

Nucleon Matrix Elements with 2+1-Flavor Clover Fermions

Rajan Gupta (PI), Tanmoy Bhattacharya, Vincenzo Cirigliano,
Yong-Chull Jang, Balint Joo, Huey-Wen Lin, Kostas Orginos,
Sungwoo Park, David Richards, Frank Winter, **Boram Yoon**

Nucleon Matrix Elements w/ $N_f = 2 + 1$ Clover Lattices

Ensemble ID	β	a (fm)	M_π (MeV)	L	T	$M_\pi L$	# of Confs.
a127m285	6.1	0.127	285	32	96	5.9	2002
a094m270	6.3	0.094	269	32	64	4.1	2469
a094m270L	6.3	0.094	269	48	128	6.2	4510
a093m220SU3	6.3	0.093	220	48	128	5.0	1500
a093m220	6.3	0.093	220	48	128	5.0	4000
a091m170	6.3	0.091	169	48	96	3.7	4012
a091m170L	6.3	0.091	170	64	128	5.1	5000
a073m270	6.5	0.073	272	48	128	4.2	4720
a072m220	6.5	0.072	220	64	192	5.2	2500
a071m170	6.5	0.071	166	72	192	4.4	5000
a070m130	6.5	0.070	128	96	192	4.4	2000
a056m280	6.7	0.056	279	64	192	4.8	5000
a056m220	6.7	0.056	220	72	192	4.5	1500

Parameters of the thirteen isotropic clover ensembles being generated by the JLab / W&M / LANL / MIT collaboration using CHROMA and QUDA Multigrid

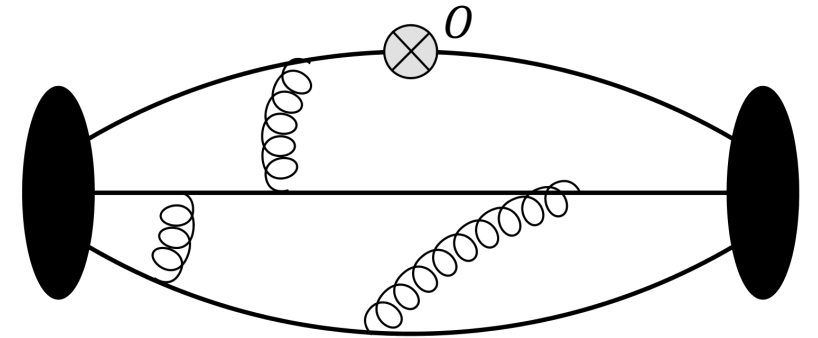
- We request **300TB** of disk space to make 1000 configurations of each ensemble **available to USQCD** under the standard non-competing arrangement

Nucleon Matrix Elements

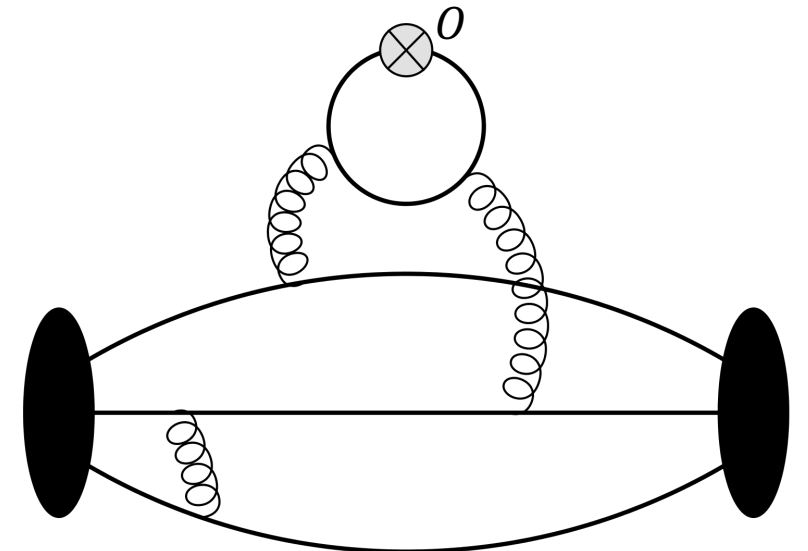
- Isovector charges $O = \bar{u}\Gamma d$
- Flavor diagonal matrix elements $O = \bar{q}\Gamma q$
- Axial vector form factors $O = \bar{u}\gamma_\mu\gamma_5 d$
- Electromagnetic form factors $O = \bar{u}\gamma_\mu d$
- Moments

$$O = \bar{q}\gamma_4\vec{D}_4 q, \quad \bar{q}\gamma_{\{3}\gamma_5\vec{D}_4\}q, \quad \bar{q}\gamma_{\{3}\gamma_5\vec{D}_4\}q$$
- Electric dipole moments

$$\langle N(q) | \bar{q}\gamma_\mu q | N(0) \rangle_{\theta, CEDM, W_{ggg}, \dots}$$

