QCD + QED studies

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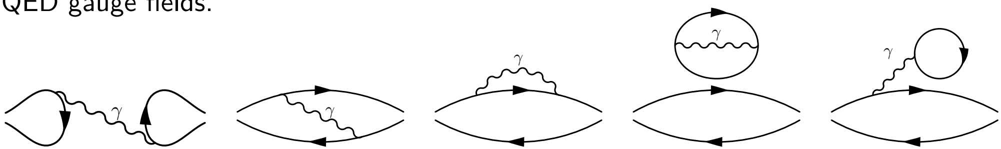
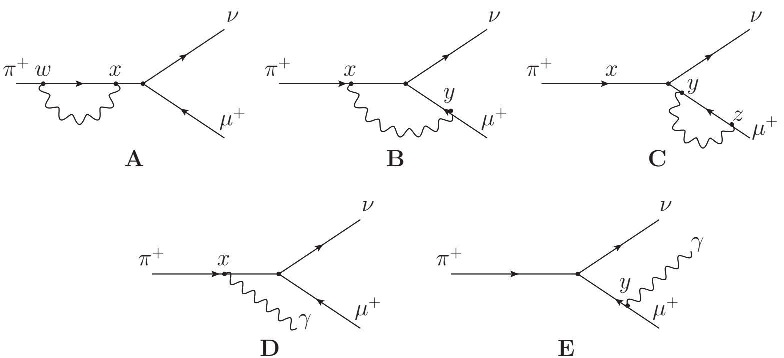
Outline

- Introduction
- QED correction to light meson mass and leptonic decay width
- Two-photon exchange contribution to the muonic-hydrogen Lamb shift
- Summary

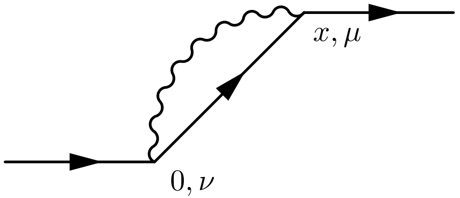
- Precision for some important observables, $m_{\pi/K}$, $f_{\pi/K}$, HVP, have been improving steadily.
- QCD + QED is needed for sub-percent accuracy. Already very relevant at present.
- Our overall strategy to include QED in lattice QCD calculations:
  Calculate the pure QCD matrix elements of local vector currents, which couple to the QED gauge fields.





Lattice QCD + QED and finite volume effects

- No massless particles in QCD → Finite volume effects for many observables are **exponentially suppressed** by the spatial lattice size L.
  - Mass of a stable particle M. Lüscher, Commun.Math.Phys. 104, 177-206 (1986)
- QED include massless photon → Use treatments similar to QCD for QED leads to **power-law suppressed** finite volume effects.
  - Mass of a stable particle in QED_L M. Hayakawa and S. Uno, Prog.Theor.Phys. (2008).



$$\Delta M(L) = \Delta M(\infty) - \frac{\kappa q^2}{8\pi}\frac{1}{L} - \frac{\kappa q^2}{4\pi m}\frac{1}{L^2} + \mathcal{O}\Big(\frac{1}{L^3}\Big)$$

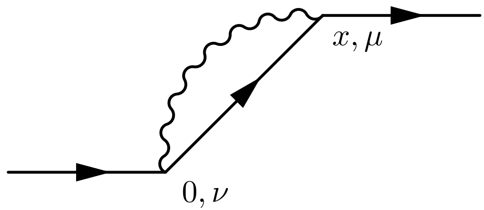
where $\kappa = 2.8372997\cdots$. S. Borsanyi et al., Science 347, 1452 (2015).

QED correction to hadron masses

$$\Delta M = \mathcal{I} = \frac{1}{2}\int d^4x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$$

$$\mathcal{H}_{\mu,\nu}(x) = \frac{1}{2M}\langle N | T J_{\mu}(x) J_{\nu}(0) | N \rangle$$

$$S^{\gamma}_{\mu,\nu}(x) = \frac{\delta_{\mu,\nu}}{4\pi^2 x^2}$$



- Evaluate the QED part, the photon propagator, in infinite volume.

