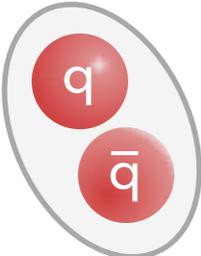


hadron spectroscopy – theory

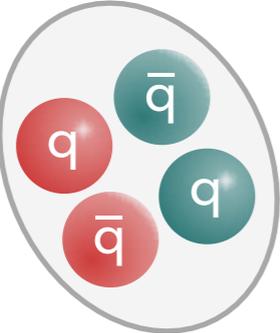
Jozef Dudek

(will focus on mesons due to limited time)

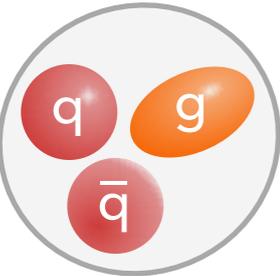
"pictures"



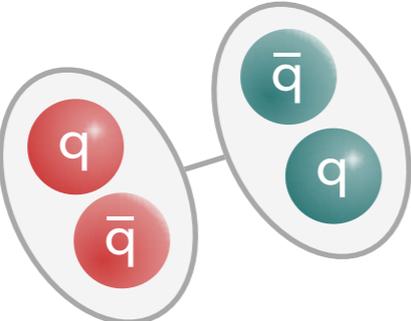
conventional meson



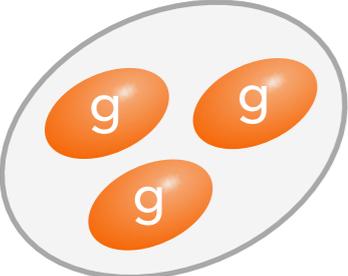
tetraquark



hybrid meson



