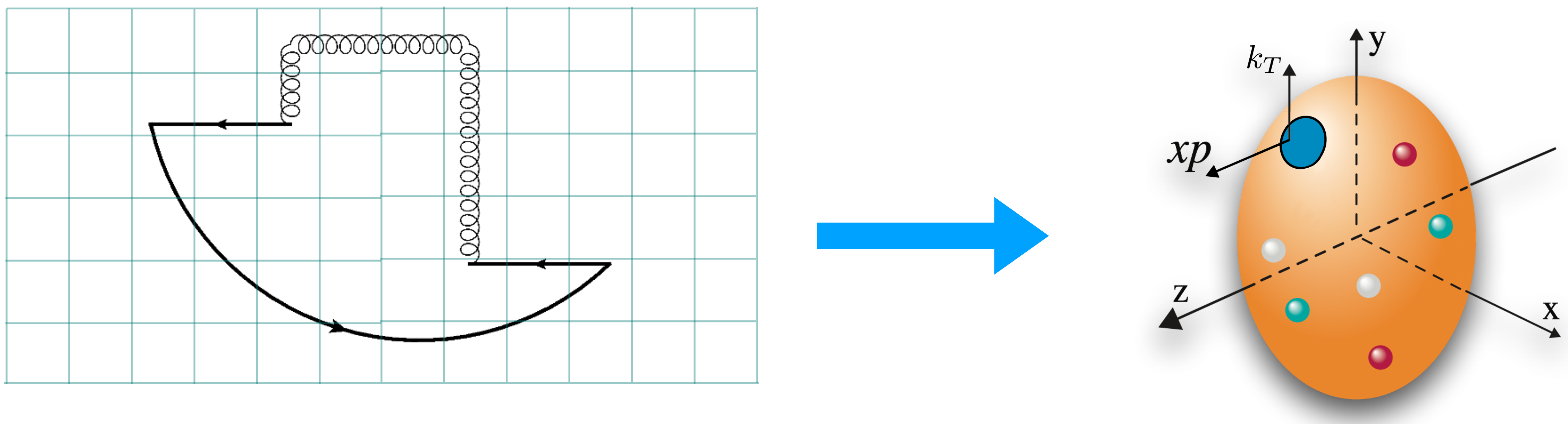
Computing the Collins-Soper Kernel

Michael Wagman



in collaboration with

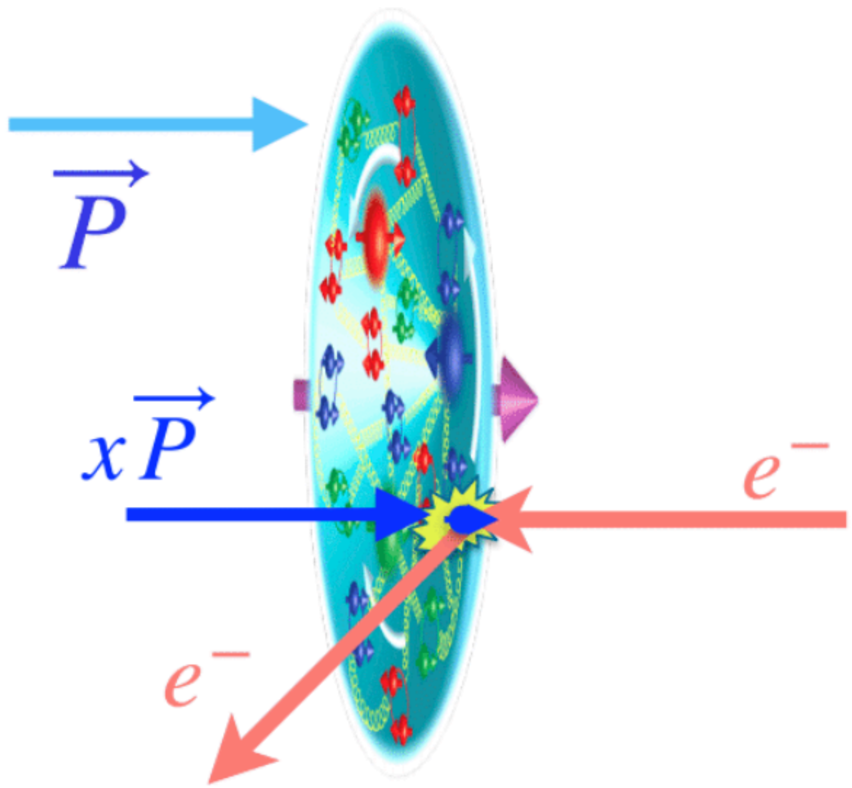
Artur Avkhadiev, Phiala Shanahan, and Yong Zhao

USQCD All Hands Meeting

April 21, 2022

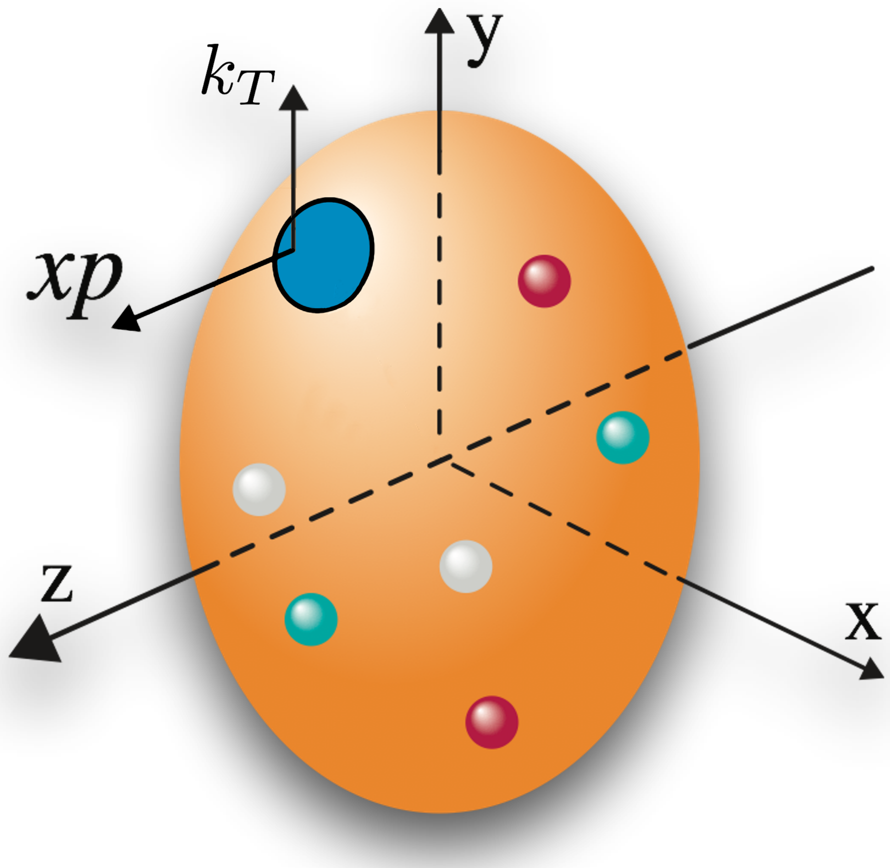
Fermilab

3D hadron structure



Our knowledge of proton structure has historically focused on collinear PDFs

$f_i(x)$ encodes probability of finding a parton of type i carrying momentum fraction x within a high-energy hadron



Hadrons further contain rich 3D structure encoded in TMDPDFs

$$f_i(x, \vec{k}_T)$$

Coordinate-space version:

$$f_i(x, \vec{b}_T) = \int d^2k_T\, e^{i\vec{k}_T \cdot \vec{b}_T}\, f_i(x, \vec{k}_T)$$

The Collins-Soper kernel

TMDPDFs depend on UV renormalization scale μ as well as a scale ζ associated with the renormalization of rapidity divergences

$$f_i^{\rm TMD}(x,\vec{b}_T,\mu,\zeta) = f_i^{\rm TMD}(x,\vec{b}_T,\mu_0,\zeta_0)$$
$$\times \exp\left[\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\gamma_\mu^i(\mu',\zeta_0)\right]\exp\left[\frac{1}{2}\gamma_\zeta^i(\mu,b_T)\ln\frac{\zeta}{\zeta_0}\right]$$

UV anomalous dimension

Collins-Soper kernel

(rapidity anomalous dimension)

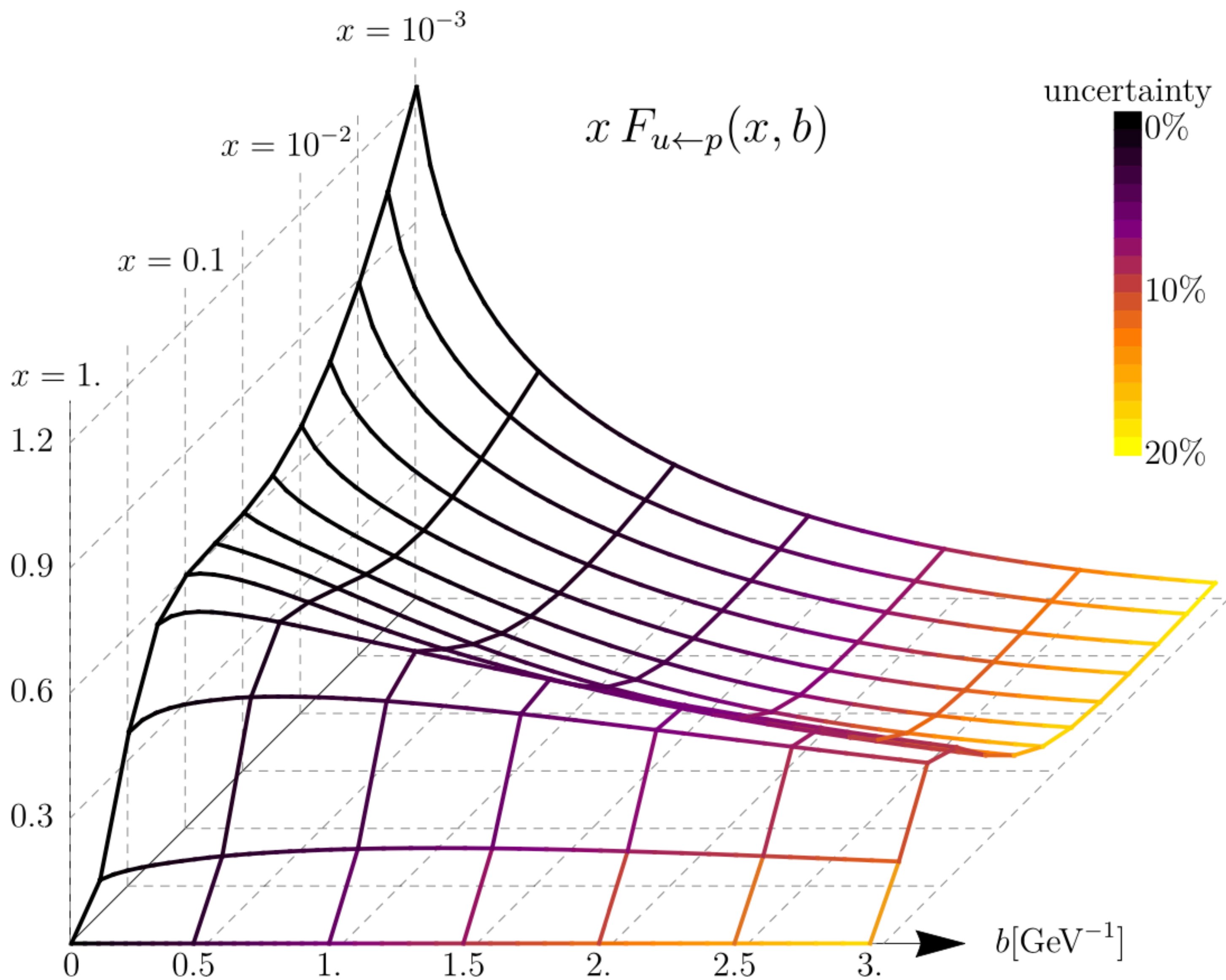
Changing hard momentum scales requires evolving TMDPDFs in μ and $\zeta \sim (2xP^z)^2$

Evolution in μ is perturbative as long as μ is large, but evolution in ζ is always nonperturbative for large b_T

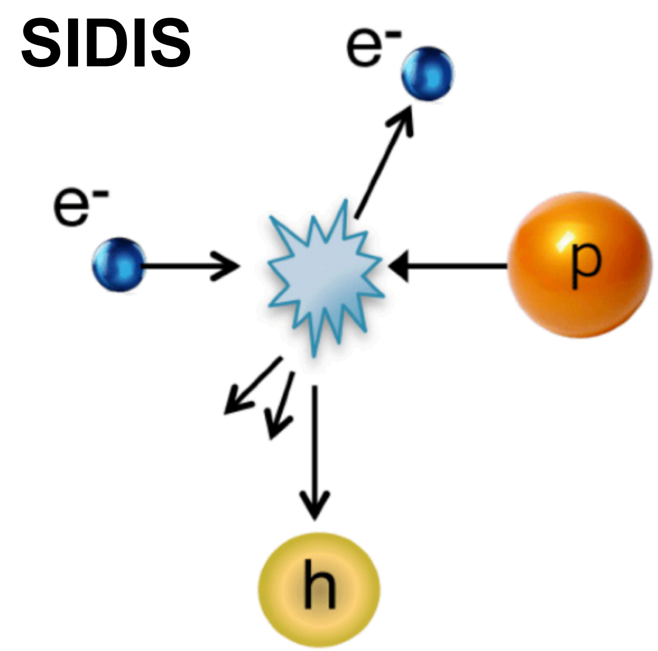
Experimental probes

TMDPDFs are needed to describe cross-sections for semi-inclusive DIS and the Drell-Yan process