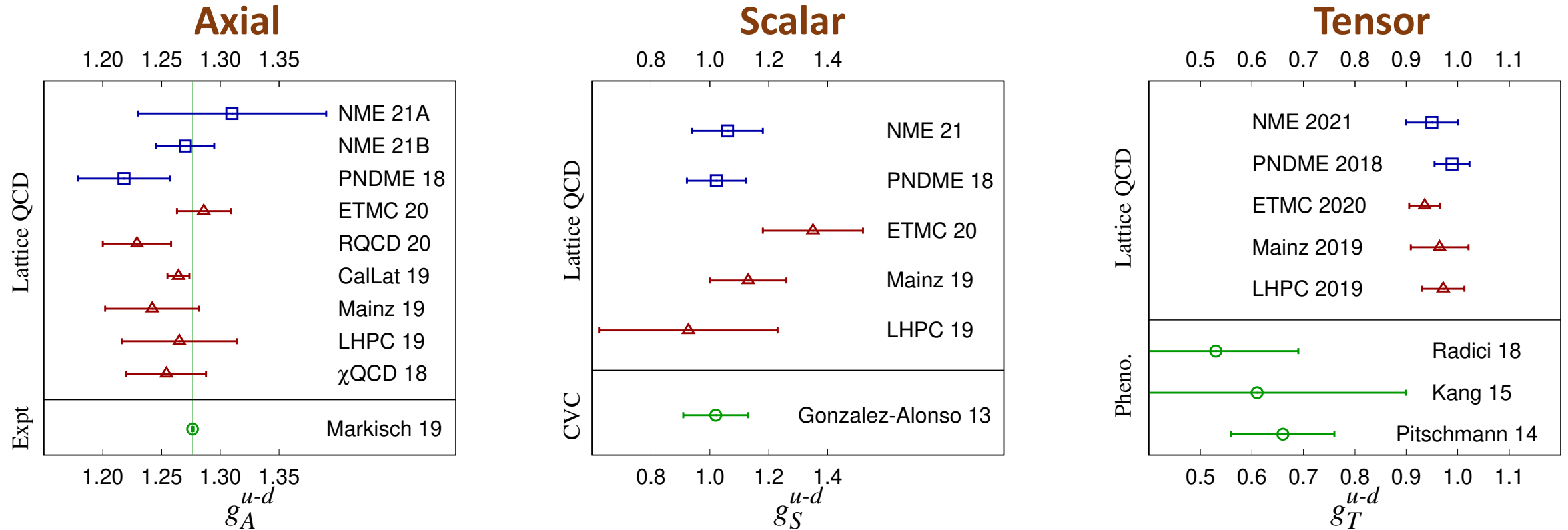


Quark-line connected diagram



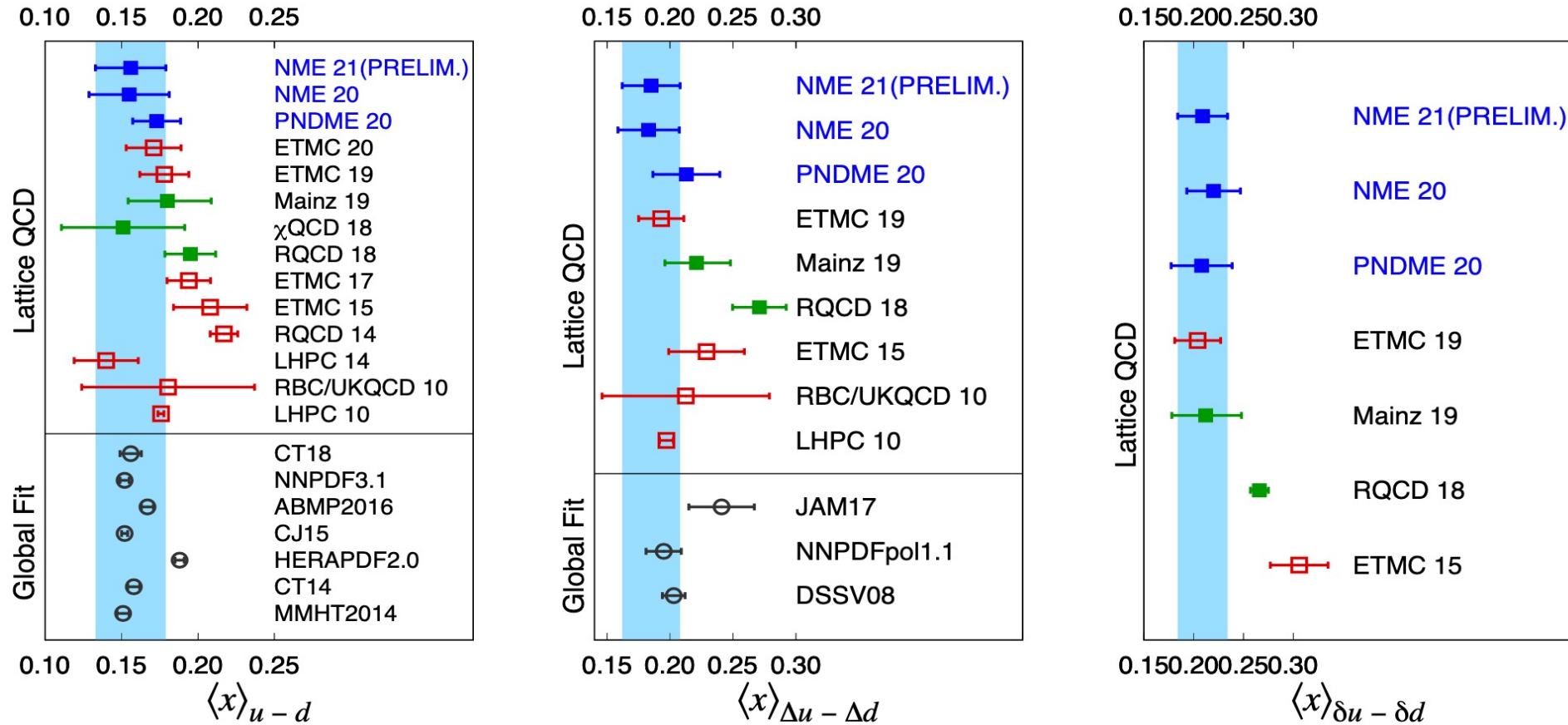
Quark-line disconnected diagram

Isovector Charges



- No disconnected diagrams: $g_{\Gamma}^{u-d} \bar{\psi}_p \Gamma \psi_n = \langle p | \bar{u} \Gamma d | n \rangle = \langle p | \bar{u} \Gamma u - \bar{d} \Gamma d | p \rangle$ in isospin limit ($m_u = m_d$)
- **Axial charge:** Nuclear beta decay, pion exchange between nucleons, nucleosynthesis, ...
- **Scalar and Tensor charges:** Novel scalar and tensor interactions in BSM [PRD98.034503(2018)]
 - CVC: Conserved vector current relation for g_S ; $g_S/g_V = (M_N - M_P)^{QCD} / (m_d - m_u)^{QCD}$
 - Pheno.: Extraction of g_T from semi-inclusive deep-inelastic scattering (SIDIS) experimental data