meson-meson molecule

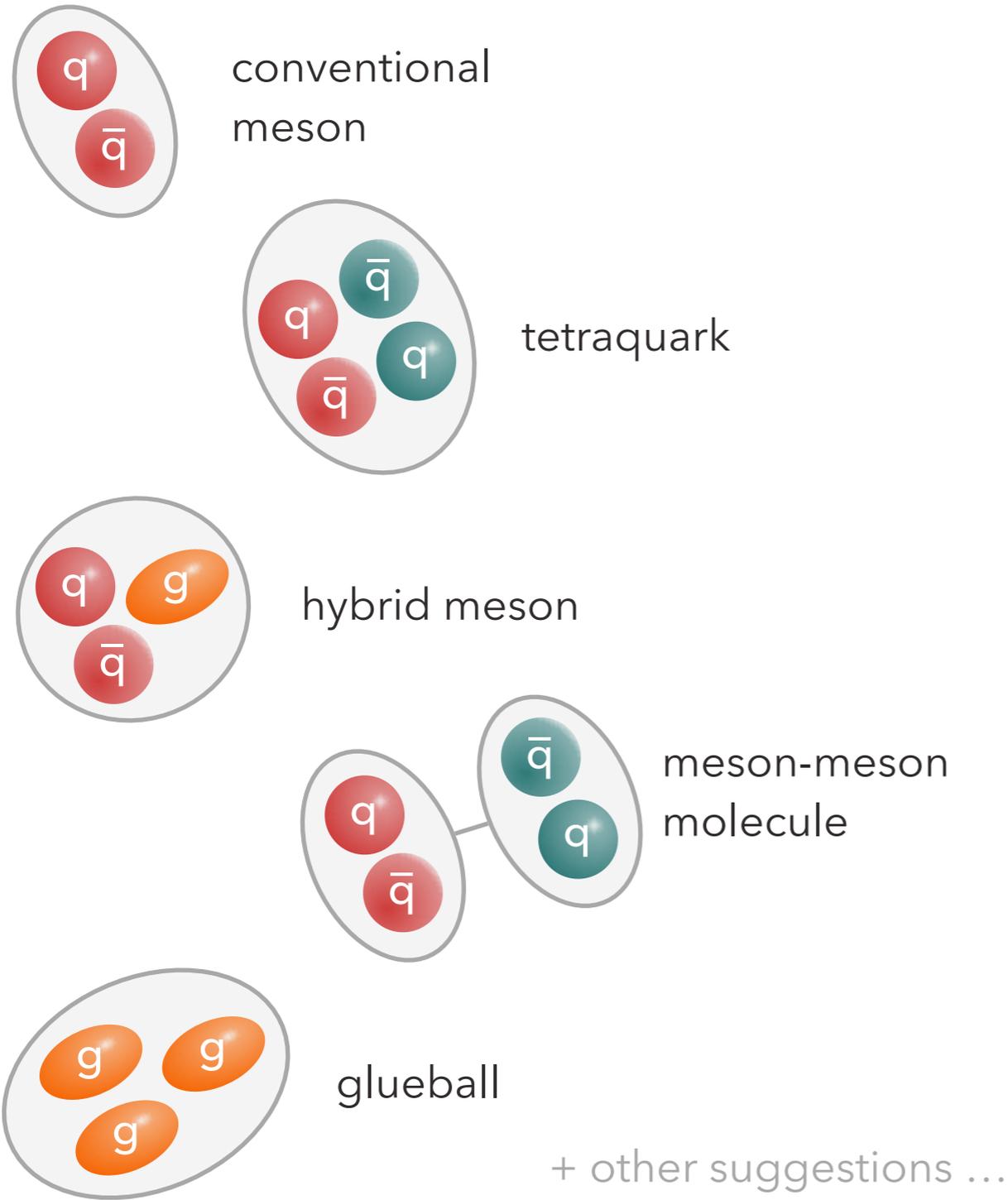


glueball

+ other suggestions ...

traditionally the preserve of **modeling**
– connection to QCD unclear

"pictures"

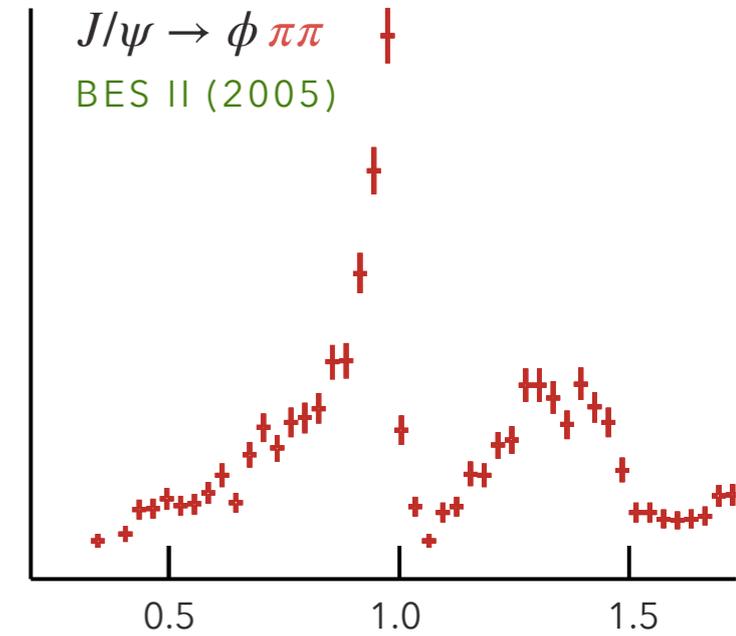
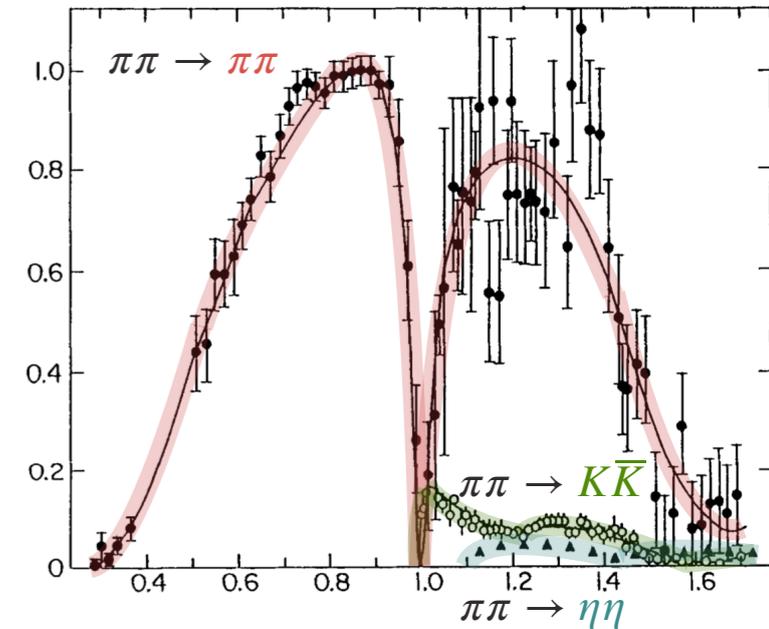


traditionally the preserve of **modeling**
 – connection to QCD unclear

"reality"