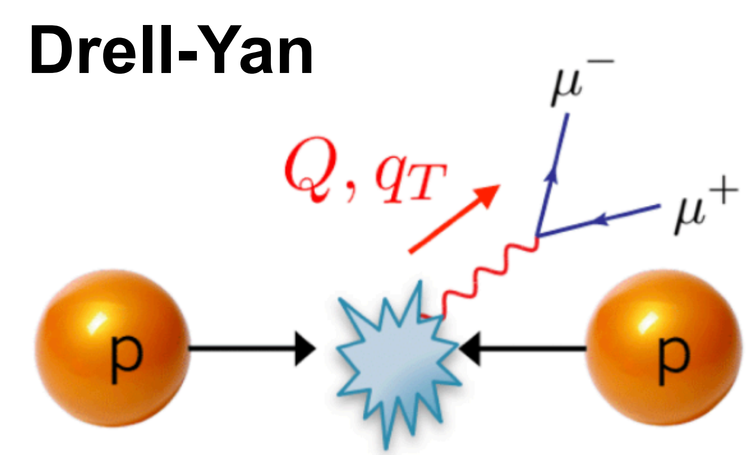
Phenomenological analysis constrain TMDPDFs precisely for small but not large b_T



Scimemi and Vladimirov, JEHP 06 (2020)



HERMES, COMPASS, JLab, …



Fermilab, RHIC, LHC, …

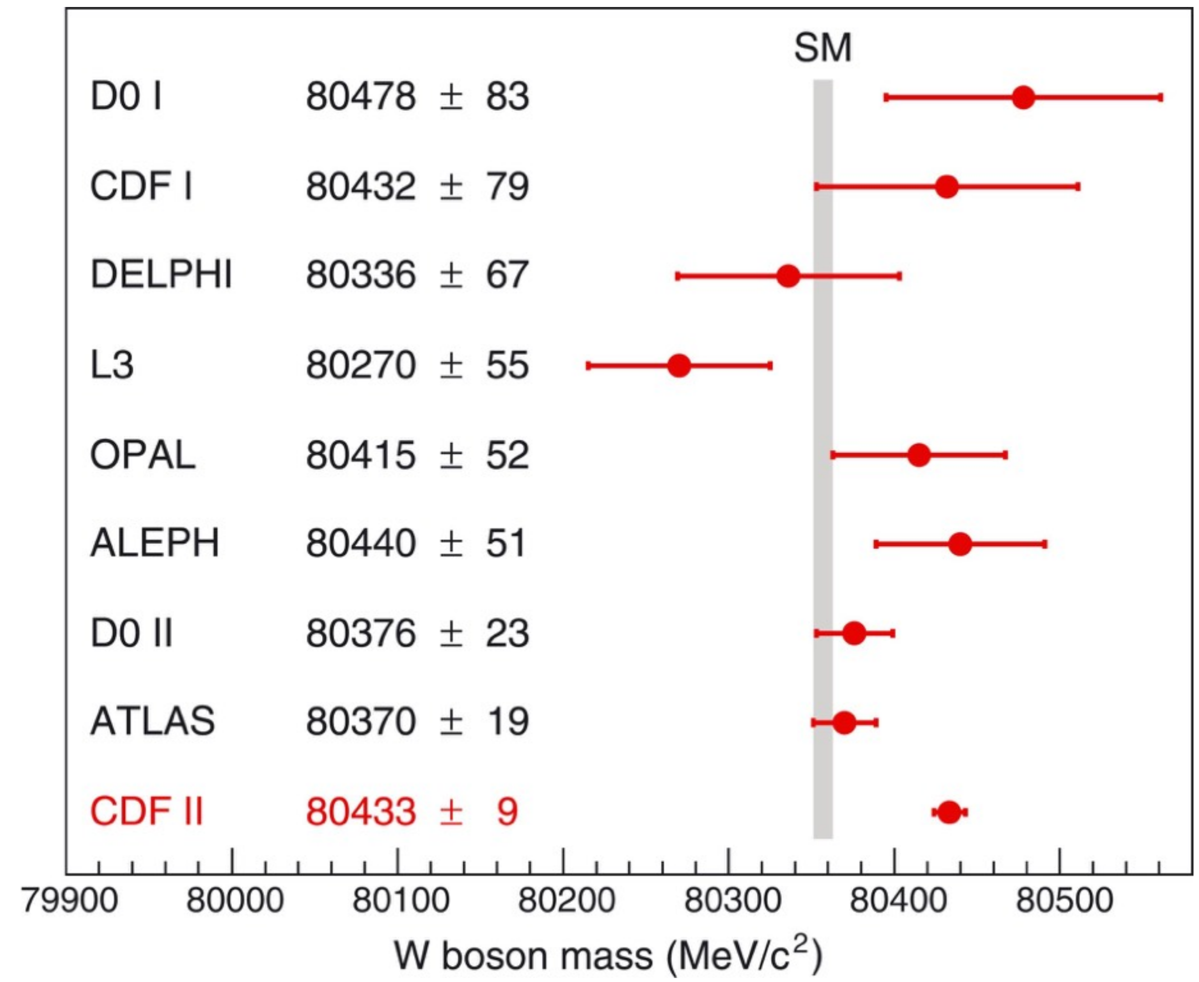
EIC will provide a wealth of data on TMDs in nonperturbative region

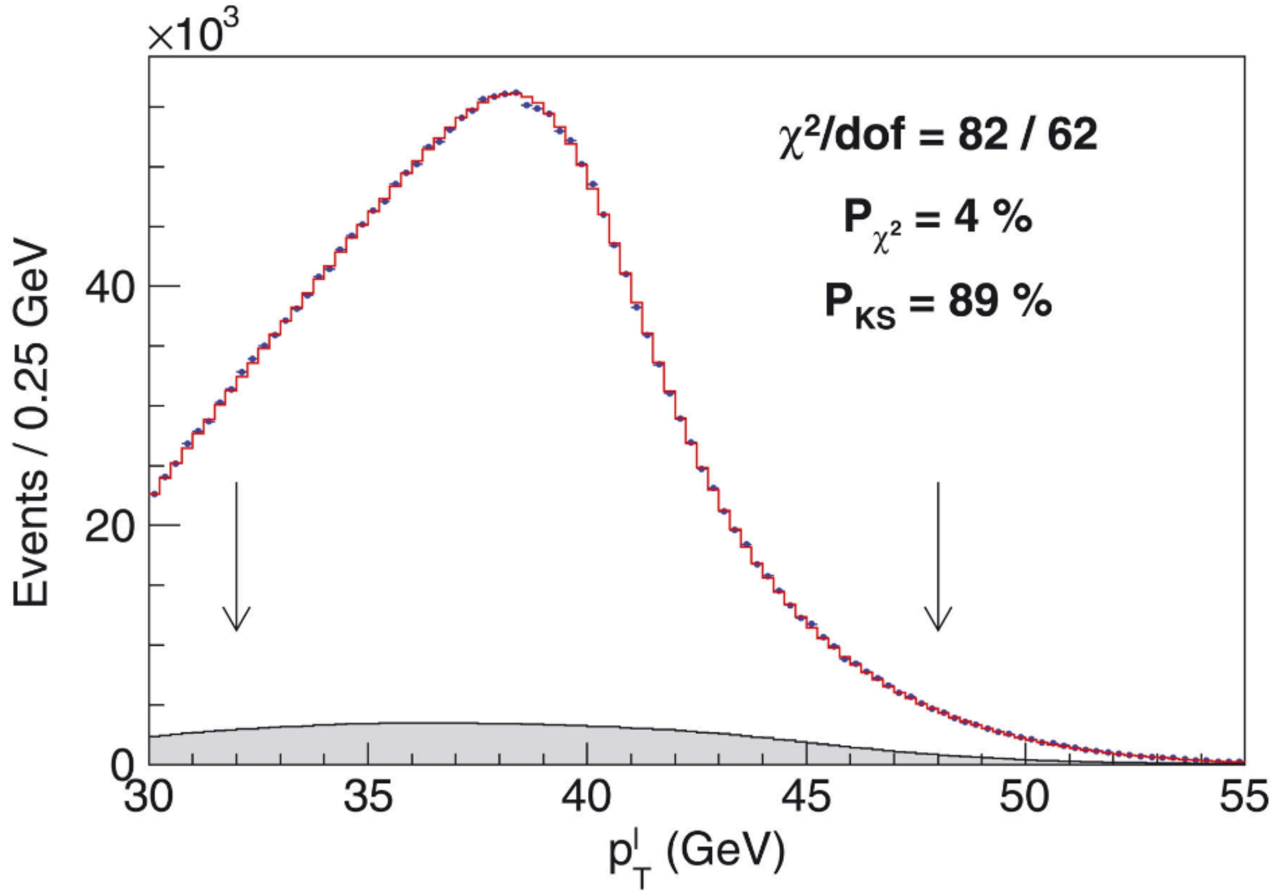
The W boson mass

Precise measurement of M_W from CDF disagrees at 7 sigma with M_W obtained from electroweak precision fits

New physics?

Robust understanding of all QCD theory uncertainties essential





Aaltonen et al [CDF], Science 376 (2022)

Measurement made by fitting shapes of transverse momentum distributions to theory predictions including resumed and nonperturbative QCD effects

Distribution shapes are insensitive to many aspects of TMDPDFs but are sensitive to flavor dependence and Collins-Soper kernel

CS kernel phenomenology

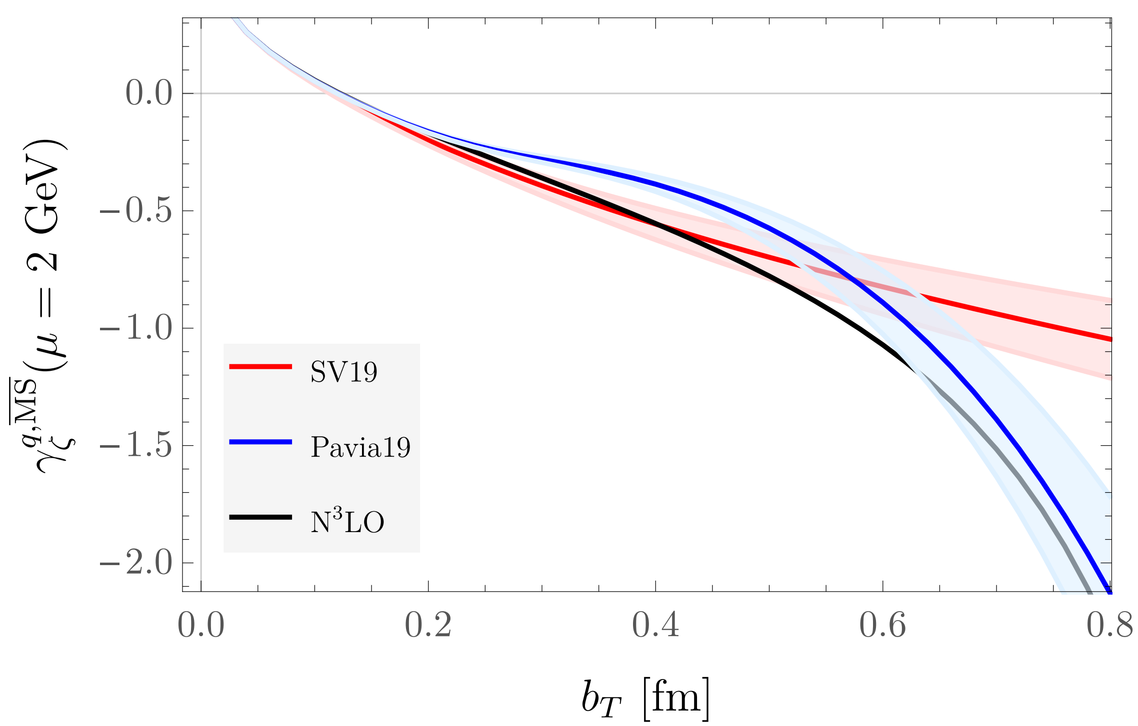
CS kernel can be extracted along with TMDPDF in global fit to DY + SIDIS data

SV19 - Scimemi and Vladimirov, JEHP 06 (2020)

(582 SIDIS + 457 DY data points)

Pavia19 - Bacchetta et al, JEHP 07 (2020)

(353 DY data points)



