Parton Distribution Functions and Amplitudes

From Ioffe-time Distributions to Structure Functions

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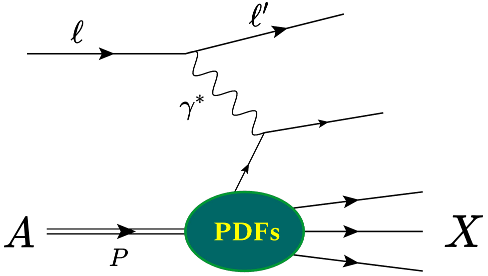
Overview/Roadmap

QCD factorization theorems separate (semi-)inclusive cross-sections into hard and long-distance (non-perturbative) components

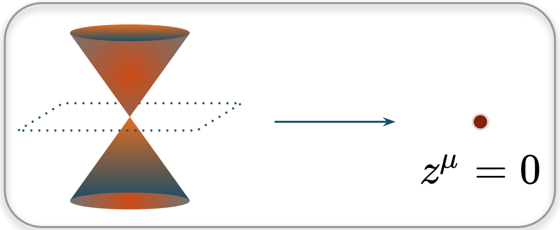
Eg. J. Collins et al., Adv. Ser. Direct. High Energy Phys. 5, 1 (1989)



$$F_i\left(x,Q^2\right)=\sum_{a=q,\bar{q},g} f_{a/h}\left(x,\mu^2\right)\otimes H_i^a\left(x,\frac{Q^2}{\mu^2},\alpha_s\left(\mu^2\right)\right)+h.t.$$

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle h\left(p\right)\right|\overline{\psi}\left(\tfrac{z}{2}\right)\gamma^+\Phi^{(f)}_{\hat{z}^-}\left(\{\tfrac{z}{2},-\tfrac{z}{2}\}\right)\psi\left(-\tfrac{z}{2}\right)\left|h\left(p\right)\right\rangle$$

- Matrix elements of space-like parton bilinears
  - generic objects accessible in LQCD sensitive to collinear structure of hadrons
- Pseudo-distributions and Distillation applied to Nucleon
  - Unpolarized quark PDFs
  - Transversity quark PDFs
  - Unpolarized gluon PDF
- Pseudo-distributions complementing global QCD analyses



From Matrix Elements in Lattice QCD to PDFs

Two popular, and related, methods to obtain PDFs
from matrix elements of space-like quantities in Lattice QCD

Additional UV singularities for space-like Wilson line

$$M^{[\Gamma]}(p,z) = \langle h(p)|\,\overline{\psi}(z)\,\Gamma\Phi^{(f)}_{\hat{z}}(\{z,0\})\,\psi(0)\,|h(p)\rangle$$

LaMET

Large Momentum Effective Theory

Quasi-PDF: Fourier transform - distribution of parton longitudinal space-like momenta

Factorizes into PDF with power corrections in $1/p_z^2$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

SDF

Short Distance Factorization

Short-distance OPE applied to matrix element

Factorizes into PDF with power corrections in z^2

V. Braun and D. Mueller, Eur.Phys.J.C 55 (2008) 349-361
A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025
Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 120 (2018) 2, 022003

Several other methods exist to extract SFs from suitable Euclidean correlations

- Hadronic tensor
  K.-F. Liu Phys.Rev.Lett. 72 (1994) 1790-1793
  J. Liang et al., Phys.Rev.D 101 (2020) 11, 114503
- "OPE without OPE"
  K.U. Can et al., Phys.Rev.D 102 (2020) 114505
  A.J. Chambers et al., Phys.Rev.Lett. 118 (2017) 24, 242001
- Auxiliary quark methods (Pion DAs & moments from OPE)
  HOPE Collab., Phys.Rev.D 105 (2022) 3, 034506
  W. Detmold and C.J.D. Lin, Phys.Rev.D 73 (2006) 014501
  G. Bali et al., Eur.Phys.J.C 78 (2018) 3, 217
  G. Bali et al., Phys.Rev.D 98 (2018) 9, 094507
- Current-current correlators
  R.S. Sufian, J. Karpie, **CE** et al., Phys.Rev.D 99 (2019) 7, 074507
  R.S. Sufian, **CE**, J. Karpie et al., Phys.Rev.D 102 (2020) 5, 054508

Towards the Unpolarized PDF from Pseudo-Distributions

A matrix element of a distinct character

$$M^{\alpha}(p,z) = \langle h(p)|\,\overline{\psi}(z)\,\gamma^{\alpha}\Phi^{(f)}_{\hat{z}}(\{z,0\})\,\psi(0)\,|h(p)\rangle = 2p^{\alpha}\mathcal{M}(\nu,z^2) + 2z^{\alpha}\mathcal{N}(\nu,z^2)$$

$$\nu \equiv p\cdot z$$

Unpolarized Light-cone PDF — **Unpolarized Pseudo-PDF**

Unpolarized leading-twist PDF defined in terms of $k^-, \mathbf{k}_\perp$ integrated parton correlator

$$p^{\alpha} = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right)$$

$$z^{\alpha} = (0, z^-, \mathbf{0}_\perp) \qquad \alpha = +$$

[Unpolarized] Ioffe-time Distribution (ITD)

$$\mathcal{M}(p^+z^-,0)_{\mu^2} \equiv Q(\nu,\mu^2) = \int_{-1}^{1} dx\; e^{i\nu x} f_{q/h}(x,\mu^2)$$

V. Braun et al., Phys.Rev.D 51 (1995) 6036-6051

Generalization of unpolarized light-cone PDF onto space-like intervals; Lorentz covariant parton momentum fraction

$$p^{\alpha} = (\mathbf{0}_\perp, p_z, E)$$

$$z^{\alpha} = (\mathbf{0}_\perp, z_3, 0) \qquad \alpha = 4$$

[Unpolarized] Ioffe-time Pseudo-distribution (pseudo-ITD)

$$\mathcal{M}(p_z z_3, z_3^2) = \int_{-1}^{1} dx\; e^{i\nu x} \mathcal{P}(x, z_3^2)$$

A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025

Unpolarized Pseudo-ITD: Lattice Implementation

JLab/WM/LANL 2+1 Flavor Isotropic Lattices

ID	a (fm)	m_π (MeV)	β	$c_{\rm SW}$	$L^3 \times T$	$N_{\rm cfg}$
E1	0.094(1)	358(3)	6.3	1.205	$32^3 \times 64$	349

(isovector combination only herein)

Parameters/Statistics

ID	$N_{\rm vec}$	$N_{\rm srcs}$	T/a	$p_z \times \left(\frac{2\pi}{L}\right)$	z/a
E1	64	4	$4, 6, \cdots, 14$ $0.38, \cdots, 1.32$ fm	$0, \pm1, \cdots, \pm6$ $0, 0.41, \cdots, 2.47$ GeV	$0, \pm1, \cdots, \pm12, \cdots$ $0, 0.094, \cdots, 1.13$ fm

➢ *Distillation* to realize correlation functions

M. Peardon et al., Phys. Rev. D80, 054506 (2009)

Reduced distribution K. Orginos et al., Phys. Rev. D 96, 094503 (2017)

