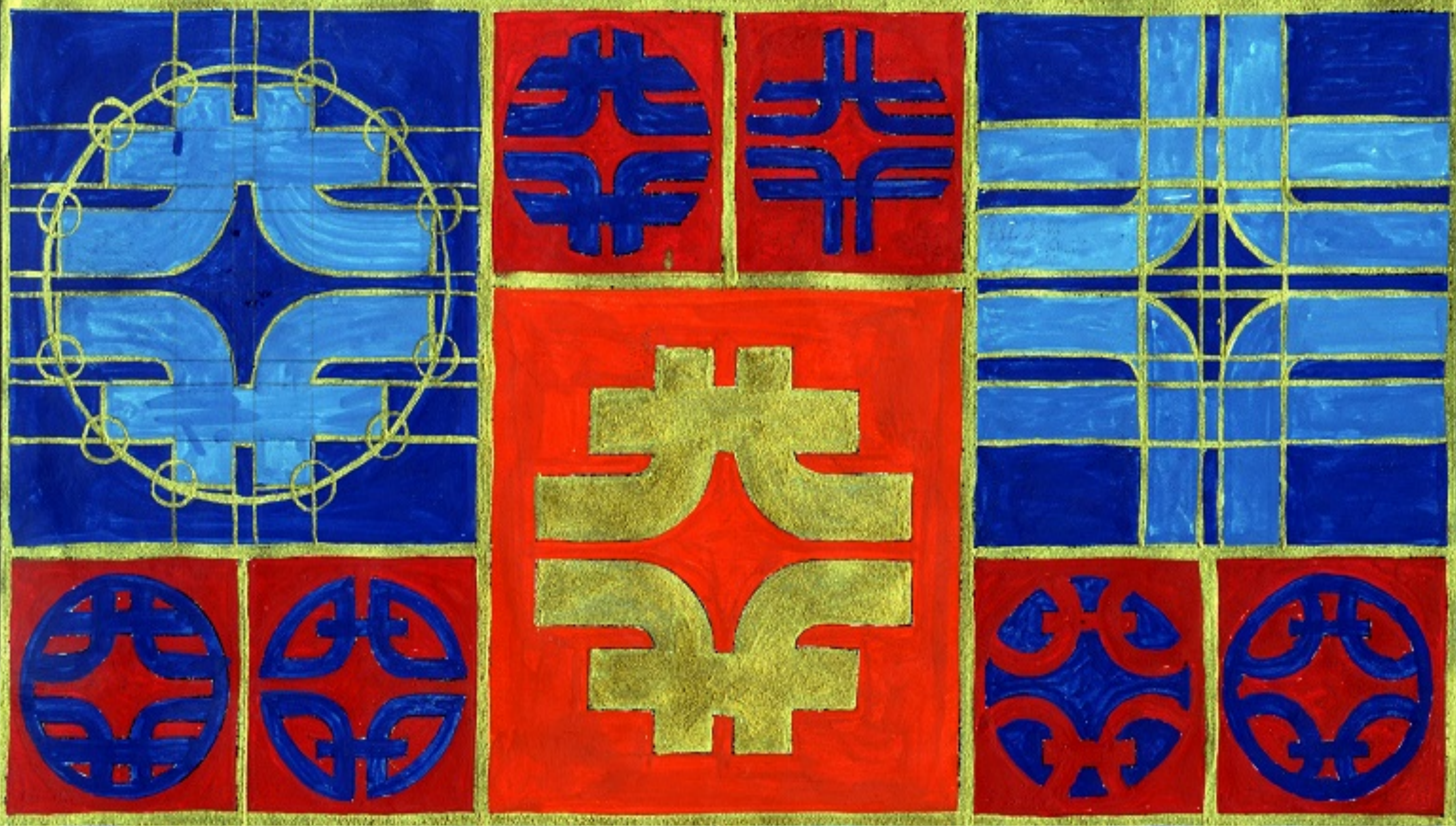


Status of B-to-D matrix elements

William I. Jay (Fermilab)

USQCD All Hands' Meeting— 30 April 2021

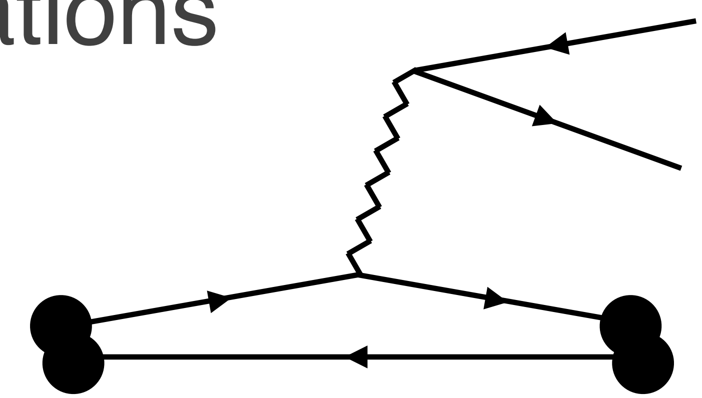
Outline

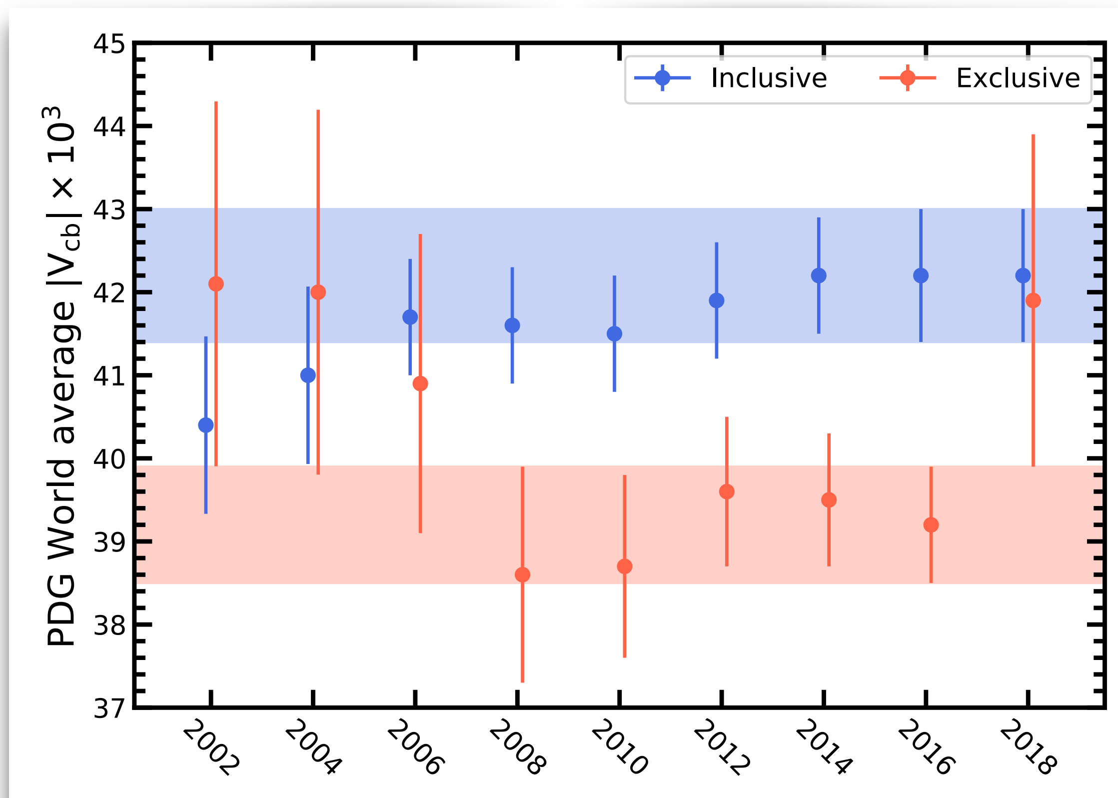
- Motivation and scope
- Kinematic setup / connection to experiment
- Approaches to heavy quarks
- Recent results (Focus on last ~ 5 years)
- Summary

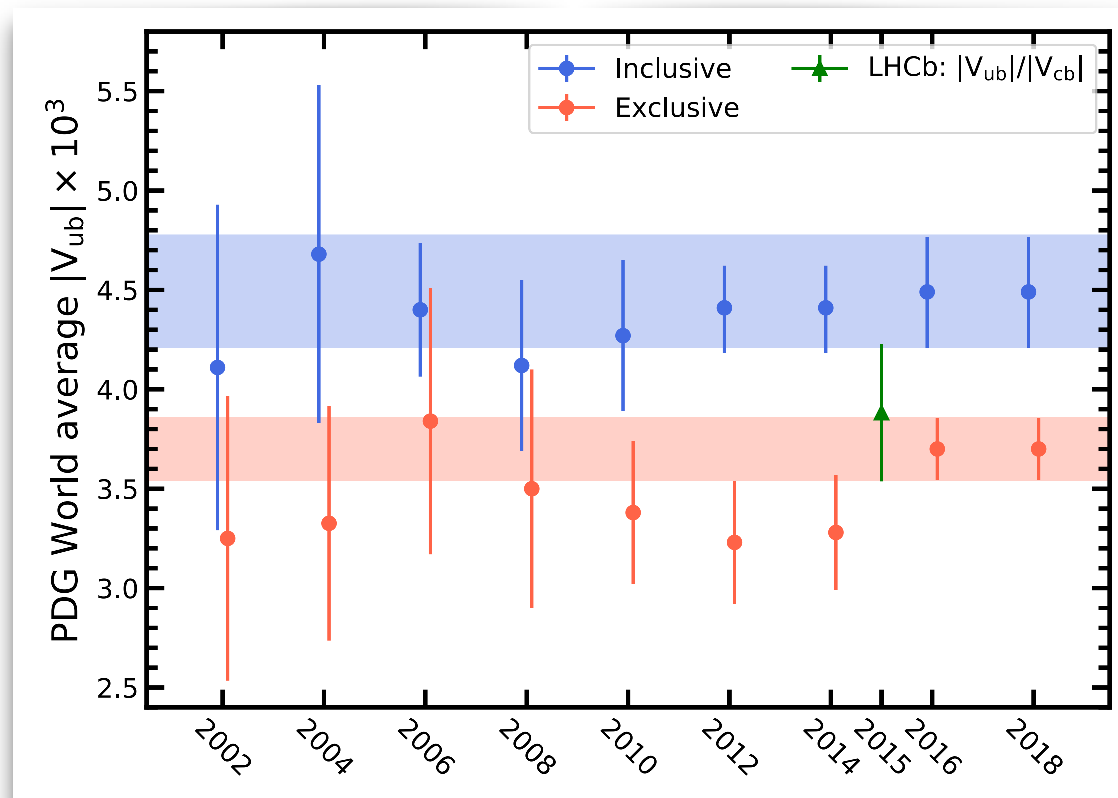
Extracting CKM matrix elements

Tension in inclusive vs. exclusive determinations

- $|V_{cb}|$ from $B \to D^* \ell \nu$ has 3.3σ tension
- $|V_{cb}|$ from $B \to D \ell \nu$ has 2.0σ tension
- $|V_{ub}|$ from $B \to \pi \ell \nu$ has 2.8σ tension







Figures: Bouchard, Cao, Owen, arXiv:1902.09412

Testing lepton universality

$$R(D) = \mathcal{B}(B \to D\tau\bar{\nu}_\tau)/\mathcal{B}(B \to D\mu\bar{\nu}_\mu)$$