- The hadronic function does not always fall exponentially in the long distance region
  $\rightarrow$ Separate the integral into two parts ($t_s \lesssim L$):

$$\Delta M = \mathcal{I} = \mathcal{I}^{(s)} + \mathcal{I}^{(l)}$$

$$\mathcal{I}^{(s)} = \frac{1}{2}\int_{-t_s}^{t_s} dt \int d^3x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$$

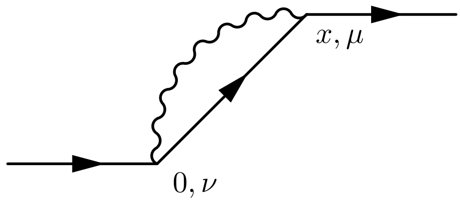
$$\mathcal{I}^{(l)} = \int_{t_s}^{\infty} dt \int d^3x\, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$$

- For the short distance part, $\mathcal{I}^{(s)}$ can be directly calculated on a finite volume lattice:

$$\mathcal{I}^{(s)} \approx \mathcal{I}^{(s,L)} = \frac{1}{2}\int_{-t_s}^{t_s} dt \int_{L/2}^{L/2} d^3x\, \mathcal{H}^{L}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$$

- For the **long distance part**, $\mathcal{I}^{(l)}$, a different treatment is required.

The infinite volume reconstruction (IVR) method



- For the long distance part, we can evaluate $\mathcal{H}_{\mu,\nu}(x)$ **indirectly** in the **infinite volume**.

$$\mathcal{I}^{(l)} = \int_{t_s}^{\infty} dt \int d^3x \, \mathcal{H}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$$

- Note that when t is large ($t > t_s$), the intermediate states between the two currents are dominated by the single particle states (possibly with small momentum). Therefore:

$$\mathcal{H}_{\mu,\nu}(x) \approx \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{2E_{\vec{p}}} \frac{1}{2M} \langle N|J_\mu(0)|N(\vec{p})\rangle \langle N(\vec{p})|J_\nu(0)|N\rangle \right] e^{i\vec{p}\cdot\vec{x}-(E_{\vec{p}}-M)t}$$

  - We only need to calculate the form factors: $\langle N(\vec{p})|J_\nu(0)|N\rangle$!
  - Values for all $\vec{p}$ are needed. Inversely Fourier transform the above relation **at** t_s!

$$\int d^3x \, \mathcal{H}_{\mu,\nu}(t_s, \vec{x}) e^{-i\vec{p}\cdot\vec{x}+(E_{\vec{p}}-M)t_s} \approx \frac{1}{2E_{\vec{p}}} \frac{1}{2M} \langle N|J_\mu(0)|N(\vec{p})\rangle \langle N(\vec{p})|J_\nu(0)|N\rangle$$

Master formula for QED correction to hadron masses

- The final expression for QED correction to hadron mass is split into two parts:

$$\Delta M = \mathcal{I} = \mathcal{I}^{(s)} + \mathcal{I}^{(l)}$$

- For the short distance part: $\mathcal{I}^{(s)} \approx \mathcal{I}^{(s,L)} = \frac{1}{2}\int_{-t_s}^{t_s} dt \int_{L/2}^{L/2} d^3x\, \mathcal{H}^{L}_{\mu,\nu}(x) S^{\gamma}_{\mu,\nu}(x)$

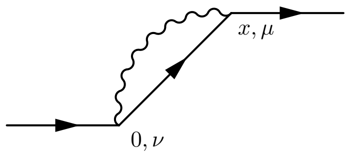
- For the long distance part: $\mathcal{I}^{(l)} \approx \mathcal{I}^{(l,L)} = \int_{-L/2}^{L/2} d^3x\, \mathcal{H}^{L}_{\mu,\nu}(t_s,\vec{x}) L_{\mu,\nu}(t_s,\vec{x})$

