Form Factors in the Partonic Regime: Computing the large-Q^2 Pion Form Factors and the Kaon Distribution Amplitude with Physical Quark Masses

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Motivations for studying pion/kaon structure

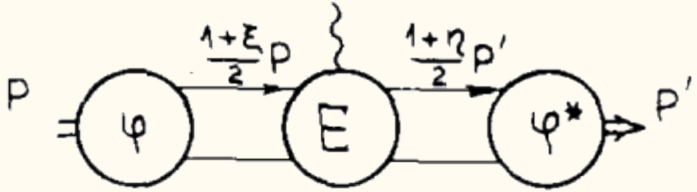
- Pion/kaon are pseudo-Goldstone bosons of chiral-symmetry breaking in QCD
    - Insights on connection between hadron mass and hadron structure
- Limited experimental knowledge of pion/kaon internal structure
- Easier to study in lattice QCD than the nucleon
    - Can achieve larger boosts due to smallness of pion mass
    - Less excited state contamination
    - Better signal-to-noise ratio
- Techniques learned can later be applied for the nucleon

Form factors in the partonic regime

- At large Q^2, form factor of pseudoscalar meson is expected to factorize into a perturbative partonic piece and the distribution amplitude $\phi(x,\mu)$

- Will be tested in various experiments
  - JLab 12 GeV upgrade
  - EIC will probe Q^2 up to 30 GeV^2

$$\langle P_1 | J_\mu | P_2 \rangle = (P_1 + P_2)_\mu F_\pi(Q^2)$$



[Efremov, Radyushkin ‘80]
[Lepage, Brodsky ‘80]

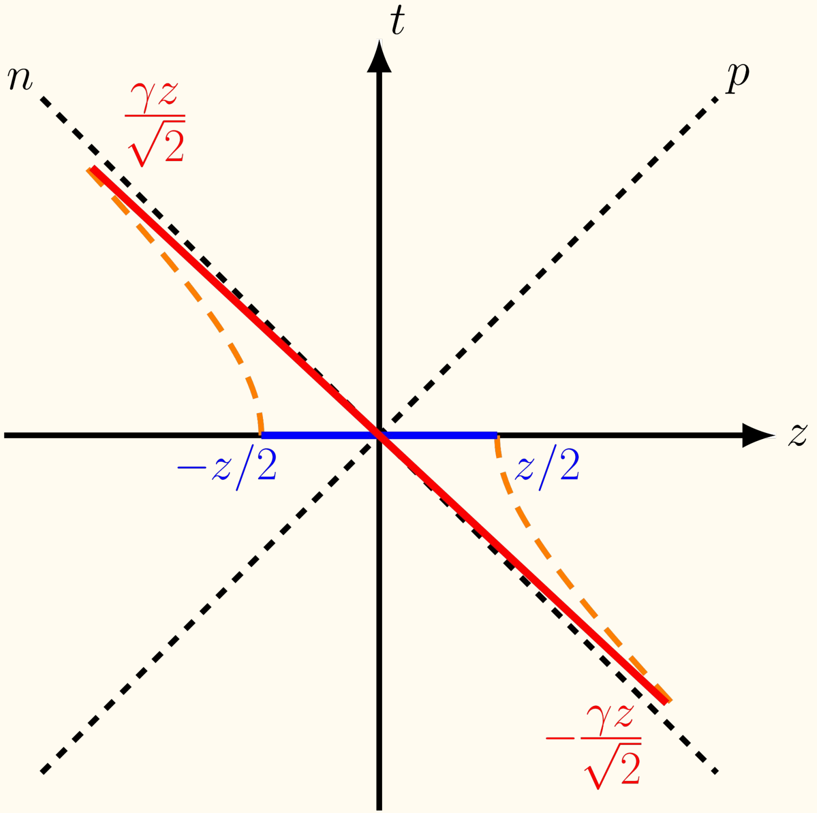
$$\langle 0 | \overline{u}(-z^-/2)\gamma^+\gamma_5 W_+ d(z^-/2) | \pi^+; P \rangle$$
$$= i f_\pi P^+ \int_{-1}^{1} \mathrm{d}x \; e^{i\pi P^+ z^-/2} \, \phi(x,\mu)$$

Distribution amplitude from lattice QCD

- Cannot calculate matrix elements separated along the light cone in lattice QCD
- Instead, calculate equal-time spatially-separated matrix element of highly-boosted hadron

$$h^B(z, P_z) = \langle 0|\overline{u}(-z/2)\gamma_z\gamma_5 W_z d(z/2)|\pi^+; P_z\rangle$$

- Can be matched to light-cone DA through Large-momentum Effective Theory or short-distance factorization



[X. Ji et al., Rev. Mod. Phys. **93**, 035005, arXiv: 2004.03543]

