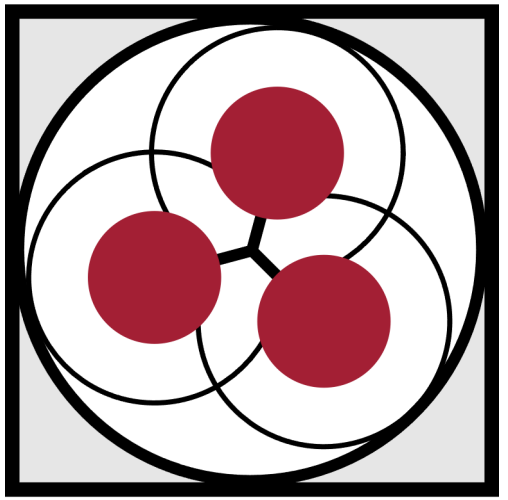

Generalized form factors of the proton (et al.)

USQCD All-Hands Meeting
Apr 22, 2022

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Outline

Generalized form factors (GFFs)

- Moments of generalized parton distributions
- Interesting subset: gravitational form factors (GFFs)

Lattice approach

Previous results (glue)

- Nucleon, pion, ρ meson, Δ baryon at $M_\pi = 450\ \mathrm{MeV}$
- [Pefkou DH Shanahan 2022]

Spatial densities of energy, pressure, shear forces

Ongoing calculation (glue + quark)

Generalized form factors (GFFs)

Interested in GFFs parametrizing hadronic matrix elements of twist-2 operators

$$\left\langle h(p')\middle|\bar{\psi}\, i\overleftrightarrow{D}\cdots i\overleftrightarrow{D}\,\Gamma\,\psi\middle|h(p)\right\rangle$$
$$\left\langle h(p')\middle|G\, i\overleftrightarrow{D}\cdots i\overleftrightarrow{D}\,\mathrm{G}\middle|h(p)\right\rangle \sim \sum_i K_i^{\mu_1\mu_2\cdots}\,\mathrm{GFF}_i(t)$$
$$\left\langle h(p')\middle|G\, i\overleftrightarrow{D}\cdots i\overleftrightarrow{D}\,\widetilde{\mathrm{G}}\middle|h(p)\right\rangle$$

Covariant derivative $\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

G : Gluon field strength tensor

Γ : Gamma matrices

K: Kinematic coefficient / Lorentz structure

p, p' : Incoming/outgoing momentum

Generalized parton distributions (GPDs)

GFFs related to Mellin moments of GPDs

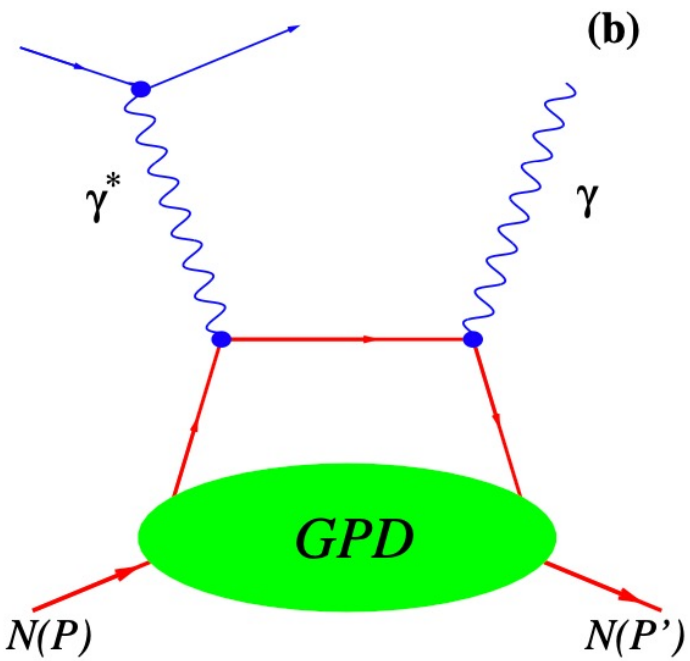
$$\int dx\, x^{n-1}\, \mathrm{GPD}(\mathrm{x}, \xi, t)$$

e.g. nucleon $n = 2$

$$\int dx\, x\, H_{q,g}(x, \xi, t) = A_{q,g}(t) + \xi^2 D_{q,g}(t)$$

$$\int dx\, x\, E_{q,g}(x, \xi, t) = B_{q,g}(t) - \xi^2 D_{q,g}(t)$$

→ relate to experiment via factorization



[Polyakov Schweitzer 2018]

Gravitational form factors (GFFs)

$a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$
$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$
$u, \bar{u}$ = Dirac spinors
$P = (p' + p)/2$
$\Delta = p' - p$
$t = \Delta^2$

$n = 2$ spin-independent generalized form factors

For (symmetric) EMT, $T^{\{\mu\nu\}} = T_g^{\{\mu\nu\}} + \sum_q T_q^{\{\mu\nu\}}$

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$

Quarks $T_q^{\{\mu\nu\}} = \bar{q}\,\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\,q$

Momentum fraction $A_{q,g}(0) = \langle x \rangle_{q,g}$ Spin fraction $J_{q,g}(0)$

$$\left\langle N(p')\middle|T_{g,q}^{\{\mu\nu\}}\middle|N(p)\right\rangle = \bar{u}(p')\left[A_{g,q}(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)Mg^{\mu\nu}\right]u(p)$$

Internal forces

$D(0)$: "the last global unknown"

Power-divergent mixing

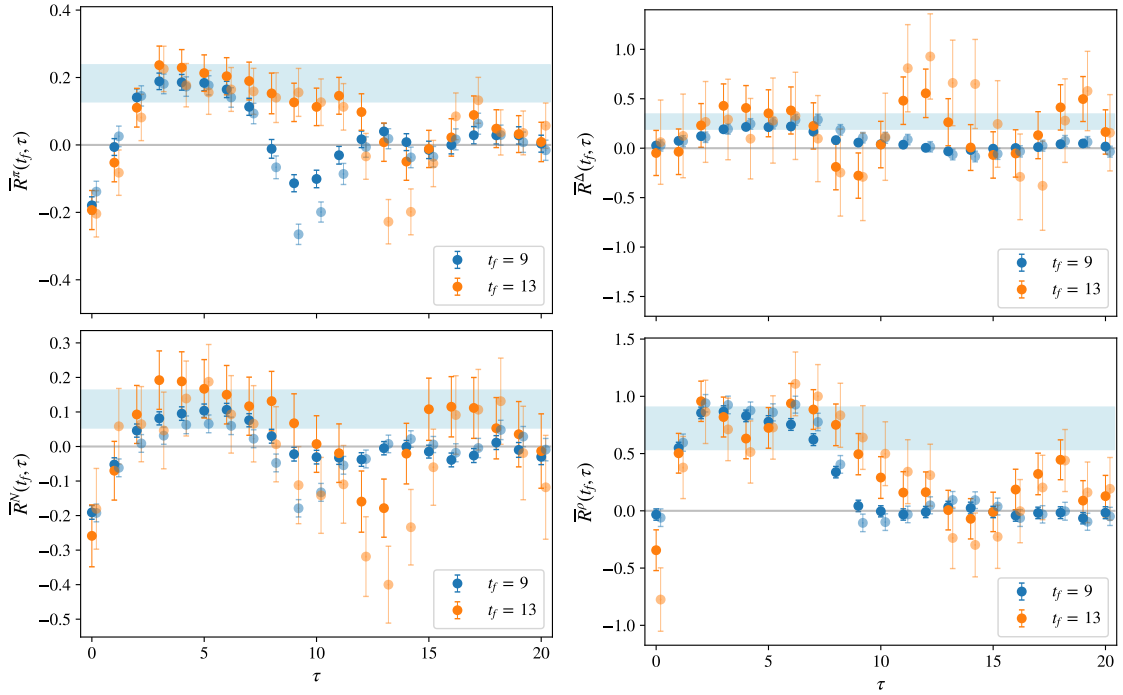
Not conserved $\sum_q \dot{c_q} + \dot{c_g} = 0$