$R(D^*)$ similar

Combined 3.1σ tension with SM prediction

$R(J/\psi)$ ~2σ

$R(K)$ ~2.6σ

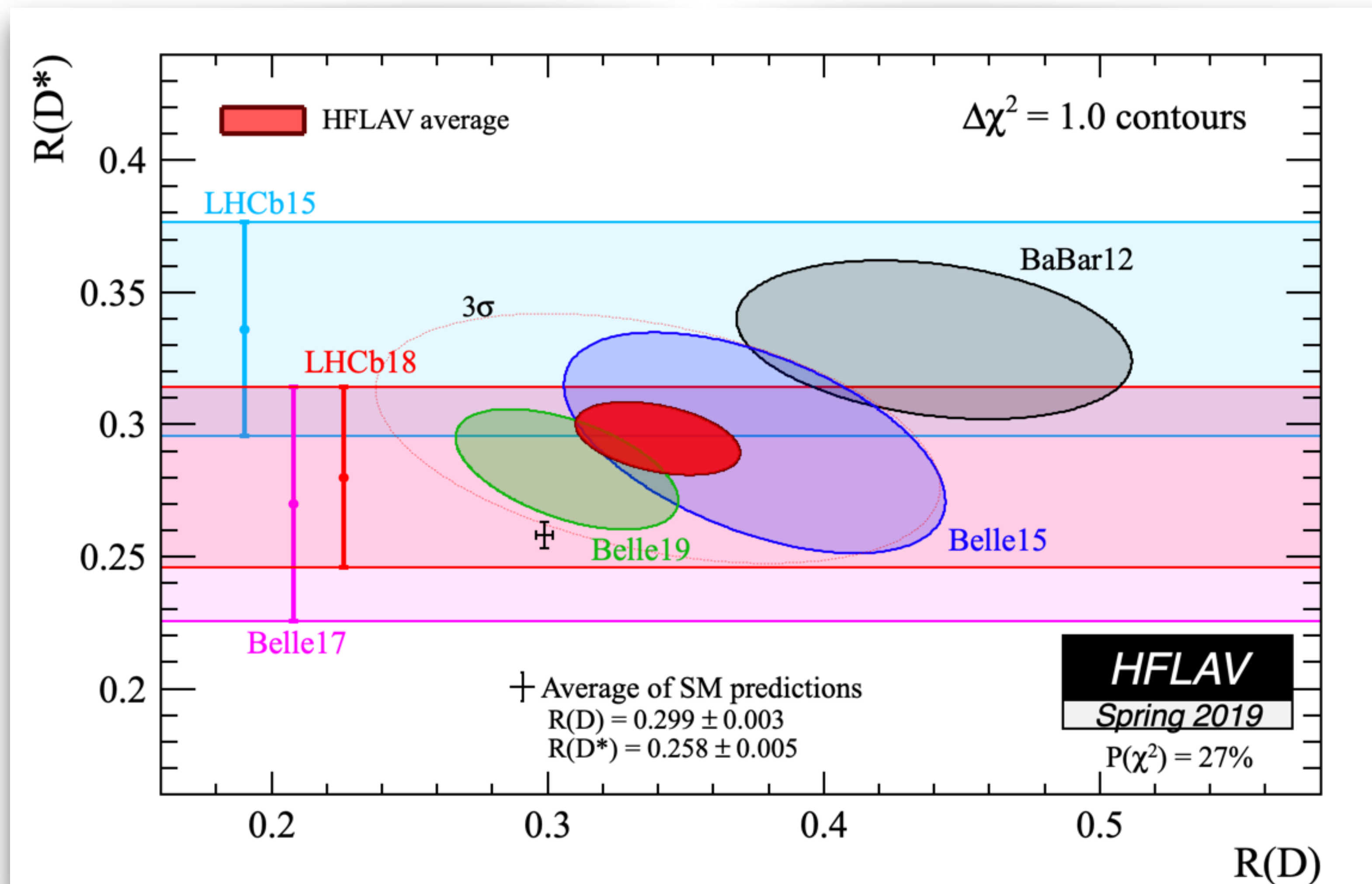


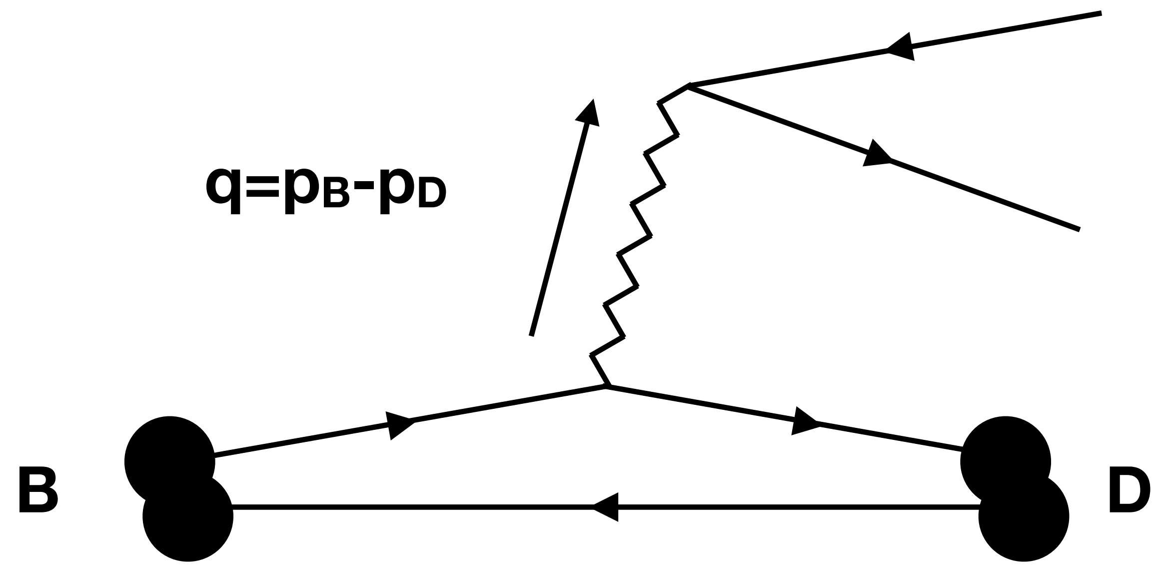
Figure: hflav.web.cern.ch

“Status of B-to-D matrix elements”

- Ultimate interest is quark-level physics: b-to-c and the CKM matrix element $|V_{cb}|$
- Many possible hadronic systems systems:
    - $B \to D \ell \nu$
    - $B \to D^* \ell \nu$
    - $B_s \to D_s \ell \nu$
    - $B_s \to D_s^* \ell \nu$
    - $B_c \to \eta_c \ell \nu$
    - $B_c \to J/\psi \ell \nu$
    - $\Lambda_b \to p \ell \nu$
    - $\Lambda_b \to \Lambda_c \ell \nu$

My focus today will be $B_{(s)} \to D_{(s)}^{(*)} \ell \nu$

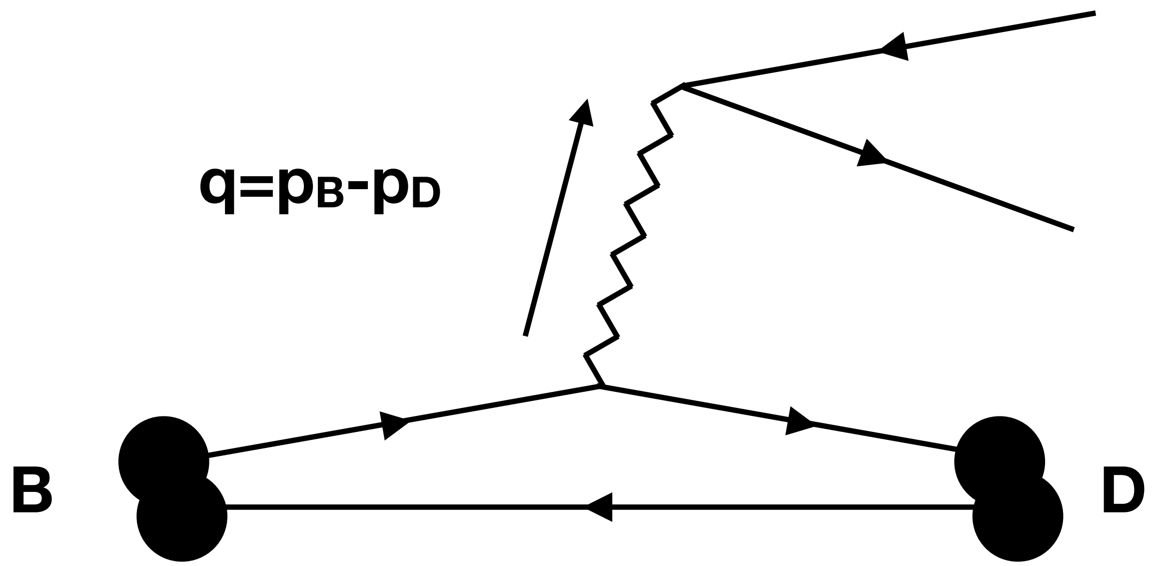
Kinematic setup — B to D



$$
\begin{aligned}
q^2 &= (p_B - p_D)^2 \\
&= M_B^2 + M_D^2 - 2 p_B \cdot p_D \\
&= M_B^2 + M_D^2 - 2 w M_B M_D
\end{aligned}
$$

$$\text{with } w \equiv v_B \cdot v_D = \frac{p_B \cdot p_D}{M_B M_D}$$

Form factors — B to D



(decay rates) ∝ (form factors) ∝ (QCD matrix elements)

$$\frac{d\Gamma}{dw} \propto \mathcal{G}(w), \mathcal{F}(w) \propto \langle B|\, J \,|D\rangle$$

For B→D **For B→D***

Form factors — B to D

$$\frac{\langle D|\, V^{\mu}\, |B\rangle}{\sqrt{m_B m_D}} = (v_B + v_D)^{\mu} h_{+}(w) + (v_B - v_D)^{\mu} h_{-}(w)$$

$$\frac{\langle D^{\star}_{\alpha}|\, V^{\mu}\, |B\rangle}{\sqrt{m_B m_{D^{\star}}}} = \epsilon^{\mu\nu\rho\sigma} v_B^{\nu} v_{D^{\star}}^{\rho} \epsilon_{\alpha}^{\star\sigma} h_V(w)$$

$$\frac{\langle D^{\star}_{\alpha}|\, A^{\mu}\, |B\rangle}{\sqrt{m_B m_{D^{\star}}}} = i\epsilon_{\alpha}^{\star\nu} \left[h_{A_1}(w)(1+w)g^{\mu\nu} - \left(h_{A_2}(w)v_B^{\mu} + h_{A_3}(w)v_{D^{\star}}^{\mu}\right) v_B^{\nu}\right]$$

$\mathcal{G}(w), \mathcal{F}(w)$ $\Leftrightarrow$ **certain linear combinations of the form factors h(w)**

The job for lattice QCD:

- Extract matrix elements
- Construct form factors
- Extrapolate to physical limit:
  - Continuum limit: $a \to 0$
  - Heavy quark limit: m_b / $m_d \to$ physical point
  - Chiral extrap/interpolation: $m_{u/d} \to$ physical point
- Parameterize results using z-expansion

The challenge of heavy quarks

Heavy quarks are hard: lattice artifacts grow like powers of (am_h) — especially tricky for masses near or above the cutoff

1. Use an effective theory for heavy quarks (b, sometimes c)
   - “FNAL interpretation,” NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching, which introduces systematic effects

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass

The challenge of heavy quarks

- Many different treatments used in the literature:

Group	Heavy valence		Sea	“Generation”
HPQCD	NRQCD	on	ASQTAD	I
HPQCD	NRQCD	on	HISQ	II
HPQCD	HISQ	on	HISQ	III
FNAL/MILC	Fermilab	on	ASQTAD	1
FNAL/MILC	Fermilab	on	HISQ	2
FNAL/MILC	HISQ	on	HISQ	3
JLQCD	Möbius DW	on	Möbius DW	
LANL/SWME	Oktay-Kronfeld	on	HISQ	
RBC/UKQCD	RHQ	on	DW	
ETMC	Twisted mass	on	Twisted mass	

