LA-UR-22-27316

# Generalized Contact Formalism – Recent Advances

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SRC collaboration meeting, MIT, August 2022

# Generalized Contact Formalism

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

**universal function**

For any short-range two-body operator $\hat{O}$ (assuming that acts on protons):

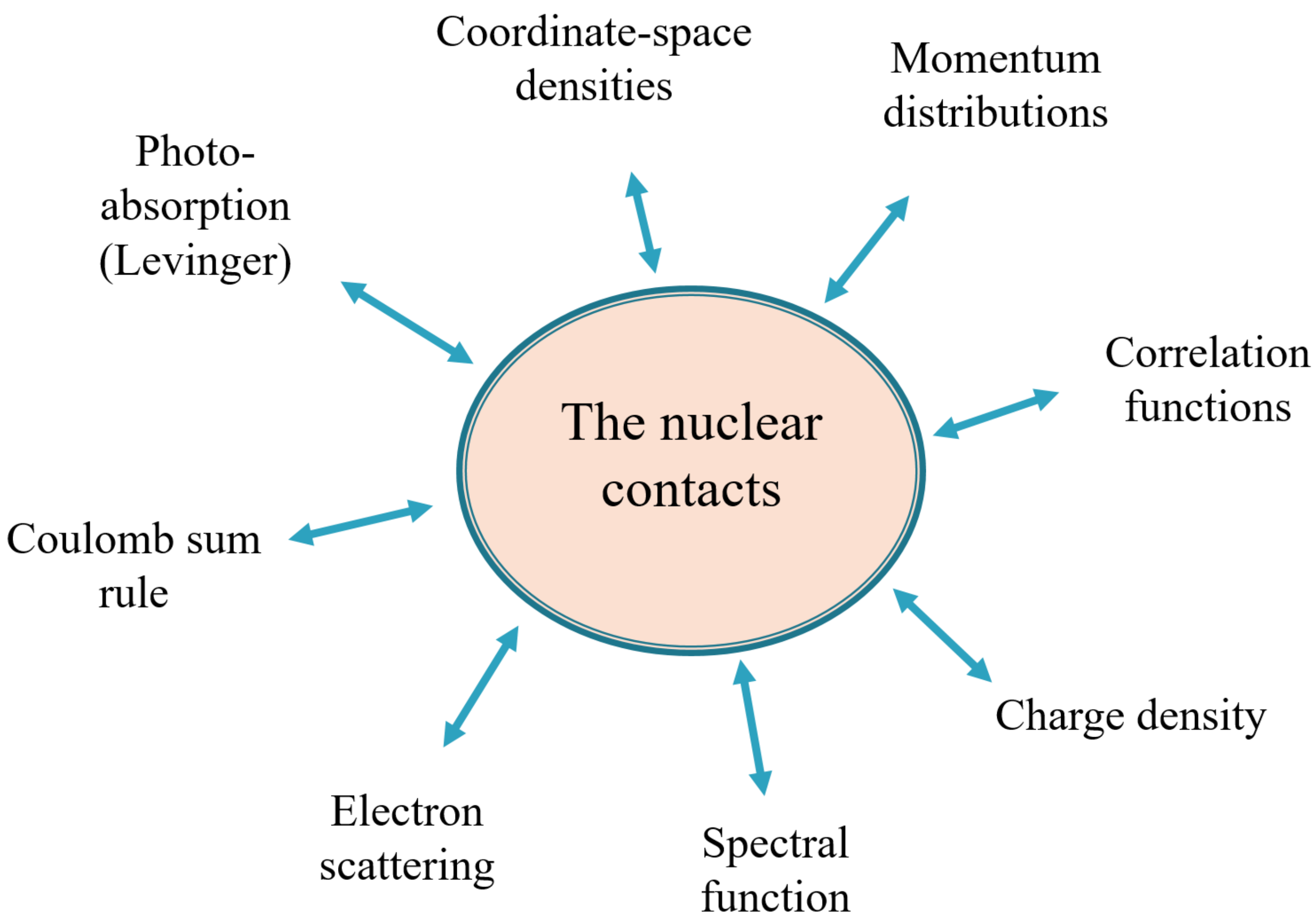
$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C_{pp}$$

- Two-body dynamics
- Universal for all nuclei
- Simply calculated

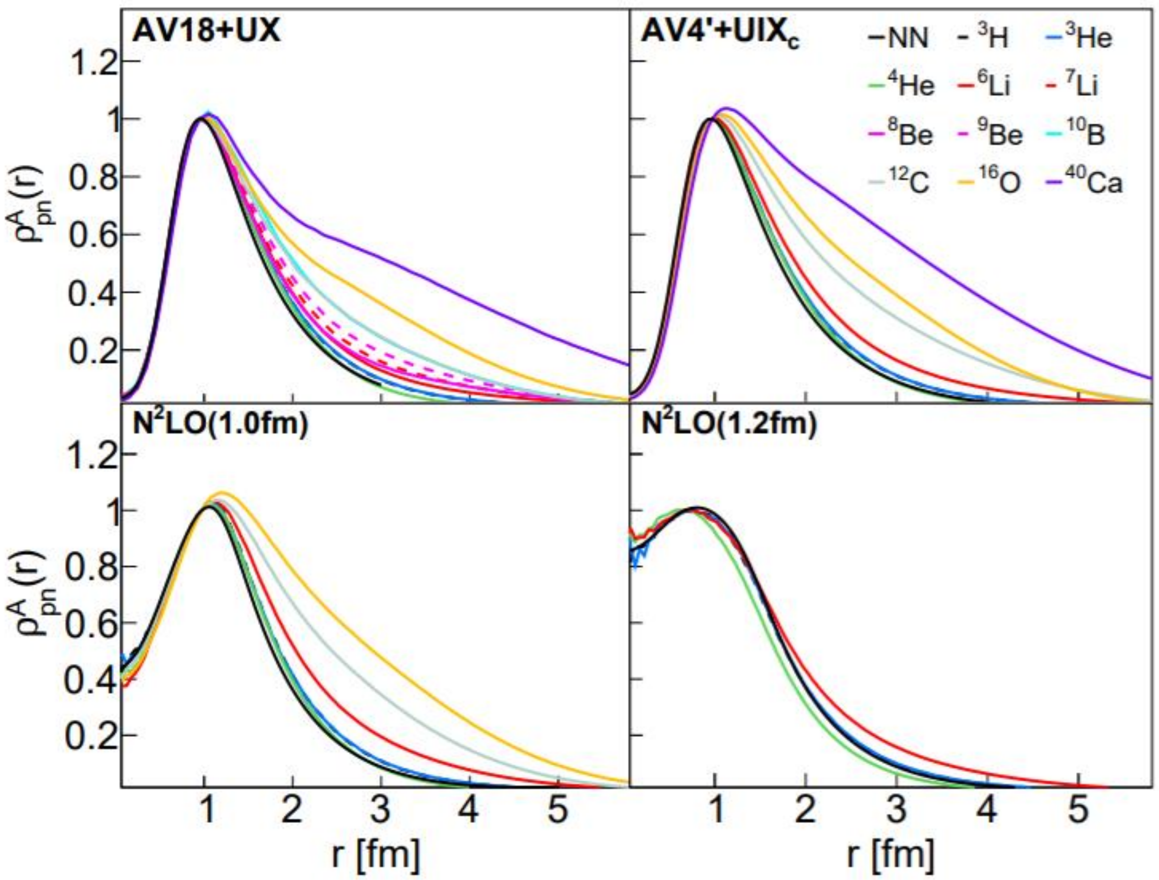
- Depends on the nucleus
- Independent of the operator
- Might be difficult to calculate directly for heavy nuclei

*RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)*

# The nuclear contact relations



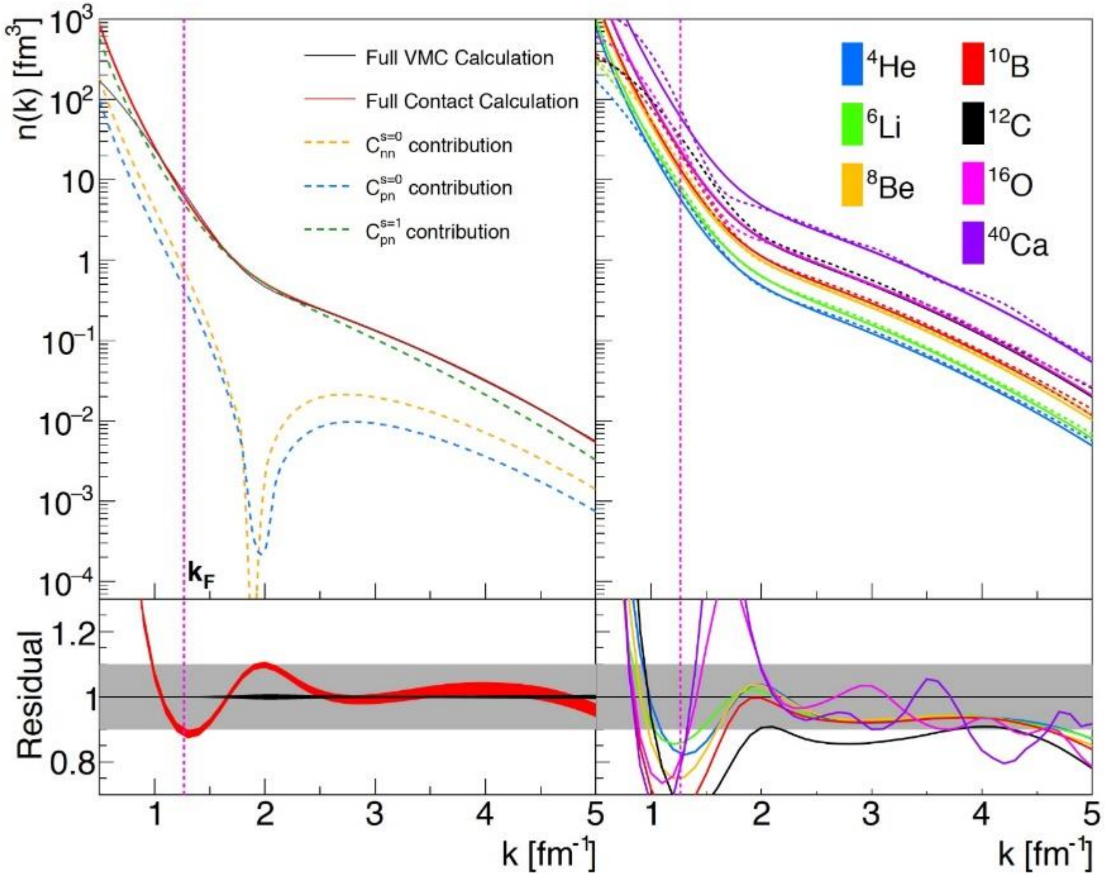
# Two-body density




*R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)*

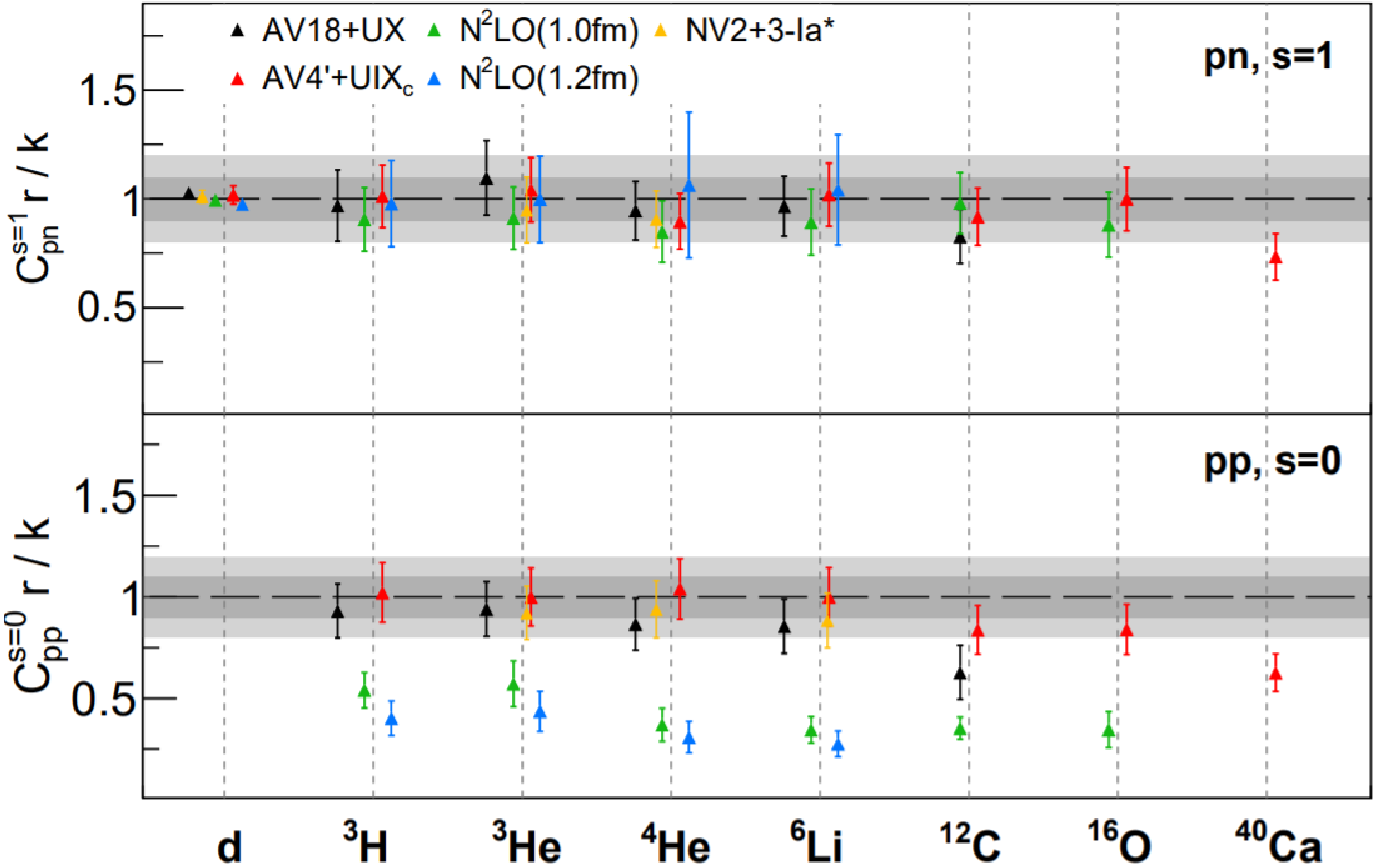
# Momentum distributions

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^d \left|\varphi_{pn}^d(k)\right|^2 + C_{pn}^0 \left|\varphi_{pn}^0(k)\right|^2 + 2C_{pp}^0 \left|\varphi_{pp}^0(k)\right|^2$$




*RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)*

# Consistency: k-space vs r-space



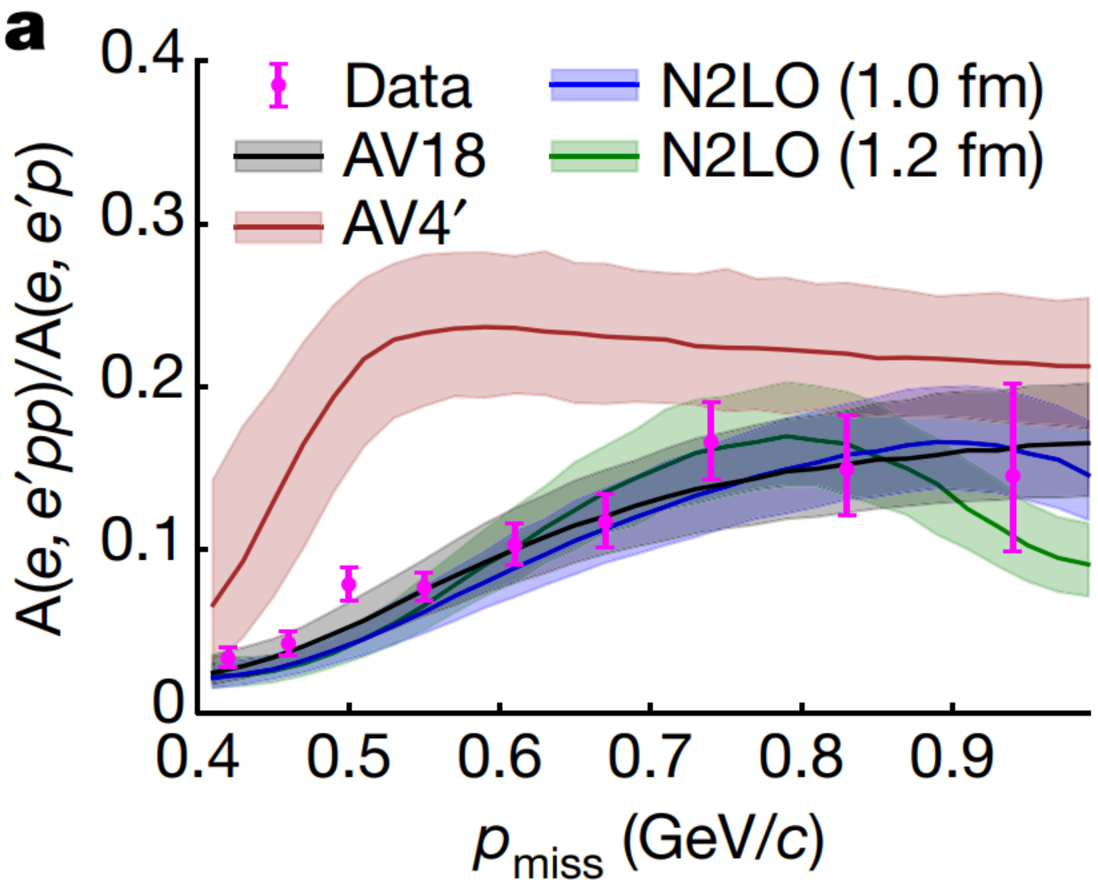

*R. Cruz-Torres, D. Lonardoni, RW, et al., arXiv: 1907.03658 [nucl-th], Nature Physics (2020)*

# Exclusive reactions – Event generator

- Detailed experimental data compared to GCF predictions

Using the VMC contact values

Direct connection to ab-initio calculations and NN-interaction models




$^{12}C$ - #pp/#p

*A. Schmidt, J.R. Pybus, RW, et al., Nature 578, 540 (2020)*

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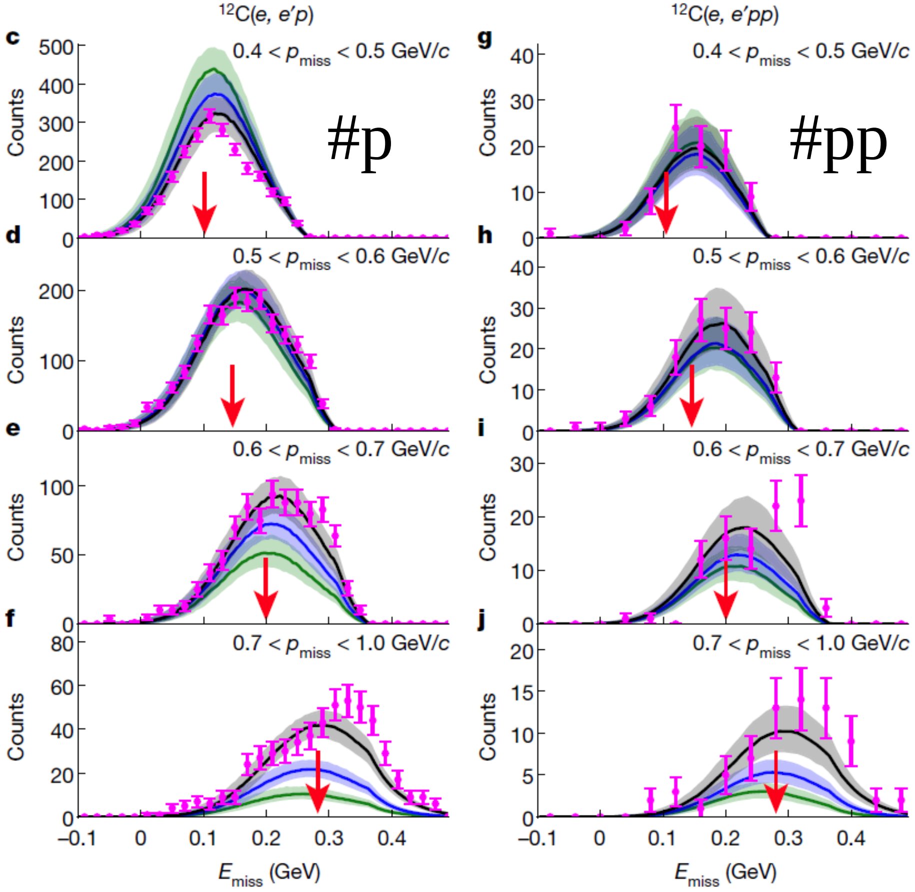


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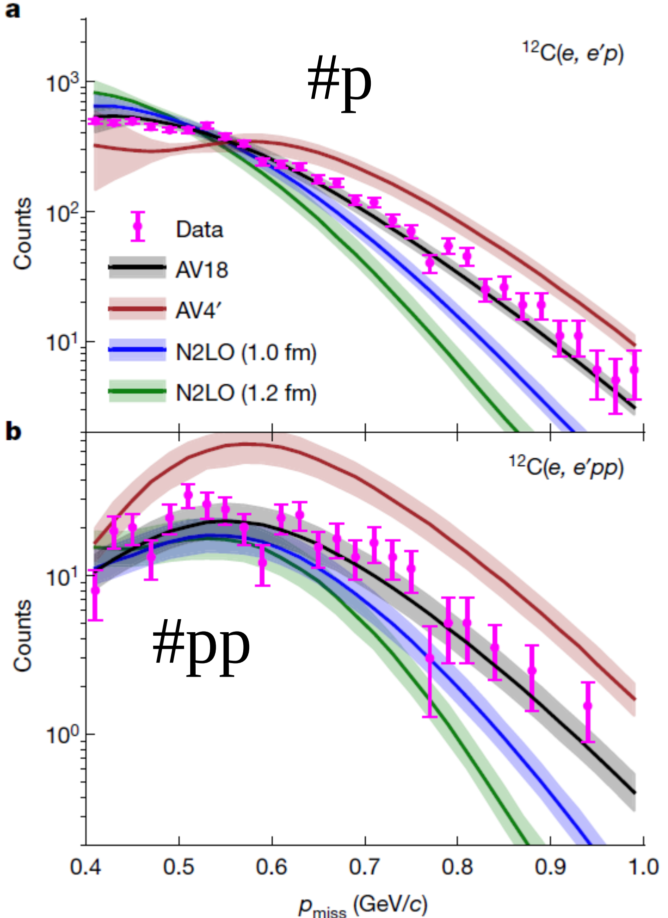
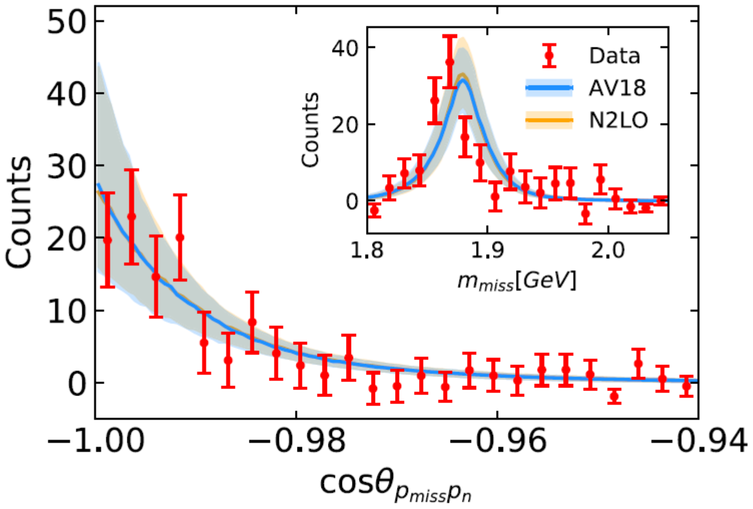
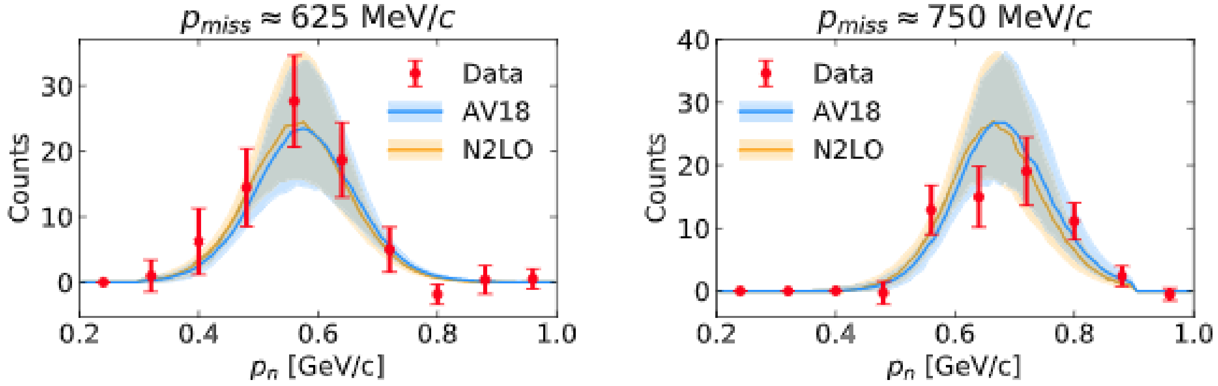
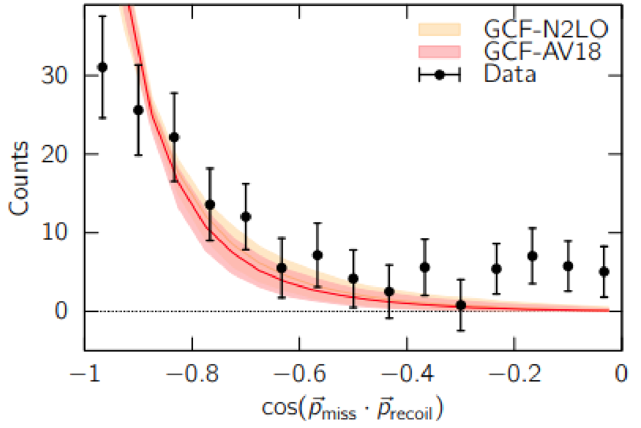