➢ cancel multiplicatively divergent factor

$$\mathfrak{M}\left(\nu, z^2\right) = \frac{M_4\left(p, z\right)/M_4\left(p, 0\right)}{M_4\left(0, z\right)/M_4\left(0, 0\right)}$$

T. Ishikawa et al., Phys.Rev.D 96 (2017) 9, 094019
X. Ji et al., Phys.Rev.Lett. 120 (2018) 11, 112001
J. Green et al., Phys.Rev.Lett. 121 (2018) 2, 022004

$$\mathfrak{M}\left(\nu, z^2\right) = C\left(z^2\mu^2, \alpha_s\left(\mu^2\right)\right) \otimes \mathcal{Q}\left(\nu, \mu^2\right) + \mathcal{O}\left(z^2\Lambda^2_{\rm QCD}\right)$$

Short-Distance Factorization

High-momenta essential

➢ modify precomputed eigenvector basis to improve high-momentum overlaps

G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
CE, R. Edwards, K. Orginos, D. Richards, PRD 103 (2021) 3, 034502

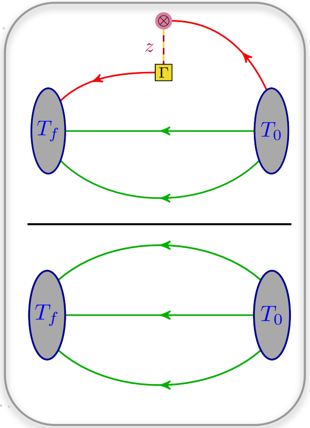
$$\xi^{(k)}_{\pm}\left(\vec{z}, T\right) \equiv e^{i\vec{\zeta}_{\pm}\cdot\vec{z}}\xi^{(k)}\left(\vec{z}, T\right)$$

Summation method - further excited-state suppression

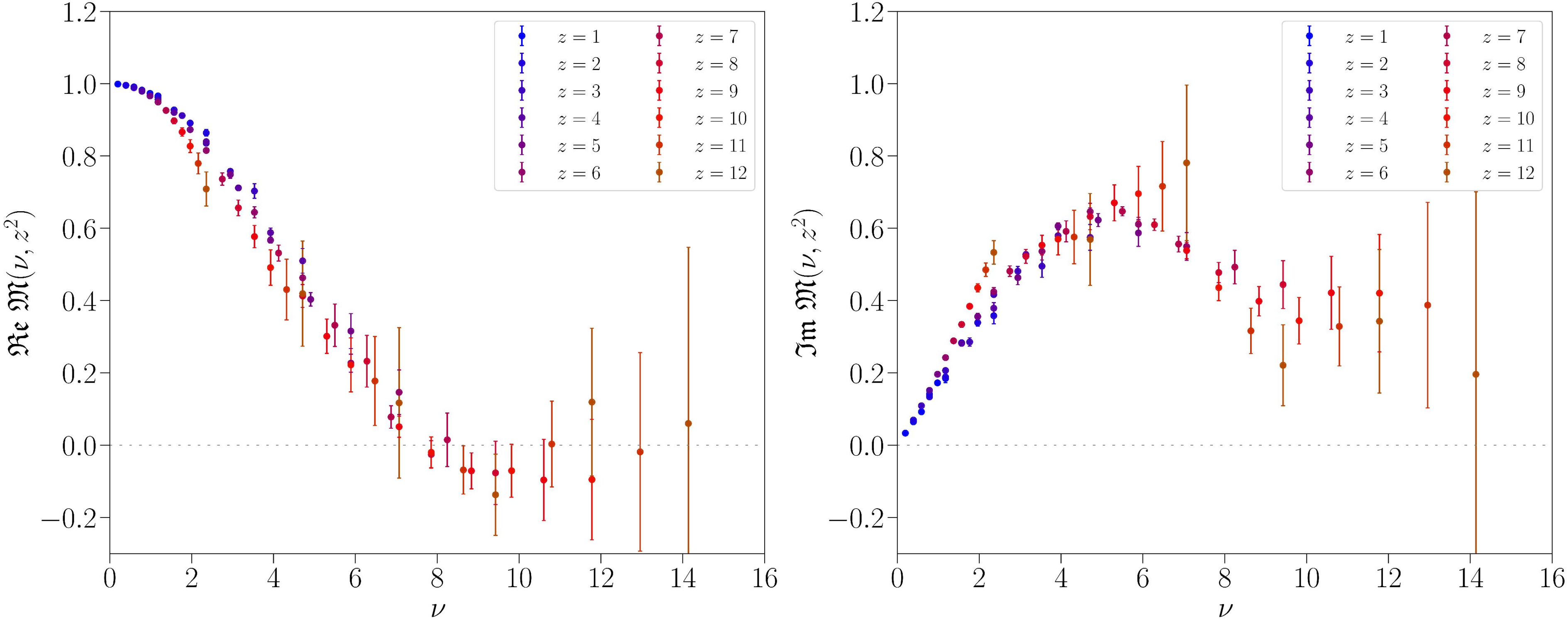
L. Maiani et al., Nucl. Phys. B293 (1987)
C. Bouchard et al., Phys. Rev. D 96, no. 1, 014504 (2017)

$$R\left(p_z, z_3; T\right) = \sum_{\tau/a=1}^{T-1} \frac{C_3\left(p_z, T, \tau; z_3\right)}{C_2\left(p_z, T\right)}$$

$$R_{\rm fit}\left(p_z, z_3; T\right) = \mathcal{A} + M_4\left(p_z, z_3\right)T + \mathcal{O}\left(e^{-\Delta ET}\right)$$



Unpolarized Reduced Ioffe-time Pseudo-Distribution



CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Determining the Unknown PDF

Ill-posed (pseudo-)ITD/PDF matching relation:

$$\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} \mathrm{d}x\ \mathcal{K}\left(x\nu, z^2\mu^2\right) f_{q/h}\left(x, \mu^2\right) + \sum_{k=1}^{\infty} \mathcal{B}\left(\nu\right)\left(z^2\right)^k$$

One choice: model parameterization

$$\int_{-1}^{1} dz\,(1-z)^{\alpha}(1+z)^{\beta} J_n^{(\alpha,\beta)}(z)\, J_m^{(\alpha,\beta)}(z) = \delta_{n,m} h_n(\alpha,\beta)$$

Change of variables

➢ polynomials span support interval of PDFs

$$f_{q/h}(x) = x^{\alpha}(1-x)^{\beta} \sum_{n=0}^{\infty} C_{q,n}^{(\alpha,\beta)} \Omega_n^{(\alpha,\beta)}(x)$$

