Triple nucleon correlations - evidence and hunting options

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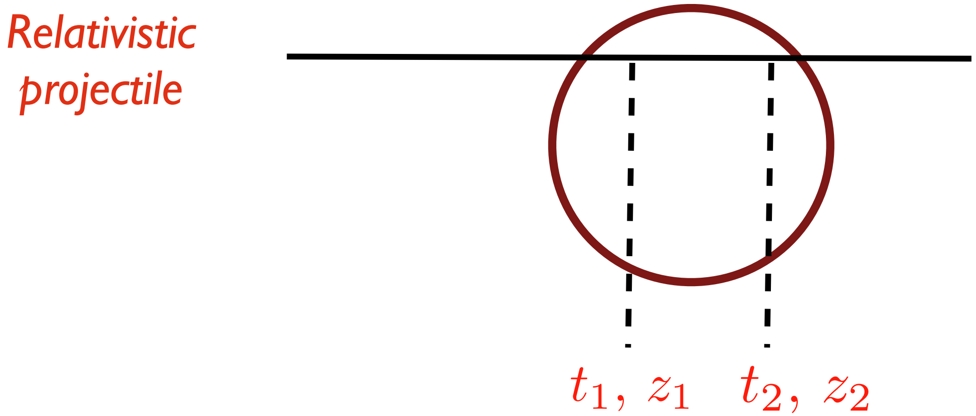
Outline

Introduction : To resolve SRC need high resolution (energy momentum transfer) to constituents of SRC—> light cone dominates

Summary of theoretical expectations for 3NSRC

Evidence for 3N SRC from analyses of hadronic and (e,e') reactions.

⇒ High energy process develops along the light cone.



$$t_1 - z_1 = t_2 - z_2$$

Similar to the perturbative QCD the amplitudes of the processes are expressed through the wave functions on the light cone. *Note: in general no benefit for using LC for low energy processes.*

Since high energy processes are dominated by interactions near light cone, their cross sections are more simply expressed through light cone wave functions (or more complicated LC objects like LC spectral function, LC decay function

$$\rho_{\mathrm{A}}^{\mathrm{p}}(\alpha, k_\perp) = \int \psi^2(\alpha_1 \ldots \alpha_A, k_{1\perp} \ldots k_{A\perp}) \prod_{i=1}^{A} \frac{\mathrm{d}\alpha_i}{\alpha_i} \mathrm{d}^2 k_{i\perp} \delta\left(1 - \frac{\sum \alpha_i}{A}\right)$$
$$\times\ \delta\left(\sum_{i=1}^{A} k_{i\perp}\right) \sum_{i=1}^{Z} \alpha_i \delta(\alpha - \alpha_i) \delta(k_{i\perp} - k_\perp).$$

Single nucleon light cone density matrix

$$\int_0^A \rho_{\mathrm{A}}^{\mathrm{N}}(\alpha, k_\perp) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^2 k_\perp = A$$

$$\int_0^A \alpha \rho_{\mathrm{A}}^{\mathrm{N}}(\alpha, k_\perp) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^2 k_\perp = \int_0^A \rho_{\mathrm{A}}^{\mathrm{N}}(\alpha, k_\perp) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^2 k_\perp \frac{\sum \alpha_i}{A} = A.$$

Example

$$F_{2A}(x, Q^2) = \sum_{\mathrm{N=p,n}} \int F_{2\mathrm{N}}(x/\alpha, Q^2) \rho_A^{\mathrm{N}}(\alpha, k_{\mathrm{t}}) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^2 k_{\mathrm{t}}.$$

If one uses a rest frame approaches - one needs to use a spectral function

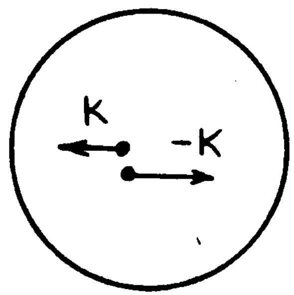
$$P_A(k,E) = \langle \psi_A | a^{+}_{\mathrm{N}}(k) \delta(E + E_{\mathrm{R}} - E_{\mathrm{fX}}) a_{\mathrm{N}}(k) | \psi_A \rangle,$$

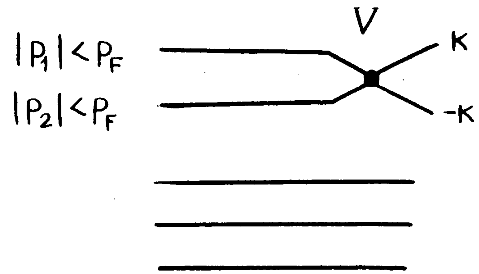
Information contained in n(k) is not sufficient/ of limited value

$$n_A(k) = \int_0^{\infty} P_A(k,E)\mathrm{d}E.$$

No correspondence between asymptotic of n(k→∞) and $\rho_A^N(\alpha \to A)$

Some resemblance between structure of diagrams for high momentum dependence of various contributions to the spectral function P(k,E) and $\rho(\alpha, p_t)$.





Properties of the spectral function P(k,E) at large nucleon momenta

$$n_A(k) \underset{k\to\infty}{\sim} \psi^2_{2\mathrm{N}}(k) \sim \psi^2_{\mathrm{D}}(k).$$

$$V(k)\big|_{k\to\infty} \sim k^{-n}, \quad n_A(k)\big|_{k\to\infty} \sim \frac{V^2(k)}{k^4}.$$

$$P_A(k,E) = \langle \psi_A | a^+_{\mathrm{N}}(k) \delta(E + E_{\mathrm{R}} - E_{\mathrm{fX}}) a_{\mathrm{N}}(k) | \psi_A \rangle, \;\; n_A(k) = \int_0^\infty P_A(k,E)\mathrm{d}E.$$

$$E(k) + E_{\mathrm{R}}(k) \sim k^2/2m.$$

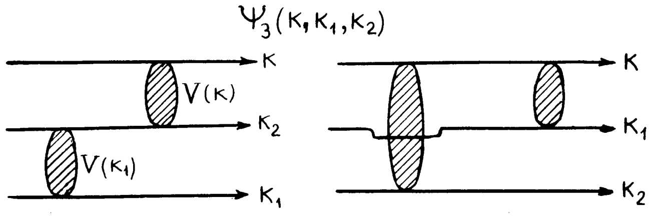
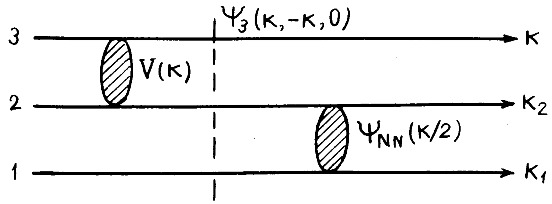


