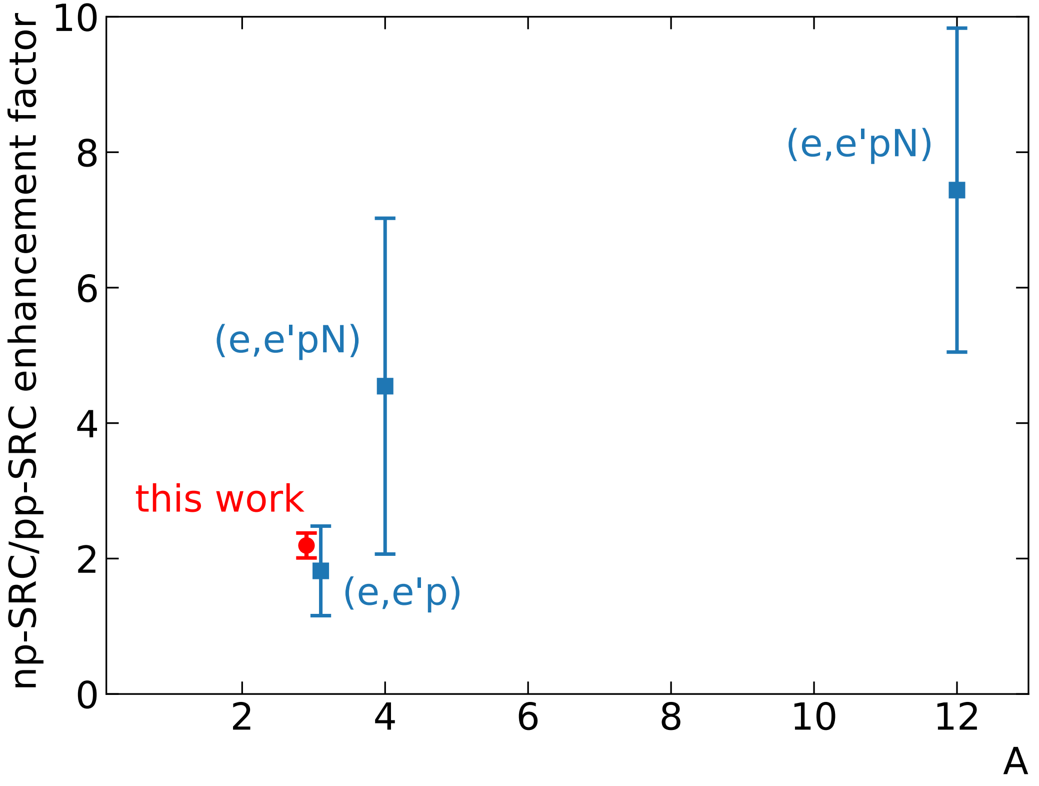
Interpretation Uncertainties in Inclusive Scattering at x>1

Axel Schmidt

2022 SRC Collaboration Meeting

August 3, 2022

Inclusive measurements as a way of learning about np dominance



In conclusion, we have presented a novel measurement on the mirror nuclei ^3H and ^3He which provided ***a clean extraction of the relative contribution of np- and pp-SRCs*** *with* ***uncertainties an order of magnitude smaller*** *than existing two-nucleon knockout measurements.*

“A precise measurement of the isospin structure of short-range correlations using inclusive scattering from the mirror nuclei ^3H and ^3He”
S. Li et al., to appear in Nature, 2022

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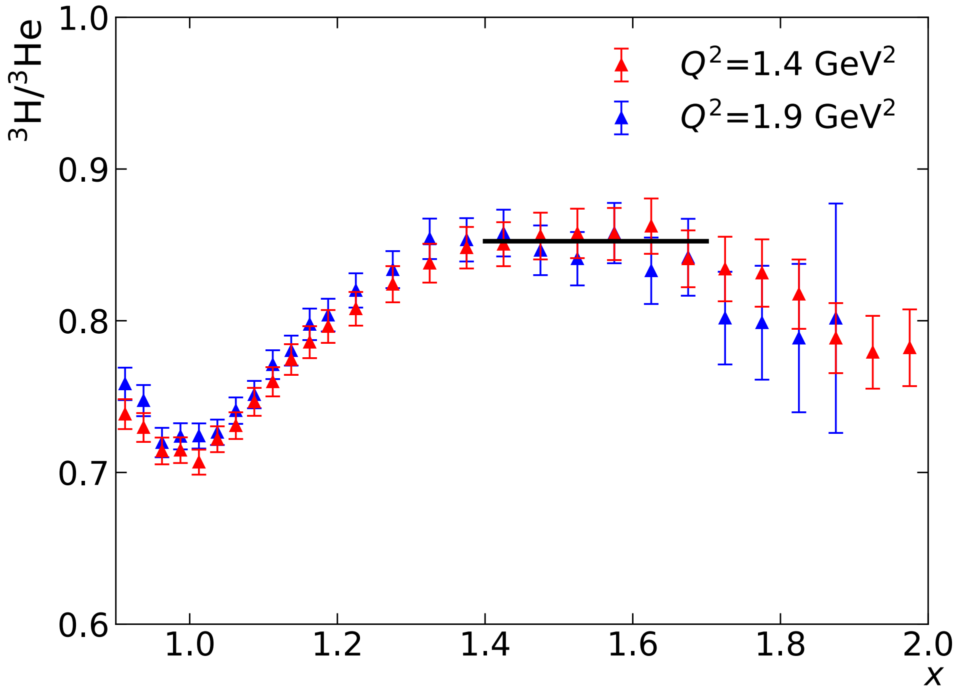
- Measure inclusive cross section ratio at x >1.5 (SRC-dominated region) for two nuclei with same (or similar) A, but different N/Z.
- Determine the value of the "plateau."



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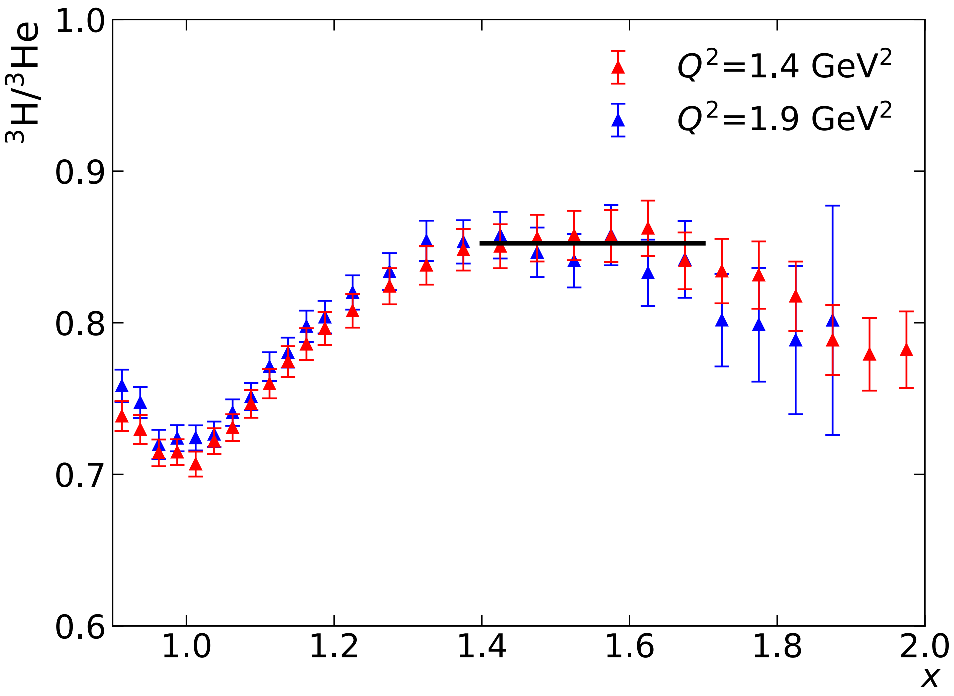

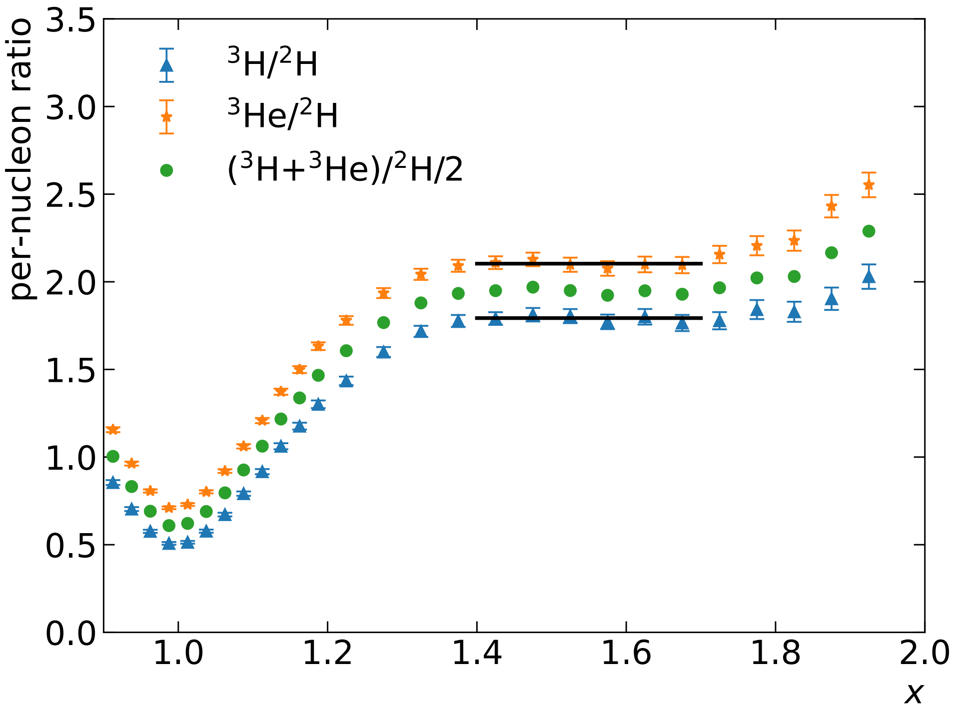

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  - Isospin multiplets are especially helpful.

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$$\alpha \equiv N^{np}_{^3H} = N^{np}_{^3He}$$
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$\frac{\sigma_{^3H}}{\sigma_{^3He}} = 0.850 \pm 0.009$, $\frac{\sigma_{ep}}{\sigma_{en}} = 2.55 \pm 0.05$ ----> $\left(\frac{\beta}{\alpha}\right) = 0.225 \pm 0.020$

1% 2% 9%

What this method fails to capture

- CM motion does residually affect the ratio.

- The plateau doesn’t have to be flat.

- The number of np and pp pairs is kind of a fuzzy quantity
  - Depends on the momentum range probed.

- Contamination from non-SRC events

Interrogating inclusive scattering with more sophisticated theory

- Spectral Function Calculations
  - Detailed calculations for light nuclei, e.g., A=3 from Kaptari and Ciofi degli Atti
- Variational Monte Carlo
  - New spectral functions that separate “correlated” and “mean field” contributions
  - 3H, 3He, 4He, 12C, for AV-18
  - Not all the kinks are worked out.
- Generalized Contact Formalism
  - Can’t address mean-field contribution
  - Allows twiddling model input parameters

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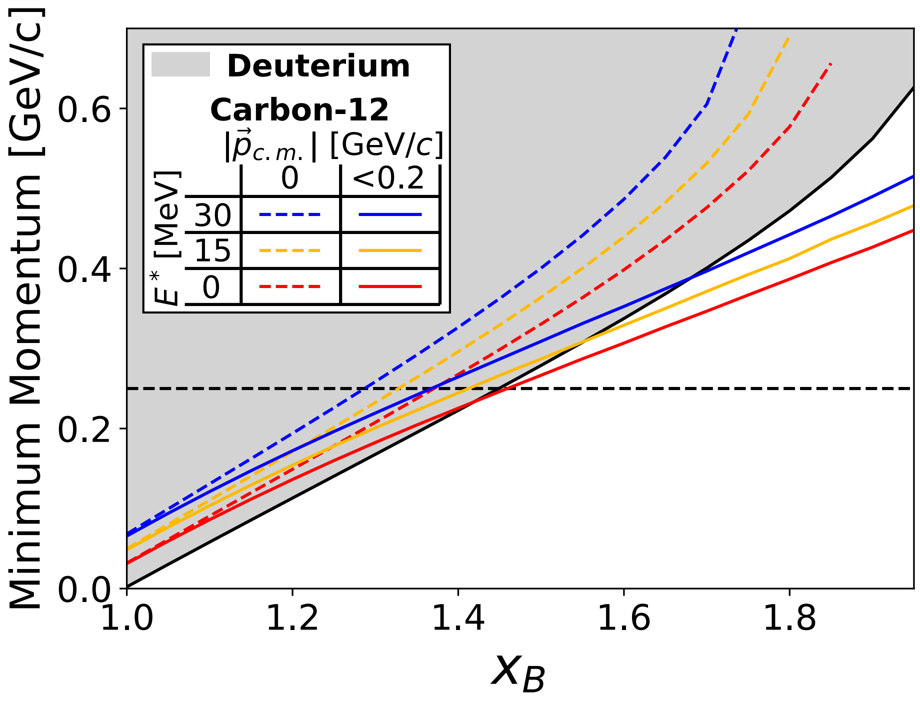
σ_{CM} and E^* affect a2 plateaus.

R. Weiss et al., PRC **103**, L031301 (2021)



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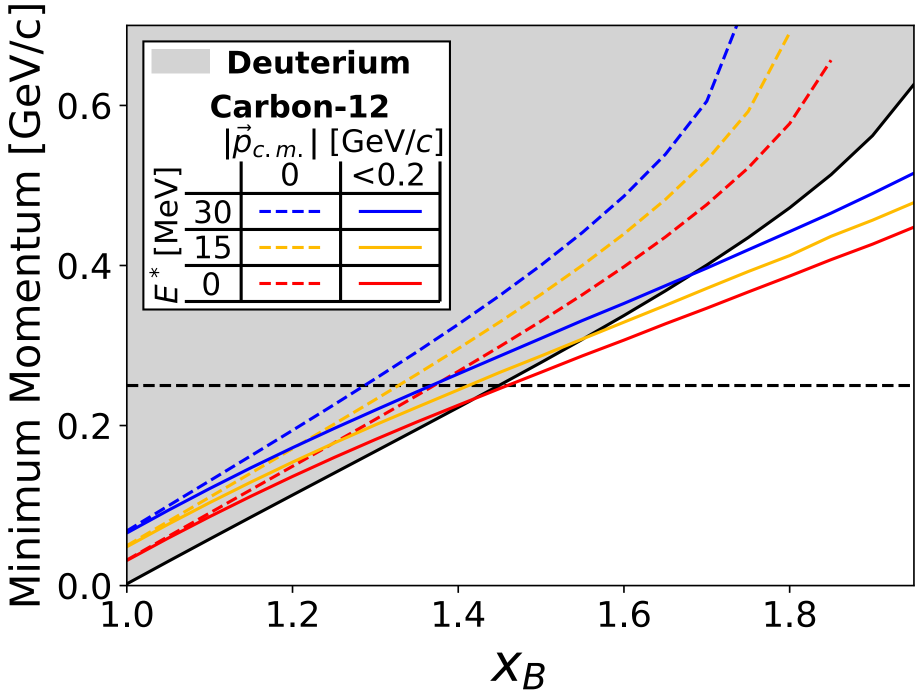

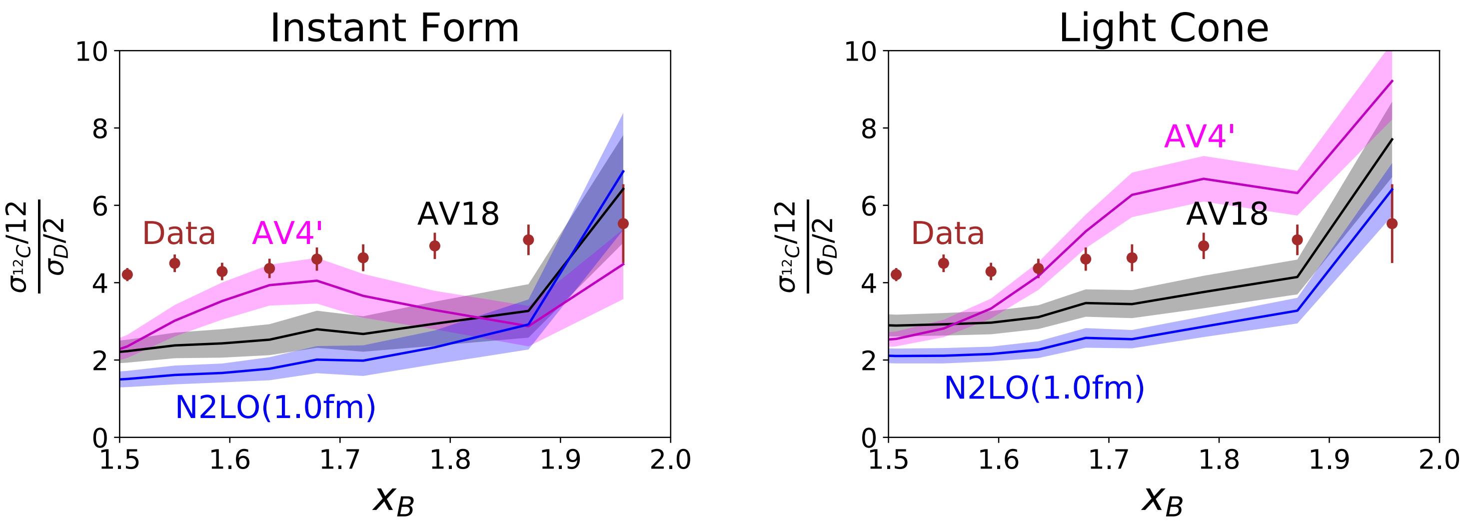

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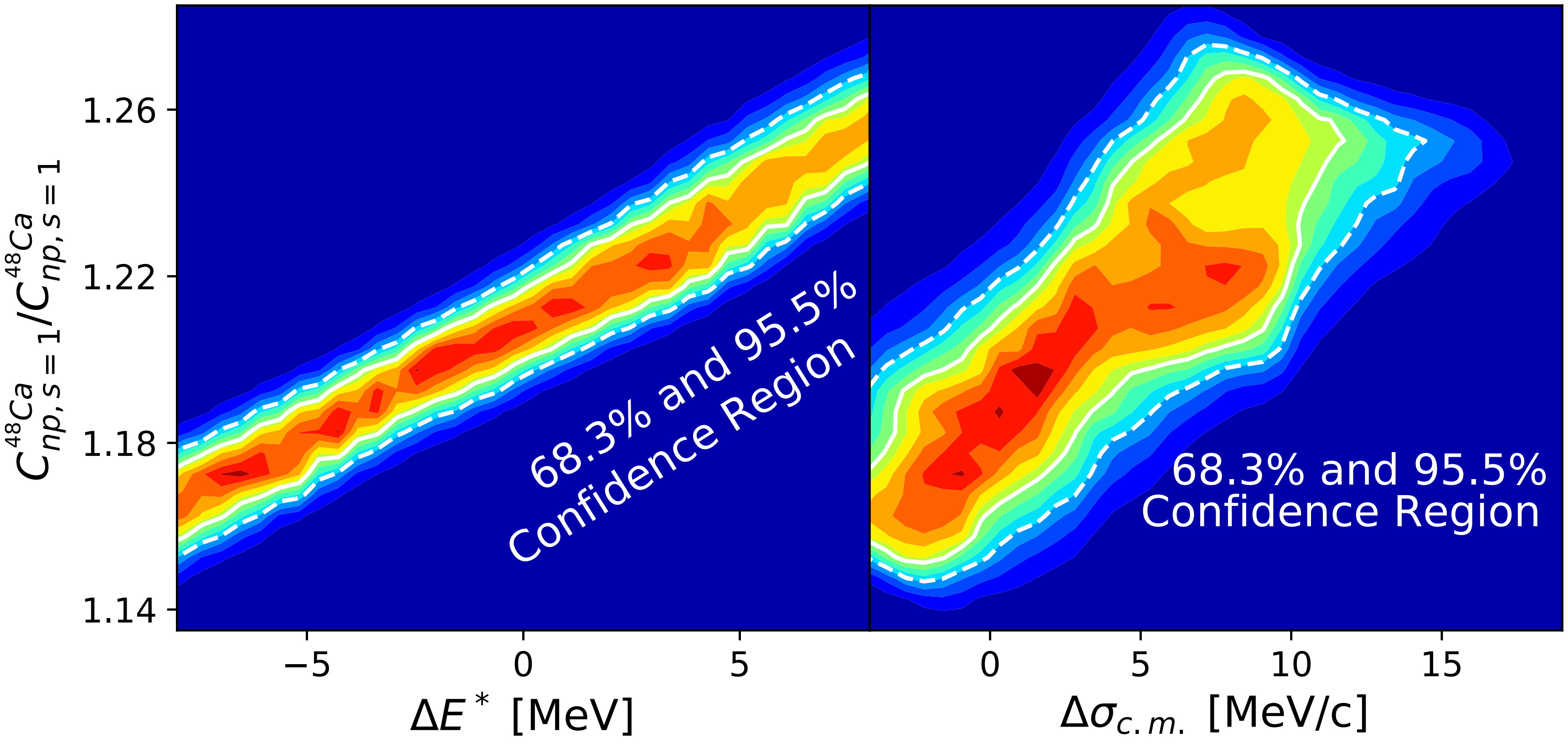

A=3 pn/pp ratio in Contact Formalism

pn/pp determined by Contacts, determined by fits to VMC distributions (by R. Cruz-Torres)