Modeling significant for

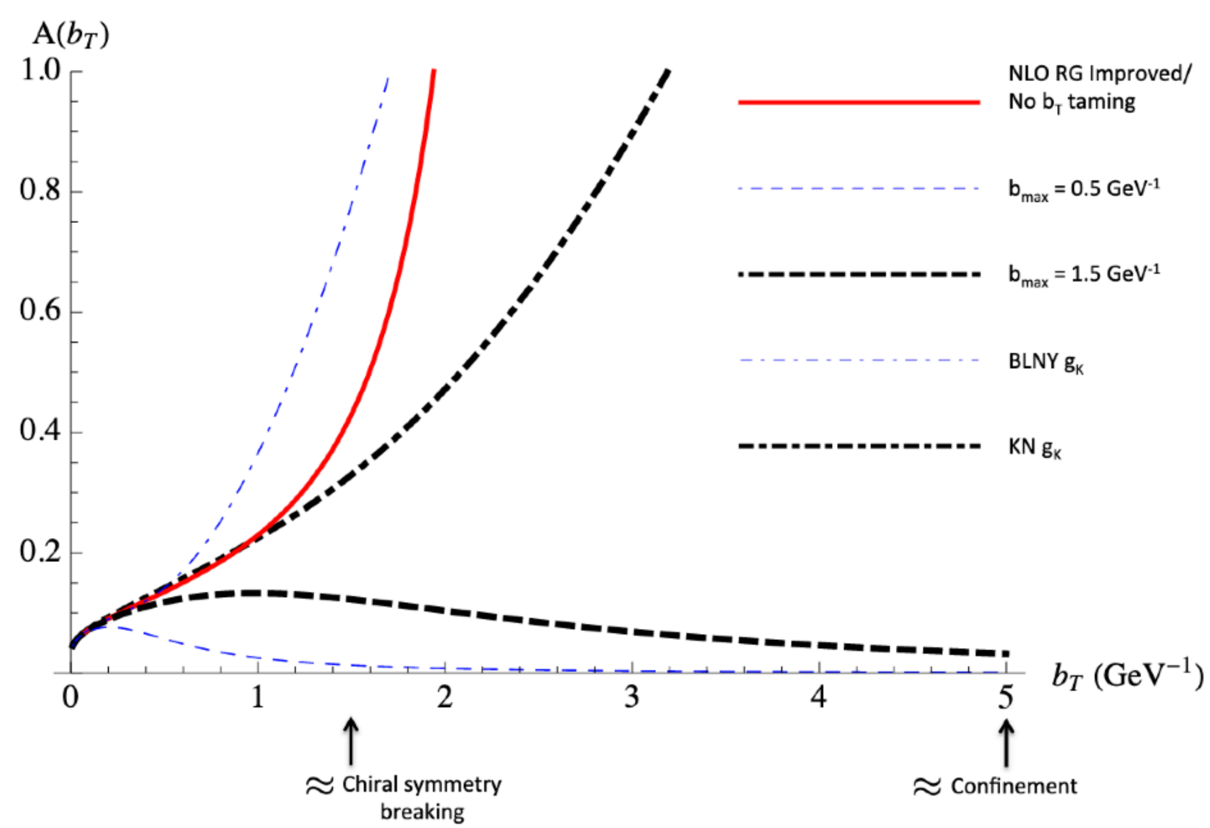
$$b_T \gtrsim 0.2\ \mathrm{fm}$$

(nonperturbative region)

Related nonperturbative function

$$A(b_T) = -\frac{b_T}{2}\frac{\partial}{\partial b_T}\gamma_\zeta^q(b_T,\mu)$$

appears in CSS factorization formula used to analyze experimental data (e.g. BLNY used by CDF)



Collins and Rogers, PRD 91 (2015)

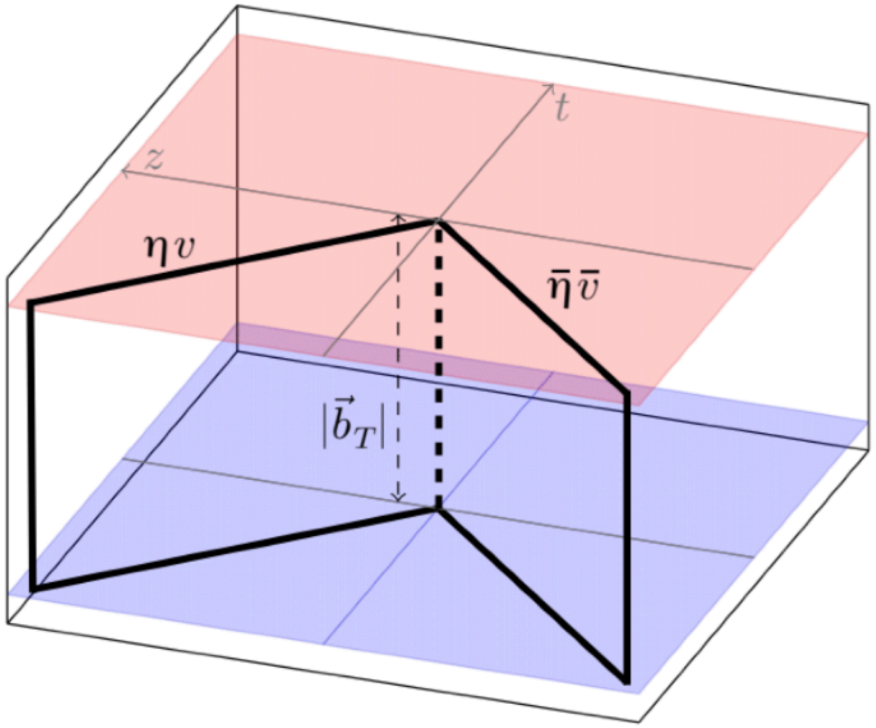
Quasi TMDPDFs

The construction of quasi TMDPDFs is more complicated than quasi PDFs

Ji, PRL 110 (2013)

TMDPDF products appearing in e.g. Drell-Yan can be expressed as convolutions of "beam functions" and "soft functions"

Soft function cannot be related to a matrix element of equal-time operator product by a Lorentz boost



Ebert, Stewart, Zhao, JHEP 1909 (2019)

Recent progress relating light-cone soft function to a large-momentum form factor that can be calculated with LQCD

Ji, Liu, and Liu, Nucl Phys B 955 (2020)

Zhang et al [LPC], PRL 125 (2020)

The CS kernel from LQCD

Ratios of TMDPDFs free from soft factors and can be calculated with LQCD

Musch et al, PRD 85 (2012) Engelhardt et al, PRD 93 (2016) Yoon et al, PRD 96 (2017)

CS kernel determination using quasi-TMDPDFs suggested

Ji, Sun, Xiong, Yuan PRD 91 (2015)

Method concretely relating CS kernel to quasi TMDPDF ratios proposed and derived

Ebert, Stewart, Zhao, PRD 99 (2019)

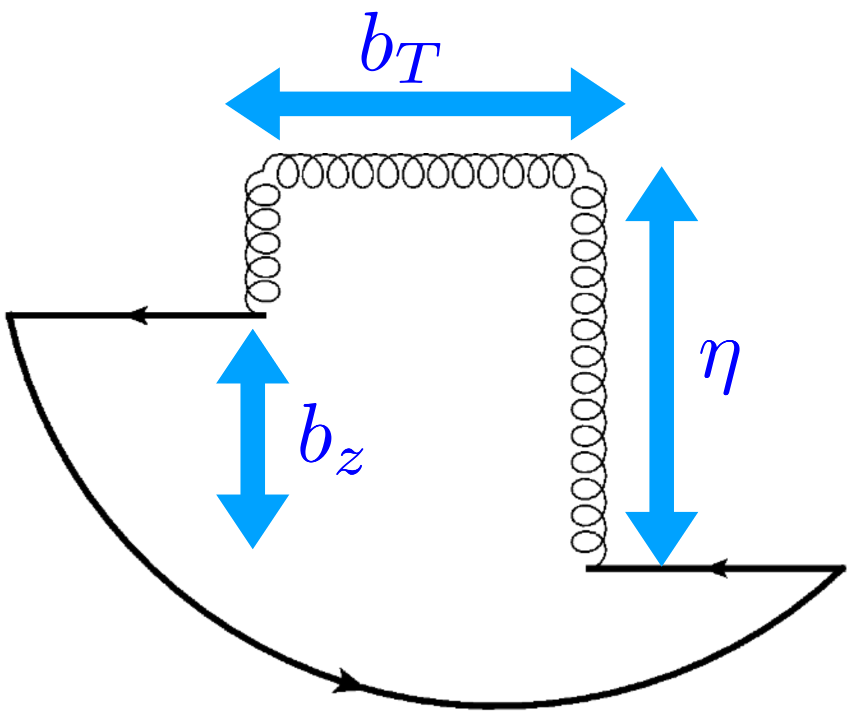
$$\gamma_\zeta^{q,\overline{\mathrm{MS}}}(b_T,\mu) = 2\zeta \frac{d}{d\zeta} \ln f_q^{\overline{\mathrm{MS}}}(x,b_T,\mu,\zeta)$$

$$= \frac{1}{\ln(p_1^z/p_2^z)} \ln \frac{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xP_2^z) \int db^z e^{ib^z x p_1^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z,b_T,\eta,\mu,p_1^z)}{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xp_1^z) \int db^z e^{ib^z x p_2^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z,b_T,\eta,\mu,p_2^z)}$$

Euclidean quasi-beam function

Quenched LQCD exploration

CS kernel property of QCD vacuum, independent of hadronic state

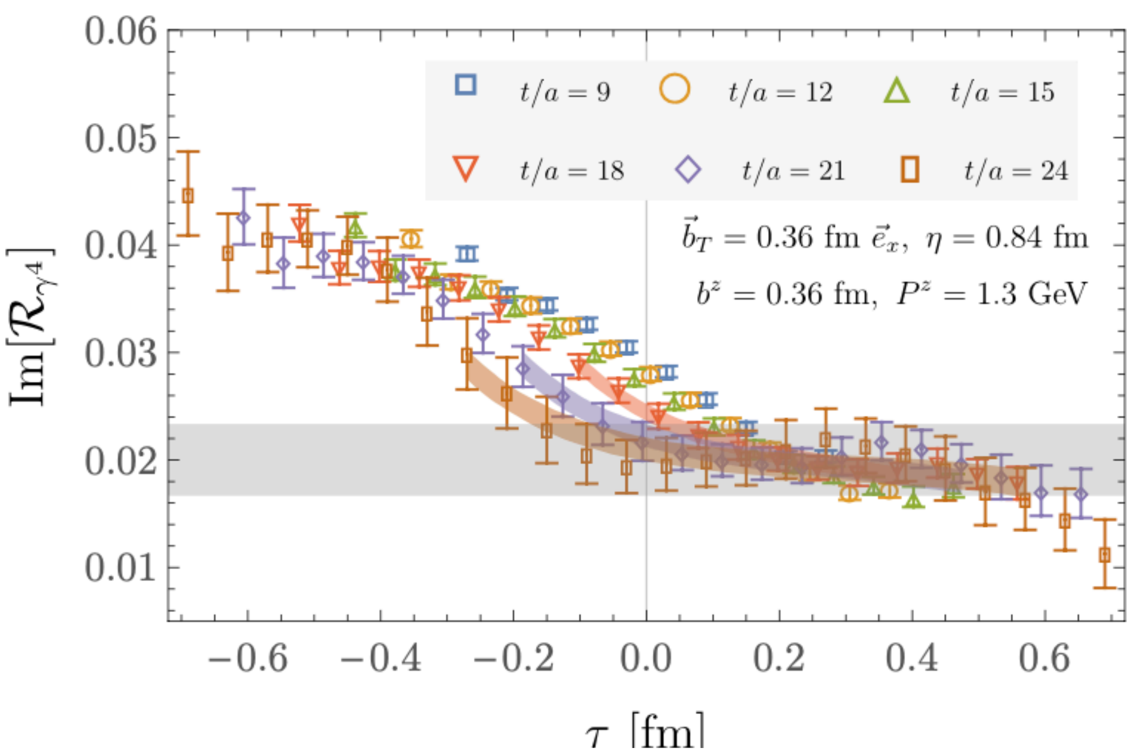


Calculate using pion state

In quenched ($N_f = 0$) QCD, exact results calculable using heavy quark probe

$$m_\pi \sim 1.2 \text{ GeV}$$

Allows high precision with only 400 quark propagator sources



Shanahan, MW, Zhao, PRD 102 (2020)