FIG. 8.8:



$$P_A(k,E)\big|_{E<\text{const.},\ k\to\infty} \sim \left(\frac{V(k/2)}{(k/2)^2}\right)^4 \sim n_A^2(k/2).$$

$3N\,SRC \ll\ 2NSRC\ for\, k \to /, \infty$

$$P_A(k,E) = \sum_{j=2}^{A} a_j(A) A P_j(k,\tilde{E}) \quad \text{at } k > k_j, \quad \tilde{E} = E + E_{\rm R}(j) - E_{\rm R}(A).$$

Numerical calculations in NR quantum mechanics confirm dominance of two nucleon correlations in the spectral functions of nuclei at k> 300 MeV/c - could be fitted by a motion of a pair in a mean field (Ciofi, Frankfurt, Simula, MS - 90). However these calculations ignored three nucleon correlations - 3p3h excitations. Relativistic effects maybe important rather early as the recoil modeling does involve k^2/m_N^2 effects.

Studies of the spectral and decay function of 3He reveal both two nucleon and three nucleon correlations - Sargsian et al 2004

Three nucleon SRCs = three nearby nucleons with large relative momenta

Since NN interaction is sufficiently singular for large momenta

$\rho_A^N(\alpha, p_t)$ can be expanded over contributions of j-nucleon correlations $\rho_j(\alpha, p_t)$

$$\rho_A^N(\alpha > 1.3, p_t) = \sum_{j=2}^{A} a_j(A)\rho_j(\alpha, p_t)$$

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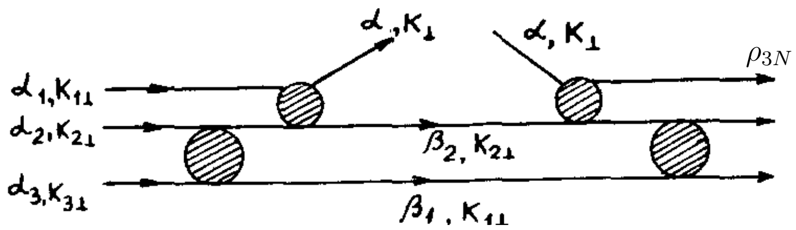
iterations of NN interactions (Plus 3N from 3N forces possible)



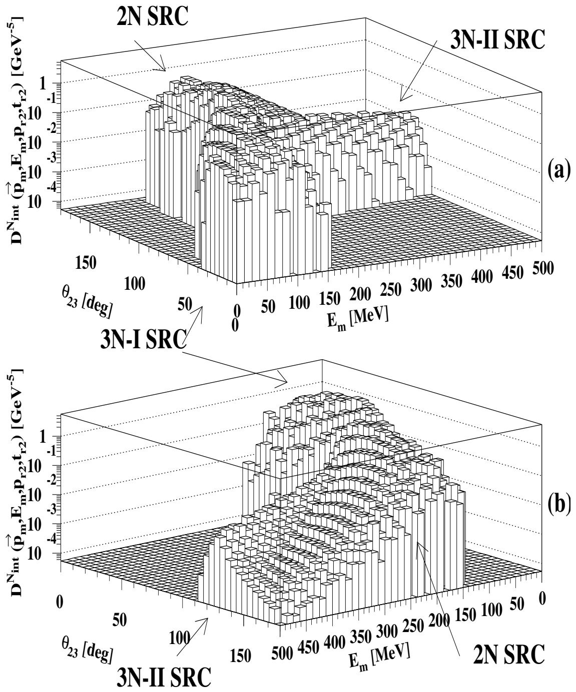
$$\rho_{3N}(\alpha_1) = \int \frac{1}{4}\left[\frac{3-\alpha_3}{(2-\alpha_3)^3}\rho_{pn}\left(\alpha_3, p_{3\perp}\right)\rho_{pn}\left(\frac{2\alpha_2}{3-\alpha_3}, p_{2\perp} + \frac{\alpha_1}{3-\alpha_3}p_{3\perp}\right) + \frac{3-\alpha_2}{(2-\alpha_2)^3}\rho_{pn}\left(\alpha_2, p_{2\perp}\right)\rho_{pn}\left(\frac{2\alpha_3}{3-\alpha_2}, p_{3\perp} + \frac{\alpha_1}{3-\alpha_2}p_{2\perp}\right)\right]\delta\left(\sum_{i=1}^{3}\alpha_i - 3\right) d\alpha_2 d^2p_{2\perp} d\alpha_3 d^2p_{3\perp},$$

$$\rho_j(\alpha, p_t)(j-\alpha)^{n(j-1)+j-2}, \ where\ \rho_j(\alpha, 0) \propto (2-\alpha)^n$$

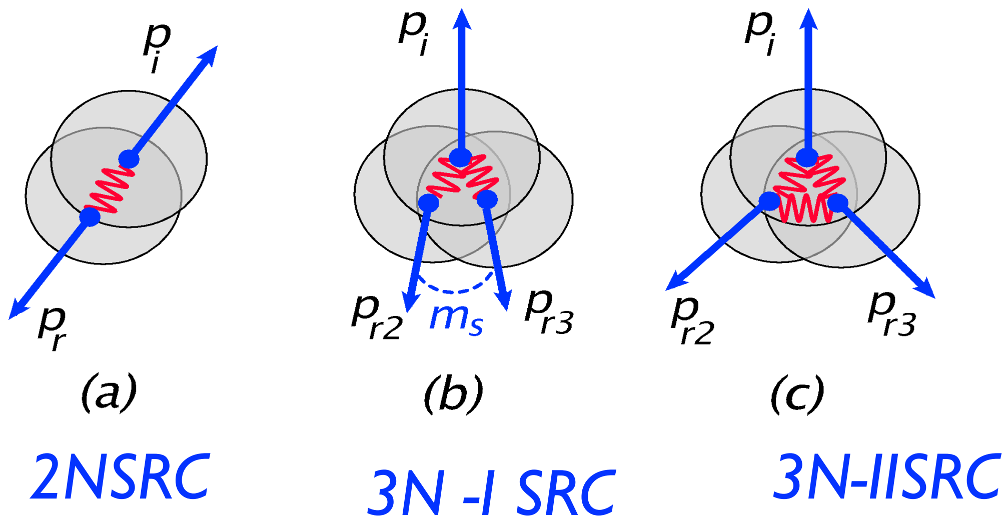
α up to 2 (3) are allowed for 2N (3N) SRC (plus small mean field corrections)