- Includes fit uncertainty

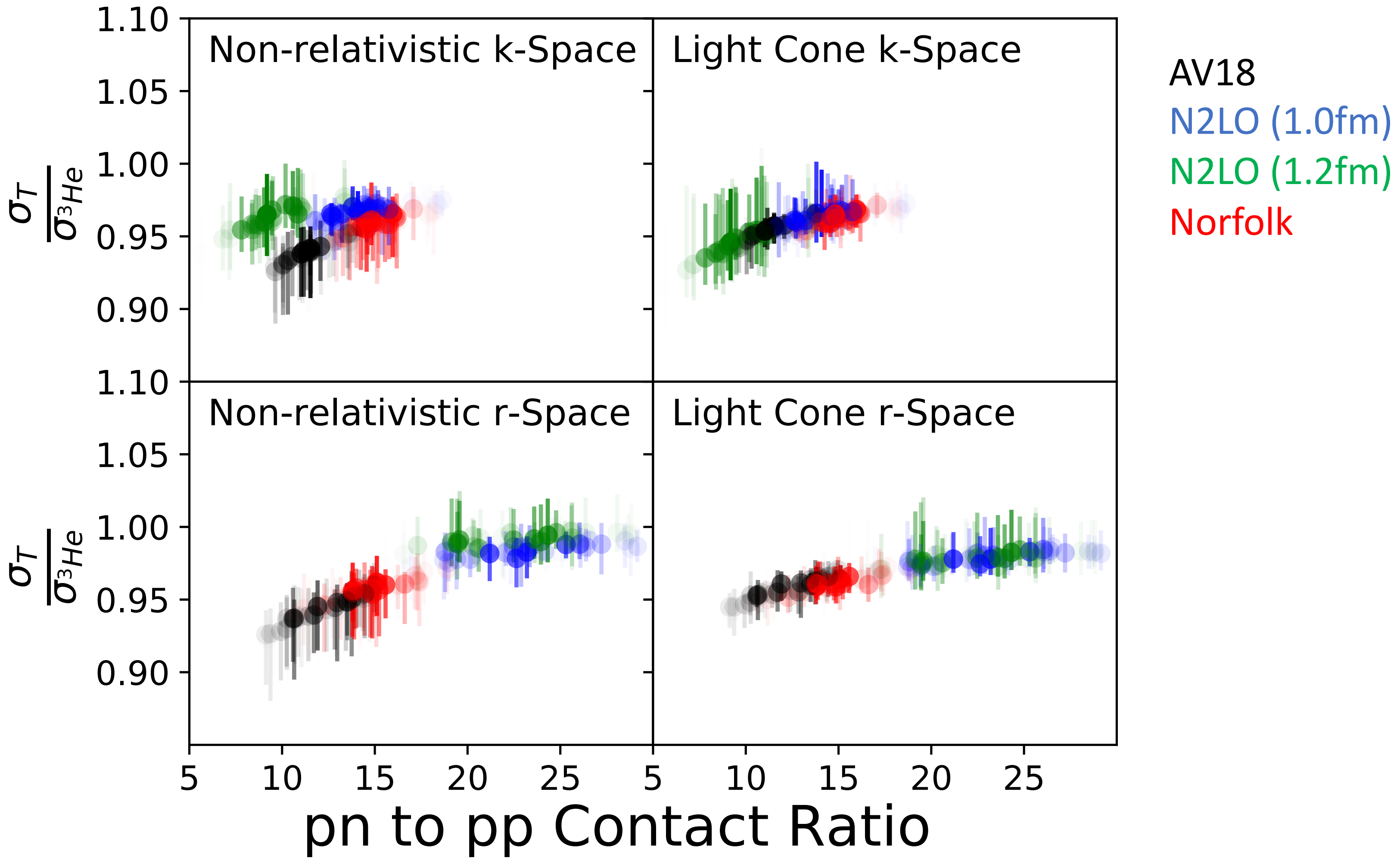

Figure by A. Denniston

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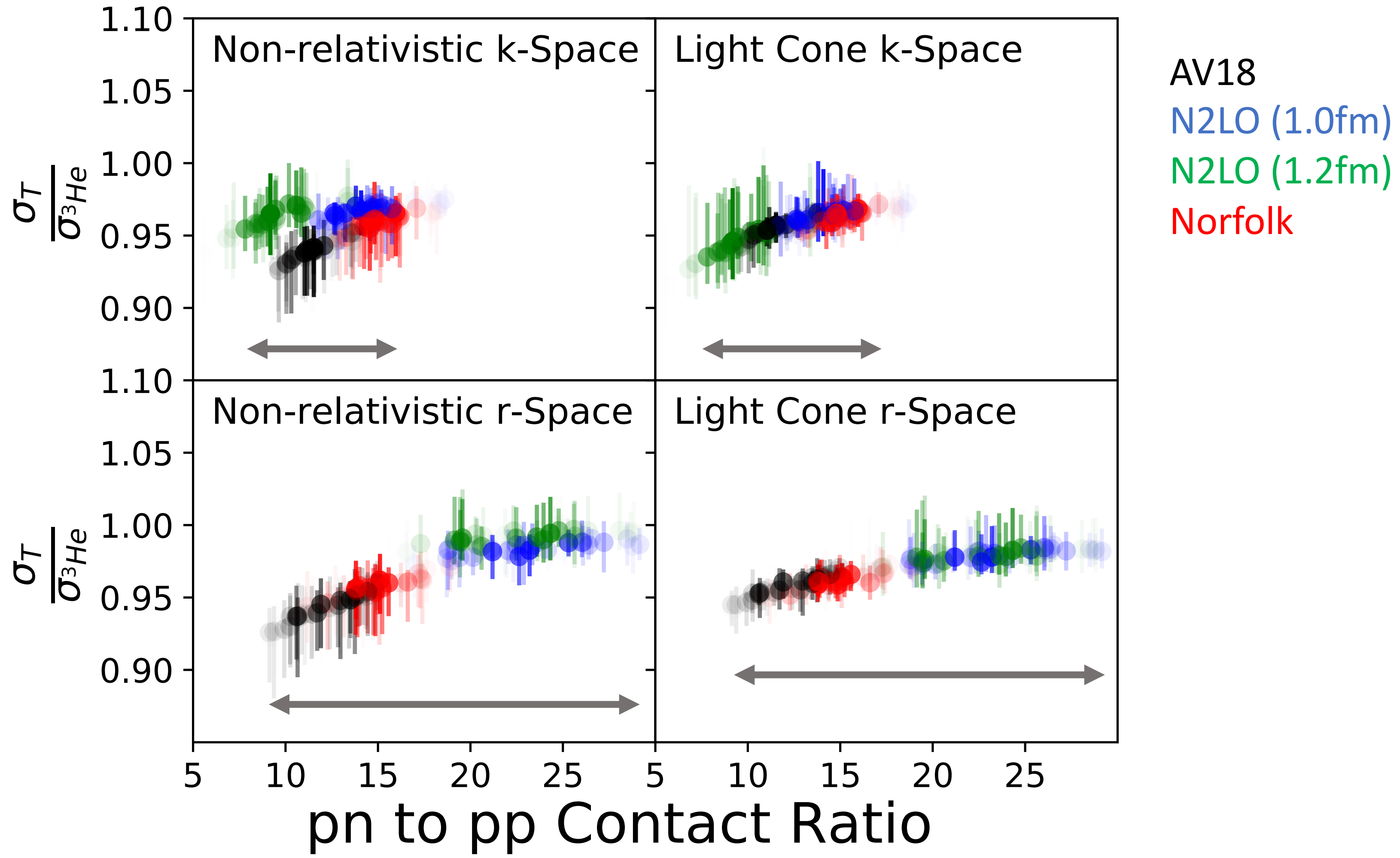
Randomly sample:

- σ_{CM}, C_{α}

(shading of the data points)



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Errors represent full range of ratio in $1.5 < x < 1.9$

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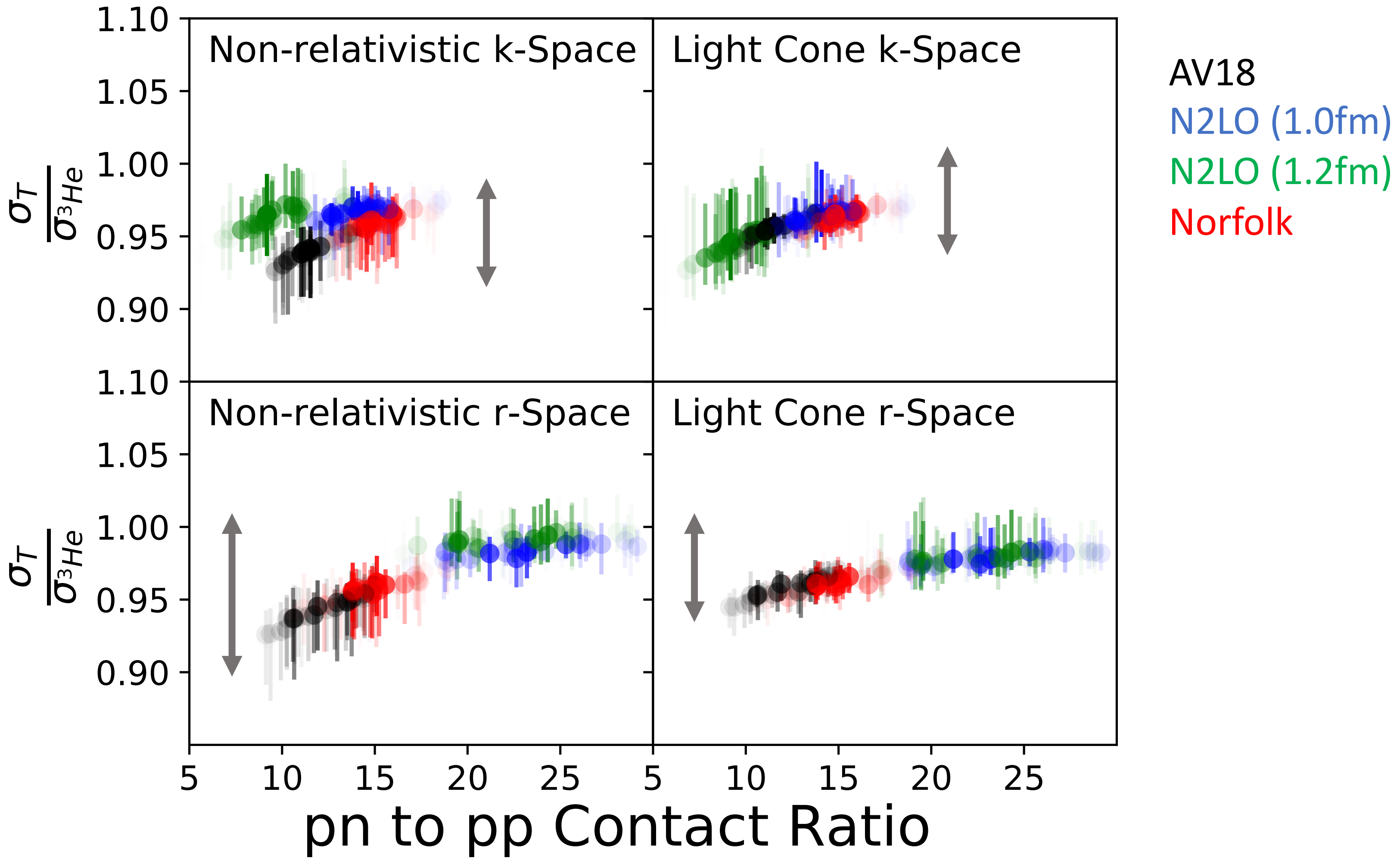

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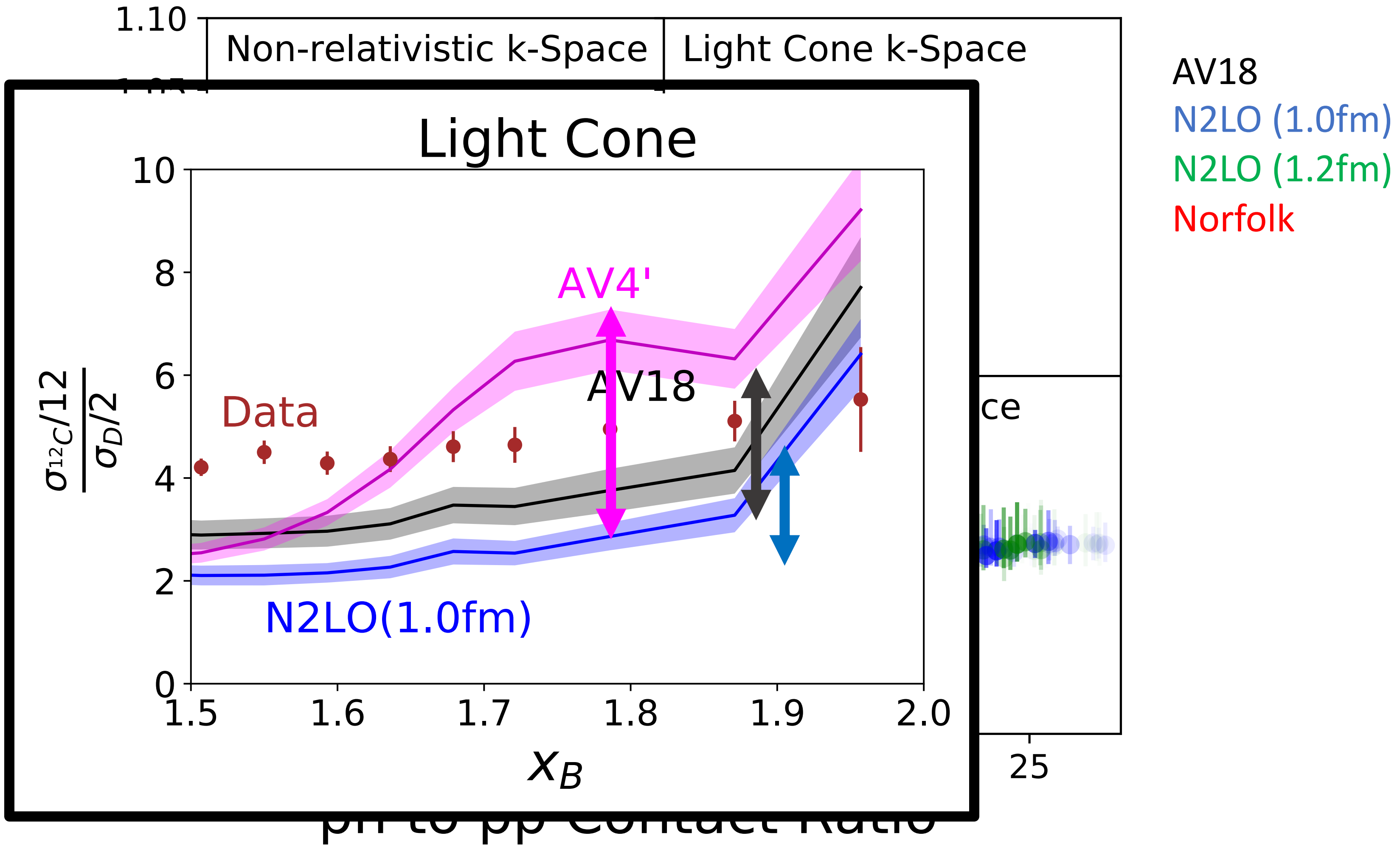

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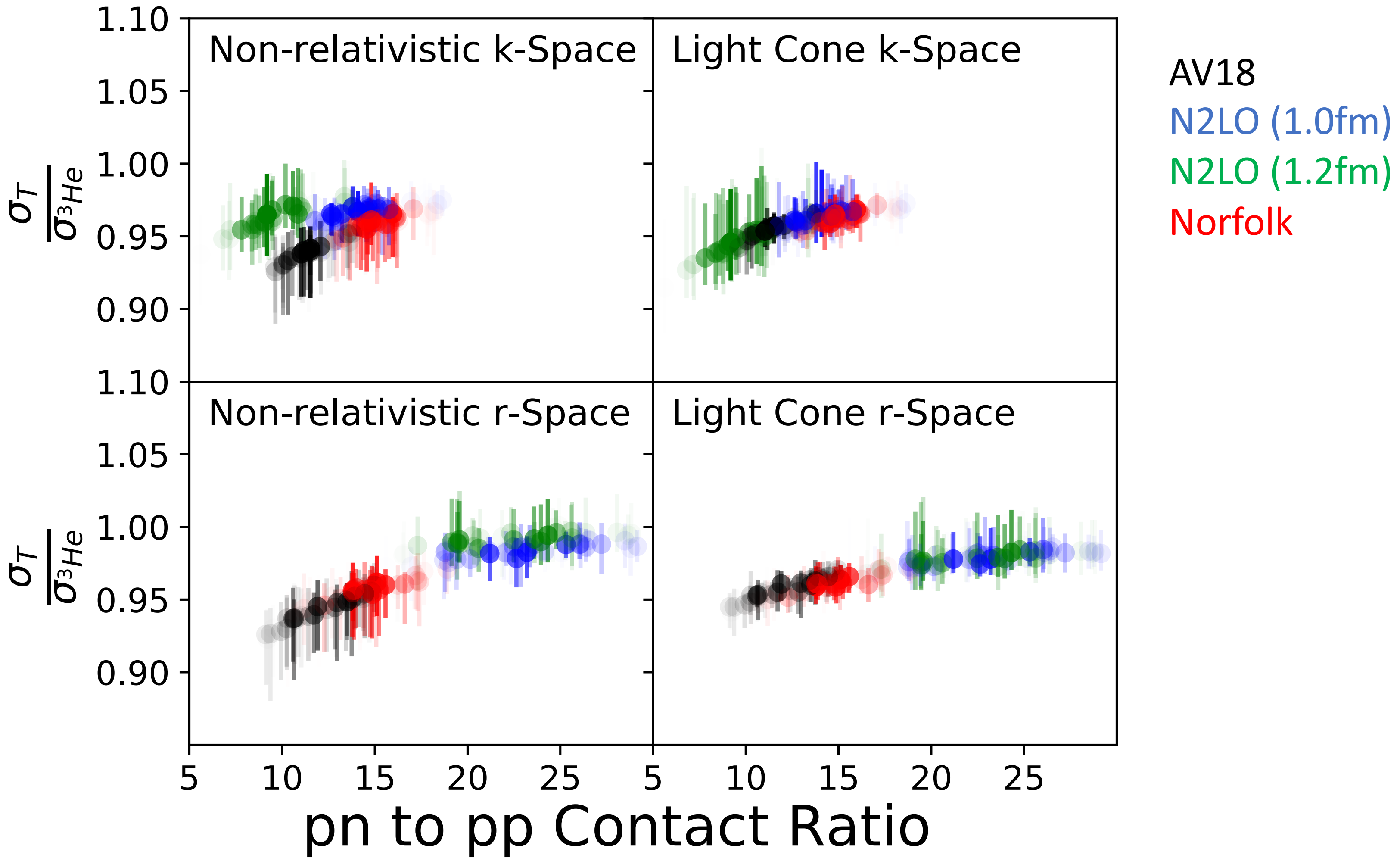
Depending on assumptions, a measured $\sigma_T/\sigma_{^3He}$, could mean a wide range of pn/pp ratios.