Extracting the GFFs

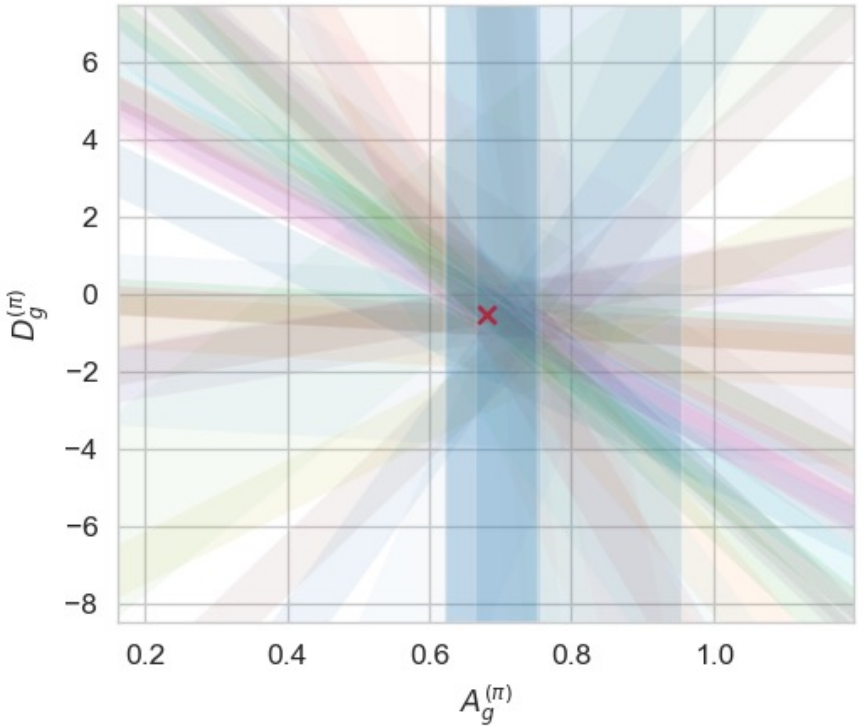
$$R(p,p';\tau,t_f) = \frac{C^{3\text{pt}}(p,p';t_f,\tau)}{C^{2\text{pt}}(p';t_f)} \sqrt{\frac{C^{2\text{pt}}(p;t_f-\tau)}{C^{2\text{pt}}(p';t_f-\tau)} \frac{C^{2\text{pt}}(p';\,t_f)}{C^{2\text{pt}}(p;t_f)} \frac{C^{2\text{pt}}(p';\,\tau)}{C^{2\text{pt}}(p;\tau)}}$$

