From walking and emergent dilatons to α_S in QCD at the Z-pole

Chik Him (Ricky) Wong

on behalf of
Lattice Higgs Collaboration (LatHC)

AHM 2022

Outline

- Our proposed research plan consists of two parts.
    - High-precision determination of $\alpha_S(m_Z)$ at the Z-boson pole in $N_f = 3$ massless QCD with β-function computed using gradient flow of lattice gauge field in applications of the scale-dependent flow of the renormalization group
    - RMT analysis in the ε-regime in the context of dilaton effective theory (dEFT) of near-conformal models
- This talk focuses on the first part of the plan. The methodology will be discussed and new results on $N_f = 0$ and $N_f = 10$ will be shown to demonstrate its challenges and feasibility. They serve as pilot tests towards the application on $N_f = 3$ QCD

Determination of $\alpha_S(m_Z)$

- Goal: High precision determination of the strong coupling $\alpha_S(m_Z)$ at the Z-pole in QCD with $N_f = 3$ massless fermions.
- The determination of $\alpha_S(m_Z)$, equivalently $r_0\Lambda_{\overline{MS}}$, requires integration of the inverse β-function up to the scale of hadron physics at strong coupling.

$$
\begin{aligned}
r_0 \cdot \Lambda_{\overline{MS}} =& (b_0\bar{g}^2)^{-b_1/2b_0^2} \\
&\cdot \exp(-1/2b_0\bar{g}^2) \cdot \exp\left(- \int_0^{\bar{g}} dx[1/\beta(x) + 1/b_0x^3 - b_1/b_0^2 x] \right)
\end{aligned}
$$

- The integration beyond the perturbative regime requires non-perturbative calculation of the β-function, which can be done on lattice

Determination of $\alpha_S(m_Z)$

- There are many ways to do such calculations [FLAG 2021]. One approach is to utilize step β-functions defined on lattices of finite physical volumes in Schrodinger functional and gradient flow schemes, matching at different scale regimes.
- We propose to define a β-function over infinite physical volume as response to infinitesimal RG scale changes, using only gradient flow on the lattice. It is expected to be feasible for the entire nonperturbative integration range without matchings.

Determination of $\alpha_S(m_Z)$

- $g^2(t)$ is defined in infinite volume as a function of continuous gradient flow time t.
- Discretization schemes of RG flow with different combinations of action, flow and energy operator ($\propto g^2$) on the lattice: WSC,WSS,SSC,SSS
- The β-function is obtained from $g^2(t)$: $\beta(g^2(t)) = t \cdot dg^2/dt$ (note the sign convention inherited from BSM studies)
- dg^2/dt is approximated by five-point stencil in t

  $$[-g^2(t+2\varepsilon)+8g^2(t+\varepsilon)-8g^2(t-\varepsilon)+g^2(t-2\varepsilon)]/(12\varepsilon) = dg^2/dt + O(\varepsilon^4)$$

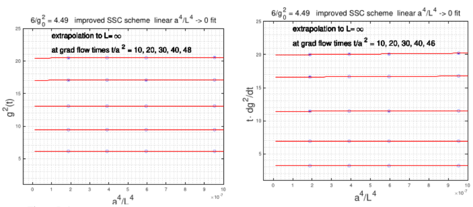
  (cross-checked by other approximations such as spline-based derivative and modified Akima interpolation method)
- An original approach was introduced in our previous study of the near-conformal $N_f = 2$ sextet model reaching the chiral limit from small fermion mass (m) deformations.[Fodor et al, 2018]. The approach was to match finite-volume step functions at massless theory with infinite-volume β-function in the chiral limit of fermion mass deformations from spontaneous χSB phase.

