Optics of magnetic spectrometers and Beamline

1. A primer on charged particle beam optics
2. Optics of the spectrometer magnets
3. Dispersed beam on target?

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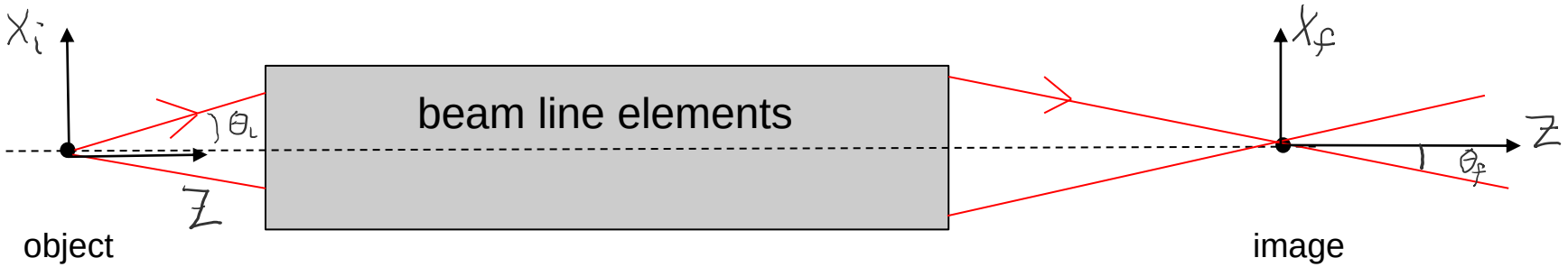
1. A primer on charged-particle beam optics

   - an intro to the subject, for newbies
   - to establish the notation I will use, for everyone

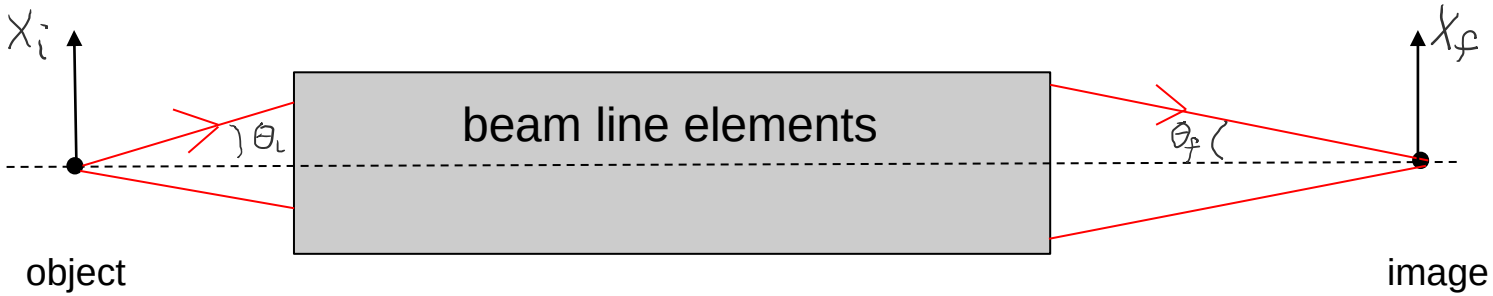
Suppose we have a set of beam line elements (quadrupole magnets, dipole magnets, solenoids, empty drift spaces) which transport a set of charged particles from some initial point “i” to a final point “f”.
This is analogous to a system of lenses transporting rays of light from an object to an image.

In two dimensions, we can define longitudinal position z and transverse position x and angle $\theta = \frac{dx}{dz}$

(small angle approximation where tan θ ~ θ)

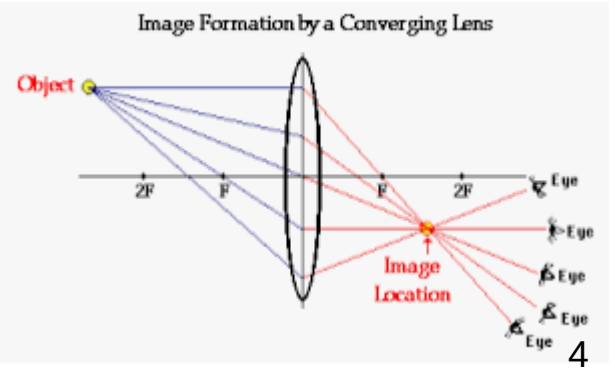


"Point-to-point focus" means that all particles emerging from a point source will intersect the target at the same final position x_f, regardless of the initial angle θ_i of the particle.



Point-to-point focus means $\frac{dx_f}{d\theta_i} = 0$

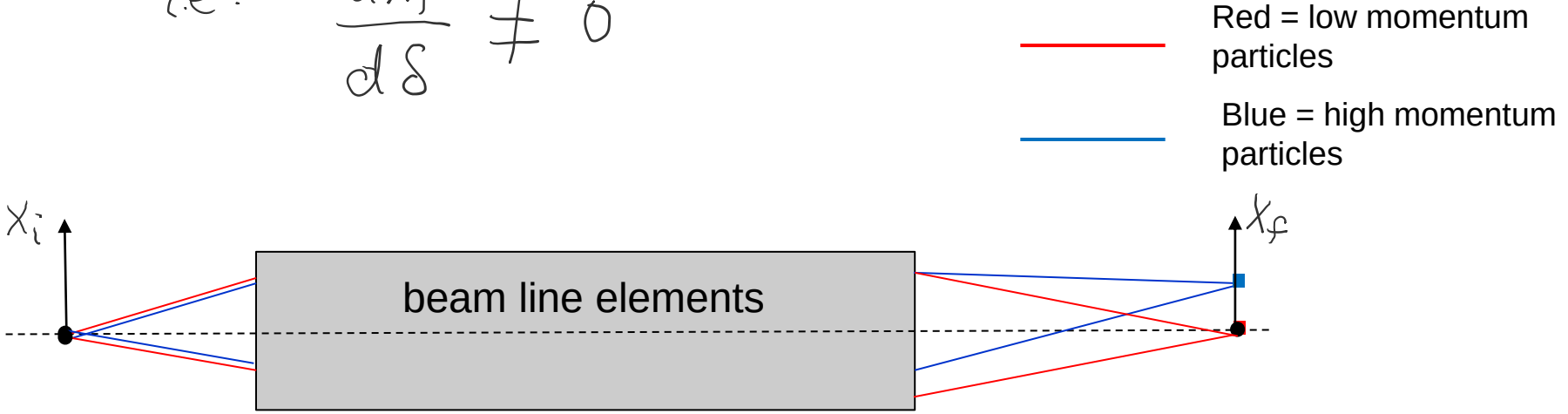
Exactly analogous to light rays through a glass lens from an object to an image!



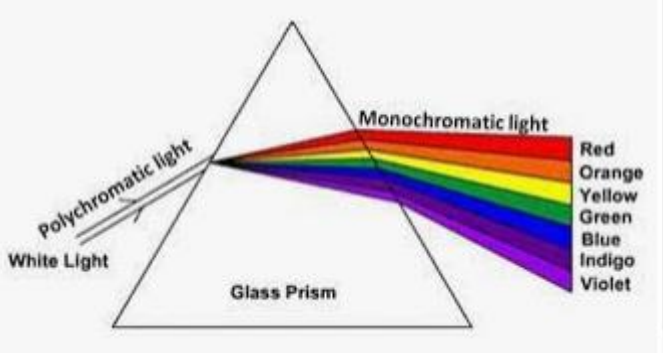
Now suppose the source emits particles of different momenta p.
Let p_0 be the central momentum of the whole range of momenta coming from the source, and let $\delta = (p - p_0)/p_0$ be the momentum deviation of any given particle, relative to the central momentum p_0.

"Dispersed focus" means that particles of different momenta will come to a point-to-point focus at different final locations x_f

i.e. $\frac{dx_f}{d\delta} \neq 0$



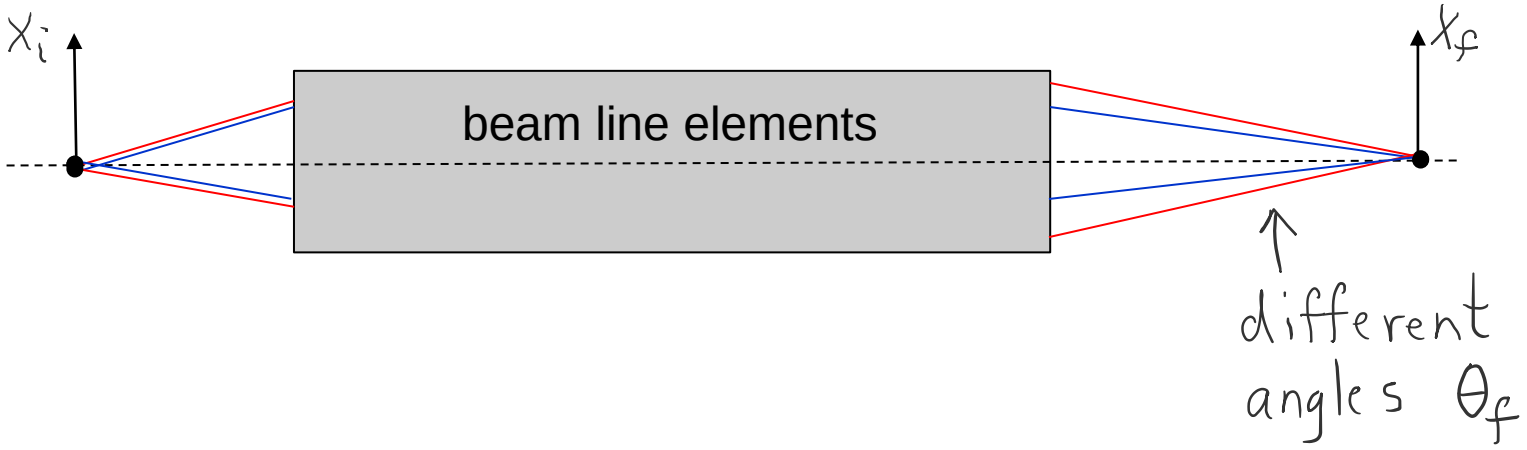
This is analogous to dispersion of polychromatic light through a glass prism – each colour focus at a different spatial location.



"Non-dispersed focus" means that particles of different momenta will intersect at the same final locations x_f

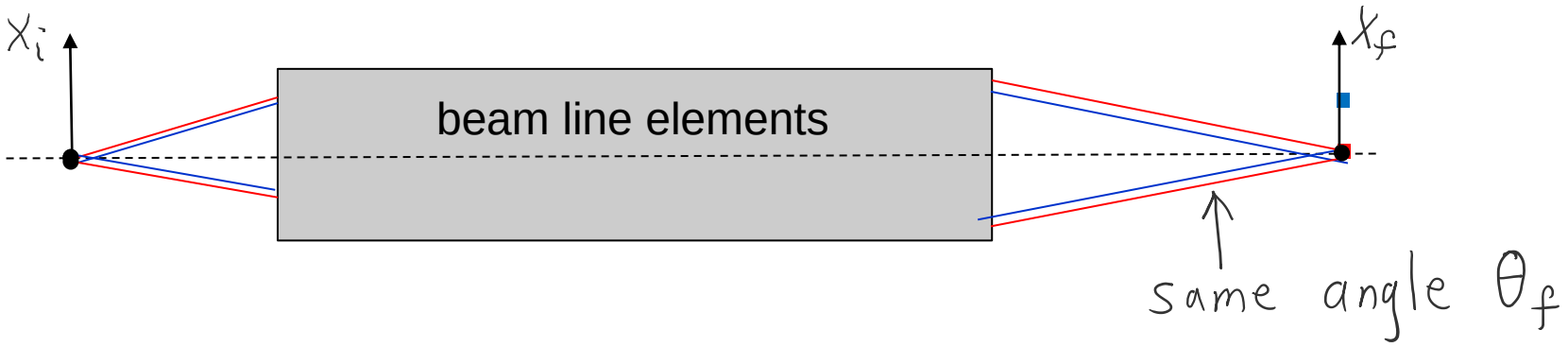
$$\text{i.e.} \quad \frac{dx_f}{d\delta} = 0$$

but it is still possible that particles of different momenta will converge at the focus with different angles, as shown below.