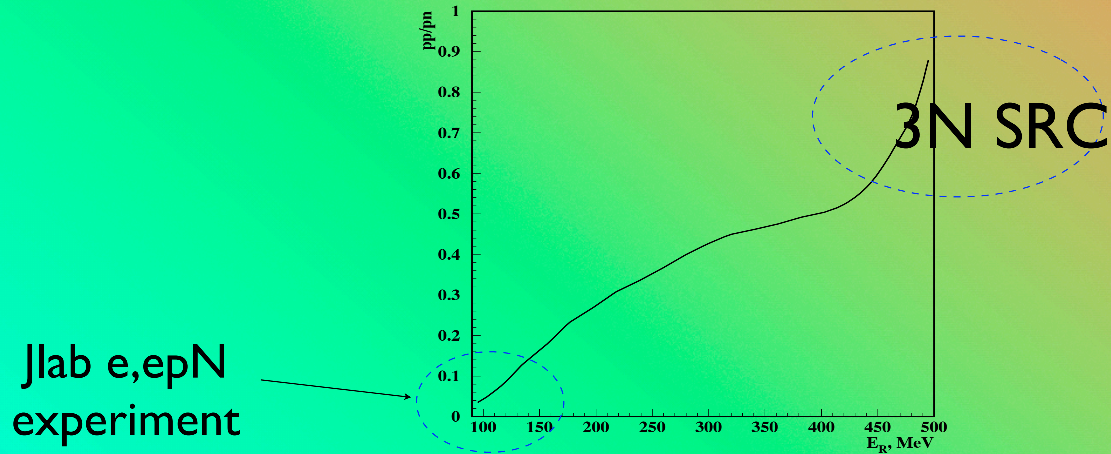
NR case large k = 2N SRC, qualitative difference relativistic and nonrelativistic dynamics

Evidence from NR calculations? *3N SRC can be seen in the structure of decay of ^3He* (Sarsgian et al 2004).

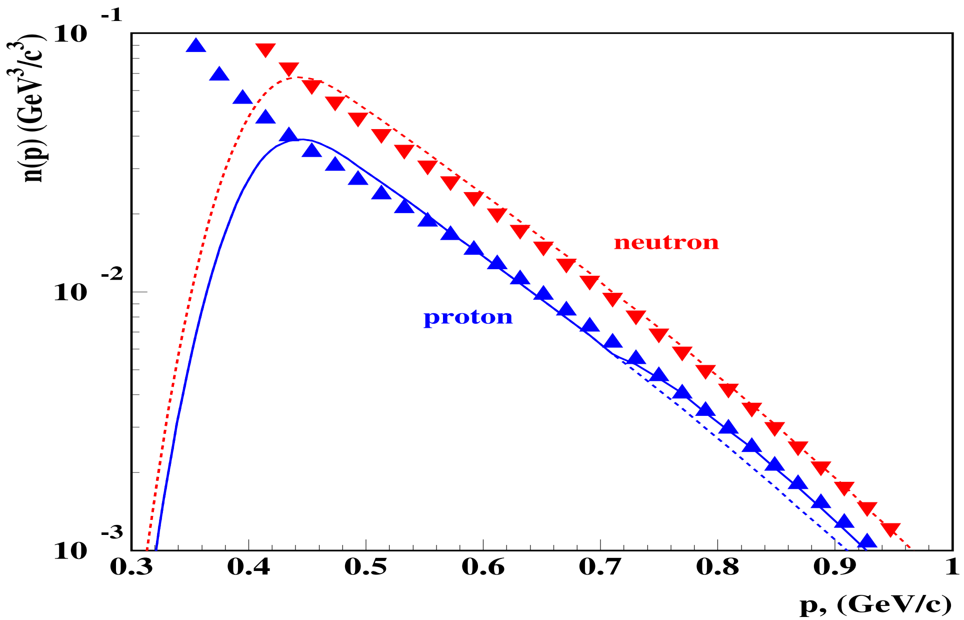


Decay function for ^{3}He nucleus calculated with the condition $p_m \geq 700$ MeV/c, and $p_{r2}, p_{r3} \geq k_F$. The θ_{23} is the relative angle between two recoil nucleons and E_m is the missing energy. Two panels show different point of views of the same figure.





Recoil energy dependence of the ratio of decay function calculated for the case of struck and recoil nucleons - p_s & p_r for struck proton and recoil proton and neutron for p_s & p_r > 400MeV/c & $180^\circ > \theta(p_s\ p_r) > 170^\circ$



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Slow onset of asymptotic regime of the ratio 3N/2N decreasing with increase of k

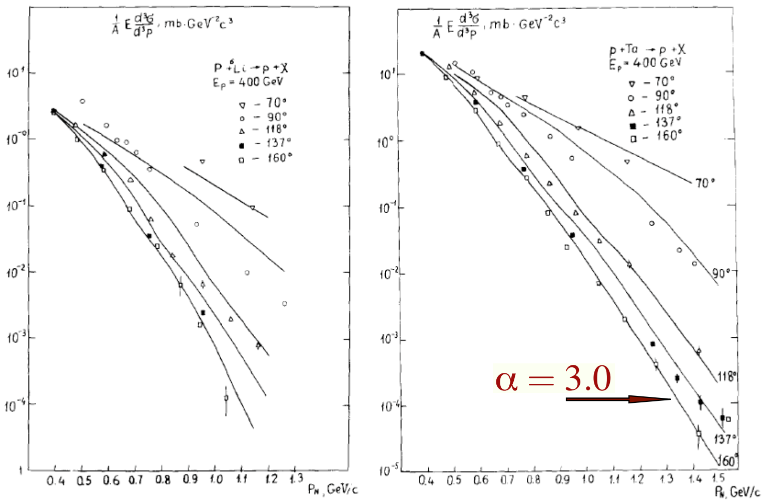
But k is a not a dynamical variable for 3N

Some of experimental evidence in historic order

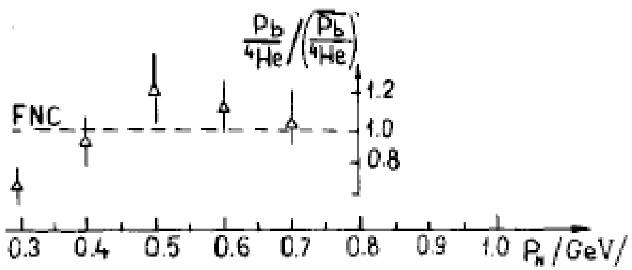
Plenty of data were described using few nucleon SRC approximation with 3N, 4N correlations dominating in certain kinematic ranges. Strength of 2N correlations is similar to the one found in (e,e’),(p,2p)

p^{6}Li→backward p+X,pTa→backward p+X

$$\frac{d\sigma(h+A\to N+X)}{\frac{d\alpha_N d^2p_t}{\alpha_N}} \kappa_h \sigma_{in}(hN)\rho_A^N(\alpha_N, d^2p_t)$$