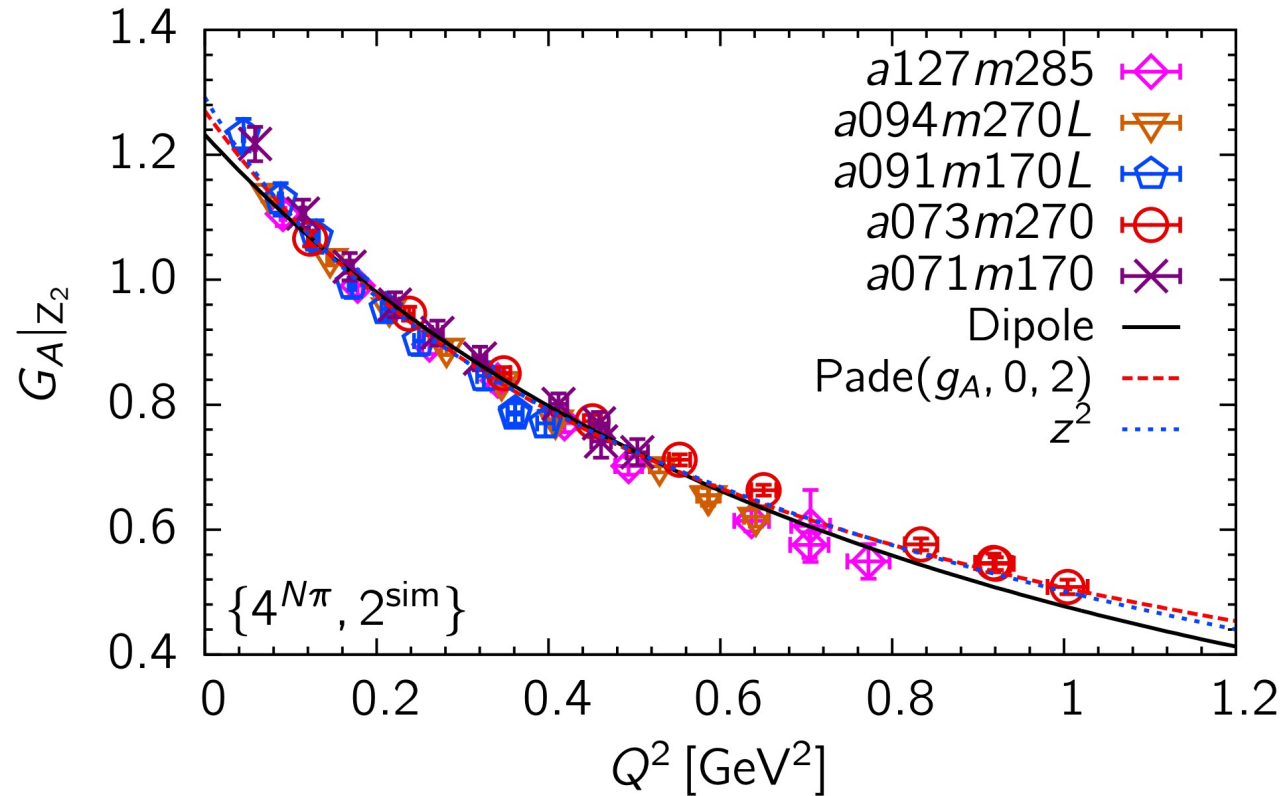
Moments



- Momentum fraction: $\langle x \rangle_q = \int_0^1 x [q(x) + \bar{q}(x)] dx$ where $q = q_\uparrow + q_\downarrow$; unpolarized
- Helicity moment: $\langle x \rangle_{\Delta q} = \int_0^1 x [\Delta q(x) + \Delta \bar{q}(x)] dx$ where $\Delta q = q_\uparrow - q_\downarrow$; longitudinally polarized
- Transversity moment: $\langle x \rangle_{\delta q} = \int_0^1 x [\delta q(x) + \delta \bar{q}(x)] dx$ where $\delta q = q_\top + q_\perp$; transversely polarized

Axial-vector Form Factor

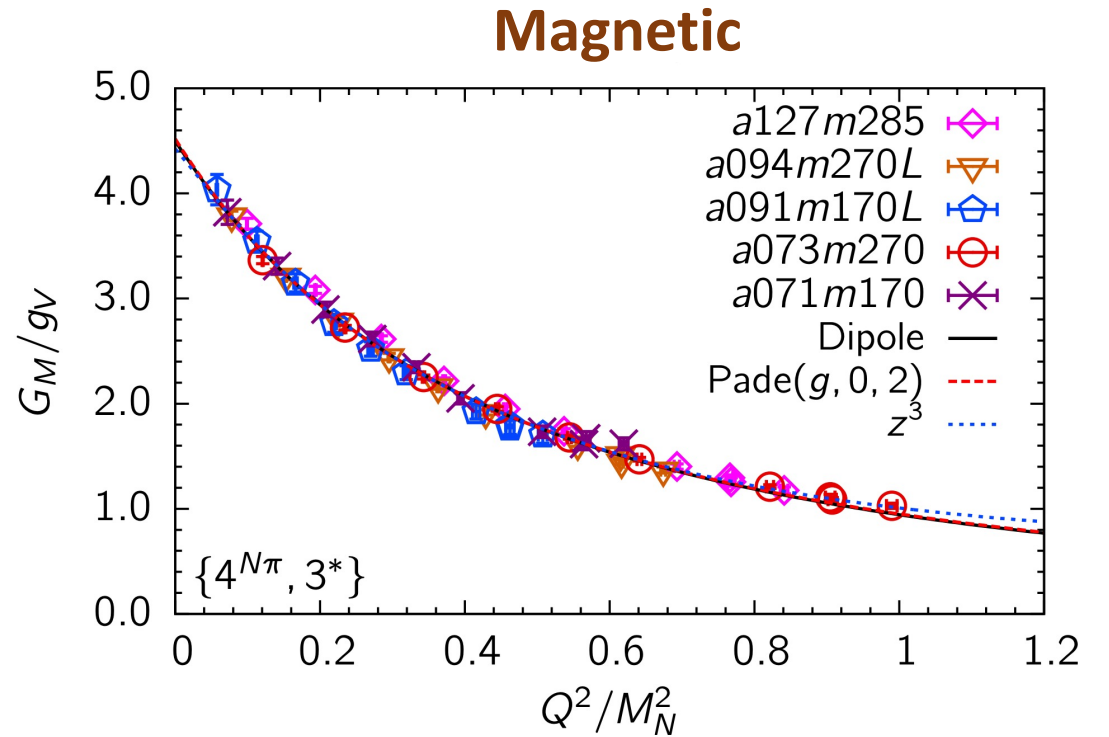
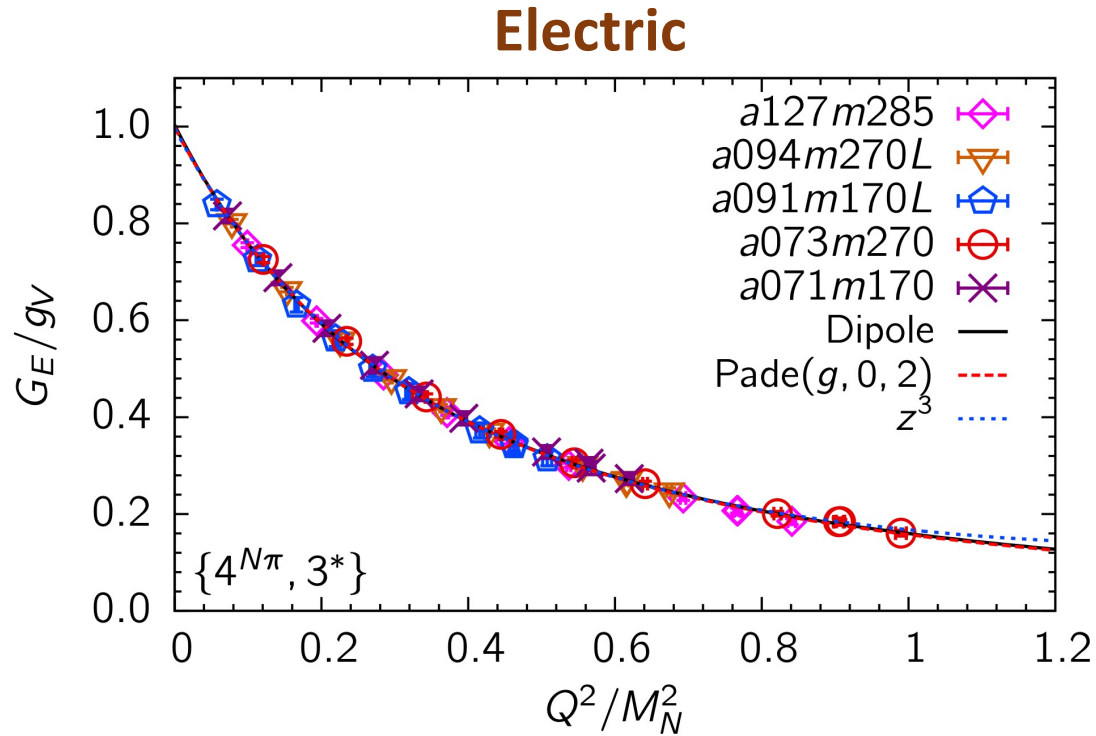
Park, et al., PRD 105, 054505 (2022)