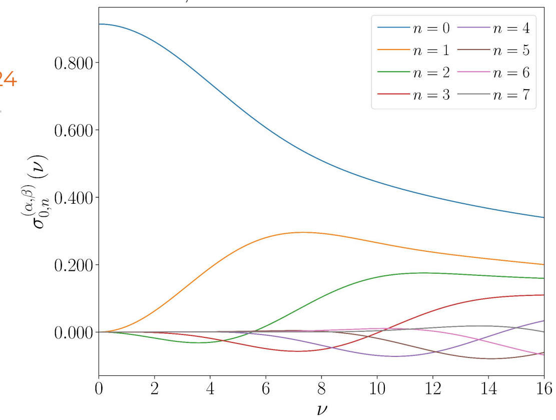
J. Karpie, K. Orginos, A. Radyushkin et al., JHEP 11 (2021) 024

$$\mathfrak{Re}\,\mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{\text{v},n}^{lt\,(\alpha,\beta)} + \Delta_{\text{corr}} \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\text{v},n}^{\Delta\,(\alpha,\beta)}$$

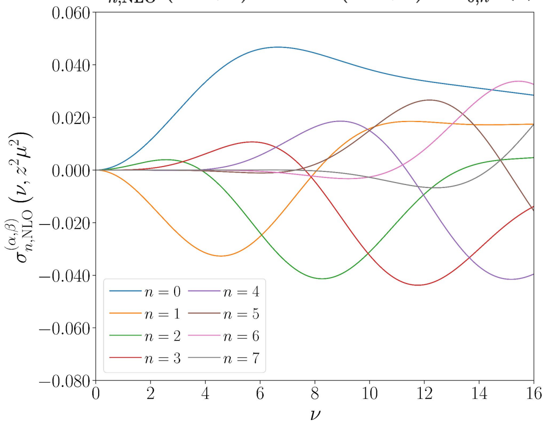
$$\mathfrak{Im}\,\mathfrak{M}_{\text{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{+,n}^{lt\,(\alpha,\beta)} + \Delta_{\text{corr}} \sum_{n=0}^{\infty} \eta_{0,n}^{(\alpha,\beta)}(\nu)\, C_{+,n}^{\Delta\,(\alpha,\beta)}$$

Δ_{corr}: $\frac{a}{|z|},\ z^2\Lambda^2_{\text{QCD}},\ z^4\Lambda^4_{\text{QCD}}$

$\sigma_{0,n}^{(\alpha,\beta)}(\nu) = \sigma_n^{(\alpha,\beta)}(\nu, z^2\mu^2)|_{\alpha_s=0}$



$\sigma_{n,\text{NLO}}^{(\alpha,\beta)}(\nu, z^2\mu^2) = \sigma_n^{(\alpha,\beta)}(\nu, z^2\mu^2) - \sigma_{0,n}^{(\alpha,\beta)}(\nu)$



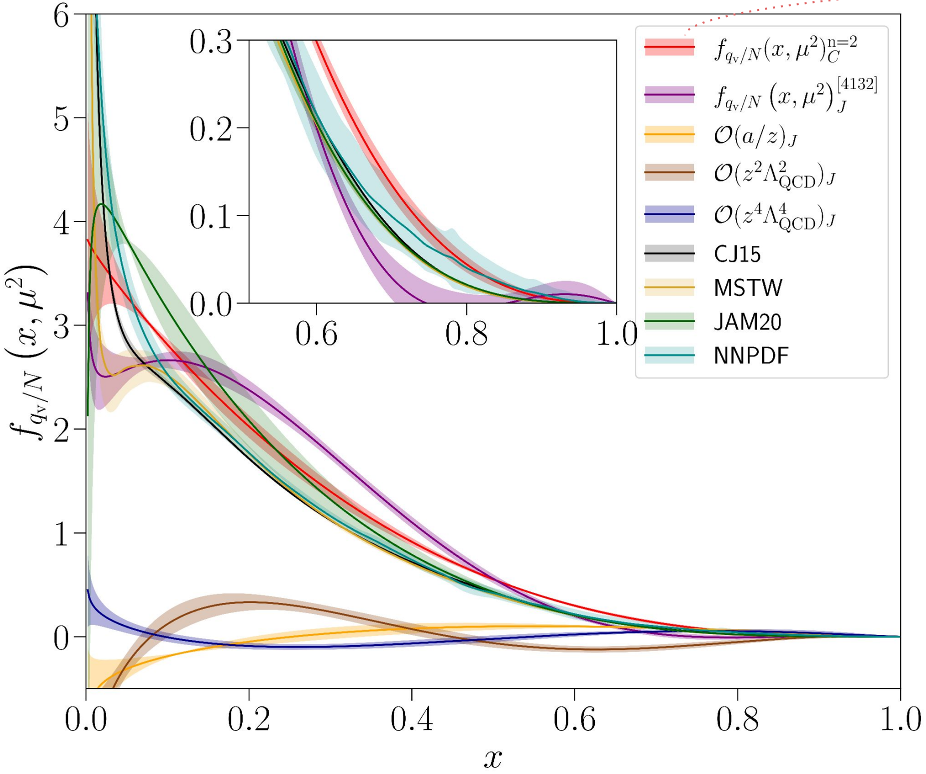
Strategy of parametric fits with Jacobi polynomials

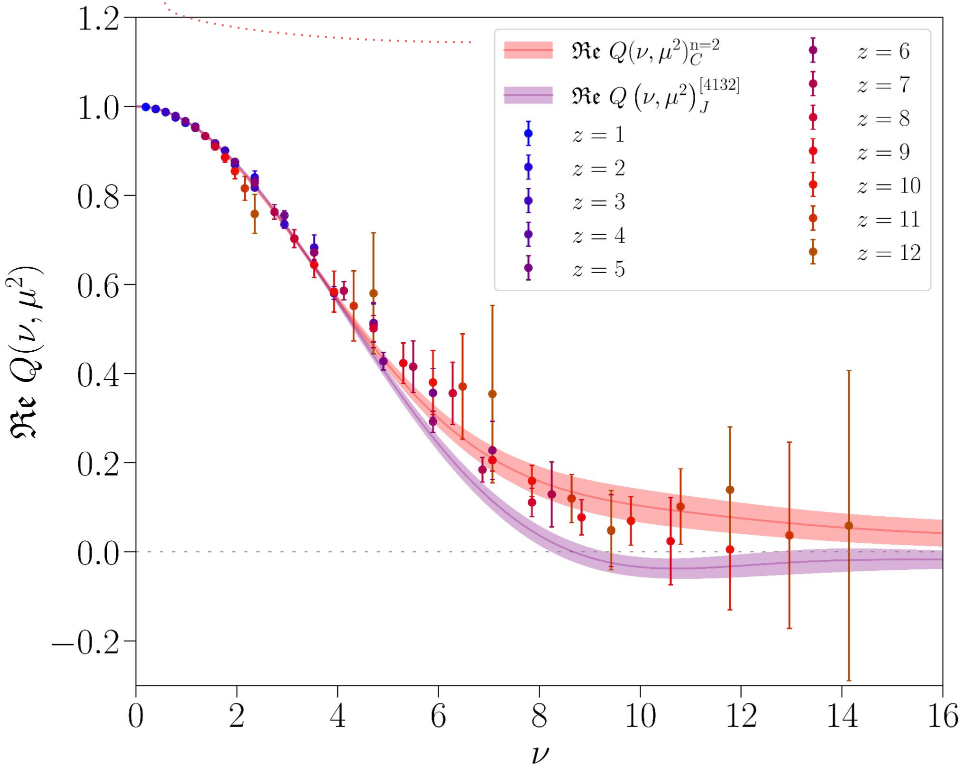
1. Bayesian fits
   a. maximizing likelihood (min. log likelihood)
   b. chi2 minimization plus prior distributions
2. Priors
   a. Log-normal and Gaussian
      i. enforce orthogonality and restrict contaminating x-space distributions
   b. low-order polynomials favored
3. Scan truncation orders to isolate basis and expansion coefficients → variable projection