Comparison of few nucleon SRC approximation with pA data at E_p^{inc}=400 GeV



Test of universality for pA→p+X spectra for backward emission at Ep= 9 GeV

Observations of (p,2pn) &(e,e’) at x>1 confirm the origin of SRC as the dominant source of the fast backward nucleons

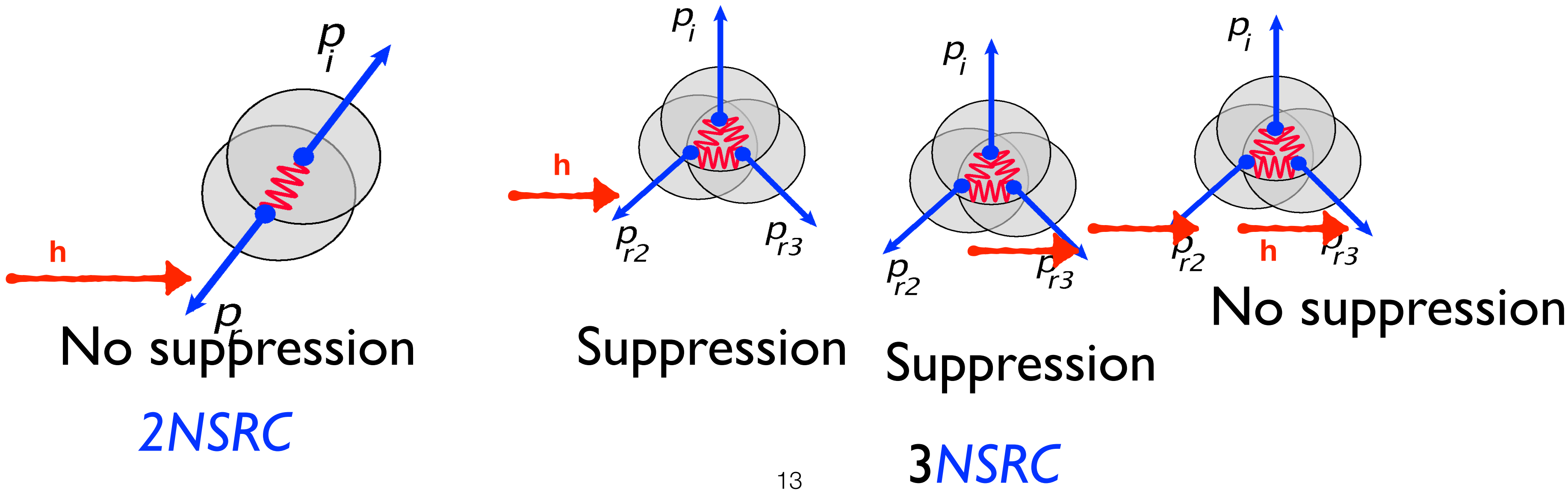
Model leading to $$\frac{d\sigma(h+A\to N+X)}{\frac{d\alpha_N d^2p_t}{\alpha_N}}\kappa_h\sigma_{in}(hN)\rho_A^N(\alpha_N,d^2p_t)$$

assumes that interaction leading to production of ward nucleon is about the same for 2N and 3N

Roughly OK - in 2N hit one nucleon- suppression factor 0.7

However in the cases DIS , backward pion production all configurations contribute - new studies are necessary.

3N - need to hit 2 forward nucleons or fsi



Scaling of the ratios of (e,e') cross sections

Qualitative idea - to absorb a large Q at x>j at least j nucleons should come close together. For each configuration wave function is determined by *__local__* properties and hence universal. In the region where scattering of j nucleons is allowed, scattering off j+1 nucleons is a small correction.

$$\sigma_{eA}(x,Q^2)_{x>1} = \sum_{j=2} A\frac{a_j(A)}{j}\sigma_j(x,Q^2) \qquad \sigma_j(x>j,Q^2)=0$$

$$a_j(A) \propto \frac{1}{A}\int d^3r \rho_A^j(r) \qquad a_2 \sim A^{0.15}; \quad a_3 \sim A^{0.22}; \quad a_4 \sim A^{0.27}$$

for A> 12 if Z=A/2

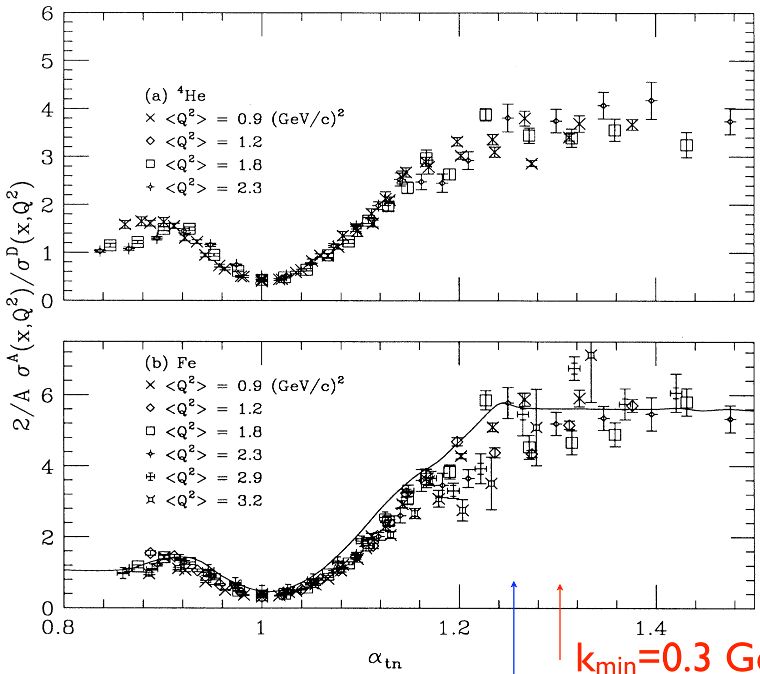
$$\sigma_{A_1}(j-1<x<j,Q^2)/\sigma_{A_1}(j-1<x<j,Q^2) = (A_1/A_2)a_j(A_1)/a_j(A_2)$$