- $G_A(Q^2) = \frac{g_A=1.270(11)}{1+5.36(20)\frac{Q^2}{4M_N^2}-0.22(81)\left(\frac{Q^2}{4M_N^2}\right)^2}$
- Axial charge radius squared
 $\langle r_A^2 \rangle^{u-d} = 0.428(53)(30) \text{ fm}^2$
- Induced pseudoscalar charge
 $g_P^* = 7.9(7)(9)$
- Pion-nucleon coupling
 $g_{\pi NN} = 12.4(1.2)$

- Input for the analysis of neutrino-nucleus interactions
- Sensitive to excited-state analysis

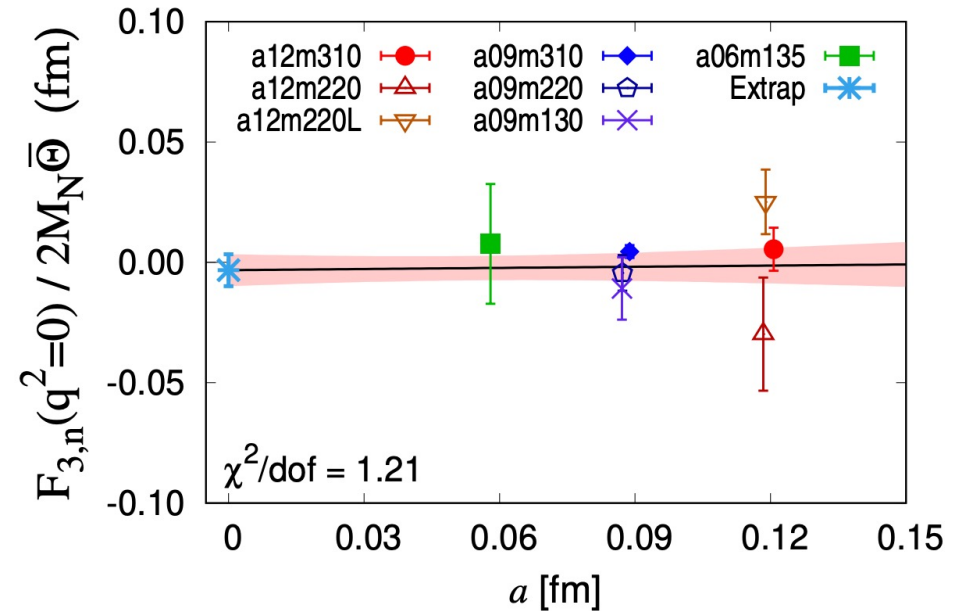
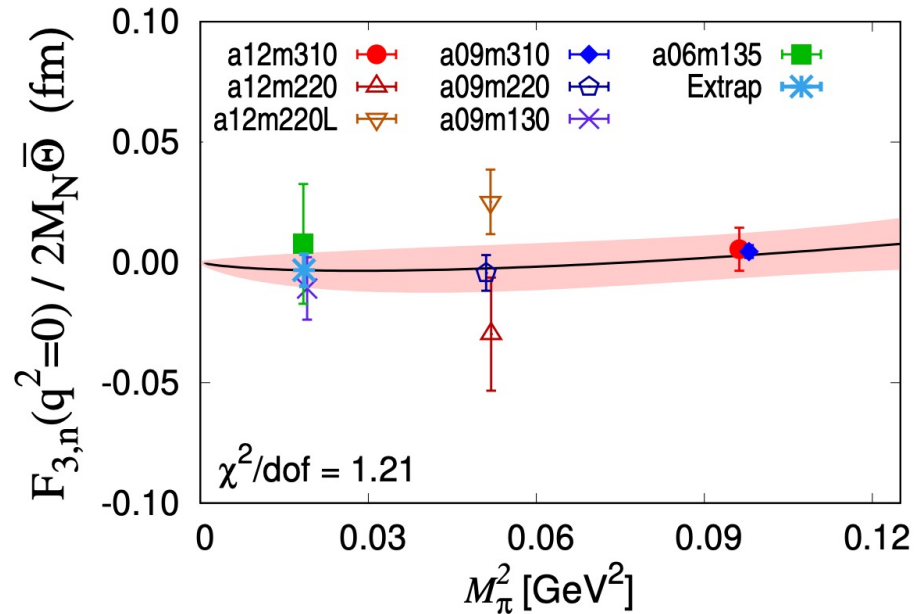
Vector Form Factor