G. Golub and V. Pereyra, SIAM Journal on Numerical Analysis 10, 413 (1973)

Unpolarized Isovector Quark PDF

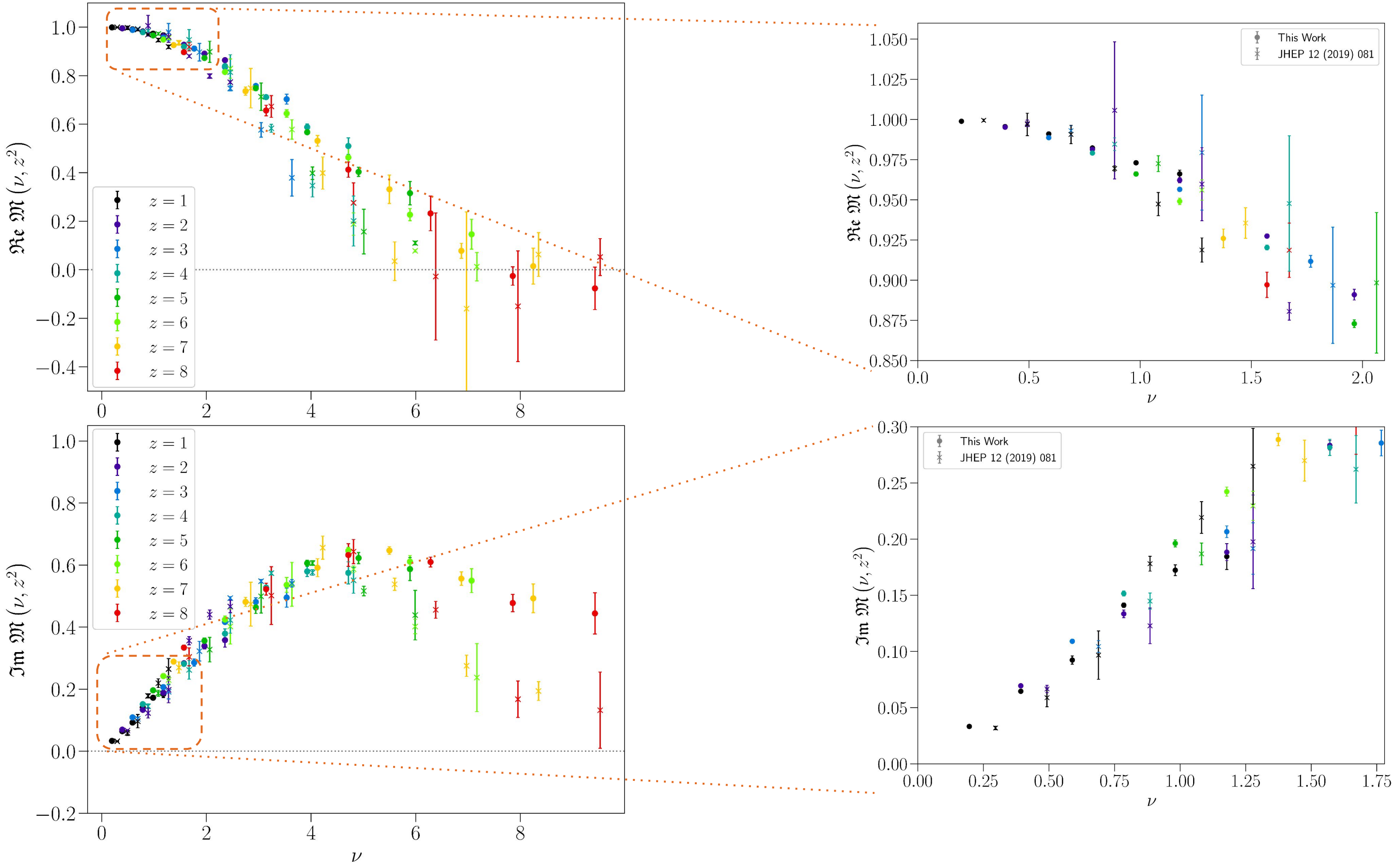
2-param Ansatz (uncorrelated)





CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Efficacy of Distillation



B. Joó et al., JHEP 12 (2019) 081
[Gaussian smearing]

$N_{\rm cfg} = 417 \quad N_{\rm src} = 8 \quad N_\zeta = 5$

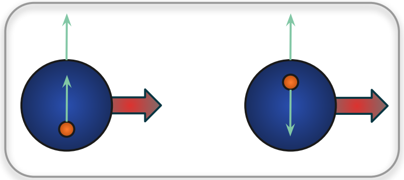
$N_{\rm inv/cfg} \simeq 8.6{\rm k}$

CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148
[Distillation]

$N_{\rm cfg} = 349 \quad N_{\rm src} = 4 \quad N_\zeta = 3$

$N_{\rm inv/cfg} \simeq 16{\rm k}$

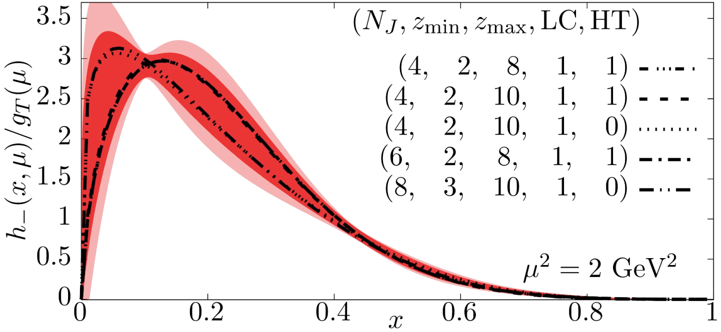
Extension to Transversity PDF



Distribution of transversely polarized quarks within transversely polarized hadron

$$M^{\alpha\beta}(p,z) = \langle h(p)|\,\overline{\psi}(z)\, i\sigma^{\alpha\beta}\gamma^5 \Phi_{\hat{z}}^{(f)}(\{z,0\})\,\psi(0)\,|h(p)\rangle$$

$$= 2(p^\alpha S_\perp^\beta - p^\beta S_\perp^\alpha)\mathcal{M}(\nu,z^2) + 2im_N^2(z^\alpha S_\perp^\beta - z^\beta S_\perp^\alpha)\mathcal{N}(\nu,z^2) + 2m_N^2(z^\alpha p^\beta - z^\beta p^\alpha)(z\cdot S_\perp)\mathcal{R}(\nu,z^2)$$



Akaike Information Criterion (AIC)