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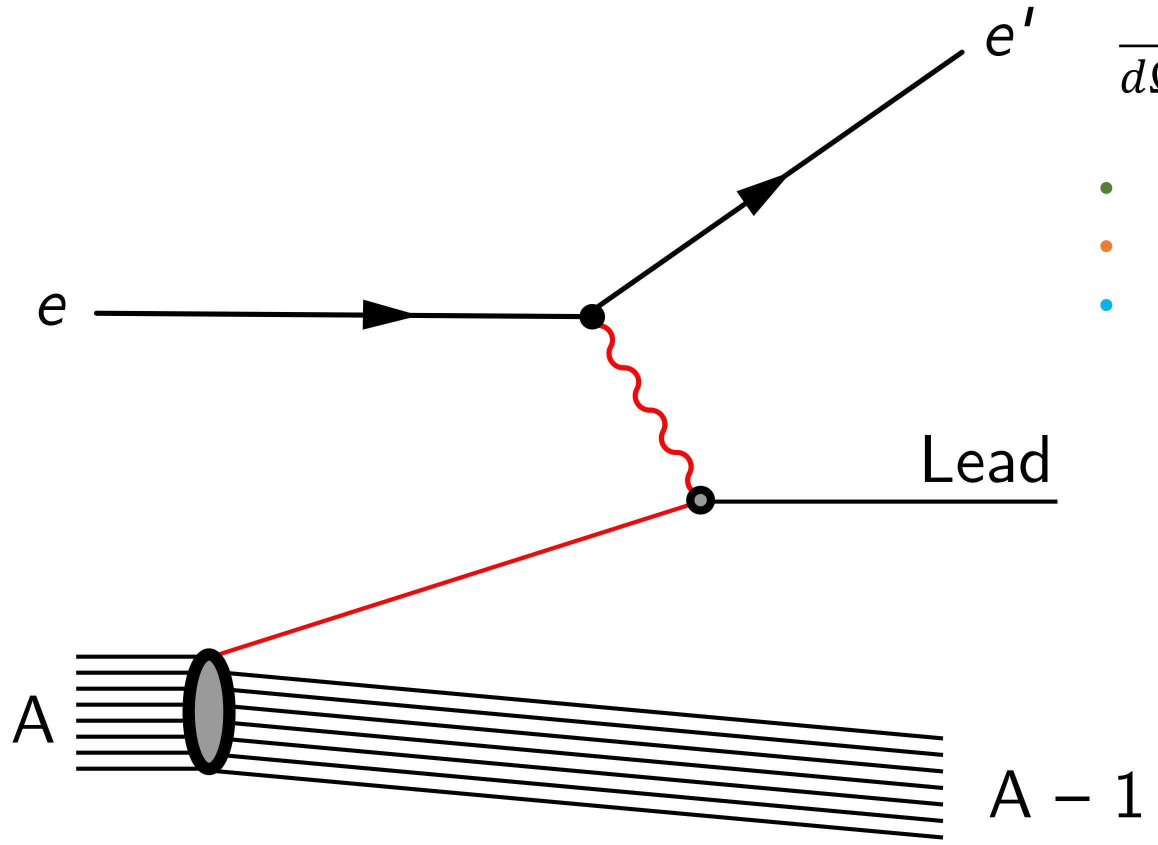
Quasi-elastic (QE) scattering in the Plane-Wave Impulse Approximation (PWIA)



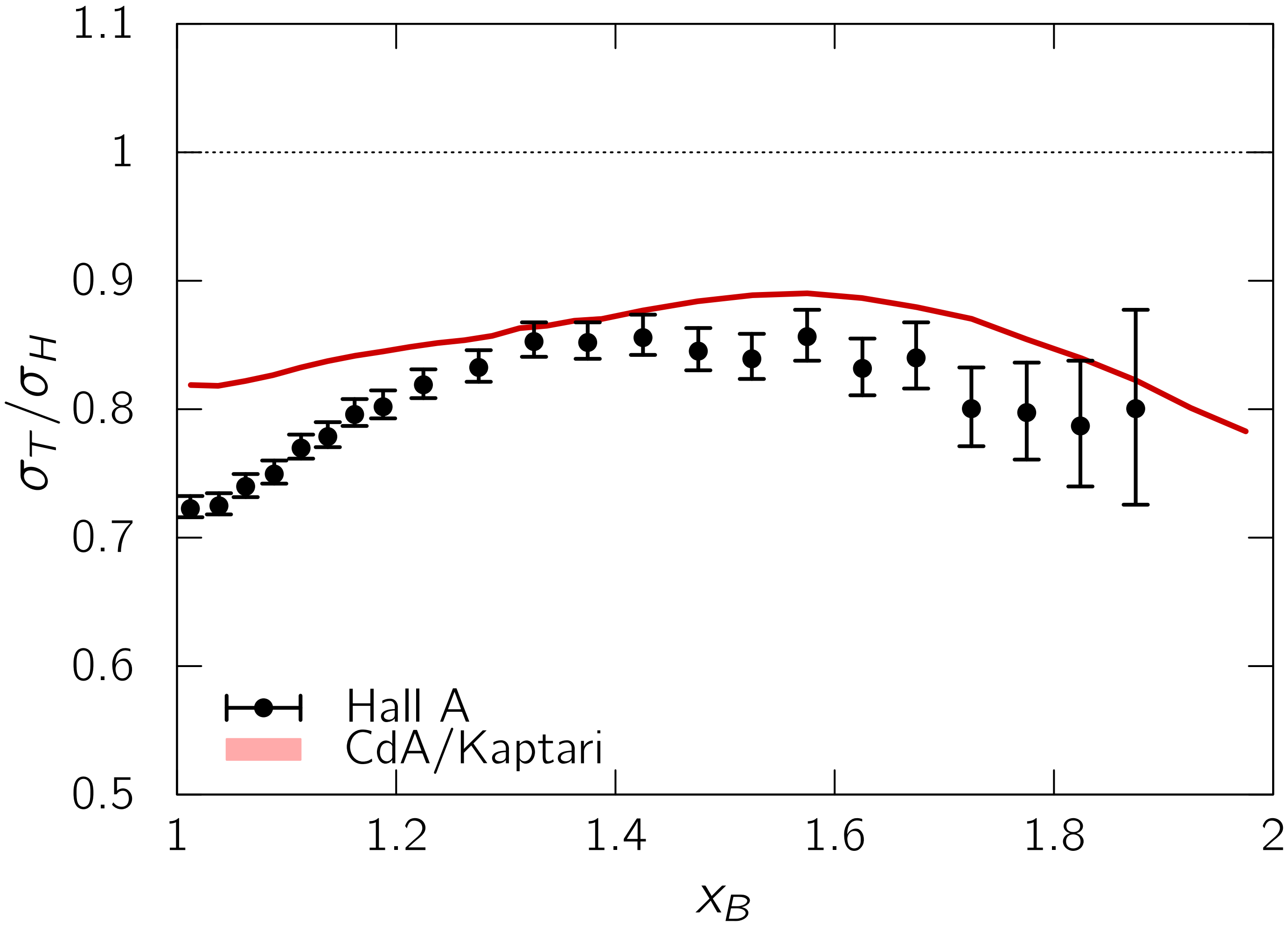
$$\frac{d^6\sigma}{d\Omega_{e'} dE_{e'} d\Omega_{Lead} dE_{Lead}} = (p_{Lead} E_{lead}) \cdot \sigma_{eN} \cdot S(E_m, p_m)$$

- Kinematic factor (i.e., a Jacobian)
- σ_{eN} is the single-nucleon cross section
- $S(E_m, p_m)$ is the “Spectral Function,” probability to find a nucleon characterized by:
  - Initial momentum, $p_m \equiv \vec{p}_{Lead} - \vec{q}$
  - Separation energy, E_m

Inclusive cross sections

Beam E = 4.33 GeV

Electron angle: 20.88˚



Inclusive cross sections

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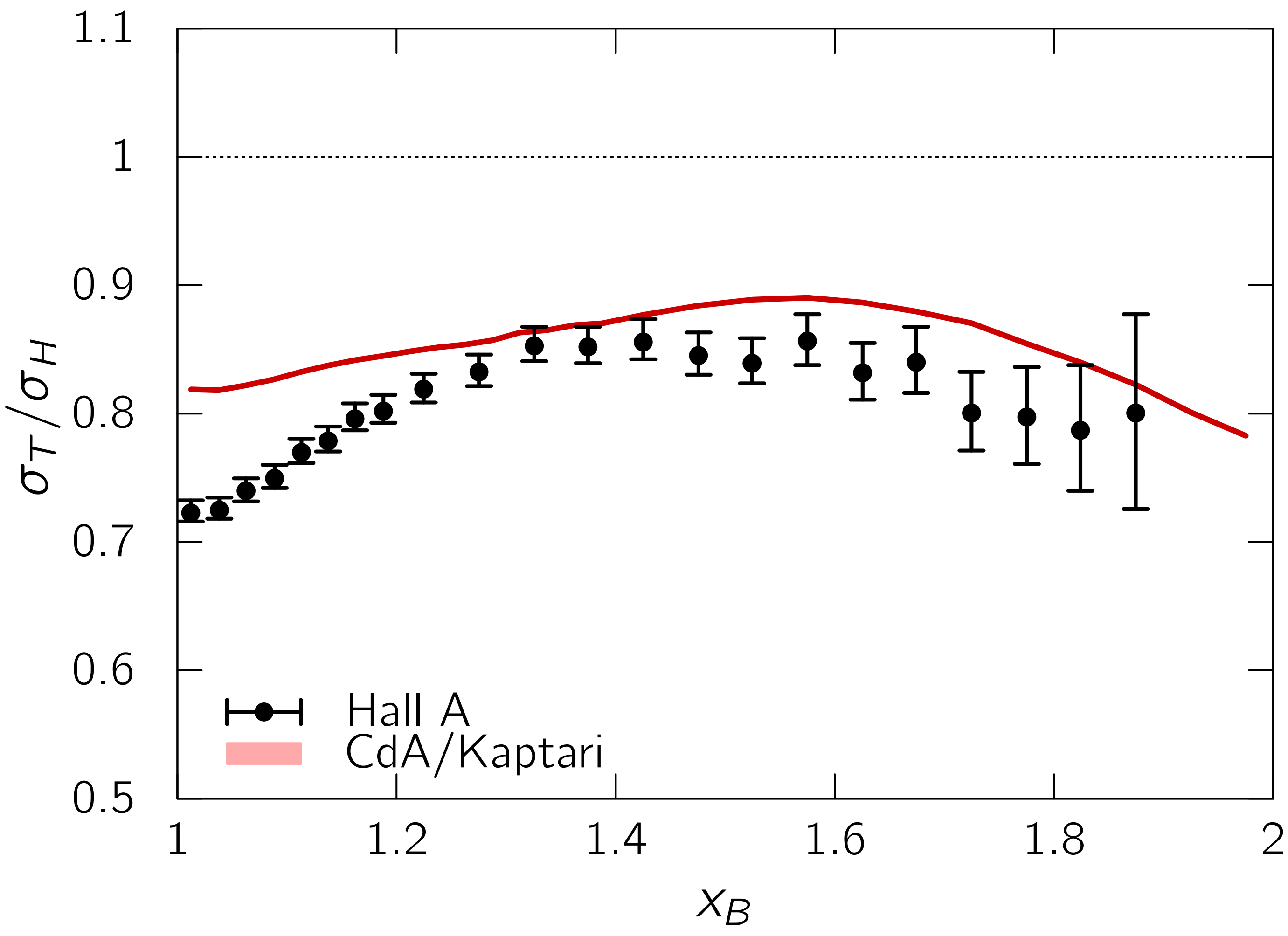
Electron angle: 20.88˚

- CK calculation is not flat.
- Also fails to reproduce x=1 region.



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Spectral Functions by Argonne VMC group

Alessandro Lovato

Noemi Rocco

So far, I have received calculations of:

- 3H
- 3He
- 4He
- 12C

$$P_{\tau_k}(\mathbf{k}, E) = \sum_n |\langle \Psi_0^A | a_k^\dagger | \Psi_n^{A-1} \rangle|^2 \delta(E + E_0^A - E_n^{A-1})$$

Evaluated in two "parts."

- Mean-field:

  e.g.:

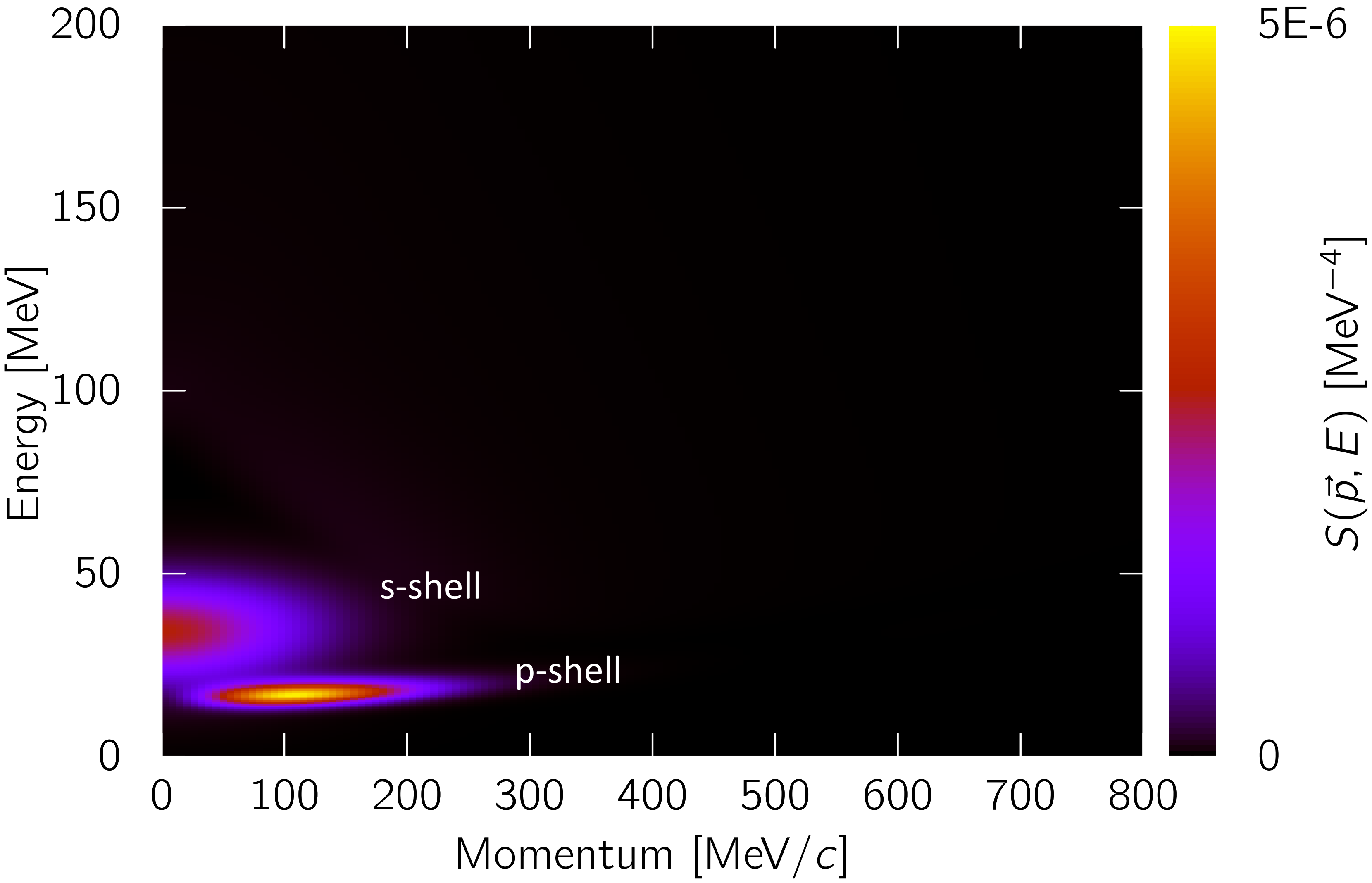
$$P_p^{\mathrm{MF}}(\mathbf{k}, E) = n_p^{\mathrm{MF}}(\mathbf{k}) \delta\left(E - B_{^4\mathrm{He}} + B_{^3\mathrm{H}} - \frac{k^2}{2m_{^3\mathrm{H}}}\right)$$

- Correlated:

  e.g.:

$$P_p^{\mathrm{corr}}(\mathbf{k}, E) = \sum_n \int \frac{d^3k'}{(2\pi)^3} |\langle \Psi_0^A | [|k\rangle \, |k'\rangle \, |\Psi_n^{A-2}\rangle]|^2 \times \delta(E + E_0^A - e(\mathbf{k}') - E_n^{A-2}) \, .$$

^{12}C Spectral Function



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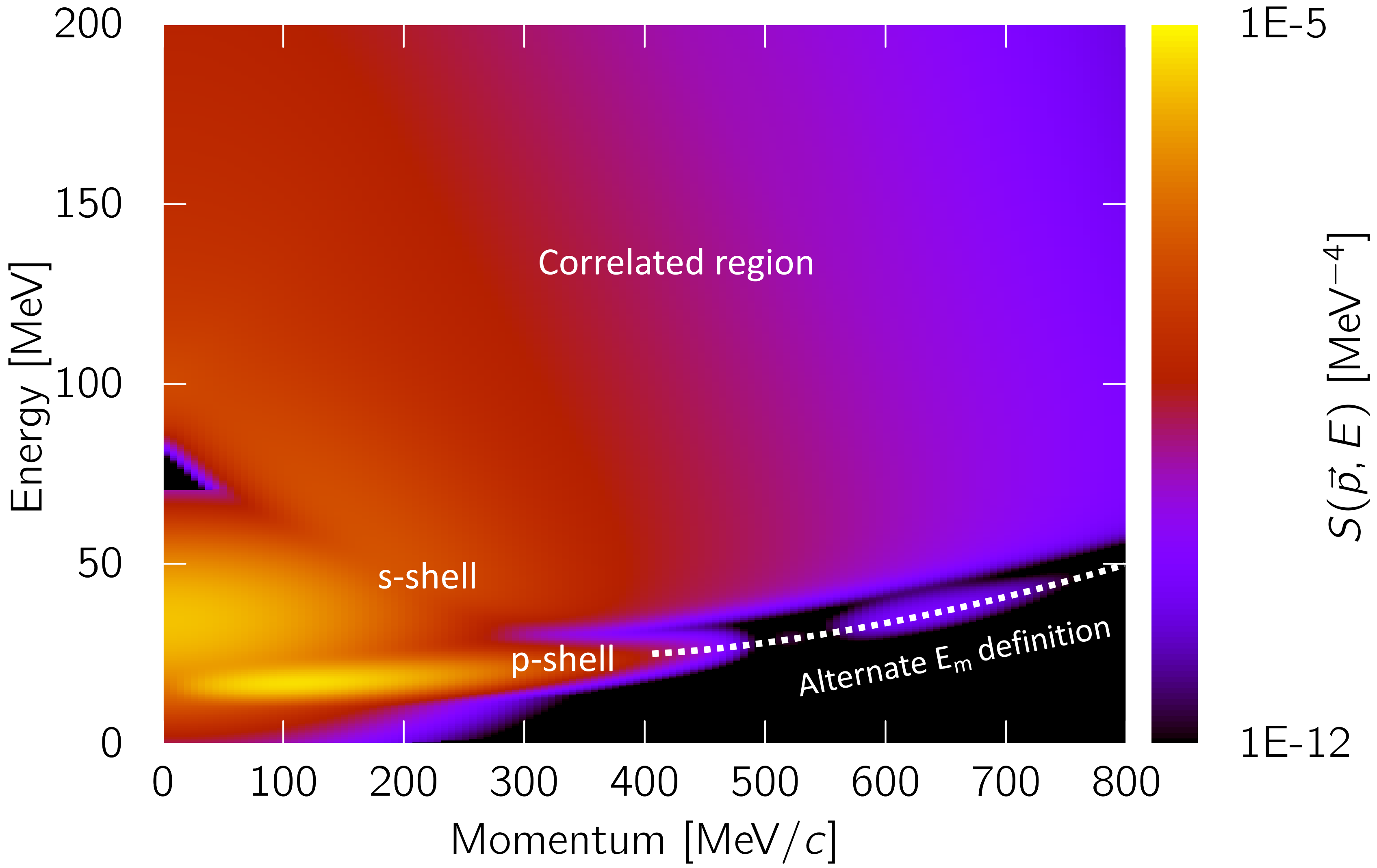
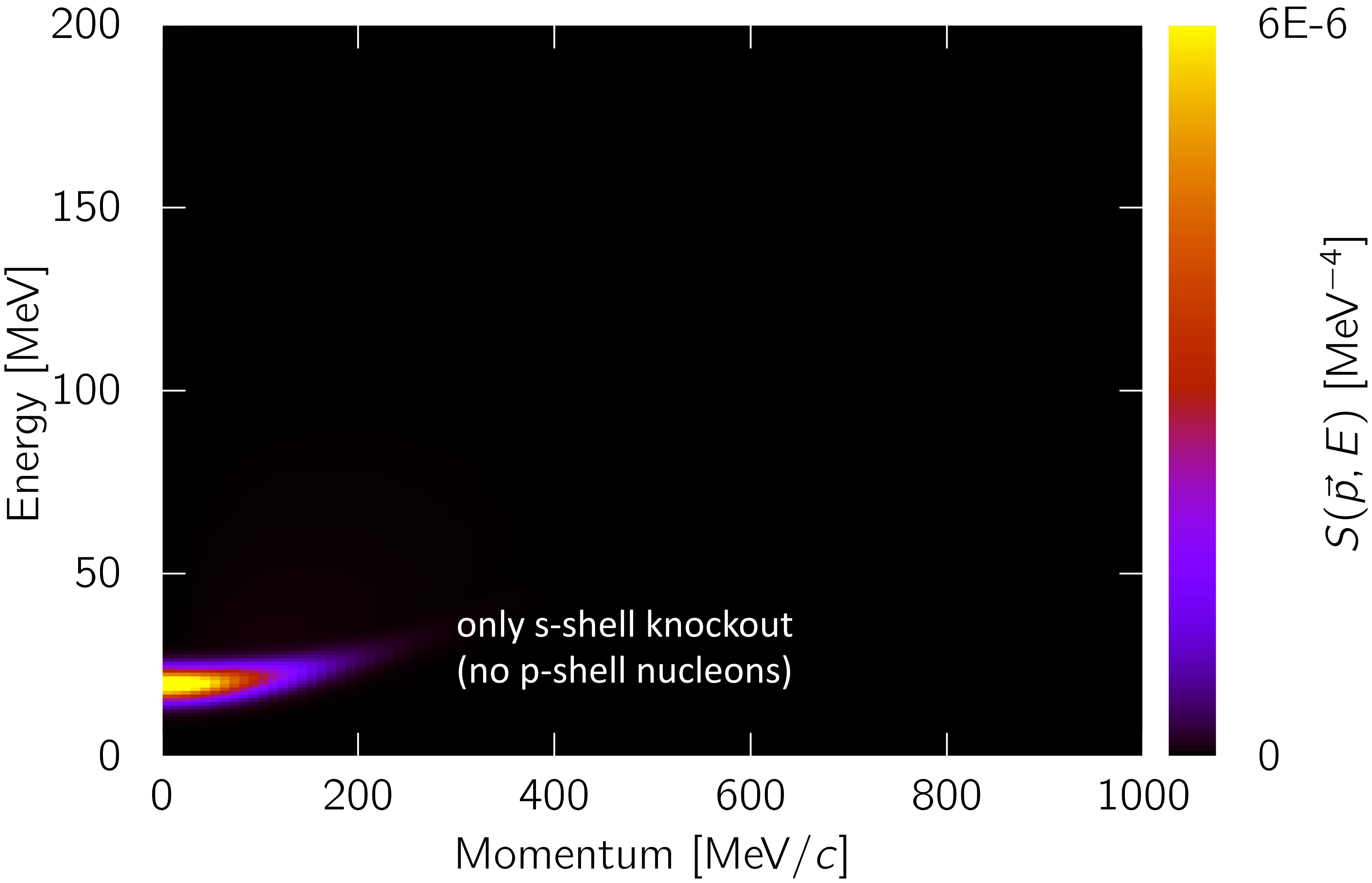