Scaling of the ratios FS80

$$\Rightarrow \frac{\sigma_{A_1}(x,Q^2)}{\sigma_{A_2}(x,Q^2)} = \frac{\int \rho_{A_1}(\alpha_{tn},p_t)d^2p_t}{\int \rho_{A_2}(\alpha_{tn},p_t)d^2p_t} = \frac{a_2(A_1)}{a_2(A_2)}\Big|_{1.6>\alpha\geq 1.3}$$

ρ- Light-cone density

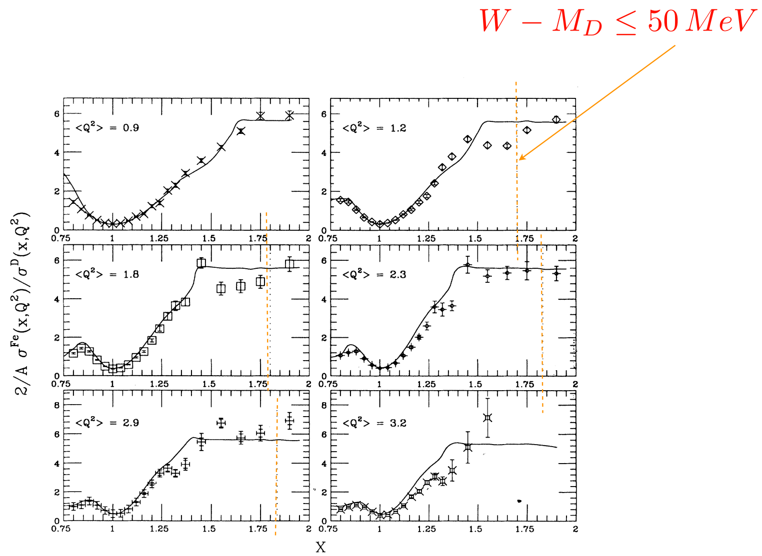
Note - local FSI interaction, up to a factor of 2 for σ(e,e'), cancels in the ratio of σ's



k_{min}=0.3 GeV

k_{min}=0.25 GeV

Frankfurt et al, 93



Masses of NN system produced in the process are small - strong suppression of isobar, 6q degrees of freedom.

Right momenta for onset of scaling of ratios !!!

Superscaling of the ratios FS88

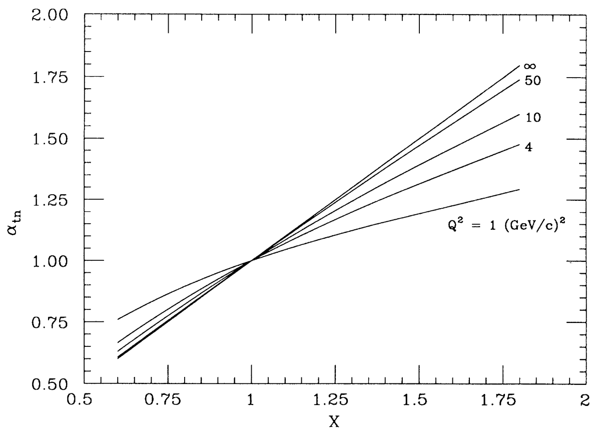
Compare the ratios for different Q^2 at x corresponding to the same momentum of nucleon in nuclei (including effect of excitation of the residual system - best done in the light-cone formalism)

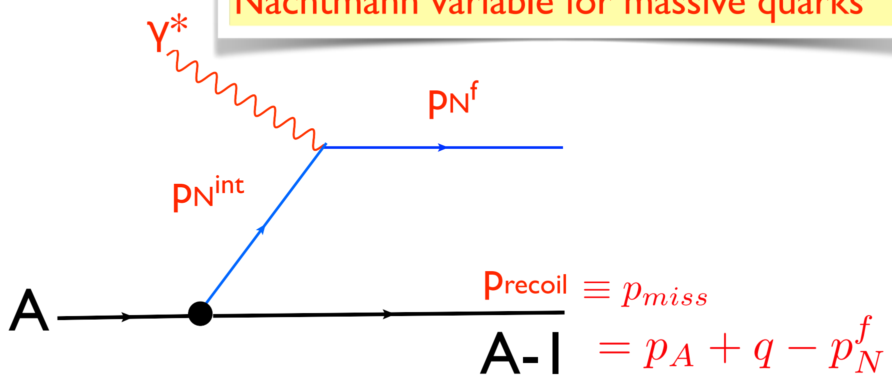
Main dependence is on "+" component (α) of p_N^{int}, allows to take "-" component in average point given by two nucleon SRC at rest

$$\alpha_{tn} = 2 - \frac{q_- + 2m_N}{2m_N}\left(1 + \frac{\sqrt{W^2 - 4m_N^2}}{W}\right)$$

$$where\ q_- = q_0 - q_3,\ W^2 = 4m_N^2 + 4q_0 m_N - Q^2$$

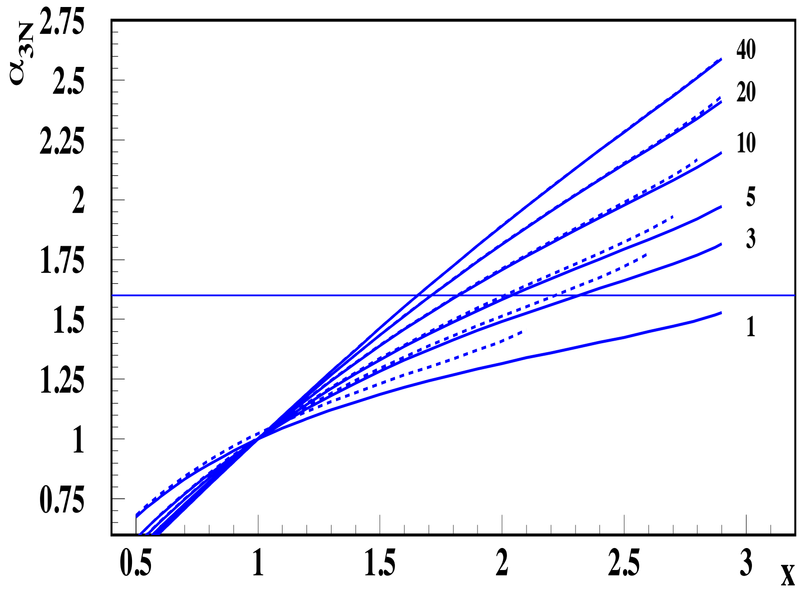
Remark for people with a QCD background: α_{tn} is rather close to Nachtmann variable for massive quarks





α_{tn} vs x for Q^2=1, 4, 10, 50, ∞. At $Q^2 \to \infty$, $\alpha_{tn} = x$

Apply the same logic for scattering off 3N SRC to calculate minimal α_{3N} for given x,Q2



From analysis of backward nucleon production: 3N starts to dominate in the LC density for $\alpha > 1.6$

Hence we (Misak, Donal, LF , MS) performed the analysis of Jlab data which cover maximal α.