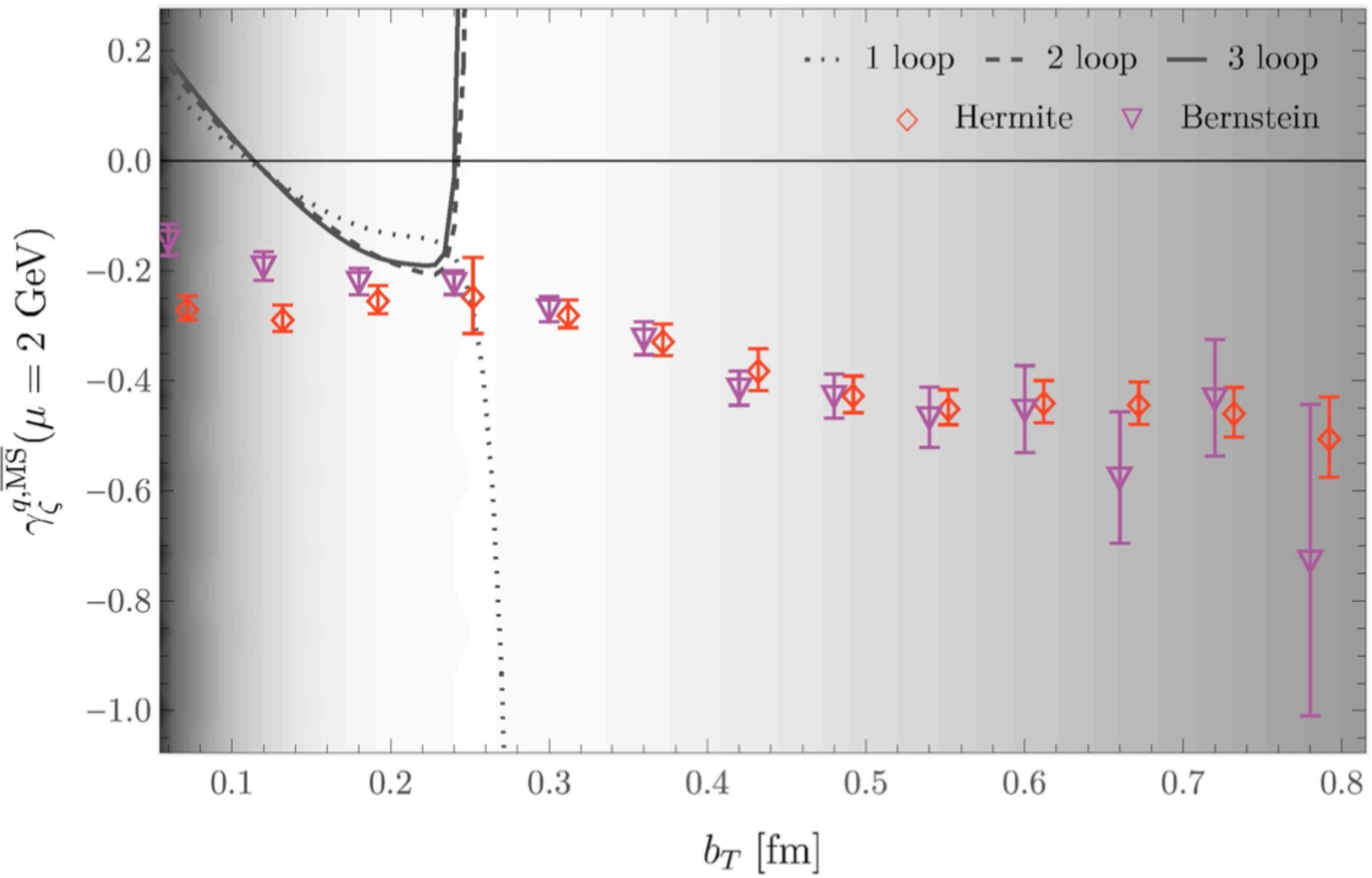
3 values of $\eta \in [0.6,\ 0.8]$ fm

3 values of $P^z \in [1.3,\ 2.6]$ GeV

All 16 Dirac structures and staple geometries b_T and b^z

35,660 bare matrix elements - robust automated fitting essential

Quenched LQCD results



Shanahan, MW, Zhao, PRD 102 (2020)

CS kernel determined precisely for b_T extending into nonperturbative regime

Fourier transform truncation effects challenging to quantify, two different models used to extrapolate beam functions outside range of data

Label	β	a [fm]	$L^3 \times T$	κ	$n_{\rm src}$	$n_{\rm cfg}$
E_{32}	6.3017	0.06	$32^3 \times 64$	0.1222	2	200

$$m_\pi = 1.2\ \mathrm{GeV}$$

$$P^z \in \{1.3,\ 1.9,\ 2.6\}\ \mathrm{GeV}$$

$$\eta \leq 0.8\ \mathrm{fm}$$

$N_f = 2+1+1$ LQCD calculation

Shanahan, MW, Zhao, PRD 104 (2021)

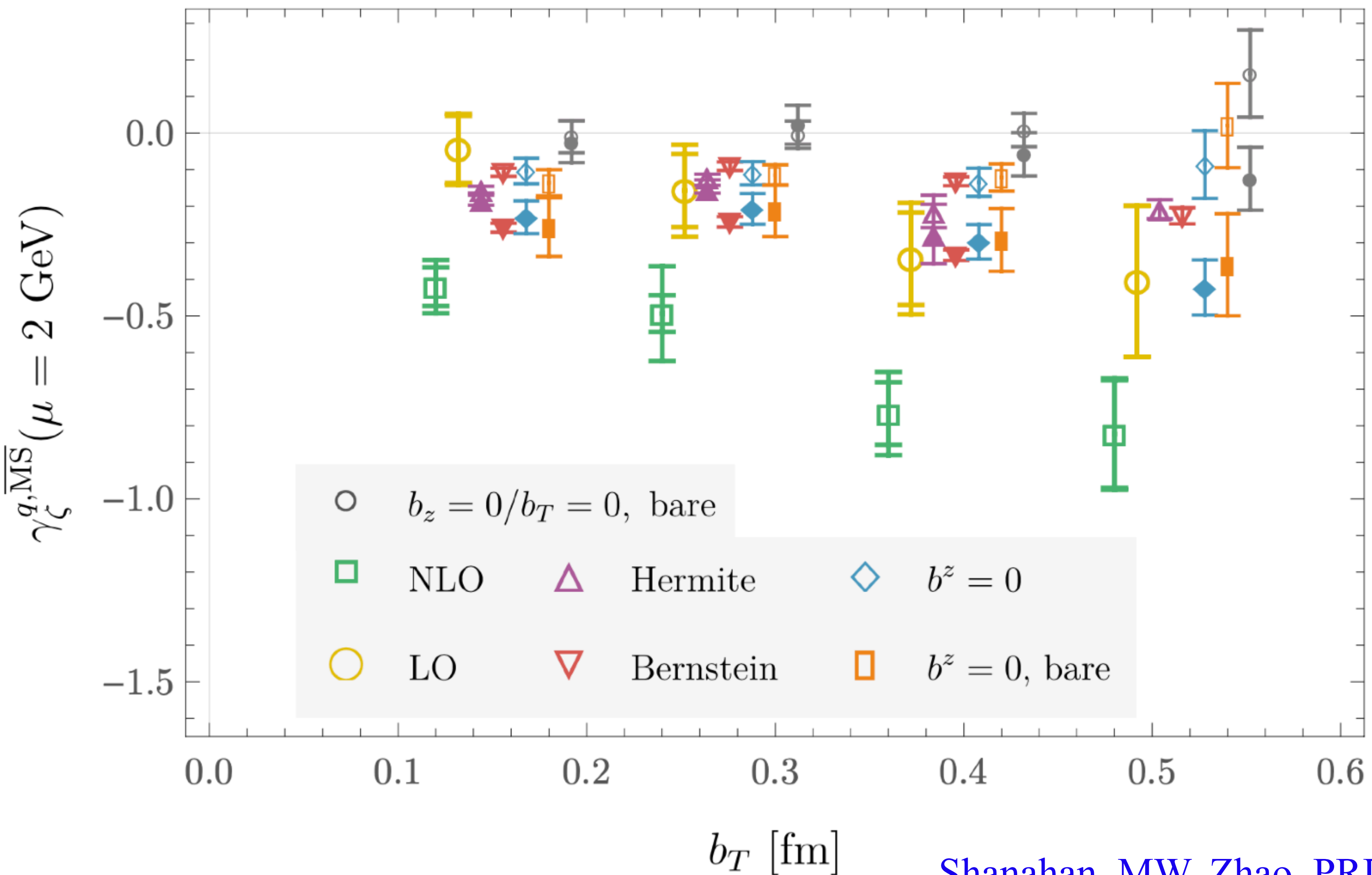
HISQ lattices from MILC, gradient flowed, with clover valence quarks

- ✓ NLO matching from quasi- to light-cone beam functions
- ✓ Demonstration of x independence of CS kernel
- ✓ $\overline{\mathrm{MS}}$ renormalization + operator mixing using RI/MOM

  (quark vs pion static quark potential correction applied)
- ✓ Fourier transformation to x - space
- ✓ Finite-momentum power corrections studied
- ✓ Unphysically heavy valence quark mass

Comparing approximations

NLO matching leads to significant effects on CS kernel determination

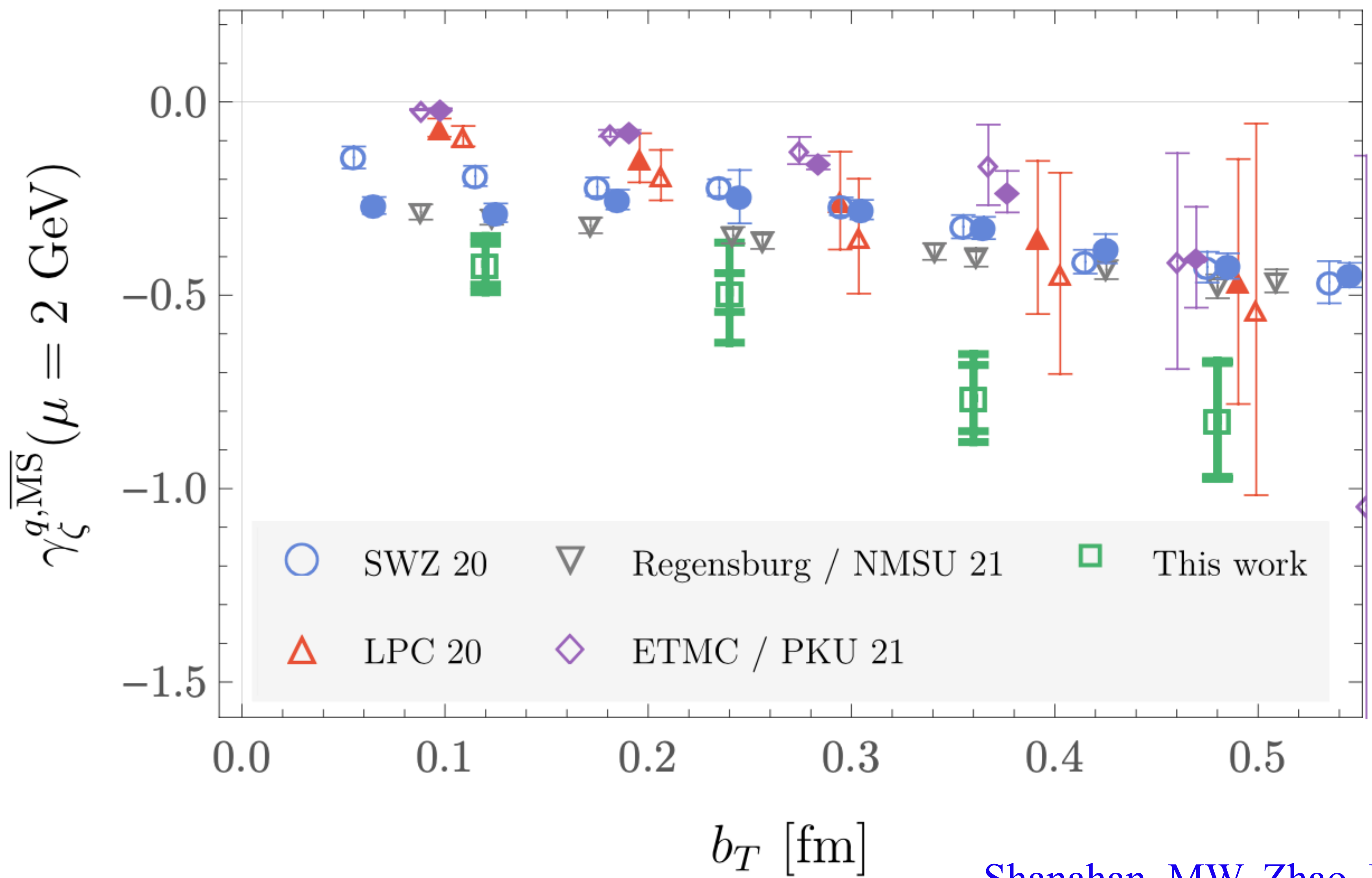


Shanahan, MW, Zhao, PRD 104 (2021)

LO results using ratios of $b^z = 0$ beam functions or the momentum-space models used in quenched calculation are consistent with LO results using average over x dependence but give smaller uncertainty estimates

Lattice comparison

Results are broadly consistent with other LQCD calculations (different actions and systematics)



Shanahan, MW, Zhao, PRD 104 (2021)