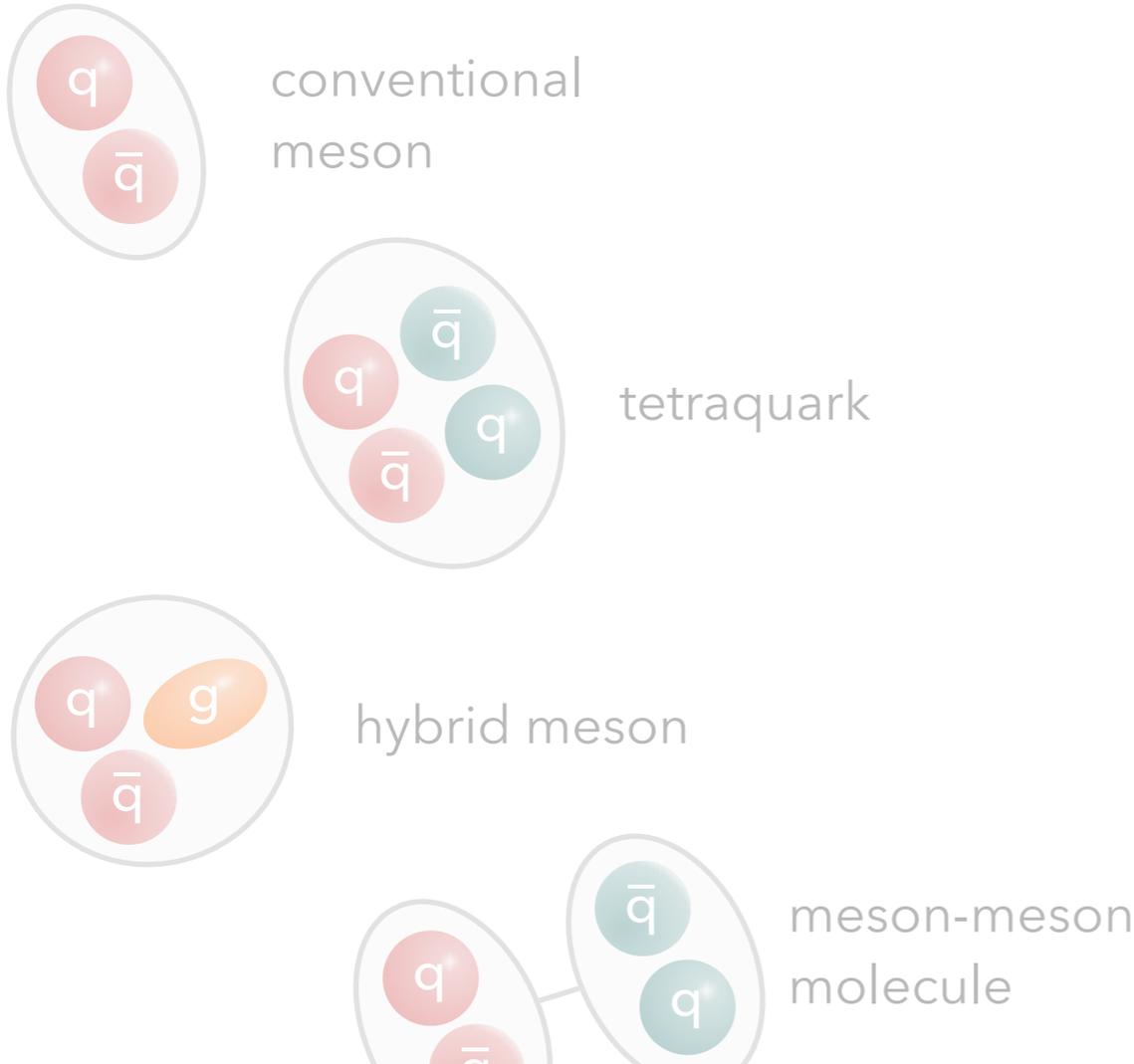
resonances in complicated production/decay

e.g. $f_0(980)$



amplitudes in terms of scattering **stable hadrons**
 – connection to QCD unclear