Park, et al., PRD 105, 054505 (2022)

- Experiments studying electron scattering off protons and neutrons
- Electric & magnetic charge radius squared: $\langle r_E^2 \rangle^{u-d} = 0.85(12)(19)\text{fm}^2$, $\langle r_M^2 \rangle^{u-d} = 0.71(19)(23)\text{fm}^2$
- Magnetic moment: $\mu^{u-d} = 4.15(22)(10)$

Electric Dipole Moment from QCD Θ -term



Bhattacharya, et al., PRD 103, 114507 (2021)

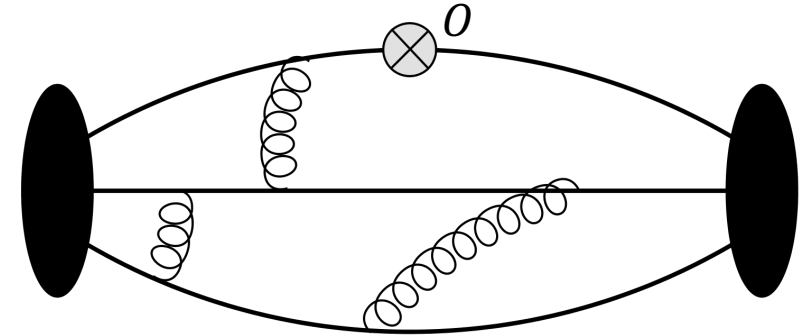
- Nucleon EDM from QCD Θ -term can be calculated using regular vector current three-point functions

$$\langle N | J_\mu^{EM} | N \rangle_\Theta \approx \langle N | J_\mu^{EM} | N \rangle_{\Theta=0} - i\Theta \left\langle N \left| J_\mu^{EM} \int d^4x \frac{G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a}{32\pi^2} \right| N \right\rangle_{\Theta=0}$$

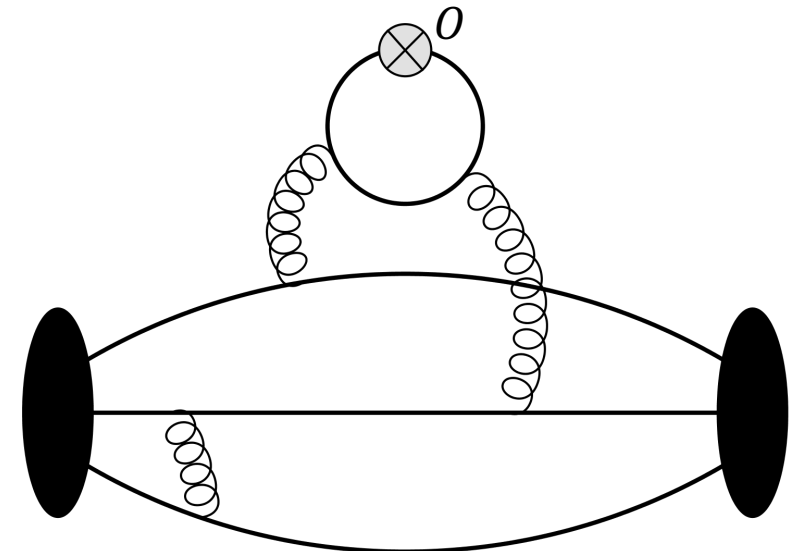
- Nonzero neutron electric dipole moment (nEDM) violates P and T, so CP
 \rightarrow nEDM is a sensitive probe of new sources of CP violation in BSM
- Necessary to understand the background SM Θ -term to the hadronic EDMs for any BSM analysis

Flavor-diagonal Charges

- $\langle N | \bar{q} \Gamma q | N \rangle$ where $q = u, d, s, \dots$
- Need to calculate computationally expensive quark-line disconnected diagrams
- Weak signal and noisy data
- Large excited-state effect
- Complicated renormalization