HPQCD

HPQCD B→D

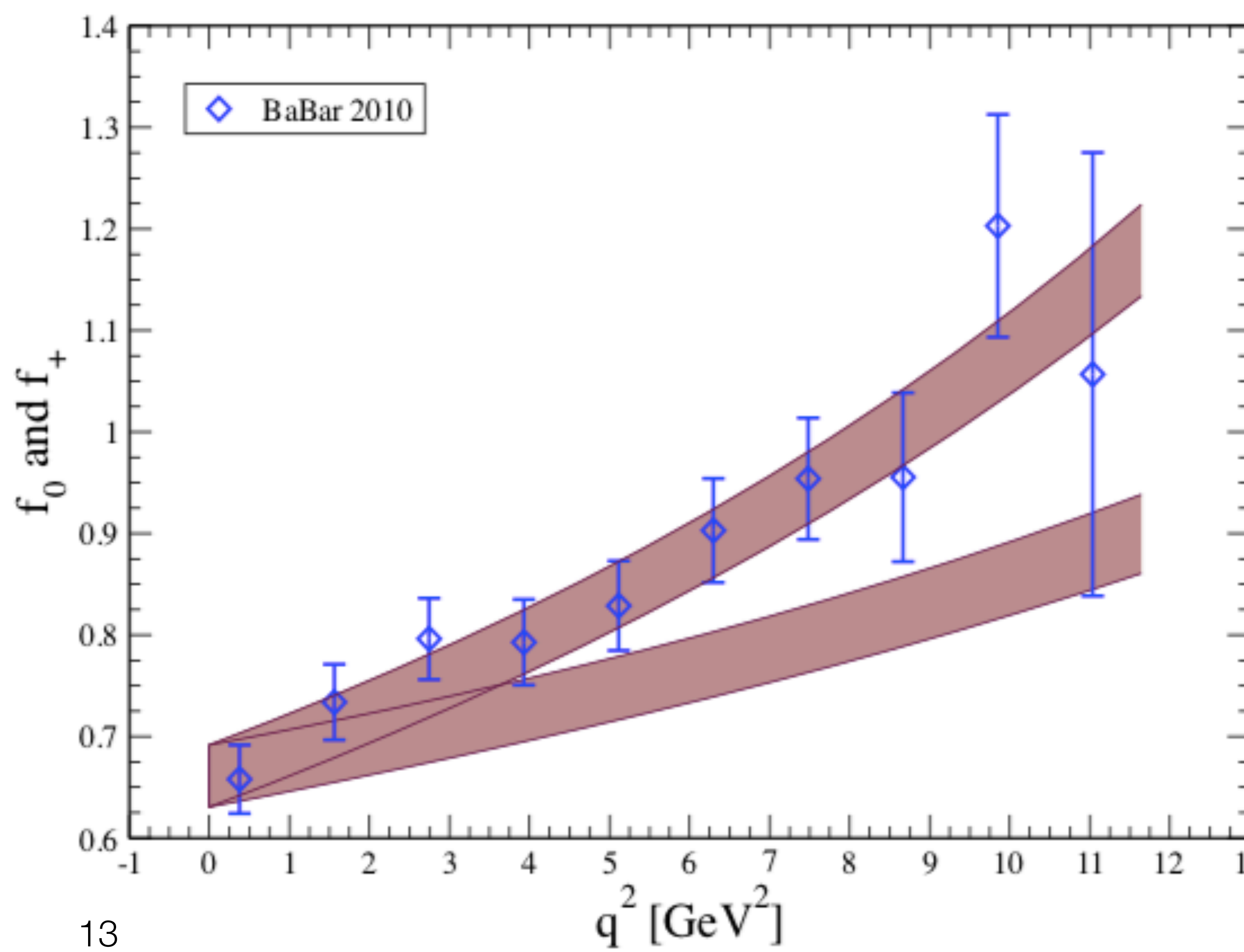
arXiv:1505.03925
PRD 92 (2015) 5, 054510

“Generation I”

- Ensembles: 5x (N_f=2+1) MILC asqtad
- Lattice spacings: 2 x in [0.09, 0.12] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- Full physical q^2
- R(D) = 0.300(8)
- G(1) = 1.035(40)

TABLE VI. Error budget table for $R(D)$.

Type	Partial errors [%]
lattice statistics	1.24
chiral extrapolation	0.28
discretization	1.08
kinematic	1.61
matching	1.03
finite size effect	0.1
total	2.54



HPQCD $B_s \to D_s$

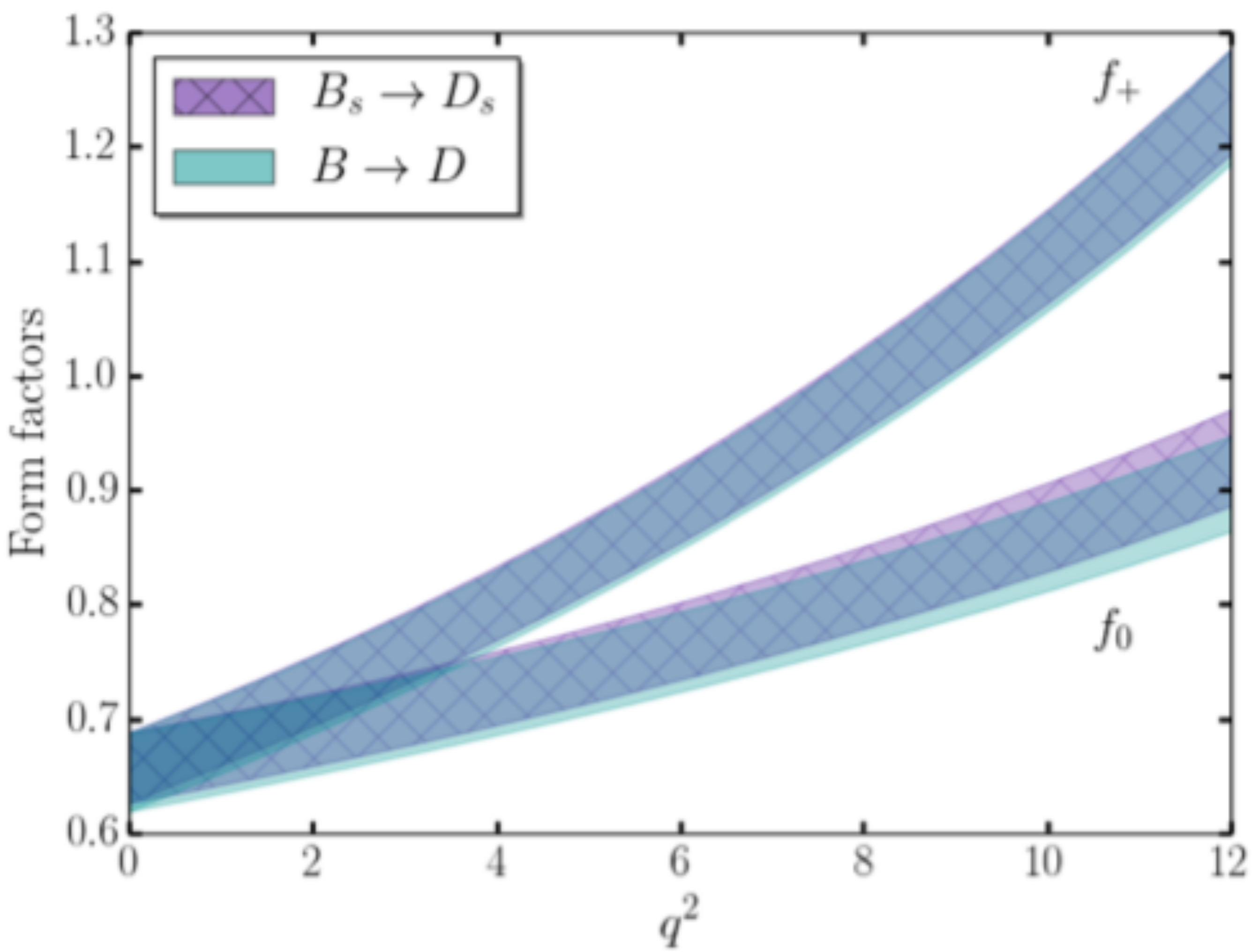
arXiv:1703.09728
PRD 95 (2017) 11, 114506

"Generation I"

- Ensembles: 5x (N_f=2+1) MILC asqtad
- Lattice spacings: [0.09, 0.12] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- Full physical q^2
- G(1)=1.068(40)

TABLE VIII. Error budget for the form factors at zero momentum transfer, $f_0(0) = f_+(0)$, for the $B_s \to D_s \ell \nu$ semileptonic decay. We describe each source of uncertainty in more detail in the accompanying text.

Type	Partial uncertainty (%)
Statistical	1.22
Chiral extrapolation	0.80
Quark mass tuning	0.66
Discretization	2.47
Kinematic	0.71
Matching	2.21
total	3.70



HPQCD $B_{(s)} \rightarrow D_{(s)}^{\star}$

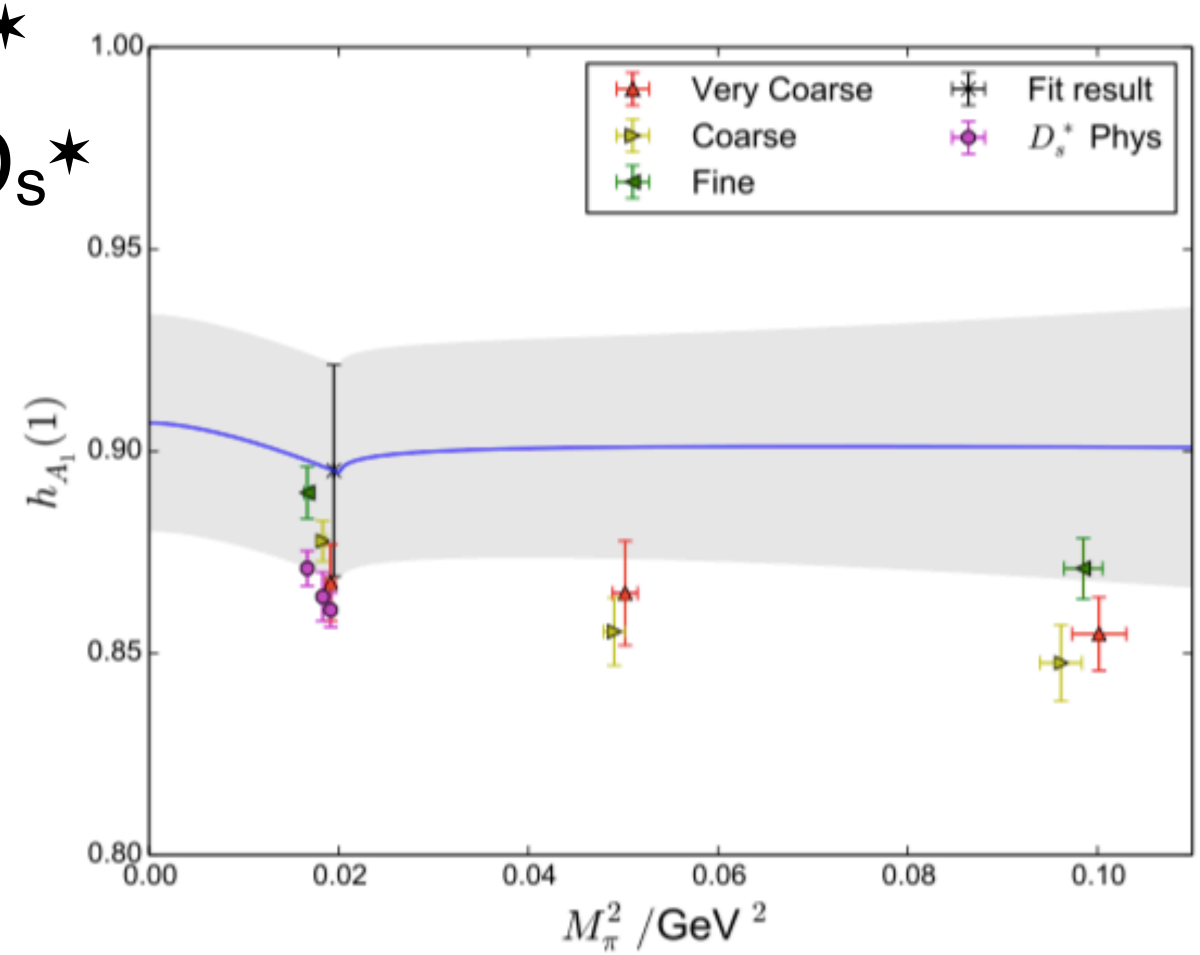
arXiv:1711.11013
PRD 97 (2018) 5, 054502

"Generation II"

- Ensembles: 8x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.09, 0.12, 0.15] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- Zero recoil only (w=1)
- $h_{A1}(1) = 0.895(10)(24)$, $B \rightarrow D^{\star}$
- $h_{A1}(1) = 0.883(12)(28)$, $B_s \rightarrow D_s^{\star}$

TABLE IX: Partial errors (in percentages) for $h_{A_1}^{(s)}(1)$.

