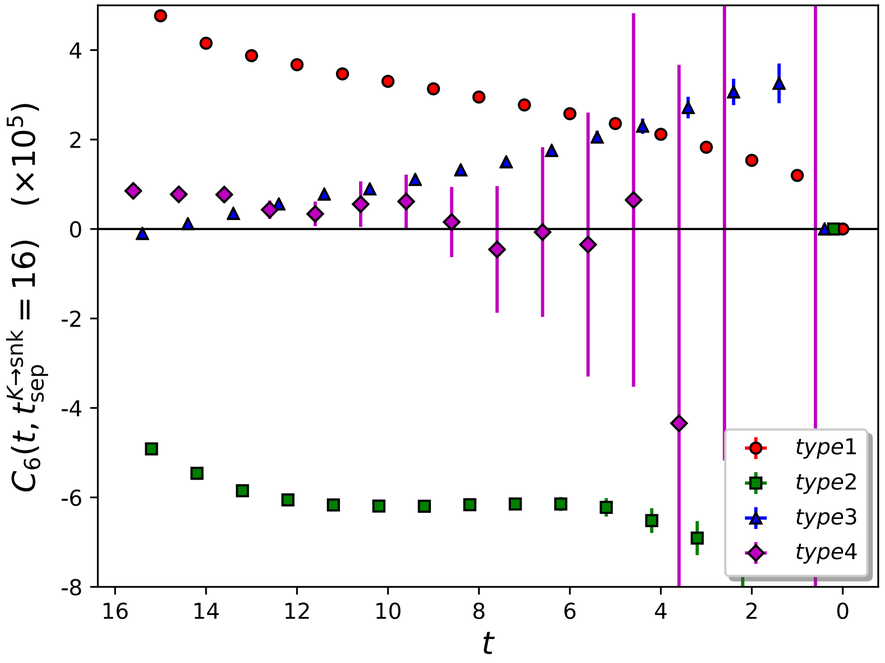


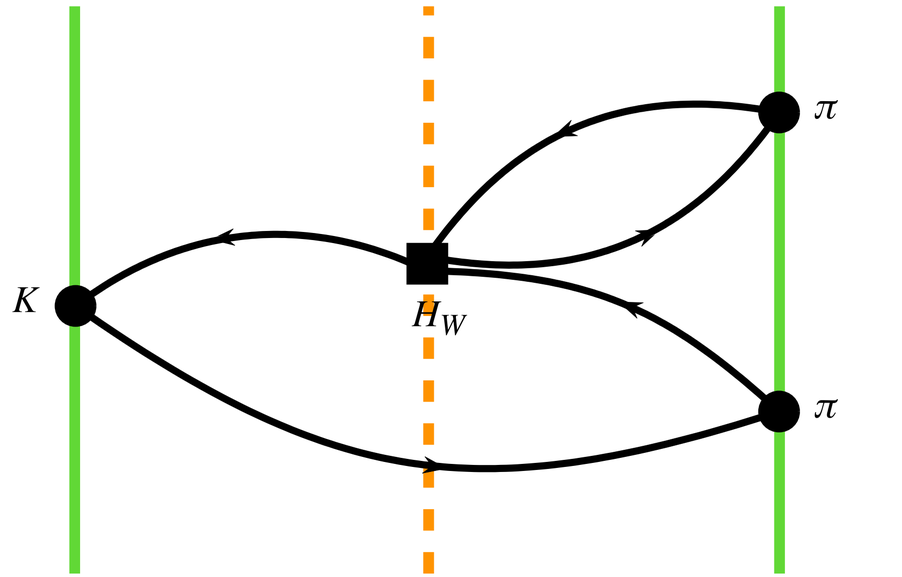
type 4



RBC/UKQCD, PRD 102,054509

Sparsening H_W

- Cost mostly promotional to volume of H_W
- G-parity calculation: summed H_W over whole 3D volume
- Plan for this time: reduce the volume of H_W ($32^3 \rightarrow 8^3$: 64x speed up) for types 1 & 2