"Achromatic focus" means that particles of different momenta
will intersect at the same final locations x_f and with the same final angles θ_f

i.e. $\frac{dx_f}{d\delta} = 0$ and $\frac{d\theta_f}{d\delta} = 0$



For the general case, the final positions, angles, and momenta are related to the initial ones, in first order, by

$$x_f = \frac{dx_f}{dx_i} x_i + \frac{dx_f}{d\theta_i} \theta_i + \frac{dx_f}{d\delta} \delta$$

↑ magnification ↑ focus ↑ dispersion

$$= R_{11} x_i + R_{12} \theta_i + R_{16} \delta \quad \text{in TRANSPORT notation}$$

$$\theta_f = \frac{d\theta_f}{dx_i} x_i + \frac{d\theta_f}{d\theta_i} \theta_i + \frac{d\theta_i}{d\delta} \delta$$

$$= R_{21} x_i + R_{22} \theta_i + R_{26} \delta_i$$

$$\delta_f = \delta_i$$

no change in particle momentum in absence of acceleration section

or in matrix notation

$$\begin{pmatrix} x_f \\ \theta_f \\ \delta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{16} \\ R_{21} & R_{22} & R_{26} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ \theta_i \\ \delta_i \end{pmatrix}$$

This is called the transfer matrix
for the beam transport system.

It relates the initial particle coordinates x_i, θ_i , δ_i
to the final particle coordinates x_f, θ_f , δ_f .

In the absence of accelerators and energy loss,
the particle momentum is a constant, so we set $\delta_i = \delta_f \equiv \delta$

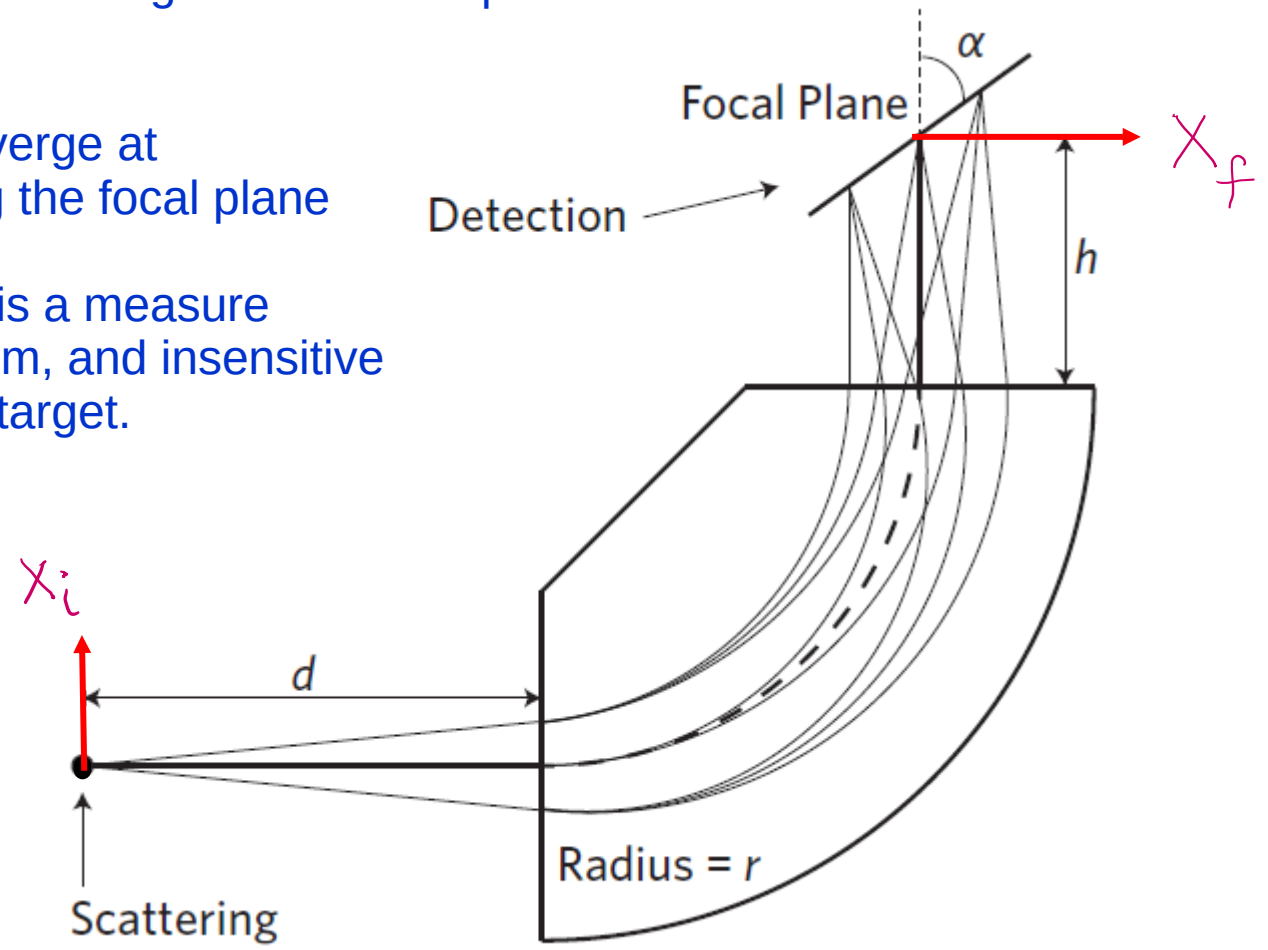
2. Optics of the spectrometer

Optics of spectrometer magnet:

We design the magnet to give point-to-point focus in the bend (vertical) direction, i.e. rays emerging at all angles from the point target location, all converge at the same point on the focal plane

Different momenta converge at different positions along the focal plane

So focal plane position is a measure of the particle momentum, and insensitive to initial angle from the target.



$$X_f = R_{11} X_i + R_{12} \theta_i + R_{16} \delta$$

$X_i = 0$ for point source

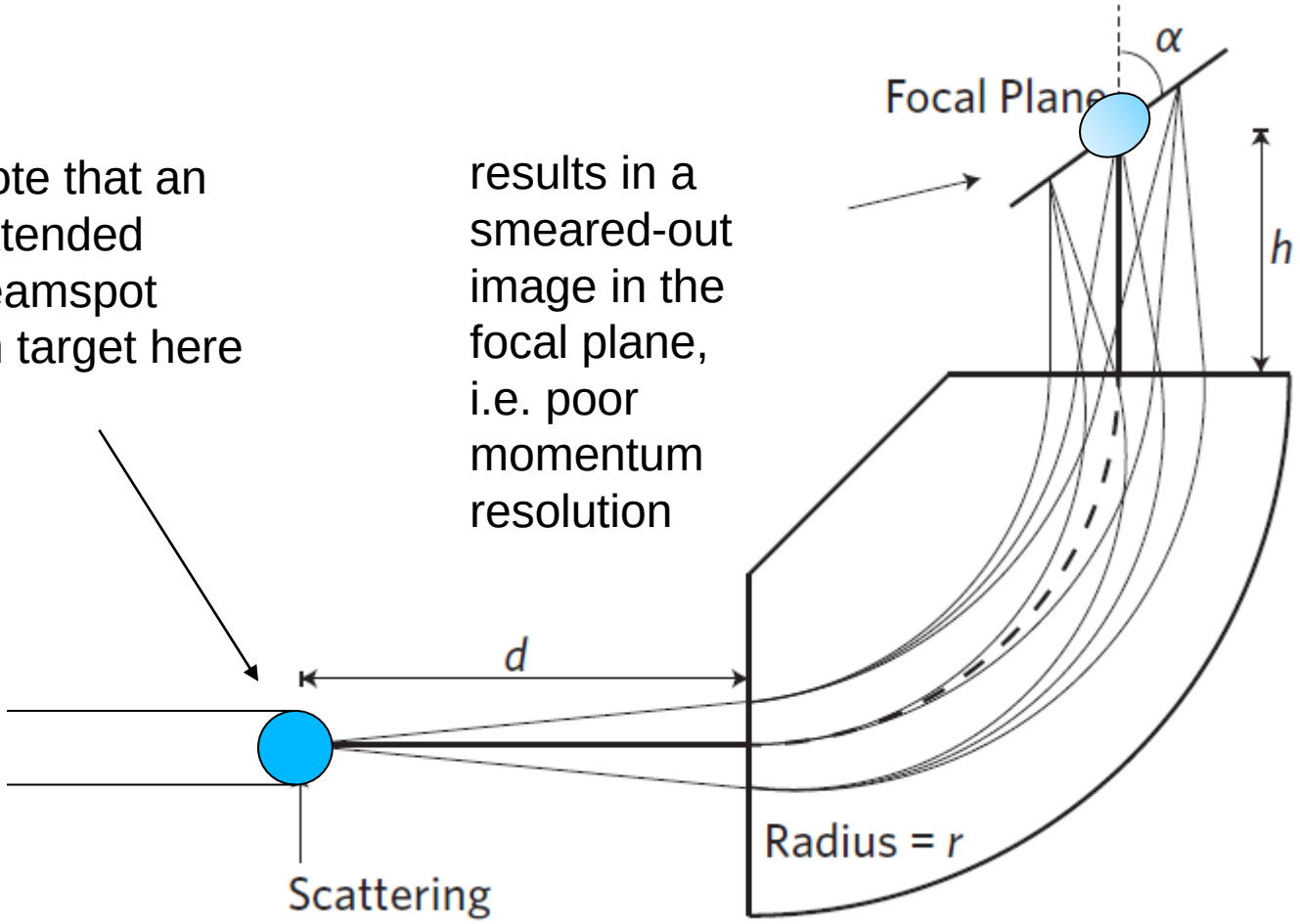
$R_{12} = 0$ for focus

so $X_f = R_{16} \delta$

X_f measures momentum deviation

Note that an extended beamspot on target here

results in a smeared-out image in the focal plane, i.e. poor momentum resolution



Since we determine momentum deviation $\delta = \frac{P - P_0}{P_0}$

using $X_f = R_{16}\,\delta$

then the uncertainty in momentum $\Delta\delta = \frac{\Delta P}{P_0}$

is given by

$$\Delta X_f = R_{16}\,\frac{\Delta P}{P_0}$$

or $$\Delta P = P_0\,\frac{\Delta X_f}{R_{16}} = P_0\,\frac{R_{11}}{R_{16}}\,\Delta X_i$$

This tells us how much momentum uncertainty ΔP is introduced if the beamspot on target has size ΔX_i

So it is important for us to know the first-order transfer matrix for the spectrometer

$$\begin{pmatrix} R_{11} & R_{12} & R_{16} \\ R_{21} & R_{22} & R_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

because it allows us to estimate things like the effect of defocusing the beam on target, the effect of multiple scattering on the mass resolution, etc.