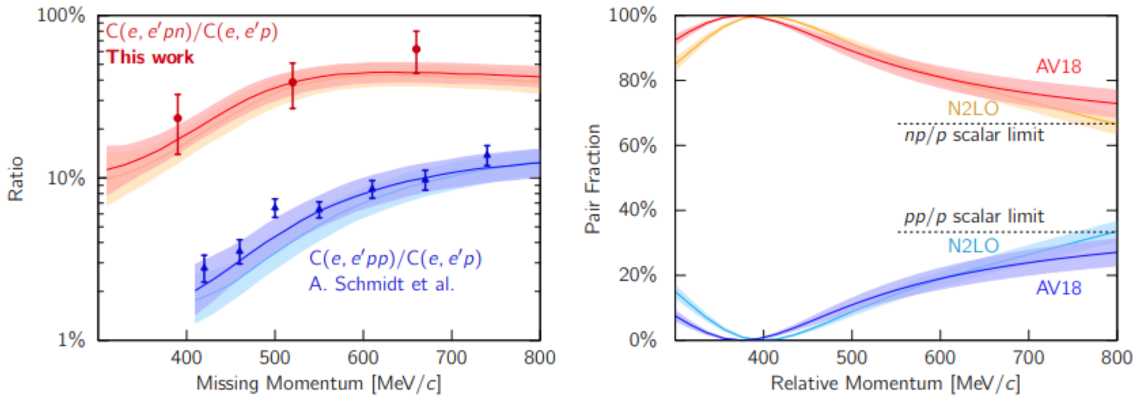
# Exclusive reactions – Event generator




*J.R. Pybus et al., PLB 805, 135429 (2020)*




*I. Korover et al., arXiv:2004.07304 (2020)*

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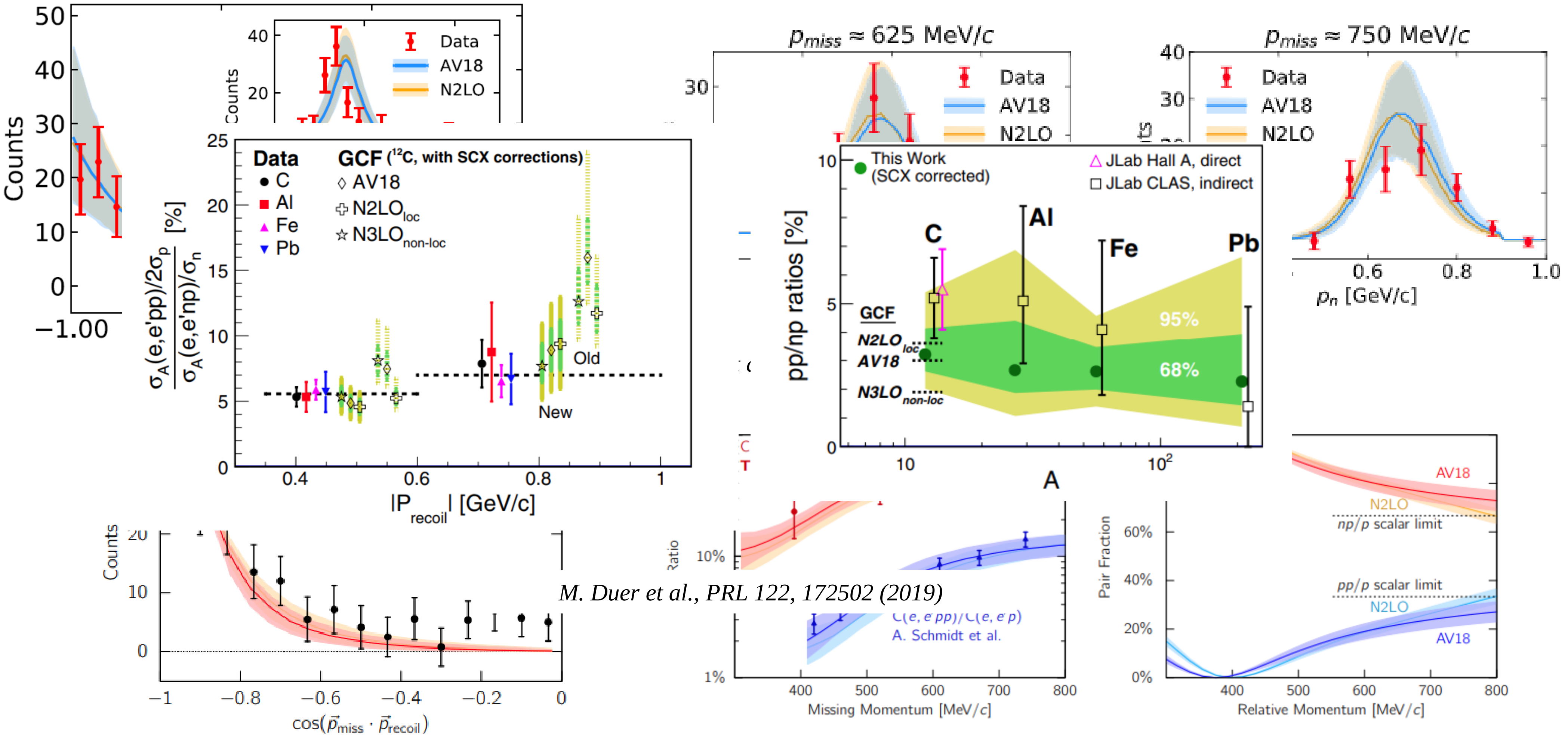



*M. Duer et al., PRL 122, 172502 (2019)*

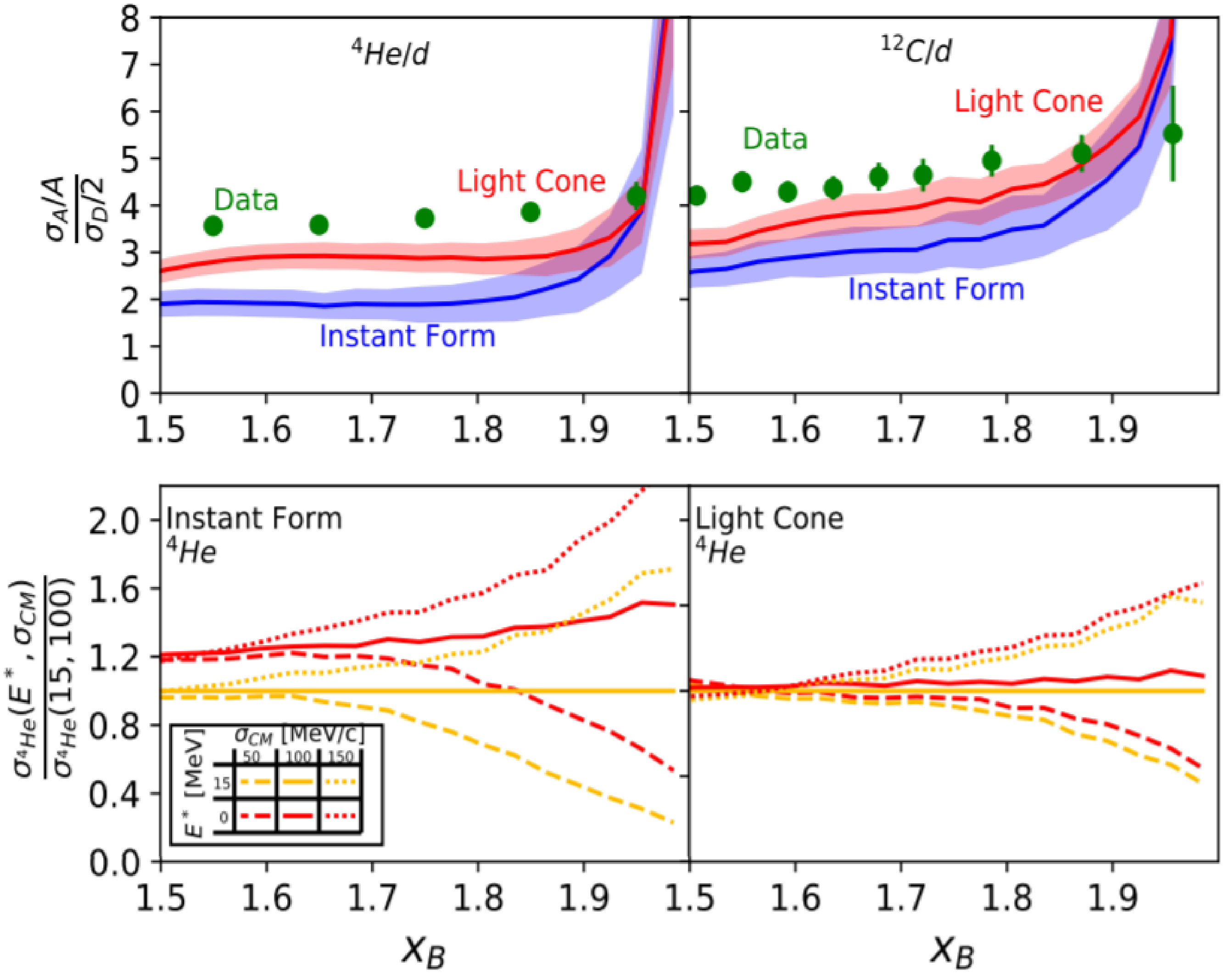
*I. Korover et al., arXiv:2004.07304 (2020)*

# Inclusive reactions

Using the VMC contact values

Possible explanations:

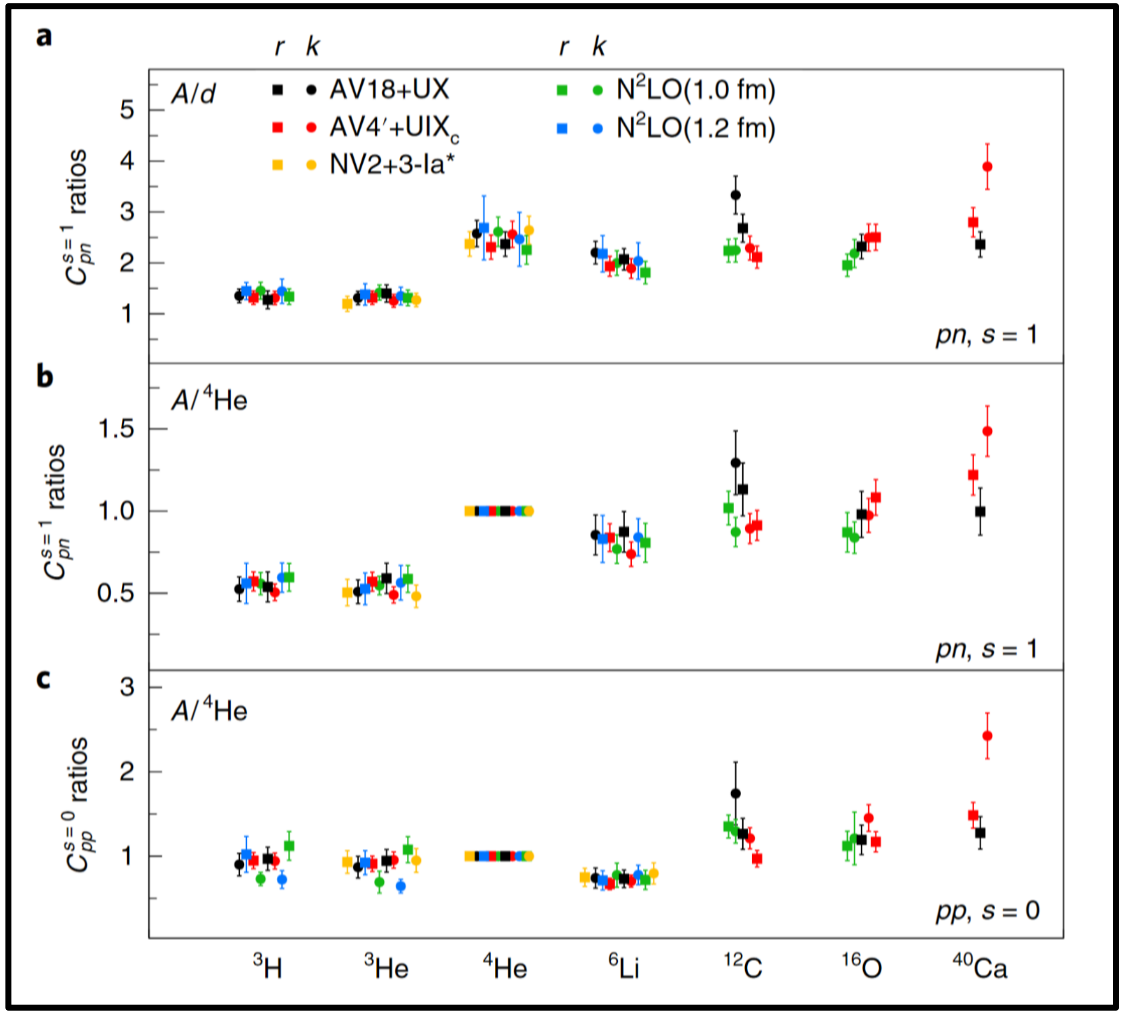
- FSI
- Non-impulse-approximation contributions
- Mean-field contribution
- Relativistic effects
- 3N SRCs?
- NN interaction / theoretical contact values




AV18

RW, A. W. Denniston et. al., *PRC 103, L031301 (2021)*

# Model independence of contact ratios




$$\frac{C^{V_1}(X)}{C^{V_1}(Y)} = \frac{C^{V_2}(X)}{C^{V_2}(Y)}$$

# Neutrinoless double beta decay

RW, P. Soriano, A. Lovato, J. Menendez, R. B. Wiringa, arXiv:2112.08146 [nucl-th]

# Neutrinoless double beta decay

$$nn \to pp + 2e$$

Measurement of the decay will provide information about:

- Majorana nature of neutrinos
- Matter dominance of the universe
- Neutrino mass
- …

**Nuclear matrix elements (NMEs) are needed**

$$^{76}\mathrm{Ge},\ ^{100}\mathrm{Mo},\ ^{130}\mathrm{Te},\ ^{136}\mathrm{Xe}$$

# NMEs - Methods

| Shell model | Quasiparticle random phase approximation | Energy density functional theory | Interacting-boson model |
| --- | --- | --- | --- |

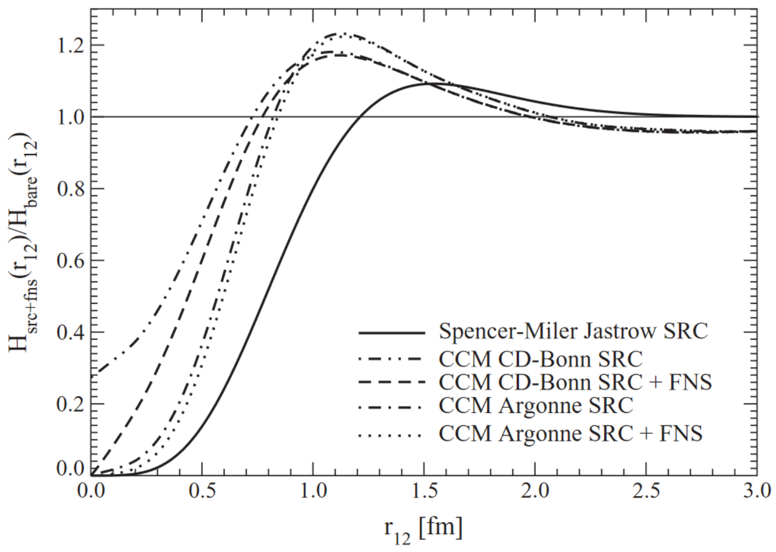
- Describe well **long-range properties** of nuclei
- Missing **short-range** correlations