Uncertainty	$h_{A_1}(1)$	$h^s_{A_1}(1)$	$h_{A_1}(1)/h^s_{A_1}(1)$
α_s^2	2.1	2.5	0.4
$\alpha_s \Lambda_{QCD}/m_b$	0.9	0.9	0.0
$(\Lambda_{QCD}/m_b)^2$	0.8	0.8	0.0
a^2	0.7	1.4	1.4
$g_{D^* D\pi}$	0.2	0.03	0.2
Total systematic	2.7	3.2	1.7
Data	1.1	1.4	1.4
Total	2.9	3.5	2.2



HPQCD $B_s \to D_s^*$

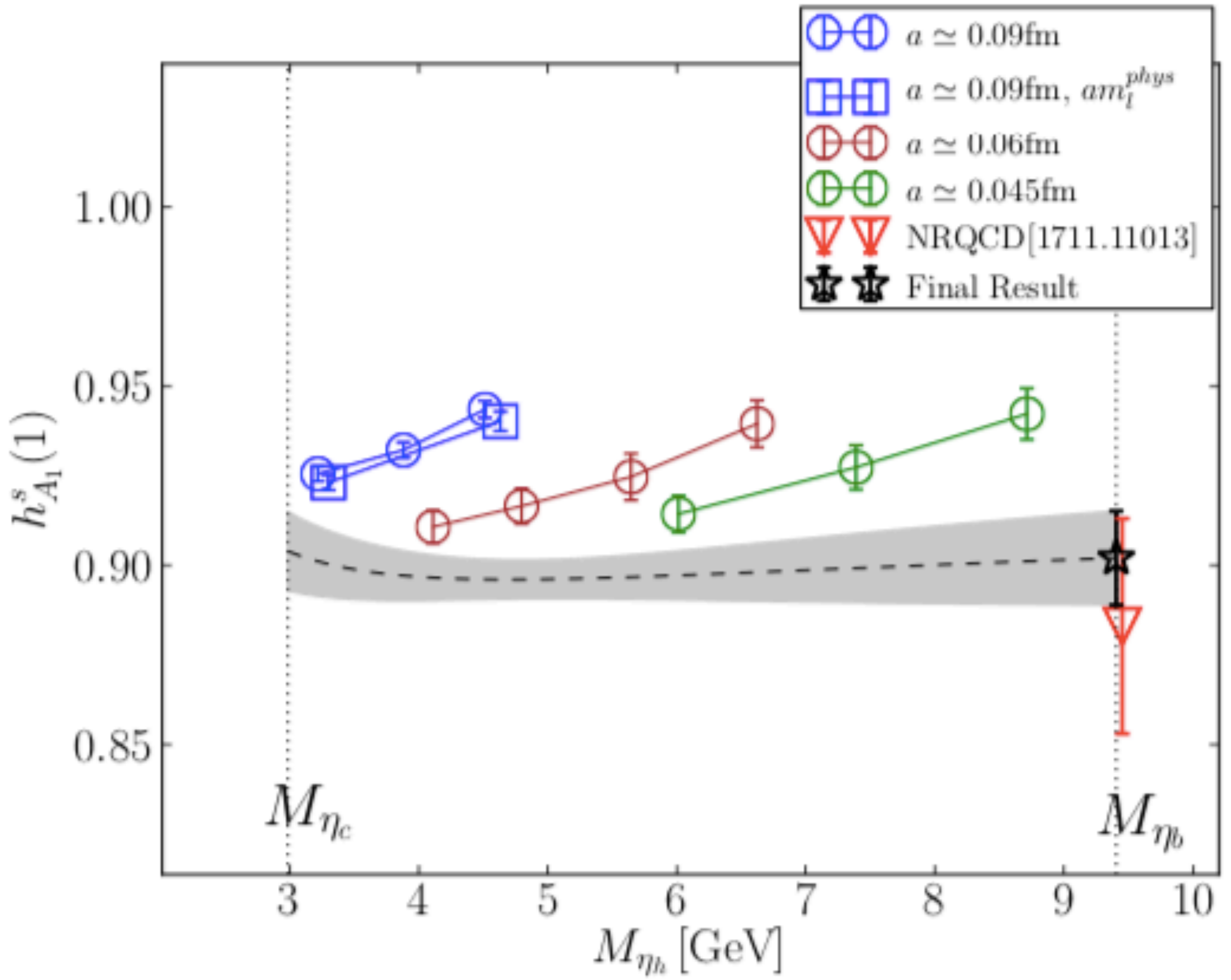
arXiv:1904.02046
PRD 99 (2019) 11, 114512

"Generation III"

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04, 0.06, 0.09] fm
- Valence quarks: all HISQ
- $m_h a < 0.8$ [close-to-physical b at 0.04 fm]
- Zero recoil only (w=1)
- $h_{A_1}^s(1) = 0.9020(96)(90)$

TABLE IV: Error budget for $h^s_{A_1}(1)$. Errors are given as a percentage of the final answer. The mass mistuning error includes that from valence strange and sea light and strange quarks; we find that taking a ± 10 MeV uncertainty in the physical value of the η_b mass has a negligible effect.

Source	% Fractional Error
Statistics $+Z_A$	1.06
$a \to 0$	0.73
$m_h \to m_b$	0.69
mass mistuning	0.20
Total	1.45



HPQCD $B_s \to D_s^*$

arXiv:1904.02046
PRD 99 (2019) 11, 114512

“Generation III”

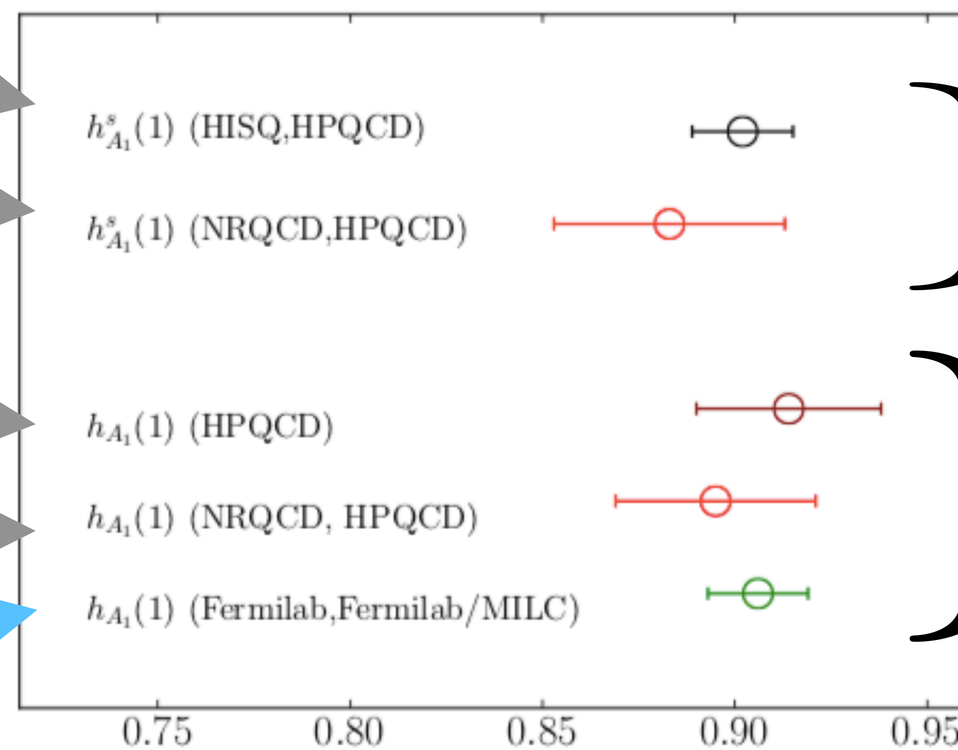
“Generation III”

“Generation II”

“Generation III”

“Generation II”

“Generation 1”



$B_s \to D_s^*$

$B \to D^*$

FIG. 6: Comparison of lattice QCD results for $h^s_{A_1}(1)$ and $h_{A_1}(1)$. Our results for $h^{(s)}_{A_1}(1)$ are marked ‘(HISQ, HPQCD)’ and for $h_{A_1}(1)$ are marked ‘(HPQCD)’. Those marked ‘(NRQCD,HPQCD)’ are from [41] and the value marked ‘(Fermilab, Fermilab/MILC)’ is from [39].

HPQCD $B_s \to D_s$

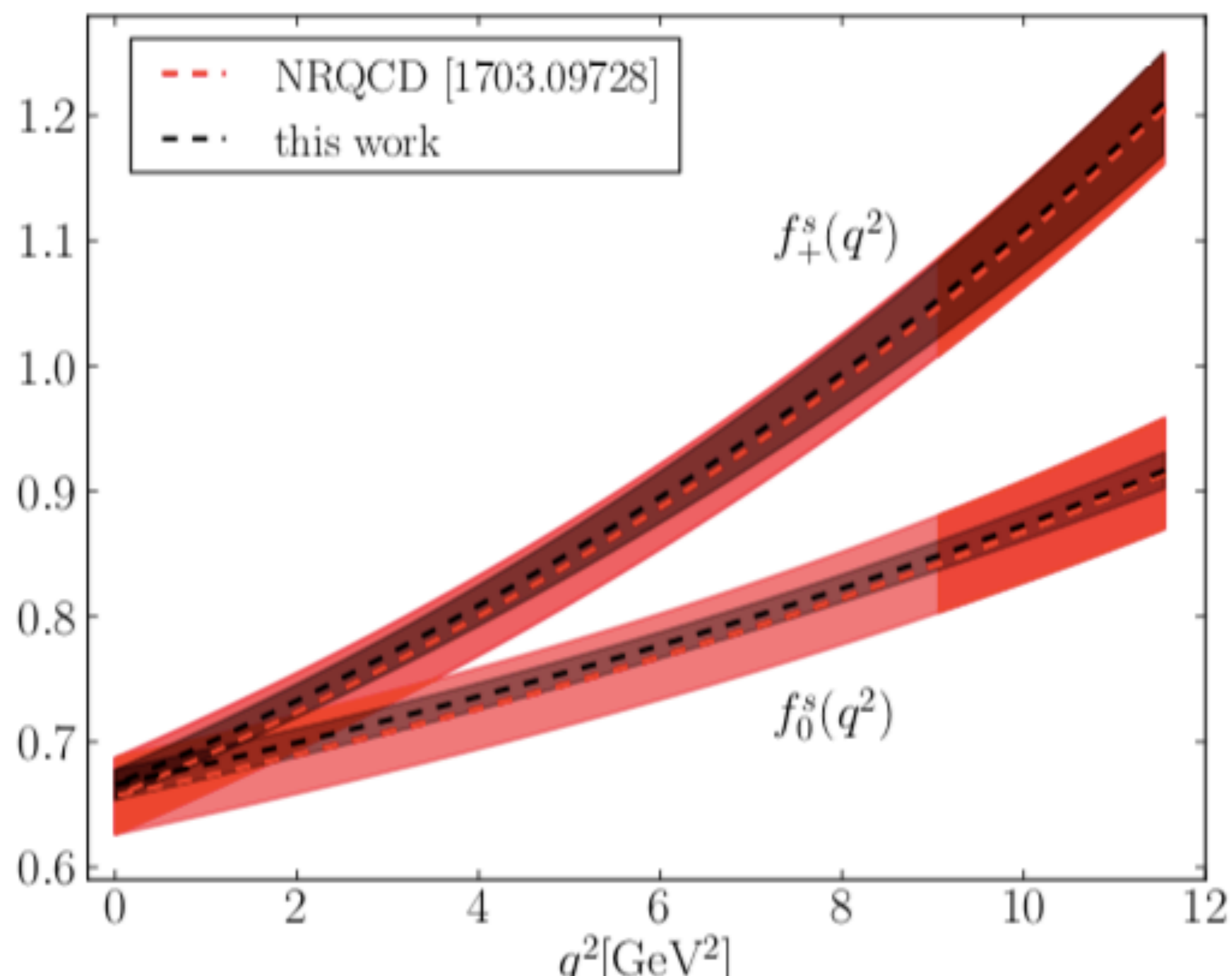
arXiv:1906.00701
PRD 101 (2020) 7, 074513

“Generation III”

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04, 0.06, 0.09] fm
- Valence quarks: all HISQ
- $m_h a < 0.8$
- Full physical q^2
- $R(D_s) = 0.2987(46)$

Source	% Fractional Error
Statistics	1.11
$m_h \to m_b$ and $a \to 0$	1.20
Quark mistuning	0.58
Total	1.73

TABLE VI: Error budget for $f_0^s(q^2_{max})$.