H. Akaike, IEEE Transactions on Automatic Control, vol.19, no.6, 716-723 (1974)

- ➢ weights assigned based on quality of fit, number of datapoints and parameters

$$w^{(m)} = \frac{e^{-\frac{1}{2}\mathrm{AIC}(m)}}{\sum_{n\in\mathrm{fit}} e^{-\frac{1}{2}\mathrm{AIC}(n)}}$$

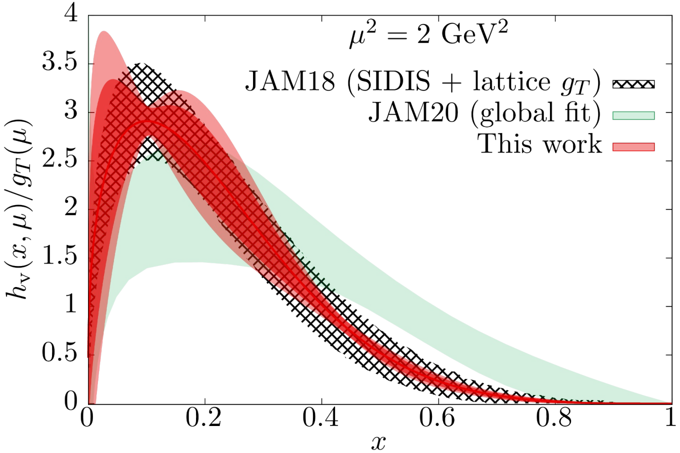
- ➢ ideally, averages away model biases for large number of models

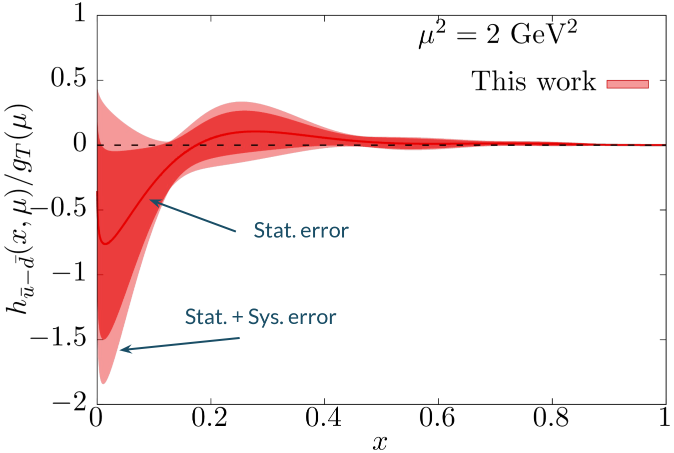
$$\mathrm{AIC}(n) = \mathcal{L}_n + 2p_n + \frac{2p_n(p_n+1)}{(d_n - p_n - 1)}$$

$$h_\pm^{\mathrm{AIC}}(x) = \sum_{m\in\mathrm{fit}} w^{(m)} h_\pm^{(m)}(x)$$

- ➢ Non-singlet antiquark distribution found to be consistent with an isospin-symmetric intrinsic sea

[HadStruc] **CE**, C. Kallidonis, J. Karpie, N. Karthik et al., Phys. Rev. D 105 (2022) 3, 034507





Nucleon Unpolarized Gluon PDF

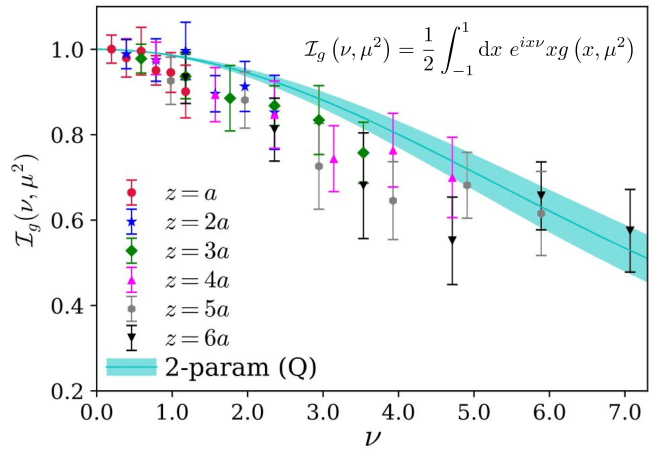
Gluonic & flavor-singlet quantities remain demanding

- ➢ well-suited for distillation
  - ○ enables thorough sampling of configurations

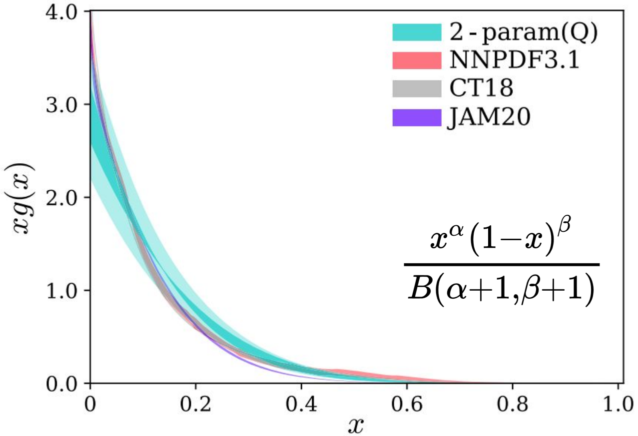
HadStruc has performed first calculation of nucleon's unpolarized gluon PDF using pseudo-distributions

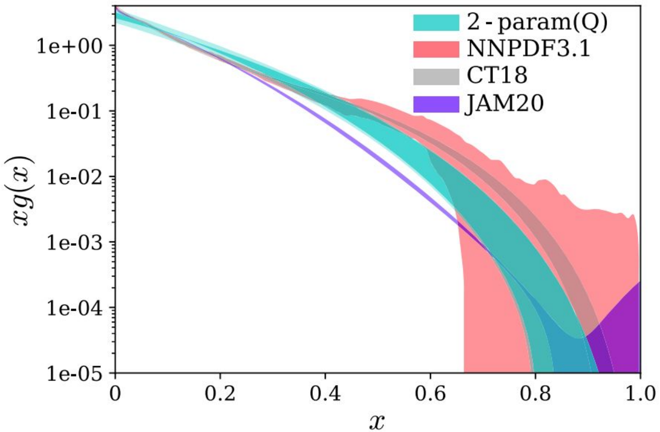
- ➢ key techniques: J. Bulava, M. Donnellan, and R. Sommer, JHEP 01 (2012) 140
  - ○ "sGEVP" analysis of expanded operator basis
  - ○ gradient flow to dampen UV fluctuations

M. Lüscher, JHEP 02 (2010) 071; 03 (2014) 092(E)
M. Lüscher and P. Weisz, JHEP 02 (2011) 051



[HadStruc] T. Khan, R.S. Sufian, J. Karpie, C. Monahan, **CE** et al., Phys. Rev. D 104 (2021) 9, 094516