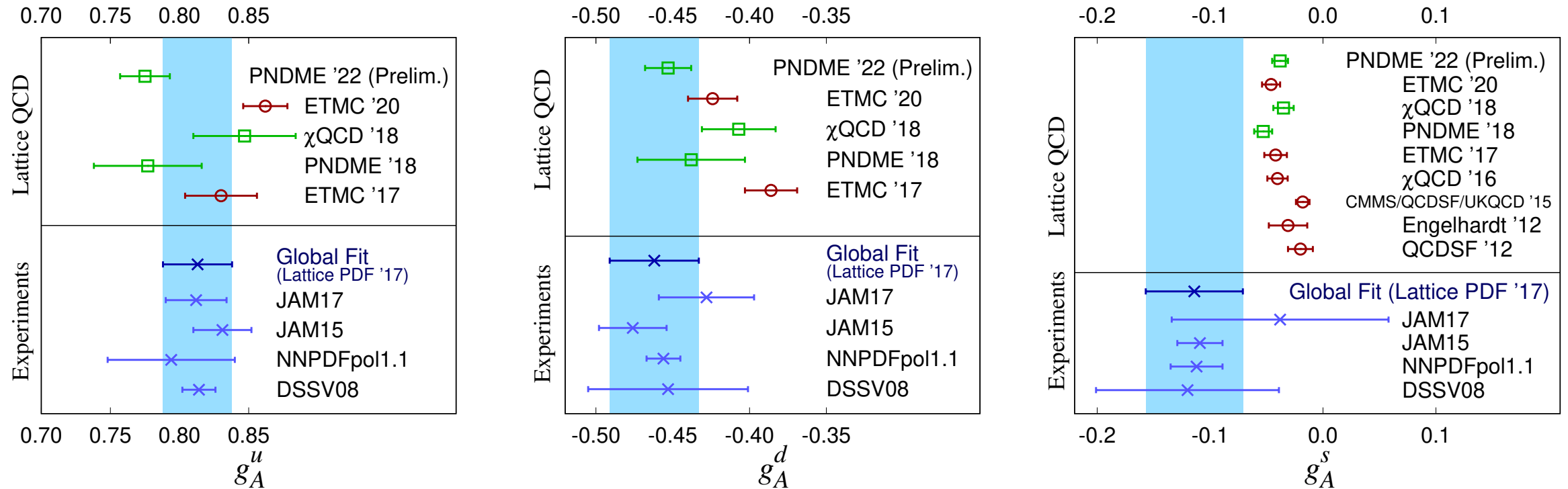


Quark-line connected diagram



Quark-line disconnected diagram

Flavor-diagonal Axial charges



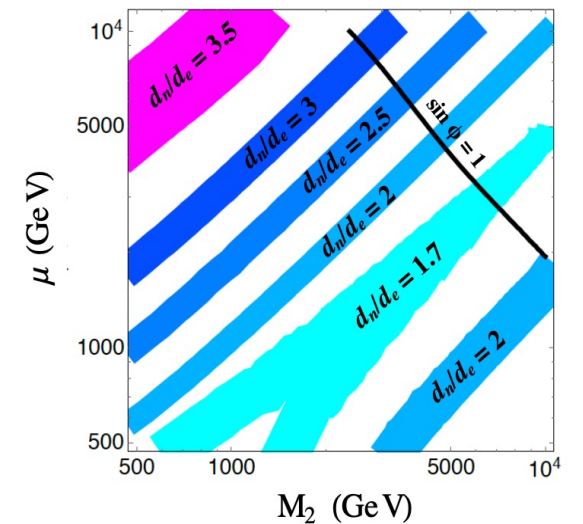
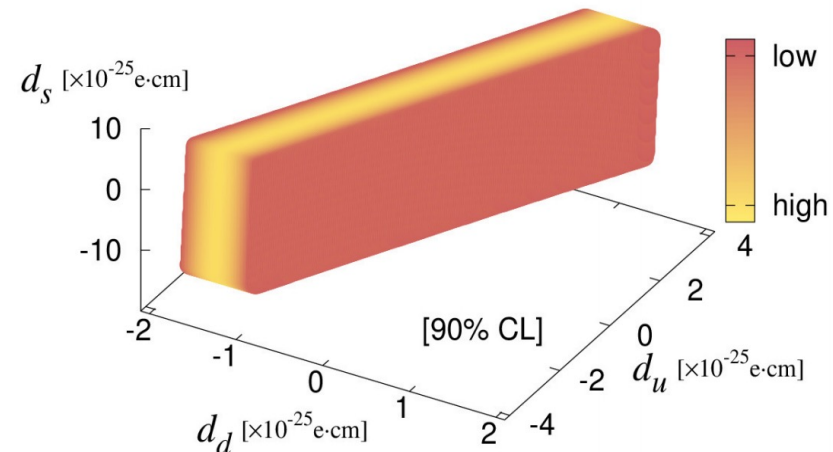
- Proton spin decomposition $\frac{1}{2} = \sum_{q=u,d,s,\dots} \left(\frac{1}{2} \Delta q + L_q \right) + J_g$. where $g_A^q \equiv \Delta q = \int_0^1 dx (\Delta q(x) + \Delta \bar{q}(x))$
- Only PNDME and χ QCD'18 data are extrapolated to continuum limit
- Charm-quark results (g_A^c) can be found at
 - PNDME: Rui Zhang's talk at Lattice 2021; ETMC: PRD 102, 054517 (2020)

Flavor-diagonal Tensor charges

- Describes quark EDM (qEDM) contribution to the neutron EDM (nEDM)
- For models in which qEDM is the dominant BSM source of CP violation

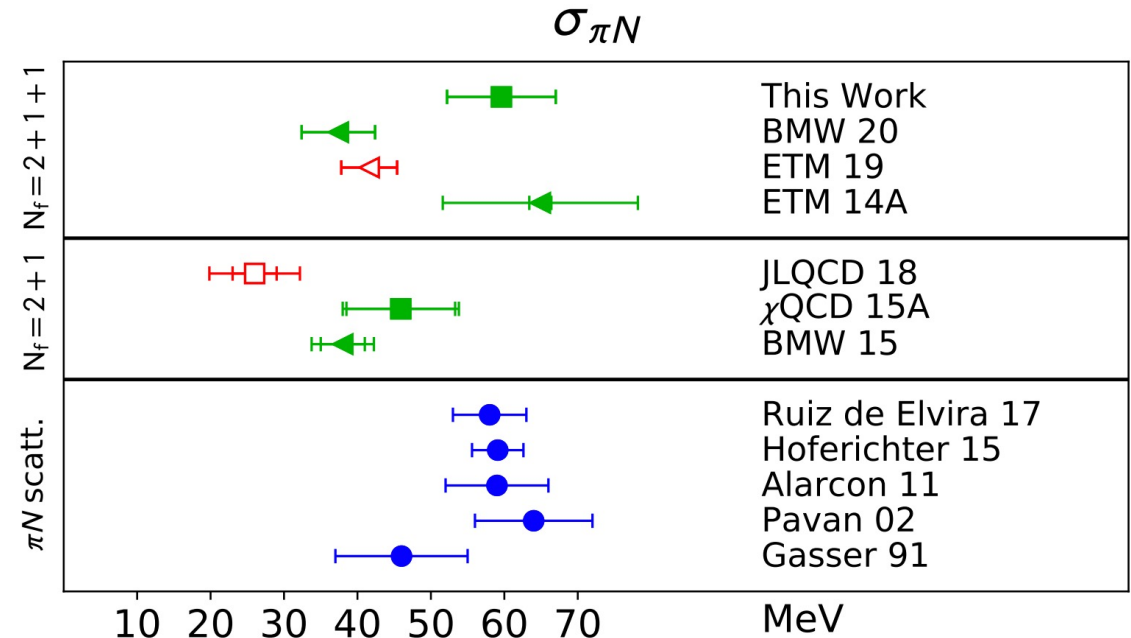
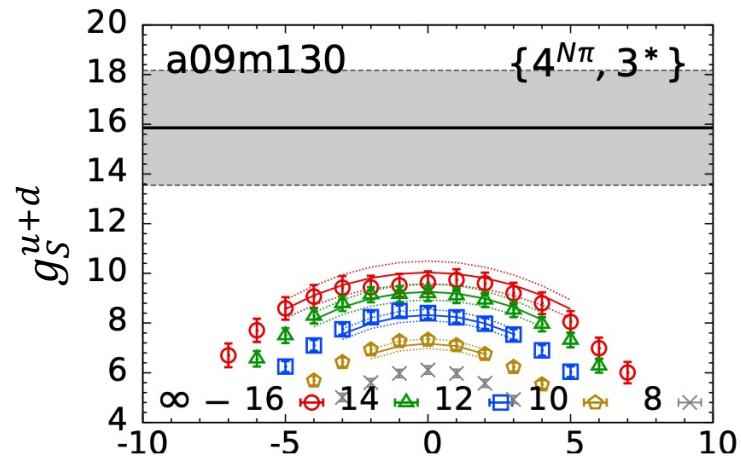
$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s + \dots$$
- Combined with current experimental bound of nEDM, it provides constraints on BSM parameters
- Strange quark tensor charge (g_T^s ; pure disconnected) is small but important in constraining BSM
- Charm-quark results (g_T^c) can be found at
 - PNDME: Rui Zhang's Lattice 2021 talk
 - ETMC: PRD 102, 054517 (2020)