$$= (\text{kinematic coeffs}) \cdot (\text{GFFs})(t) + (\text{excited states})$$

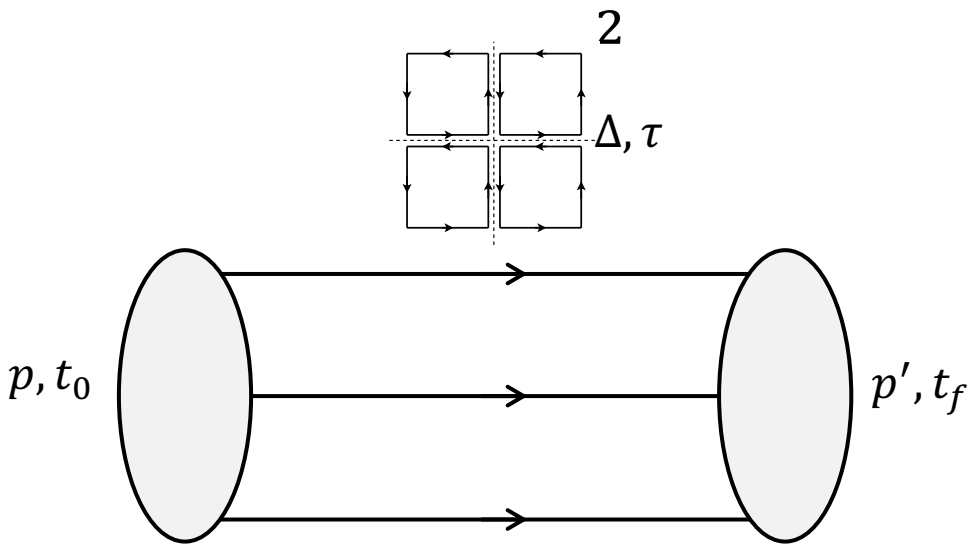
Example ratios



Linear constraints on GFFs



Glue three-point functions



Gluons are **noisy**

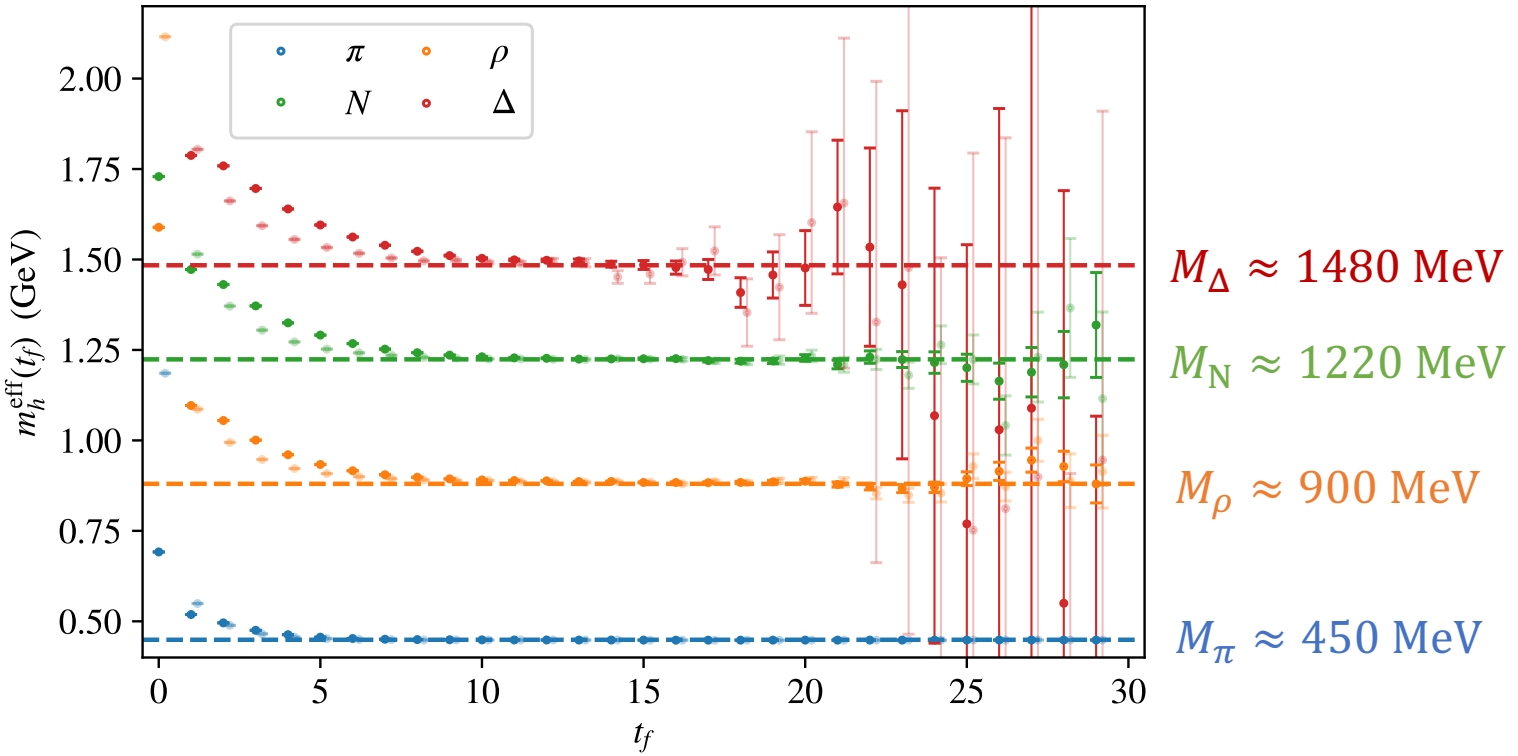
- Construct improved $G_{\mu\nu}$ with flowed links
- High statistics:
  - $O(1000)$ configs
  - 100s meas/config
  - Multiple source & sink smearings
  - All spin channels
  - Many p, p'
  - Complete hypercubic irrep bases
  - Fit full τ, t_f dependence

Results so far

Ensemble:

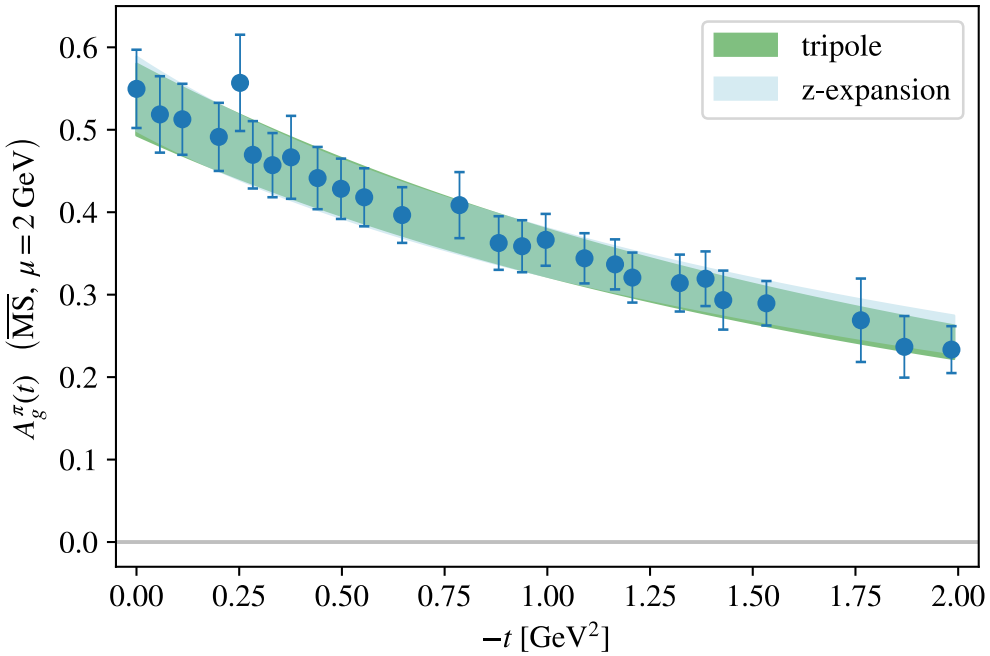
- $32^3 \times 96$ lattice, $M_\pi L \sim 8.5$
- Lüscher-Weisz gauge action
- 2+1 stout-smeared clover fermions
- $a = 0.1167(16)$ fm
- $M_\pi = 450(5)$ MeV
  - $\rightarrow \rho, \Delta$ are stable
- 2820 configs, ≈ 235 sources/config
- 2 source/sink smearings

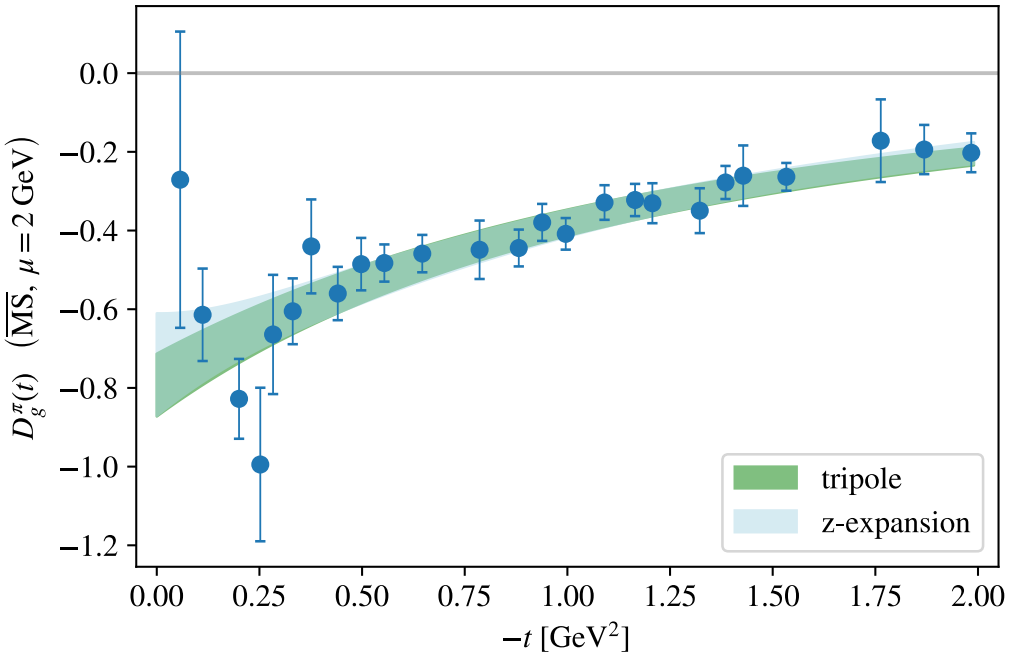
[Pefkou DH Shanahan 2022]