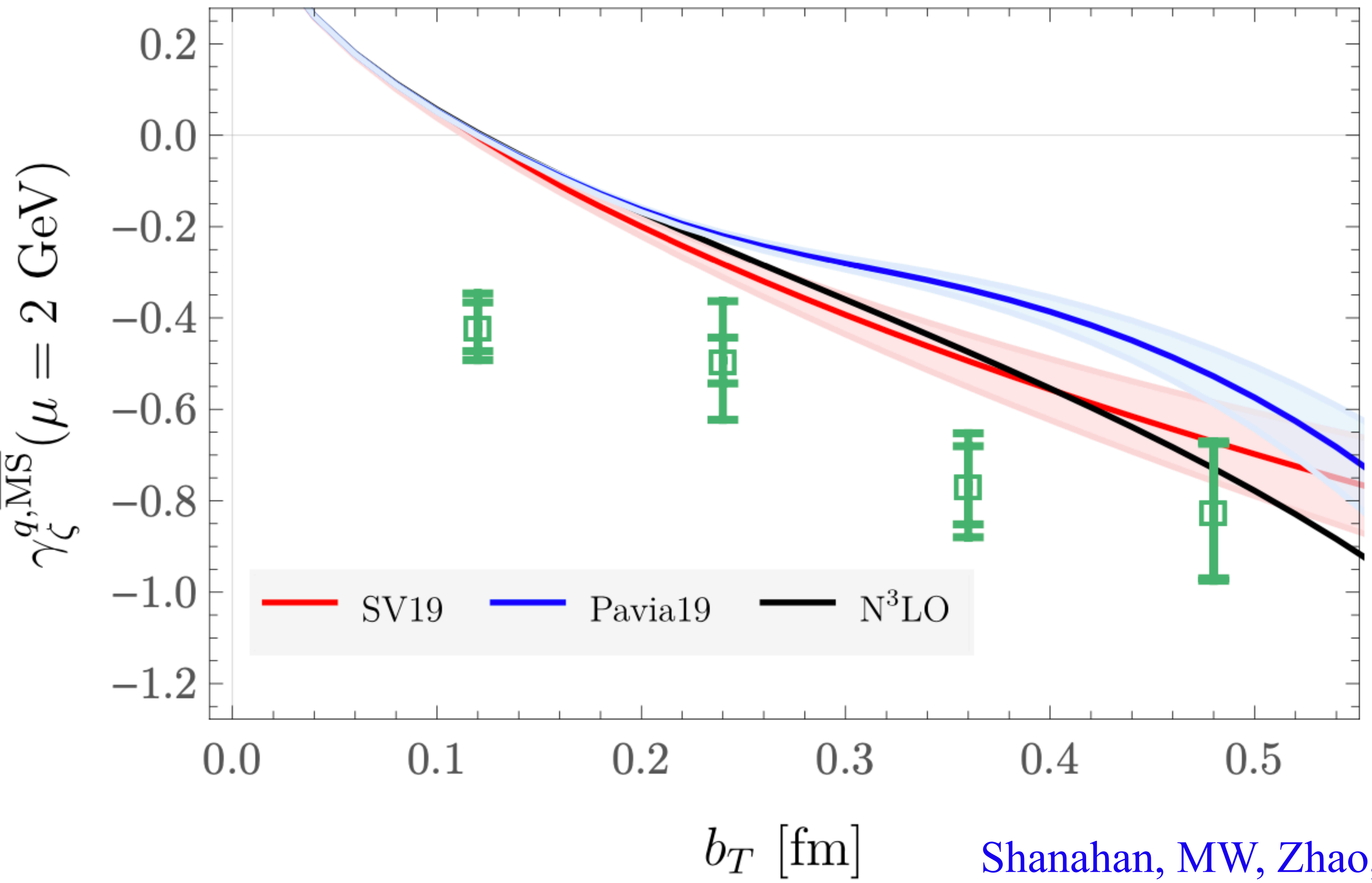
Differences with previous LO calculations (SWZ 20, LPC 20, ETMC / PKU 21) consistent with differences between Fourier transform schemes

See also Chu et al [LPC] arXiv:2204.00200

Phenomenological comparison

Current LQCD results can also be compared with phenomenology

Warning - no continuum extrapolation, unquantified systematics remain



Shanahan, MW, Zhao, PRD 104 (2021)

Lattice artifacts at small b_T ? Underestimated Fourier transform systematics?
Further studies needed!

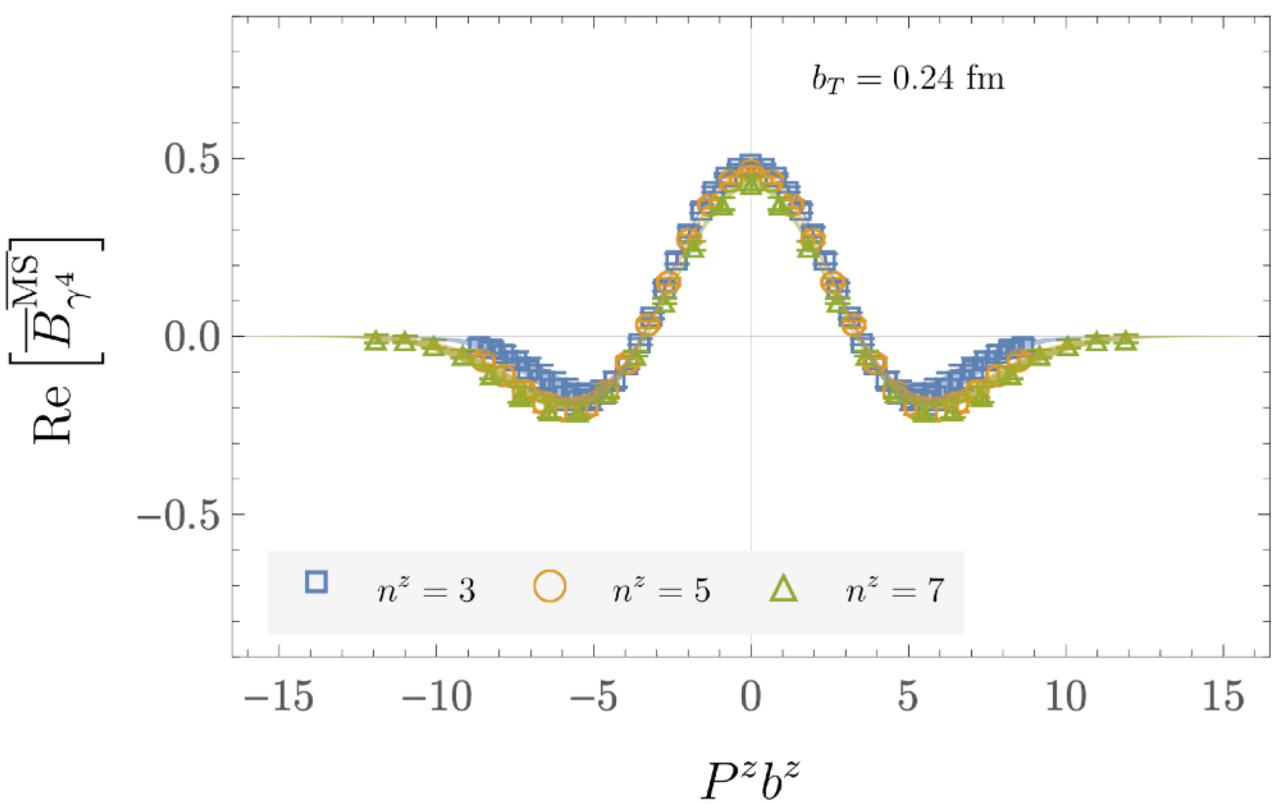
Large-distance extrapolation

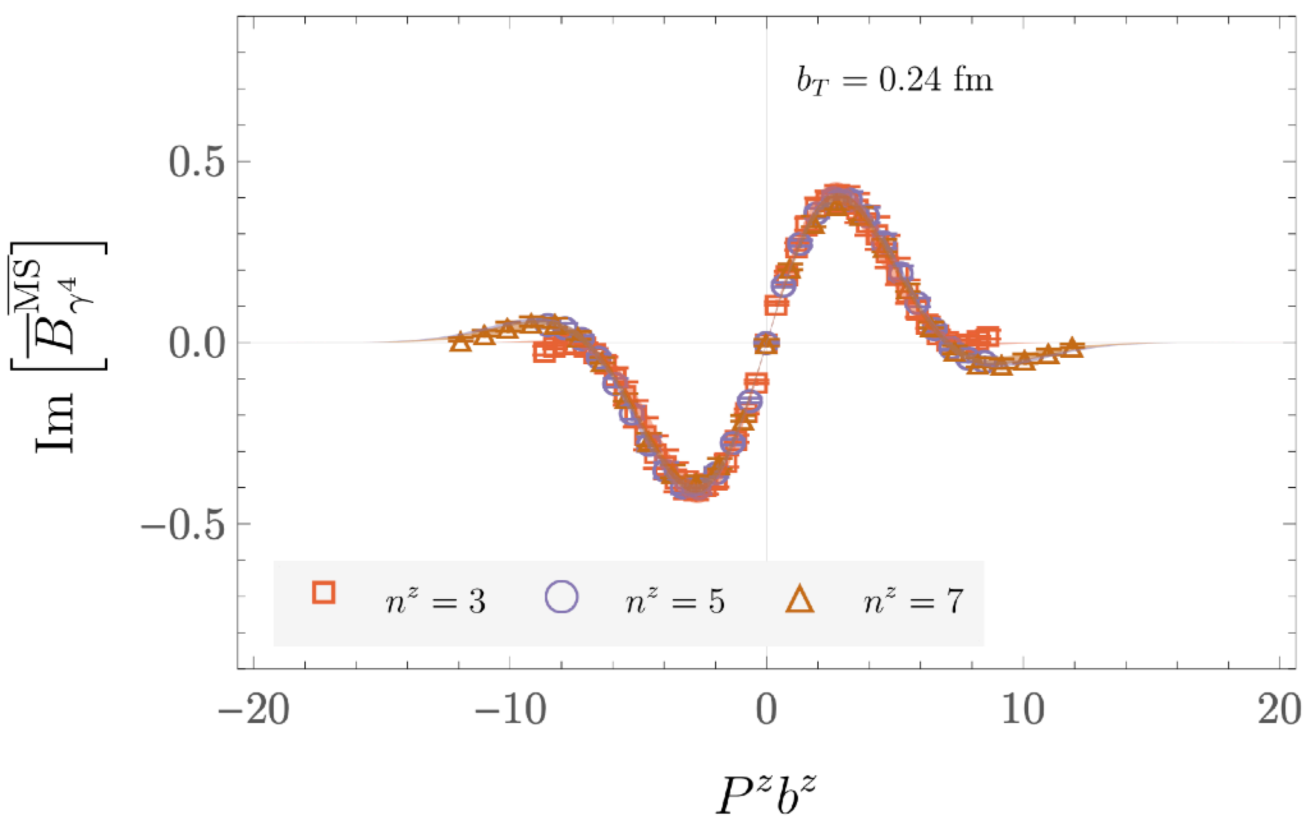
Fourier transforming data from a finite interval is a formally ill-posed problem

LQCD results span a finite range of $b^z,\ P^z,\ b \cdot P = b^z P^z$

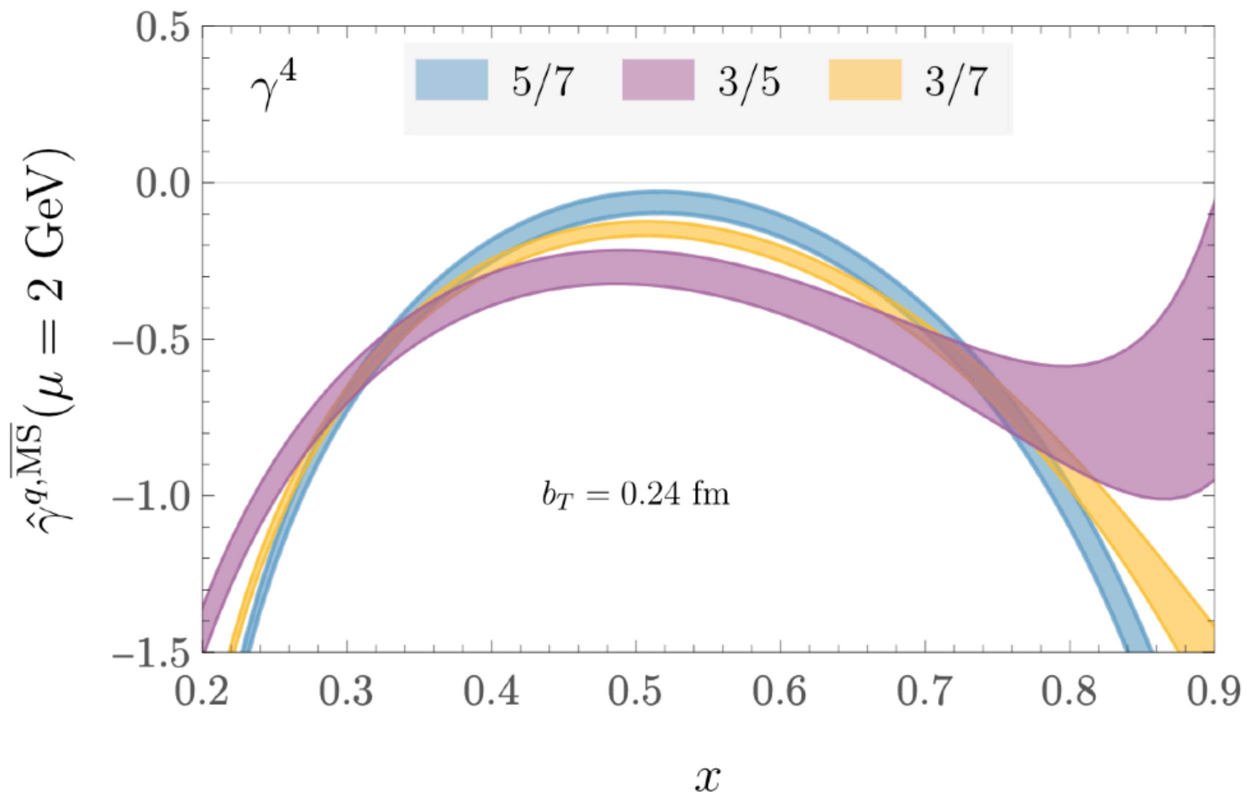
Fourier conjugate to x

Fits performed independently for each $b_T,\ P^z$ to analytic model in order to extrapolate to larger b^z