^{4}He Spectral Function



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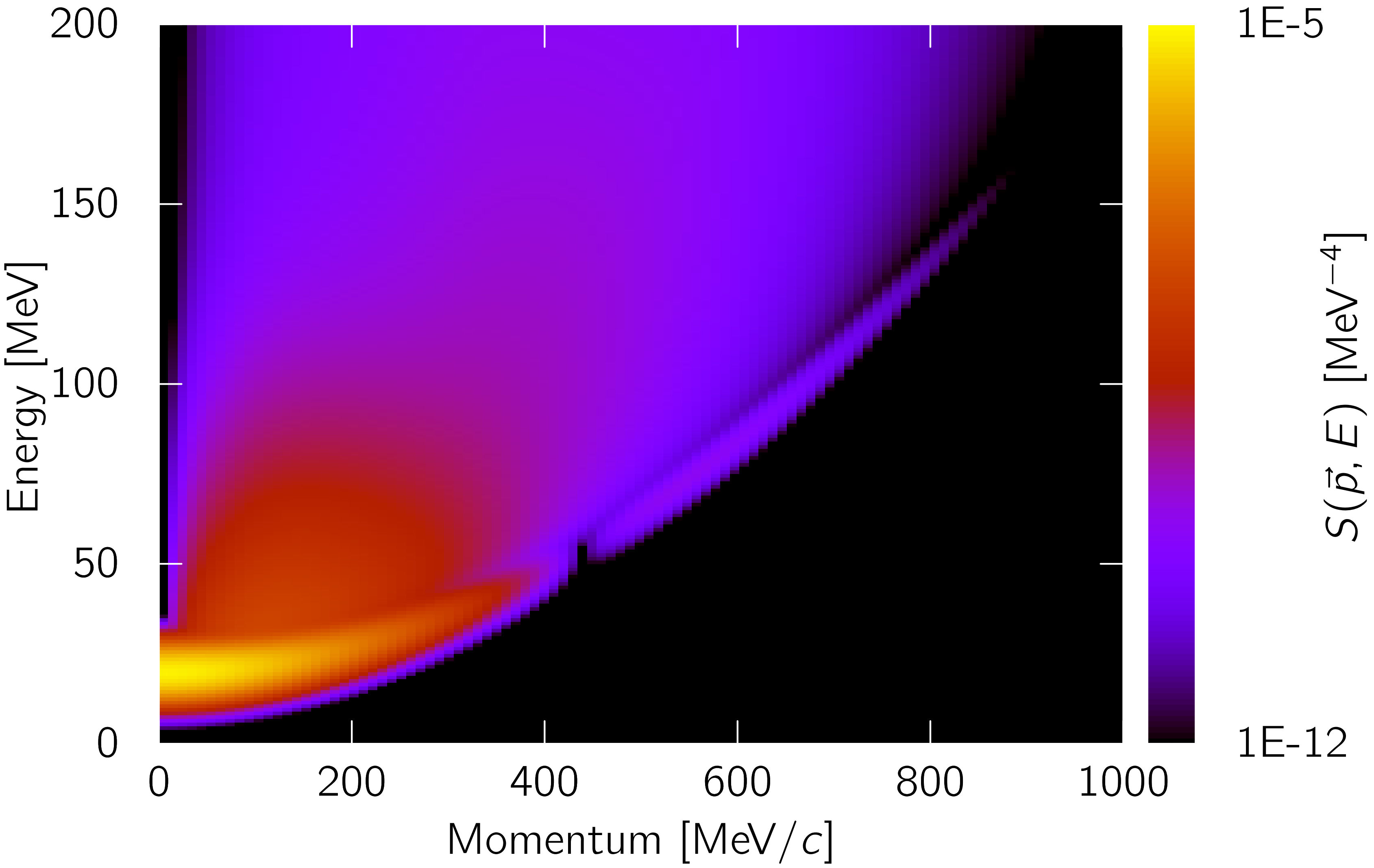

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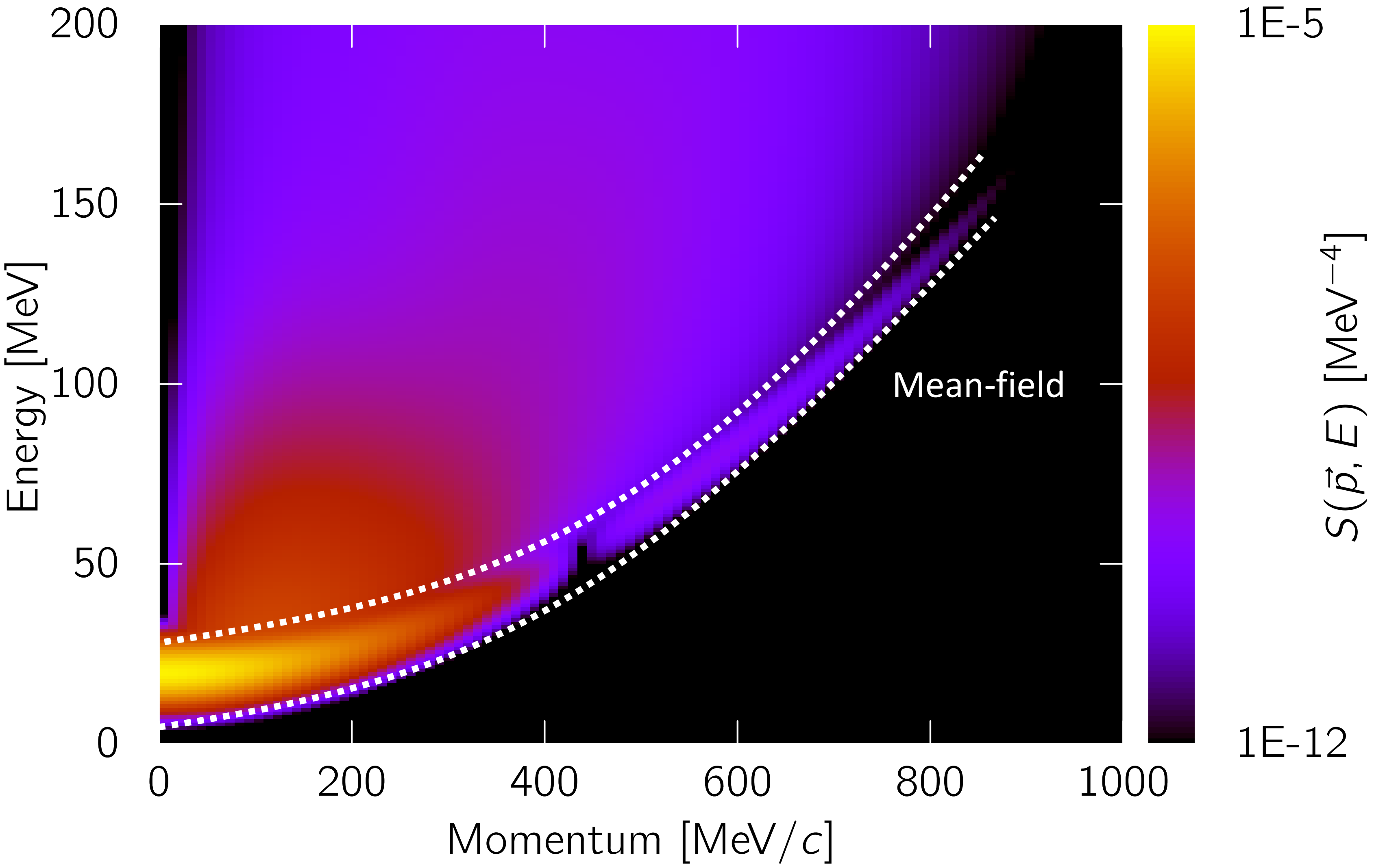

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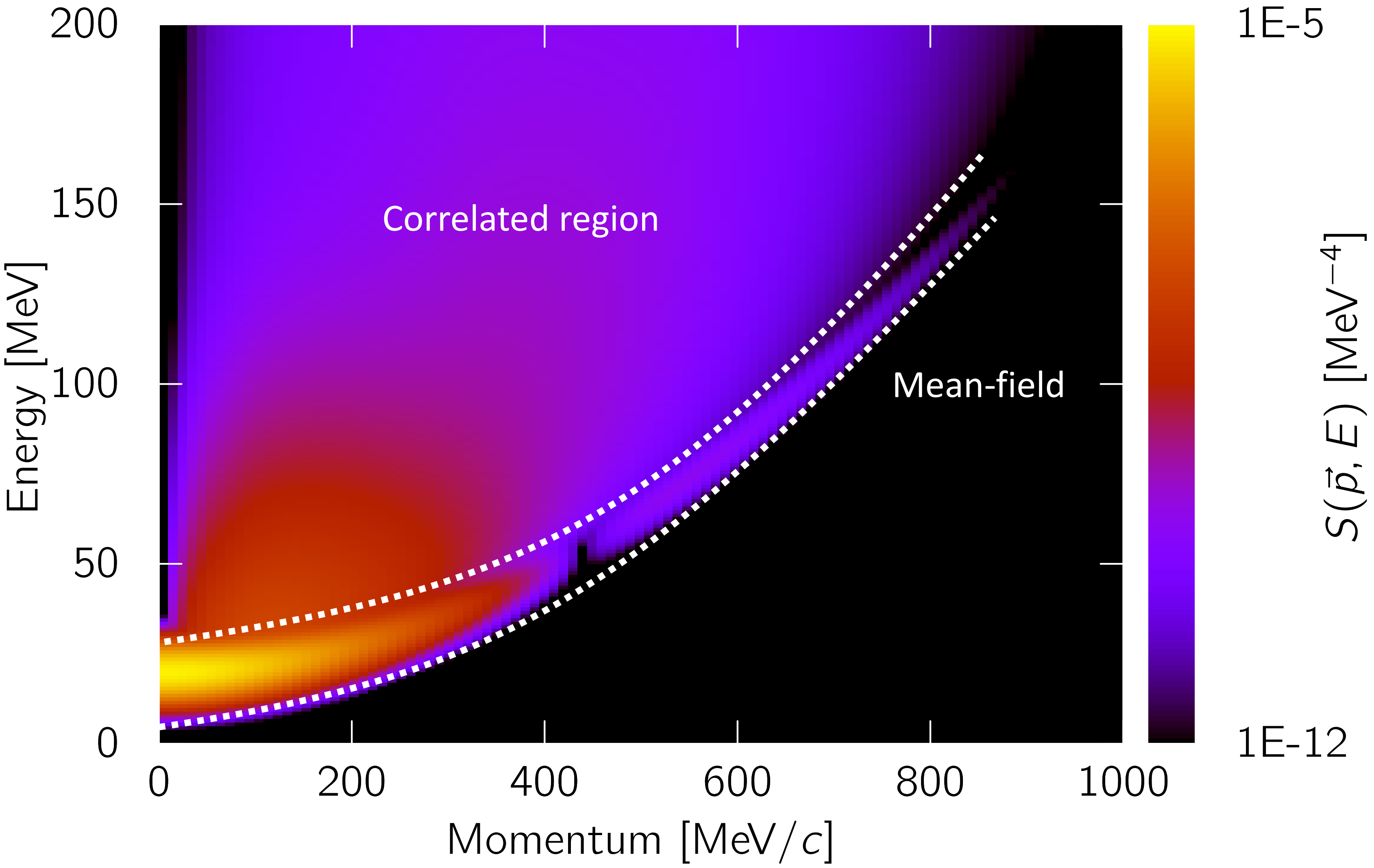


He-4 Normalization

$$\int \frac{d^3 p_m}{(2\pi)^3} dE_m S_p(p_m, E_m) = Z$$

Mean-Field Integral:	1.673786
Correlated Integral:	0.326078
Total Integral:	1.999867

The calculation predicts the spectroscopic factor, i.e., the occupancy of the s-shell state.

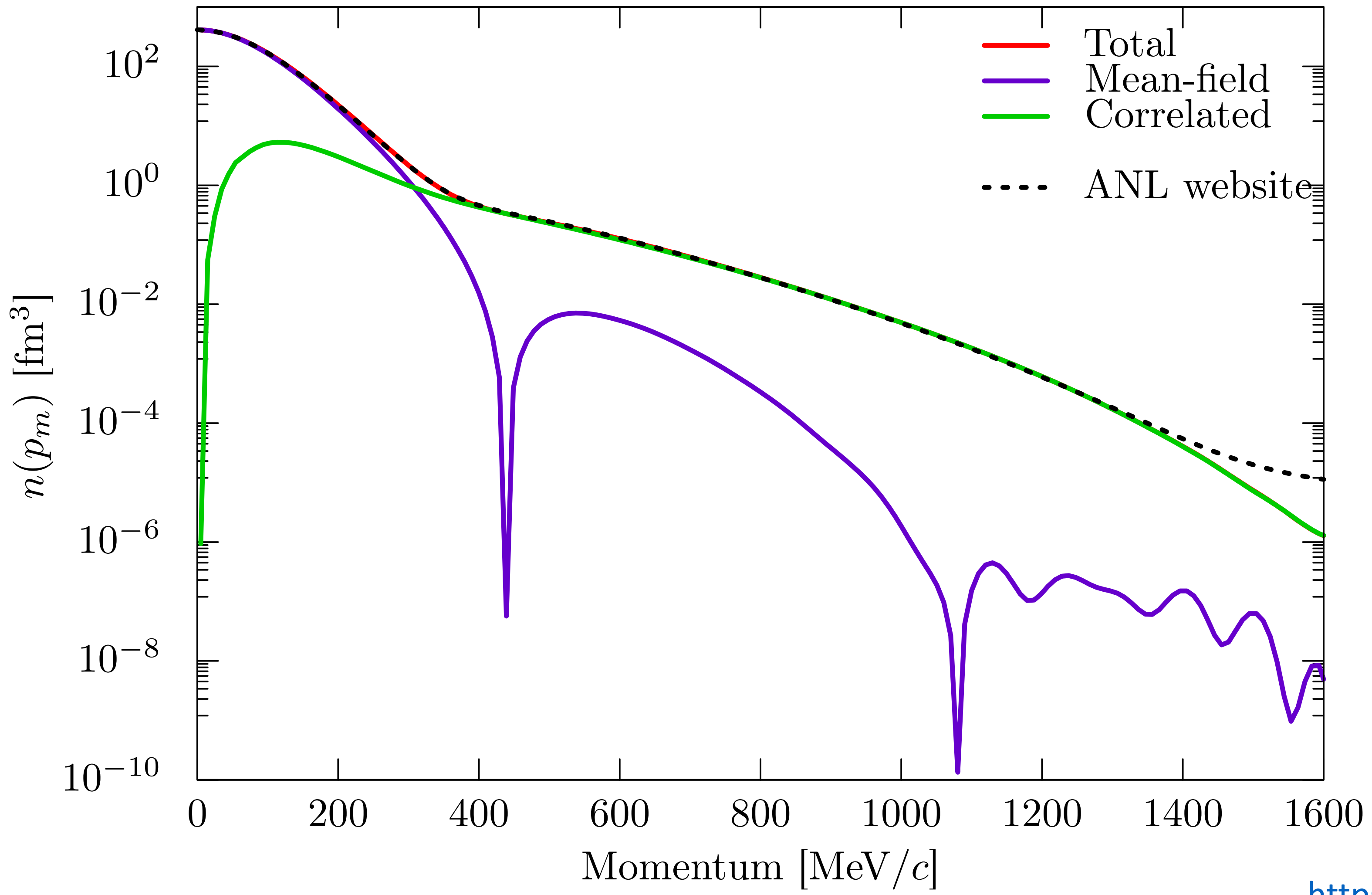
The correlated part is just the difference.

C-12 Normalization

$$\int \frac{d^3 p_m}{(2\pi)^3} dE_m S_p(p_m, E_m) = Z$$

Mean-Field Integral:	4.610938	76.7%
Correlated Integral:	1.398381	23.3%
Total Integral:	6.009318	

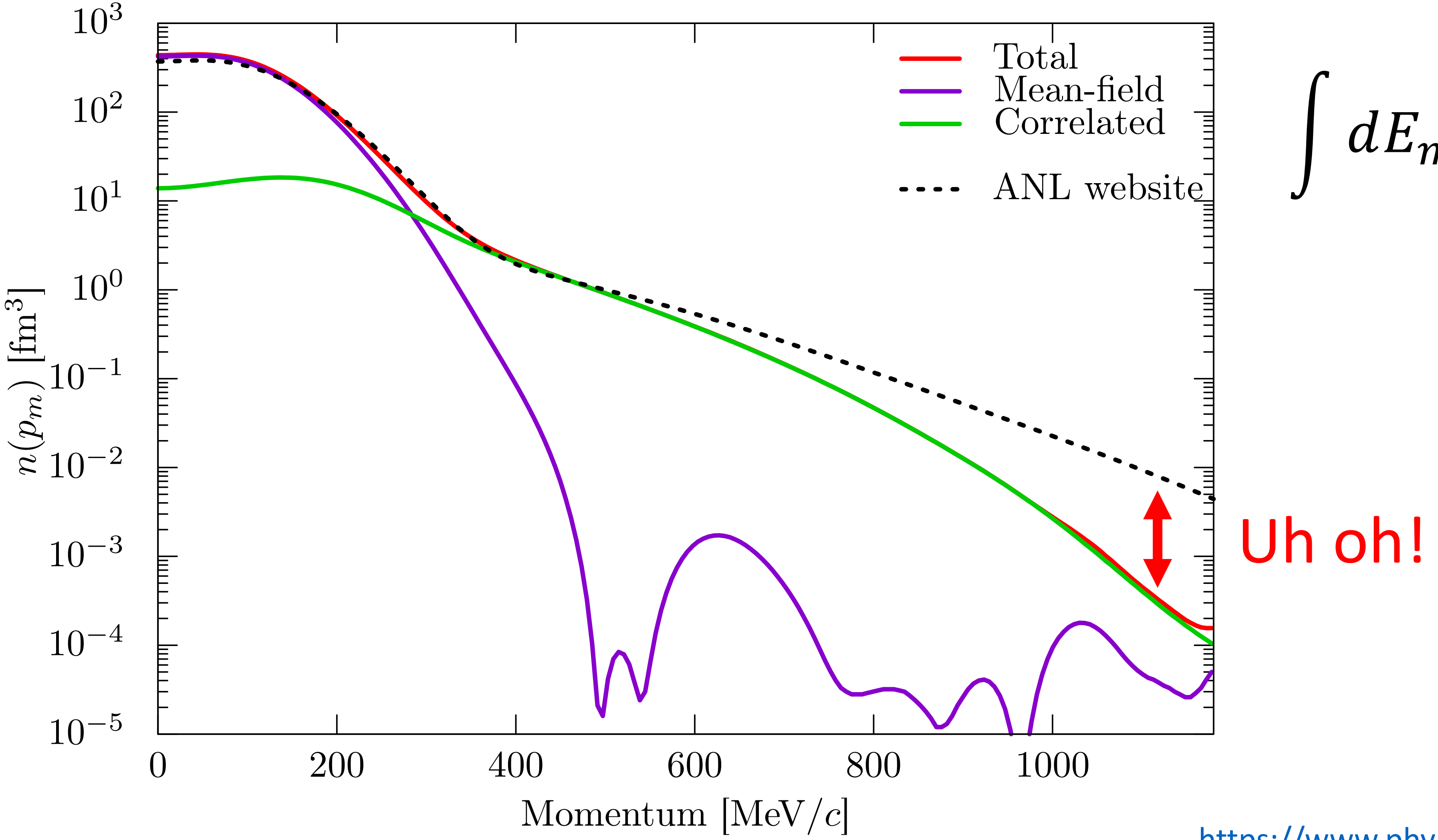
He-4 Momentum Distributions



$$\int dE_m \, S(p_m, E_m) = n(p_m)$$

https://www.phy.anl.gov/theory/research/momenta/

C-12 Momentum Distributions



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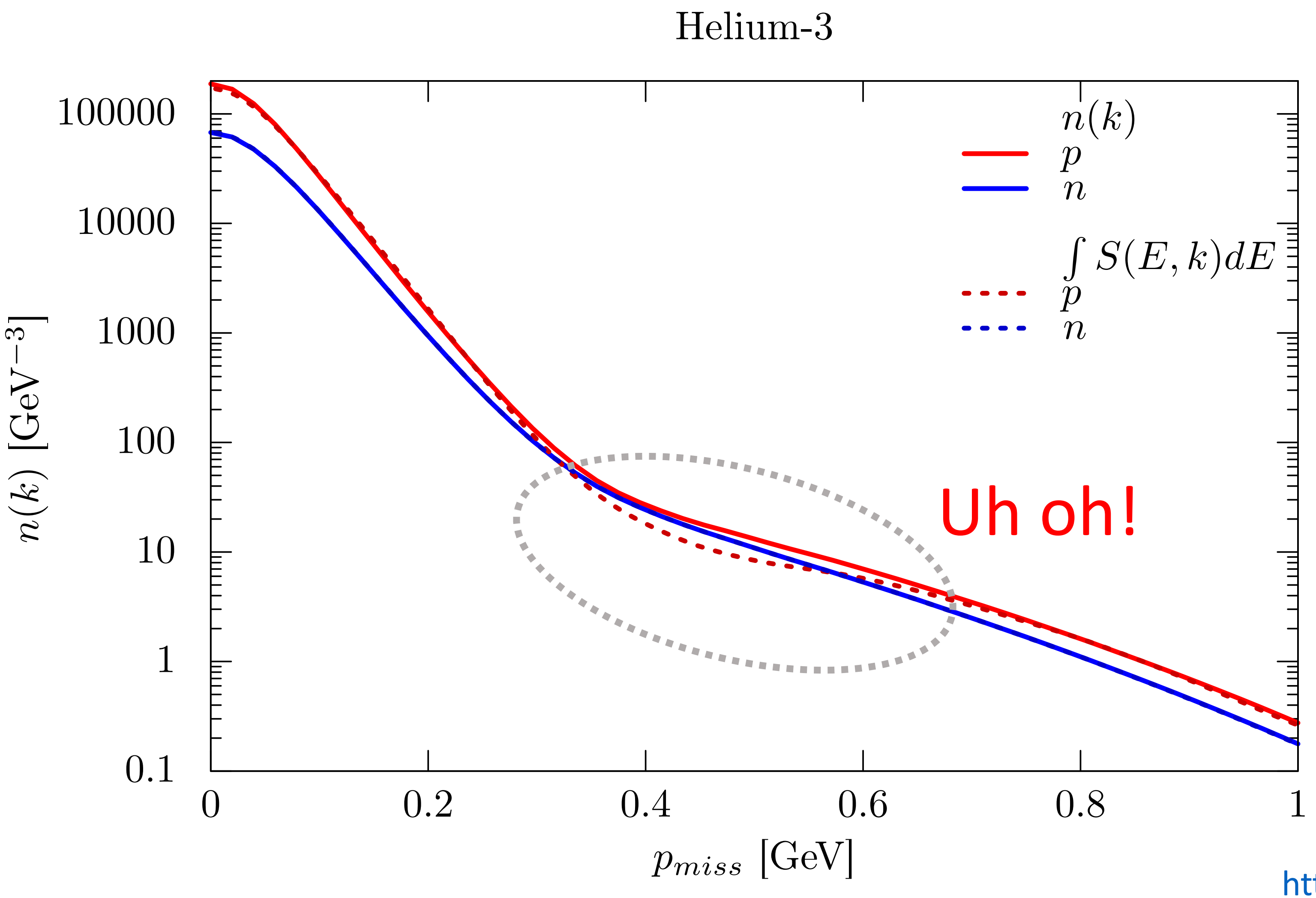
A=3 Momentum Distributions