- For Feynman gauge:

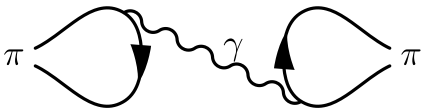
$$S^{\gamma}_{\mu,\nu}(x) = \frac{\delta_{\mu,\nu}}{4\pi^2 x^2} \qquad L_{\mu,\nu}(t_s,\vec{x}) = \frac{\delta_{\mu,\nu}}{2\pi^2}\int_0^{\infty} dp \frac{\sin(p|\vec{x}|)}{2(p+E_p-M)|\vec{x}|} e^{-pt_s}$$

- Only use $\mathcal{H}^{L}_{\mu,\nu}(t,\vec{x})$ within $-t_s \le t \le t_s$.

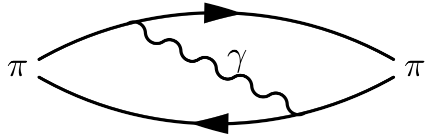
- Choose $t_s = L/2$, **finite volume errors and the ignored excited states contribution to $\mathcal{I}^{(l)}$ are both exponentially suppressed by the spatial lattice size L.**



Pion mass splitting: $\Delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$



Disconnected diagram



Connected diagram

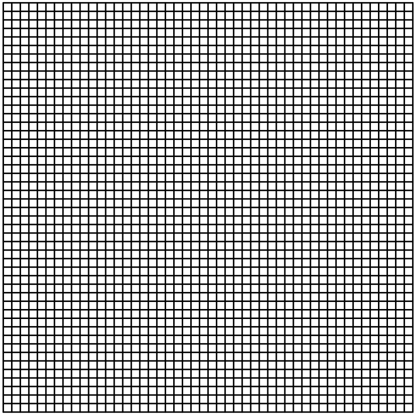
- Coulomb gauge fixed wall sources are used to interpolate the pion interpolating operators.
- Fixed time separation between the vector current operator and the closest pion interpolating operators: $t_{\text{sep}} \approx 1.5\text{fm}$.

$$\mathcal{H}^L_{\mu,\nu}(t,\vec{x}) = L^3 \frac{\langle \pi(t+t_{\text{sep}}) J_\mu(t,\vec{x}) J_\nu(0) \pi^\dagger(-t_{\text{sep}})\rangle_L}{\langle \pi(t+t_{\text{sep}})\pi^\dagger(-t_{\text{sep}})\rangle_L^{[*]}}$$

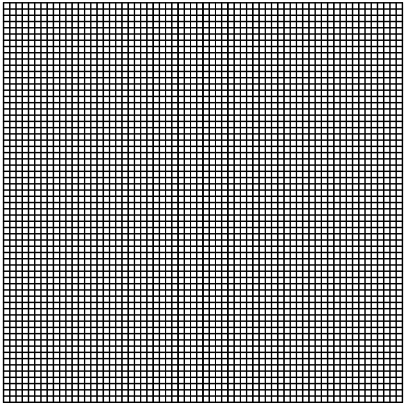
- Diagrams are similar to the $\pi^- \to \pi^+ ee$ neutrinoless double beta $(0\nu 2\beta)$ decay. D. Murphy and W. Detmold (2018), Tuo, Feng, and Jin (2019)
- At $\mathcal{O}(\alpha_{\text{QED}}, (m_u - m_d)/\Lambda_{\text{QCD}})$, all UV divergence are canceled. The two diagrams are the only diagrams contributing to $m_{\pi^\pm} - m_{\pi^0}$. RM123 (2013)
- In particular, the pion mass splitting at leading order does not depend on $m_u - m_d$.

[*]: Around the world effects corrected two point correlation function.