	g_T^u	g_T^d	g_T^s
PNDME '22 (Prelim.)	0.775(22)	-0.208(12)	-0.0016(9)
ETMC '20	0.729(22)	-0.208(8)	-0.0027(6)
PNDME '18	0.784(28)(10)	-0.204(11)(10)	-0.0027(16)



PRL 115, 212002 (2015)
PRD 98, 091501 (2018)

Flavor-diagonal Scalar Charge



- Nucleon σ -terms

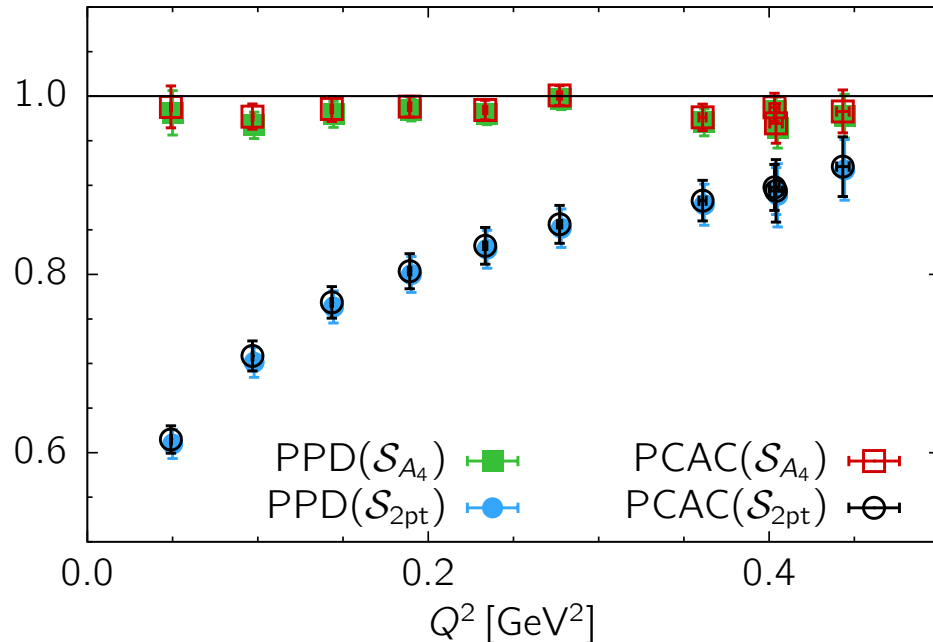
$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle = m_{ud} (g_S^u + g_S^d \equiv g_S^{u+d}), \quad \sigma_s = m_s \langle N | \bar{s}s | N \rangle = m_s g_S^s$$

- Large contributions from disconnected diagrams & excited-state (ES) effects
- **Critical to include $N\pi$ ES terms** to remove ES contamination

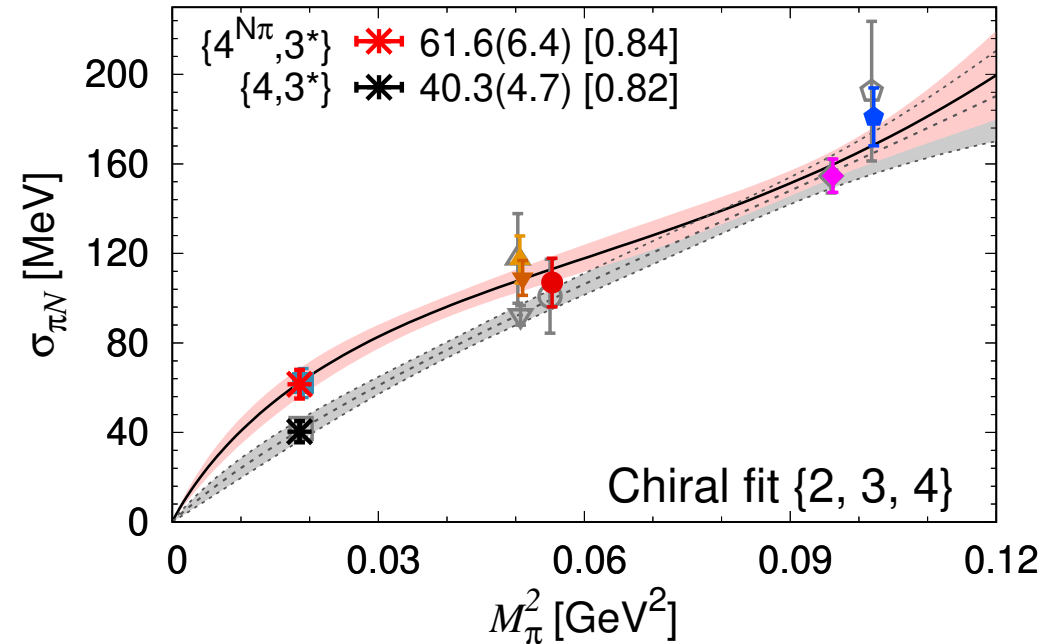
- Phenomenological estimates using πN -scattering data: $\sigma_{\pi N} \sim 60$ MeV
- Lattice results without $N\pi$ ES-terms: $\sigma_{\pi N} \sim 40$ MeV
- Lattice result with $N\pi$ ES-terms: $\sigma_{\pi N} = 59.6(7.4)$ MeV

Removing Excited-state Contamination

- Tower of possible nucleon excited states; which contribute significantly?
 - Radial excitations: $N(1440)$, $N(1710)$, ... & Multi-hadron states: $N(\mathbf{p})\pi(-\mathbf{p})$, $N(\mathbf{0})\pi(\mathbf{0})\pi(\mathbf{0})$, ...
- Excited state fits to C_{2pt} gives large first ES mass $M_1 \gtrsim M_{N(1440)}$
- But various evidences advocate $M_1 \ll M_{N(1440)}$
 - PCAC relation between G_A , \tilde{G}_P , and G_P is much better satisfied with smaller M_1
 - Nucleon σ -term results are consistent with χ PT when $M_1 \sim M_{N\pi, N\pi\pi}$