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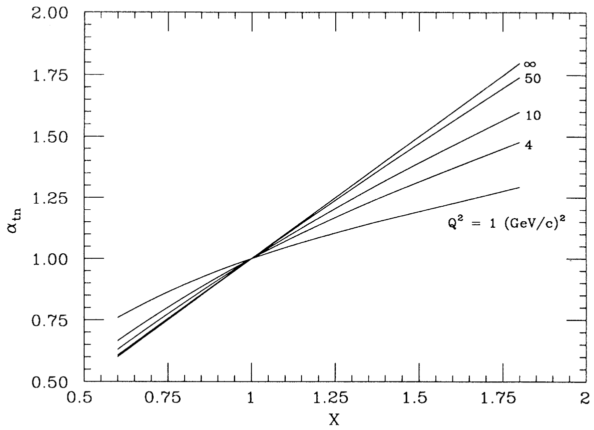

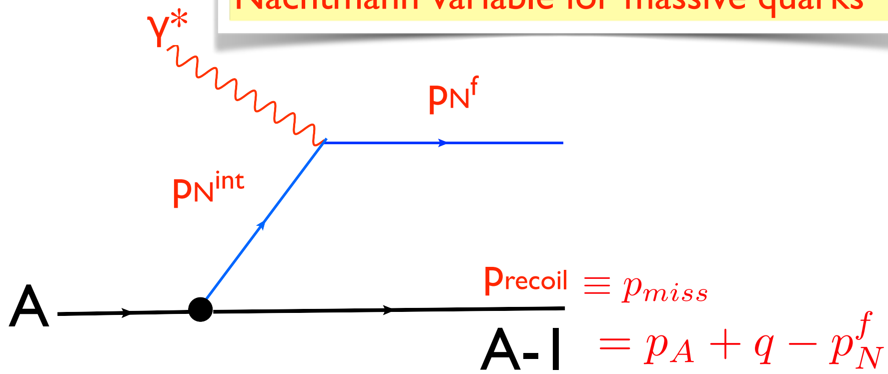

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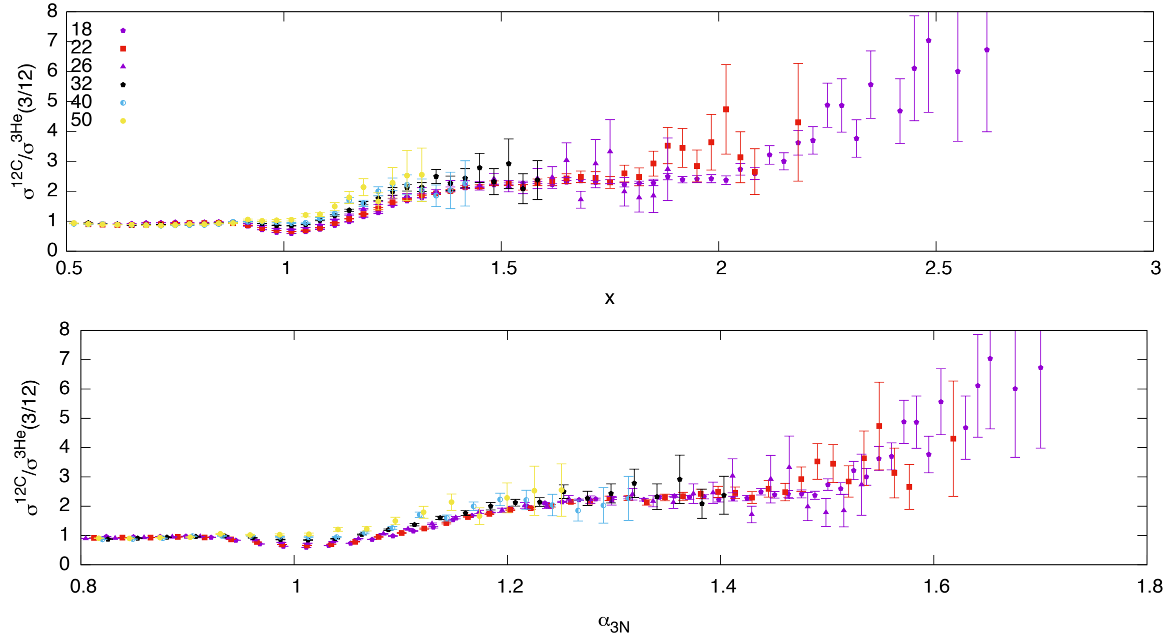
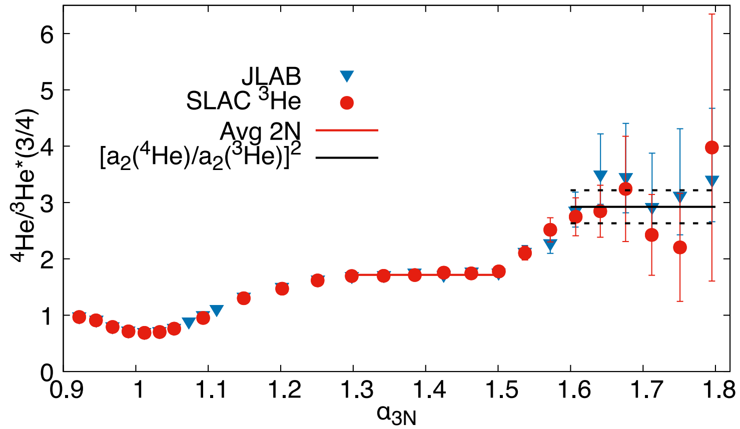


The x and α_{3N} dependences of the per-nucleon ratios of 12C/3He for different angles with Q^2 ranging from 2.5 —7.5 GeV^2 (at x = 1) against x (top) and α_{3N} . Only data with relative errors less than 0.5 are shown.

Problem - 3He data have problems, Donal spend long hours to find best ways to combine Jlab and SLAC 3He data



Onset of 3N dominance at α~ 1.6 ?