48I



64I



- Domain wall fermion action (preserves Chiral symmetry, no $\mathcal{O}(a)$ lattice artifacts).
- Iwasaki gauge action.
- $M_\pi = 135$ MeV *, $L = 5.5$ fm box, $1/a_{48I} = 1.73$ GeV, $1/a_{64I} = 2.359$ GeV.

*: Valence pion mass. Slightly different from the 139 MeV unitary pion mass used in the ensemble generation.

RBC/UKQCD, PRD [arXiv:1411.7017]

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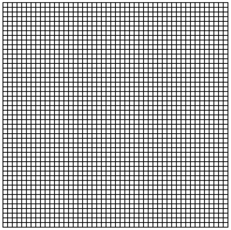

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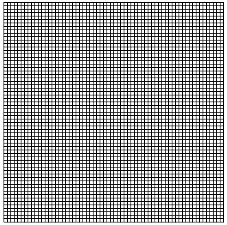
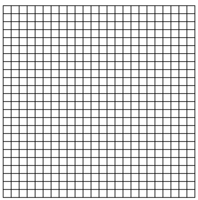
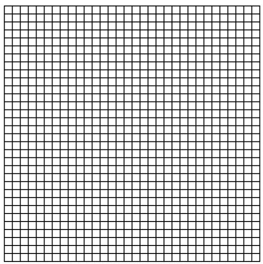


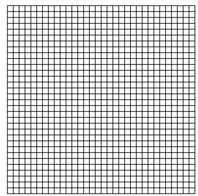
24D



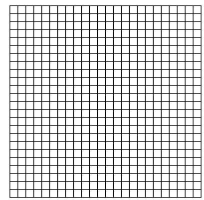
32D



32Dfine



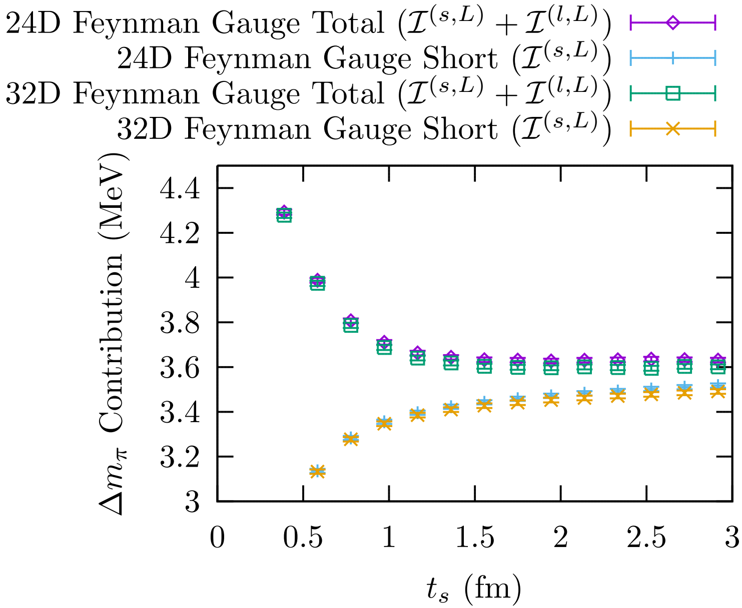
24DH



- For 24D, 32D, 32Dfine, $M_\pi \approx 140$ MeV
- For 24DH, $M_\pi \approx 340$ MeV

RBC/UKQCD, PRD [arXiv:1411.7017]

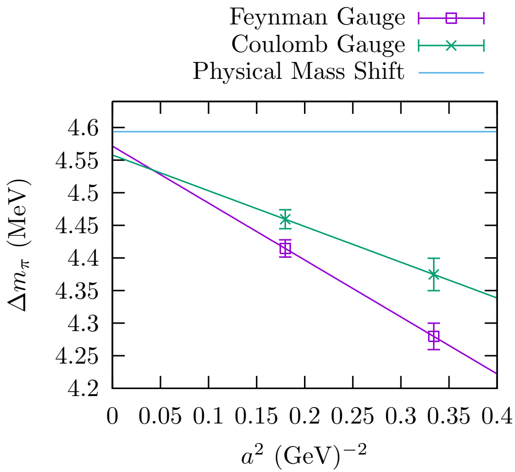
Finite volume effects and t_s dependence of Δm_π



- The difference between 32D and 24D is $-0.035(16)$MeV. This is consistent with a scalar QED calculation, which yields -0.022MeV.