$$\int dE_m\, S(p_m, E_m) = n(p_m)$$

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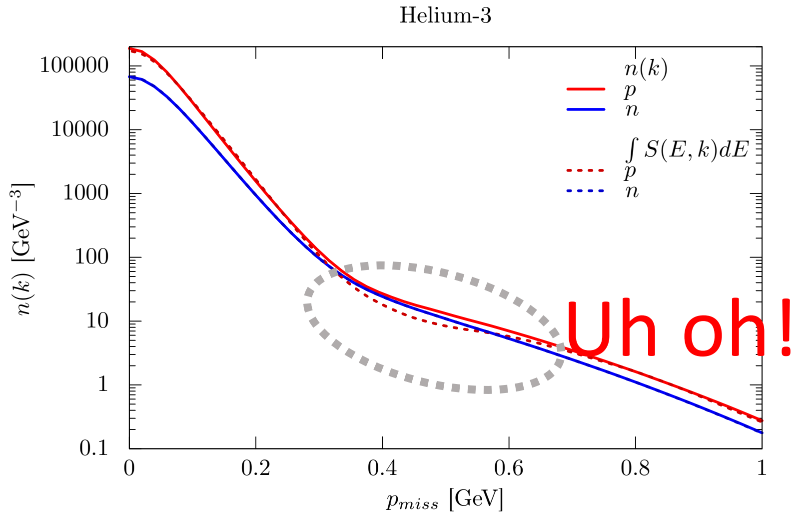
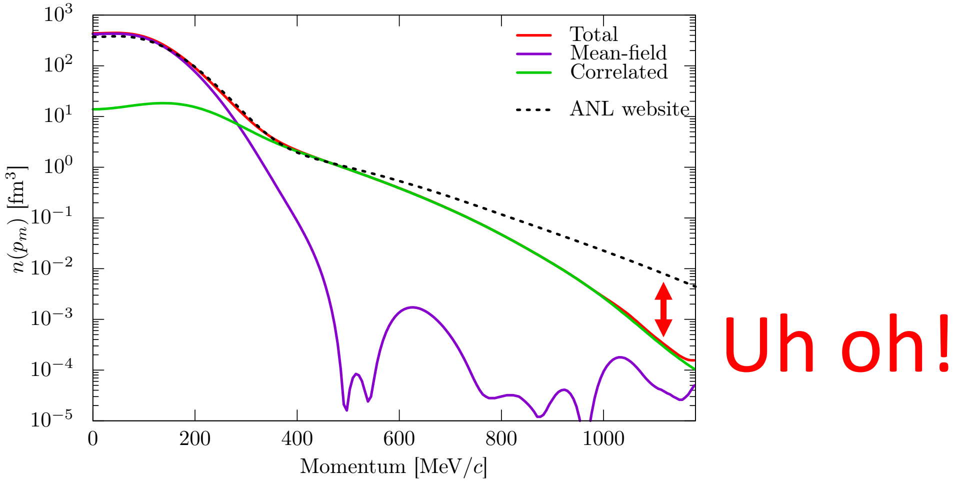
Why are the VMC Spectral functions inconsistent?

- Noemi and Alessandro “re-used” a calculation in which not all of the information was saved.
- They assume a form for the distribution of angles between q and Q, and tune to approximate 1d momentum distribution
  - Clearly not perfect.





Inclusive Scattering at x > 1

Event Generator: https://github.com/schmidta87/QE_Generator

$$\frac{d\sigma}{d\Omega_e dx_B dE_m dp_m d\phi_{qN}} = \frac{E_{e'}\omega}{E_e x_B q} \cdot E_N p_m \cdot \sigma_{eN} \cdot S(p_m, E_m)$$

Jacobian from $dE_{e'} \rightarrow dx_B$

Jacobian from $dE_N d\cos\theta_N \rightarrow dE_m\, dp_m$

Possible additional Jacobian based on the different missing energy conventions

$$\mathcal{J} = \frac{E_{m,1} + m_A - m_N}{E_{m,2} + m_A - m_N}$$

I am just neglecting this, because I'm not sure it's needed and it's very close to 1.

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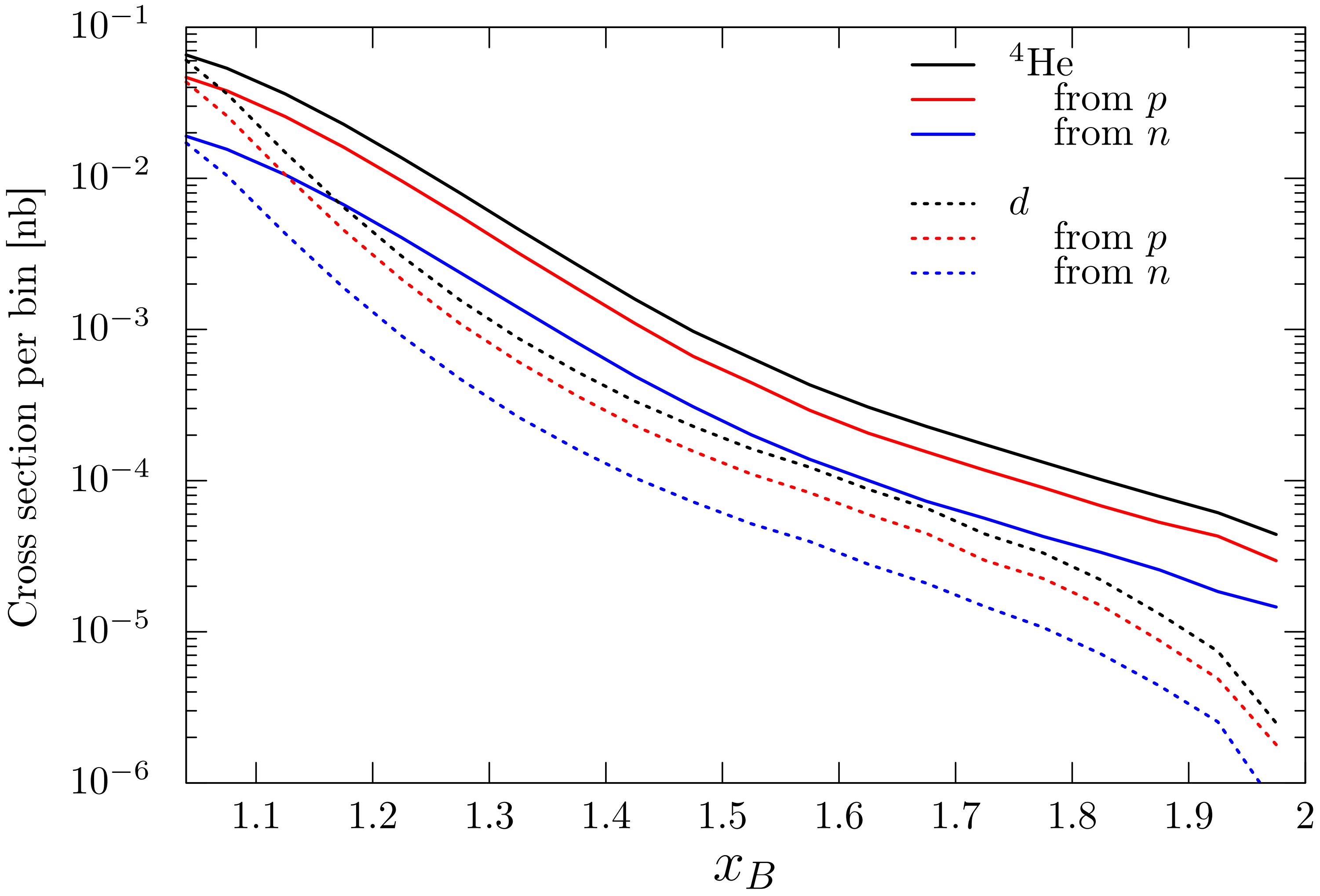
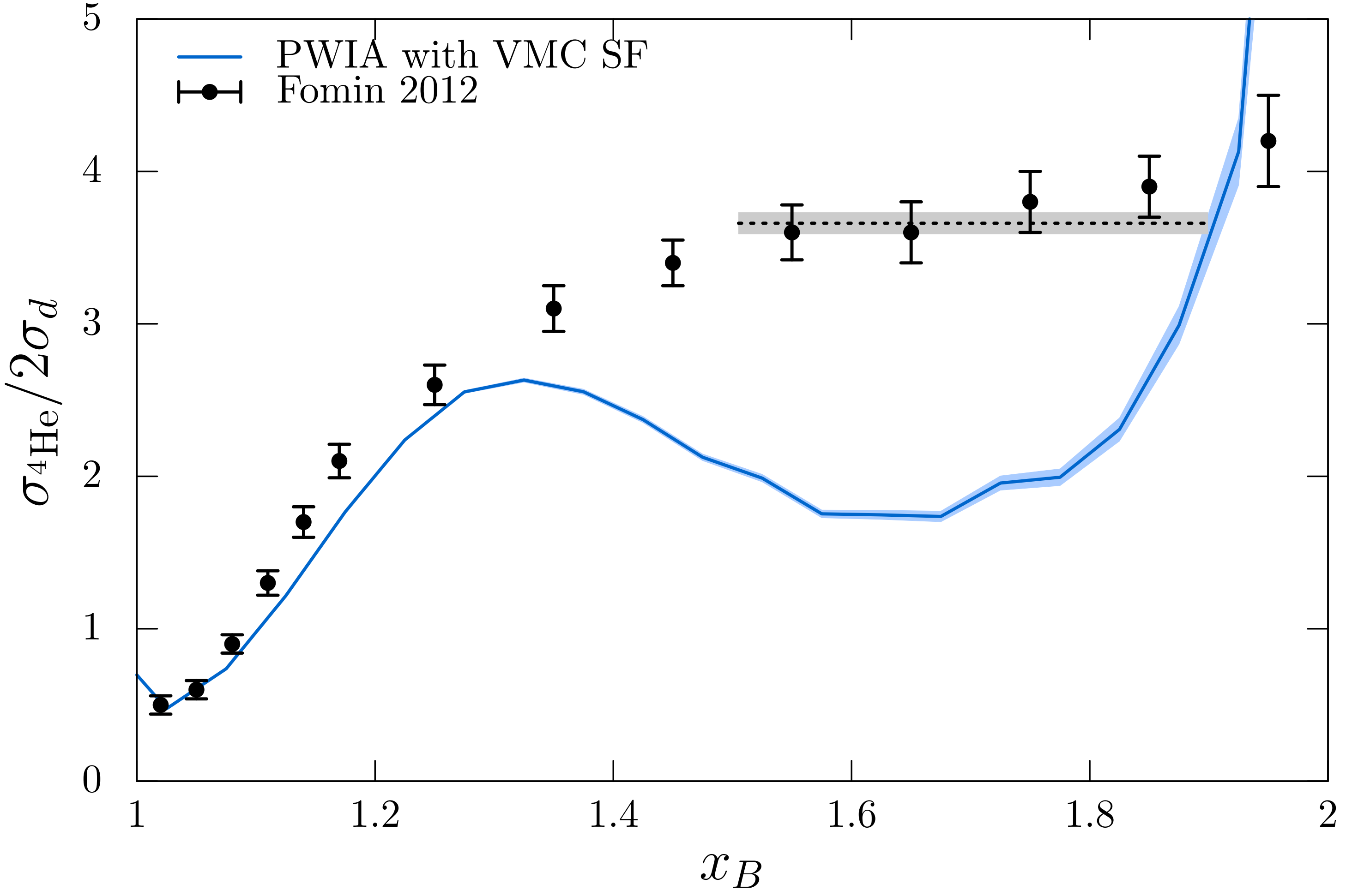
- $A(e,e')$, integrating over all possible leading protons and neutrons.
- Kinematics corresponding to Fomin et al., PRL (2012)
  - 5.766 GeV beam
  - 18° electron scattering angle
  - Spectra plotted in terms of x_B

Helium / Deuterium Yields



Helium / Deuterium Ratio



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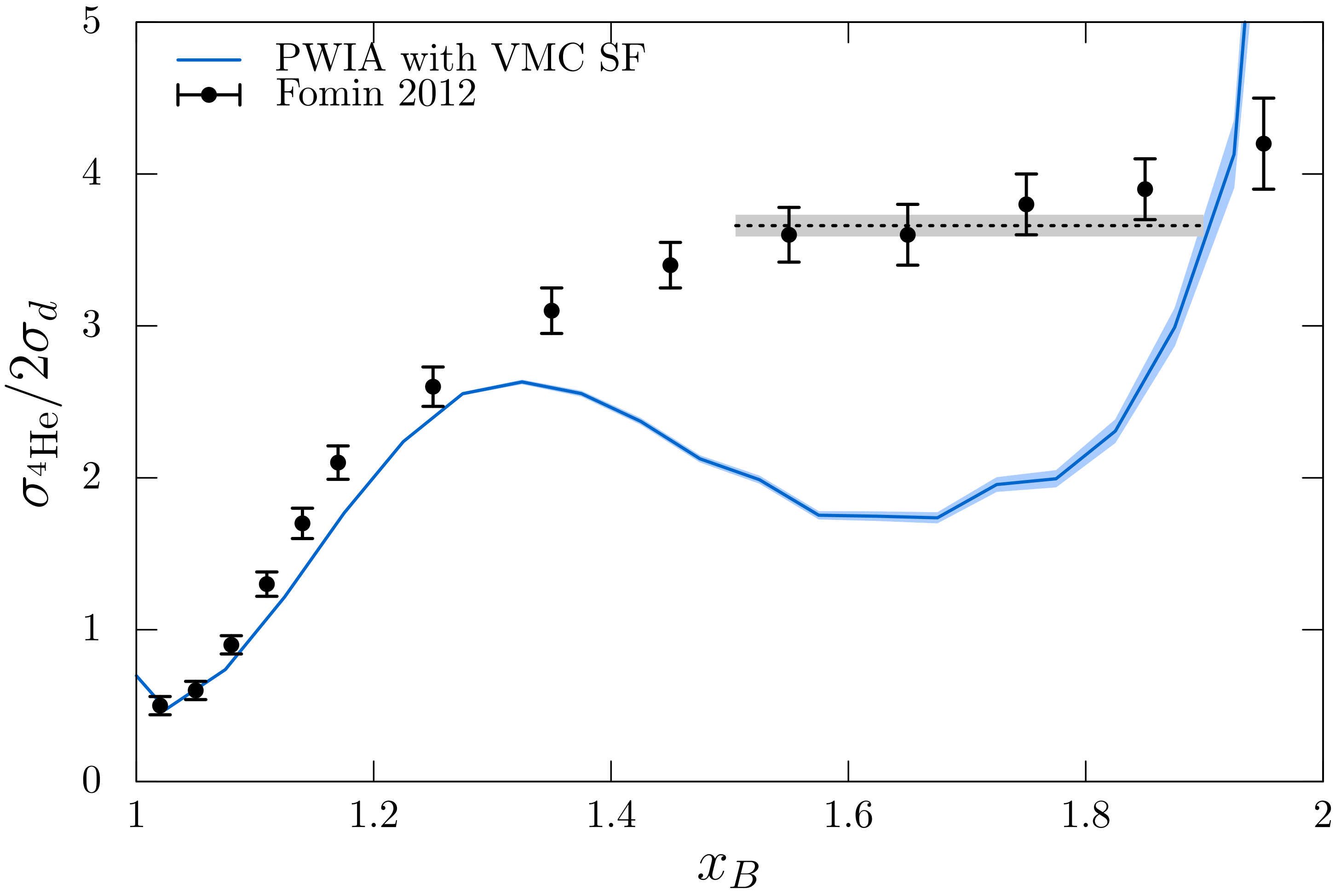
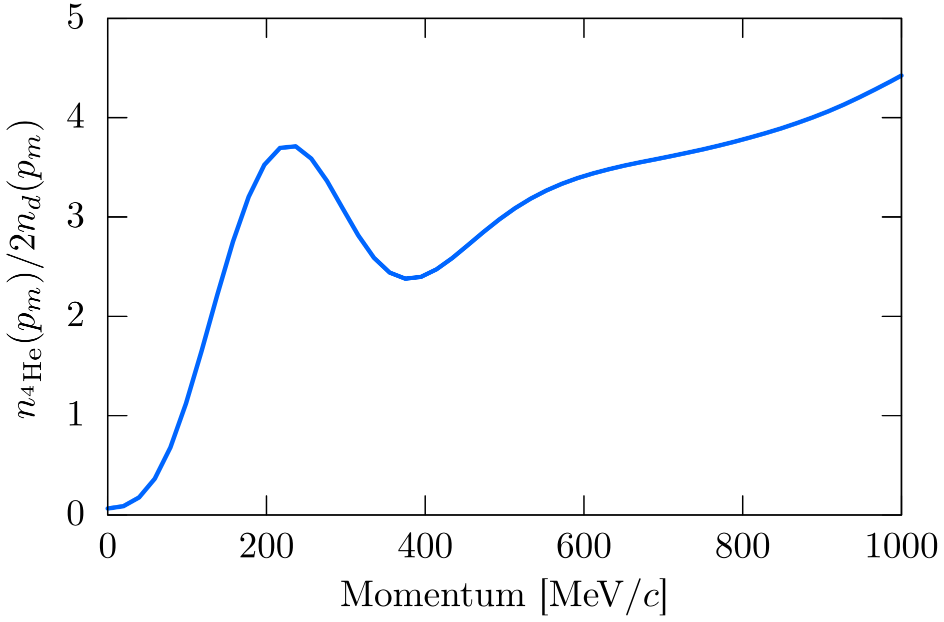
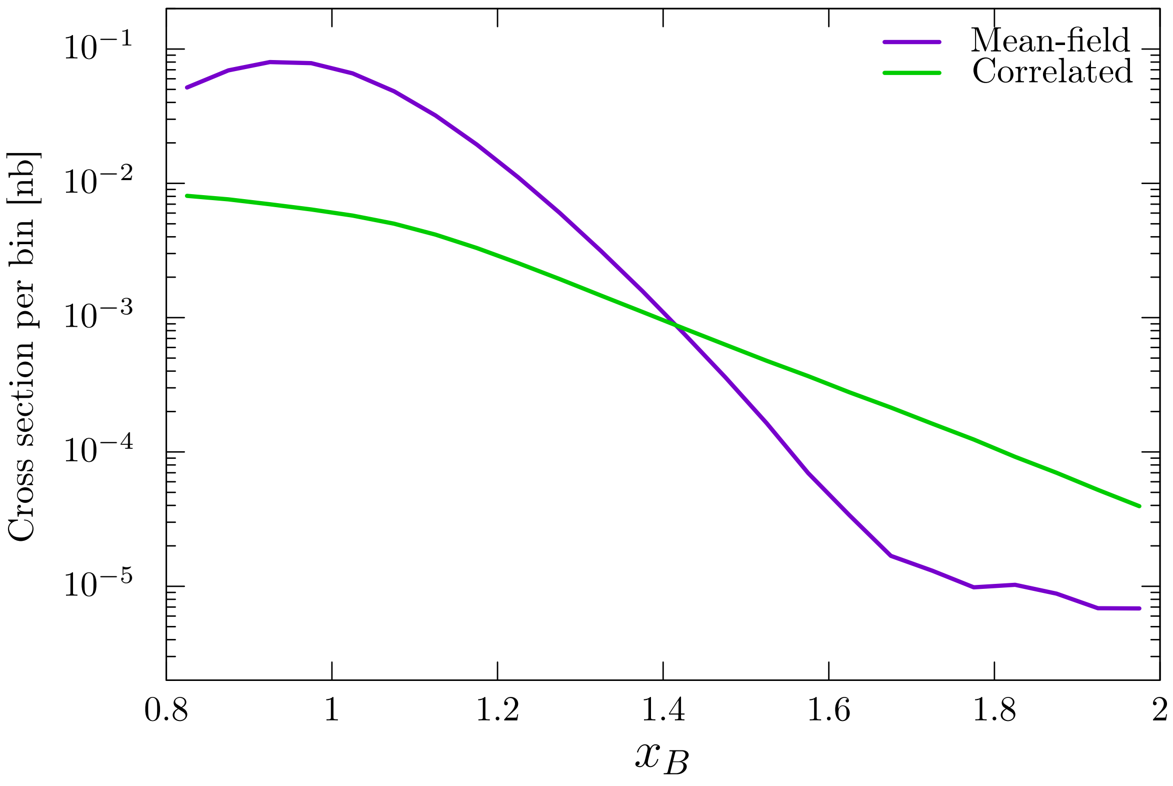
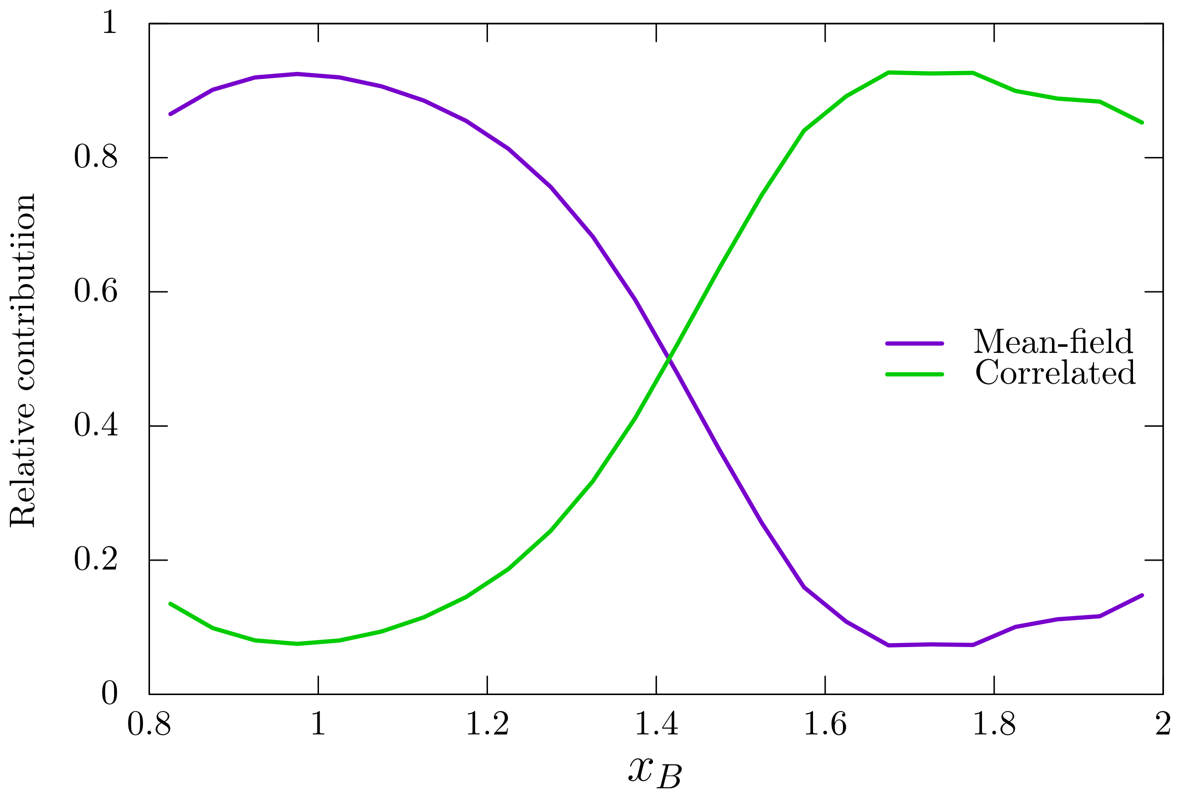