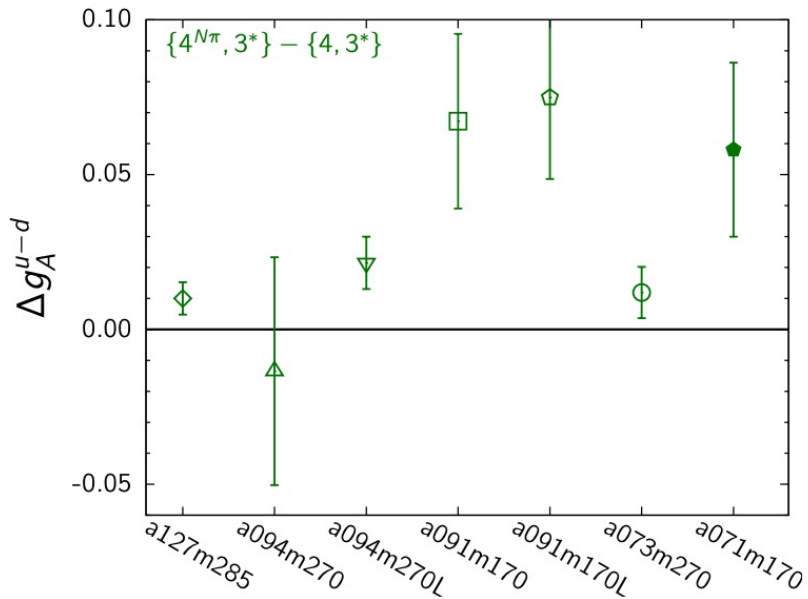
Jang, et al., PRL 124, 072002 (2020)



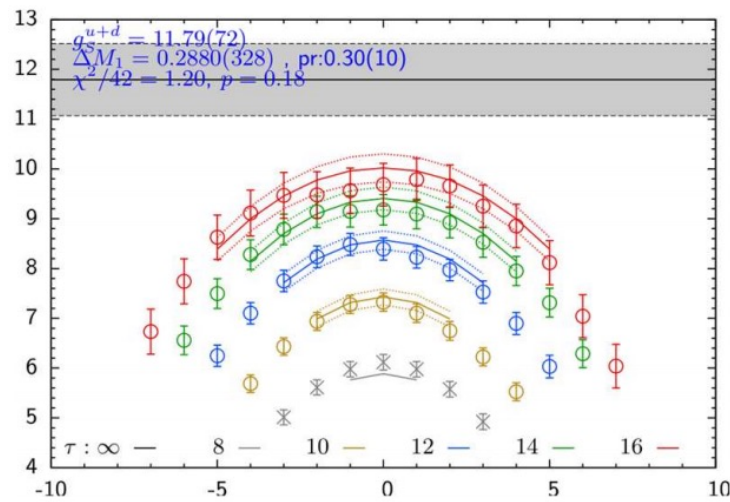
Gupta, et al., PRL 127, 242002 (2021)

Effect of $N\pi$ Excited States

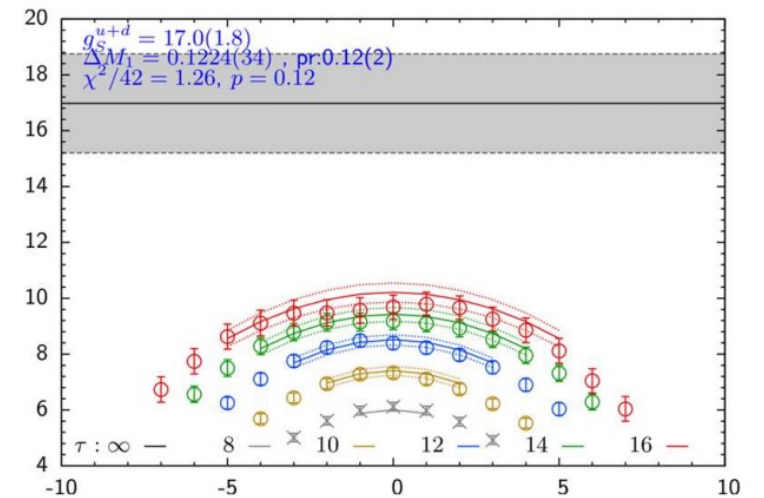
- Excited state fit results with and without $N\pi$ -state is huge:
 - Smaller $M_1 \rightarrow$ smaller mass gap $\Delta M_1 = M_1 - M_0 \rightarrow$ longer extrapolation in $t \rightarrow \infty, (\tau - t) \rightarrow \infty$ ($C_{3pt}^\Gamma(t, \tau)/C_{2pt}(\tau) = g_\Gamma^{u-d} + O(e^{-\Delta M_1 t}, e^{-\Delta M_1(\tau-t)}, e^{-\Delta M_1 \tau})$)
- But χ^2 of 3pt fits are not sensitive to the low-lying excited-state mass
- Larger effect with smaller pion mass



Park, et al., PRD 105, 054505 (2022)



Gupta, et al., PRL 127, 242002 (2021), Sungwoo Park's talk at Lattice 2021



Proposed Calculation of Disconnected

Ensemble ID	β	a (fm)	M_π (MeV)	L	T	$M_\pi L$	# of Confs.
a093m220	6.3	0.093	220	48	128	5.0	1000
a072m220	6.5	0.072	220	64	192	5.2	1000

- Published results with 7 ensembles; targeting full analysis of all 13 ensembles
- We request 26M Sky-core-hours of computer time to carry out [light- and strange-quark disconnected diagram calculations](#) on 1000 configurations of [a093m220](#) and [a072m220](#) ensembles

Summary

- Nucleon matrix elements play an important role in analysis of experimental data and probing new BSM physics
- Lattice QCD is providing precise estimates
- Removing excited state contamination requires evaluating the contribution of $N\pi$ excited states
- We request **300TB** of disk space to make 1000 configurations for each of the 13 ensembles available to USQCD under the standard non-competing arrangement
- We request **26M Sky-core-hours** of computer time to carry out light- and strange-quark disconnected diagram calculations on 1000 configurations on both a093m220 and a072m220 ensembles