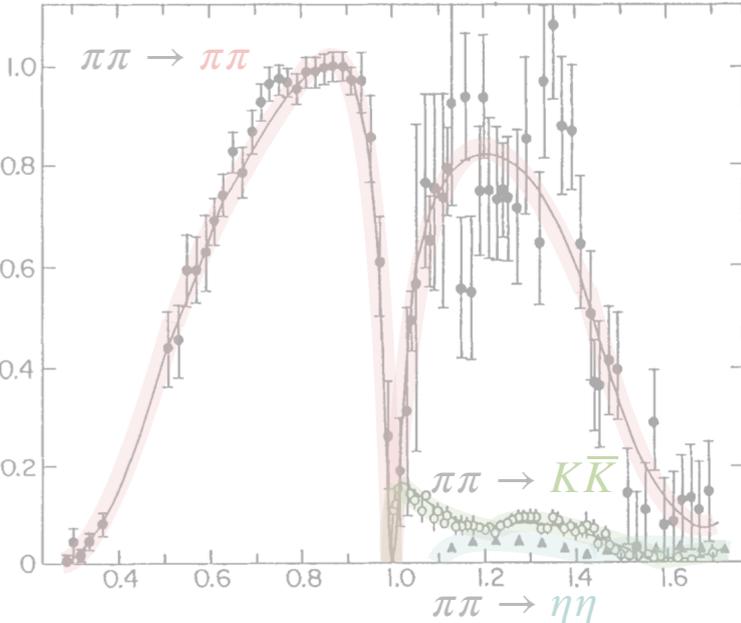
"pictures"



"reality"

resonances in complicated production/decay

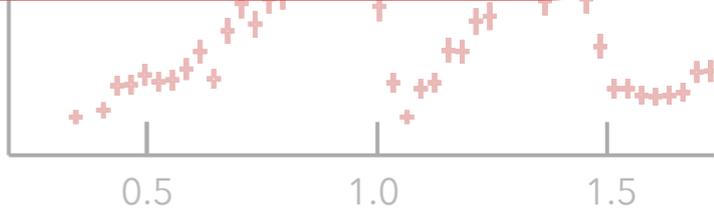
e.g. $f_0(980)$



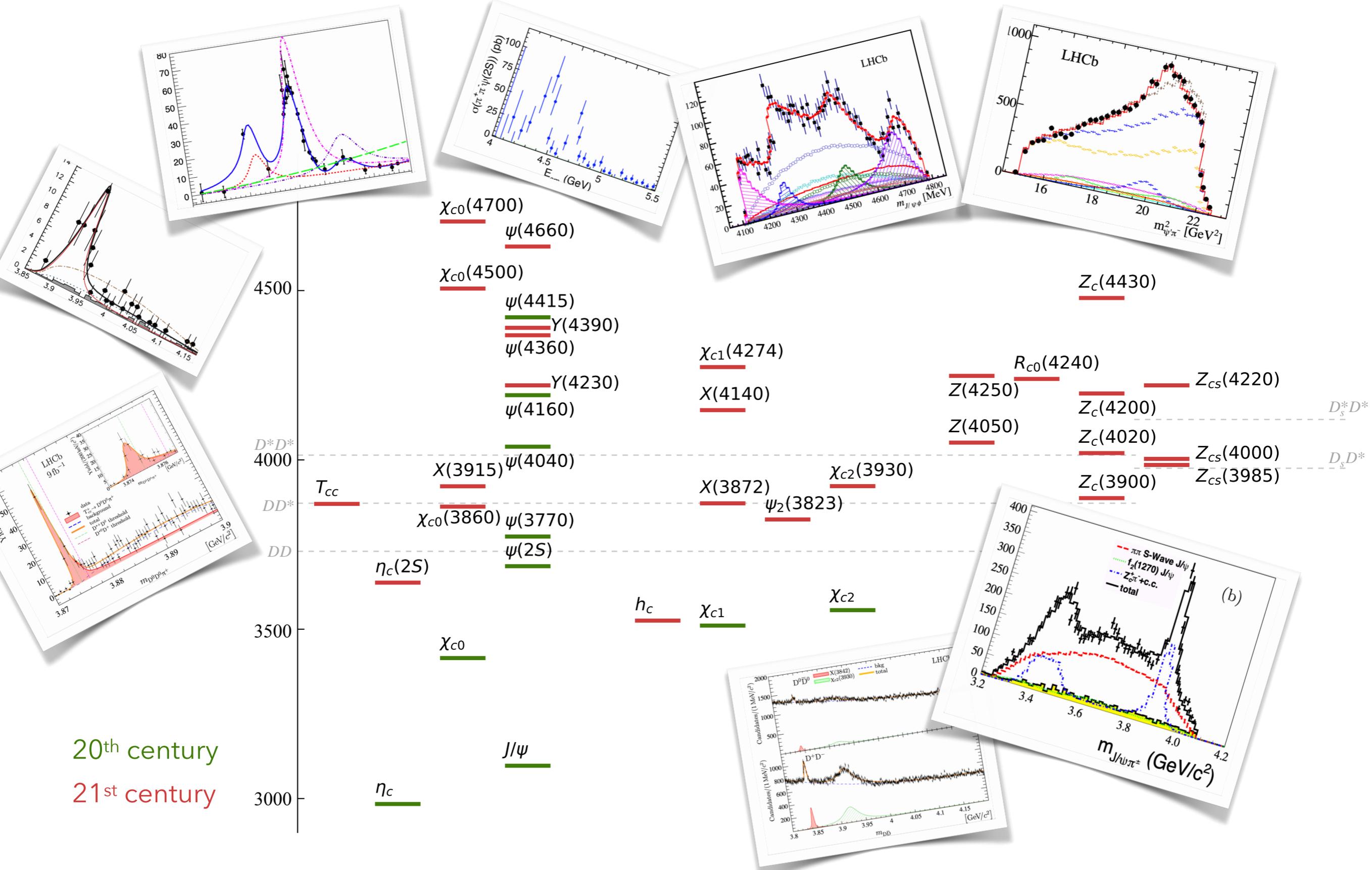
$J/\psi \rightarrow \phi \pi\pi$
BES II (2005)