Results: pion

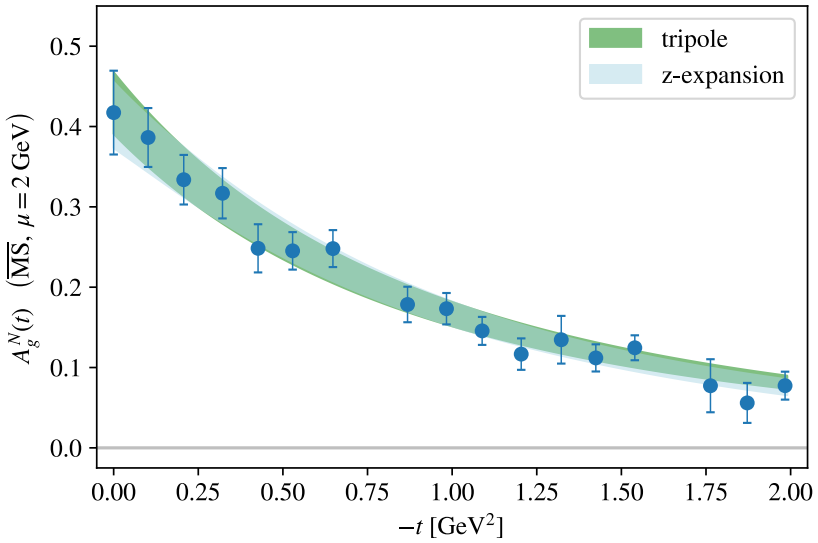
$$\left\langle \pi(p') \middle| T_g^{\{\mu\nu\}} \middle| \pi(p) \right\rangle = A_g(t)\, 2P^\mu P^\nu + D_g(t) \frac{1}{2} (\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) + \bar{c}_g(t) 2M^2 g^{\mu\nu}$$

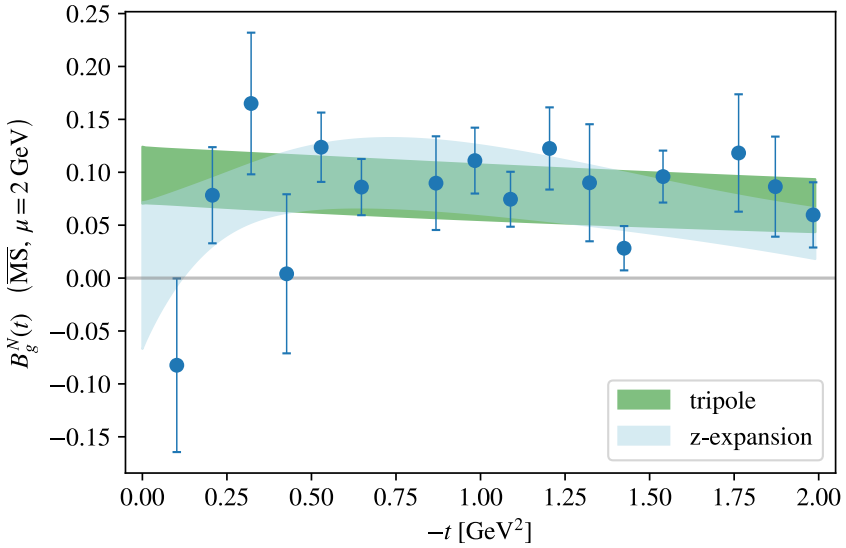


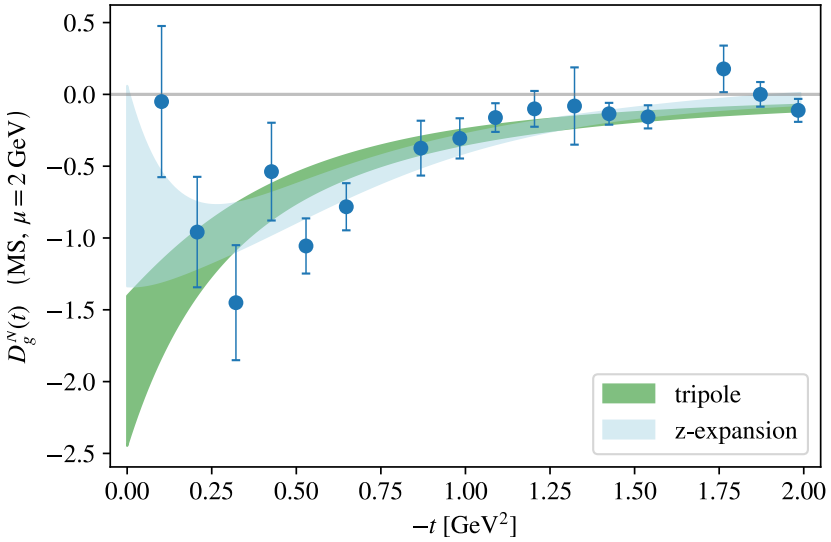


Results: nucleon

$$\left\langle N(p',s')\left|T_g^{\{\mu\nu\}}\right|N(p,s)\right\rangle = \bar{u}(p',s')\left[A_g(t)\,\gamma^{\{\mu}P^{\nu\}} + B_g(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_g(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_g(t)Mg^{\mu\nu}\right]u(p,s)$$