Determination of $\alpha_S(m_Z)$

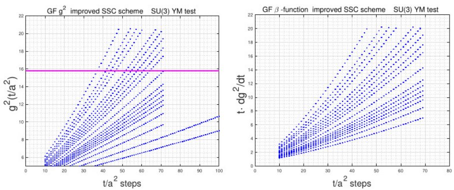
- Alternative approach: (variant 1)
  - $g^2(t/a)$ and $\beta \equiv t \cdot dg^2/dt$ are measured on ensembles in a set of $\{6/g_0^2,\ L/a\}$ generated at $m = 0$.
  - At each $\{6/g_0^2\}$, the infinite-volume limit ($a^4/L^4 \to 0$) of $\{g^2, \beta\}$ as functions of t/a^2 is taken (denoted here by $\{g_\infty^2(t/a^2), \beta_\infty(t/a^2)\}$)
  - A set of target values of infinite-volume continuum coupling $\{g^2\}$ within the range of integration is chosen. At each target value of g^2 , the corresponding $\{\beta_\infty, t/a^2\}|_{g_\infty^2 = g^2}$ is obtained by interpolating in t/a^2 in each $\{6/g_0^2\}$
  - At each g^2, the continuum limit $t/a^2 \to 0$ of β_∞ is taken.
- The proposed plan and the following tests are done on symanzik-improved gauge action with 4-step $\rho = 0.15$ stout-smeared staggered fermions in periodic boundary conditions for gauge links and anti-periodic boundary conditions for fermions.

Test on $N_f = 0$ (in collaboration with S. Borsanyi)

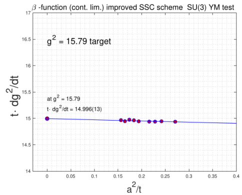
- $L/a = \{32, 36, 40, 48, 64\}$, $6/g_0^2 = \{4$ to $8.6\}$



⇒

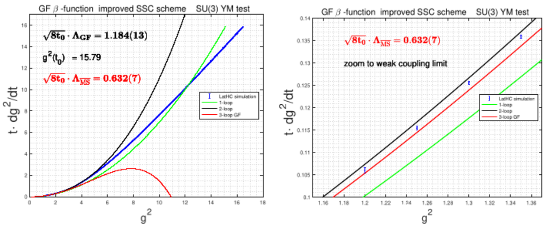


⇒



⇒

Test on $N_f = 0$



$$\sqrt{8t_0} \cdot \Lambda_{GF} = (b_0\bar{g}^2)^{-b_1/2b_0^2} \cdot \exp(-1/2b_0\bar{g}^2) \cdot \exp\left(-\int_0^{\bar{g}} dx[-1/\beta(x) + 1/b_0x^3 - b_1/b_0^2x]\right)$$

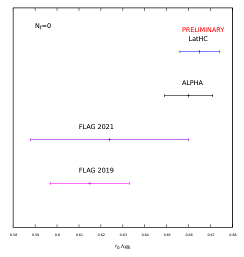
- Integration broken into two parts ($\bar{g}^2(t_0) = 15.79$) :
  - $x^2 = 0$ to 1.2: three-loop value of β-function
  - $x^2 = 1.2$ to $\bar{g}^2(t_0)$: numerical integration based on spline fit to the data
- $\Lambda_{GF}/\Lambda_{\overline{MS}} = 1.873$[Harlander et al, 2016; Artz et al, 2019]

Test on $N_f = 0$

- Our result has high accuracy comparable with the work[Dalla Brida et al, 2019] of ALPHA collaboration using step function, also agree in values
- LatHC: $\sqrt{8t_0} \cdot \Lambda_{\overline{MS}} = 0.632(7)$ (ALPHA: $0.6227(98)$)
- Our results are preliminary. We aim at demonstrating
  - consistency across different discretization schemes of the RG flow
  - that our analysis properly took into account the well-known topological issues of the lattice ensembles.

Test on $N_f = 0$

- Using $\sqrt{8t_0}/r_0 = 0.948(7)$[Luescher, 2014], our result converts into: $r_0 \cdot \Lambda_{\overline{MS}} = 0.665(9)$ (ALPHA, converted differently: $0.660(11)$)
- World average significantly lower:
  FLAG 2019: $r_0 \cdot \Lambda_{\overline{MS}} = 0.615(18)$
  FLAG 2021 (including ALPHA's result): $r_0 \cdot \Lambda_{\overline{MS}} = 0.624(36)$



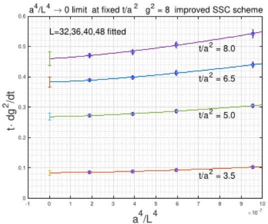
- This recent unresolved issue in the $N_f = 0$ model add incentive for an independent determination of $r_0 \cdot \Lambda_{\overline{MS}}$ in $N_f = 3$ QCD