FNAL/MILC

FNAL/MILC B→D*

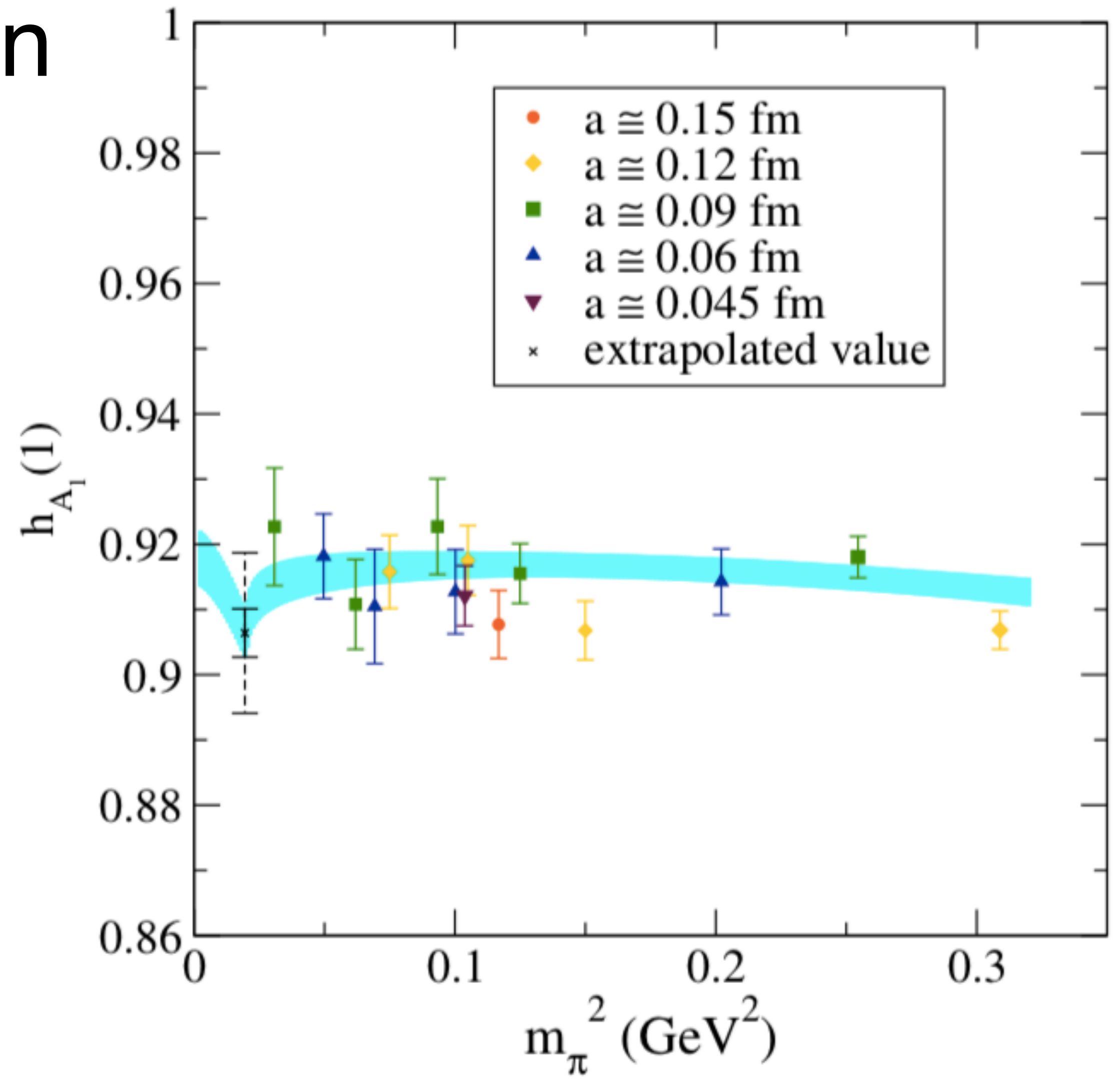
arXiv:1403.0635
PRD 89 (2014) 11, 114504

"Generation 1"

- Ensembles: 15x (N_f=2+1) MILC asqtad
- Lattice spacings: 5 x in [0.045 - 0.15] fm
- Light valence: asqtad staggered
- Heavy b/c: FNAL interpretation
- Zero recoil only (w=1)
- $h_{A1}(1)$=F(1) = 0.906(4)(13)

TABLE X. Final error budget for $h_{A_1}(1)$

Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^* D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%



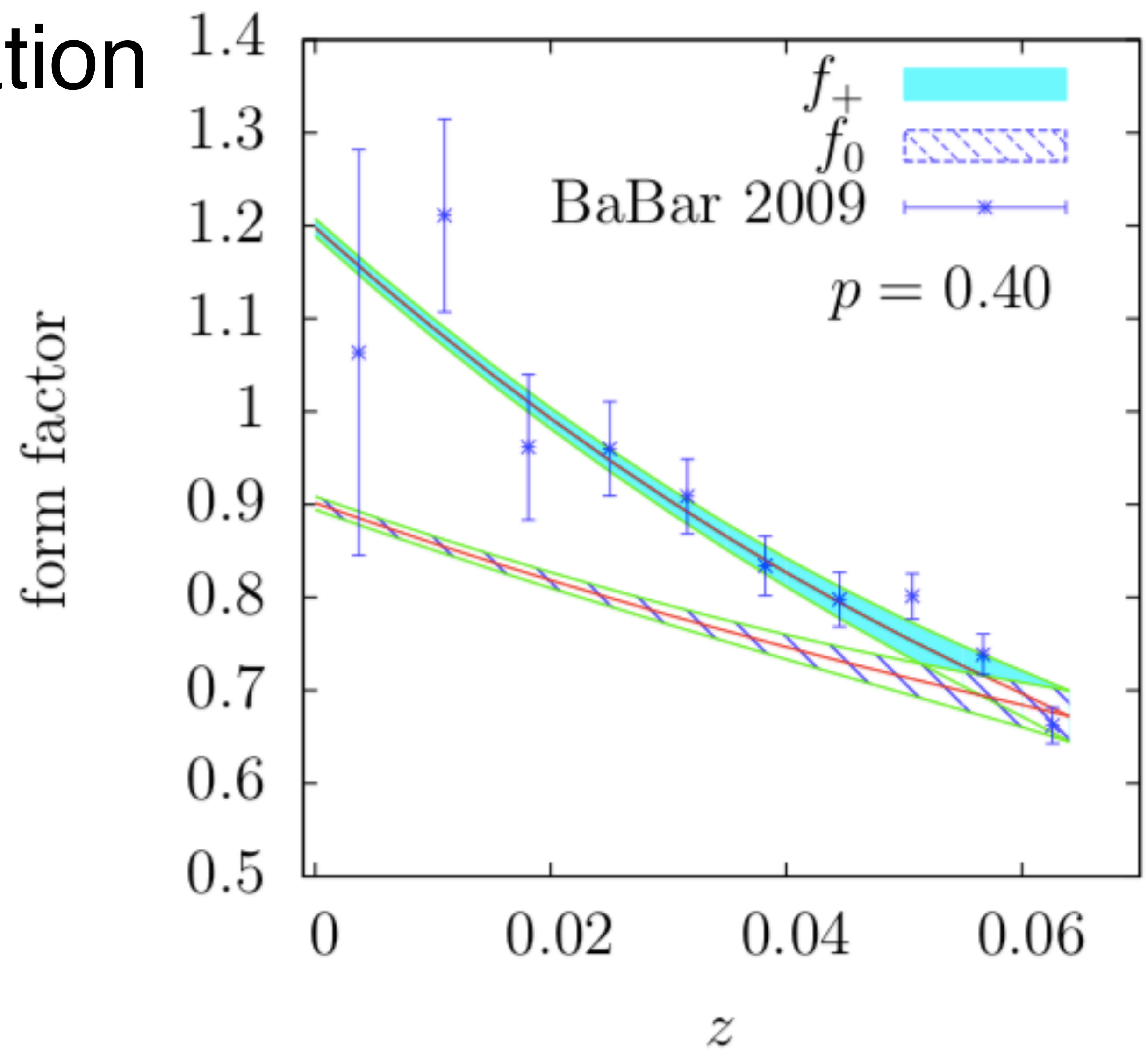
FNAL/MILC B→D

arXiv:1503.07237
PRD 92 (2015) 3, 034506

"Generation 1"

- Ensembles: 14x (N_f=2+1) MILC asqtad
- Lattice spacings: 4 x in [0.045 - 0.12] fm
- Light valence: asqtad staggered
- Heavy b/c: FNAL interpretation
- Full physical q^2
- R(D) = 0.299(11)
- G(1) = 1.054(4)(8)

Source	$f_+(\%)$	$f_0(\%)$
Statistics+matching+χPT cont. extrap.	1.2	1.1
(Statistics)	(0.7)	(0.7)
(Matching)	(0.7)	(0.7)
(χPT/cont. extrap.)	(0.6)	(0.5)
Heavy-quark discretization	0.4	0.4
Lattice scale r_1	0.2	0.2
Total error	1.2	1.1



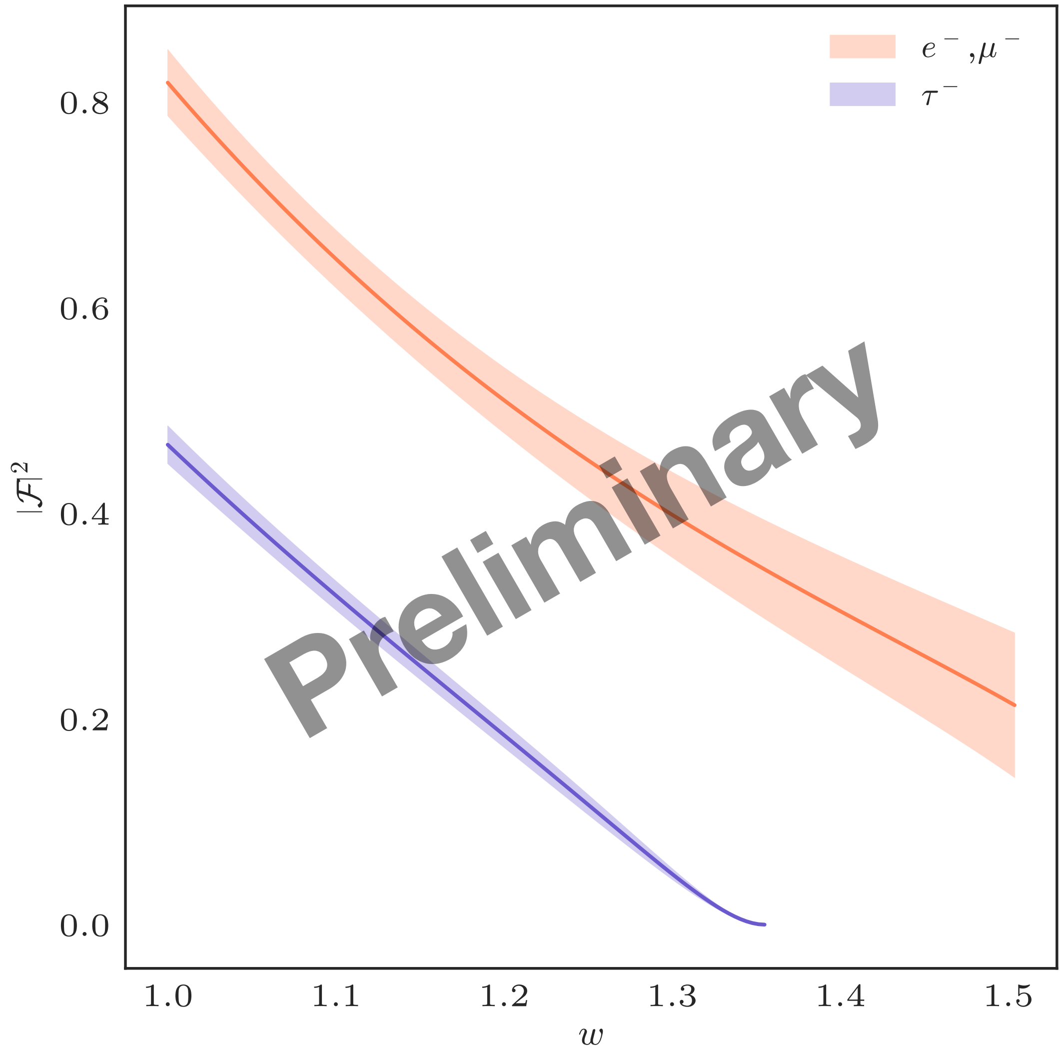
FNAL/MILC B→D*

arXiv:1710.09817
EPJ Web Conf. 175 (2018) 13003

"Generation 1"

- Ensembles: 5x (N_f=2+1) MILC asqtad
- Lattice spacings: 5x in [0.045 - 0.15] fm
- Light valence quarks: asqtad
- Heavy valence charm/bottom: FNAL interpretation

- *New results coming soon!* Alex Vaquero is giving the Fermilab theory seminar on May 20 — stay tuned for unblinded results!