Most precise and complete (i.e. sampling) determination of x-dependent gluon PDF

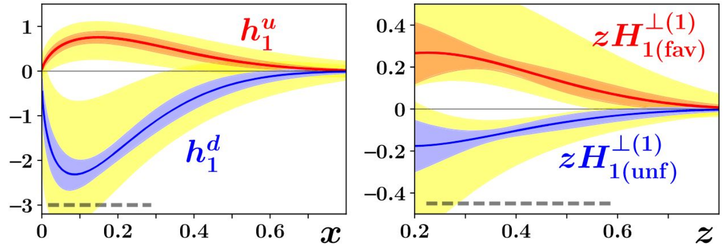
Z.-Y. Fan et al., Phys. Rev. Lett. 121, 242001 (2018)
Z. Fan, R. Zhang, and H.-W. Lin, Int. J. Mod. Phys. A 36, 2150080 (2021)

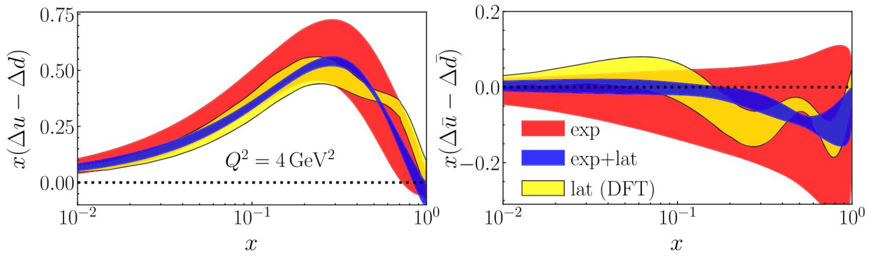
Capable of complementing phenomenological extractions

- ➢ a central focus of EIC!

Complementarity with Global Analyses

Constraints provided by lattice QCD (LQCD):





J. Bringewatt, N. Sato, W. Melnitchouk et al., Phys. Rev. D 103 (2021) 1, 016003

H.-W. Lin, W. Melnitchouk, A. Prokudin et al., Phys.Rev.Lett.120 152502 (2018)

Pion PDFs extracted in a MC global QCD analysis

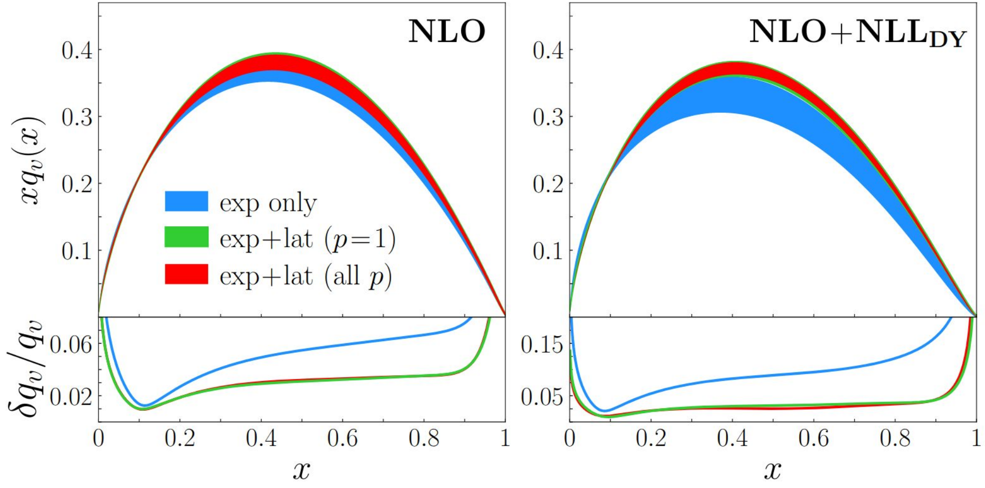
- ➢ experimental data
- ➢ reduced Ioffe-time pseudo-distributions & current-current (CC) matrix elements
  - ○ CC systematics limit impact

Each dataset can inform errors of counterpart

- ➢ PDF uncertainties guided by matrix elements
- ➢ systematics inherent to LQCD calculation guided by experiment

Pseudo-distributions dramatically affect PDF

$$\sim (1-x)^{\beta_{\mathrm{eff}}} \qquad \beta_{\mathrm{eff}} \simeq 1.0 - 1.2$$



[JAM/HadStruc] P.C. Barry, **CE**, J. Karpie et al., arXiv: 2204.00543 [hep-ph] (in review)

Closing Remarks

Hadronic structure accessible from certain lattice calculable matrix elements

- ➢ short-distance factorization

Isovector twist-2 quark PDFs of Nucleon

a[fm] \ m_π[MeV]	$f_{q_\pm/N}\left(x,\mu^2\right)$	$\Delta f_{q_\pm/N}\left(x,\mu^2\right)$	$h_{q_\pm/N}\left(x,\mu^2\right)$
0.094(1) / 358(3)	Published	Preliminary	Published
0.094(1) / 278(4)	Analyzing	Preliminary	Analyzing
0.091(2) / 170(5)	Ongoing	Ongoing	Ongoing

- ➢ systematic effects can be reliably addressed

Statistical precision afforded by use of distillation

- ➢ unpolarized gluon PDF of nucleon
- ➢ extending trajectories to expose systematics [USQCD]

DA and flavor-singlet PDF program underway

Off-forward matrix elements

- ➢ short-distance factorization + distillation [GPDs]