# Shell model + correlation functions

$$M = \langle SM | f(r)\, \hat{O}\, f(r) | SM \rangle$$

- Correlation function - Main features:
  - reduction at short distances
  - peak around 1 fm
  - $f(r) \rightarrow 1$ for r $\rightarrow \infty$

$$f(r) = 1 - ce^{-ar^2}(1 - br^2)$$




F. Simkovic et al., PRC 79, 055501 (2009)

- Extracted for example from coupled-cluster calculations:

$$|\Psi\rangle = e^{\hat{T}} |\Phi\rangle$$

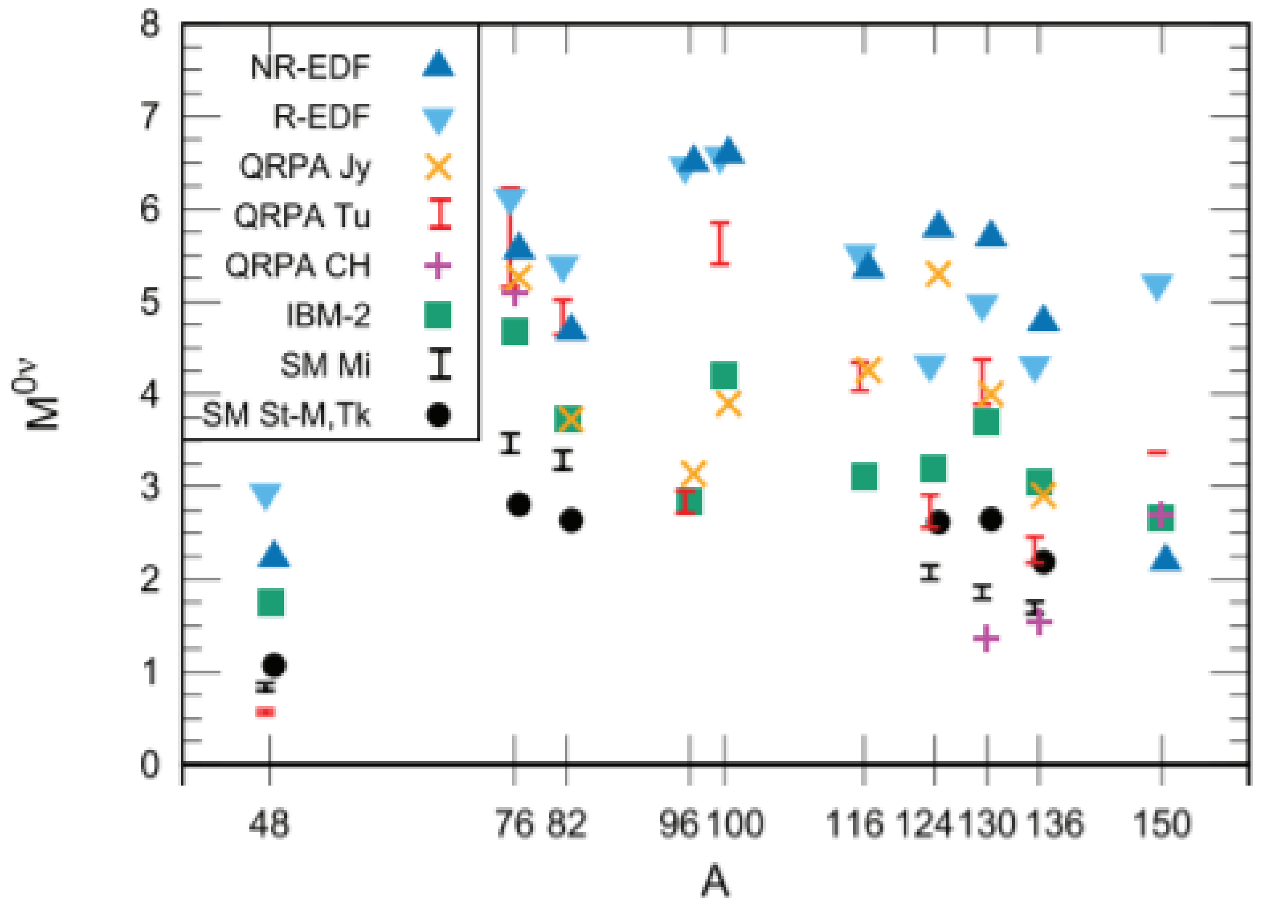
- Possible inconsistencies:

$$|\mathrm{SM}\rangle \neq |\Phi\rangle$$

More consistent approaches – evolved effective operator (Coraggio, Engel,…)

# Neutrinoless double beta decay

Nuclear matrix elements




J. Engel and J. Menendez, Rep. Prog. Phys. 80 046301 (2017)

Very different values of matrix elements

# NMEs – ab-initio methods

Based on single-particle basis expansion:

Coupled Cluster

VS-IMSRG

IMSRG + GCM

- All applied for $^{48}Ca$
- $^{76}Ge$ and $^{82}Se$ using VS-IMSRG
- Relatively small values of NMEs
- Used with “soft” interactions → possibly larger contribution of two-body nuclear currents

# NMEs – ab-initio methods

Quantum Monte Carlo (Variational Monte Carlo):

- Very accurate
- Can be applied to both “soft” and “hard” (local) interactions
- Captures well short-range dynamics
- Limited to $A \leq 12$ nuclei for $0\nu2\beta$

X.B. Wang, A.C. Hayes, J. Carlson, G.X. Dong, E. Mereghetti, S. Pastore, R.B. Wiringa, PLB 798, 134974 (2019)

# Our approach: GCF-SM method

| Quantum Monte Carlo | + | Shell Model | + | Generalized Contact Formalism |
|---|---|---|---|---|
| Accurate solution for light nuclei | | Long-range physics | | Short-range physics |

RW, P. Soriano, A. Lovato, J. Menendez, R. B. Wiringa, arXiv:2112.08146 [nucl-th]

# NMEs and transition densities

Light Majorana neutrino exchange mechanism

$$M^{0\nu} = \langle \Psi_f | O^{0\nu} | \Psi_i \rangle$$

$$O^{0\nu} = O_F^{0\nu} + O_{GT}^{0\nu} + O_T^{0\nu} + O_S^{0\nu}$$

$$4\pi r^2 \rho_F(r) = \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \tau_a^+ \tau_b^+ | \Psi_i \rangle$$

$$C_\alpha^{0\nu}(r) \equiv (8\pi R_A) 4\pi r^2 \rho_\alpha(r) V_\alpha^{0\nu}(r)$$

$$M_\alpha^{0\nu} = \int_0^\infty dr\, C_\alpha^{0\nu}(r)$$

# The GCF-SM method

$$\rho_\alpha(r)$$

$r < 1$ fm

GCF

$r > 1$ fm

Shell model

# GCF-SM: Short distances ($r < 1$ fm)

- Fermi density for example:

$$\rho_F(r) = \frac{1}{4\pi r^2} \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \tau_a^+ \tau_b^+ | \Psi_i \rangle$$

- New contacts

$$C(f,i) = \frac{A(A-1)}{2} \langle A(f) | A(i) \rangle$$

$$\rho_F(r) \to \frac{1}{4\pi} |\phi(r)|^2 C(f,i)$$

$$\rho_{GT}(r) \to -\frac{3}{4\pi} |\phi(r)|^2 C(f,i)$$

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The values of the contacts are needed

# Model independence of contact ratios

- For $0\nu 2\beta$:

$$\frac{C^{AV18}(f_1, i_1)}{C^{AV18}(f_2, i_2)} = \frac{C^{SM}(f_1, i_1)}{C^{SM}(f_2, i_2)}$$

$$C^{AV18}(f_1, i_1) = \frac{C^{SM}(f_1, i_1)}{C^{SM}(f_2, i_2)} C^{AV18}(f_2, i_2)$$

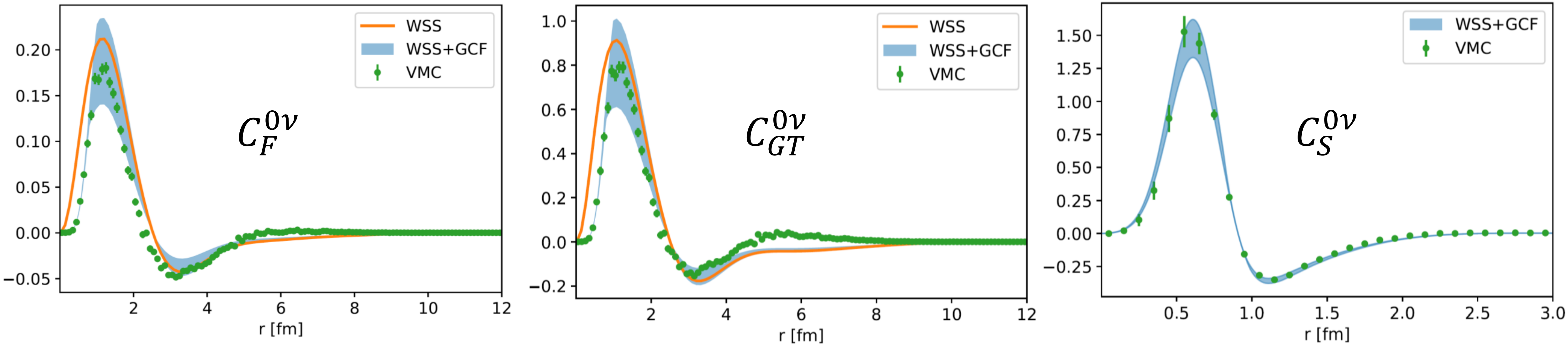
- For example

Exact QMC calculations

$$C^{AV18}\left({}^{76}\text{Ge} \rightarrow {}^{76}\text{Se}\right) = \frac{C^{SM}\left({}^{76}\text{Ge} \rightarrow {}^{76}\text{Se}\right)}{C^{SM}\left({}^{12}\text{Be} \rightarrow {}^{12}\text{C}\right)} C^{AV18}\left({}^{12}\text{Be} \rightarrow {}^{12}\text{C}\right)$$

# Validation using light nuclei (AV18)

Using $^{6}He \rightarrow {}^{6}Be$ and $^{10}Be \rightarrow {}^{10}C$ to "predict" $^{12}Be \rightarrow {}^{12}C$




Short distances - GCF

Long distances – Shell model

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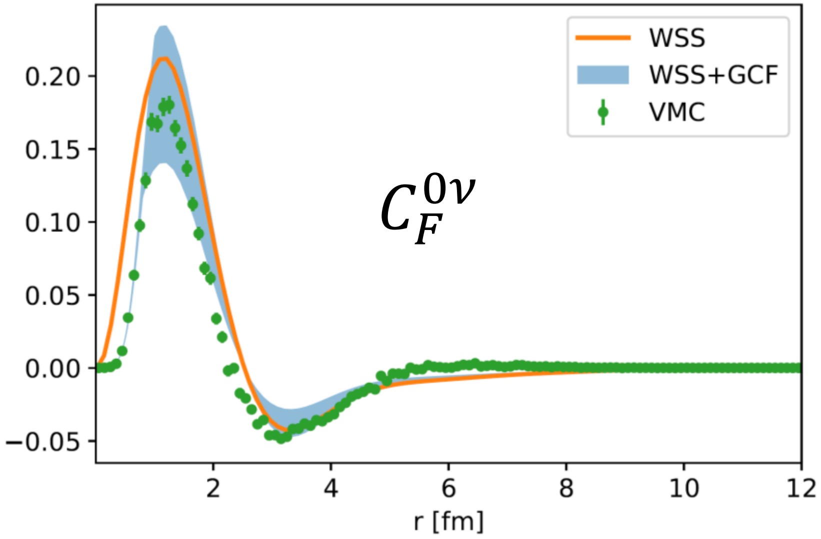
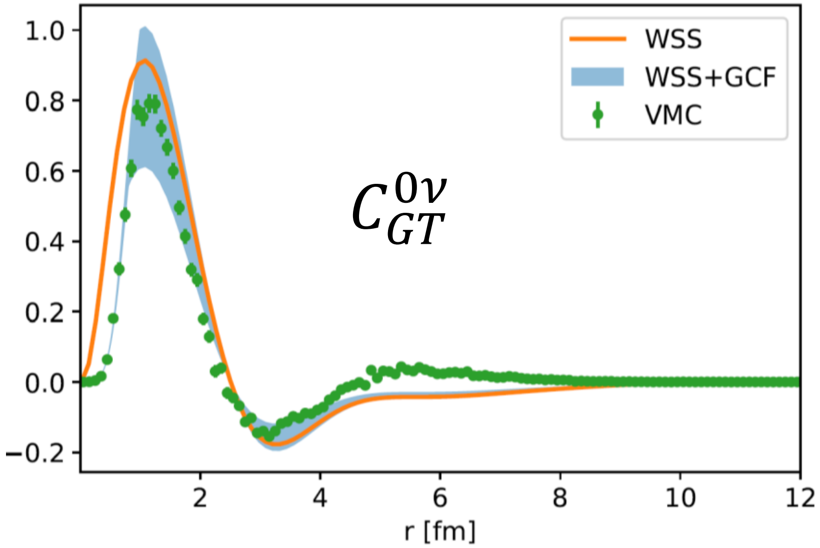
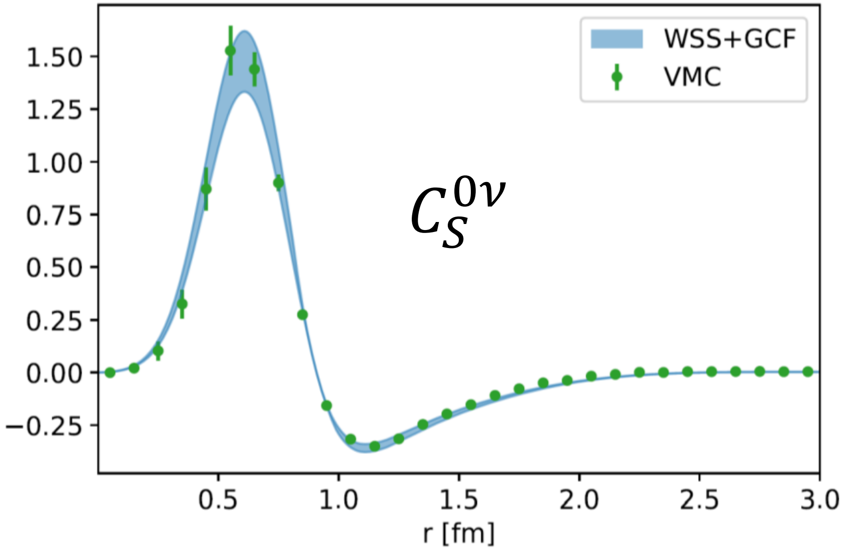