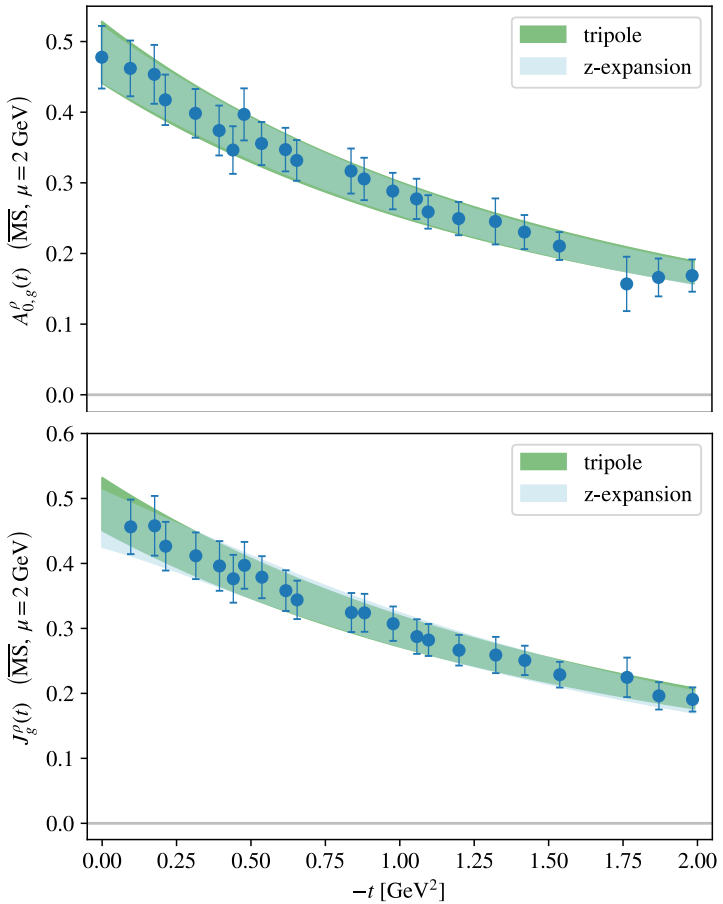
$$B(t) = 2J(t) - A(t)$$

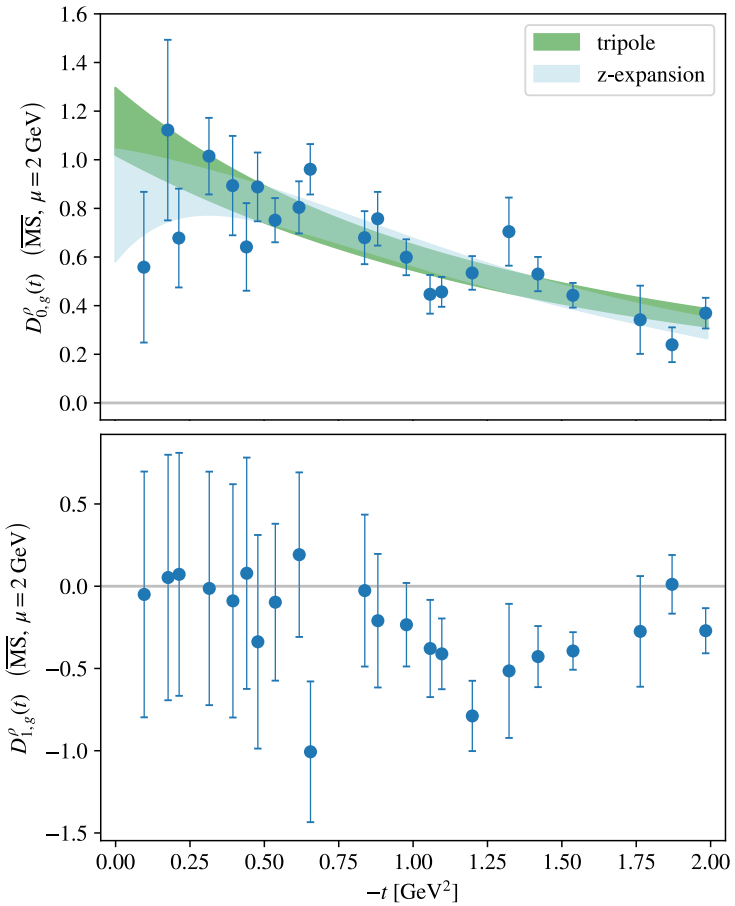
Results: ρ meson

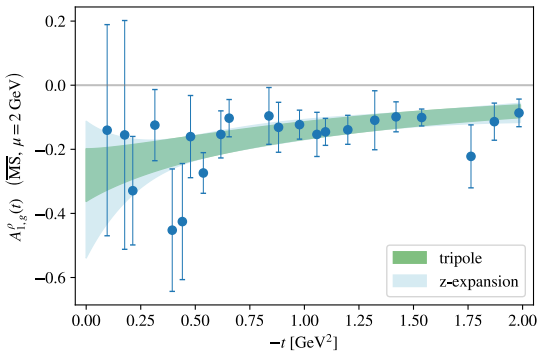
$$\left\langle \rho(p',\lambda') \middle| T_g^{\{\mu\nu\}} \middle| \rho(p,\lambda) \right\rangle = E^*_{\alpha'}(p',\lambda') \Big\{ 2P^{\{\mu}P^{\nu\}} \left[-g^{\alpha'\alpha} A_{0g}(t) + \frac{\mathrm{P}^{\alpha'}P^{\alpha}}{M^2} A_{1g}(t) \right] + \frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2) \left[g^{\alpha'\alpha}\, D_{0g}(t) + \frac{\mathrm{P}^{\alpha'}\mathrm{P}^{\alpha}}{M^2} D_{1g}(t) \right]$$

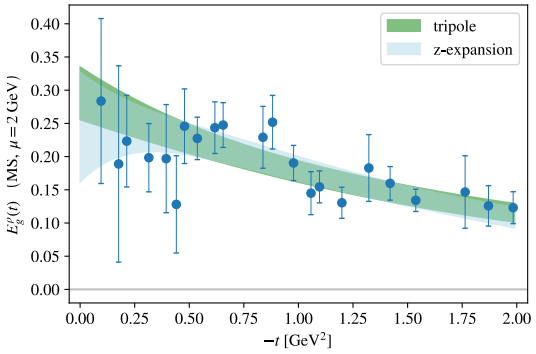
$$+ J_g(t)\, 8P^{\{\mu}g^{\nu\}\{\alpha'}\mathrm{P}^{\alpha\}} + E_g(t) \left(g^{\alpha\{\mu}g^{\nu\}\alpha'}\Delta^2 - 2\, g^{\alpha'\{\mu}\Delta^{\nu\}}P^{\alpha} + 2\, g^{\alpha\{\mu}\Delta^{\nu\}}P^{\alpha'} - 4g^{\mu\nu}P^{\alpha}P^{\alpha'} \right)$$

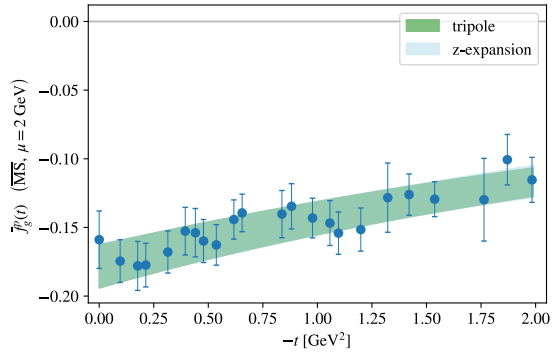
$$+ M^2 \left(2g^{\alpha'\{\mu}g^{\nu\}\alpha} - \frac{1}{2} g^{\alpha\alpha'}g^{\mu\nu} \right) \bar{f}_g(t) + g^{\mu\nu} \left[g^{\alpha\alpha'} M^2 \bar{c}_{0g}(t) + P^{\alpha}P^{\alpha'} \bar{c}_{1g}(t) \right] \Big\} E^*_{\alpha}(p,\lambda)$$