Feng, Jin, and **Michael Riberdy**, [arXiv:2108.05311]

Continuum extrapolation of Δm_π

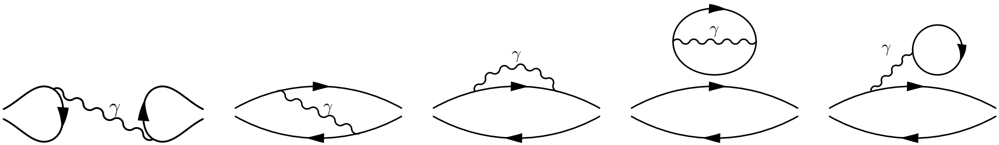


	Disc (MeV)	Conn (MeV)	Total (MeV)
Feyn	0.051(9)(22)	4.483(40)(28)	4.534(42)(43)
Coul	0.052(2)(13)	4.508(46)(42)	4.560(46)(41)
Coul-t	0.018(1)(4)	1.840(22)(39)	1.858(22)(41)

Finite volume corrections calculated with the difference of the 32D and 24D ensembles are already included.

Feng, Jin, and **Michael Riberdy**, [arXiv:2108.05311]

Hadronic matrix elements - meson mass



Done:	π^0	π	π, K	π, K (noisy)	
To do:		K			π, K

Other applications using these matrix elements:

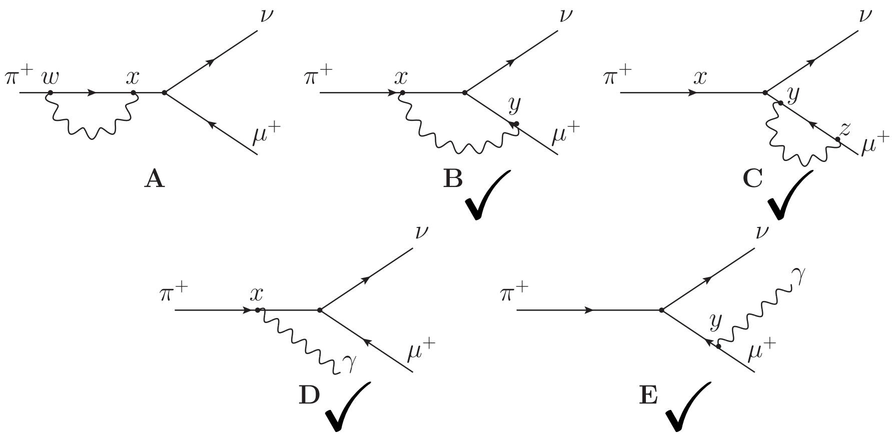
- Pion electric polarizabilities.

  Feng, Golterman, Izubuchi, Jin, [arXiv:2201.01396]

- The γW-box contribution to the pion β decay.

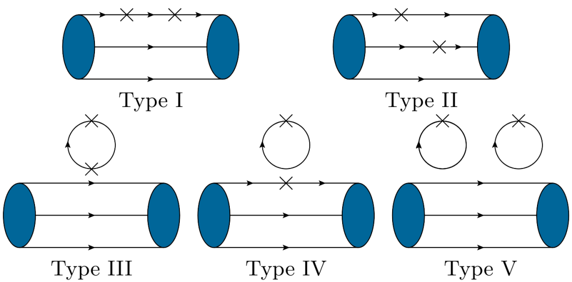
  Feng, Gorchtein, Jin, Ma and Seng [arXiv:2003.09798]

  Ma, Feng, Gorchtein, Jin and Seng [arXiv:2102.12048]



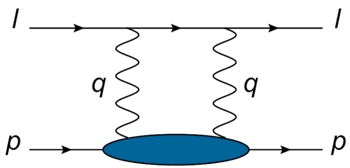
- Diagram B and D share the same matrix elements.
- Diagram C and E only depends the pure QCD decay constants.
- Diagram A is currently being worked on.
- Plan to use the IVR method to calculate the QED corrections to the meson leptonic decay width, which is needed to determine the CKM matrix elements $|V_{us}|$.