VMC Momentum Distribution Ratio

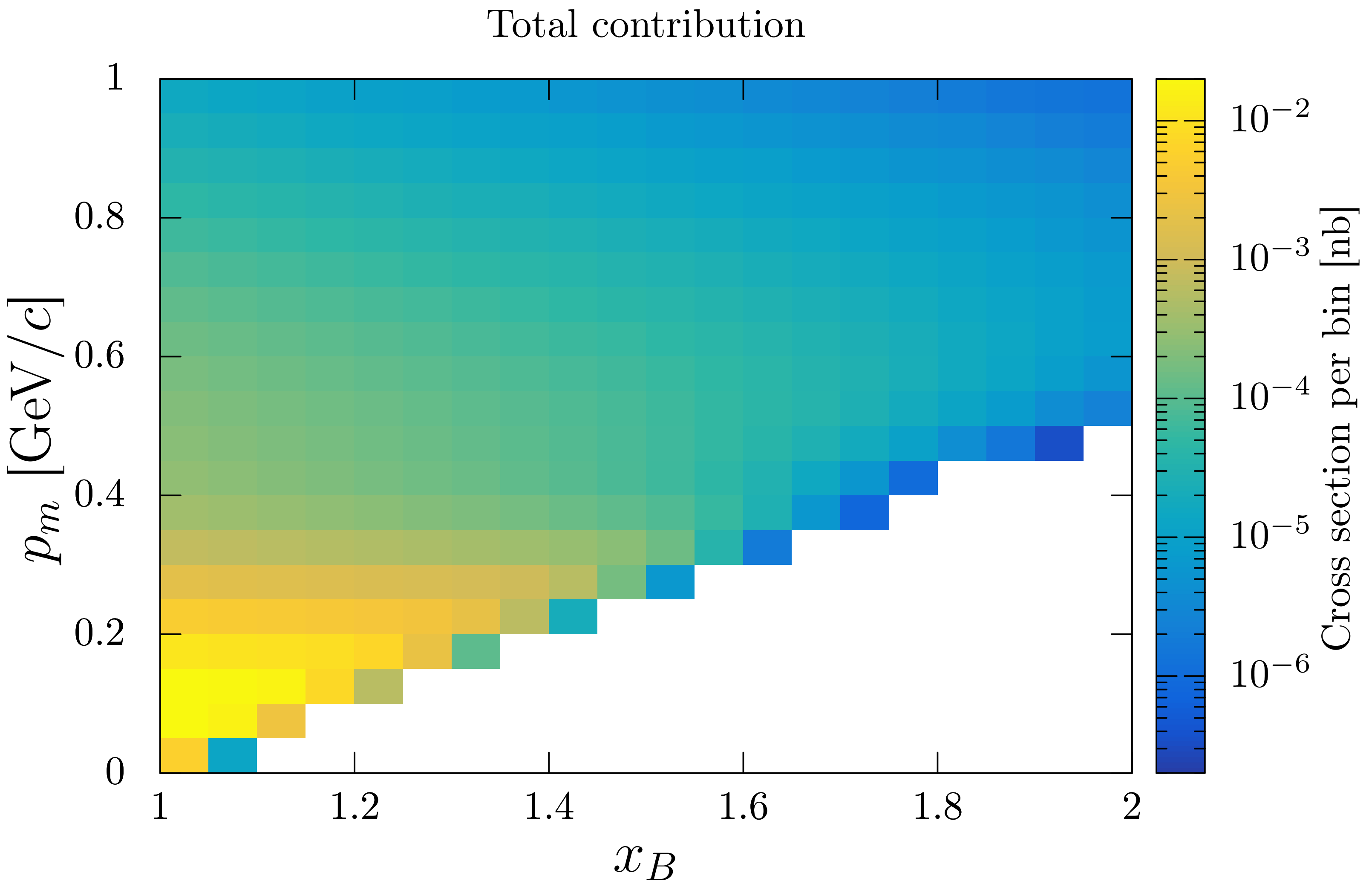


Breakdown by MF/SRC

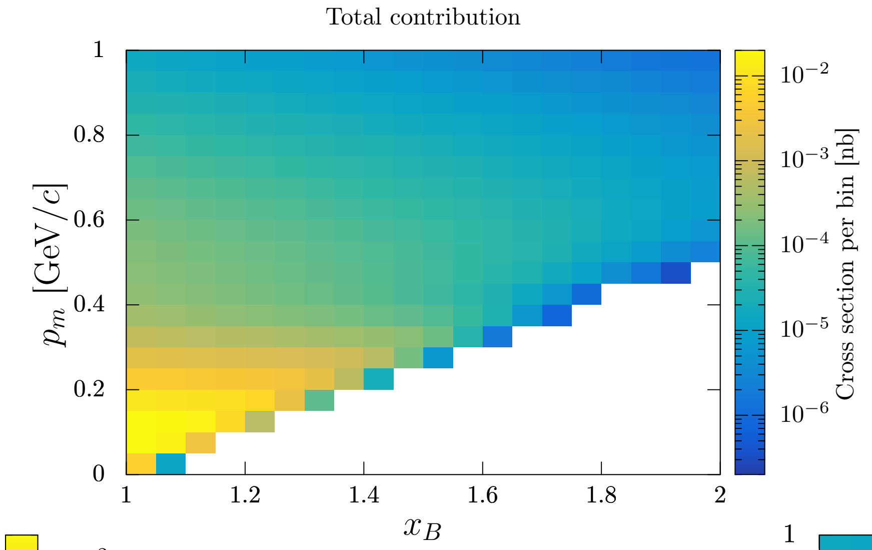


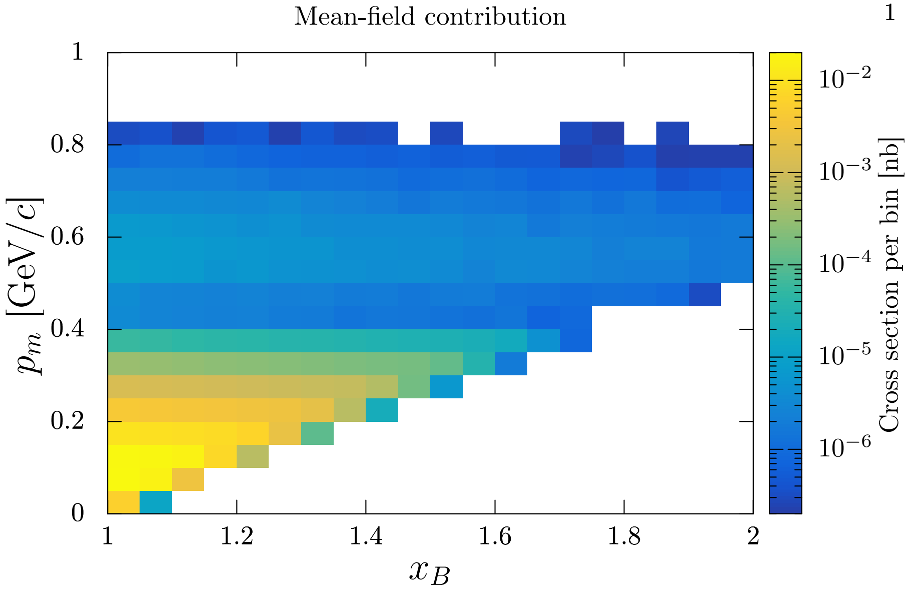


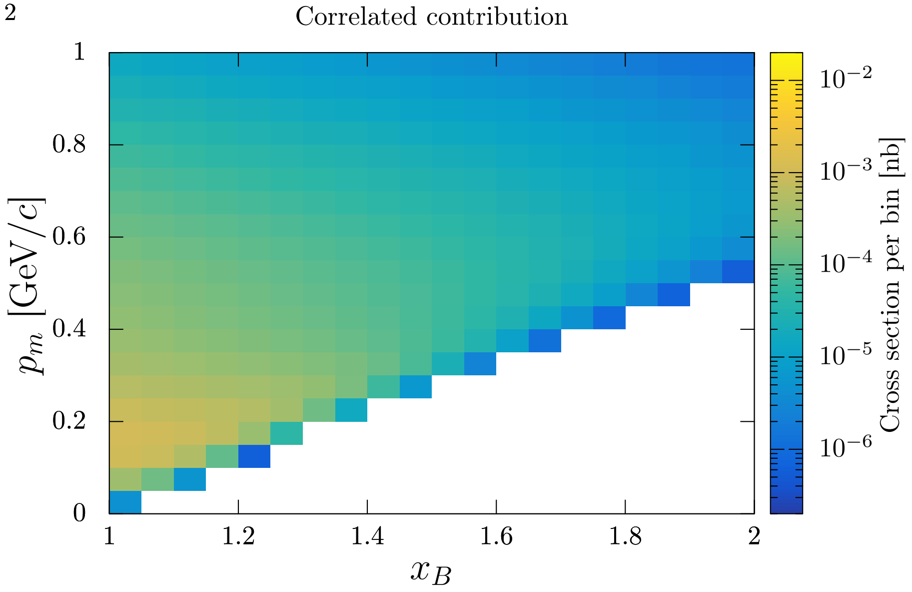
The $x_B \leftrightarrow p_m$ correspondence



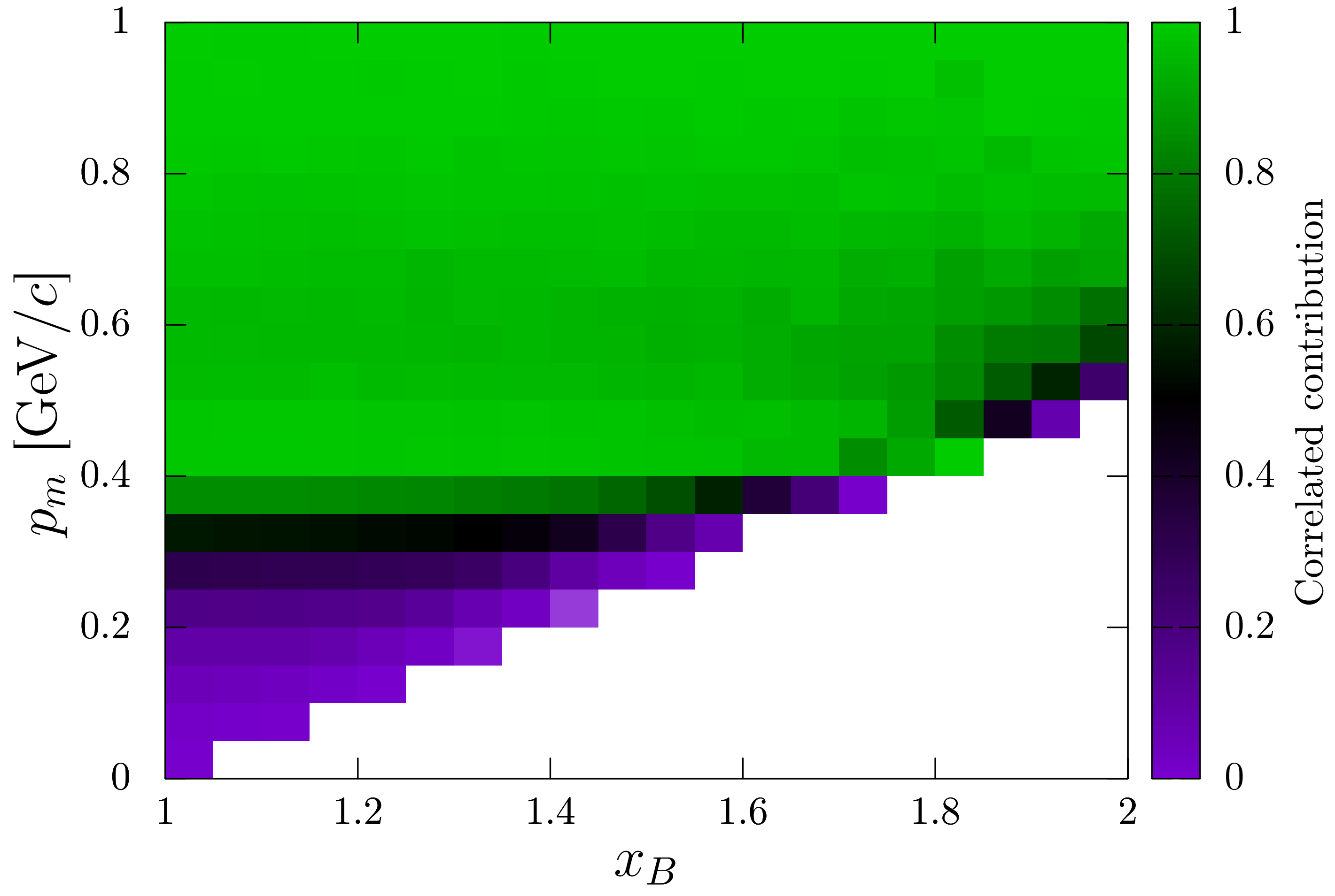
The $x_B \leftrightarrow p_m$ correspondence



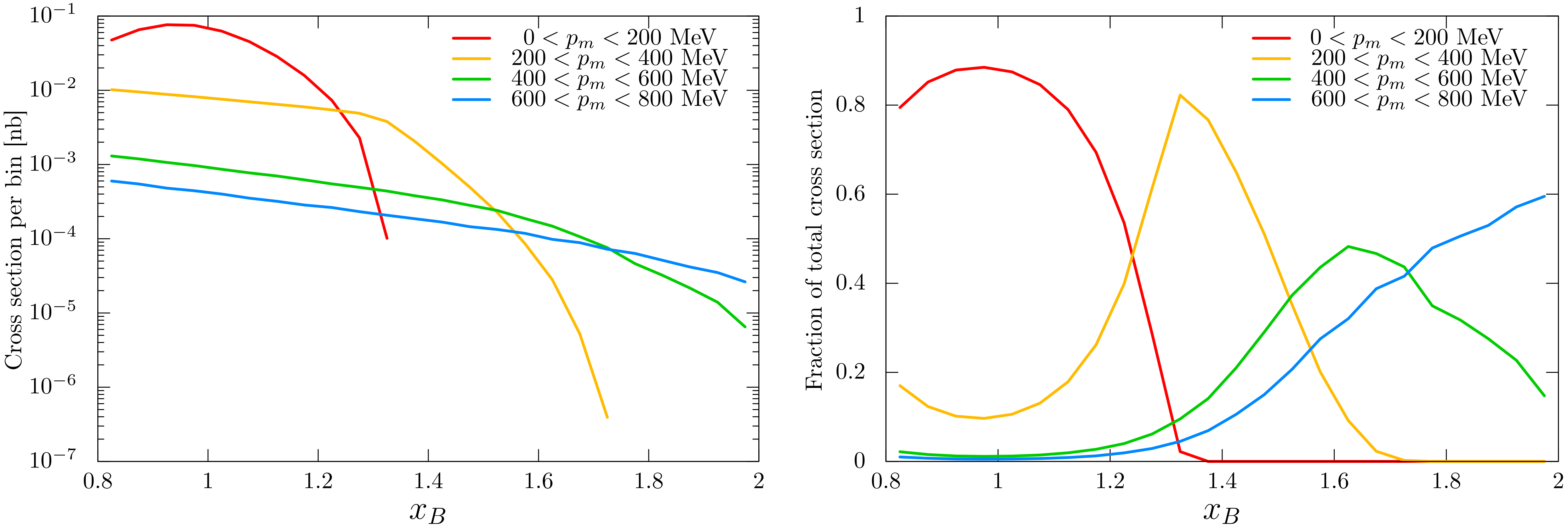




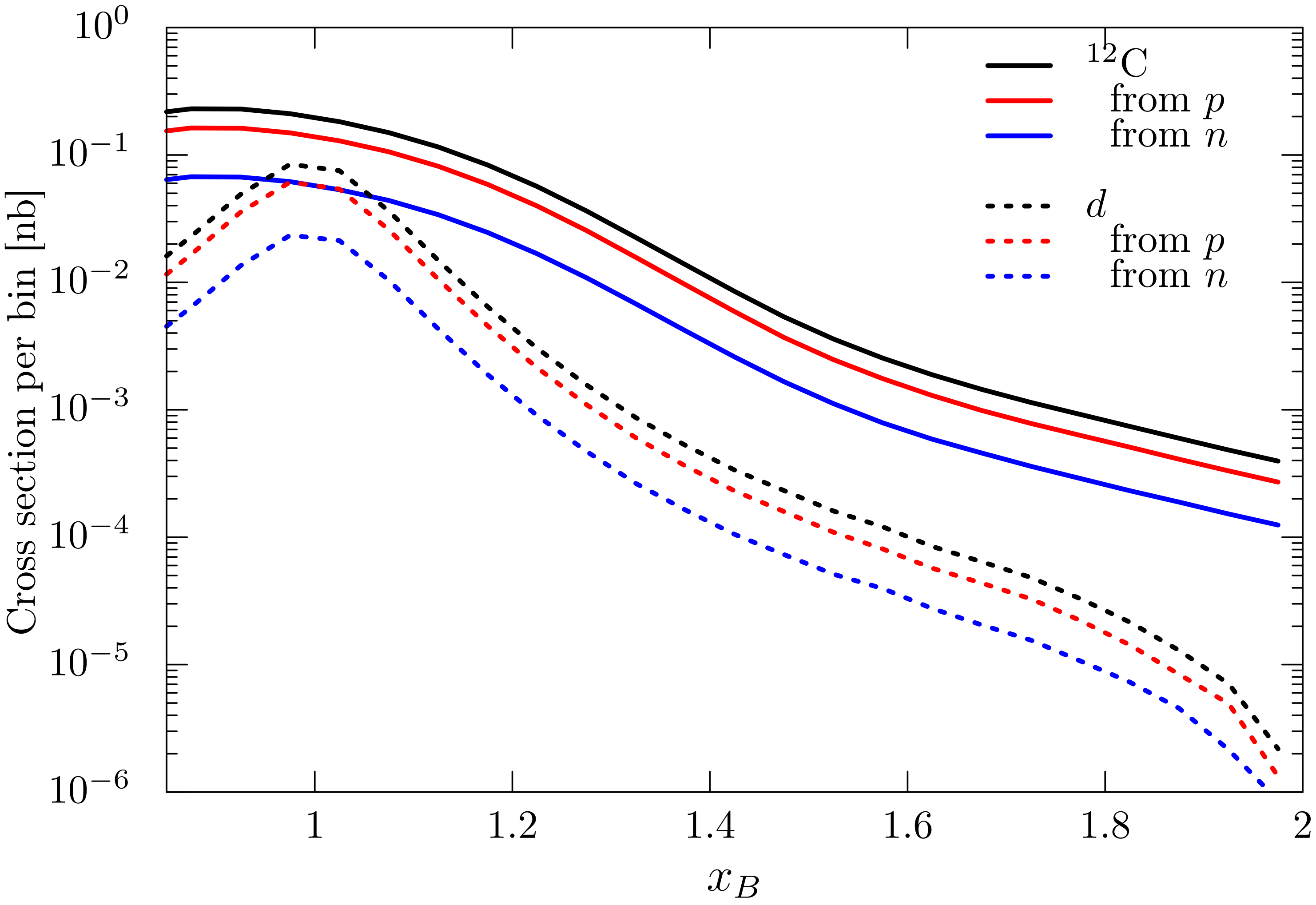
The $x_B \leftrightarrow p_m$ correspondence