HadStruc Collaboration

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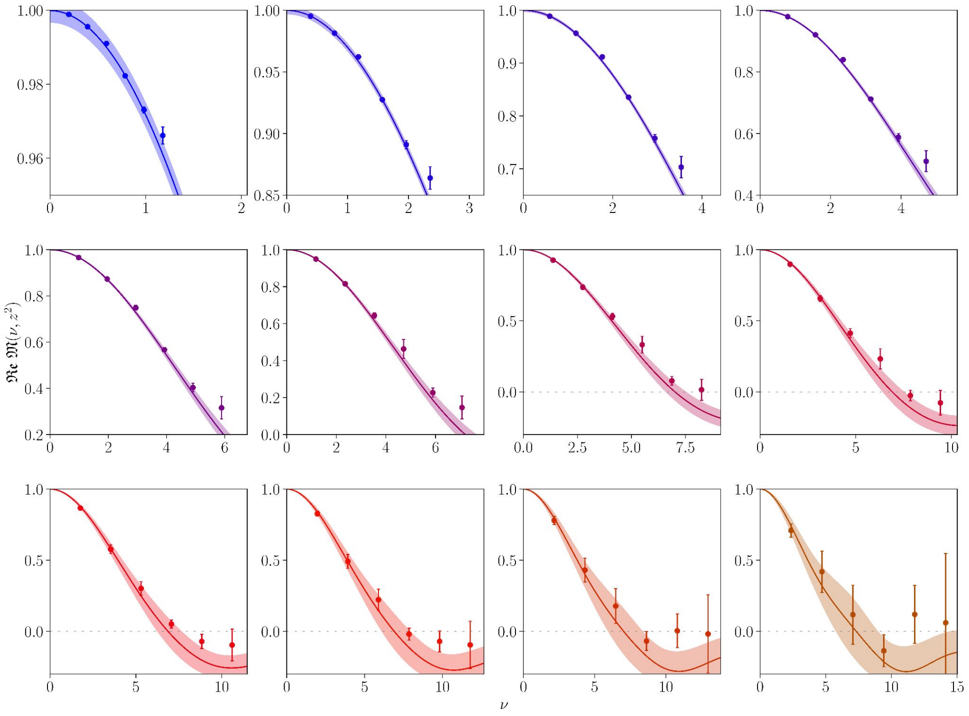
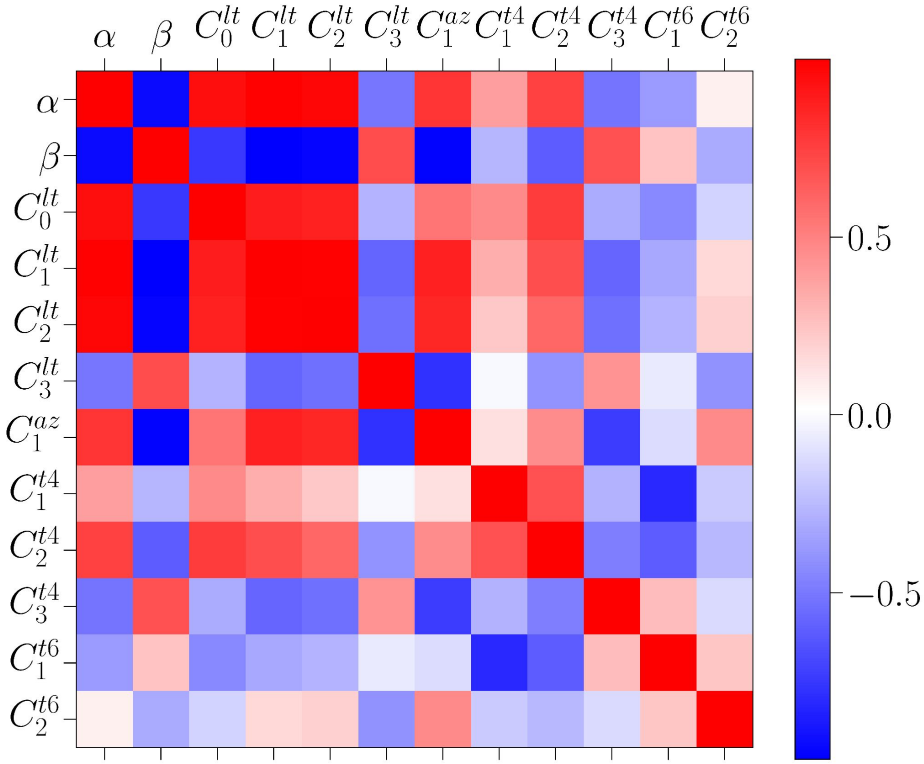
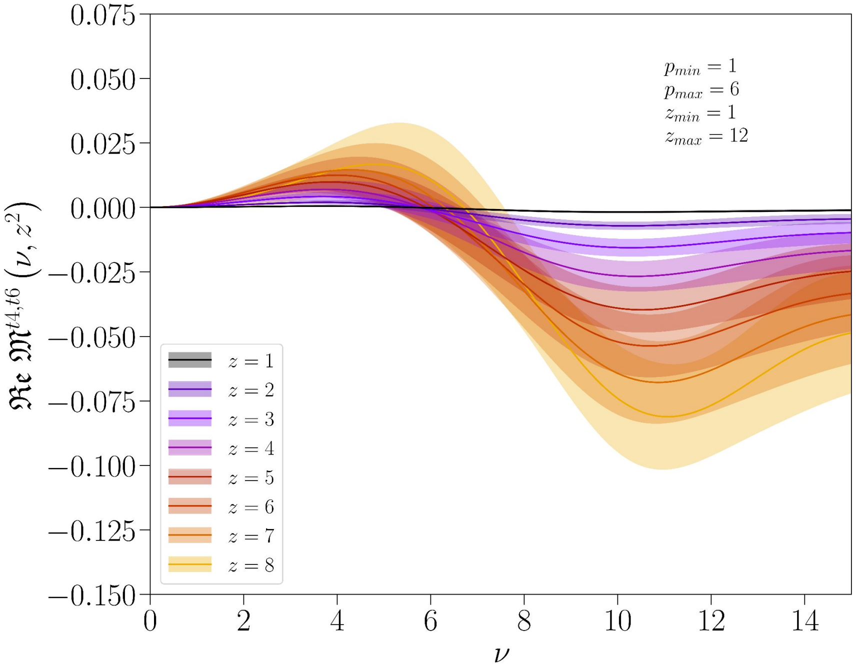
Optimal Fit for Unpolarized Valence Quark PDF

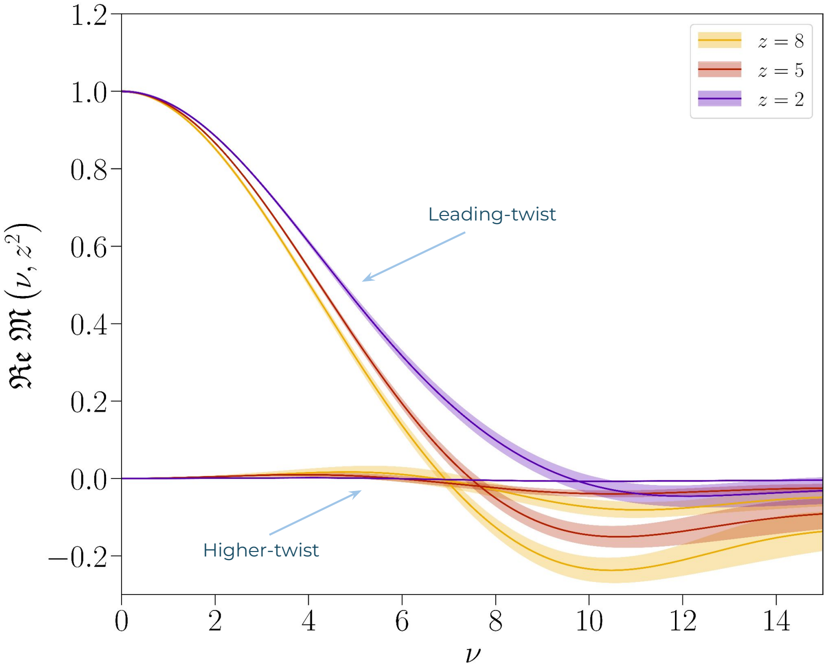




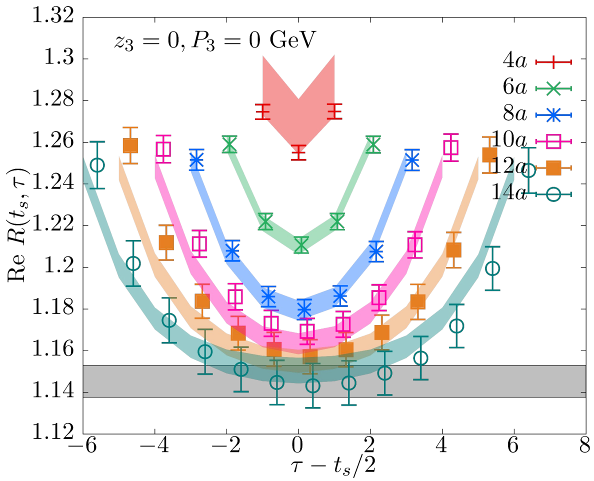
CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