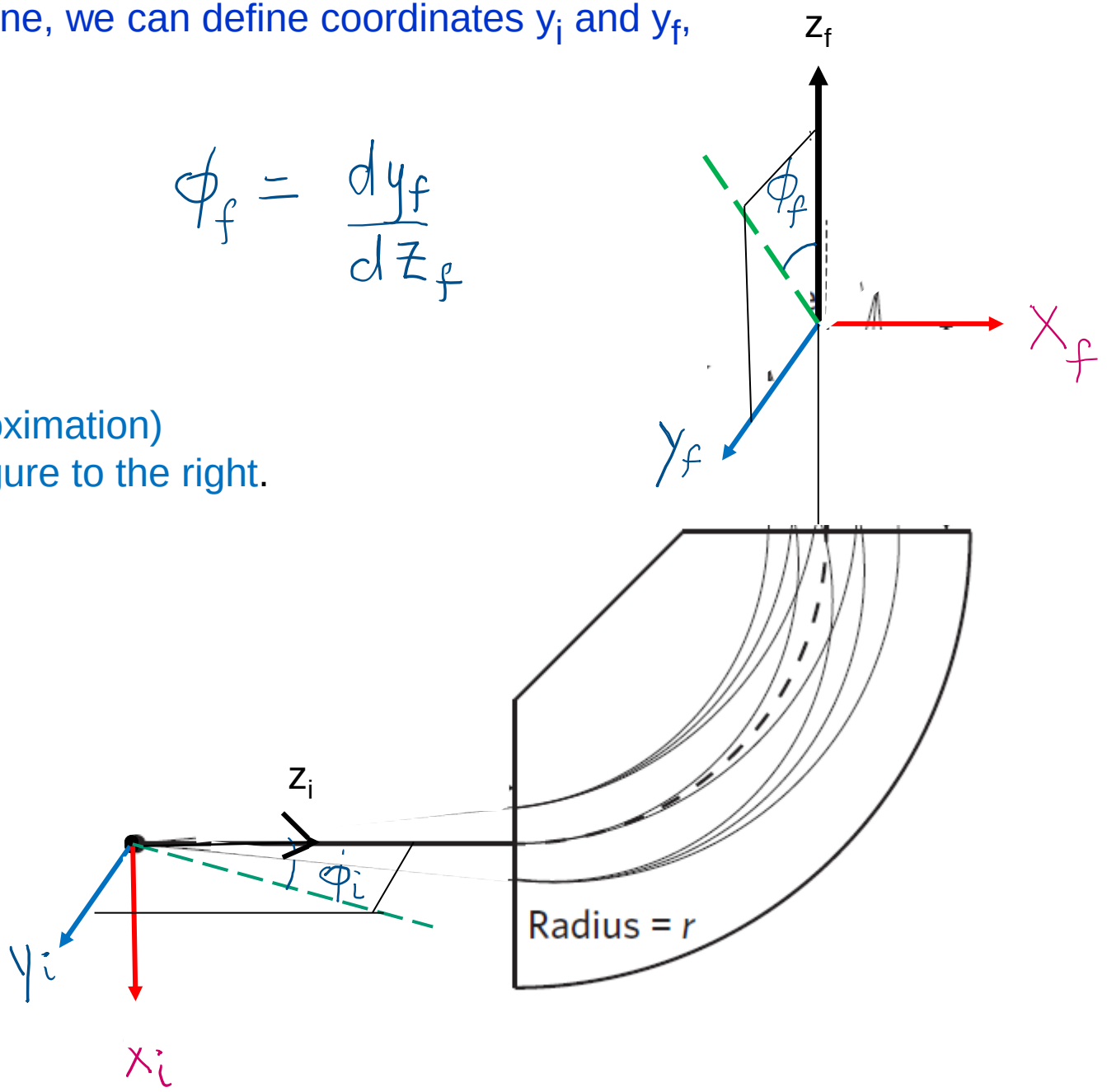
Q1: What is the 1st order transfer matrix for the current spectrometer design?

Q2: How large is the vertical beamsize on target and how much is it affecting the mass resolution of the experiment?

In the non-bend plane, we can define coordinates y_i and y_f, and angles

$$\phi_i = \frac{dy_i}{dz_i} \qquad \phi_f = \frac{dy_f}{dz_f}$$

(small-angle approximation)
as shown in the figure to the right.

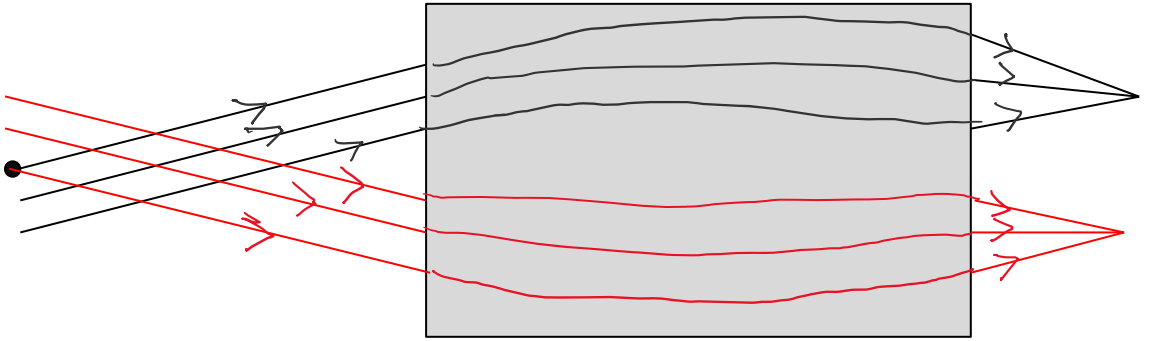


Similar to what we had in the dispersive (bend) plane, we define a transfer matrix for the non-bend plane coordinates:

$$\begin{pmatrix} y_f \\ \phi_f \end{pmatrix} = \begin{pmatrix} R_{33} & R_{34} \\ R_{43} & R_{44} \end{pmatrix} \begin{pmatrix} y_i \\ \phi_i \end{pmatrix}$$

Q3: What is the transfer matrix for the non-bend plane, in the current spectrometer design?

In the orthogonal (non-bend, or horizontal) direction,
the focusing condition of the spectrometer is parallel-to-point like this:



where the final position is a measure of the initial angle,
and insensitive to the initial position

$$y_f = R_{33}\, y_i + R_{34}\, \phi_i$$

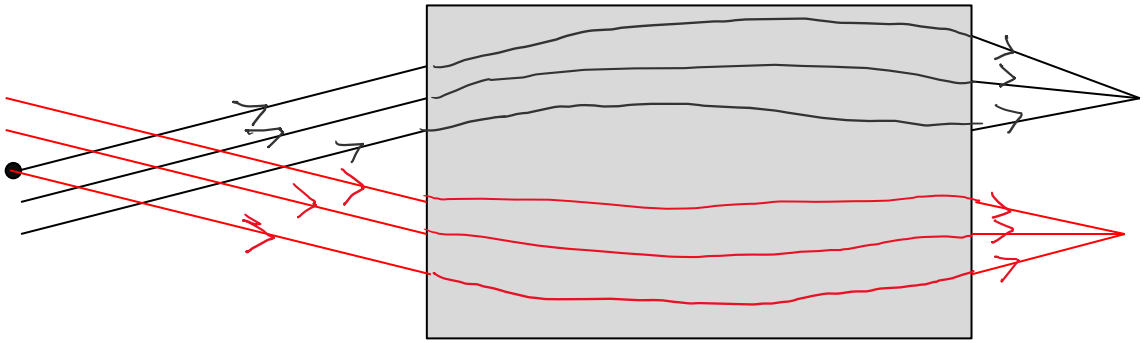
Set to zero by optics of spectrometer

$$\text{so} \quad \phi_i = \frac{y_f}{R_{34}}$$

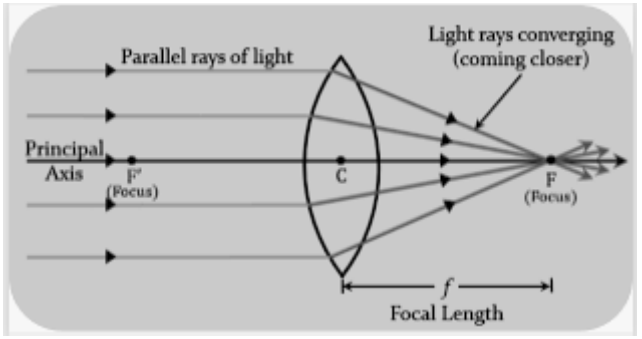
This is how we determine the horizontal plane scattering angle of the outgoing e^+ or e^- - important for reconstructing the invariant mass of the lepton pair.

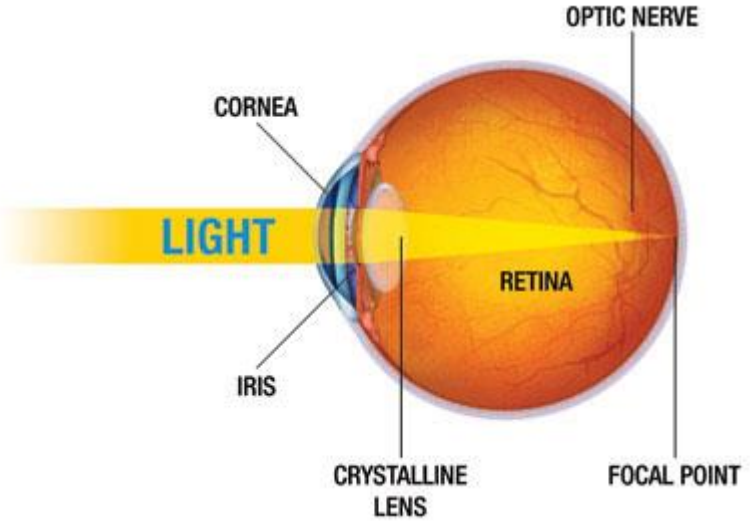
We determine the horizontal scattering angle in this way, by measuring the final position, because the focal plane detectors allow us to measure final positions y_f very accurately (~ 0.1 mm with a drift chamber), but measuring final angles ϕ_f is vulnerable to multiple scattering through the material of the focal plane detectors.

This parallel-to-point focus condition, where the final position is determined by initial angle