FNAL/MILC $B_{(s)} \rightarrow D_{(s)}^{(*)}$

- 2nd Generation:

  "Generation 2"

  - FNAL on (N_f=2+1+1) MILC HISQ
  - Plan: joint correlated analysis on $B \rightarrow D$ and $B \rightarrow D^*$
  - Analysis underway by Alex Vaquero

- 3rd Generation:

  "Generation 3"

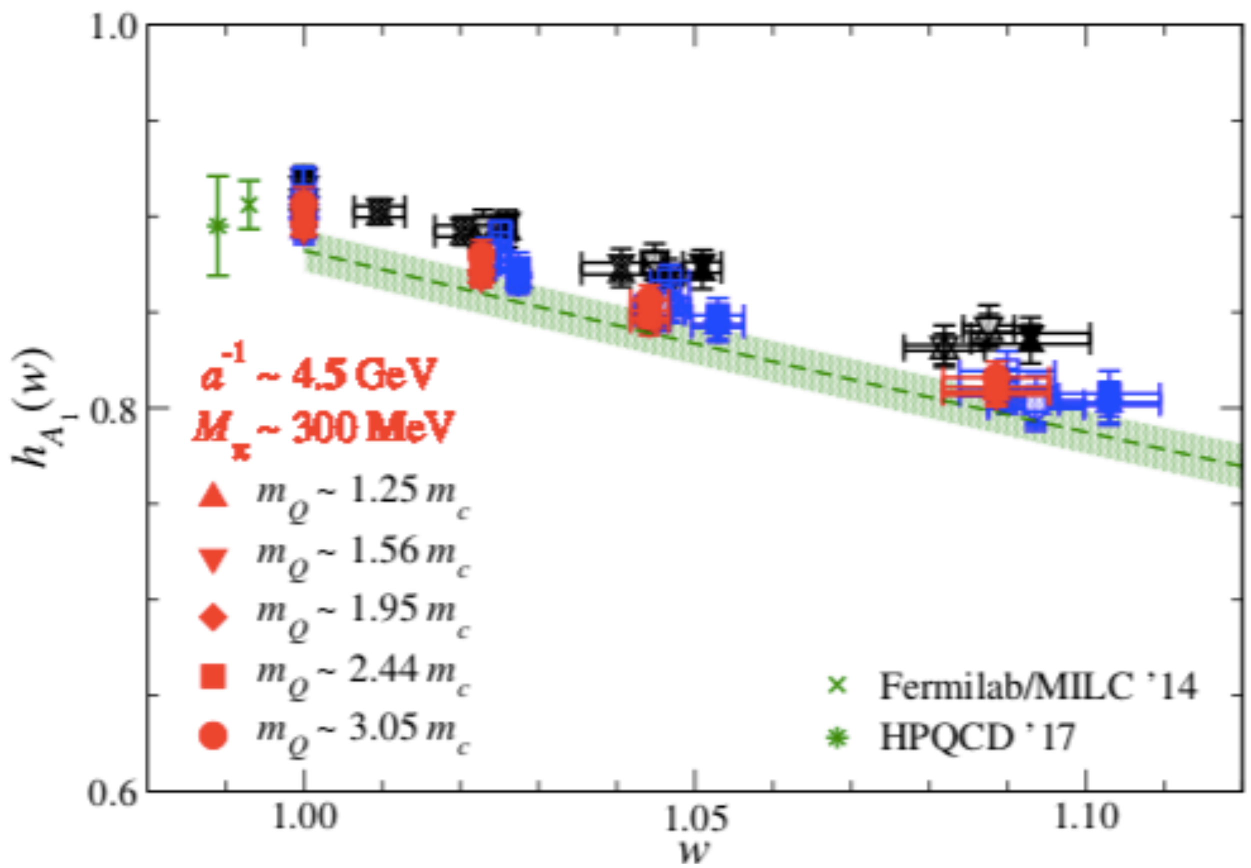
  - HISQ on (Nf=2+1+1) MILC HISQ
  - Complete set: scalar, vector, and tensor currents
  - Broad range of momenta across kinematic range
  - $B_{(s)} \rightarrow D_{(s)}$ [+ many others, e.g., $B_{(s)}/D_{(s)} \rightarrow K/\pi$)
  - Analysis underway by WJ, Andrew Lytle

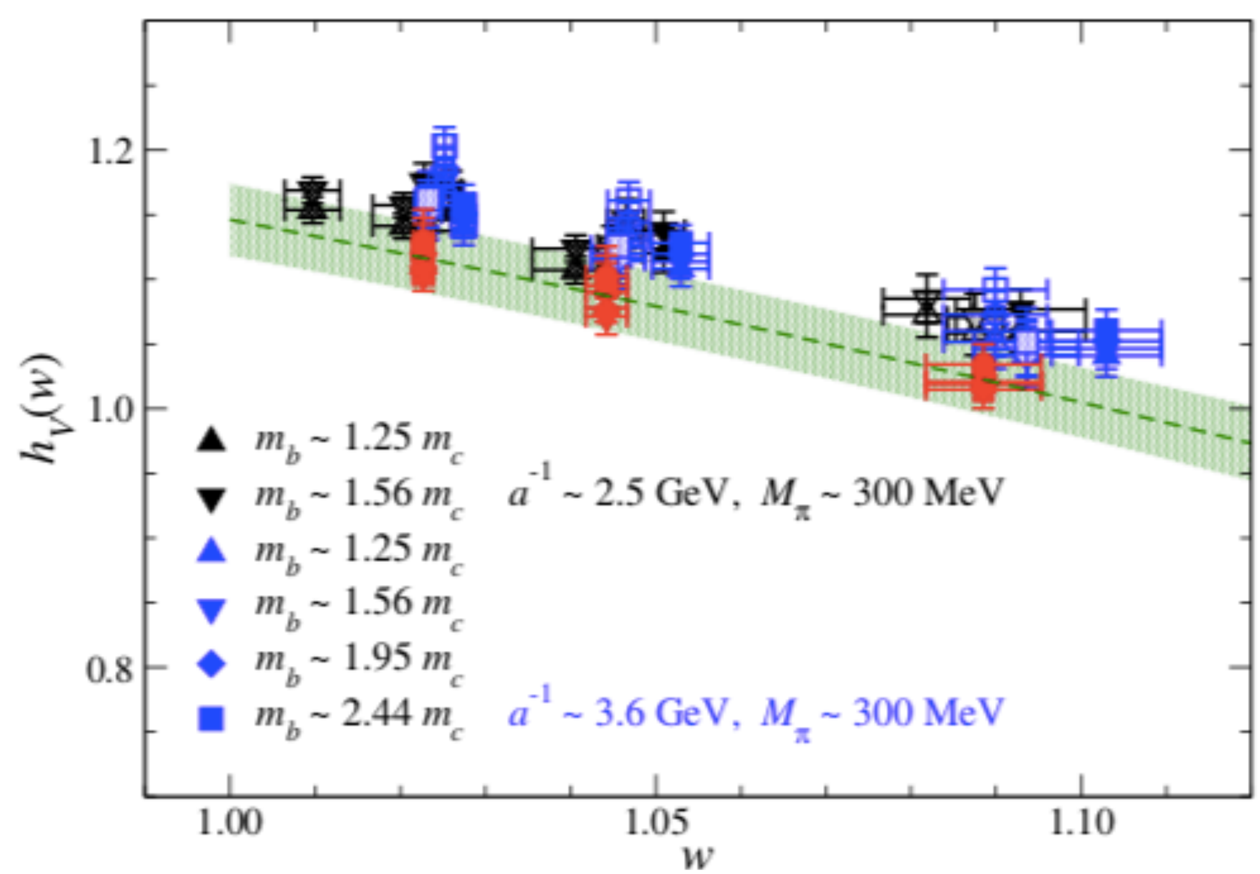
Ongoing work from other groups

JLQCD B→D(*)

arXiv:1912.11770
PoS LATTICE2019 (2019) 139

- Ensembles: 8 x (Nf=2+1) Möbius domain wall
- Lattice spacings: ~ [0.044, 0.055, 0.08] fm
- Valence quarks: Möbius domain wall (light+heavy)
- Work up to $m_h = 2.4\ m_c$

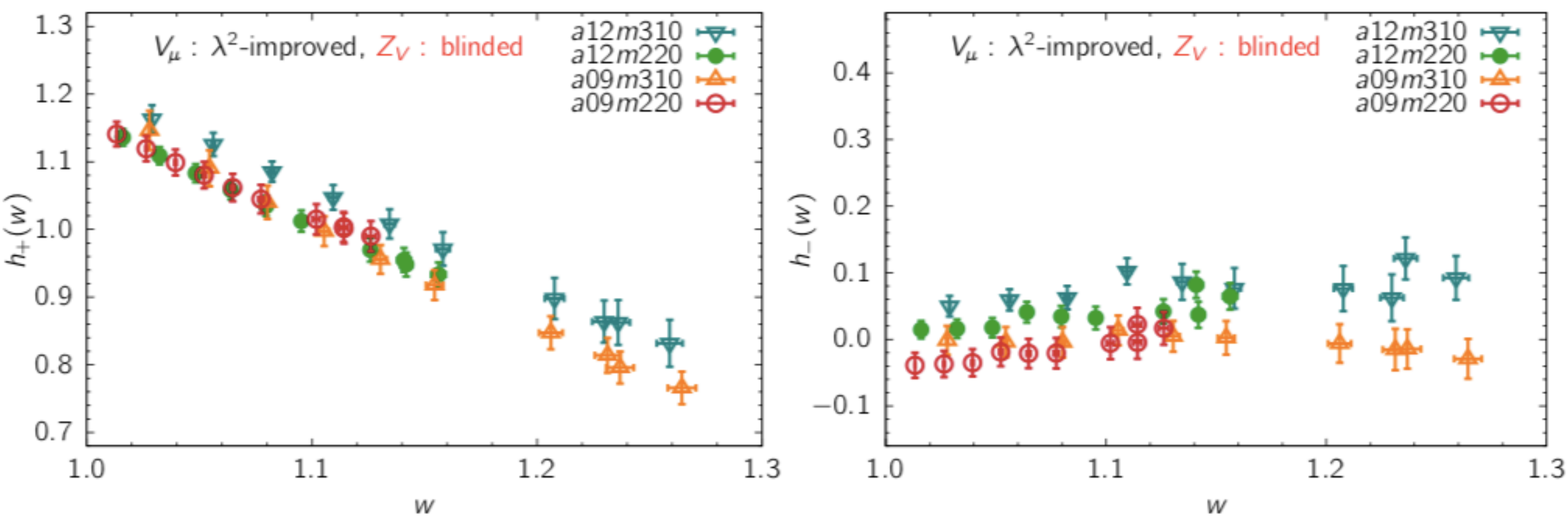




LANL/SWME B→D(*)

arXiv:2003.09206
PoS LATTICE2019 (2020) 056

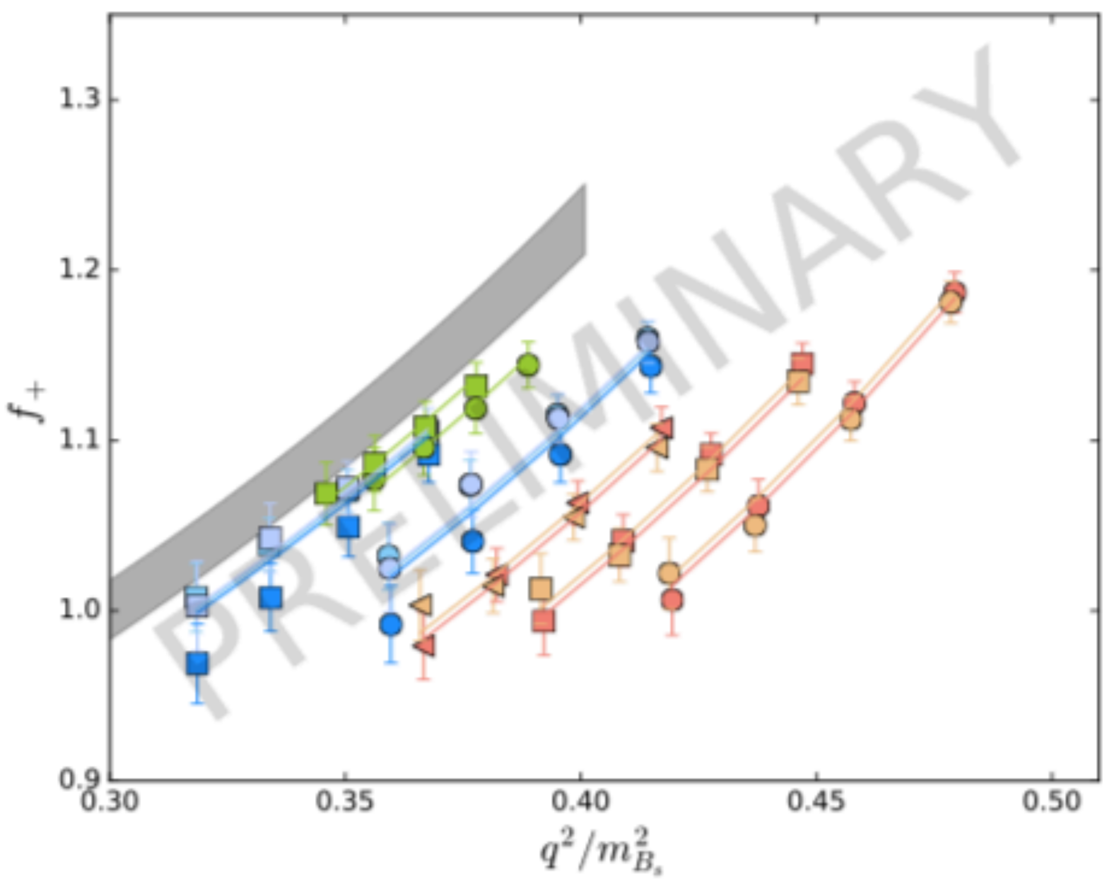
- Ensembles: 4x (Nf=2+1+1) MILC HISQ
- Lattice spacings: [0.09, 0.12] fm
- Light valence quarks: HISQ
- Heavy valence: Oktay-Kronfeld (OK) action