CS kernel systematics



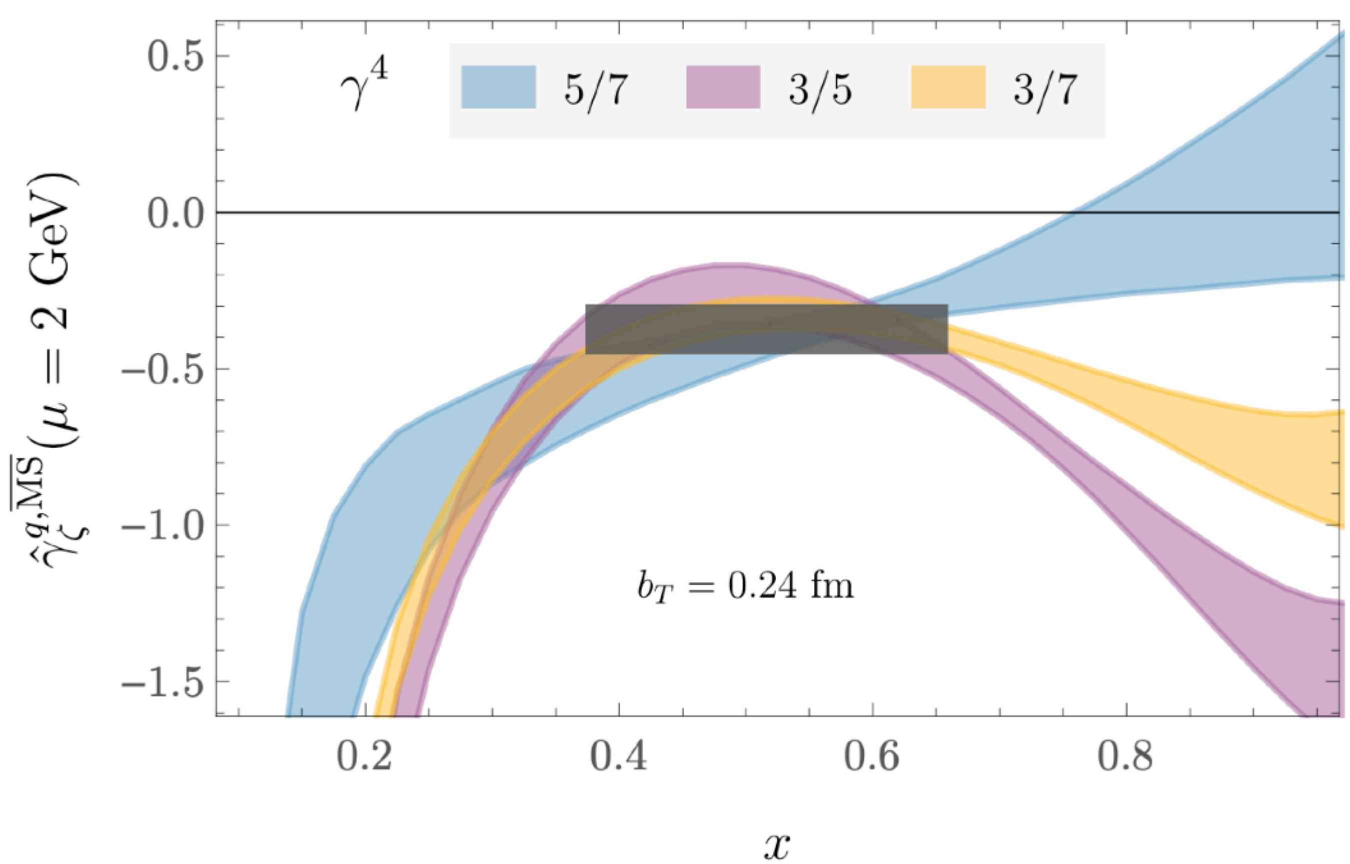
Discrete Fourier transform leads to significant x dependence of (asymptotically flat) CS kernel estimate

Differences between estimates with different momentum pairs visible

Fourier transforming the analytically extrapolated model leads to smaller (though still visible) x and P^z dependence

"Plateau region" identified by automated search for overlap between different P^z pairs

Fits of $1/P^z$ artifacts also attempted

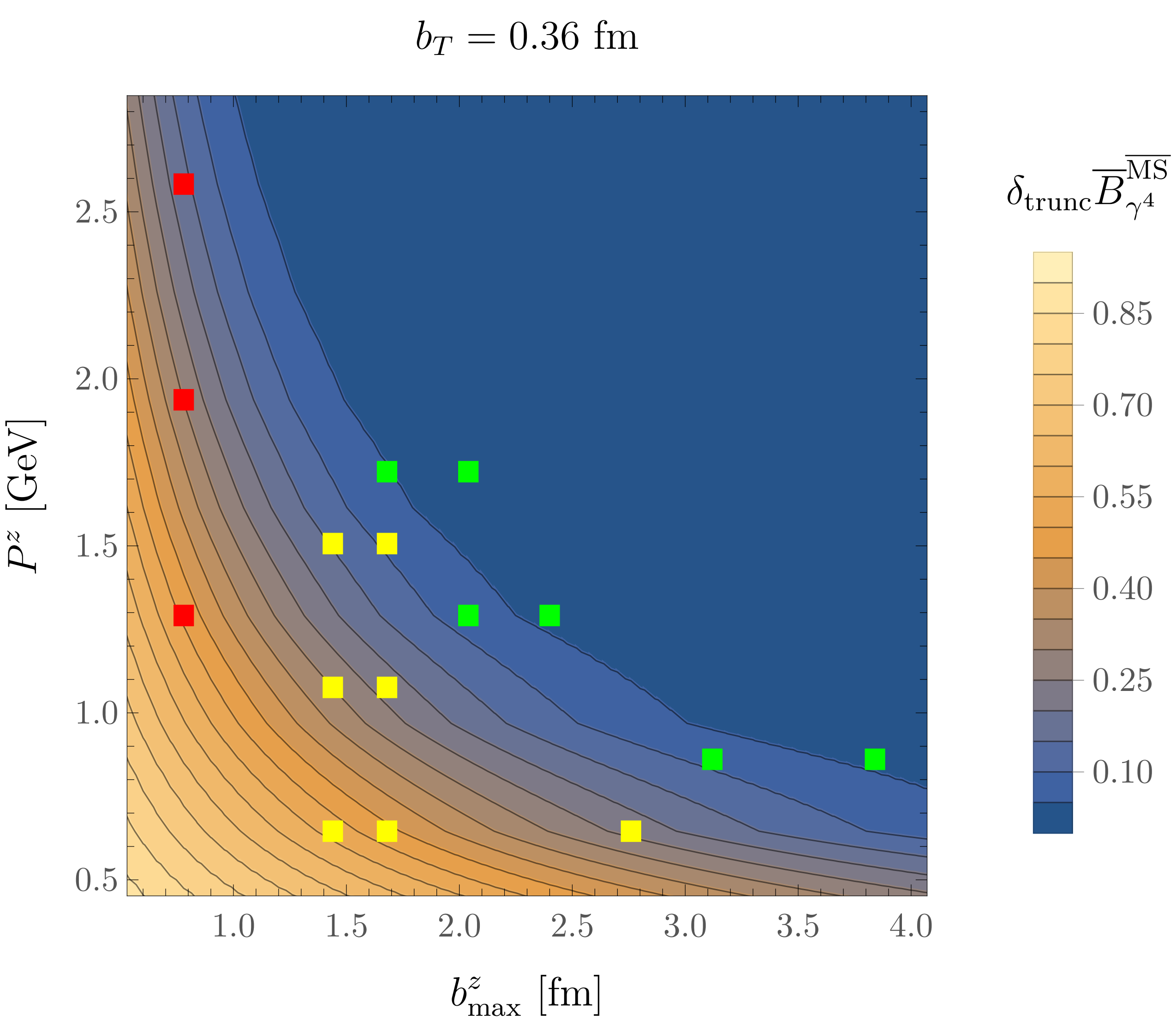


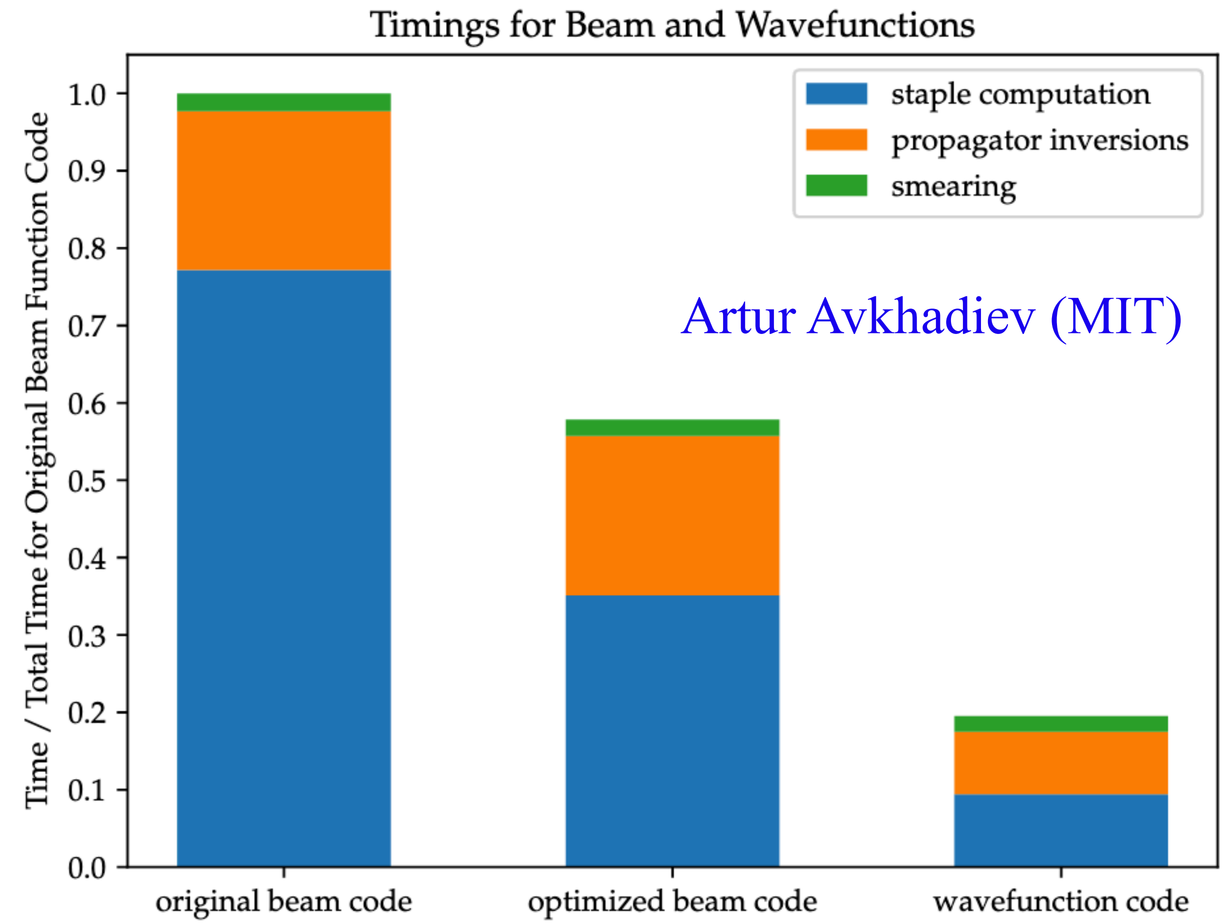
Proposed calculations

TMDPDF evolution effects can be determined from ratios of TMDWFs analogous to distribution amplitudes

$$\tilde{\psi}(b^z, b_T, \eta, P^z) \propto \langle 0|\mathcal{O}(b^z, b_T, \eta)|\pi(P^z)\rangle$$

Preliminary studies suggest TMDWFs will enable significantly more efficient CS kernel calculations