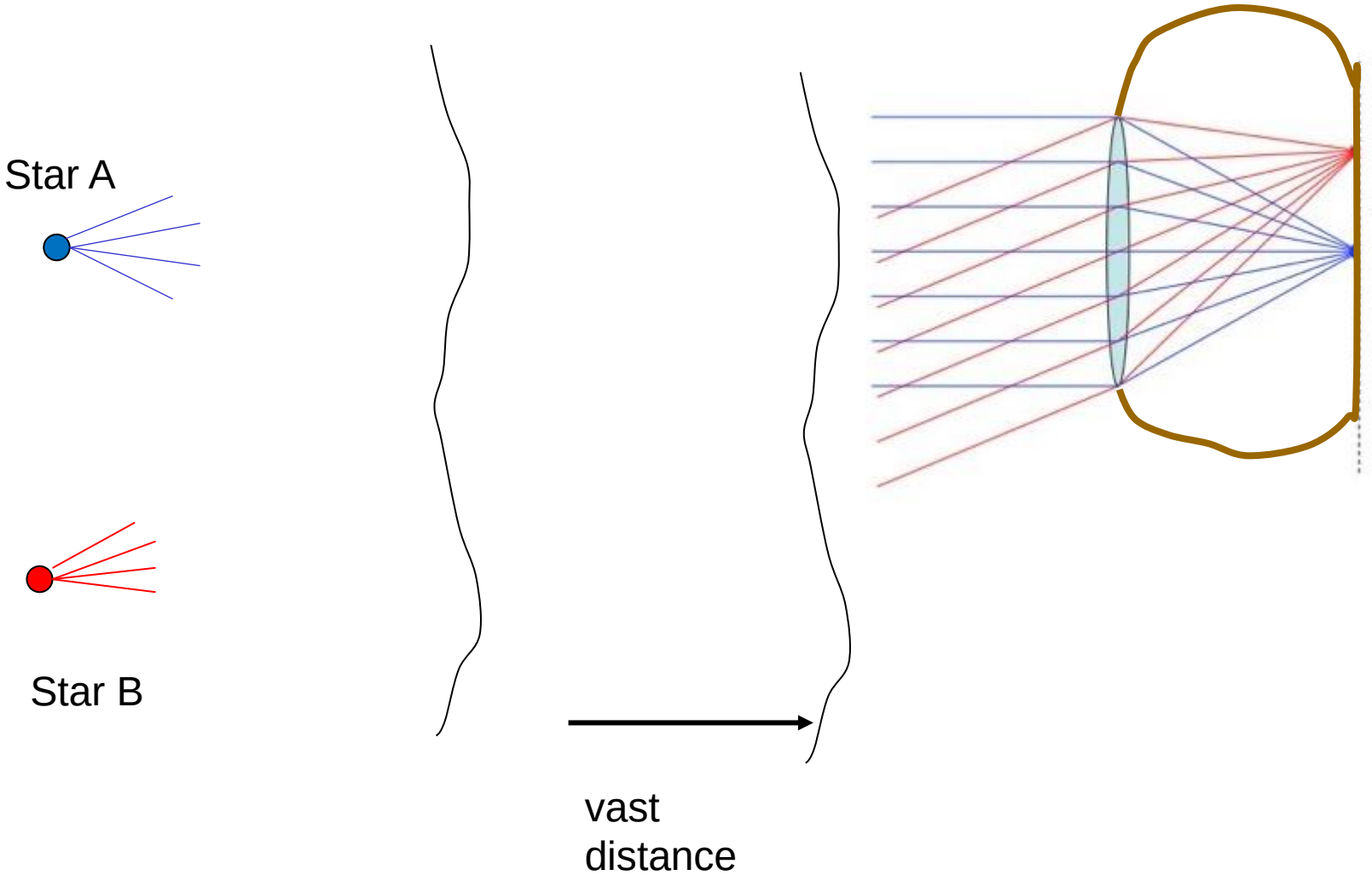


is exactly the condition when we look at distant stars with our eyes. The rays of light from a distant star are coming from infinity and are thus parallel:





but the light rays from two stars at different positions in the sky enter the lens at different angles, and focus at different positions on the retina.
This is parallel-to-point focusing!

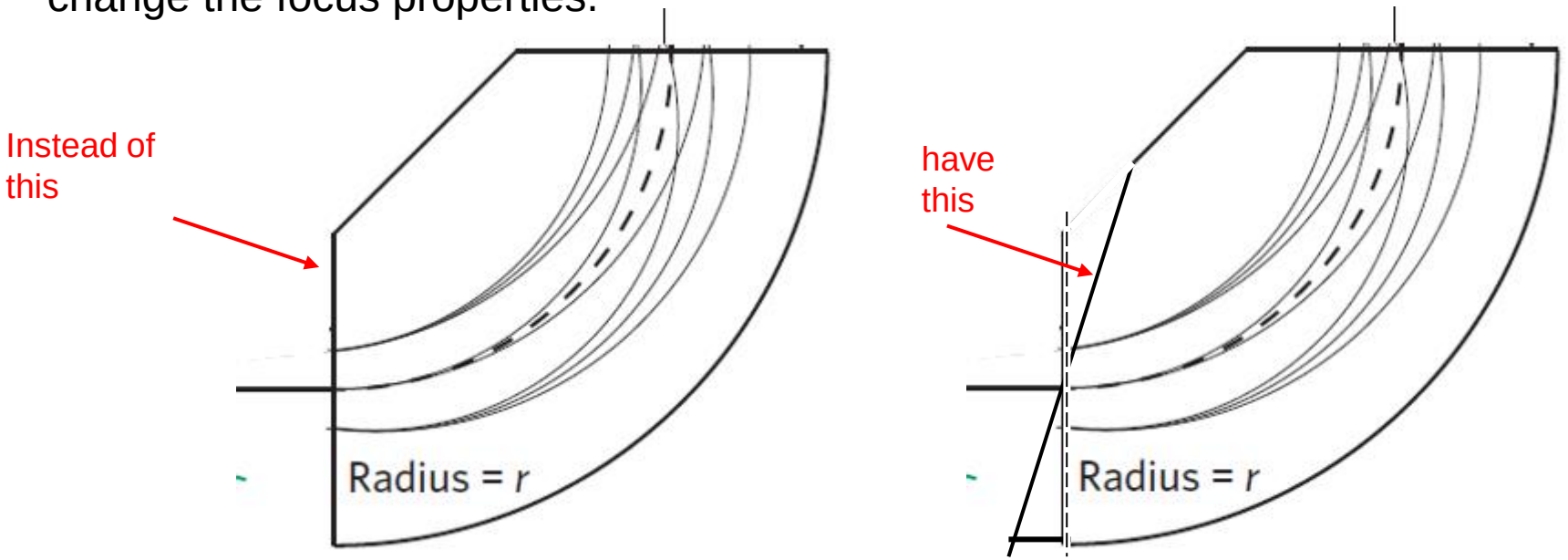


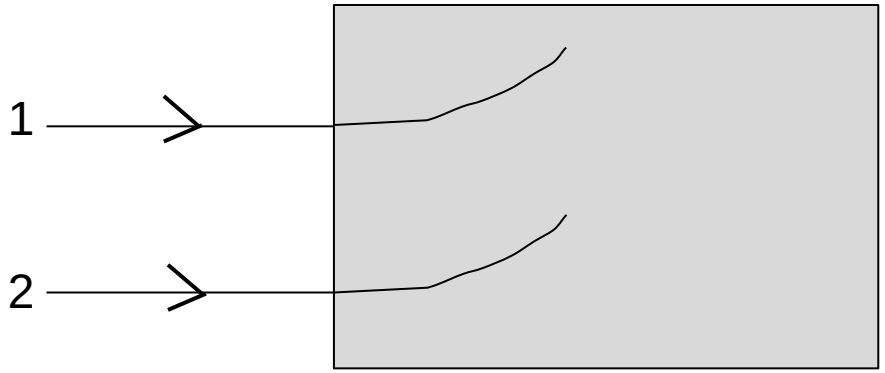
So the optical design of the spectrometer demands

$R_{12} = \frac{dx_f}{d\theta_i} = 0$ point-to-point focus in the dispersion plane

$R_{33} = \frac{dy_f}{dy_i} = 0$ parallel-to-point focus in the non-dispersion plane

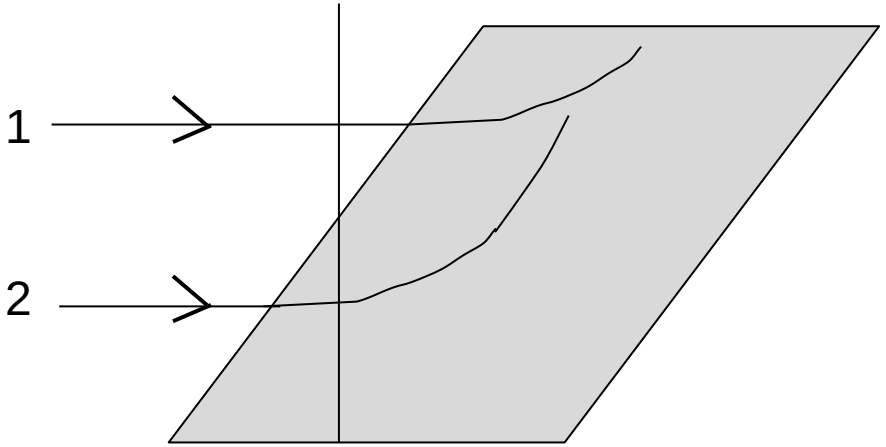
What physical parameters of the spectrometer magnet do we have available to adjust, in order to achieve these optical parameters?
The angles of the entrance and exist pole edges can be adjusted to change the focus properties.





Zero pole edge rotation:
Rays 1 and 2 enter the magnetic field at the same time,

Rays 1 and 2 remain parallel at all times



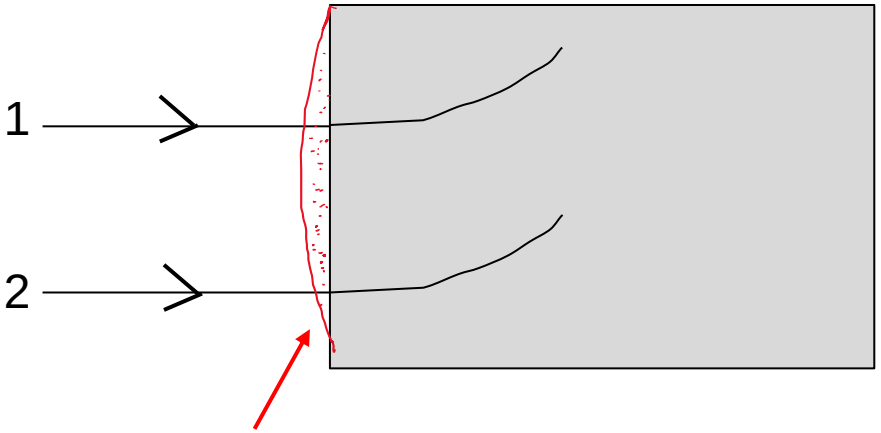
Positive pole edge rotation:
Ray 2 enters the magnetic field earlier, starts bending upward earlier

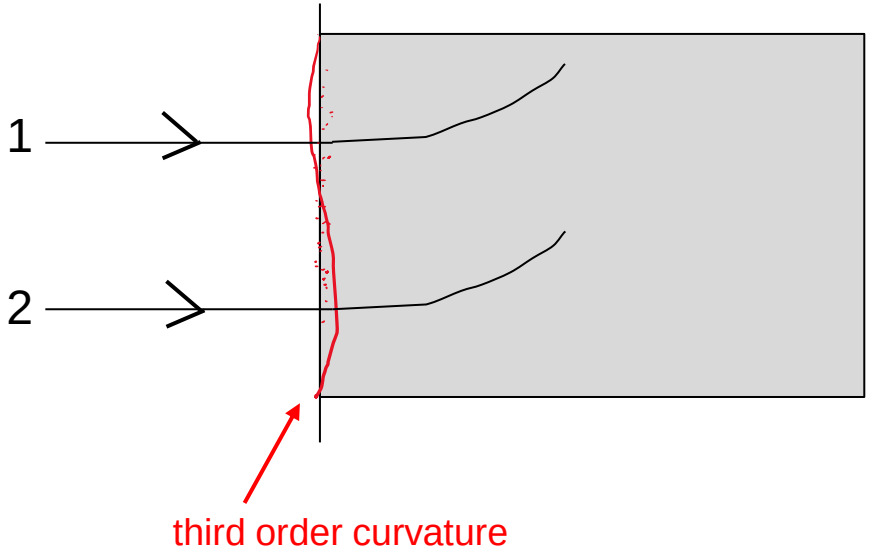
Effect is to make rays 1 and 2 converge

Positive pole edge rotation adds focusing in the bend plane.

We have two physical parameters to adjust (the pole edge rotation angles at the magnet entrance and exit), and two optical constraints $R_{12}=0$ $R_{33}=0$. It should be possible to find a unique solution.

Q4: In the current spectrometer magnet design, what are the pole edge rotation angles for the entrance and exit?