Hadronic matrix elements - baryon mass



Application of these matrix elements:

- Two-photon exchange contribution to the muonic-hydrogen Lamb shift.
  Yang Fu, Feng, Jin and Lu, [arXiv:2202.01472]
- γW-box contribution to neutron β decay.
- Proton and neutron polarizabilities.



$$\begin{aligned} H_1(\vec{x}, t) &= \langle p | \, \mathrm{T}[j_0(\vec{x}, t) j_0(0)] | p \rangle, \\ H_2(\vec{x}, t) &= \langle p | \, \mathrm{T}[\vec{j}(\vec{x}, t) \cdot \vec{j}(0)] | p \rangle. \end{aligned}$$

$$\Delta E_{\mathrm{TPE}} = -\sum_{i=1,2} \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}^3\vec{x} \, w_i(\vec{x}, t) H_i(\vec{x}, t) - C$$

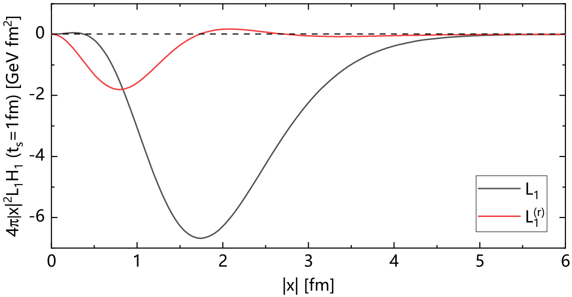
where C comes from the IR divergence subtraction, which depends on the proton charge radius.

Introduce t_s sufficiently large for ground-state dominance:

(single nucleon state dominance, non-zero momentum states are allowed)

$$\Delta E_{\mathrm{TPE}} = -\sum_{i=1,2} \int_{-t_s}^{t_s} \mathrm{d}t \int \mathrm{d}^3\vec{x} \, w_i(\vec{x}, t) H_i(\vec{x}, t) - \sum_{i=1,2} \int \mathrm{d}^3\vec{x} \, L_i(\vec{x}, t_s) H_i(\vec{x}, t_s) - C$$

Yang Fu, Feng, Jin and Lu, [arXiv:2202.01472]

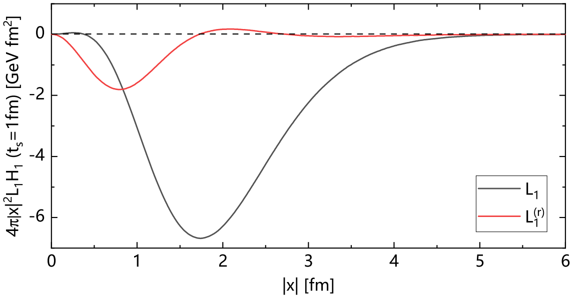


$$L_1^{(r)}(\vec{x}, t_s) = L_1(\vec{x}, t_s) - c_0 L_0(\vec{x}, t_s) - c_r L_r(\vec{x}, t_s).$$

$$L_0(\vec{x}, t_s) = \frac{1}{2M}, \quad L_r(\vec{x}, t_s) = \frac{1}{4M}\left(\vec{x}^2 - \frac{3+6Mt_s}{2M^2}\right).$$

$$G_E^2(0) = \int \mathrm{d}^3\vec{x}\, L_0(\vec{x}, t_s) H_1(\vec{x}, t_s), \quad \langle r_p^2 \rangle = \int \mathrm{d}^3\vec{x}\, L_r(\vec{x}, t_s) H_1(\vec{x}, t_s),$$