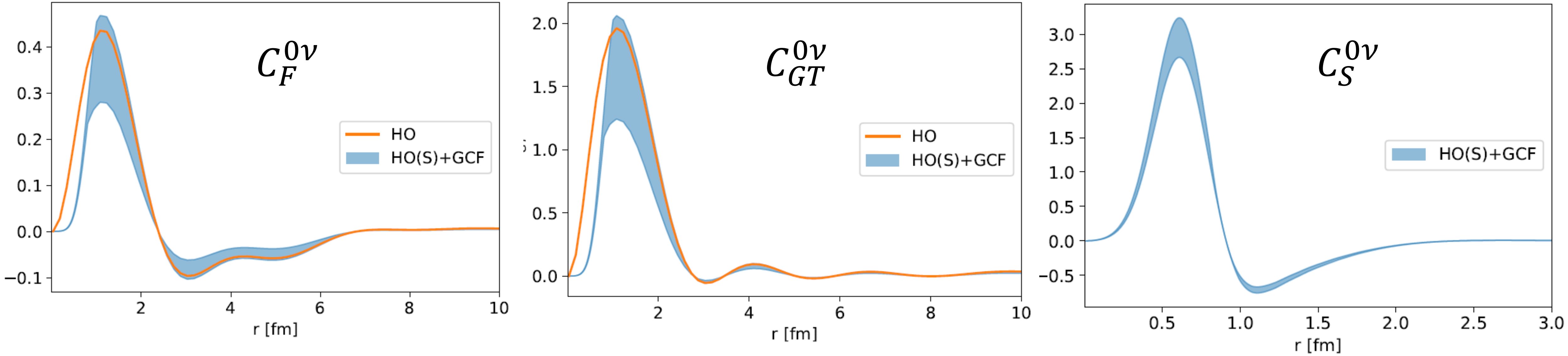

Short distances - GCF

Long distances – Shell model

Uncertainty band: 10% on the contact value + varying matching point $(0.8 - 1\ \text{fm})$

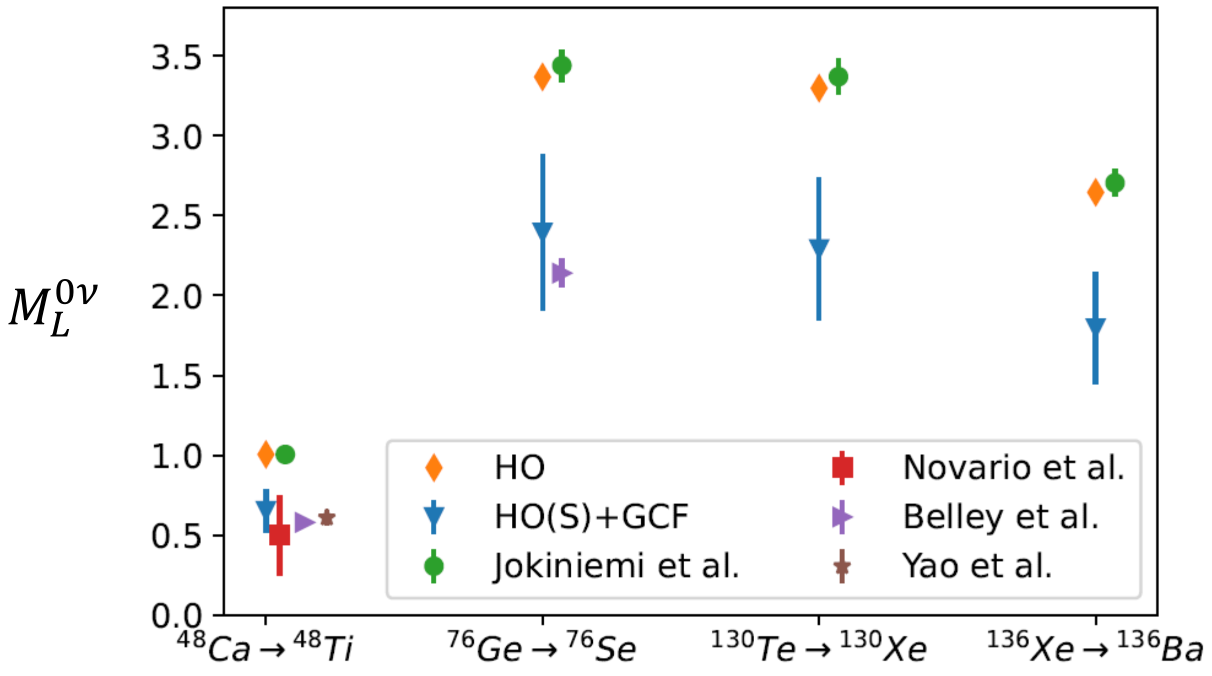
# Results – heavy nuclei (AV18)

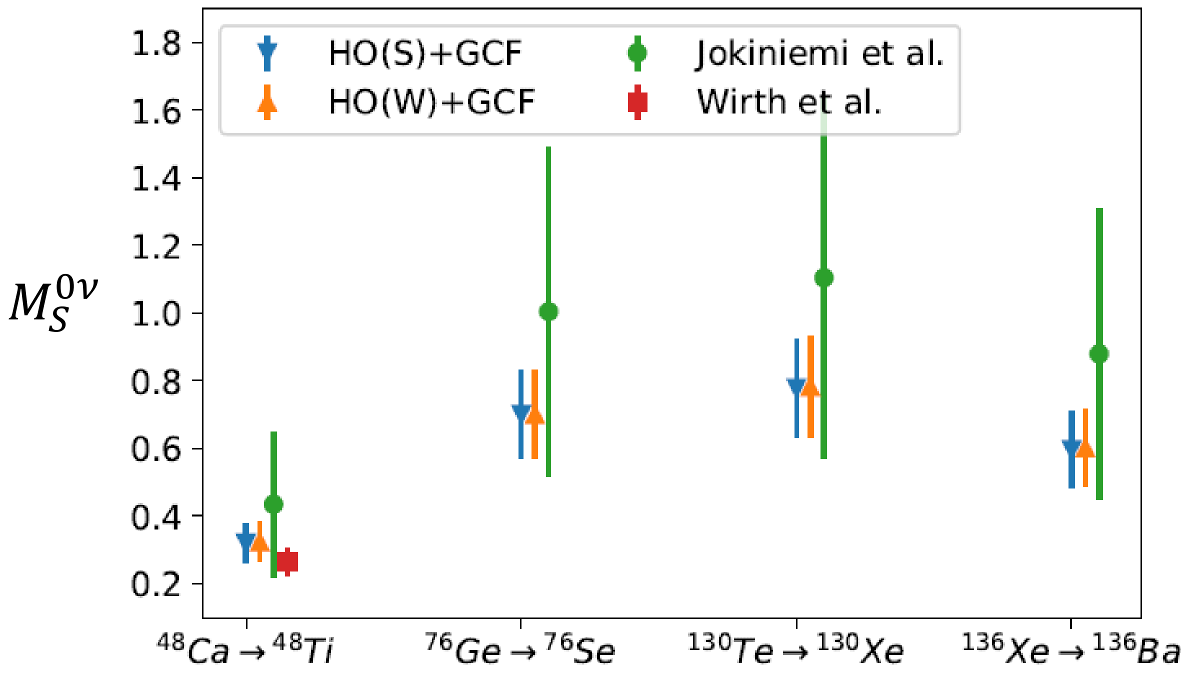
- Transition densities (using $A = 6, 10, 12$ to predict heavy nuclei):




$^{76}Ge \rightarrow {}^{76}Se$

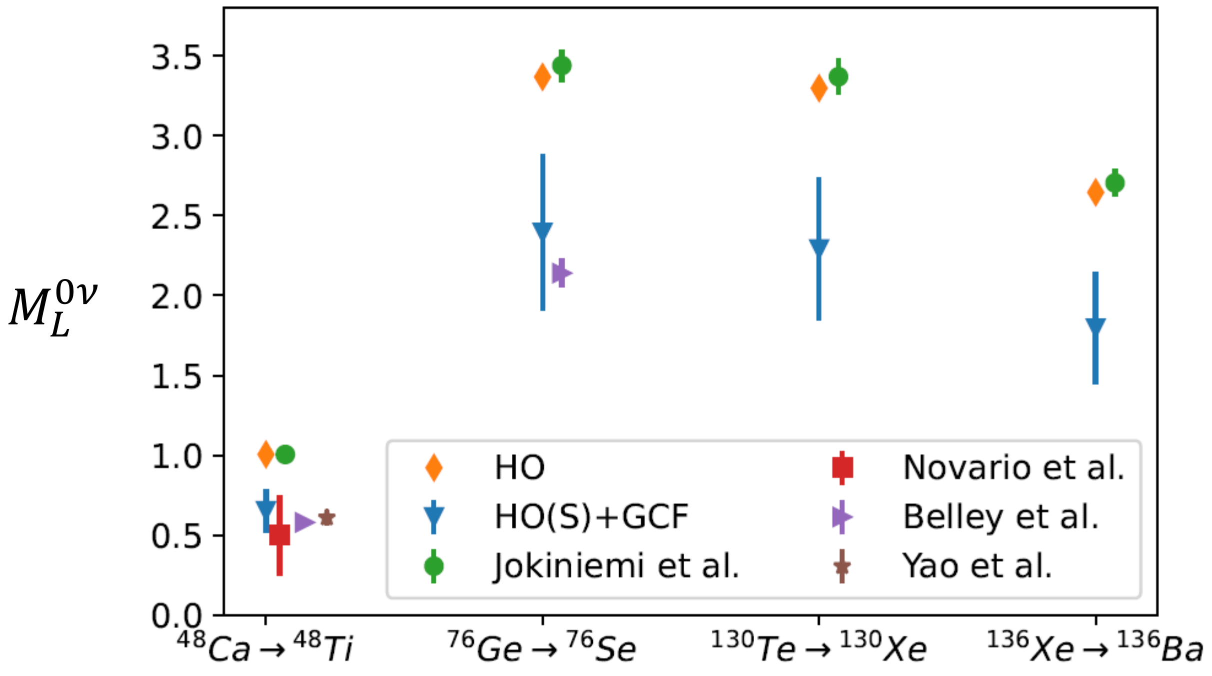
# Results – heavy nuclei (AV18)




Significant reduction due to SRCs

# Results – heavy nuclei (AV18)




Significant reduction due to SRCs

Next:

- Model and cutoff dependence - chiral interactions
- Include 3N-SRCs and other corrections
- Detailed comparison with other methods (using the same interaction)
- Tensor matrix elements

# Isospin symmetry

RW, A. Lovato, R. B. Wiringa, arXiv:2206.14235 [nucl-th]

# Isospin symmetry

- Derived relations based on isospin symmetry involving two-body densities

$$\rho_{t,t_z}(r) = \frac{A(A-1)}{2}\frac{1}{4\pi r^2}\langle\Psi\left|\delta(r-r_{12})\hat{P}_{12}^{t,t_z}\right|\Psi\rangle$$

- For $T = 0$ nuclei:

$$\rho_{1,1}(r) = \rho_{1,0}(r) = \rho_{1,-1}(r)$$

- For $T = 1/2$ nuclei:

$$2\rho_{1,0}(r) = \rho_{1,1}(r) + \rho_{1,-1}(r)$$

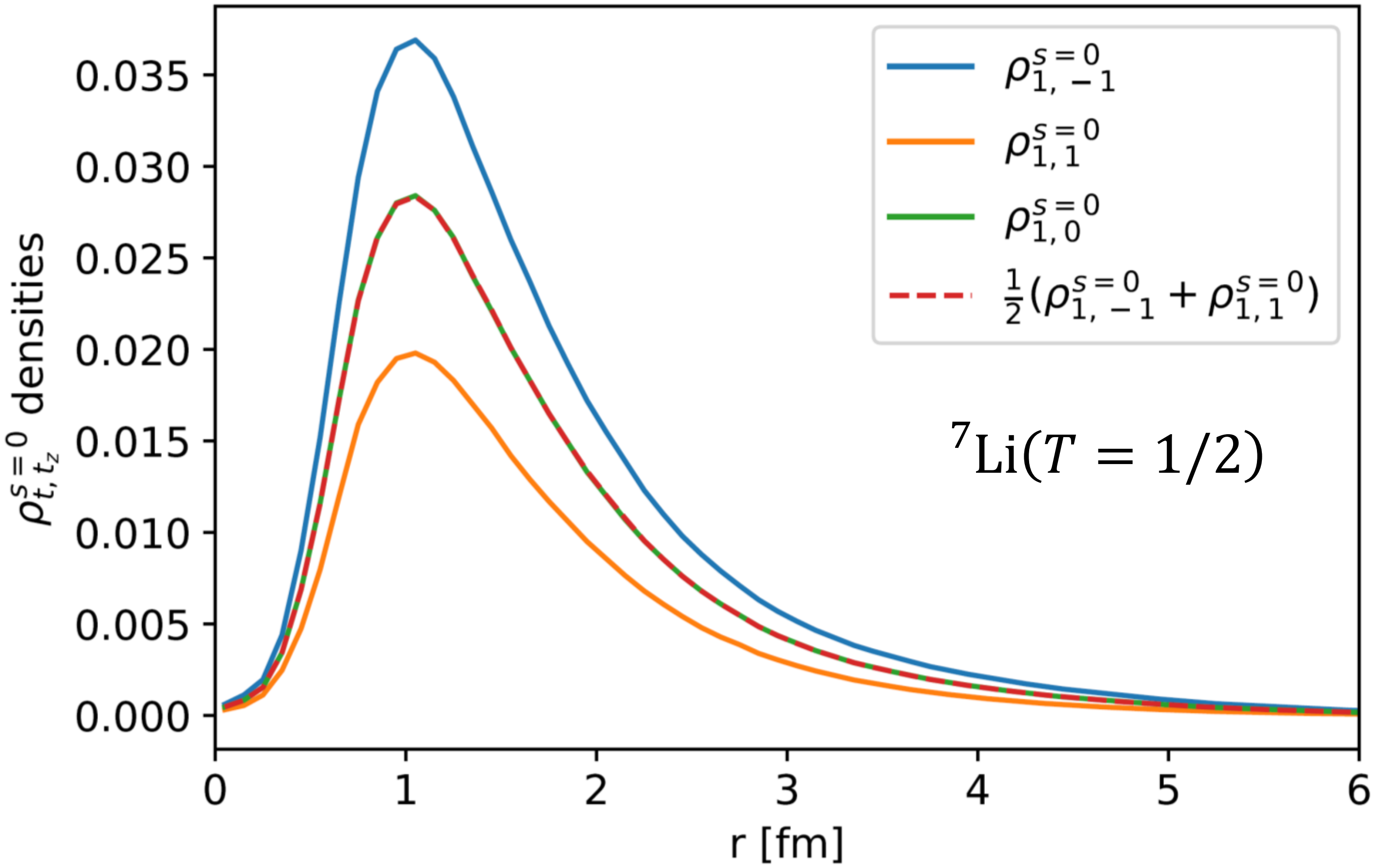
- For $T \geq 1$ nuclei:

No relation exist!