The α_{3N} dependence of the inclusive cross section ratios for 4He to 3He, triangles - JLAB data [6, 49], circles - ratios when using a parameterization of SLAC 3He cross sections [12, 15]. The horizontal line at 1.3 < α_{3N} < 1.5 identifies the magnitude of the 2N-SRC plateau. The line for α_{3N} > 1.6 is Eq.(27) with a 10% error introduced to account for the systematic uncertainty in $a_2(A,Z)$ parameters across all measurements. The data correspond to Q2 ~ 2.5 GeV2 at x = 1, α_{3N} = 1. The figure is from Sargsian, Day,Frankfurt MS 2019

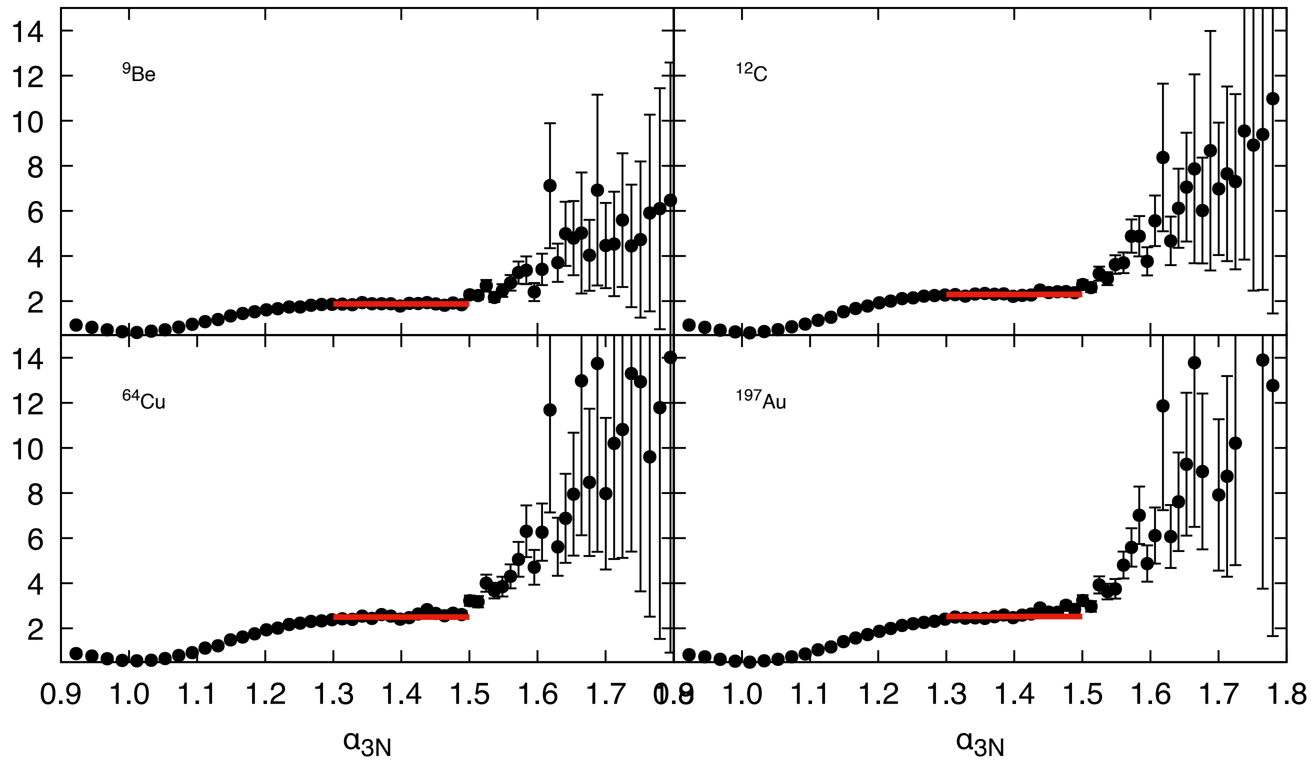
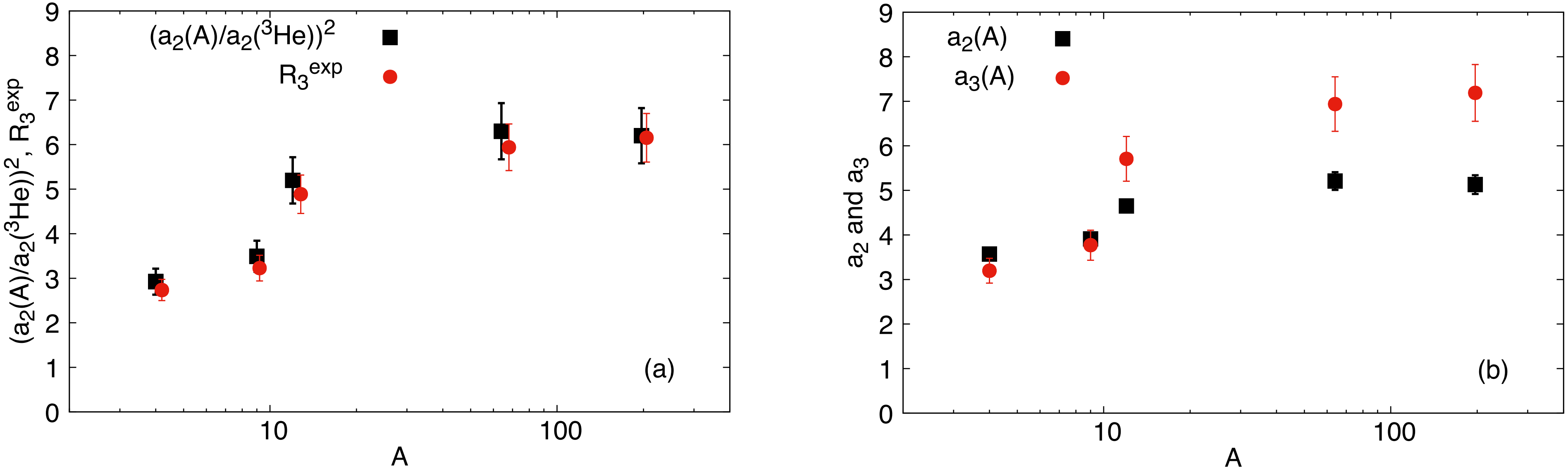


FIG. 16: Per-nucleon cross section ratios for ^{9}Be, ^{12}C, ^{64}Cu, ^{197}Au to ^{3}He. Horizontal lines indicating $\frac{a_2(A)}{a_2(^3He)}$ in the 2N-SRC region.

$$R_3(A,Z) \approx \frac{54}{56} R_2(A,Z)^2 \approx R_2(A,Z)^2.$$



(a) The A dependence of the experimental evaluation of R_3 compared with the prediction of Eq.27.
(b) The A dependence of $a_3(A,Z)$ parameter compared to $a_2(A,Z)$ of Ref.[6].