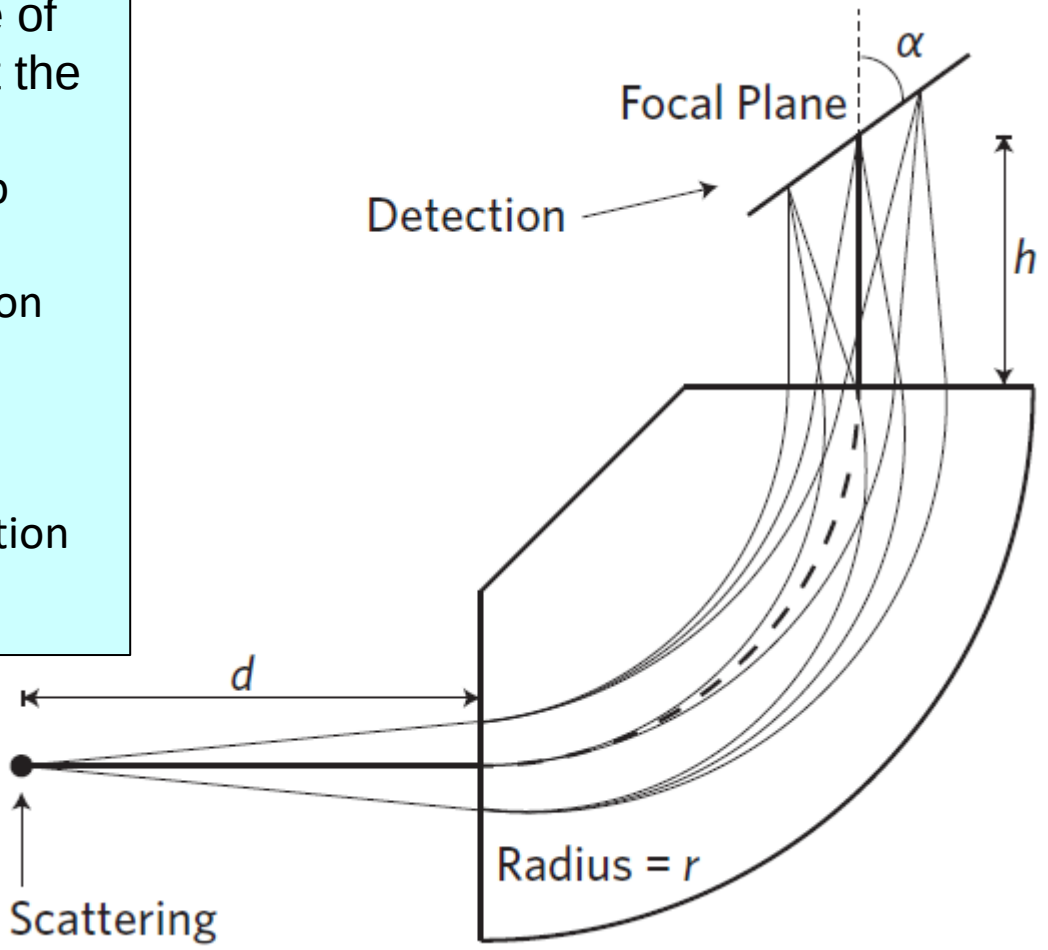


The pole edges don't have to be straight … for better momentum resolution, the pole edges could have 2nd order, 3rd order, etc. corrections for higher order optical aberrations. This applies to both the entrance and exit pole edges. The aim is to get the sharpest possible focus at the focal plane.

Q5: Were any of these higher-corrections used in the current spectrometer magnet design to improve resolution?

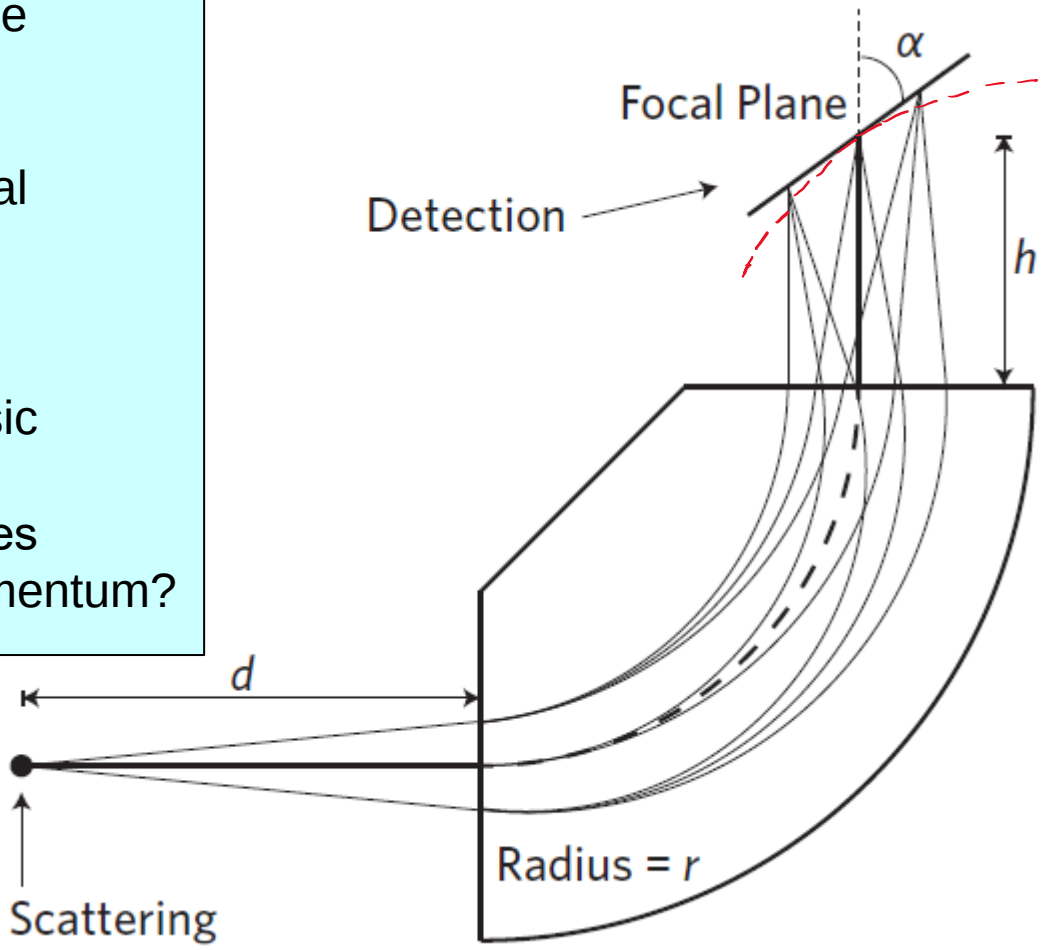
Q6: If I had a point source of monoenergetic particles at the target location, what is the momentum resolution Δp/p at the focal plane? (i.e. the intrinsic momentum resolution of each spectrometer).

How much would it help the overall invariant mass resolution if we could improve this?

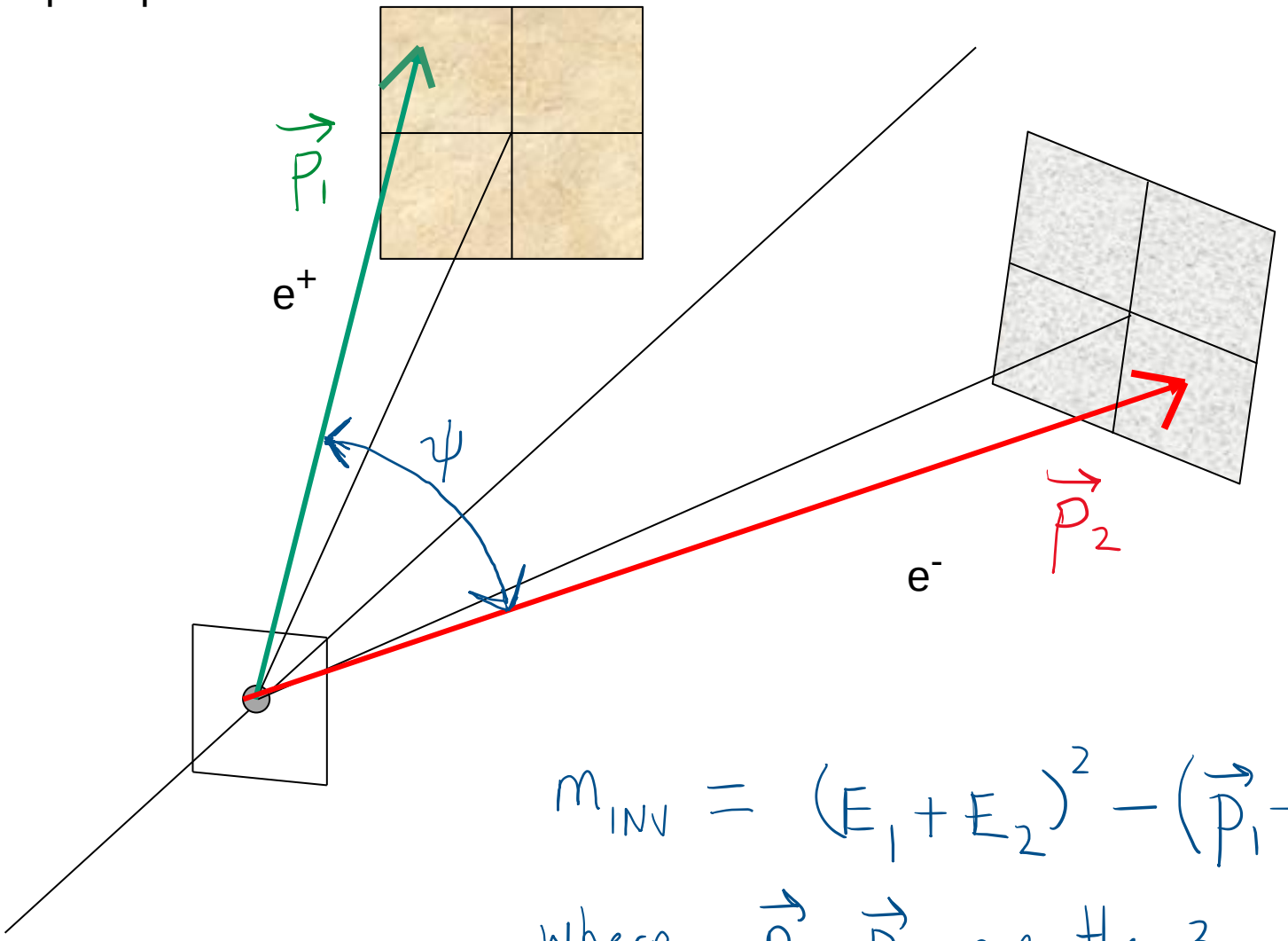


Q7: What is the focal plane tilt angle α ?

Q8: How planar is the focal "plane"? Or is the focal curved as shown by the dashed red line?
How much does the intrinsic momentum resolution Δp/p degrade as one moves away from the central momentum?