Equivalent relations hold in momentum space

# Isospin symmetry

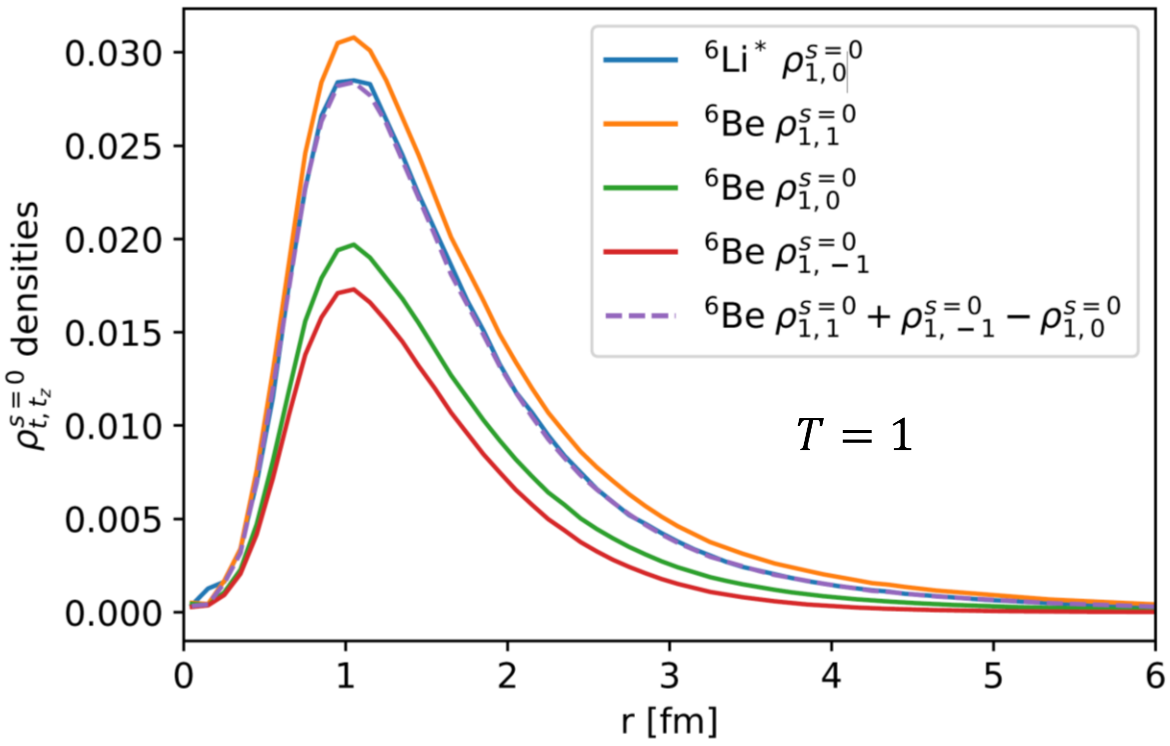
- For $T = 1/2$ nuclei: $$2\rho_{1,0}(r) = \rho_{1,1}(r) + \rho_{1,-1}(r)$$



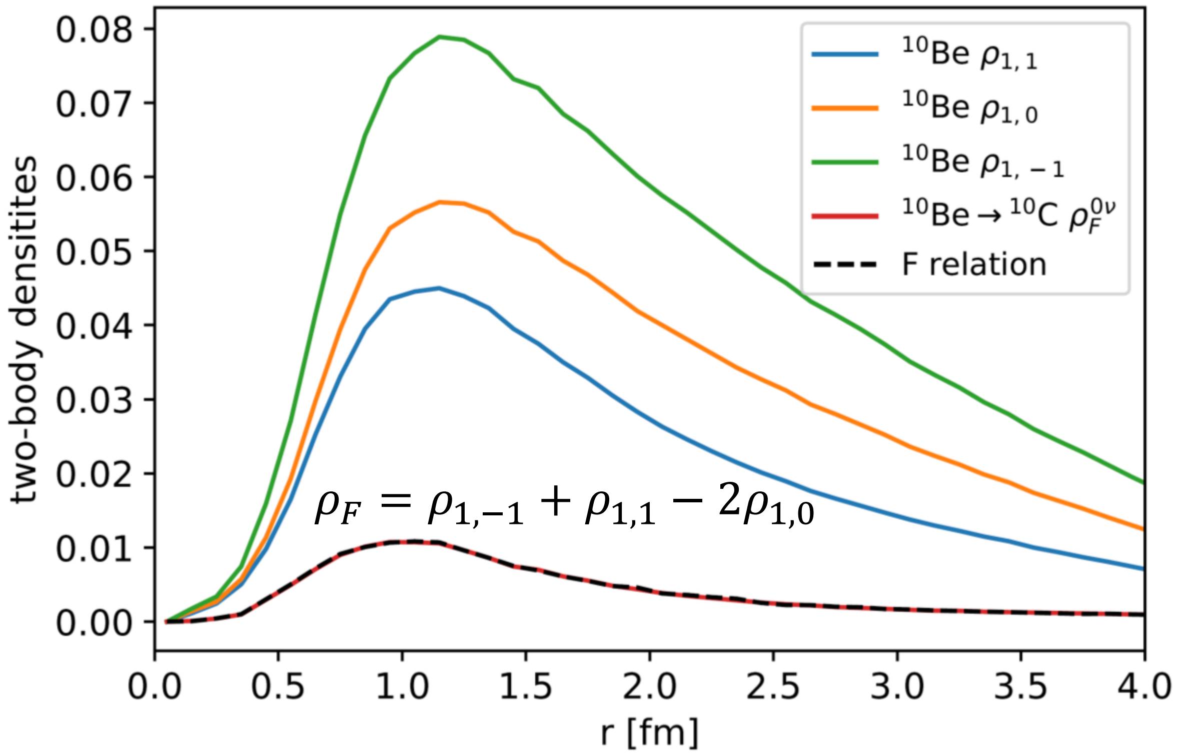
# Isospin symmetry

- Additional relations:

Connecting nuclei in the same isospin multiplet




Connection to $0\nu\beta\beta$ ($\Delta T = 0$ )



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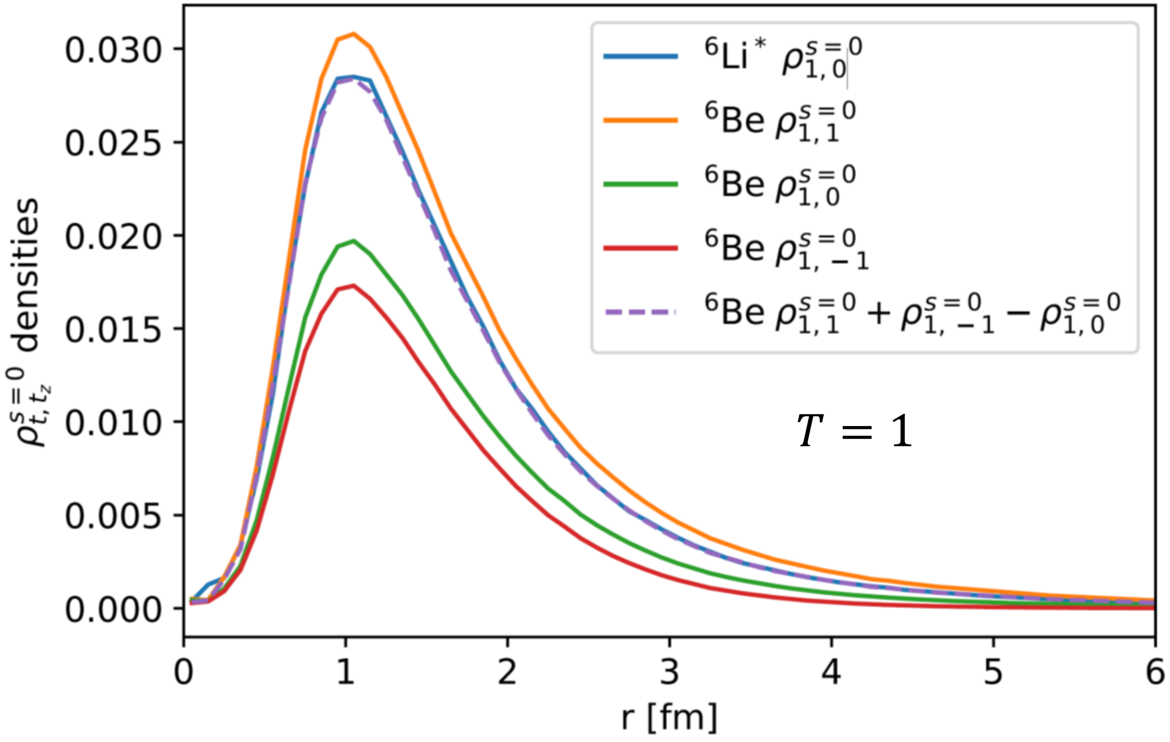



Useful for:

- Benchmarking codes
- Reducing cost of calculation
- Studying isospin-breaking effects
- Obtaining information on excited states using ground-state calculations
- SRC studies

# Isospin symmetry - SRCs

- For $T = 1/2$ nuclei: $2\rho_{1,0}(r) = \rho_{1,1}(r) + \rho_{1,-1}(r)$

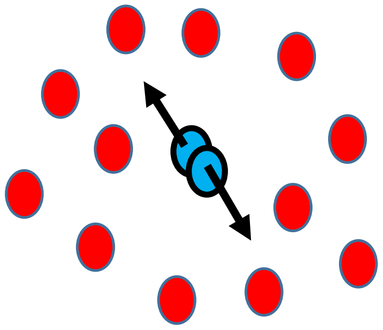
$$\Rightarrow \quad 2C_{t_z=0}^{t=1} = C_{t_z=1}^{t=1} + C_{t_z=-1}^{t=1}$$

Can help in disentangling contribution from different channels in experiments

- $0\nu\beta\beta$ $(\Delta T = 0)$ $\rho_F(r) = \rho_{1,-1}(r) + \rho_{1,1}(r) - 2\rho_{1,0}(r)$

$$\Rightarrow \quad C(f,i) = C_{t_z=-1}^{t=1} + C_{t_z=1}^{t=1} - 2C_{t_z=0}^{t=1}$$

# Isospin symmetry - SRCs



- Information regarding the spectator $(A-2)$ subsystem
- For $pn$ $t=0$ SRC pair: The $A-2$ system must have the same $T$ as $\Psi$
- For $t=1$ pairs there can generally be three values of $T^{A-2}$
- Based on isospin symmetry we get

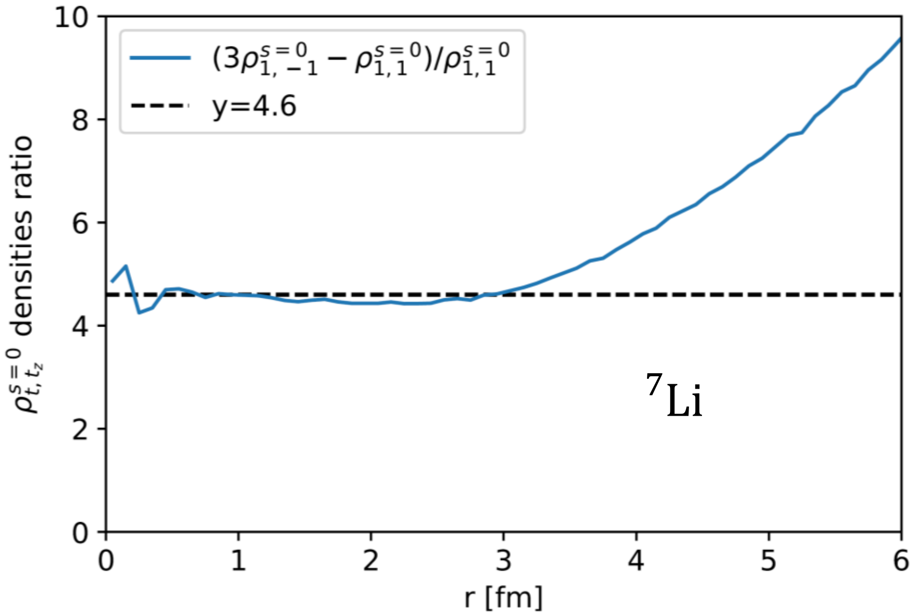
$$C_{t_z}^{t=1} = \frac{A(A-1)}{2} \sum_{T^{A-2}} \left| \langle T^{A-2}\, T_z - t_z\; 1\; t_z | T\; T_z \rangle \right|^2 \langle A(T^{A-2}) | A(T^{A-2}) \rangle$$

We can express the probability that the $A-2$ system is in $T^{A-2}$-state using the contacts