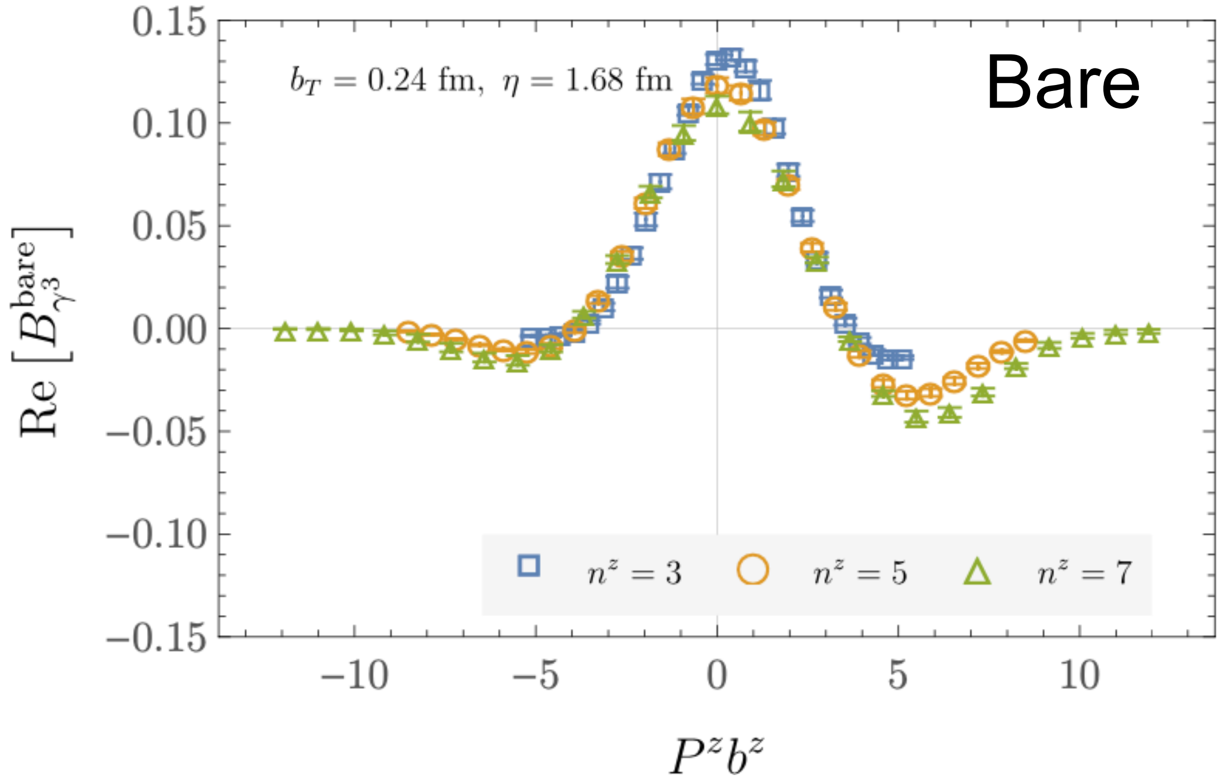


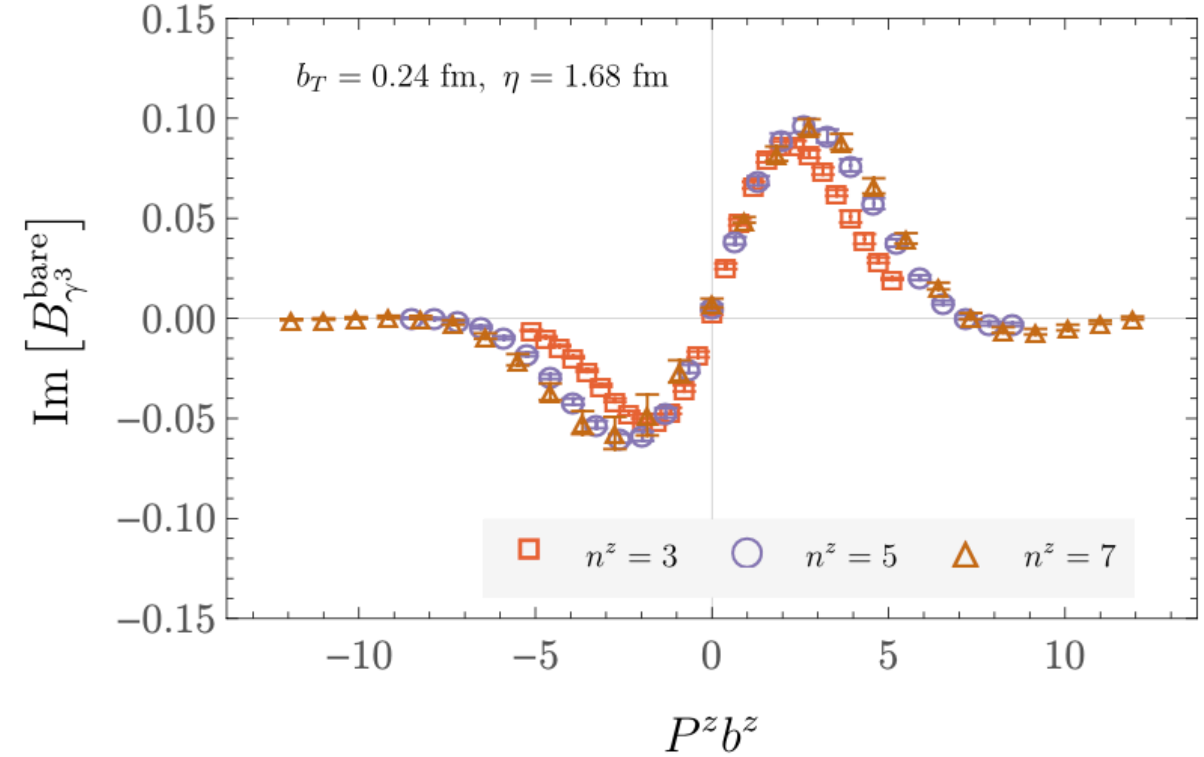
New formal and code developments provide several improvements:

- Larger $b^z P^z$ with similar truncation effects for all P^z
- Robust nonlocal operator renormalization
- Approximately physical quark masses