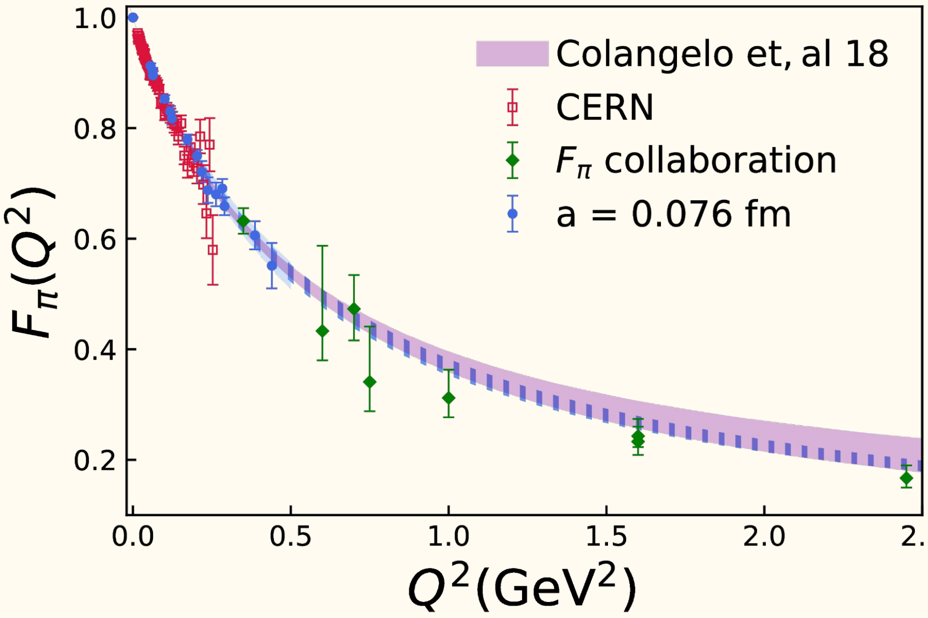
Theoretical Framework:
[V. Braun, D. Müller ‘07]
[X. Ji ’13]
[A. Radyushkin ’17]

Lattice Setup

- Mixed fermion action
  - Sea quark action: $N_f = 2+1$ HISQ with physical quark masses, $L^3 \times T = 64^3 \times 64$, $a = 0.076$ fm
  - Valence quark action: $N_f = 2 + 1$ Wilson-Clover with physical quark masses, 1-HYP smeared gauge links
- Use momentum smearing for quarks to achieve better overlap with boosted hadrons
- Pion form factor
  - $P_z^f = 3,4,5,6$ and $t_{sep} = 6,8,10,12$
- Kaon distribution amplitude
  - Two momentum smearings

Previous calculation of small-Q^2 pion form factor

- $F_\pi(Q^2)$ at $Q^2 < 0.5\ \text{GeV}^2$ from our previous USQCD allocation
- This allocation will cover $0.80\ \text{GeV}^2 \leq Q^2 \leq 12.85\ \text{GeV}^2$
- Momentum smearing tuned for Breit Frame ($P_\text{f} = -P_\text{i}$), giving Q^2=2.34, 4.16, 6.50, 9.35 GeV^2



[X. Gao *et al.*, Phys. Rev. D **104**, 114515, arXiv:2102.06047]

Pion distribution amplitude

- Use renormalization group invariant ratios

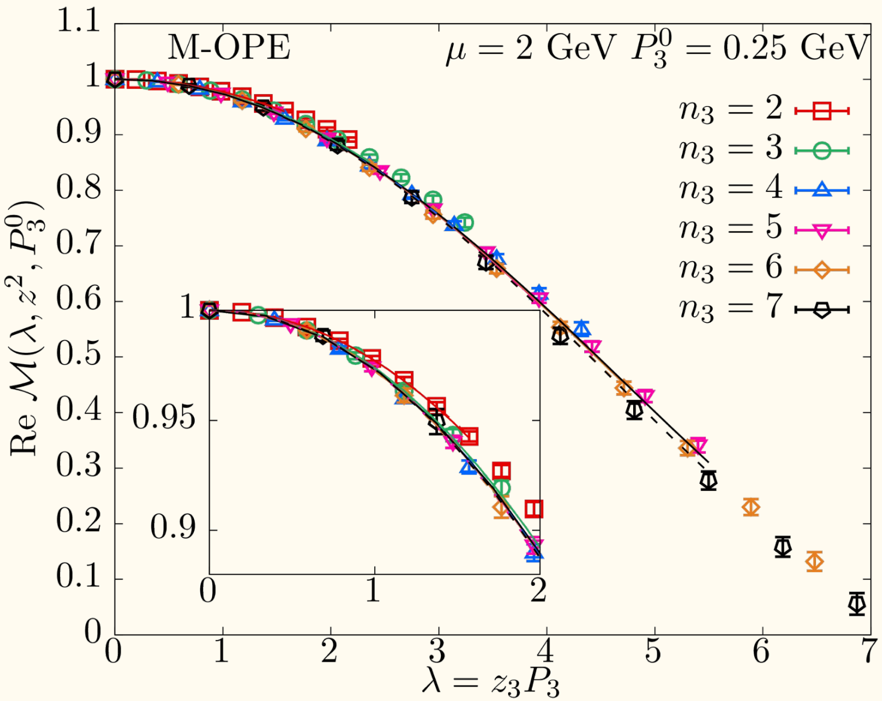
$$\mathcal{M}(\lambda, z^2, P^0) = \left(\frac{h^B(z_3, P_3)}{h^B(z_3, P_3^0)}\right)\left(\frac{h^B(0, P_3^0)}{h^B(0, P_3)}\right)$$