in what configurations does QCD bind quarks and gluons ?

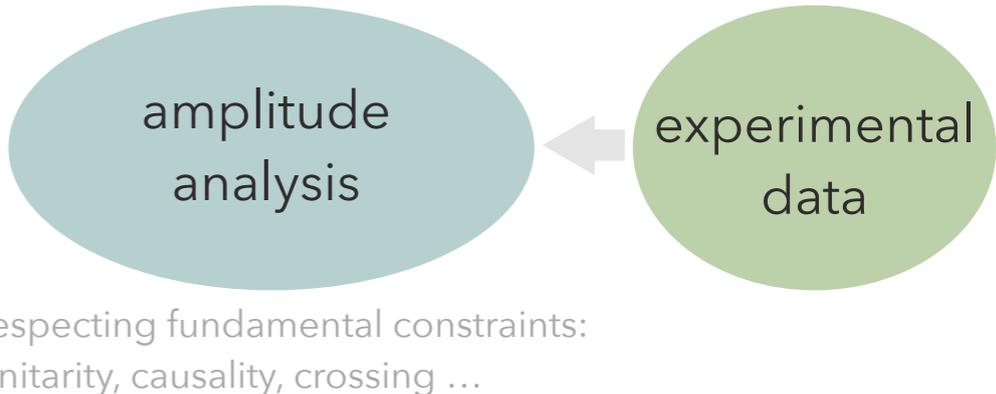
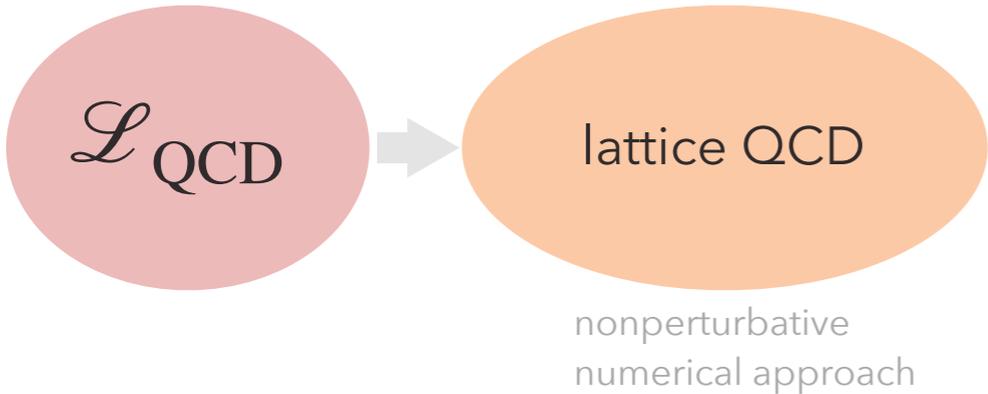
can complex and varied experimental amplitudes be explained with QCD ?