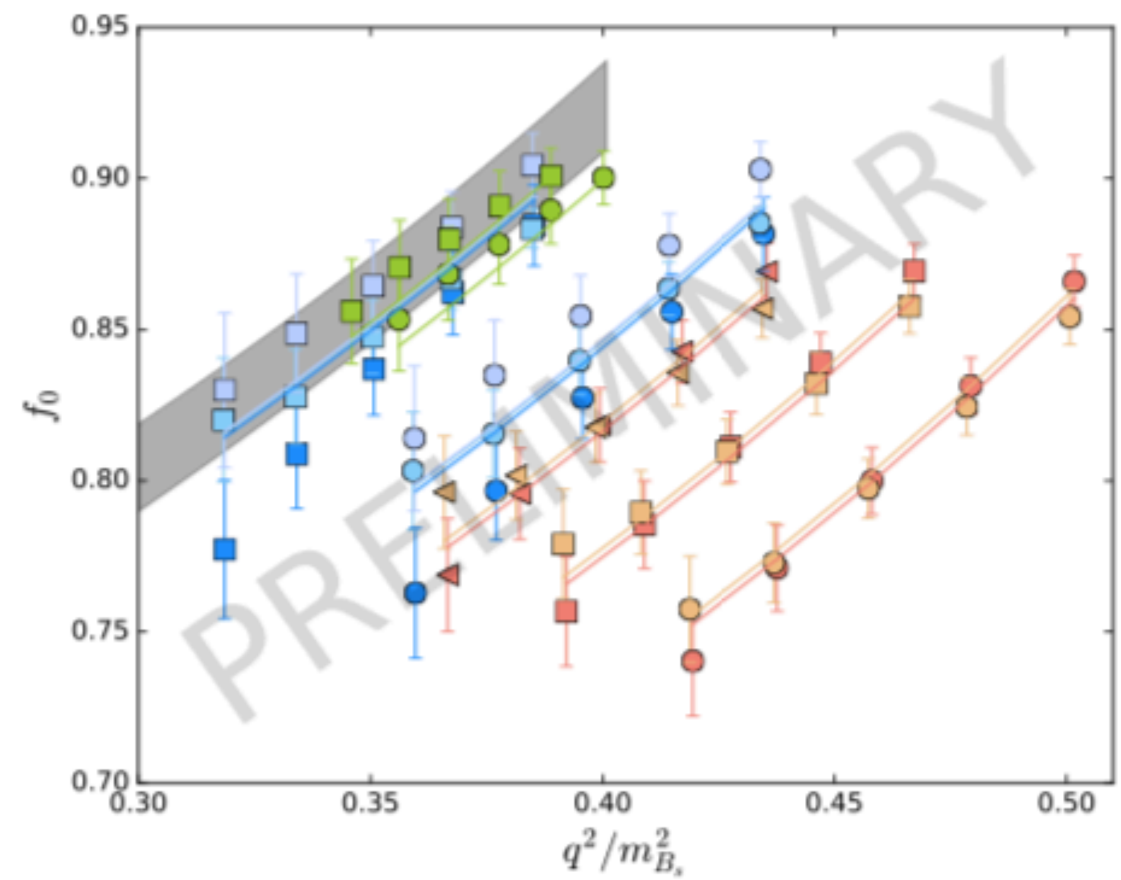


RBC/UKQCD $B_s \to D_s$

arXiv:1903.02100
PoS LATTICE2018 (2019) 290

- Ensembles: 5x (Nf=2+1) domain wall
- Lattice spacings: [0.07, 0.08, 0.11] fm
- Light valence quarks: domain wall
- Valence charm: Möbius domain wall
- Valence “bottom”: RHQ





Summary

- Vibrant community effort to calculate B-to-D matrix elements
- Improved experimental measurements at LHCb and Belle II require commensurate improvements from LQCD
- Recent movement toward simulations with relativistic light-quark actions for charm and bottom
- Push to extend results for $B \to D^*$ away from zero recoil (w=1)
- Future: QED corrections

Acknowledgements

FNAL/MILC All-HISQ working group:

- Carleton DeTar
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- Elvira Gamiz
- Zech Gelzer
- Steve Gottlieb
- Andreas Kronfeld
- Andrew Lytle
- Jim Simone

Special thanks:

- Alex Vaquero

  indico.fnal.gov/event/46246/
  Snowmass workshop
  “Theory meets experiment on $|V_{ub}|$ and $|V_{cb}|$

- Andrew Lytle

  Plenary review talk
  PoS LATTICE2019 (2020)
  arXiv:2004.01132

Backup

More results

HPQCD $B_s \to D_s$

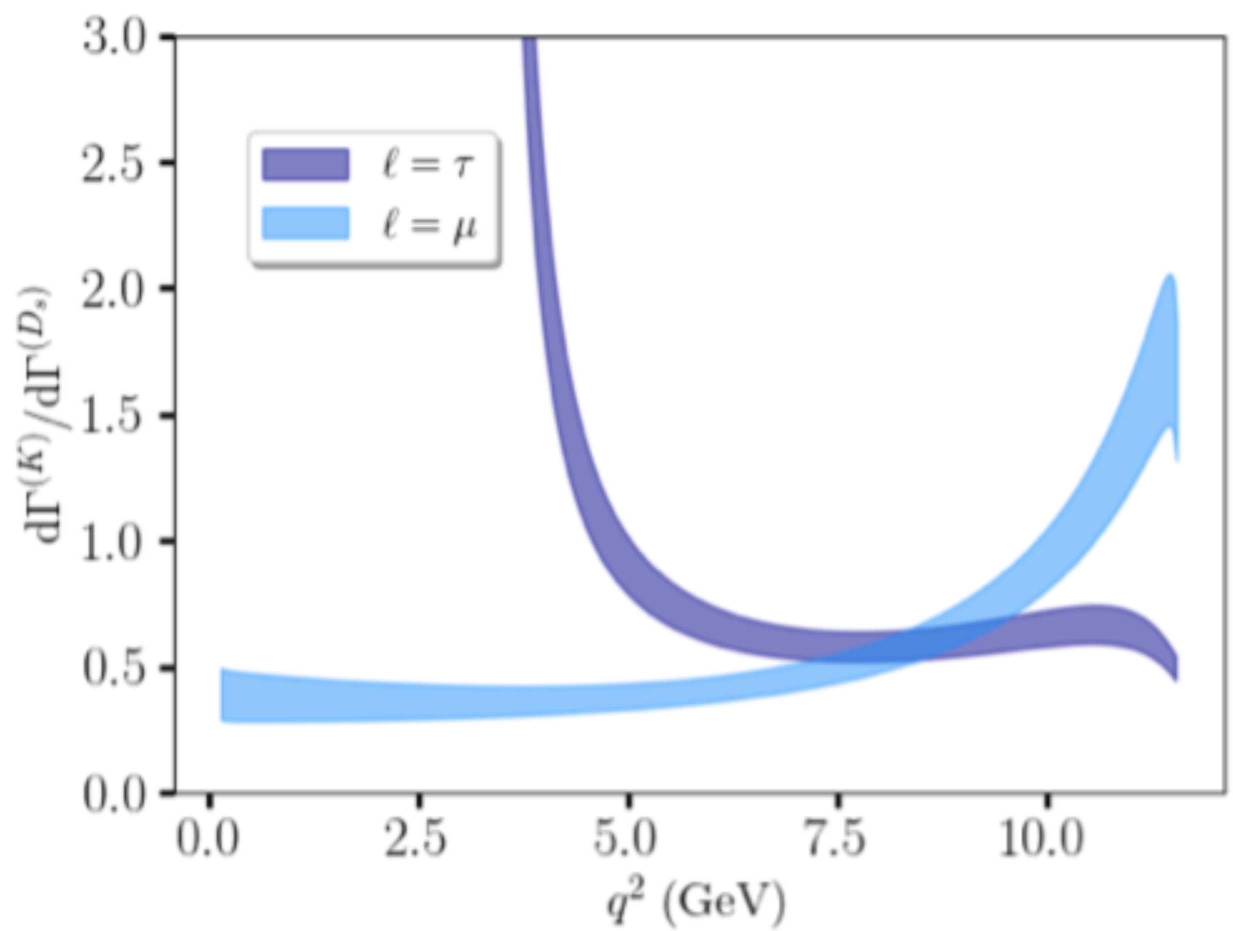
arXiv:1808.09285
PRD 98 (2018) 11, 114509

"Generation I"

- Ensembles: 5x (N_f=2+1) MILC asqtad
- Lattice spacings: [0.09, 0.12] fm
- Light valence quarks: asqtad/HISQ
- Heavy valence: HISQ (charm), NRQCD (bottom)
- Full physical q^2
- $R(K)/R(D_s) = 2.02(12)$
- $f_0^{(K)}(0)/f_0^{(D_s)}(0) = 0.507(66)$

TABLE VI. Error budget for the form factor ratios at zero momentum transfer, Eq. (22). We describe each source of uncertainty in more detail in the accompanying text.

Type	Partial uncertainty (%)
Statistical	6.63
Chiral extrapolation	0.89
Quark mass tuning	2.18
Discretization	4.16
Kinematic	9.31
Matching	0.28
Total	13.03



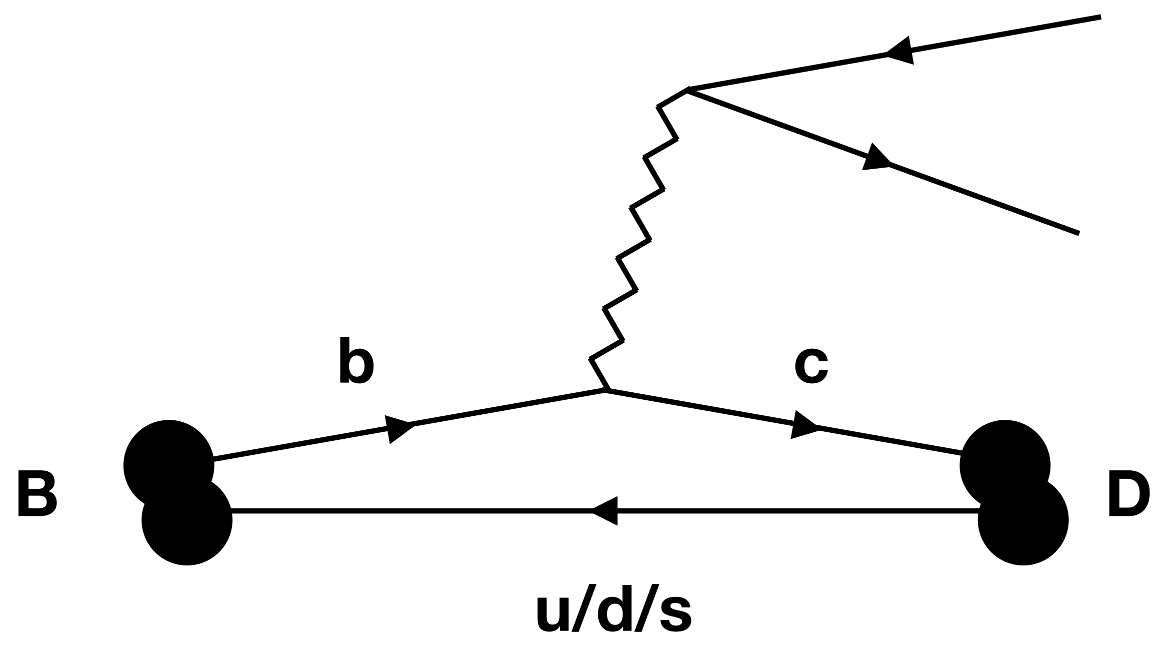
ETMC $B_{(s)} \to D_{(s)}$

arXiv:1310.5238
EPJ C 74 (2014) 5, 2861

- Ensembles: 3 x (Nf=2) ETMC twisted-mass
- Lattice spacings: 4 x in [0.05 - 0.1] fm
- Valence quarks: maximally twisted-mass
- Used scaling function to *interpolate* between the static limit and heavy masses near m_c
- G(1) = 1.033(95), $B \to D$
- G(1) = 1.052(46), $B_s \to D_s$