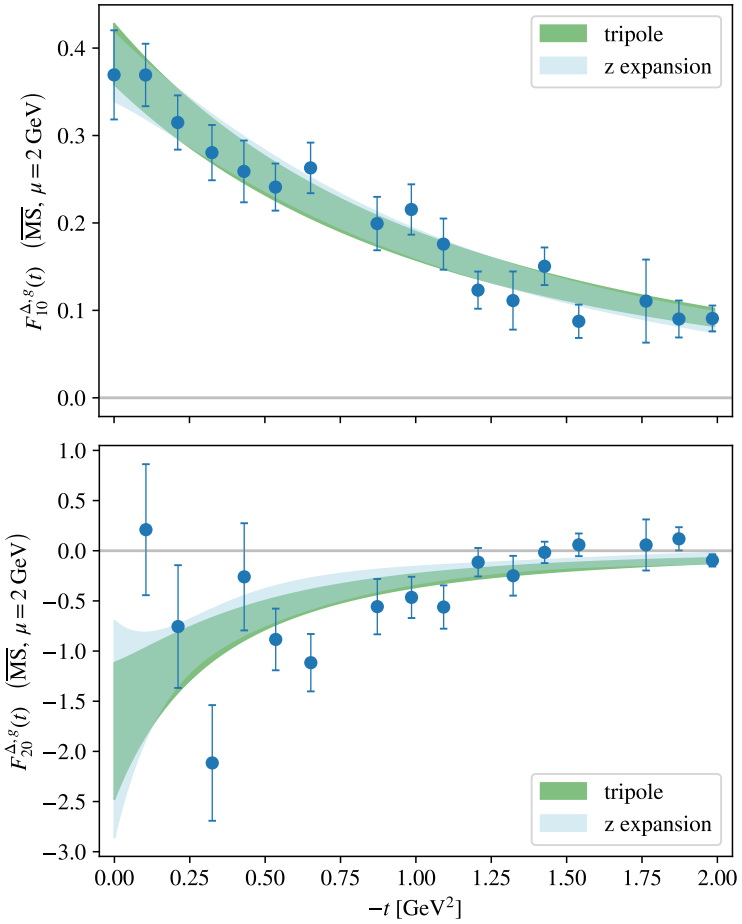


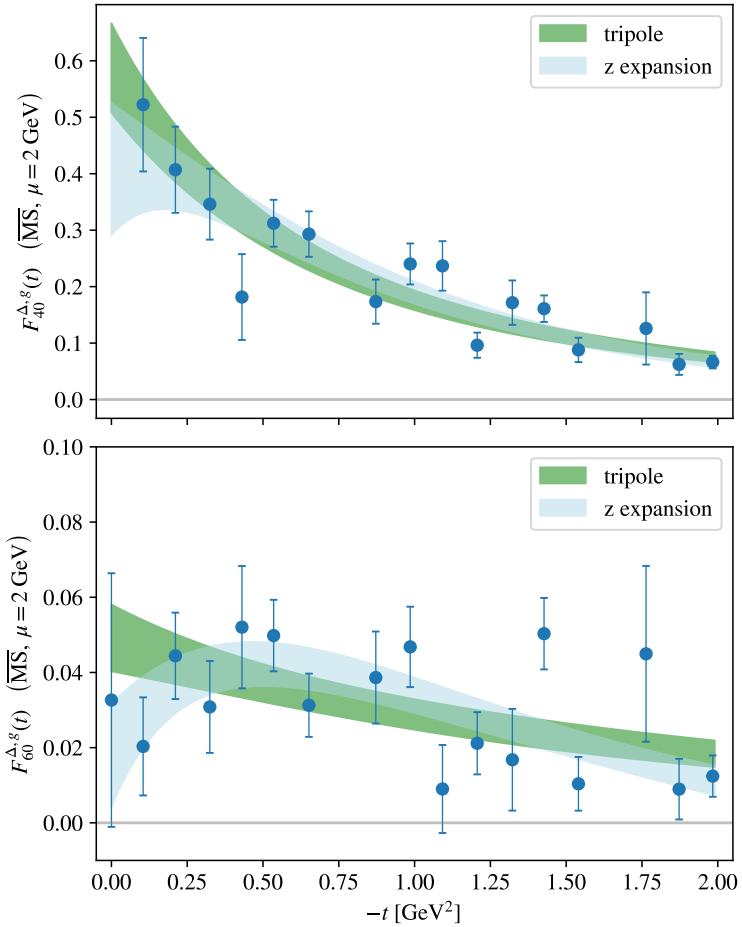


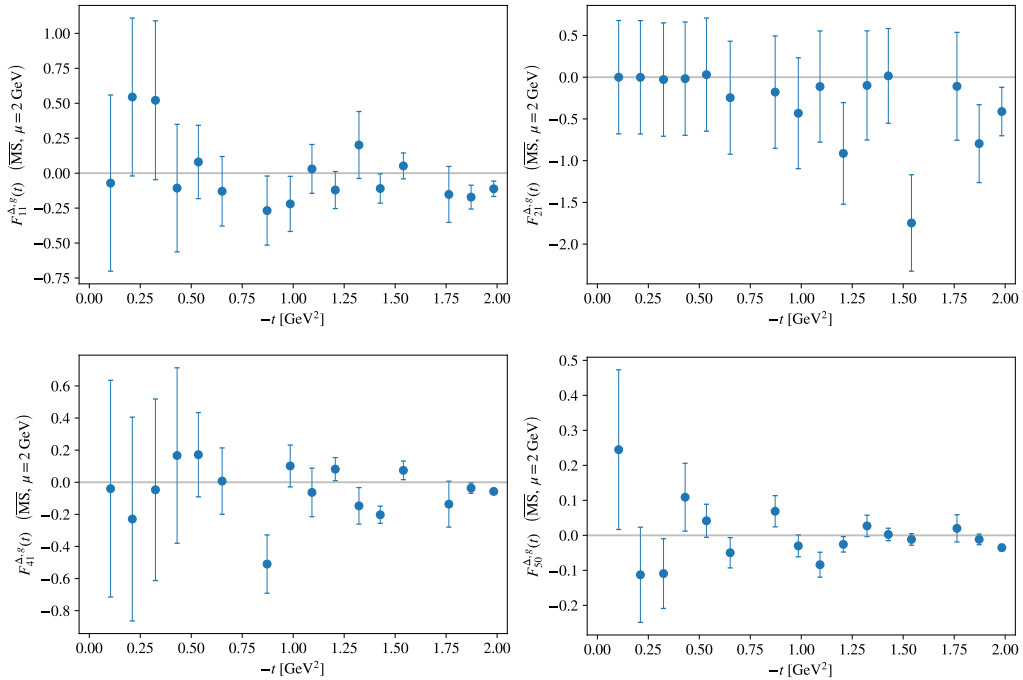
[conventions: Polyakov, Sun [1912.08749](1912.08749)]

Results: Δ baryon

$$\langle \Delta(p',\xi')|T_g^{\mu\nu}|\Delta(p,\xi)\rangle = \bar{u}_{\alpha'}(p',\xi')\left[\frac{P^\mu P^\nu}{M}\left(-g^{\alpha\alpha'}F_{10}^g(t)+\frac{\Delta^\alpha\Delta^{\alpha'}}{2M}F_{11}^g(t)\right)+\frac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{4M}\left(-g^{\alpha\alpha'}F_{20}^g(t)+\frac{\Delta^\alpha\Delta^{\alpha'}}{2M}F_{21}^g(t)\right)\right.$$
$$+Mg^{\mu\nu}\left(-g^{\alpha\alpha'}F_{30}^g(t)+\frac{\Delta^\alpha\Delta^{\alpha'}}{2M}F_{31}^g(t)\right)+\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{M}\left(-g^{\alpha'\alpha}F_{40}^g(t)+\frac{\Delta^{\alpha'}\Delta^\alpha}{2M^2}F_{41}^g(t)\right)$$
$$\left.+\frac{2}{M}\left(\Delta^{\{\mu}g^{\nu\}\{\alpha'}\Delta^{\alpha\}}-g^{\mu\nu}\Delta^\alpha\Delta^{\alpha'}-g^{\alpha'\{\mu}g^{\nu\}\alpha}\Delta^2\right)F_{50}^g(t)-2g^{\alpha'\{\mu}g^{\nu\}\alpha}MF_{60}^g(t)\right]u_\alpha(p,\xi)$$