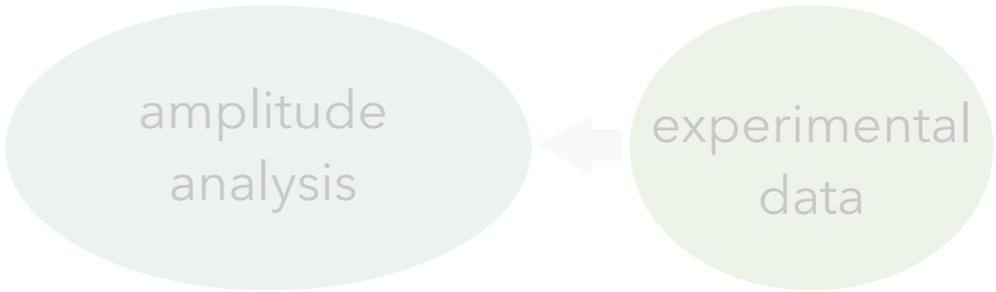
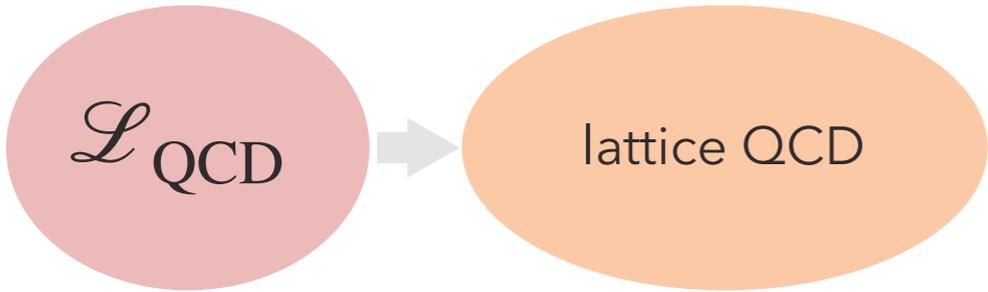


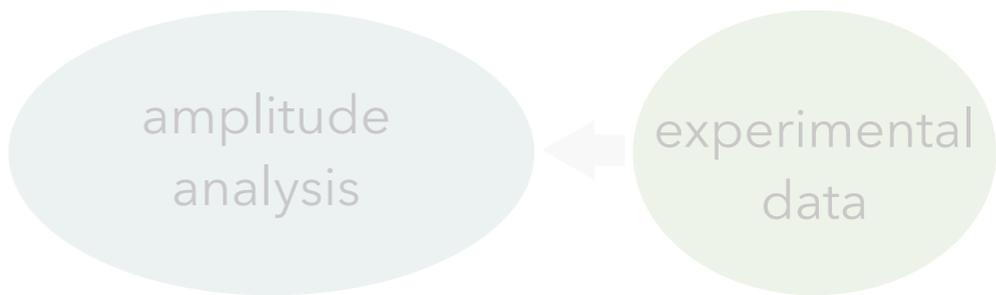
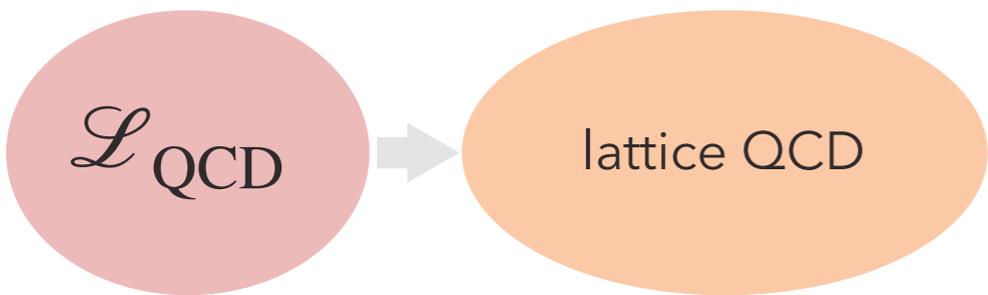
renewed motivation – the XYZ explosion