Yang Fu, Feng, Jin and Lu, [arXiv:2202.01472]



$$L_1^{(r)}(\vec{x}, t_s) = L_1(\vec{x}, t_s) - c_0 L_0(\vec{x}, t_s) - c_r L_r(\vec{x}, t_s).$$

The subtraction coefficients c_0, c_r are obtained by minimizing

$$I(c_0, c_r) = \int_{R_{\text{min}}}^{R_{\text{max}}} \mathrm{d}|\vec{x}| \; \left(4\pi|\vec{x}|^2\right) \left|L_1^{(r)}(\vec{x}, t_s)\right|^2.$$

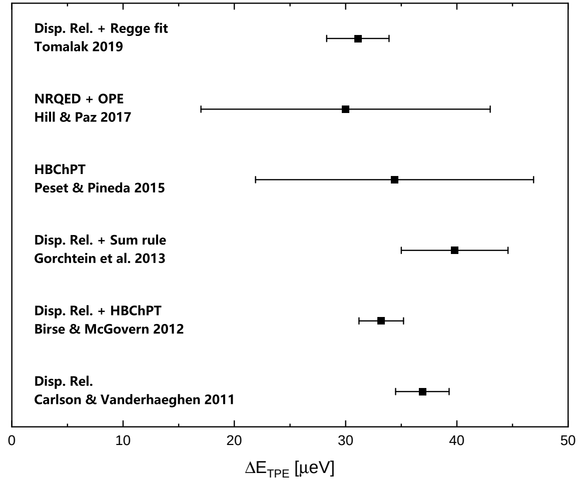
where $R_{\text{min}} = 1$ fm and $R_{\text{max}} = 3$ fm. Use $L_1^{(r)}(\vec{x}, t_s)$ instead of $L_1(\vec{x}, t_s)$ to obtain ΔE_{lat}

$$\Delta E_{\text{TPE}} = 0.77\mu\text{eV} + 93.72\mu\text{eV} \cdot \langle r_p^2 \rangle/\text{fm}^2 - \Delta E_{\text{lat}}.$$

Yang Fu, Feng, Jin and Lu, [arXiv:2202.01472]

Lattice calculation on the 24D ensemble ($M_\pi = 140$ MeV, $a^{-1} = 1.015$ GeV).

$$\begin{aligned}\Delta E_{\text{TPE}} &= 0.77\mu\text{eV} + 93.72\mu\text{eV}\cdot\langle r_p^2\rangle/\text{fm}^2 - \Delta E_{\text{lat}} \\ &= 0.77\mu\text{eV} + 93.72\mu\text{eV}\cdot\langle r_p^2\rangle/\text{fm}^2 - 29.7(4.9)\mu\text{eV} \\ &= 37.4(4.9)\mu\text{eV}\end{aligned}$$



Yang Fu, Feng, Jin and Lu, [arXiv:2202.01472]

Summary

- Pion mass splitting obtained $m_{\pi^\pm} - m_{\pi^0} = 4.534(42)(43)\text{MeV}$ (Feynman gauge), in good agreement with the experimental value $4.5936(5)\text{MeV}$.
  Feng, Jin, and **Michael Riberdy**, [arXiv:2108.05311]
- Two-photon exchange contribution to the muonic-hydrogen Lamb shift calculated $\Delta E_{\text{TPE}} = 37.4(4.9)\mu\text{eV}$ on 24D ensemble ($M_\pi = 140$ MeV, $a^{-1} = 1.015$ GeV).
  Yang Fu, Feng, Jin and Lu, [arXiv:2202.01472]
- Also working on other applications: polarizabilities, γW-box contribution to β decays, etc.
- Calculations of QED corrections to kaon mass and leptonic decay are in progress.

Thank You!