# Isospin symmetry - SRCs

- For example, for $T = 1/2$, $T_z = -1/2$ nucleus - when a $nn$ SRC pairs is formed the $T^{A-2}$ probabilities obey:

$$\frac{P_{nn}\left(T^{A-2} = 1/2\right)}{P_{nn}\left(T^{A-2} = 3/2\right)} = \frac{3C^{t=1}_{t_z=-1} - C^{t=1}_{t_z=1}}{C^{t=1}_{t_z=1}}$$



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- Only the pair need to be detected to extract information on the spectators
- Can be useful to model the $A - 2$ system (relevant for example for spectral function models)
- Can possibly be tested in inverse-kinematics experiments (if $A - 2$ state is identified using gamma detection)

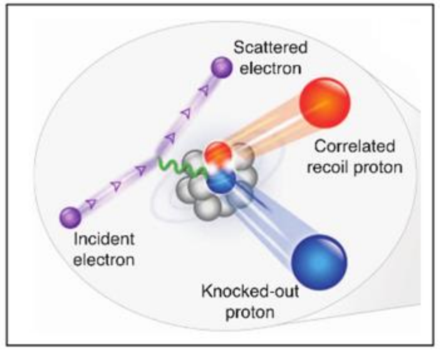
# Future work

- Next order corrections to the GCF
  - Systematic expansion
  - Three-body correlations

- Electron scattering:
  - Beyond the spectral function PWIA description (coherent contributions + FSI)
  - Relativistic effects

- GCF + SM: $0\nu\beta\beta$, single-beta decay, spectral function…

# Back up




# Electron-scattering experiments

- Cross sections can be calculated using **spectral function** (PWIA)

$$S(\boldsymbol{p_1}, \epsilon_1) = \sum_s \sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left| \langle f_{A-1} | a_{\boldsymbol{p_1},s} | \psi_0 \rangle \right|^2$$

- with the GCF:

$$S^p(\boldsymbol{p_1} > k_F, \epsilon_1) = C_{pn}^1 S_{pn}^1(\boldsymbol{p_1}, \epsilon_1) + C_{pn}^0 S_{pn}^0(\boldsymbol{p_1}, \epsilon_1) + 2C_{pp}^0 S_{pp}^0(\boldsymbol{p_1}, \epsilon_1)$$

→ We can calculate cross sections for all nuclei if contact values are known (or fit contact values to experiment)

# Inclusive reactions

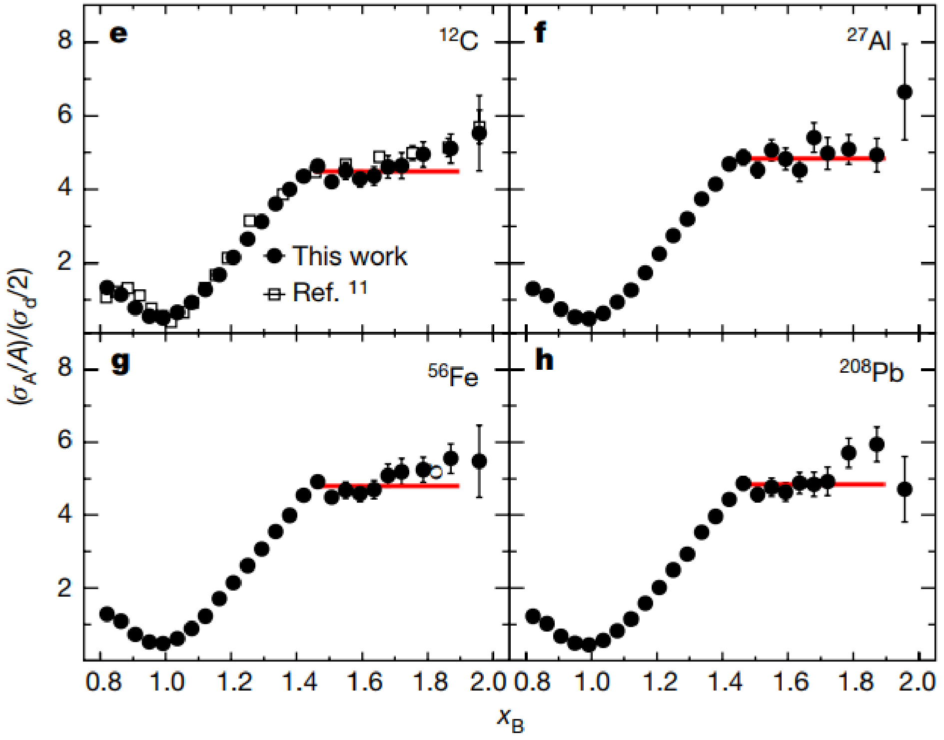
Traditional interpretation of $a_2$:

The number of deuteron-like correlated pairs in nucleus A relative to the deuteron

Interpretation might be affected by:

- CM motion of the pair
- Excitation energy of the $A-2$ system
- Contribution of non-deuteron-like pairs