We checked that R_{3exp} does not change when we vary minimal value of x and hence α_3

Further studies are necessary of LC scaling of the ratios, etc. Recoil structure more complicated than in 2N case

Possible evidence for 3N SRC from a correlation experiment

p A→ p (backward) + p (backward) +X

measurements of Bayukov et al 86

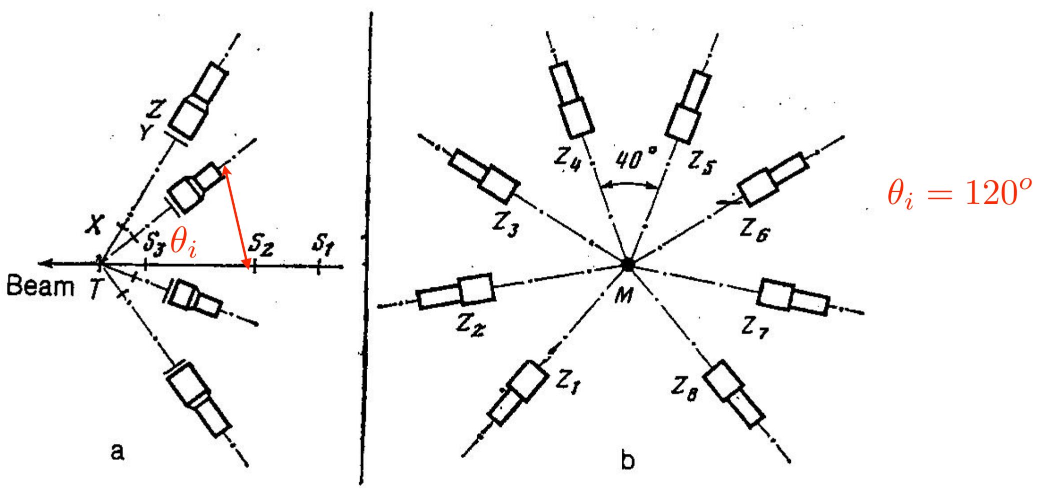
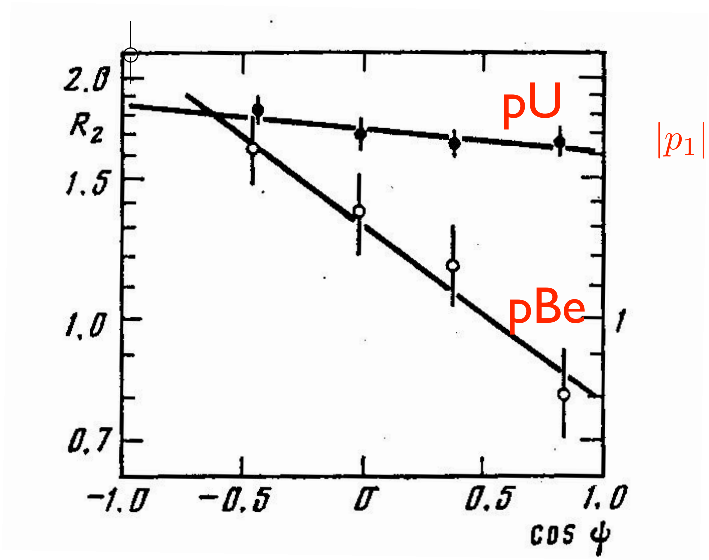


FIG. 1. Diagram of apparatus. (a)—Side view, (b)—view along the beam direction. Only the Z counters are shown.

$$p_i \approx 0.5\,GeV, \alpha \approx 1.4, p_t \approx .25\,GeV$$

$$R_2 = \frac{1}{\sigma_{pA}^{in}} \frac{d\sigma(p + A \to pp + X)/d^3p_1 d^3p_2}{d\sigma(p + A \to p + X)/d^3p_1 d\sigma(p + A \to p + X)/d^3p_2}$$



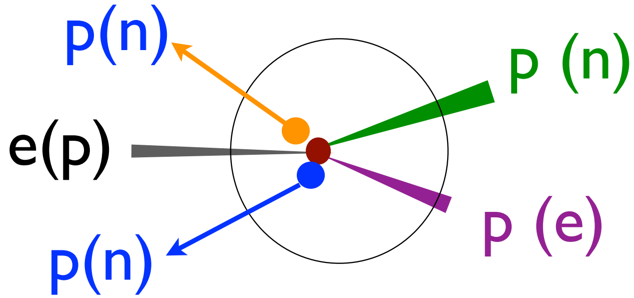
$|p_1| = |p_2| \approx 500 MeV/c$

Curves are the experimental fit.

the pattern of ψ dependence of R_2 can be reproduced

Study 3N correlations in A(e,e' p +2 backward nucleons) &A(p,p' p +2 backward nucleons).

Fast nucleus kinematics - nucleons with momenta >1.4 p_N Can inverse kinematics help? Lumi ?



Start with ^{3}He, followed by ^{4}He, C. Expectations:

(a) $\alpha_{1\,Back.Nucl} + \alpha_{2\,Back.Nucl} + \alpha_{3\,Forw.Nucl} \approx 3$

(b) ppn ~ nnp >> nnn, ppp

(c) e+A → e+ 2N +X stronger angular dependence and larger $R_2(\psi=-180^\circ)$ than in pA (for the case when in pA A is large enough for $\psi=-180^\circ$ not be close to 0.

Reminder:* *for the neutron star dynamics mostly isotriplet nn, nnn,.. SRC are relevant.

Summary - discoveries through precision and through new processes

- ☛ Precision theoretical and experimental studies of the lightest nuclei, including relativistic dynamics

- ☛ Tests of realistic modeling of FSI

- ☛ Tests of factorization (comparing electron, photon, nucleon- nucleus SRC sensitive processes.

  Example: 3N - to release $\alpha=1.7$ nucleon without strong suppression due to fsi need to hit both recoiling nucleons with $\alpha_2 \sim \alpha_3 \sim 0.65$. No such suppression for production of say pion in the projectile - $\alpha=1.7$ interaction.

- ☛ Tests of dynamic assumptions of LC many nucleon approximation (Misak's talk).

- ☛ Separation of S and D waves in SRCs

- ☛ Calculating and looking for 3N SRC

- ☛ Including in the calculation of WF explicitly Δ-isobars and looking for them and other non-nucleonic degrees of freedom in nuclei