Determination of $\alpha_S(m_Z)$

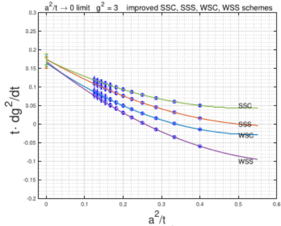
- A dense scan of $6/g_0^2$ is costly for fermionic simulations $\Rightarrow$
- Alternative approach: (variant 2)
  - $g^2(t/a)$ and $\beta \equiv t \cdot dg^2/dt$ are measured on ensembles in a set of $\{6/g_0^2,\ L/a\}$ generated at $m = 0$.
  - A set of target values of infinite-volume continuum coupling $\{g^2\}$ within the range of integration is chosen. At each target value of g^2, a set of target values of $\{t/a^2\}|_{g^2}$ is chosen. β is obtained for each combination of $\{g^2, t/a^2, L/a\}$ by interpolating among measurements in $\{6/g_0^2\}$
  - At each t/a^2, the infinite-volume limit ($a^4/L^4 \to 0$) of β is taken while holding g^2 and t/a^2 fixed. (denoted here by $\beta_\infty(t/a^2)$)
  - At each g^2, the continuum limit $t/a^2 \to 0$ of β_∞ is taken.
- This approach was first tested earlier in the $N_f = 2$ QCD [Hasenfratz et al] and $N_f = 12$ models [Fodor et al]

Test on $N_f = 10$

- $L/a = \{32, 36, 40, 48\}$, $6/g_0^2 = \{2.6, 2.7, \ldots, 4.1, 4.5, 5.0, 6.0 \ldots 8.0\}$
- For each $\{L/a, 6/g_0^2\}$, β at each target $\{g^2, t/a^2\}$ is obtained with fourth order polynomial interpolation



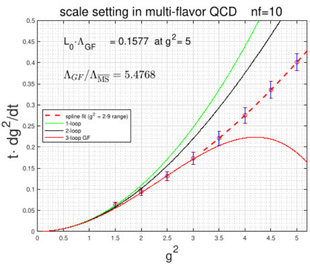
$\Rightarrow$



$\Rightarrow$

Test on $N_f = 10$

- Integration broken into two parts (we set $\bar{g}^2(t_0) = 5$ here) :
  - $x^2 = 0$ to 3: three-loop value of β-function
  - $x^2 = 3$ to $\bar{g}^2(t_0)$: numerical integration based on spline fit to the data
- $\Lambda_{GF}/\Lambda_{\overline{MS}} = 5.4768$ [Harlander et al, 2016; Artz et al, 2019]
- We successfully expressed $\Lambda_{\overline{MS}}$ of the theory in terms of a scale set implicitly at flow time t_0 . This holds promise for the challenge to apply the method to the strong coupling for $N_f = 3$ QCD.
- Proposed Plan : Apply one or both of the variants of the method on $N_f = 3$ QCD to determine $r_0 \cdot \Lambda_{\overline{MS}}$ at high accuracy



$$\sqrt{8t_0} \cdot \Lambda_{GF} = (b_0 \bar{g}^2)^{-b_1/2b_0^2}$$
$$\cdot \exp(-1/2b_0\bar{g}^2) \cdot \exp\left(-\int_0^{\bar{g}} dx[-1/\beta(x) + 1/b_0 x^3 - b_1/b_0^2 x]\right)$$

Study of dilaton in near-conformal models

- In the framework of dilaton effective field theories of near-conformal gauge theories, a parametric form of the dilaton potential interpolating between model choices, and the suggested taste breaking effects is recently proposed[Golterman& Shamir 2016,2017; Appelquist et al 2017,2018; Golterman; Shamir & Neil 2020]
- We investigated this in the *p*-regime on the sextet model and the result is not sensitive enough to determine the dilaton potential in such framework. Therefore we propose to investigate the predictions of dEFT, in particular the suggested taste-breaking effects, in the ε-regime using RMT analysis.