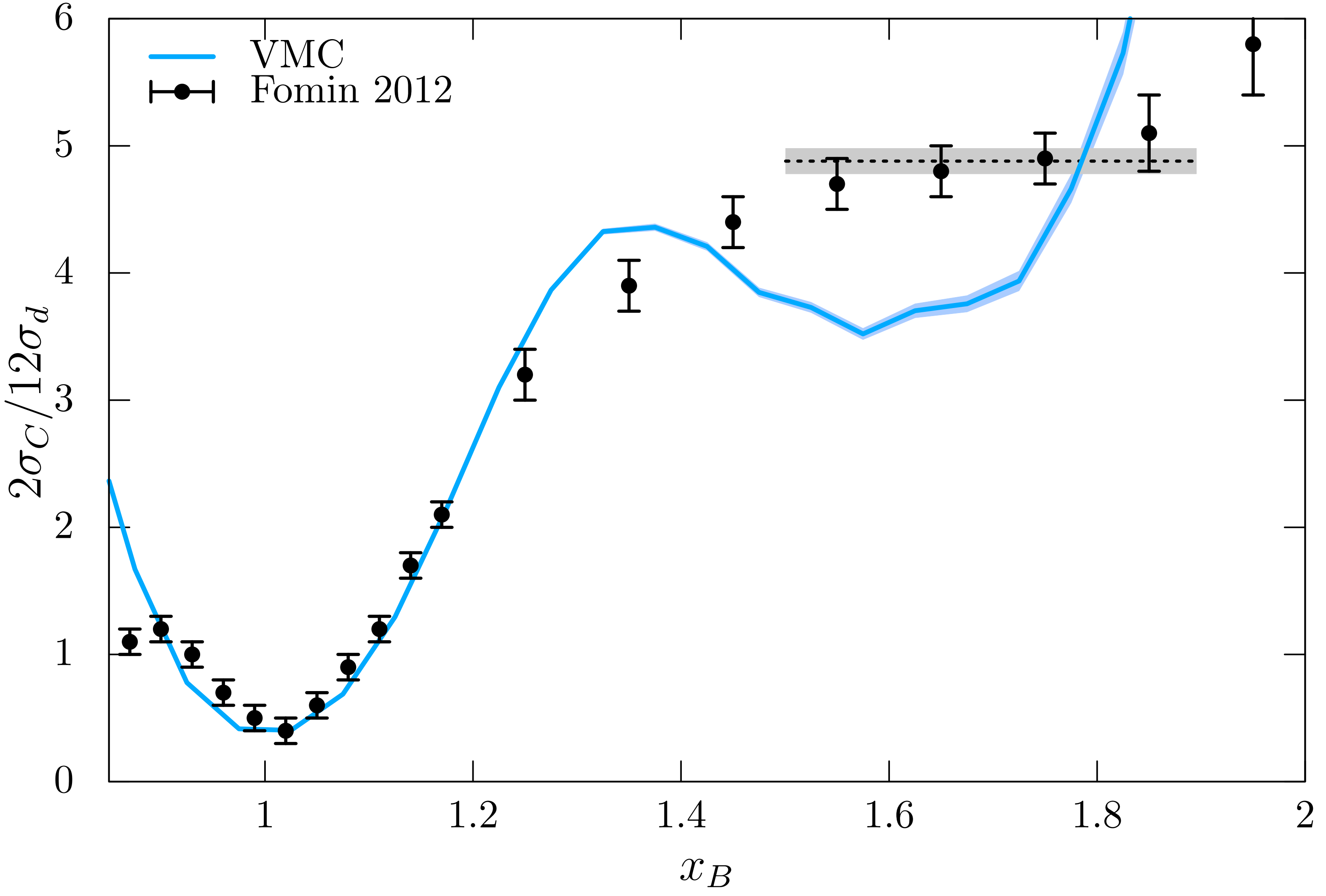
The $x_B \leftrightarrow p_m$ correspondence



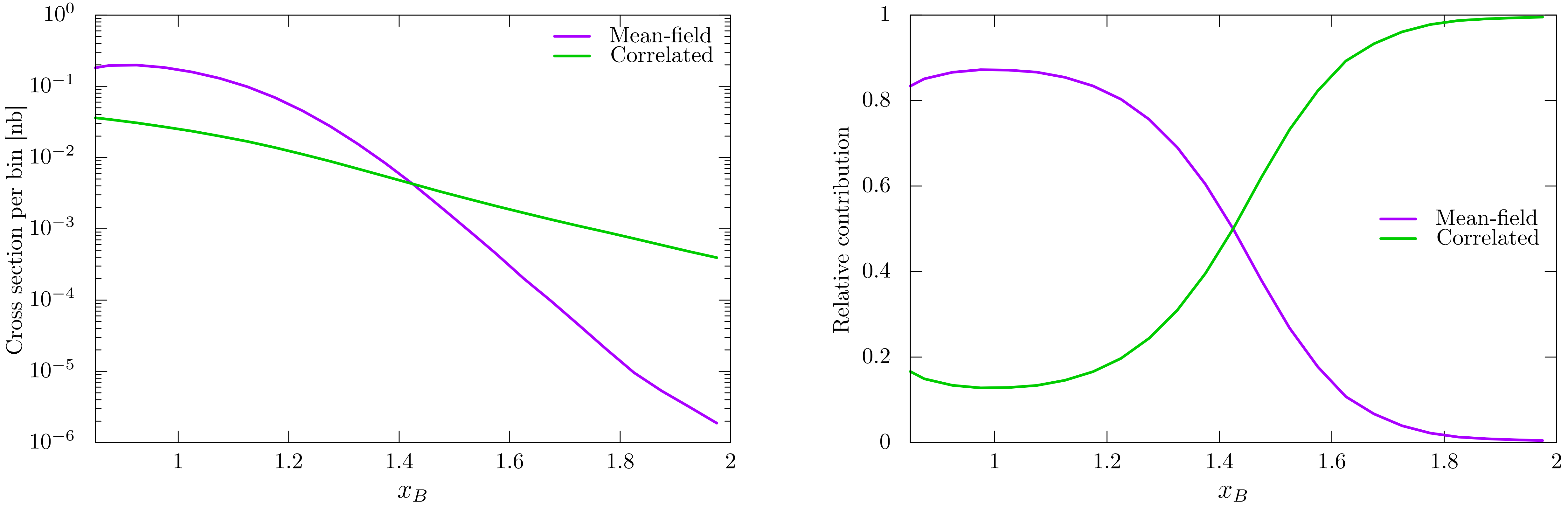
Carbon / Deuterium Yields



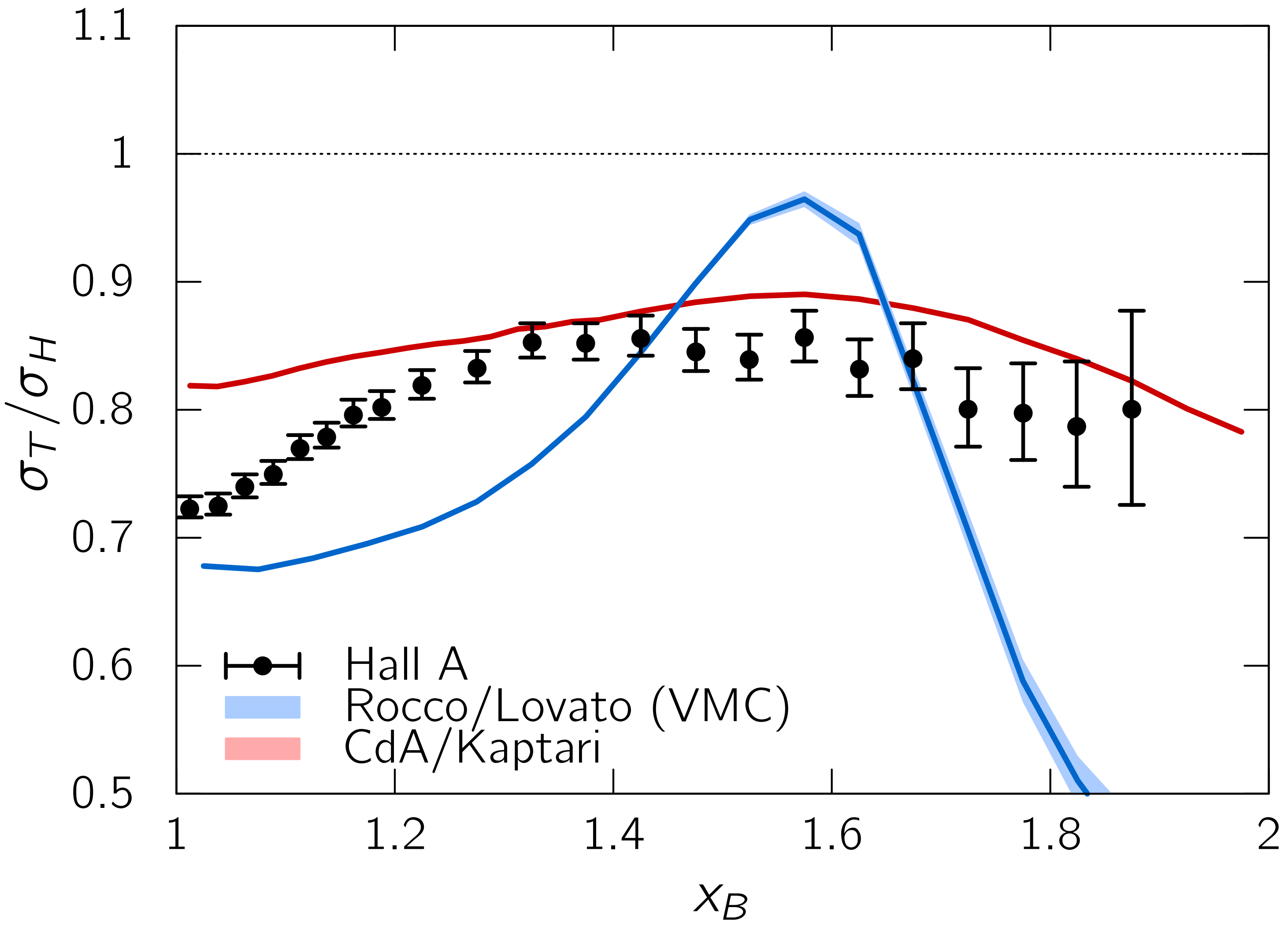
Carbon / Deuterium Ratio



Breakdown by MF/SRC

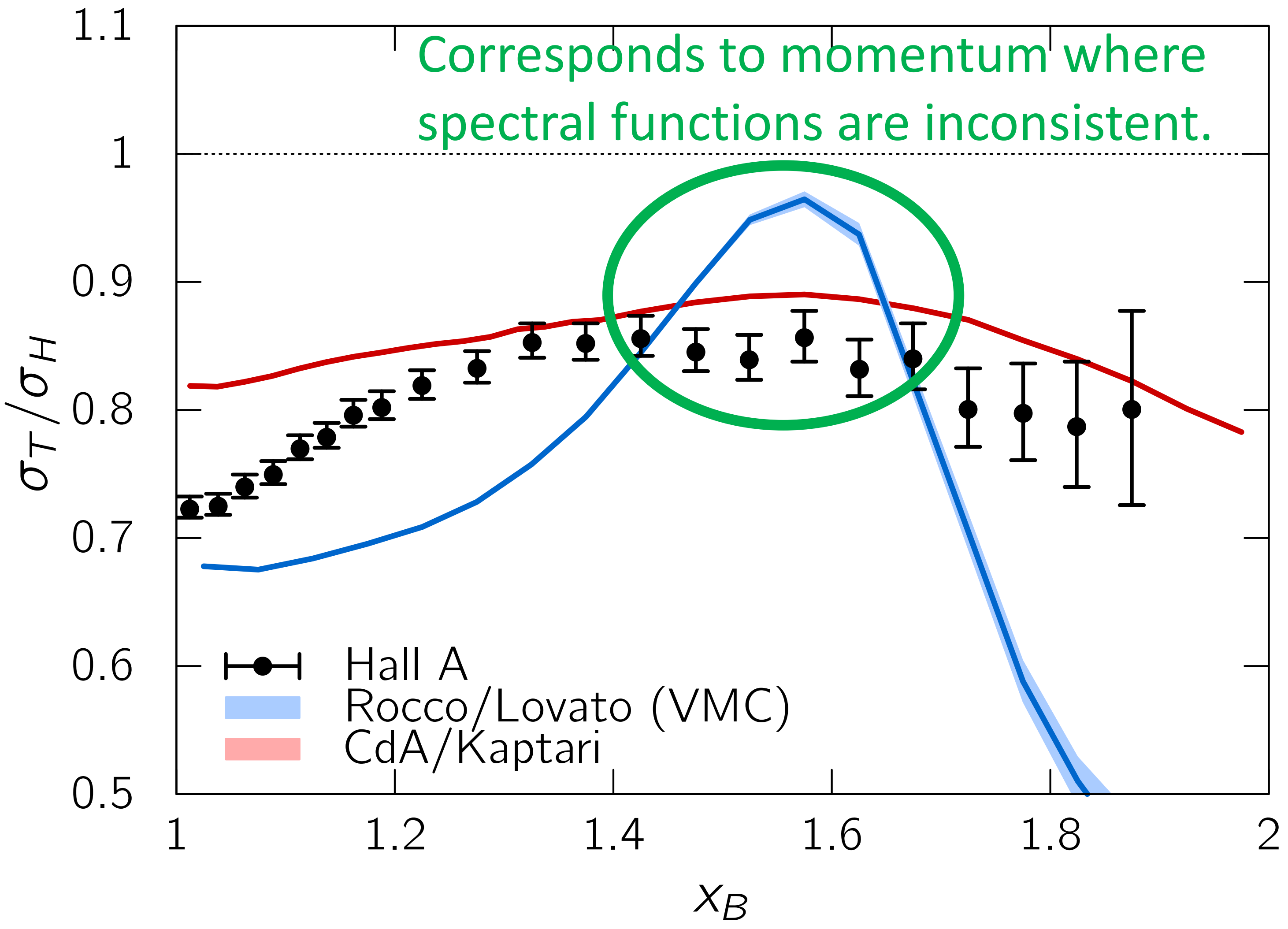


A=3 Ratio



1.1
1
0.9
0.8
0.7
0.6
0.5
σ_T/σ_H
1
1.2
1.4
1.6
1.8
2
x_B
Hall A
Rocco/Lovato (VMC)
CdA/Kaptari

A=3 Ratio



Corresponds to momentum where spectral functions are inconsistent.
1.1
1
0.9
0.8
0.7
0.6
0.5
σ_T/σ_H
1
1.2
1.4
1.6
1.8
2
x_B
Hall A
Rocco/Lovato (VMC)
CdA/Kaptari

Summary
Inclusive ratios are sensitive to:

- CM motion and E^*, which may not be precisely known.
- Flatness of plateaus
- The momentum range probed
- Contamination from non-SRC events

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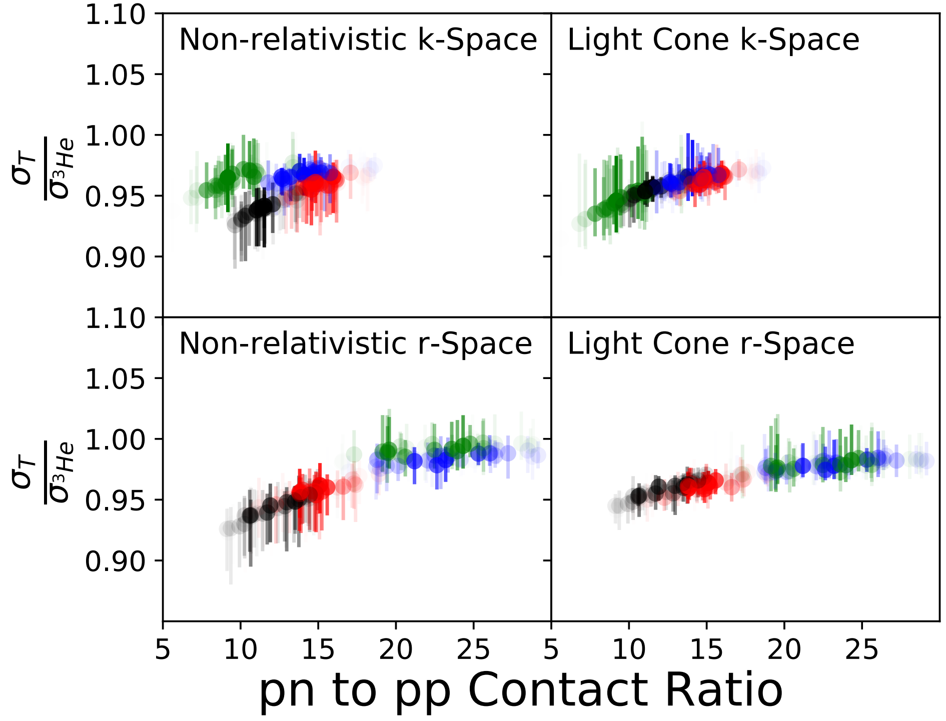

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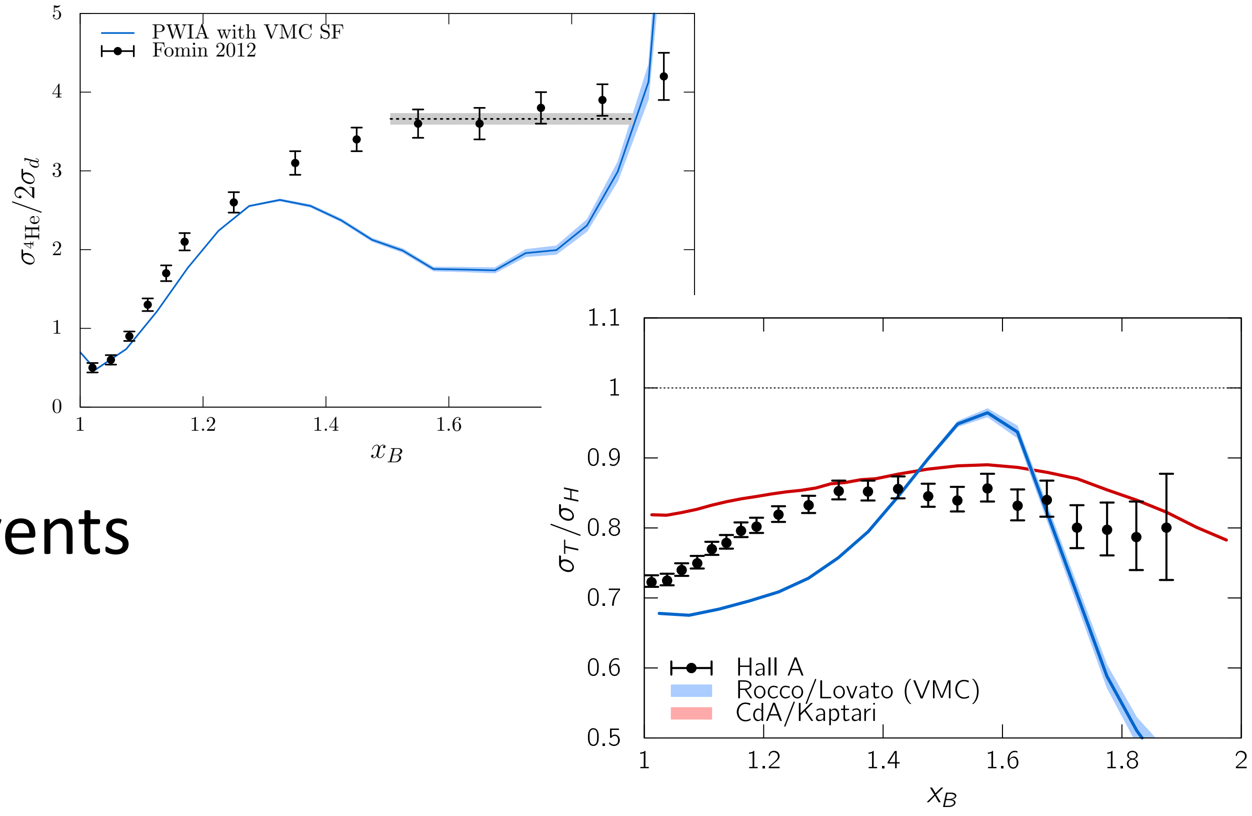


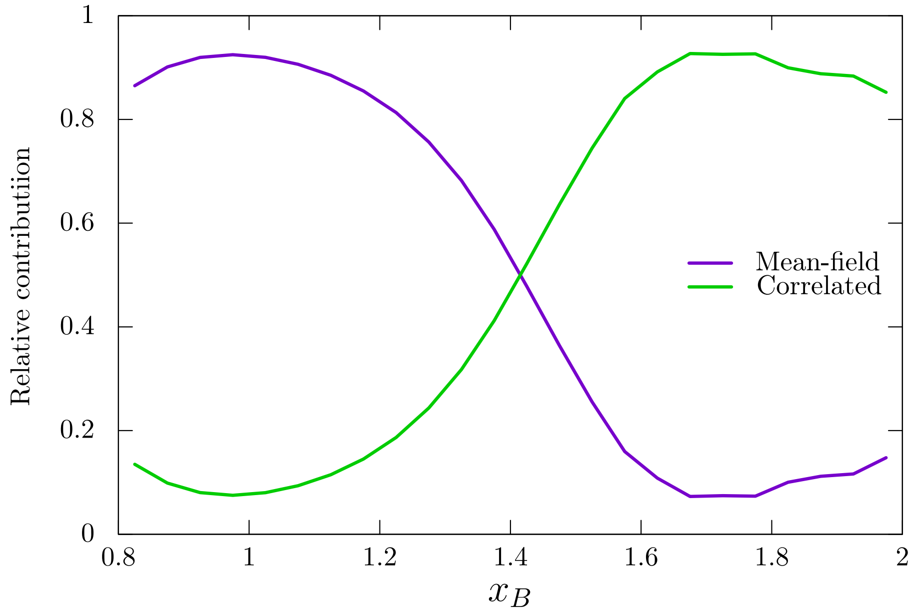
Summary
Inclusive ratios are sensitive to:

- CM motion and E^*, which may not be precisely known.
- Flatness of plateaus
- **The momentum range probed**
- Contamination from non-SRC events

Do a study in which we vary Q2, vary xB range, show that the same contacts or same spectral function can lead to different "plateau" value.