[conventions: Kim, Sun 2011.00292]

Energy, pressure, and shear force densities

1. Parametrize $T_{\mu\nu}(t)$ with GFFs
2. Fourier transform $T_{\mu\nu}(t) \rightarrow T_{\mu\nu}(r)$
3. Identify e.g.

$$T_{\mu\nu}(r) = \begin{bmatrix} T_{tt}(r) & \\ & T_{ij}(r) \end{bmatrix} = \begin{bmatrix} \epsilon(r) & \\ & \left(\frac{r_i r_j}{r^2} - \frac{1}{d}\delta_{ij}\right) s(r) + \delta_{ij} p(r) \end{bmatrix}$$

→ Spatial densities

energy $\epsilon(r)$

pressure $p(r)$

shear forces $s(r)$

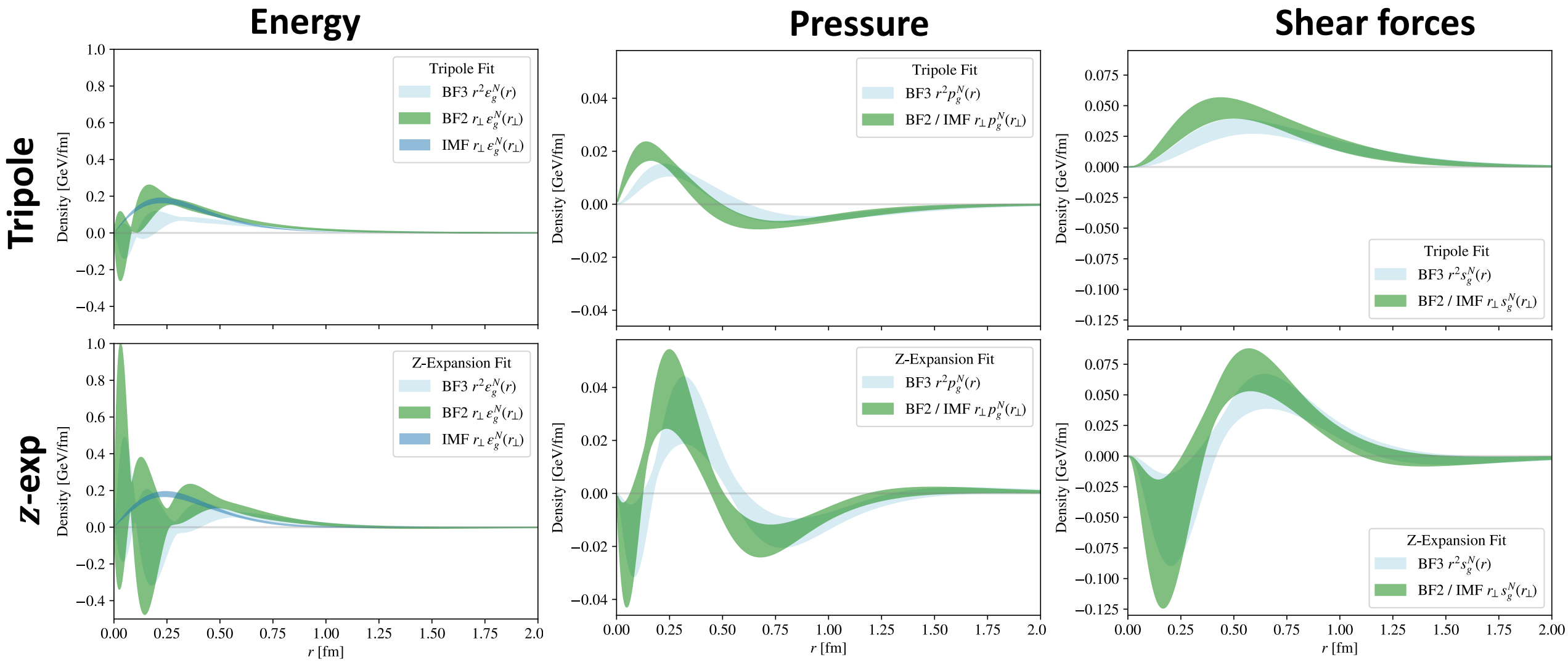
Note: frame dependent

$$T^{\mu\nu}_{i,\mathrm{BF3}}(r) = \int \frac{d^3\Delta e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}}{2P^0(2\pi)^3} \langle h(p,s)| T_i^{\mu\nu} |h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{BF2}}(r) = \int \frac{d^2\Delta_\perp e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)| T_i^{\mu\nu} |h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{IMF}}(r) = \int \frac{d^2\Delta_\perp e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)| T_i^{\mu\nu} |h(p',s')\rangle\big|^{P_z\rightarrow\infty}_{\mathbf{P}\cdot\boldsymbol{\Delta}=0}$$

Results: nucleon densities



In progress

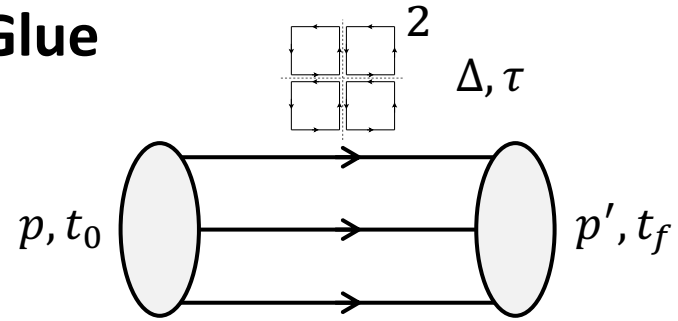
Full set of quark + glue GFFs for N, π on a different ensemble:

- Lüscher-Weisz gauge action
- 2+1 stout-smeared clover fermions
- $M_\pi = 170\ \mathrm{MeV}$
- $a = 0.091\ \mathrm{fm}$
- $48^3 \times 96$

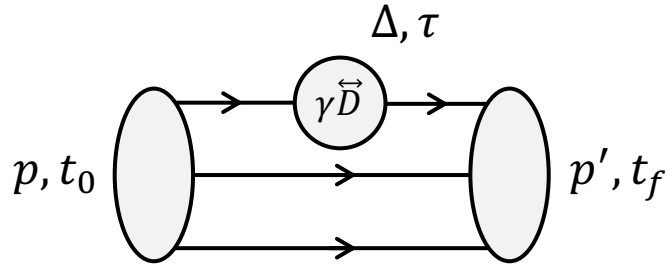
Estimated final stats:

- Glue: ~ 1800 configs × 512 sources
- Connected: ~ 1000 configs × 16 sources
- Disconnected: WIP

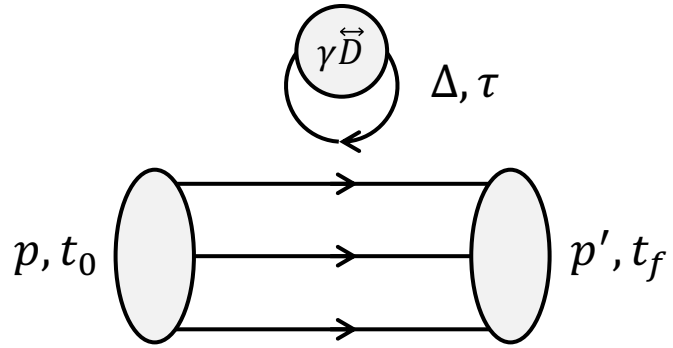
Glue



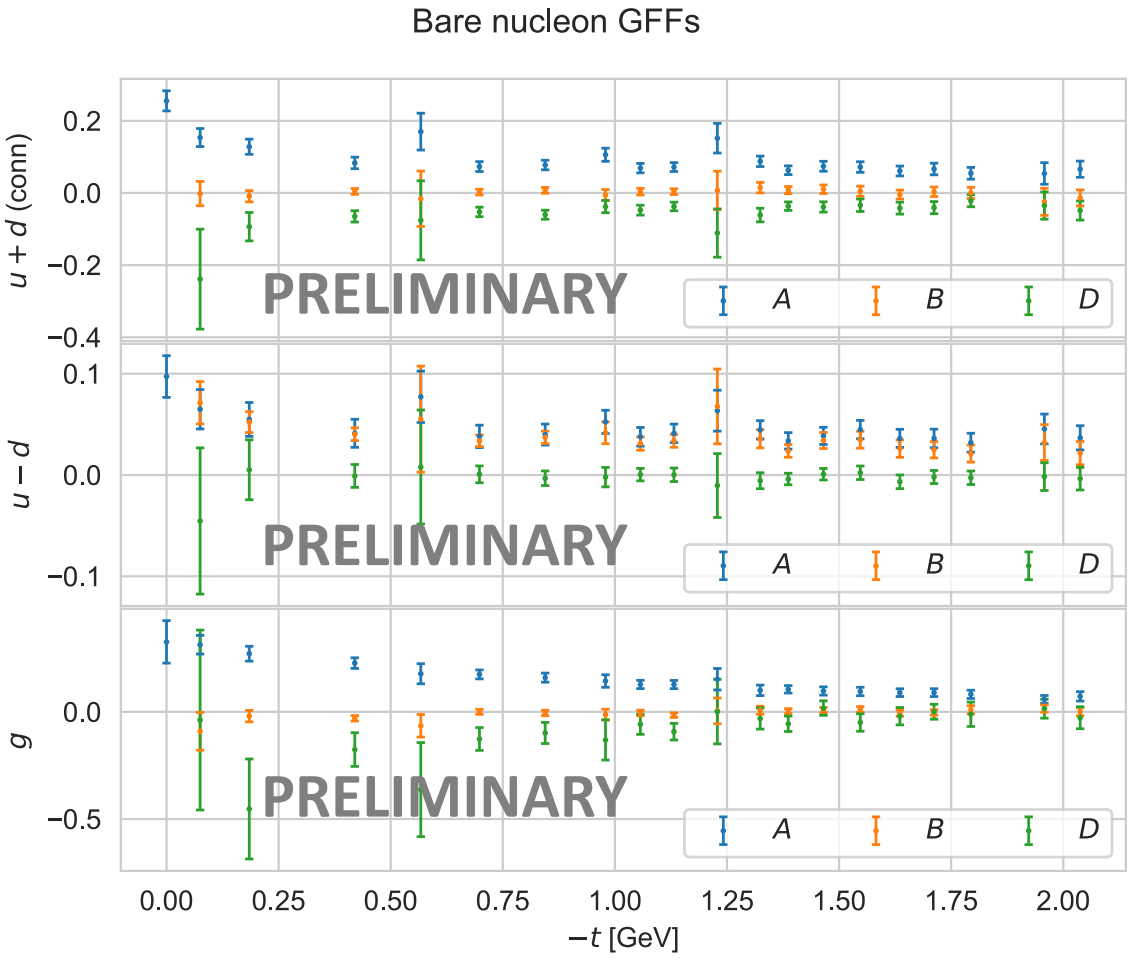
Connected quark $(u - d)$



Disconnected quark $(u + d, s)$



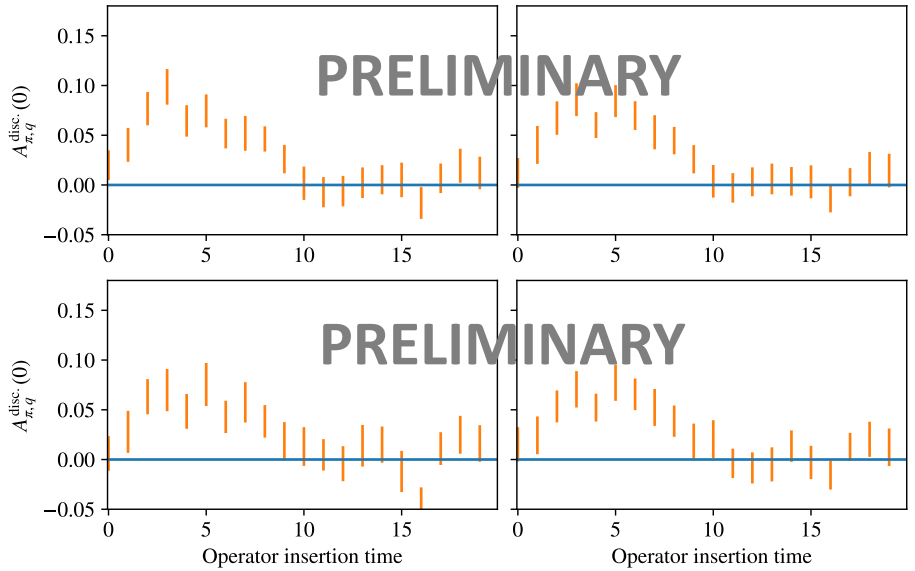
Preliminary results



Note:

- Partial datasets ($\lesssim 50\%$)
- Preliminary analysis
- Not renormalized

Example disconnected pion ratios



Ongoing/Future

So far:

- Gluon GFFs of the N, π, ρ, Δ at $m_\pi = 450\ \mathrm{MeV}$

Upcoming:

- Gravitational form factors of the N, π at $m_\pi = 170\ \mathrm{MeV}$
- Complete calculation of energy, pressure, shear force densities

Current proposal:

- Full set of lowest-lying quark and gluon GFFs: axial, tensor structures

Future:

- More ensembles → full control over systematics