A2A propagators, V & W vectors

$$D_{A2A}^{-1} = \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle \phi_l| + \frac{1}{N_h} \sum_{h=1}^{N_h} \underbrace{\left(D^{-1} - \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle \phi_l| \right)}_{D_{defl}^{-1}} |\eta_h\rangle \langle \eta_h|$$

$$= \sum_{i=1}^{N_l+N_h} |V_i\rangle \langle W_i|$$

- V & W vectors

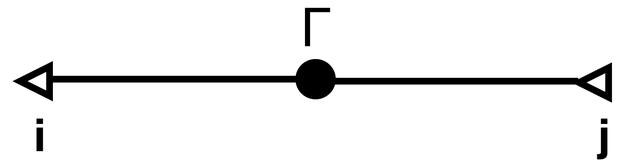
$$1 \le i \le N_l \;\Rightarrow\; \underline{|V_i\rangle = \frac{1}{\lambda} |\phi_i\rangle, \quad |W_i\rangle = |\phi_i\rangle}$$

$$N_l + 1 \le i(= N_l + h) \le N_l + N_h \;\Rightarrow\; \underline{|V_i\rangle = \frac{1}{N_h} D_{defl}^{-1} |\eta_h\rangle, \quad |W_i\rangle = |\eta_h\rangle}$$

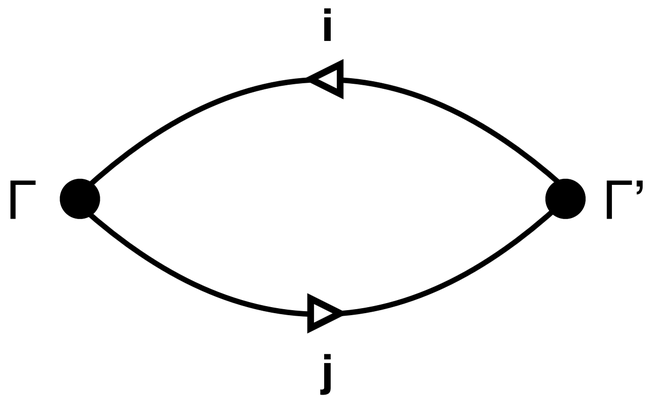
Meson fields

- Spin & color contractions leaving mode indices i, j
- Easily summed over time slice → savable data size
- Multiplied with any other meson fields to construct correlation functions

meson field



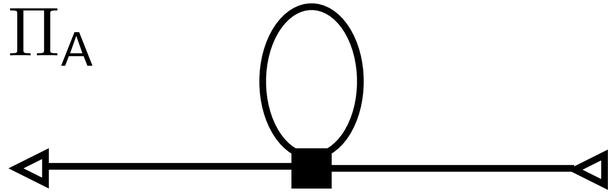
$$\Pi_{\Gamma,ij}(t) = \langle W_i|\Gamma|V_j\rangle_t$$

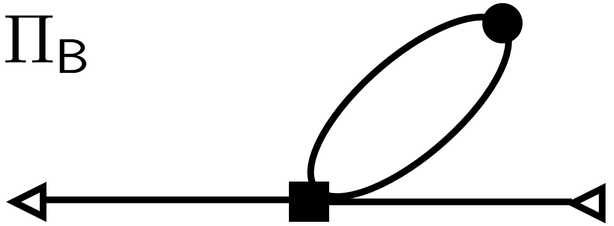


$$\Pi_{\Gamma,ij}(t)\Pi_{\Gamma',ji}(t')$$

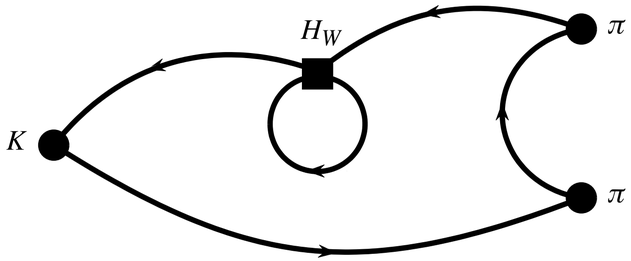
4-quark fields & contractions

1. 4-quark operator fields
   - too inefficient to leave 4 mode indices
   - generate partially contracted ones Π_A & Π_B
2. Contractions of mode indices





Example



$$= \sum_{i,j,k,l} \Pi_{\pi,ij}(t_{\pi_1})\Pi_{\pi,jk}(t_{\pi_2})\Pi_{K,kl}(t_K)\Pi_{A,li}(t_{H_W})$$

Summary

- Purpose
  - New independent calculation of K → ππ decays
  - Periodic-BC study gives prospect of introducing QED/IB effects
- Ambitious goal: continuum limit of K → ππ amplitudes & ε’
- With requested 13 M KNL core hours, we expect remaining calculation on 32^3 x 64 lattice can be done with 200 confs or more