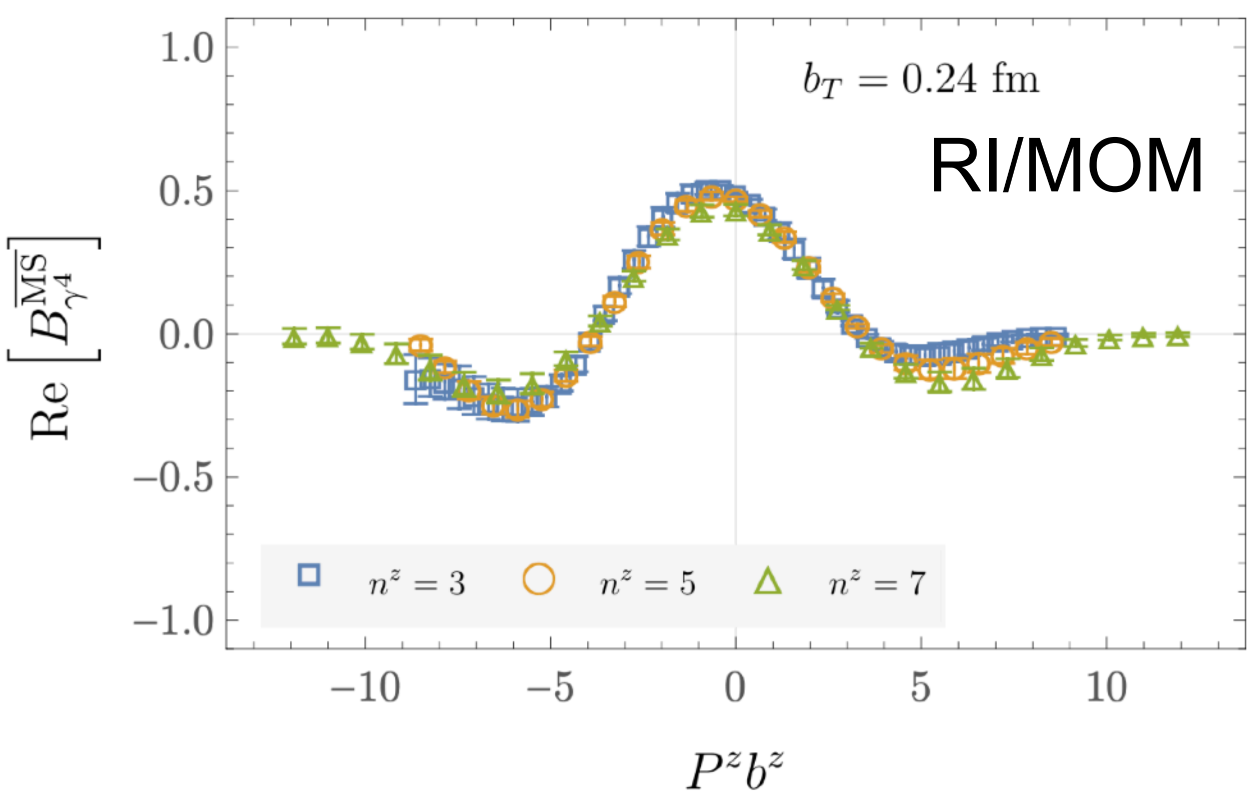
Backup

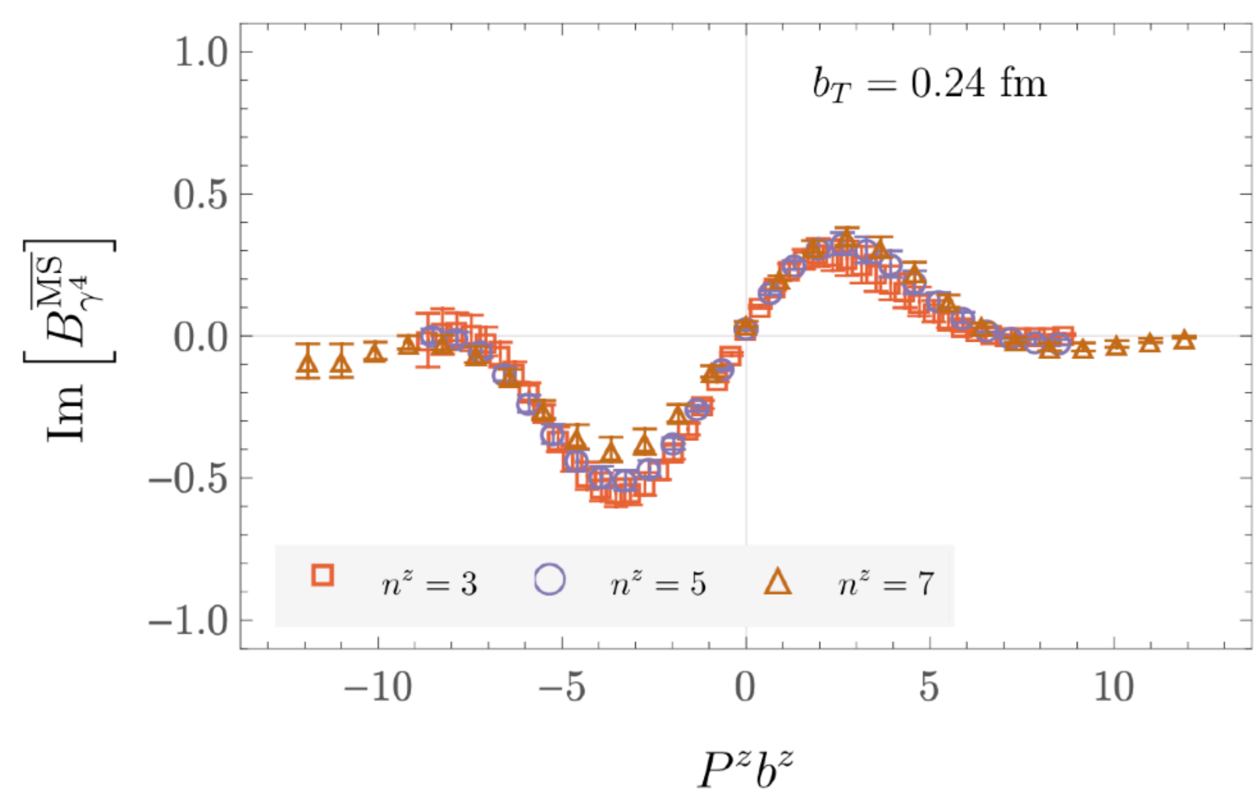
Trouble with RI/MOM





Asymmetry visible in beam functions after RI/MOM renormalization





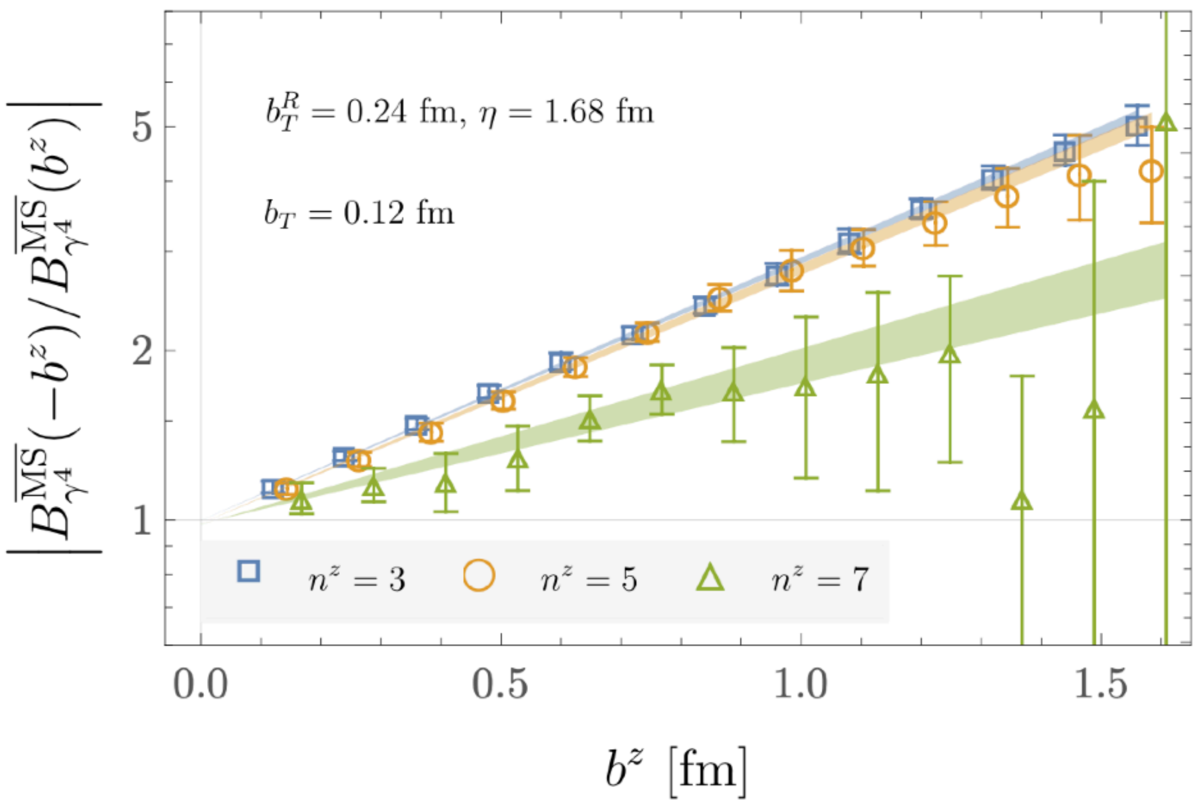
Beam function asymmetry

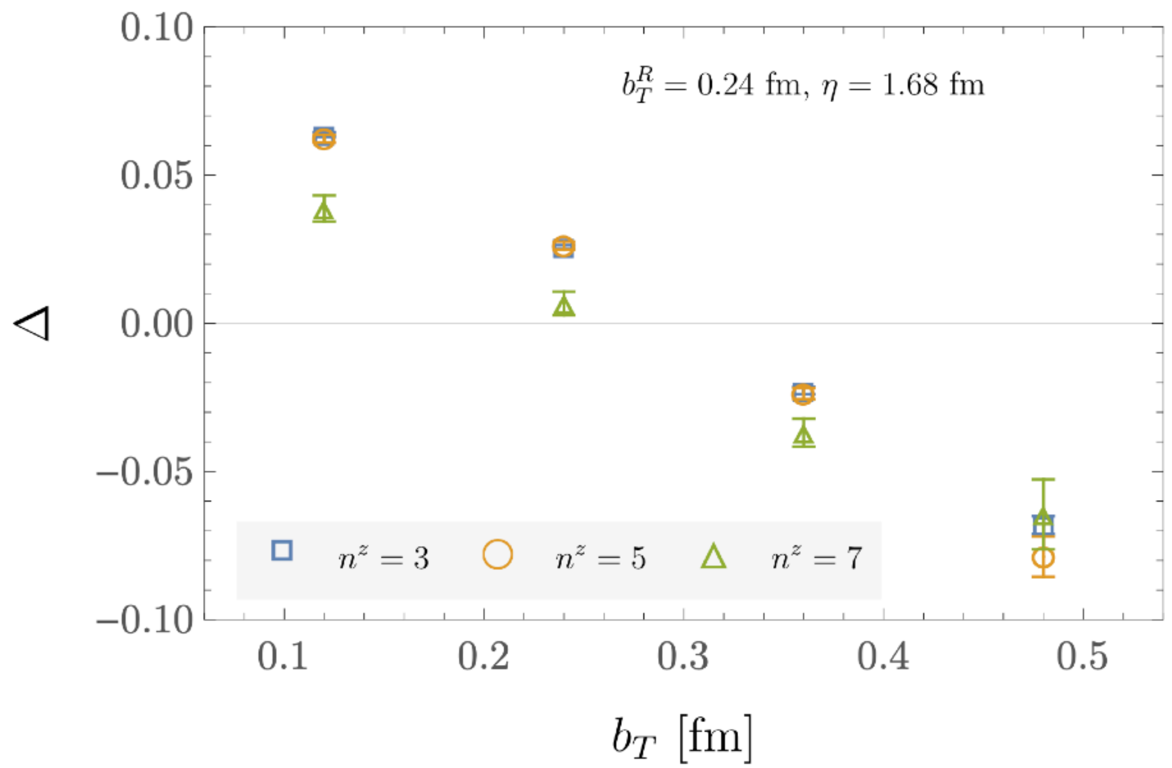
Asymmetry visible after RI/MOM renormalization could arise from state-dependence of static quark potential

State dependence of static quark potential also visible in previous calculations

Zhang et al [χQCD], PRD 104 (2021)

Huo et al [LPC], Nucl. Phys. B 969 (2021)





Correction for difference in static quark potentials applied

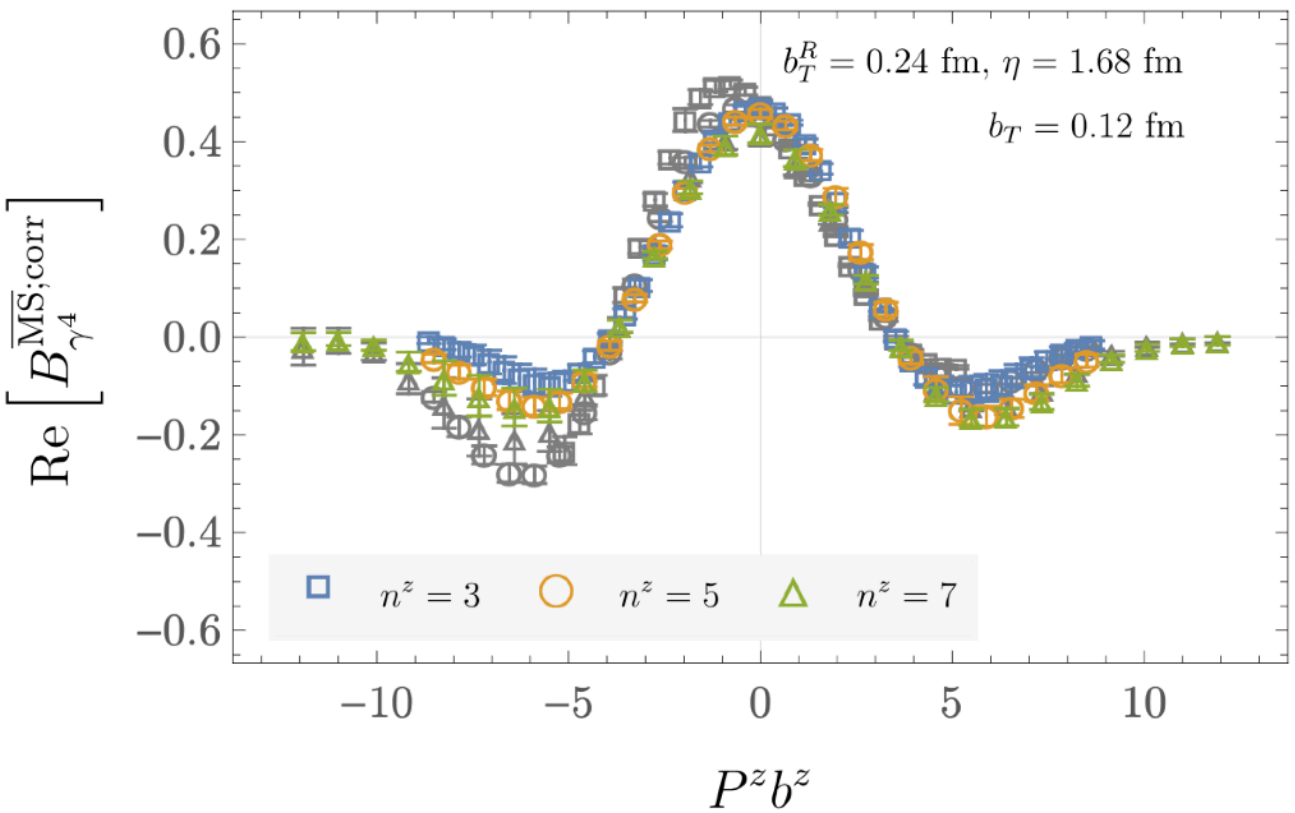
$$B^{\overline{\text{MS}};\text{corr}}_{\gamma_4}(b^z, b_T) = e^{\Delta(b_T)|b^z|} B^{\overline{\text{MS}}}_{\gamma_4}(b^z, b_T)$$

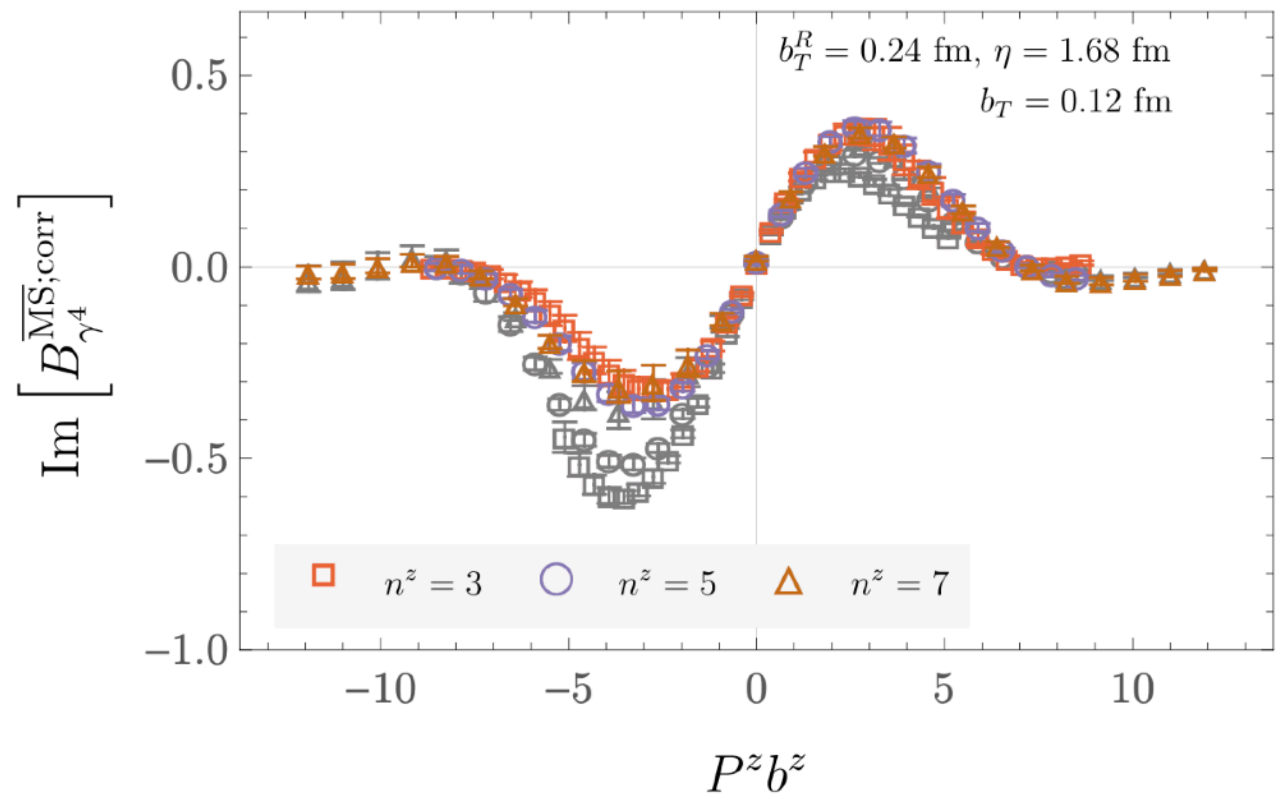
Roughly linear trend in b_T observed

$$\Delta(b_T) = V(b_T)_{\text{quark}} - V(b_T)_{\text{pion}} \sim \sigma\, b_T$$

Asymmetry correction

After correcting for state dependence of static quark potential, expected (anti)symmetrization of beam function emerges





Extrapolation to large η (by a constant) and averaging over choice of b_T^R used in renormalization performed after including corrections independently

Systematics included to reflect variation in η and b_T^R reduced after corrections included