- Use leading twist OPE

$$h^{\mathrm{tw2}}(\lambda, z^2, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m \rangle$$

- Extract Mellin moments from fit to ratio

$$\mathcal{M}^{\mathrm{tw2}}(\lambda, z^2, P^0) = \frac{h^{\mathrm{tw2}}(\lambda, z^2, \mu)}{h^{\mathrm{tw2}}(\lambda_0, z^2, \mu)}$$



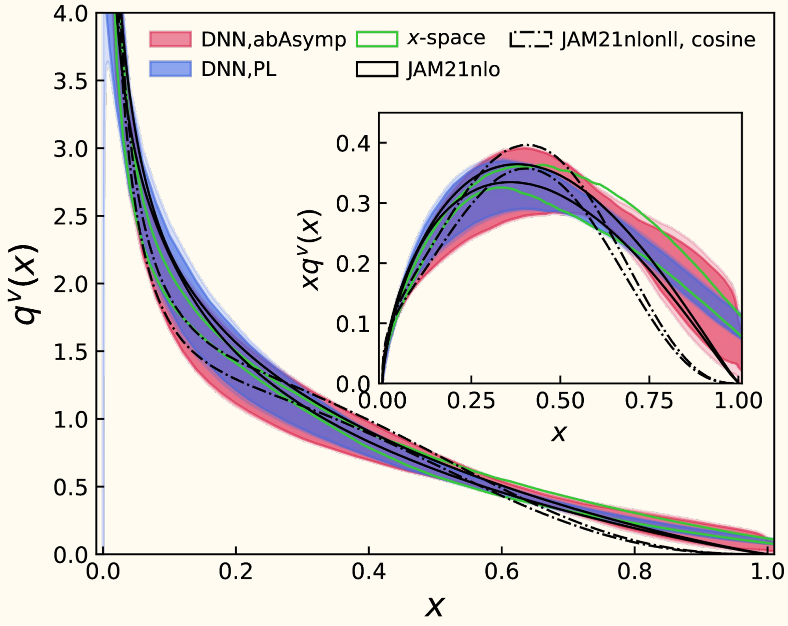
[In preparation]

Pion valence PDF using various methods

- Prefer model independent strategies
- Obtain moments from OPE (like for DA)
- Hybrid renormalization with x-space matching at NNLO
  [X. Gao et al., Phys. Rev. Lett. **128**, 142003]
- Use a Deep Neural Network (with short-distance factorization) to avoid truncating in Mellin moments and choosing a specific model

$$h^R(z, P_z, \mu) = \int_{-1}^{1} d\alpha \mathcal{C}(\alpha, \mu^2 z^2) Q(\alpha\lambda, \mu)$$

$$+\mathcal{O}(z^2\Lambda_{QCD}^2)$$



[In preparation]

Request totals

- Software:
  - Uses the Qlua package
  - Inversions done with multi-grid solvers
  - Interfaces with QUDA
- Statistics:
  - 350 configs
  - 1 exact and 32 sloppy solves per config
- Requested resources:
  - 140k K80 hours
  - 25 TB disk space
  - 50 TB archival storage

Timings using 16 nodes of the BNL IC cluster (32 K80 cards)

Extended source/sink creation	t_{ext}	10 sec
MG u/d-quark inversion sloppy	$t_{u,inv}$	26 sec
MG u/d-quark inversion exact	$t_{u,inv}$	50 sec
MG s-quark inversion sloppy	$t_{s,inv}$	18 sec
MG s-quark inversion exact	$t_{s,inv}$	27 sec
contraction for pion 2pt function	$t_{\pi,2pt}$	7.6 sec
contraction for kaon 2pt function	$t_{K,2pt}$	10 sec
contraction for 3pt function for FF	$t_{FF,3pt}$	4 sec
contraction for quasi-DA 2pt function	$t_{DA,2pt}$	40 sec

Conclusions

- Propose to calculate large-Q^2 pion form factor and kaon distribution amplitude as a continuation of pion GPD calculations from last year
  - Will have all quantities needed to test large-Q^2 factorization
- Our collaboration has made significant progress toward the understanding of the pion internal structure
  - Valence PDF of the pion using new methods
- We will utilize the various methods described here to the data acquired from this new allocation