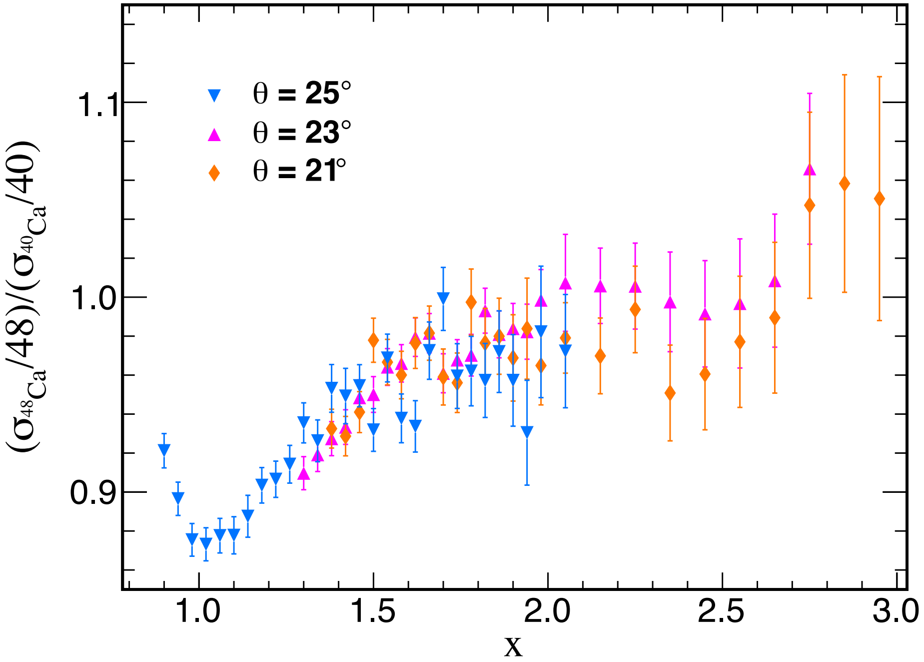
Summary
Inclusive ratios are sensitive to:

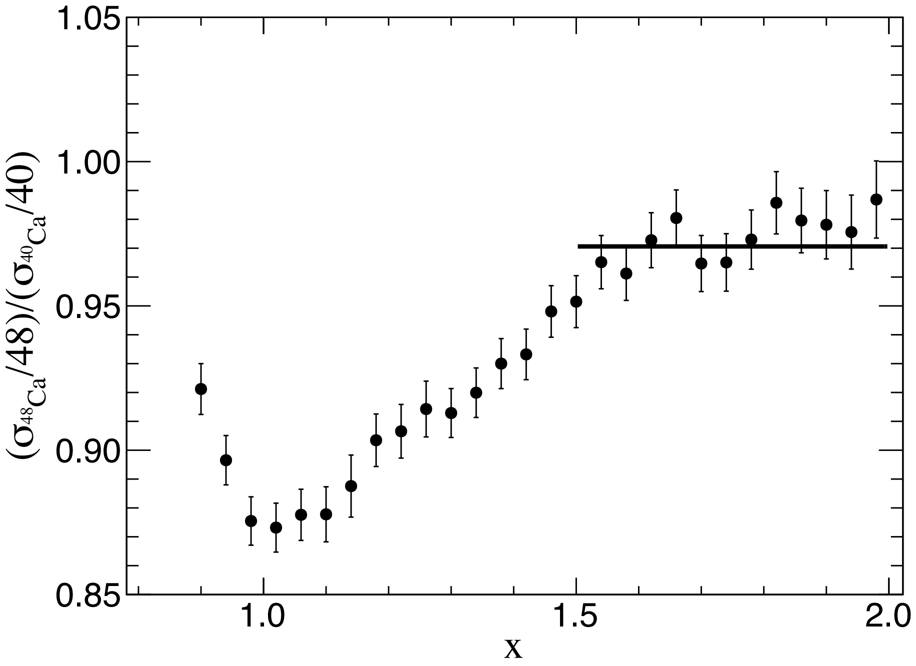
- CM motion and E^*, which may not be precisely known.
- Flatness of plateaus
- The momentum range probed
- **Contamination from non-SRC events**



BACK-UP

Dien's Ca-40/Ca-48 result





"Novel observation of isospin structure of short-range correlations in calcium isotopes"
D. Nguyen et al., Phys. Rev,. C 102, 064004 (2020)

Caution: Missing energy can be defined a couple of different ways.

Most common approach:

$$E_m \equiv \omega - T_{Lead} - T_{A-1}$$

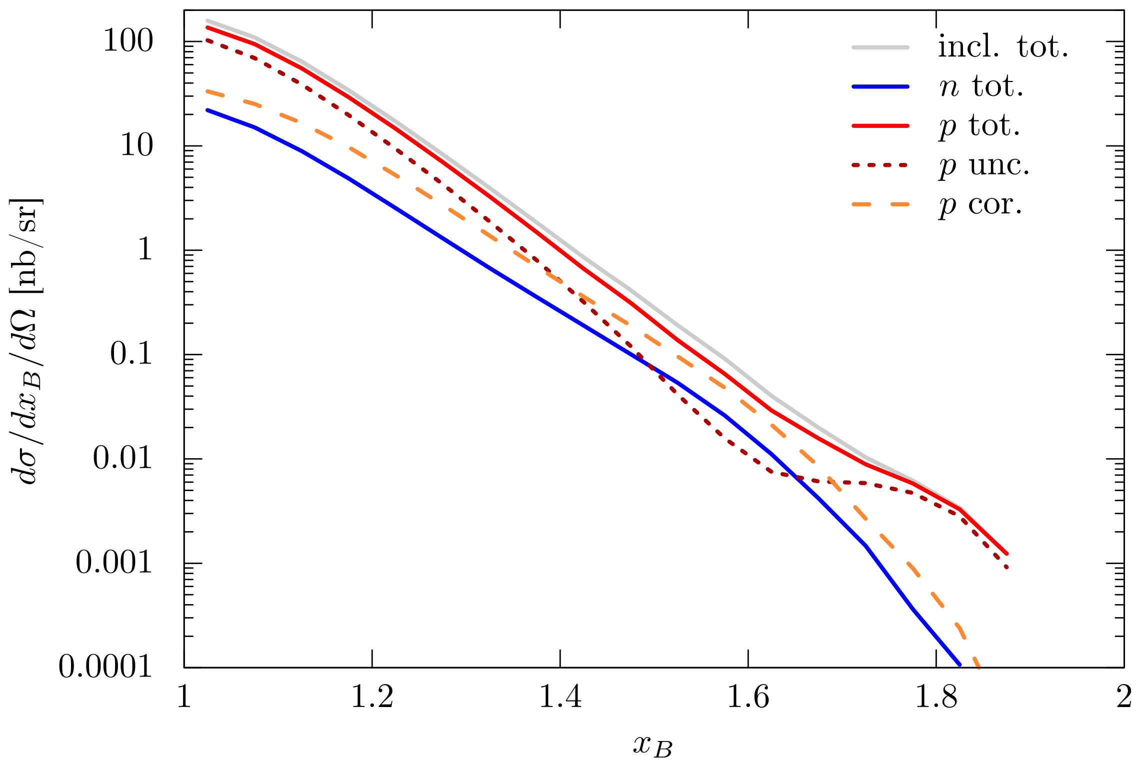
Missing energy of two-body break-up states is fixed

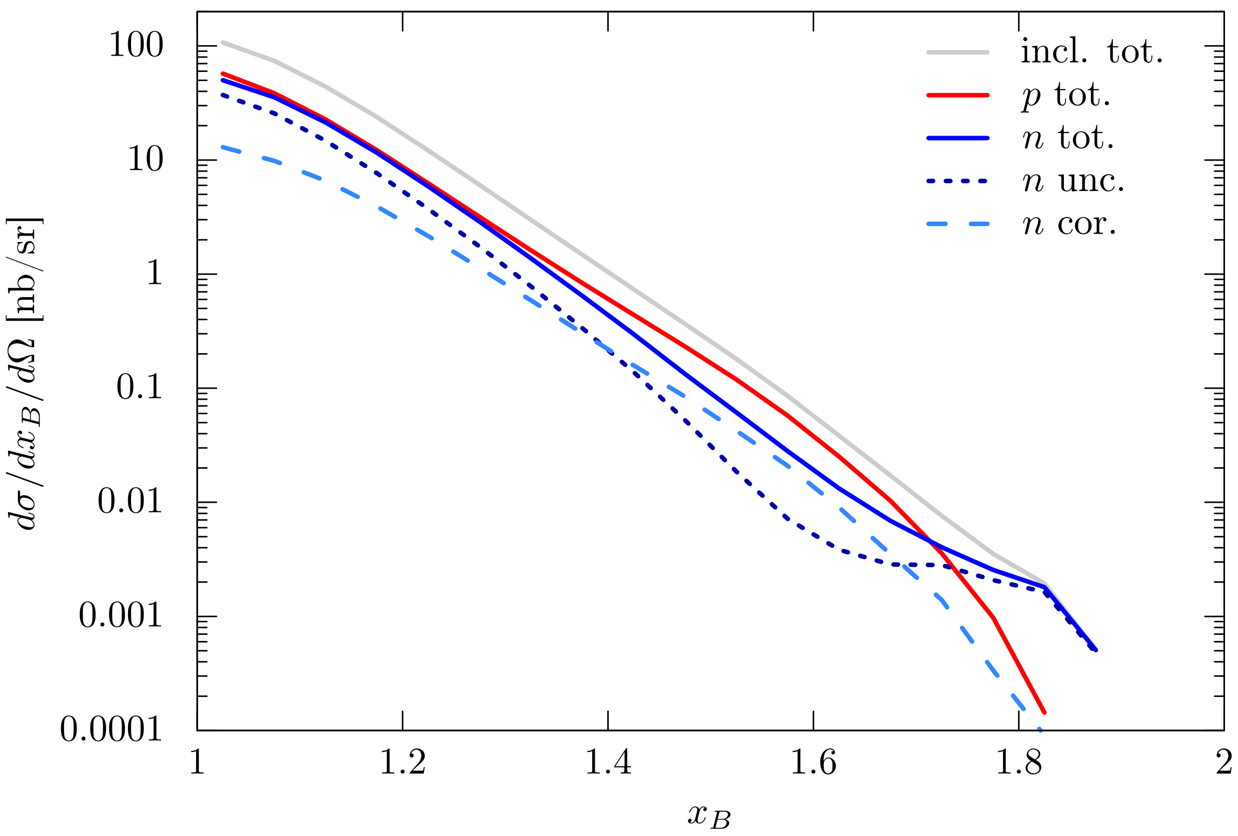
An alternative approach:

$$E_m \equiv \omega - T_{Lead}$$

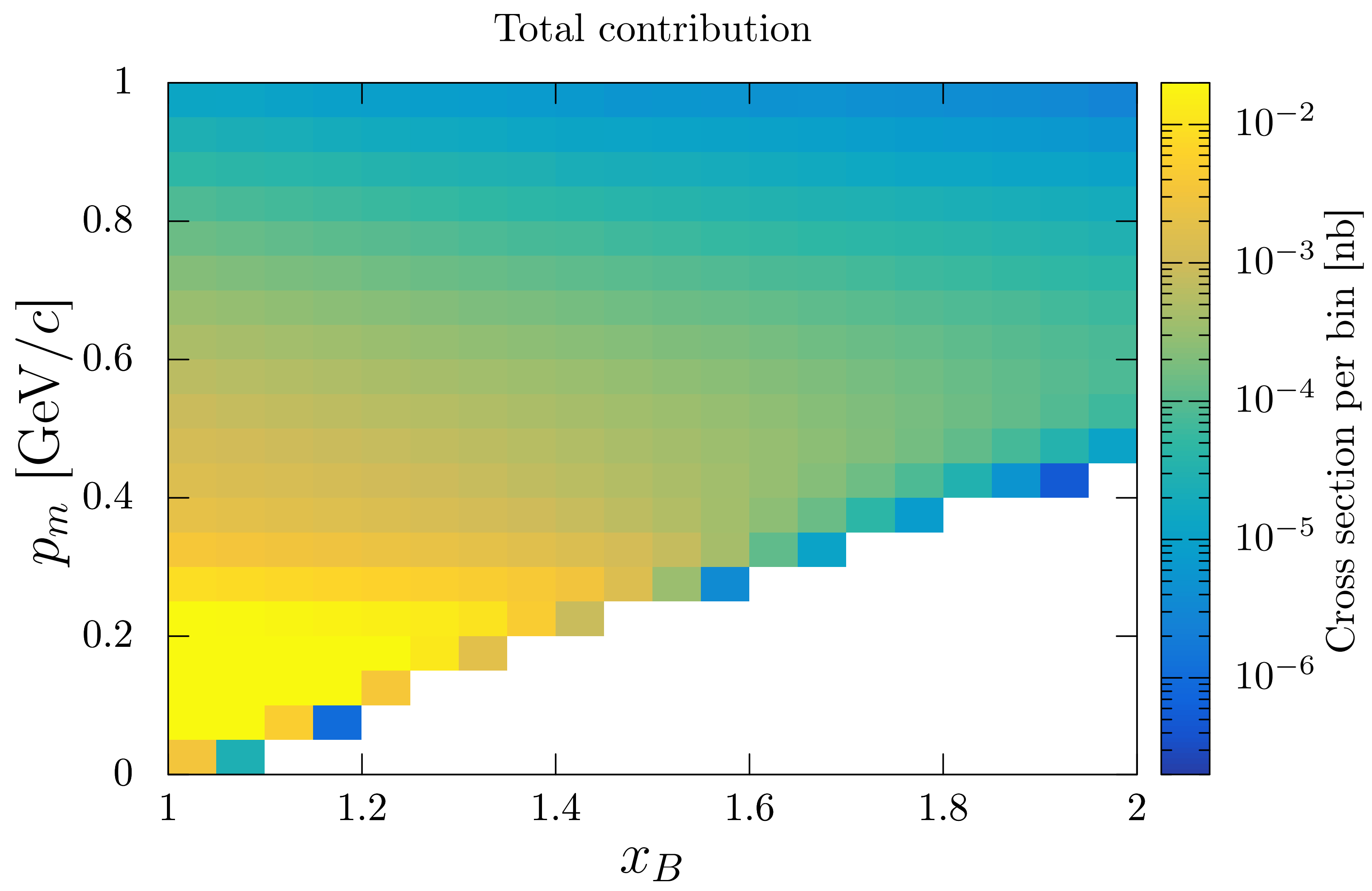
Missing energy of two-body break-up states changes with p_m

Absolute Cross Sections for A=3

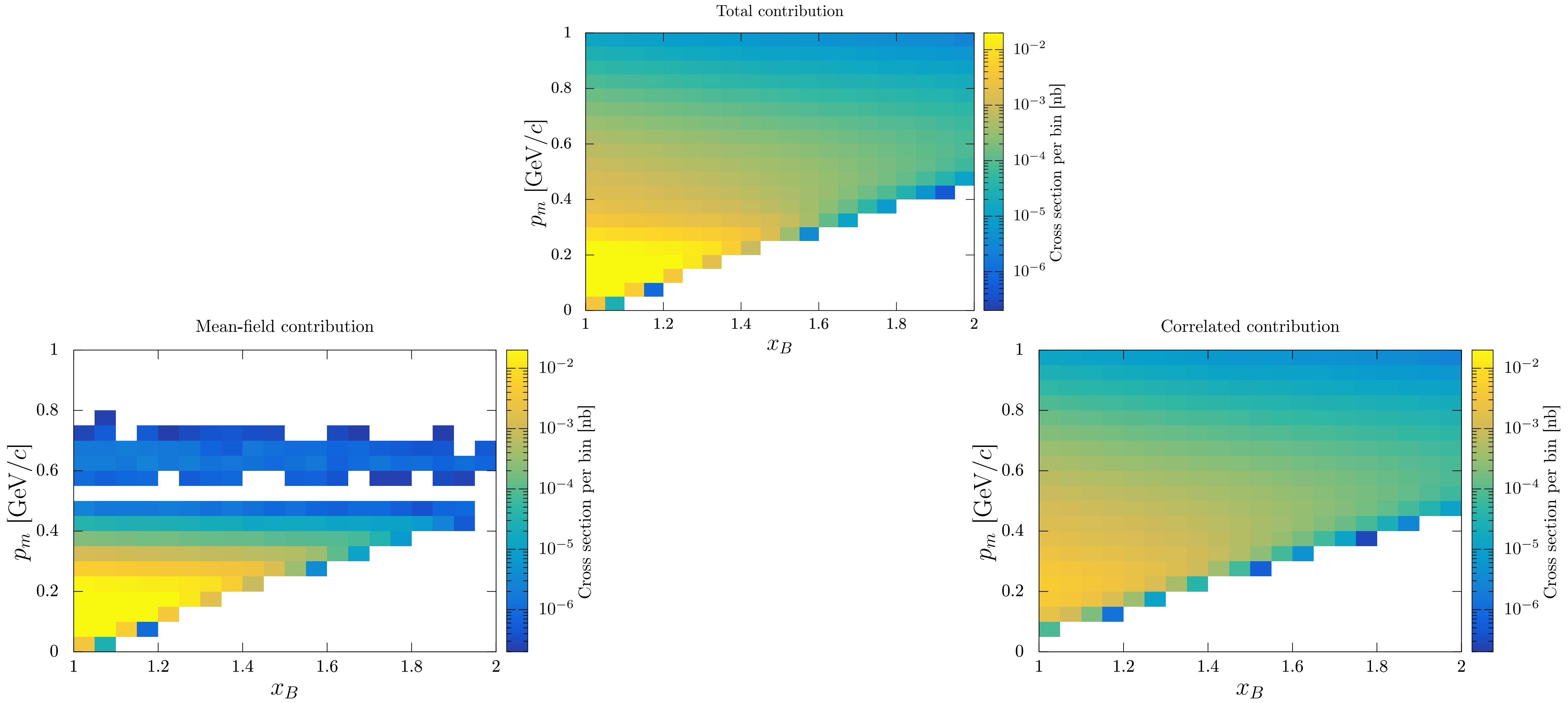




C(e,e'): the $x_B \leftrightarrow p_m$ correspondence



C(e,e’): the $x_B \leftrightarrow p_m$ correspondence



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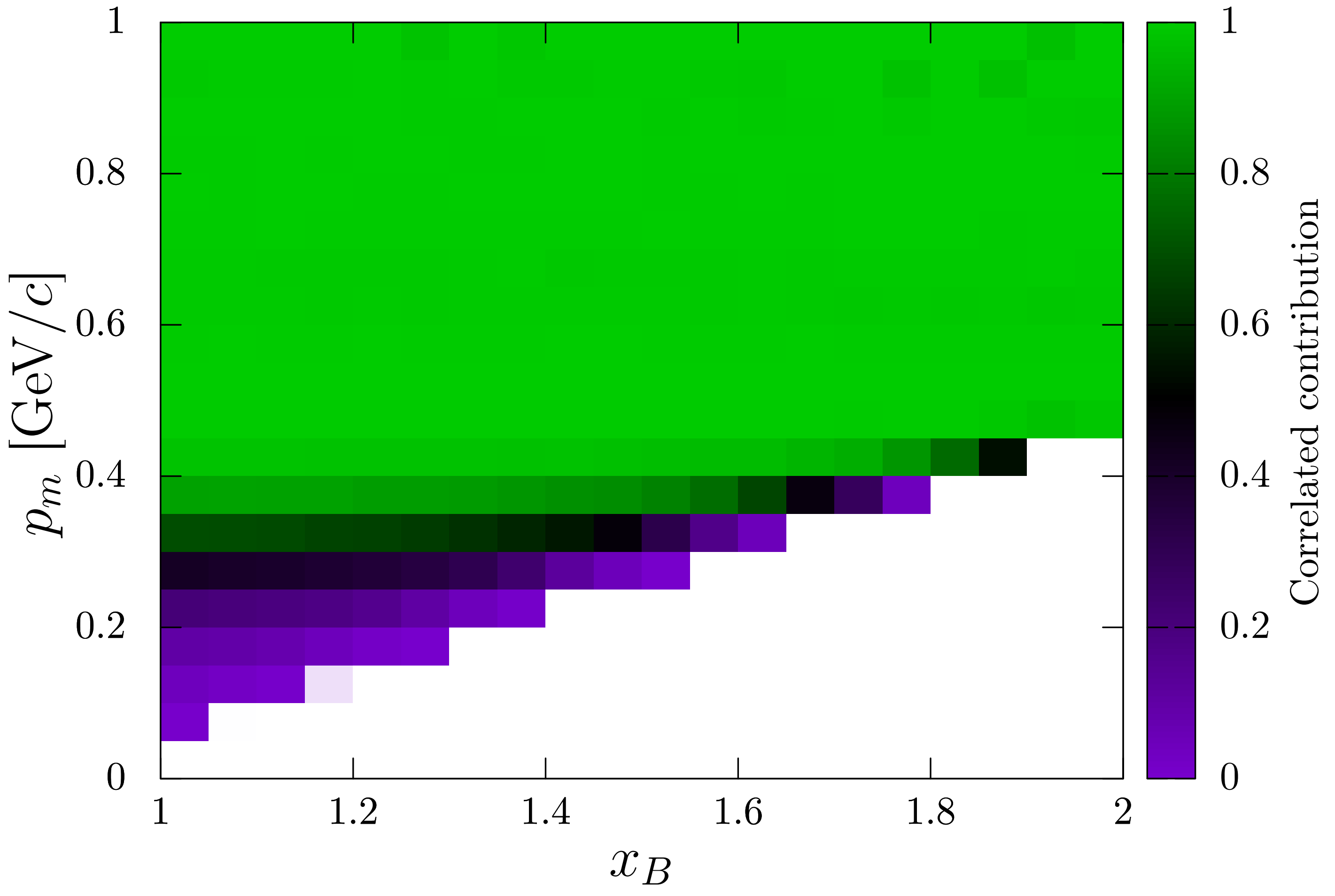

C(e,e'): the $x_B \leftrightarrow p_m$ correspondence