In the Darklight experiment, we measure the momenta and angles of the two leptons, one in each spectrometer, and then reconstruct the invariant mass of each lepton pair.



$$m_{INV} = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

where $\vec{p}_1$, $\vec{p}_2$ are the 3-momentum vectors of the two leptons

$$m_{INV} = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

$$= (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)\cdot(\vec{p}_1 + \vec{p}_2)$$

$$= E_1^2 + 2E_1E_2 + E_2^2 - [\,|\vec{p}_1|^2 + 2\vec{p}_1\cdot\vec{p}_2 + |\vec{p}_2|^2\,]$$

For highly relativistic electrons $E_1 \approx |\vec{p}_1|$

so $m_{INV} = 2E_1E_2 - 2\vec{p}_1\cdot\vec{p}_2$

$= 2E_1E_2 - 2E_1\hat{p}_1 \cdot E_2\hat{p}_2$ where $\hat{p}$ denotes unit vector

$= 2E_1E_2(1 - \hat{p}_1\cdot\hat{p}_2)$

$= 2E_1E_2(1 - \cos\psi)$

ψ = opening angle between e^+ and e^- trajectories

The uncertainty in the invariant mass has contributions from the uncertainties in the energies of the two leptons, and the uncertainty in the angle between the two leptons:

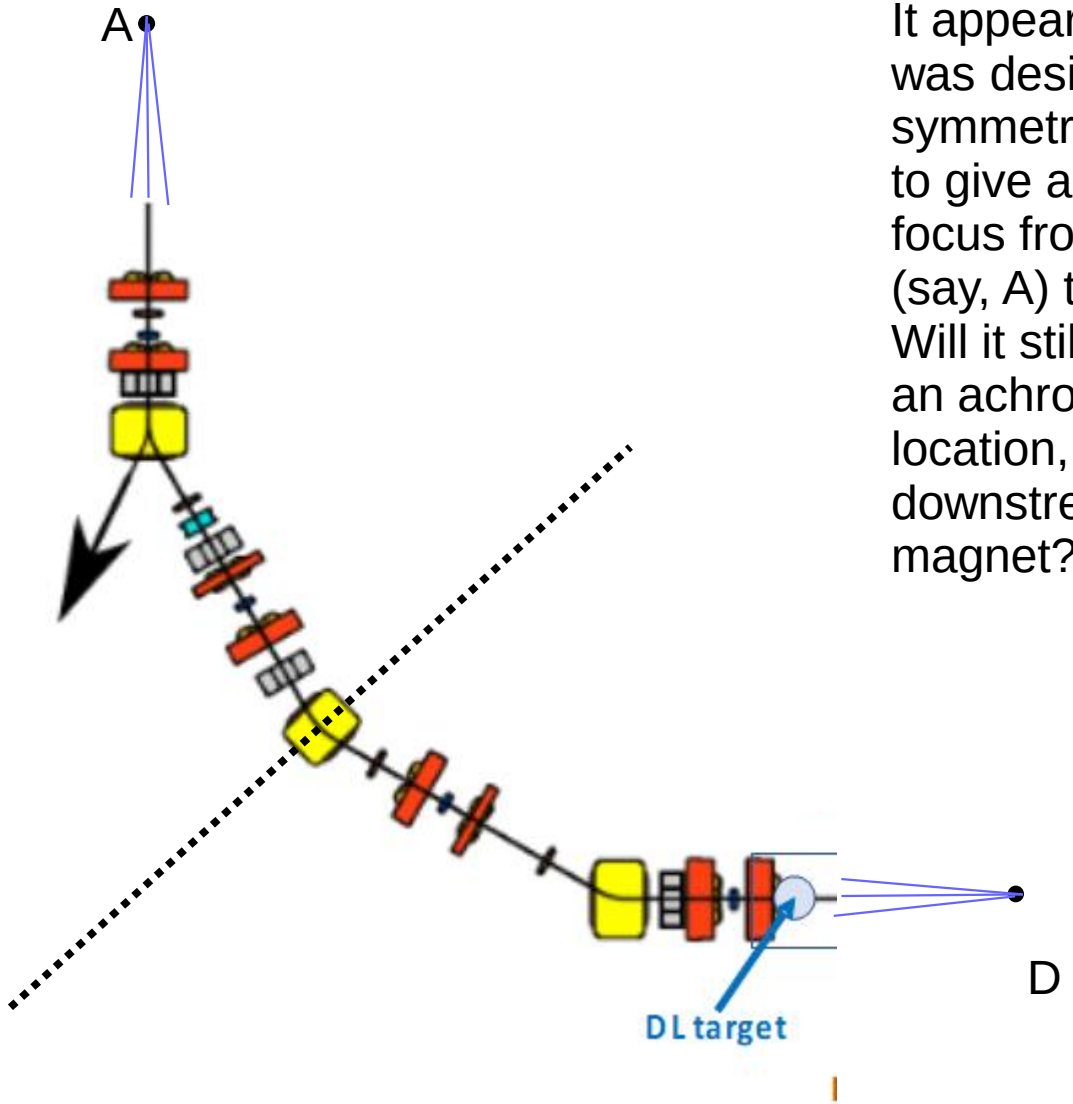
$$\Delta m_{INV} = 2\,\Delta E_1\, E_2 (1-\cos\psi) + 2E_1\,\Delta E_2 (1-\cos\psi) + 2E_1 E_2 (\sin\psi)\Delta\psi$$

3 terms added in quadrature

Since the two spectrometers are identical, the first two terms are the same.

Q9: What is the size of the individual terms? Does one term dominate over the other? Can we work harder to reduce the dominant term, to achieve better mass resolution?

3. Dispersed beam on target?



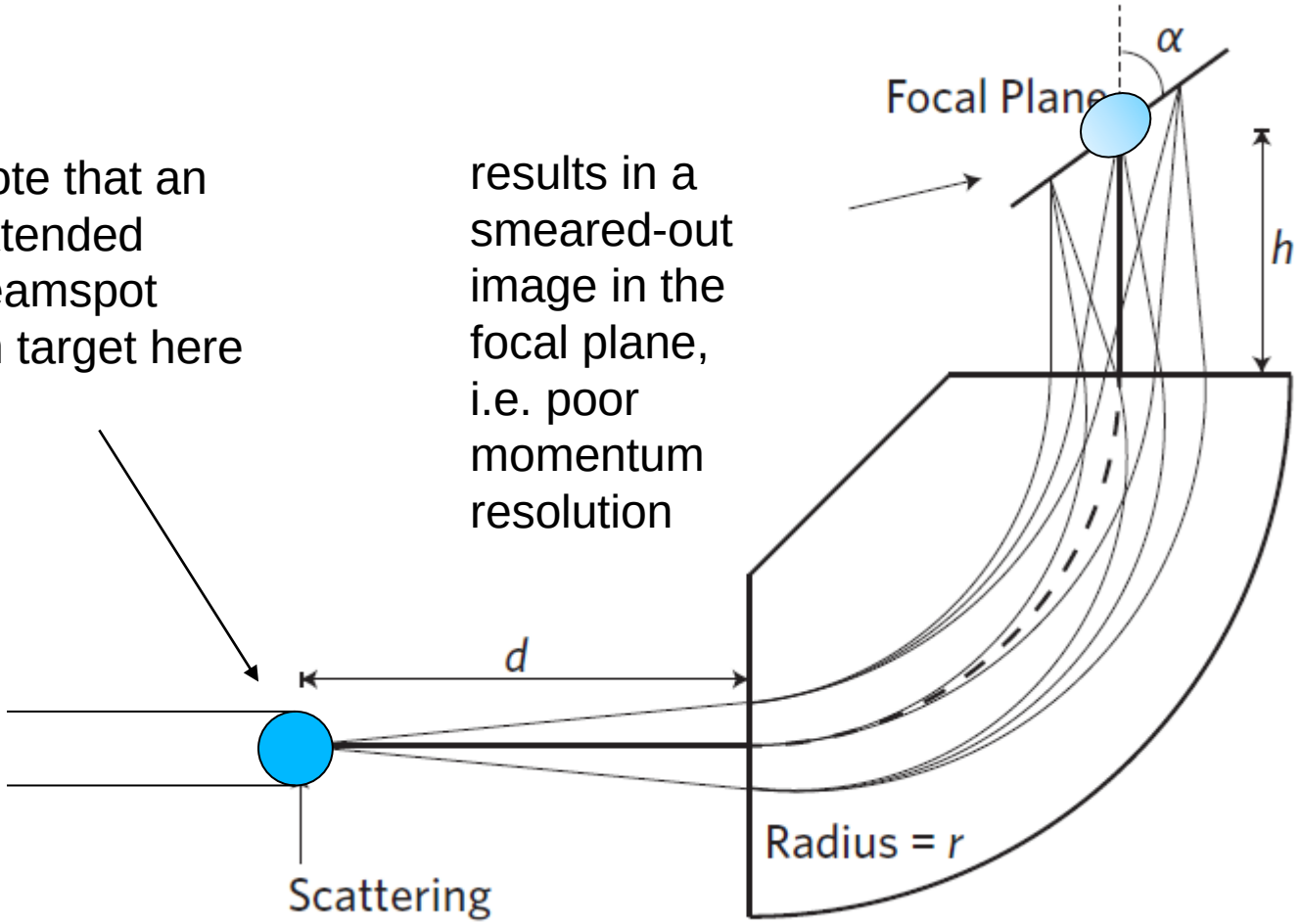
My Beamline optics question on March 18, 2022:

It appears that the beam transport was designed with a mirror reflection symmetry about the dashed line, to give a point-to-point achromatic focus from some point upstream (say, A) to the beam dump (D)
Will it still be possible to achieve an achromatic focus at the DL target location, located such a short distance downstream of the last quadrupole magnet?

Recall our earlier discussion

Note that an extended beamspot on target here

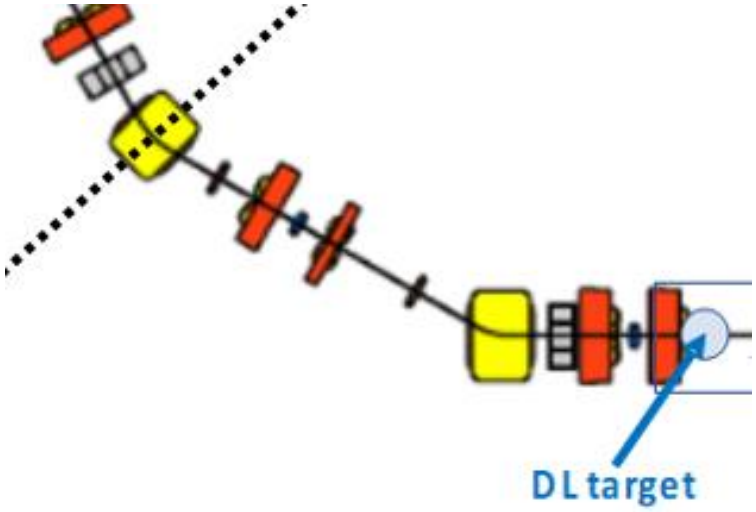
results in a smeared-out image in the focal plane, i.e. poor momentum resolution



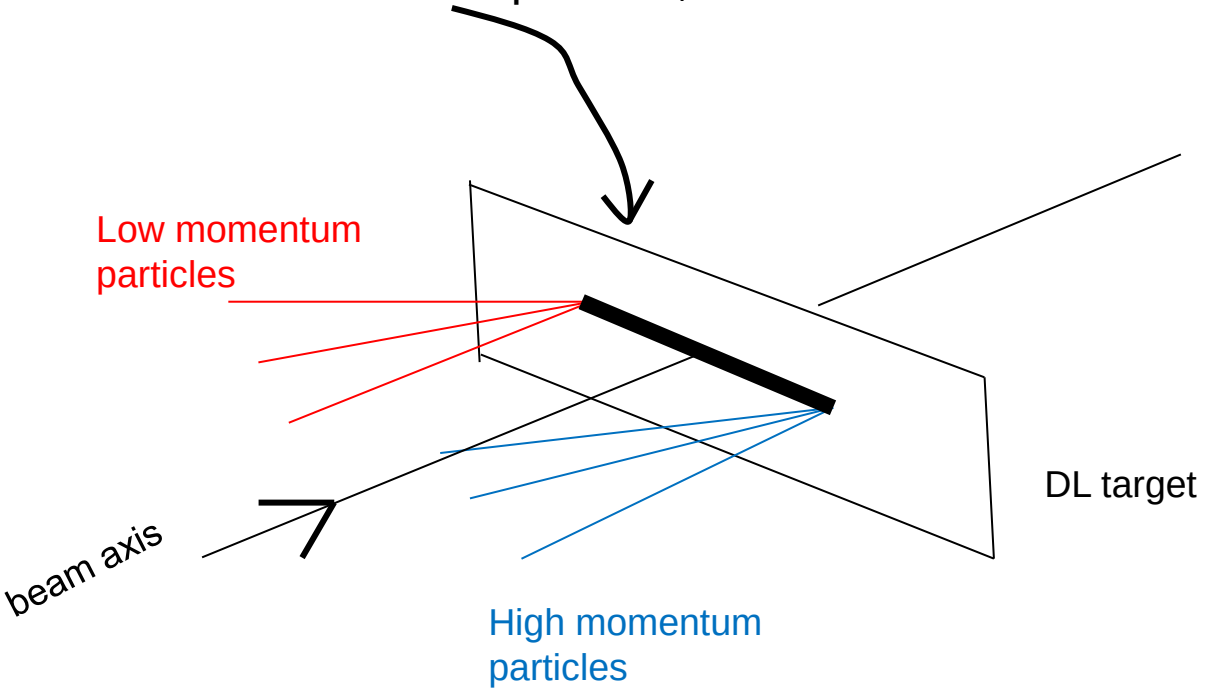
It is therefore important that the beam spot size on the target be as small as possible in the vertical direction, because a large vertical beam size results directly in poorer momentum resolution.