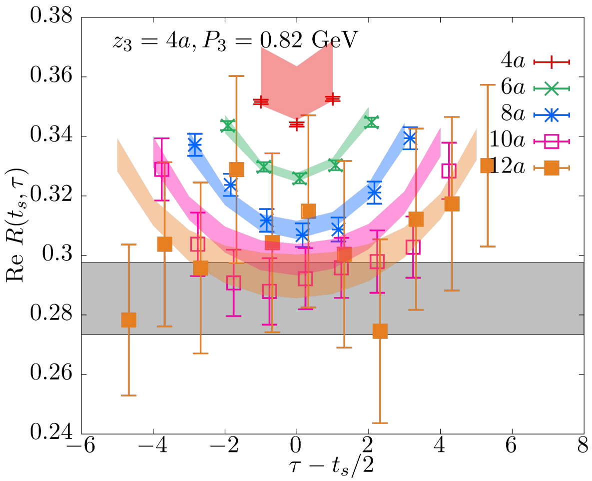
Parameterized Higher-Twist Contamination (Unpol.)

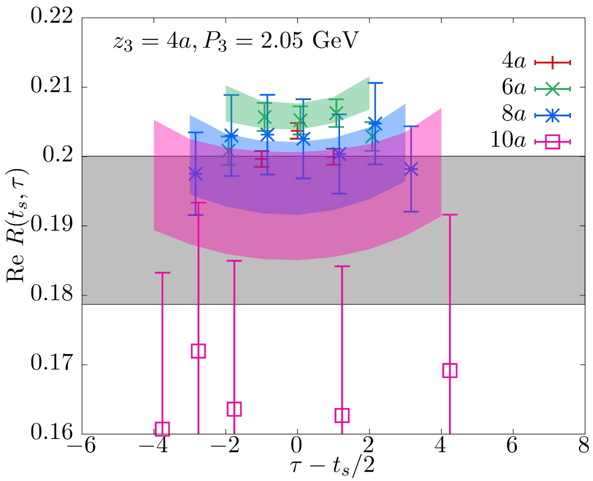




Selected Transversity Matrix Element Fits







Further details on Akaike Information Criterion (AIC)

H. Akaike, IEEE Transactions on Automatic Control, vol.19, no.6, 716-723 (1974)

$$h_{\pm}^{\mathrm{AIC}}(x) = \sum_{m\in\mathrm{fit}} w^{(m)} h_{\pm}^{(m)}(x) \qquad \Delta_{\pm}^{\mathrm{AIC}}(x) = \sqrt{\sum_{m\in\mathrm{fit}} w^{(m)} \left[h_{\pm}^{(m)}(x) - h_{\pm}^{\mathrm{AIC}}(x)\right]^2} \qquad w^{(m)} = \frac{e^{-\frac{1}{2}\mathrm{AIC}(m)}}{\sum_{n\in\mathrm{fit}} e^{-\frac{1}{2}\mathrm{AIC}(n)}} \qquad \mathrm{AIC}(n) = \mathcal{L}_n + 2p_n + \frac{2p_n(p_n+1)}{(d_n - p_n - 1)}$$

➢ weights assigned based on quality of fit, number of datapoints and parameters

Nucleon Interpolators with Distillation

Excited-state contamination

- ➢ optimize operator/state overlaps - saturate correlation functions at early temporal separations

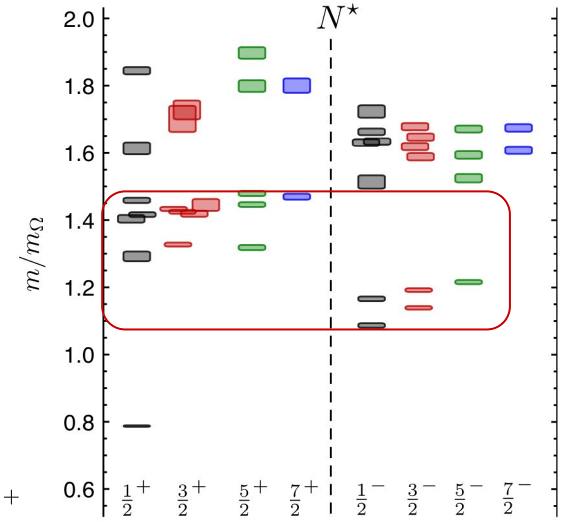
Generic light-quark nucleon interpolator smeared with distillation

$$\mathcal{O}_i(t) = \epsilon^{abc}\left(\mathcal{D}_1 \Box u\right)_a^\alpha \left(\mathcal{D}_2 \Box d\right)_b^\beta \left(\mathcal{D}_3 \Box u\right)_c^\gamma (t)\, S_i^{\alpha\beta\gamma}$$

Dirac structure/covariant derivatives

Discretized continuum-like interpolators of definite permutational symmetries

$$\mathcal{O}_B = \left(\mathcal{F}_{\mathcal{P}(\mathrm{F})} \otimes \mathcal{S}_{\mathcal{P}(\mathrm{S})} \otimes \mathcal{D}_{\mathcal{P}(\mathrm{D})}\right)\{q_1 q_2 q_3\} \qquad \left(N_M \otimes \left(\tfrac{1}{2}^+\right)_M^1 \otimes D_{L=1,A}^{[2]}\right)^{J^P=\frac{1}{2}^+} \equiv N^2 P_A \tfrac{1}{2}^+$$



R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)

(Generally) Continuum spins reducible under octahedral group

Canonical subductions

- ➢ spinors/derivatives combined into object of definite spin/parity

$$\mathcal{O}^{\{J\}}_{^n\Lambda,r} = \sum_m S^{J,m}_{^n\Lambda,r}\, \mathcal{O}^{\{J,m\}}$$

R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)
J. Dudek and R. Edwards, Phys. Rev. D85, 054016 (2012)

Helicity subductions

C. Thomas, et al., Phys. Rev. D85, 014507 (2012)
C. Thomas, private communication

- ➢ boost breaks (double-cover) octahedral symmetry to little groups

$$\left[\mathbb{O}^{J^P,\lambda}(\vec{p})\right]^\dagger = \sum_m \mathcal{D}^{(J)}_{m,\lambda}(R)\left[O^{J^P,m}(\vec{p})\right]^\dagger$$

- ➢ subduce into little groups

$$\left[\mathbb{O}^{J^P,|\lambda|}_{\Lambda,\mu}(\vec{p})\right]^\dagger = \sum_{\hat{\lambda}=\pm|\lambda|} S^{\tilde{\eta},\hat{\lambda}}_{\Lambda,\mu}\left[\mathbb{O}^{J^P,\hat{\lambda}}(\vec{p})\right]^\dagger$$