$a_2$



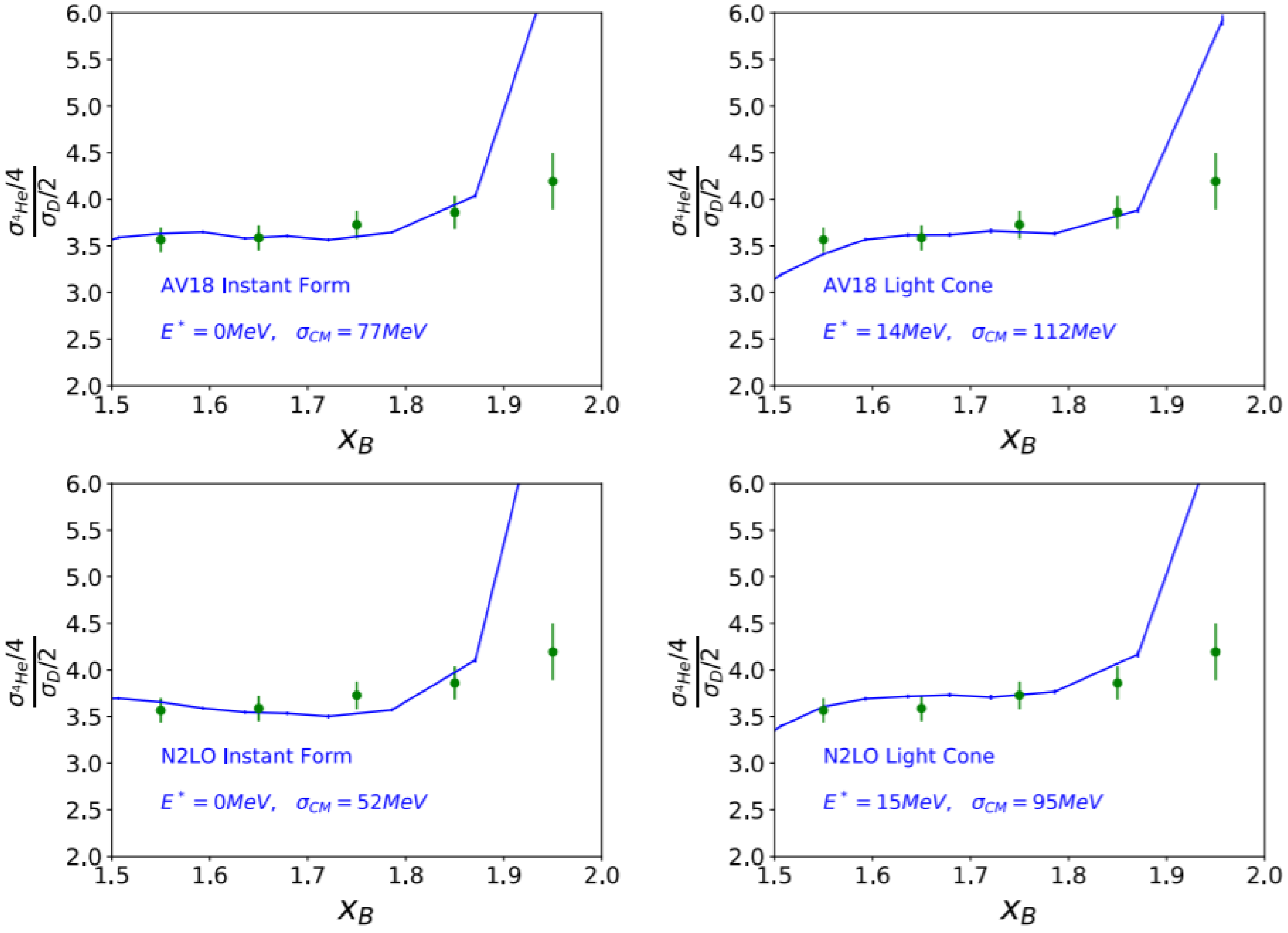

$x_B = Q^2/2m\omega$

B. Schmookler et. al. (CLAS Collaboration) *Nature 566, 354–358 (2019)*

# Inclusive reactions

$^4$He - AV18

Fitting contact values



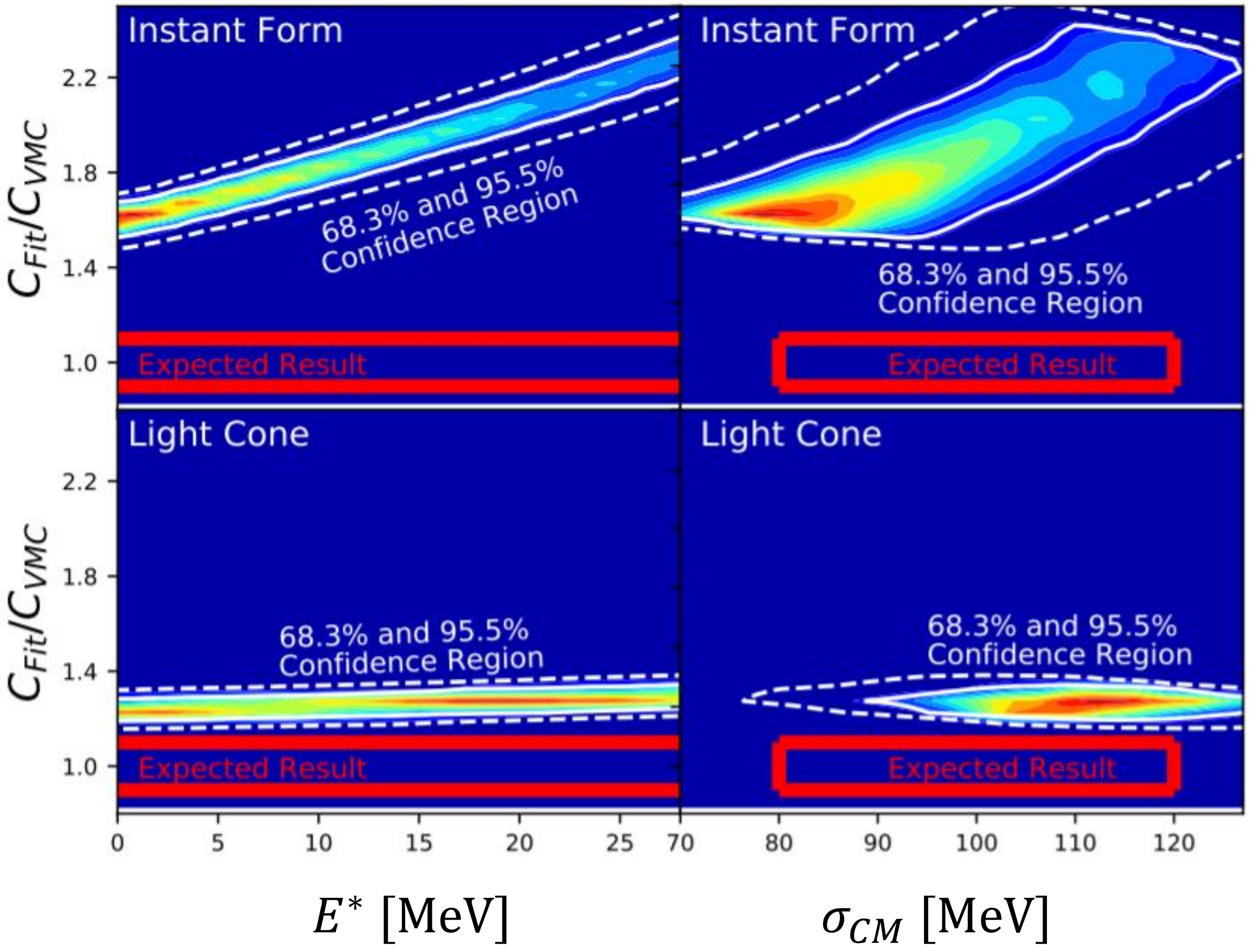

RW, A. W. Denniston et. al., *PRC 103, L031301 (2021)*

# Inclusive reactions

$^4$He - AV18

Fitting contact values

Interpretation depends on $E^*$ and $\sigma_{CM}$



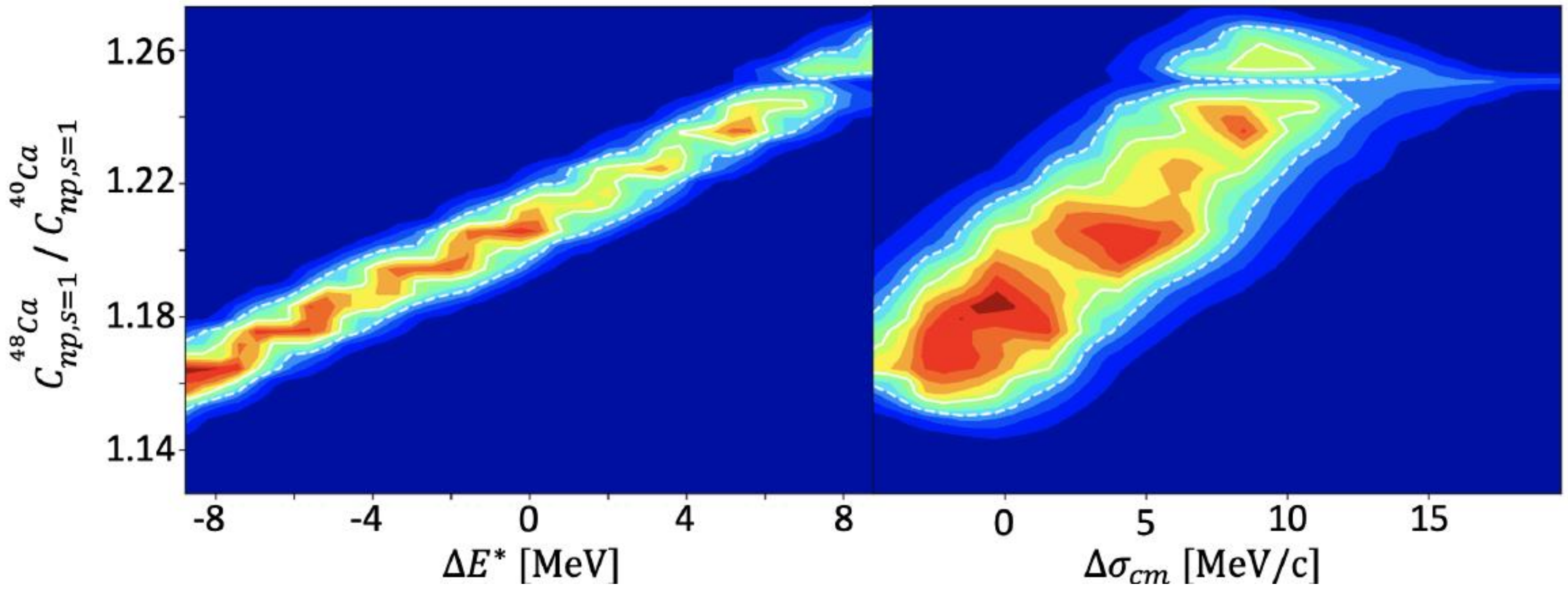

RW, A. W. Denniston et. al., *PRC 103, L031301 (2021)*

# Inclusive reactions

$^{48}\text{Ca}/^{40}\text{Ca}$
AV18
Light cone

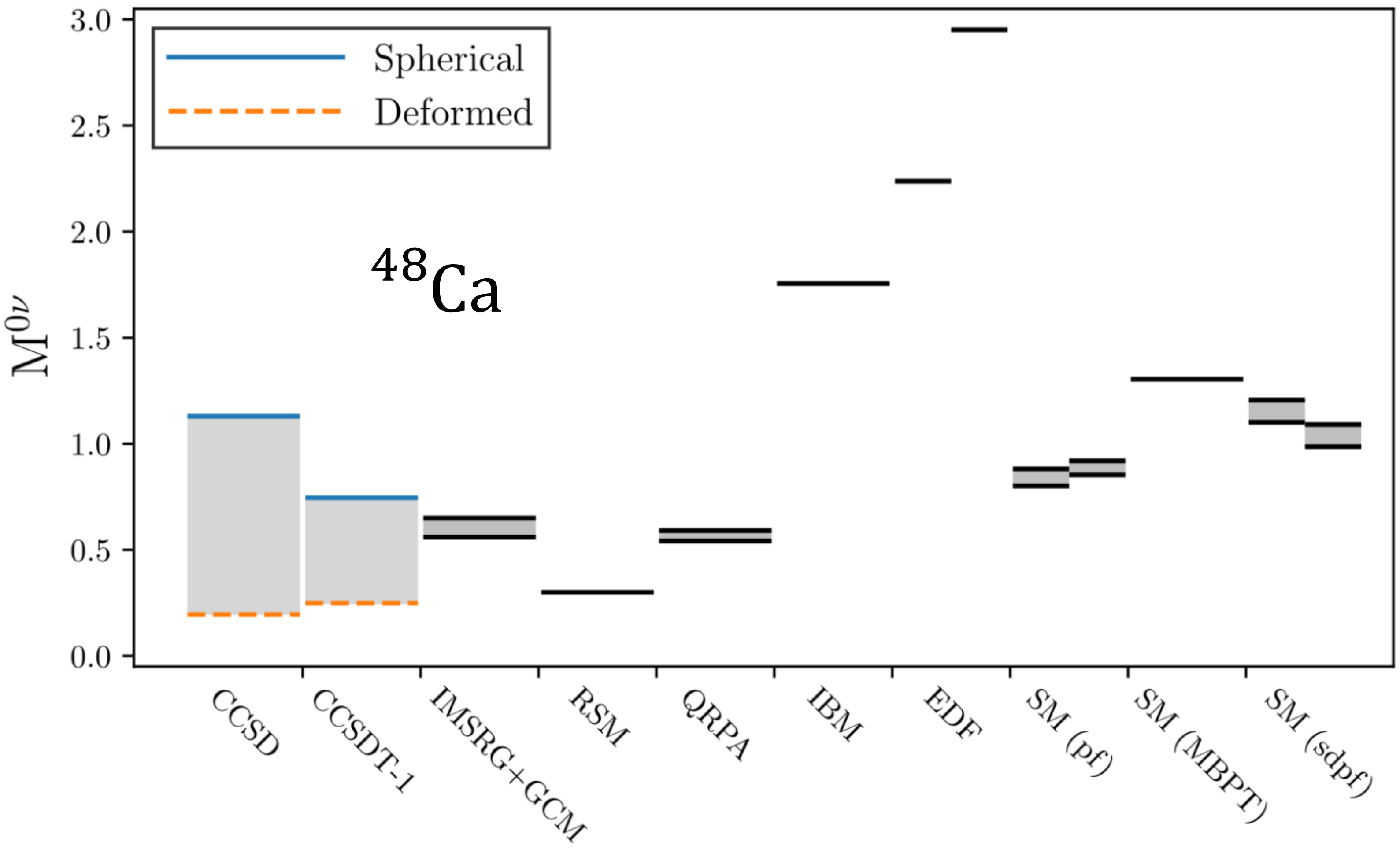
Fitting contact values

Interpretation depends on $E^*$ and $\sigma_{CM}$




RW, A. W. Denniston et. al., *PRC 103, L031301 (2021)*

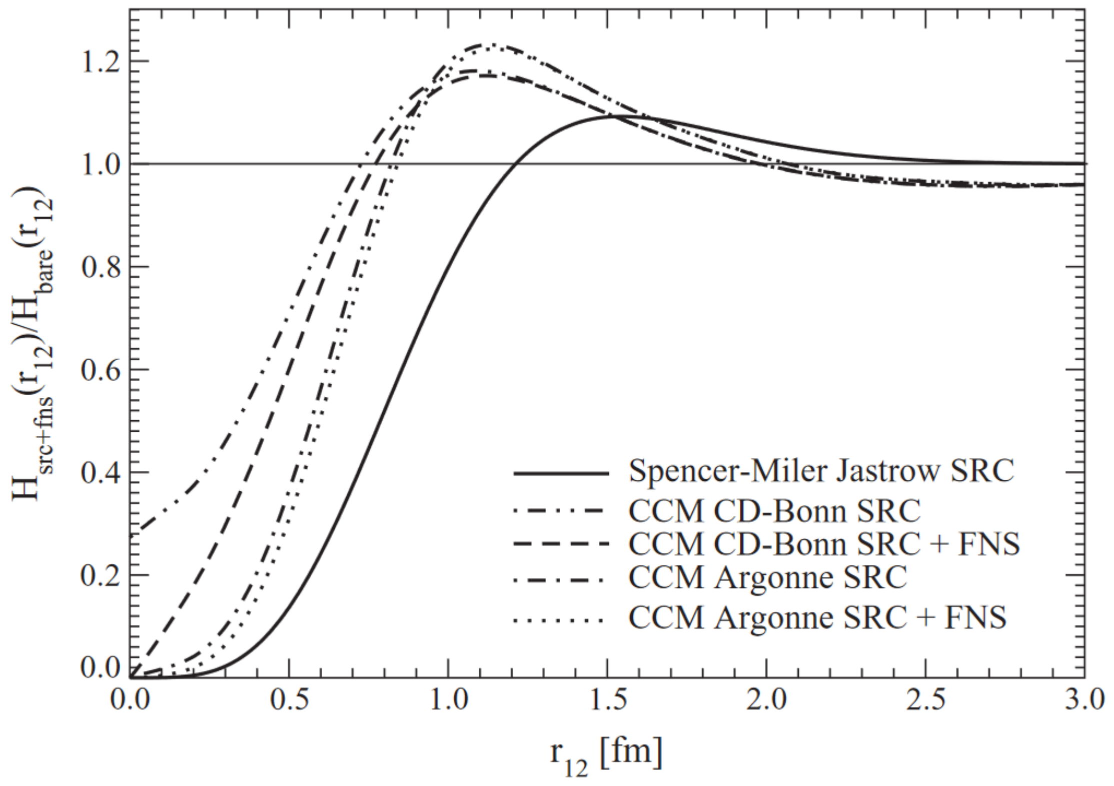
# NMEs – ab-initio methods




S. Novario, et al., PRL 126, 182502 (2021)

# Comparison to previous works

- Studies using correlation functions find smaller SRC effect:



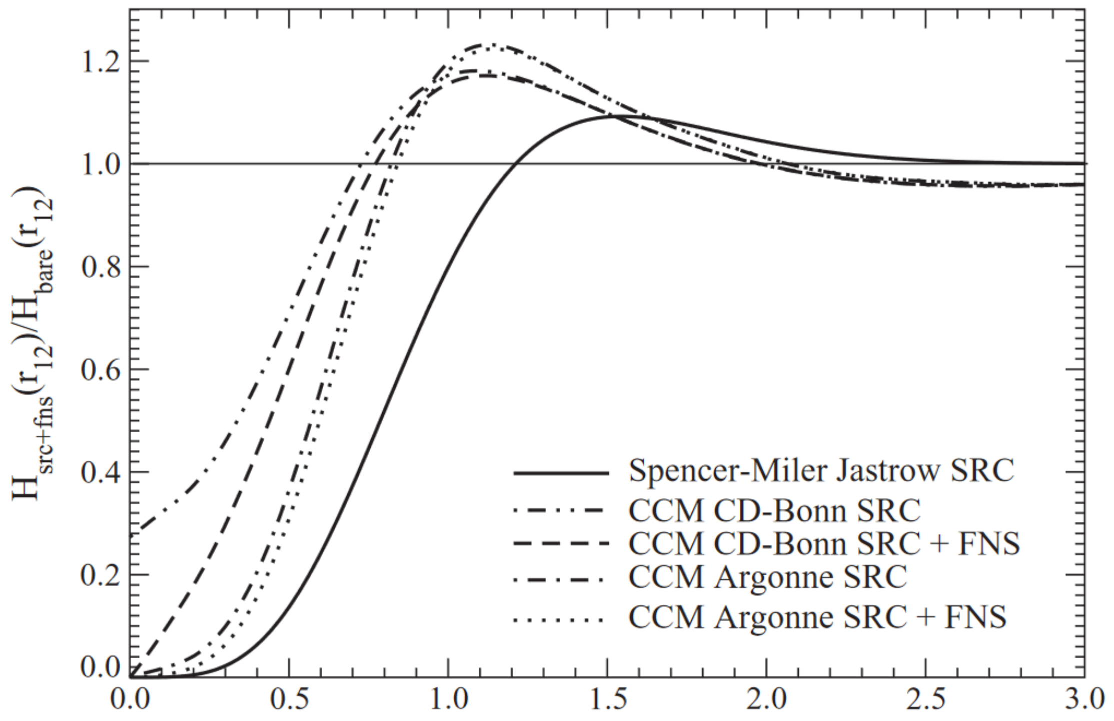

- Peak around $r = 1$ fm leads to the small effect
- Peak argued to be necessary to conserve isospin symmetry

J. Engel et al., PRC 83, 034317 (2011)

$$\int_0^\infty \rho_F(r)dr = \langle\psi_f| \sum_{a<b} \tau_a^+ \tau_b^+ |\psi_i\rangle = 0$$

# Comparison to previous works

- Studies using correlation functions find smaller SRC effect:



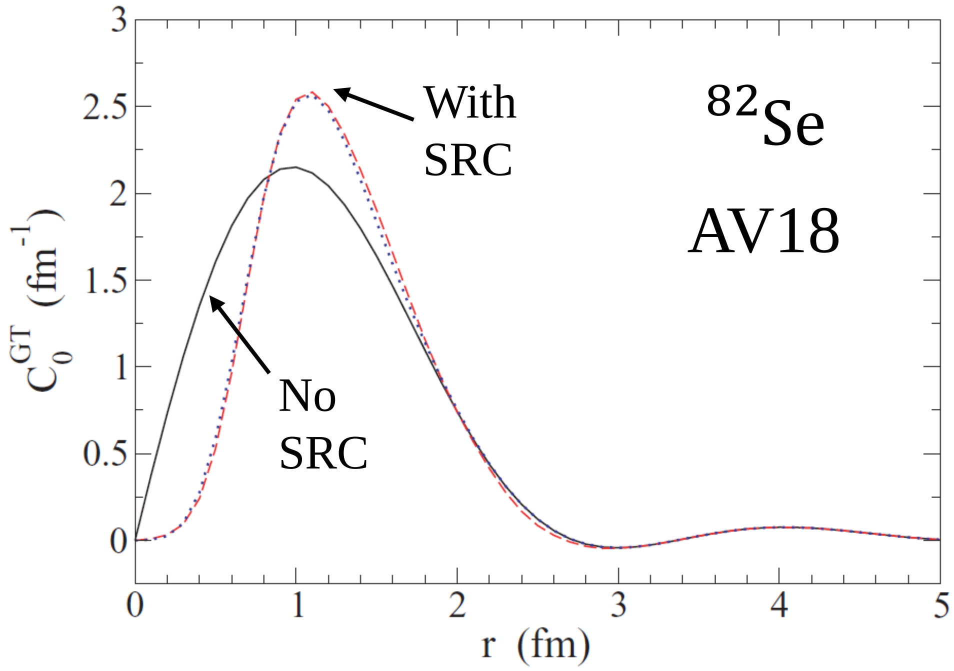

- Peak around $r = 1$ fm leads to the small effect
- Peak argued to be necessary to conserve isospin symmetry

J. Engel et al., PRC 83, 034317 (2011)

Isospin symmetry can be conserved without the peak due to the SM rescaling

# Comparison to previous works

- Other studies found small effect as well
- For example: (diagrammatic perturbation theory to construct an effective shell-model operator)




J. Engel and G. Hagen, PRC 79, 064317 (2009)

Inconsistent with QMC calculations