In the horizontal direction, it is not so critical, but the wider the beam is horizontally, the more likely that multiple-scattered particles will hit the beampipe downstream.

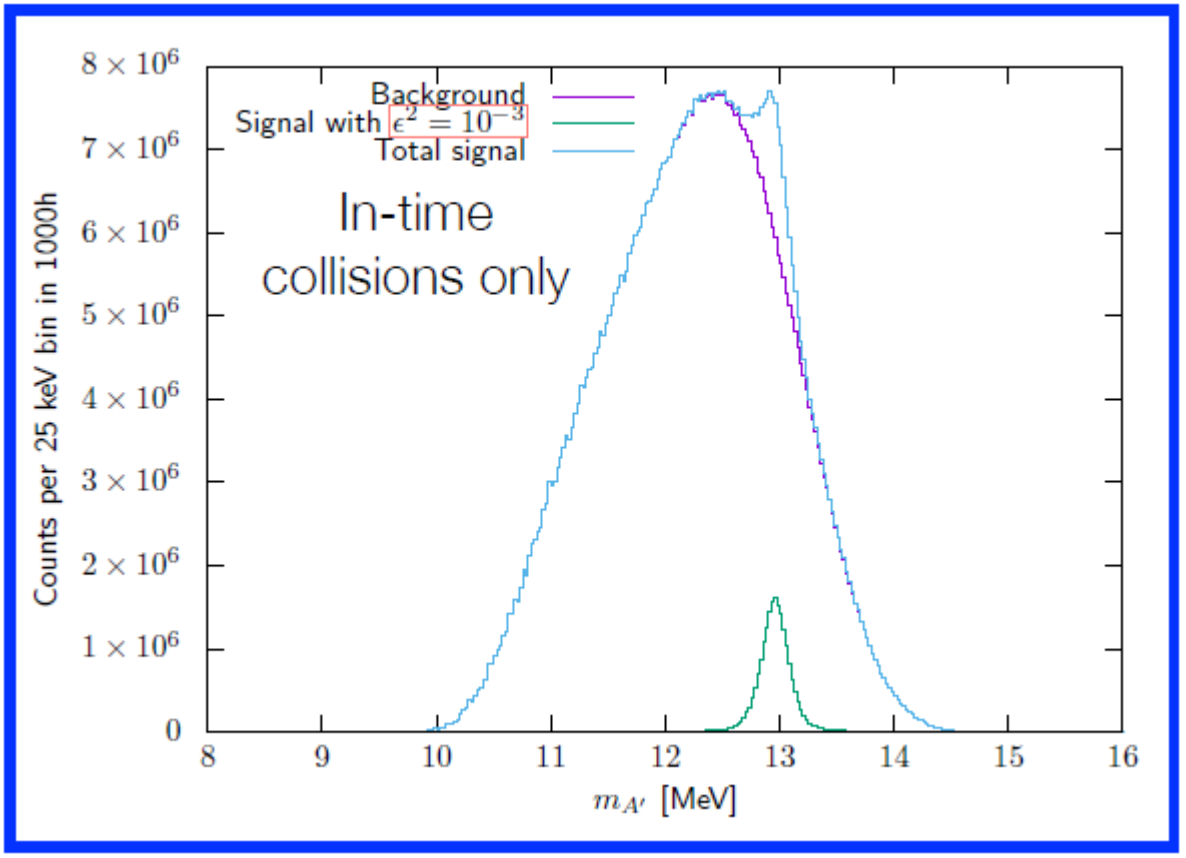
However, there might be an advantage to having a momentum dispersed focus at the target, despite the wider beam size on target. This will be described in the next slides.



Suppose we had a momentum-dispersed focus on the DL target, so that incident particle momentum is correlated with horizontal position, like this:



In Darklight, the signal is sitting on a huge background.

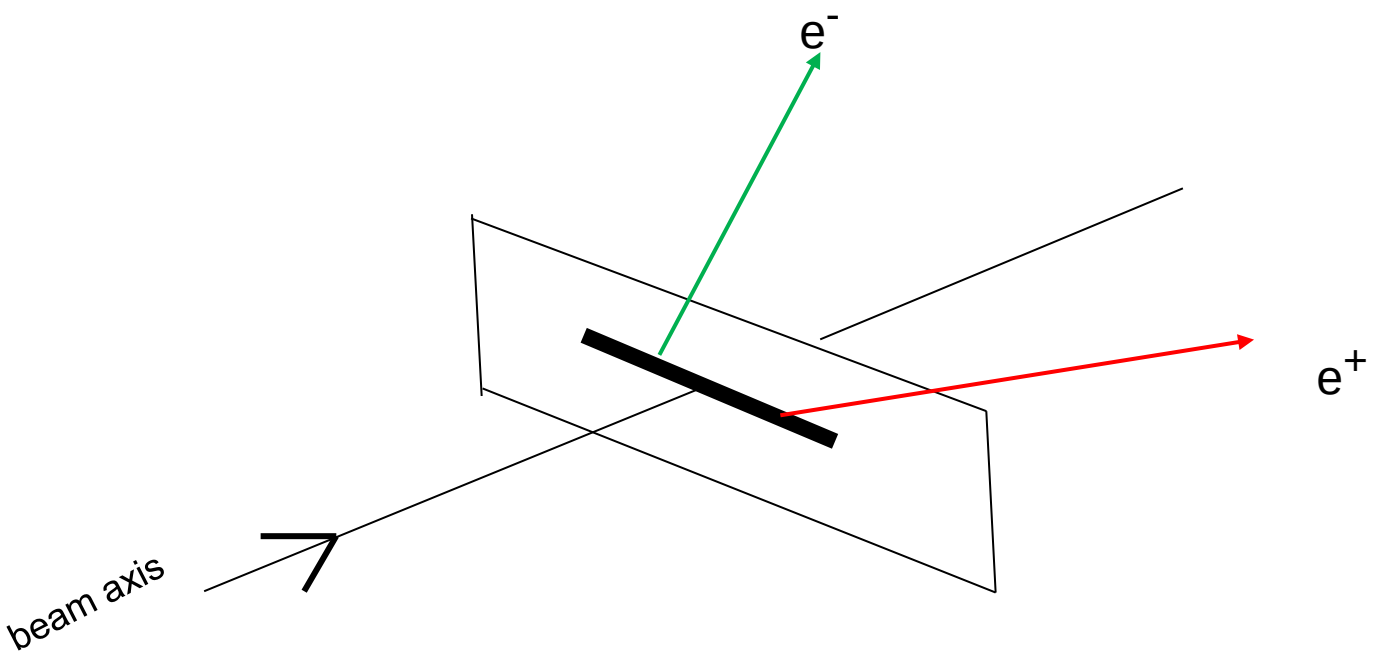


By contrast, true e+ e- coincidences from decay of a boson will have the two leptons emerging from the same horizontal position on the target.

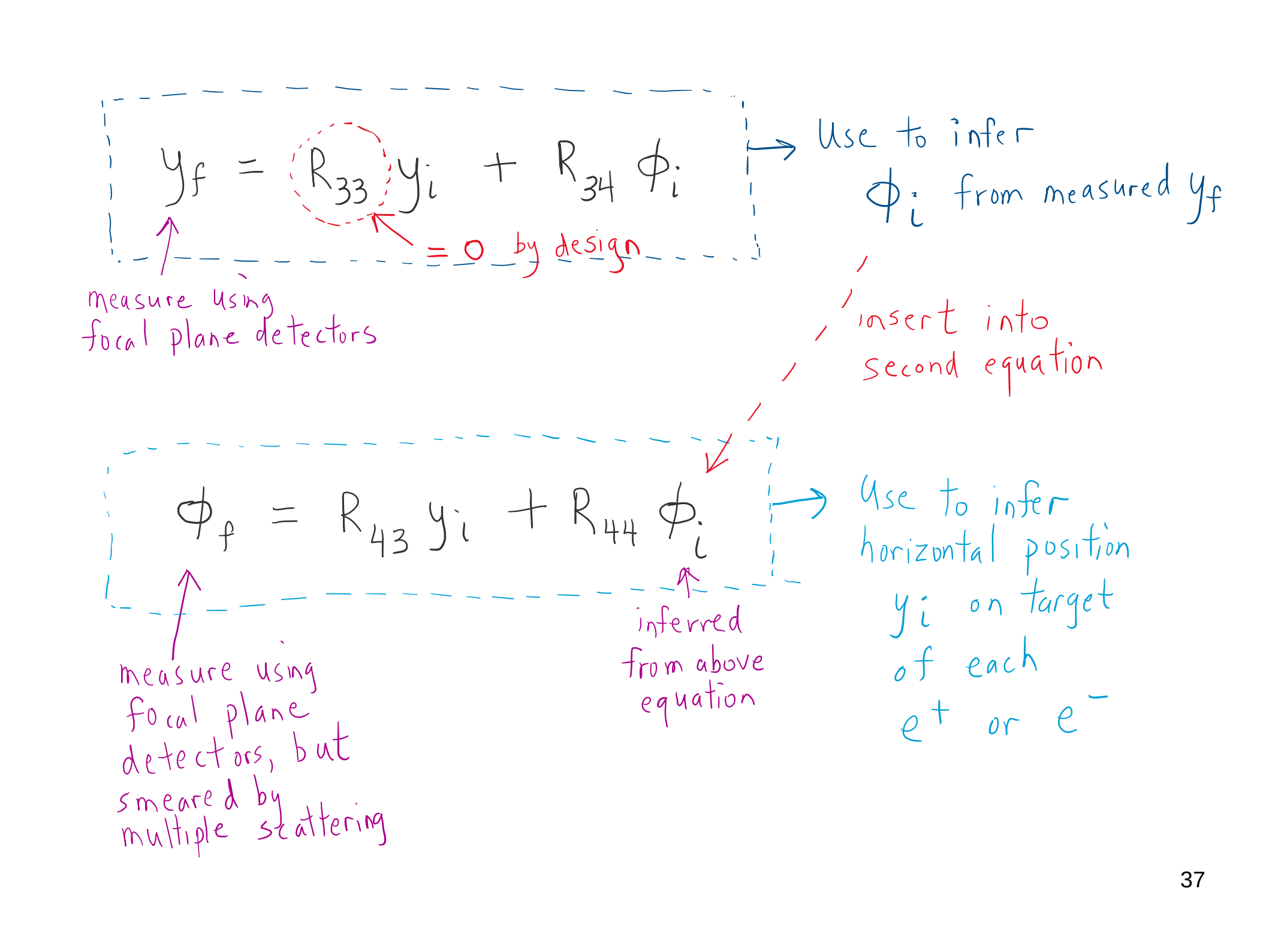
This figure shows the situation for $\varepsilon^2 = 10^{-3}$, but we need to reach $\varepsilon^2 = 10^{-6}$ for a competitive experiment, i.e. S/B ratio 1000x worse than depicted here.

The background is overwhelmingly accidental coincidences of unrelated e^+ and e^-'s. For a dispersed beam on the target, this means accidental coincidences will come from e^+ and e^-'s emerging from different horizontal positions on the target.