Pedagogy / background

Kinematic setup — B to D



Decaying pseudoscalars

- $B^{\pm}$, B^0 ~ 5.3 GeV
- B_s ~ 5.4 GeV

Note: vectors are unstable at the physical point

- $D_{(s)}^{\star} - D_{(s)} \gtrsim M_\pi$

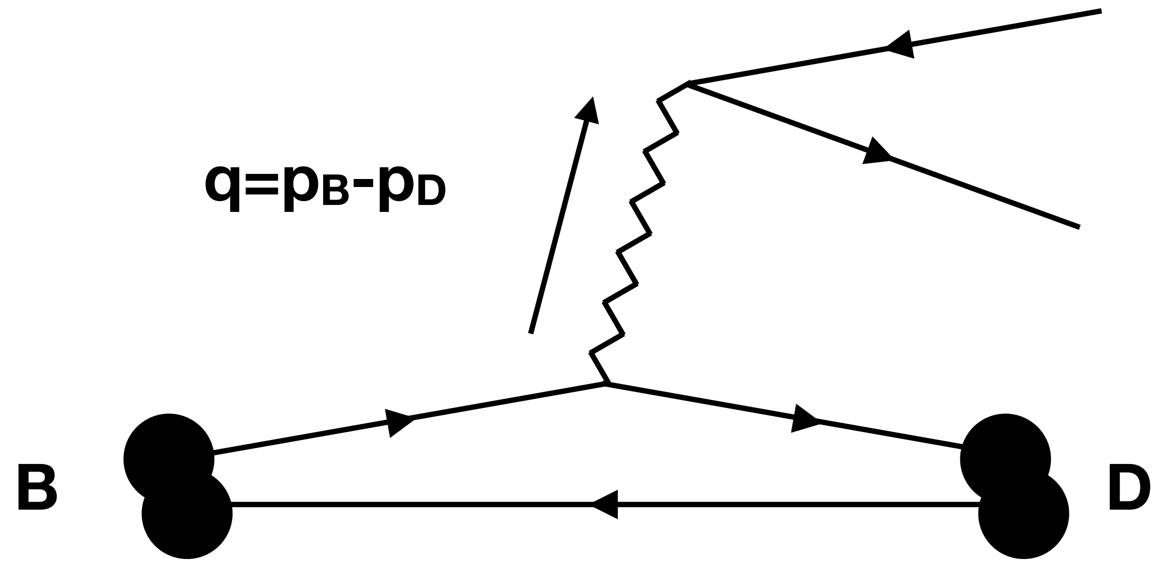
Daughter pseudoscalars

- $D^{\pm}$, D^0 ~ 1.9 GeV
- D_s ~ 2.0 GeV

Daughter vectors:

- $D^{\star\pm}$, $D^{\star 0}$ ~ 2.0 GeV
- $D^{\star}_s$ ~ 2.1 GeV

Kinematic setup — B to D



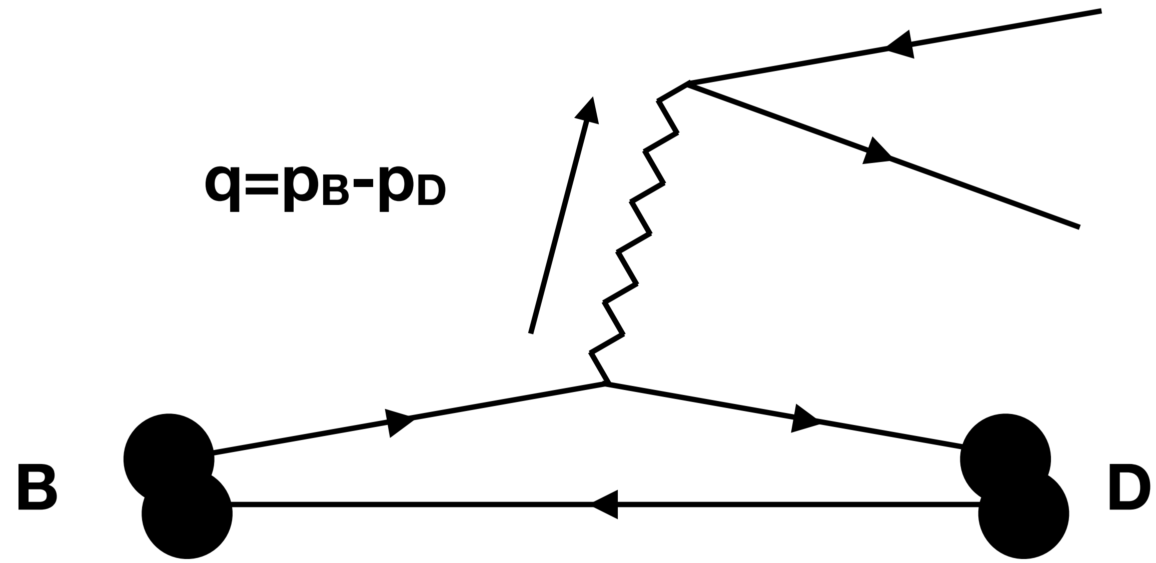
- Momentum transfer q^2:
  - $q^2 \to 0 \Leftrightarrow$ daughter meson carries away most energy
  - $q^2 \to q^2_{max}$

    $\Leftrightarrow$ zero recoil by daughter meson

    $\Leftrightarrow$ leptons carry away most energy

    $\Leftrightarrow$ $w = 1$ $(= v_B \cdot v_D)$

Kinematic setup — B to D

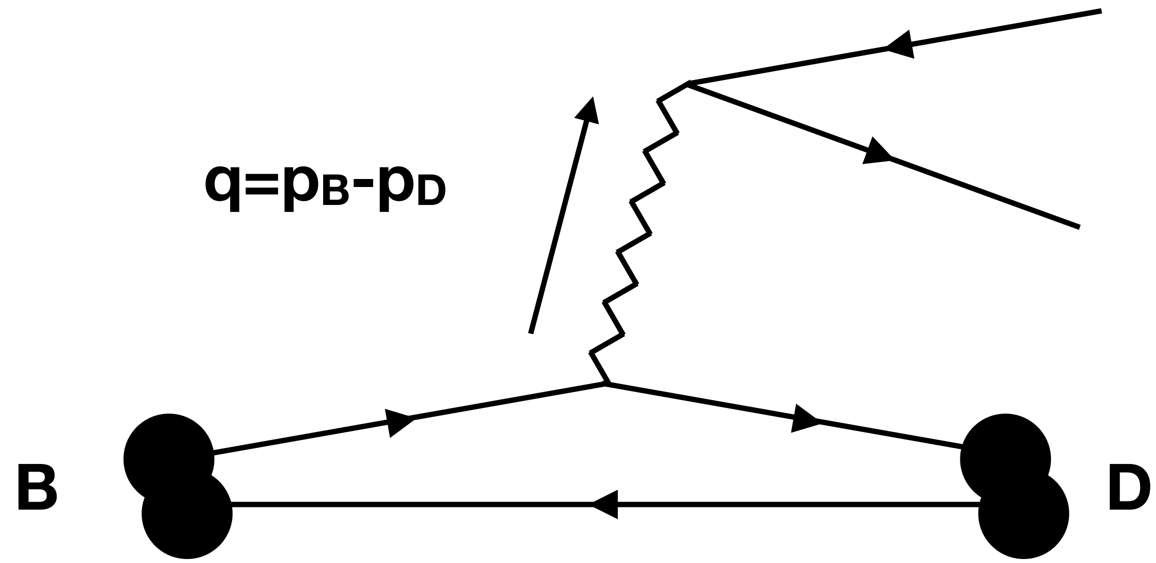


$$\frac{d\Gamma}{dw} = (\text{known}) \times |V_{cb}|^2 \times (\text{phase space}) \times (\text{form factors})$$

$$\frac{d\Gamma(B \to D)}{dw} = (\text{known})|V_{cb}|^2 (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

$$\frac{d\Gamma(B \to D^\star)}{dw} = (\text{known})|V_{cb}|^2 (w^2 - 1)^{1/2} \chi(w) |\mathcal{F}(w)|^2$$

Kinematic setup — B to D [pseudoscalar]



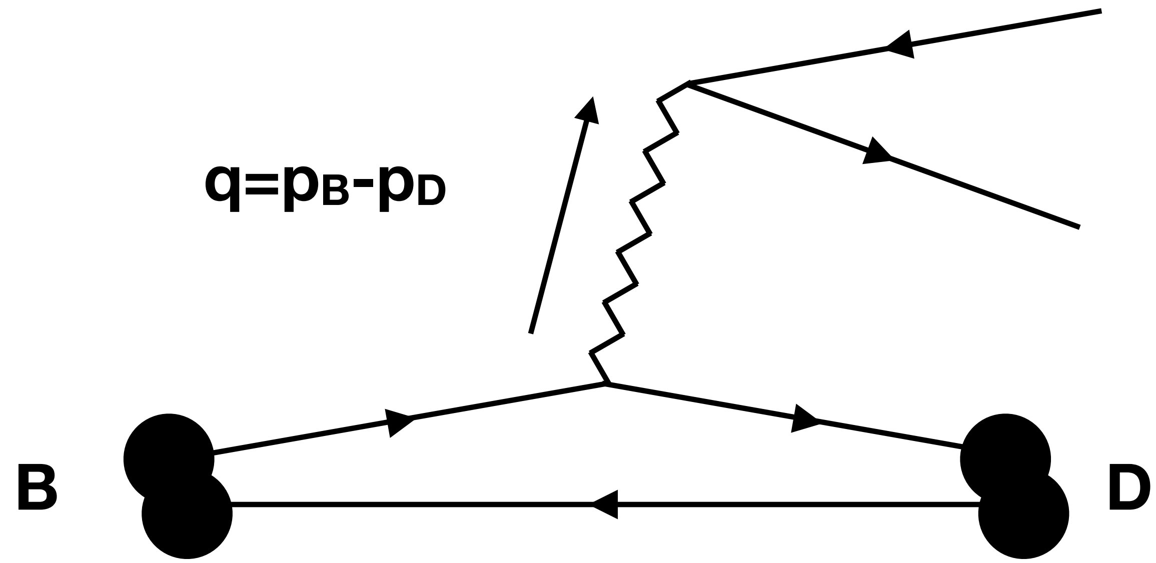
$$\frac{d\Gamma}{dw} = (\text{known}) \times |V_{cb}|^2 \times (\text{phase space}) \times (\text{form factors})$$

Case 1: $B_{(s)} \rightarrow D_{(s)}$ [Decay to a pseudoscalar]

$$(\text{phase space}) \propto \left|\mathbf{p}_{D_{(s)}}\right|^3 \sim (w^2 - 1)^{3/2}$$

- Phase-space suppression at low recoil ⟺ high q^2
- Experimental precision lowest where lattice is best

Kinematic setup — B to D [vector]



$$\frac{d\Gamma}{dw} = (\text{known}) \times |V_{cb}|^2 \times (\text{phase space}) \times (\text{form factors})$$

Case 2: $B_{(s)} \to D_{(s)}^\star$ [Decay to a vector]

$$(\text{phase space}) \propto \left|\mathbf{p}_{D^\star_{(s)}}\right| \sim (w^2 - 1)^{1/2}$$

- Much better experimental efficiency for vectors
- Experimental precision lowest where lattice is best

Connecting theory and experiment

- Much focus on the zero-recoil (w=1) limit:
  - Lattice calculations are most precise here
  - Certain expressions simplify at zero recoil
  - Experimental data is phase-space suppressed

- To bridge this gap, experiments parameterize form factors using the z-expansion, which exploits
  - Analyticity
  - Unitarity

Boyd, Grinstein, Lebed, PRL 74, 4603 (1995) [hep- ph/9412324]
Boyd, Grinstein Lebed, PRD 56, 6895 (1997) [hep-ph/9705252]
Grinstein, Kobach, Phys. Lett. B 771, 359 (2017) [arXiv:1703.08170]

Connecting theory and experiment

- Change variables using conformal map:

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} - \sqrt{2}}$$

- Expand in new variable:

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_n a_n^{(i)} z^n$$

Physical decay kinematics:
$0 < z < 0.06$

Generic form factor

Accounts for resonant (=analytic) structure of process

Weighting function (unitarity constraint)

Bounded:
$\sum |a_n|^2 < 1$