Most of the large background comes from e+ and e- coming from different horizontal locations along the dispersed beam spot on target.



If we could reconstruct, on an event-by-event basis, the horizontal positions of the e^+ and e^-, then we could suppress this accidental background, because the two leptons would appear to come from different locations on the target. How do we reconstruct the horizontal position on target?


$y_f = R_{33} y_i + R_{34} \phi_i$
= 0 by design
measure using focal plane detectors
Use to infer ϕ_i from measured y_f
insert into second equation
$\phi_f = R_{43} y_i + R_{44} \phi_i$
measure using focal plane detectors, but smeared by multiple scattering
inferred from above equation
Use to infer horizontal position y_i on target of each e^+ or e^-

This gives a way to determine the horizontal position of origin of each e+ and e-.

E.g. If the beam spot were dispersed by 1 cm on the DL target

and we could determine the horizontal position y_i of each e+ and e- to 2 mm accuracy,

then we could reduce that accidental background by a factor of 5.

The question is, could this still be done with sufficient accuracy in the presence of the multiple scattering which smears out the measurement of the final non-bend plane angle ϕ_f ?

This depends on the optical design of the spectrometer providing us with a sufficiently large R_{43} value, since

$$\phi_f = R_{43}\, y_i + R_{44}\, \phi_i$$

or

$$y_i = \frac{1}{R_{43}} \left(\phi_f - R_{44}\, \phi_i \right) = \frac{1}{R_{43}} \left(\phi_f - R_{44} \frac{y_f}{R_{34}} \right)$$

Continued on next slide

$$y_i = \frac{1}{R_{43}}\left(\phi_f - R_{44}\frac{y_f}{R_{34}}\right)$$

can measure very accurately using focal plane detectors

can measure with focal plane detectors but smeared by multiple scattering

The uncertainty in the reconstruction of the horizontal position at the target is given by

$$\Delta y_i = \frac{1}{R_{43}}\Delta\phi_f$$

due to multiple scattering in focal plane detectors

An optical design which maximizes R_{43} will reduce the uncertainty in the reconstructed horizontal beam position y_i .

Just suppose (and I don’t know if this is realistic)
we can disperse the beam horizontally to 10 mm wide

and we can reconstruct the horizontal position at the target to 2 mm accuracy.

Then the accidental background can be reduced by a factor of 5.

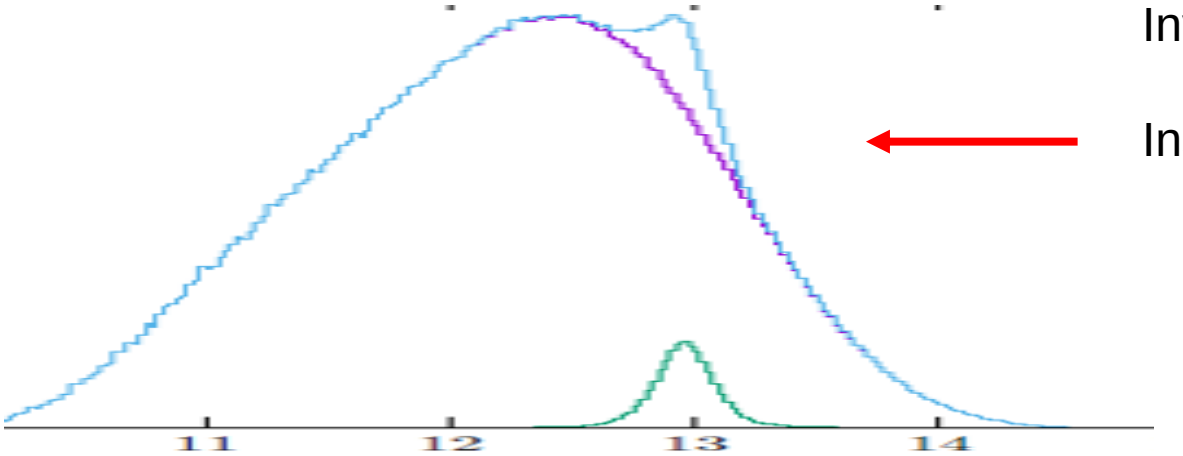
Being able to reconstruct the initial horizontal position on target has another advantage:
it allows us to correct, event-by-event, the smearing of the invariant mass resolution due to the energy spread of the incident beam.

The momentum spread in the electron beam is $\frac{\Delta p}{p} \sim 0.3\%$

so for a 30 MeV beam, this is 90 keV
50 MeV beam, this is 150 keV

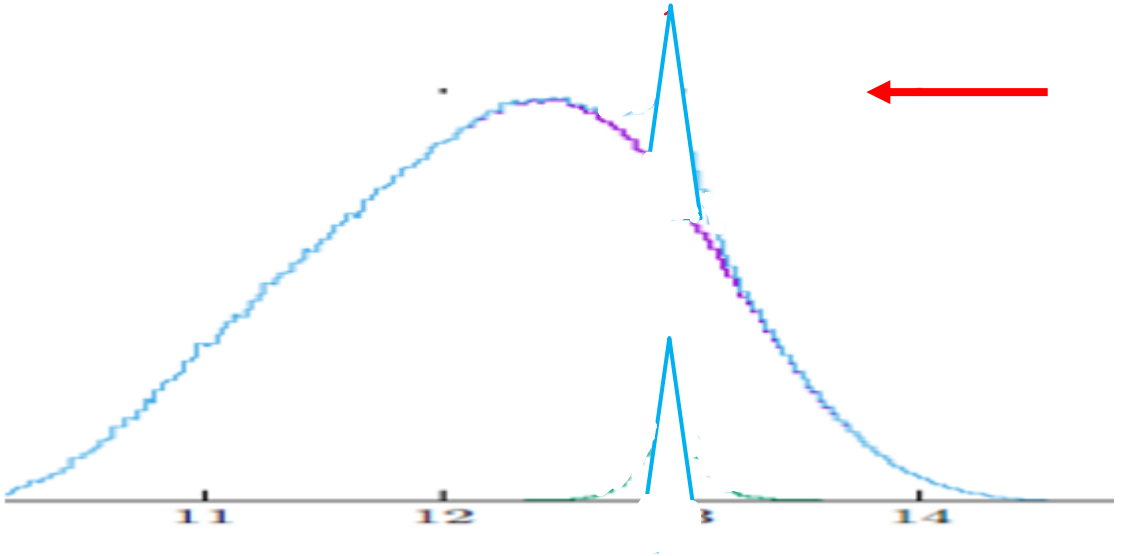
This uncertainty in beam energy must be added to the other factors which limit the invariant mass of the e^+ e^- pairs that we detect.
This is small compared to the observed ~1 MeV width of the Atomki resonance but most of that observed width is instrumental. The intrinsic width of the X17 resonance might be very small, if the decay into e+ e- were the dominant decay mode.

If the intrinsic width were small, then better resolution helps to make the signal show up over the background.



Suppose we could achieve
2-fold improvement in
Invariant mass resolution

Instead of this



we see this.

Signal to background
Improves by factor of 2

It was previously said that we don't need great momentum resolution in the spectrometer, that $\Delta p/p \sim 10^{-3}$ is sufficient. Why is this?
If we could

1) achieve better overall invariant mass resolution by a factor of 2 by improving the intrinsic resolution of each spectrometer,

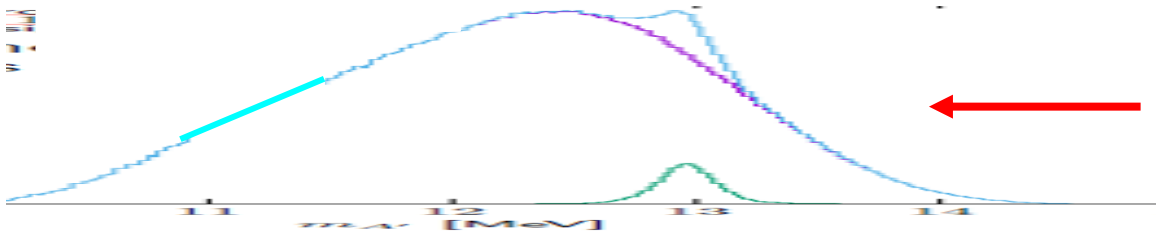
2) improve the overall mass resolution some more by using a horizontally dispersed beam and correcting for the variation in incident beam energy event-by-event

and

3) reduce the accidental background by tracing back to the target position in each spectrometer and eliminate events where the two leptons don't match positions

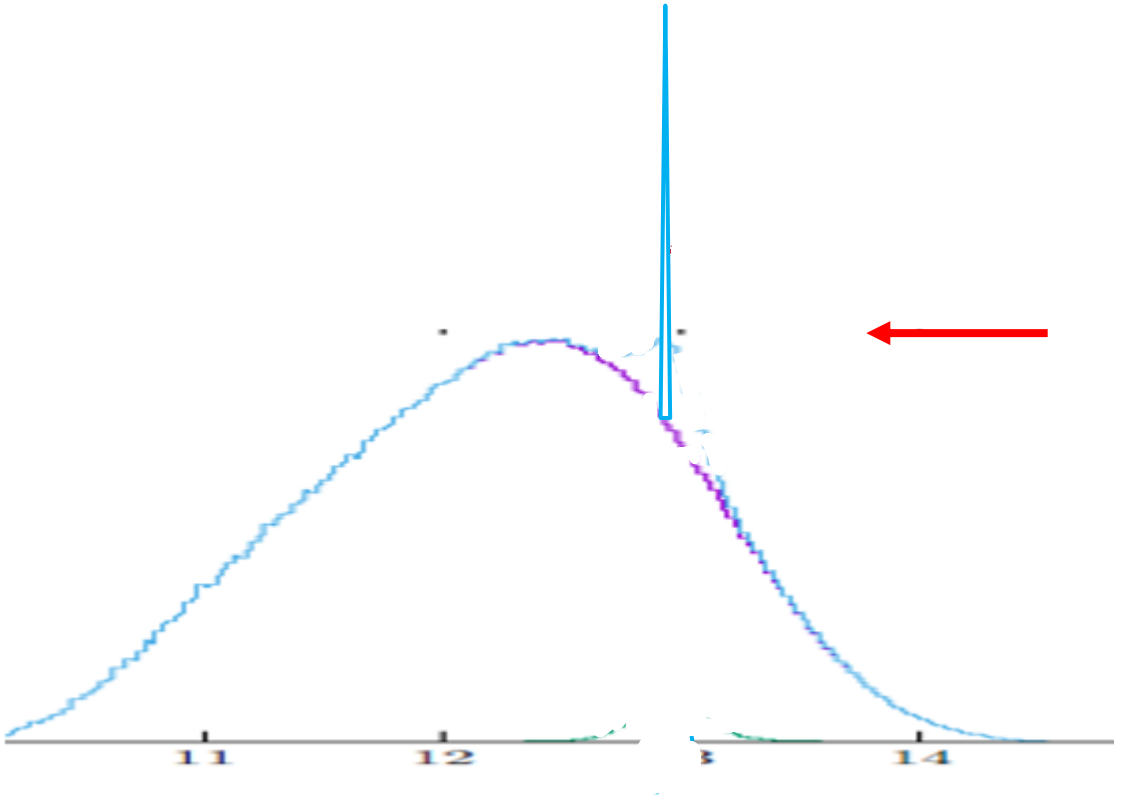
it may be possible to improve the signal to background by an order of magnitude.

Worth further study?



Suppose we could achieve
10-fold improvement in
Invariant mass resolution

Instead of this



we see this.

Signal to background
Improves by factor of 10

Overall big question:

Q10: Mass resolution is limiting the sensitivity of the experiment.
What is limiting the mass resolution in the proposed setup?
Is there anything we can do with regards to beamline or spectrometer design to improve this?

Is it worth putting more effort into this?