20th century
21st century

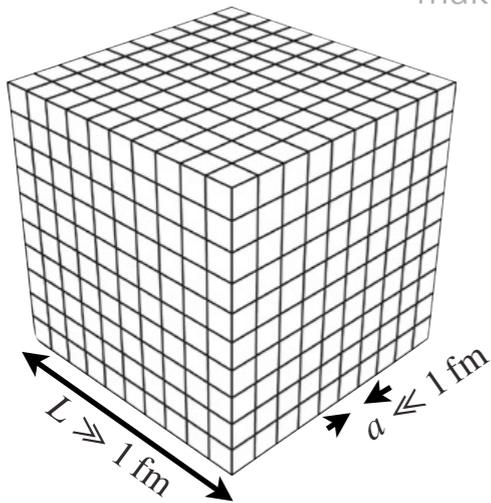




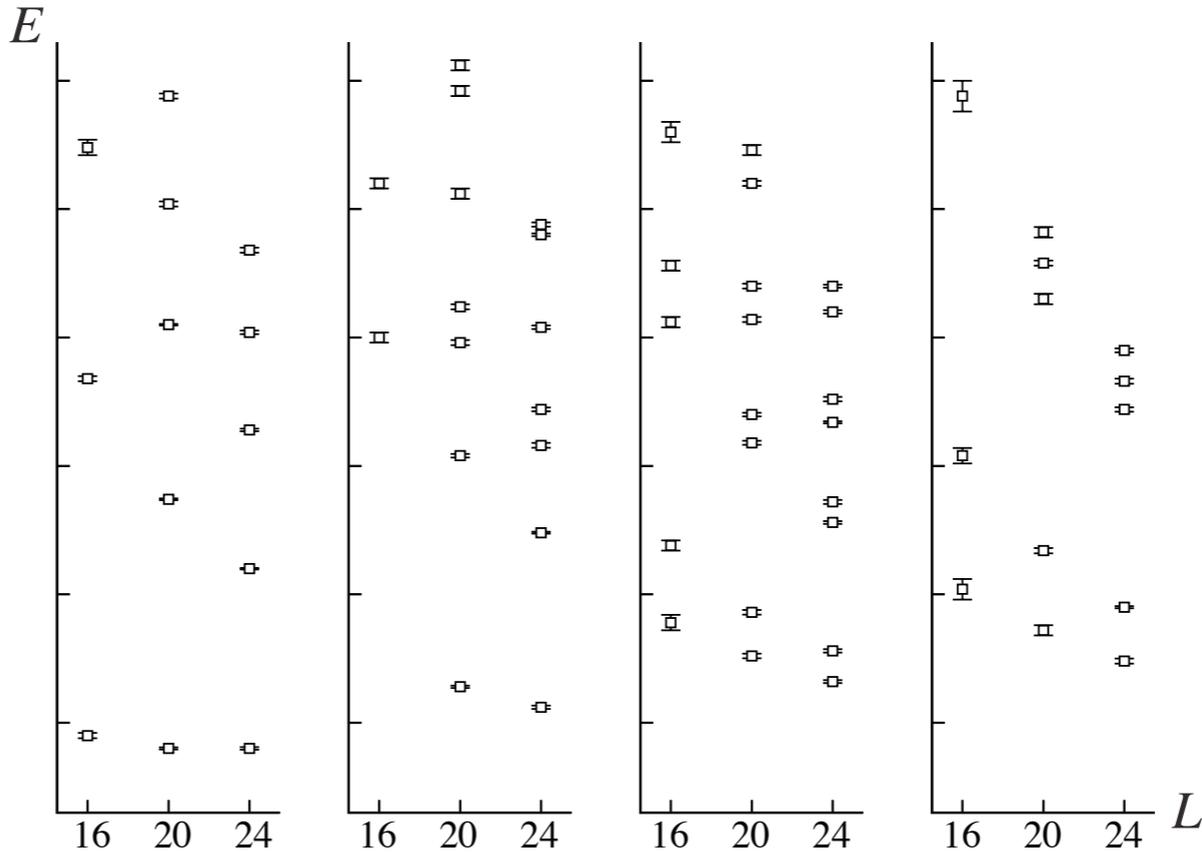


discretize in a finite space-time

making only controlled approximations



extract **discrete spectra** from computed **correlation functions**



volume dependence maps to **scattering amplitudes**

rigorous relation between
discrete spectrum in a finite-volume, $E_n(L)$
and coupled-channel scattering amplitudes, $\mathbf{t}(E)$

$E_n(L)$ are discrete solutions of

$$0 = \det \left[\mathbf{t}(E) + \mathbf{F}(E, L) \right]$$

known functions

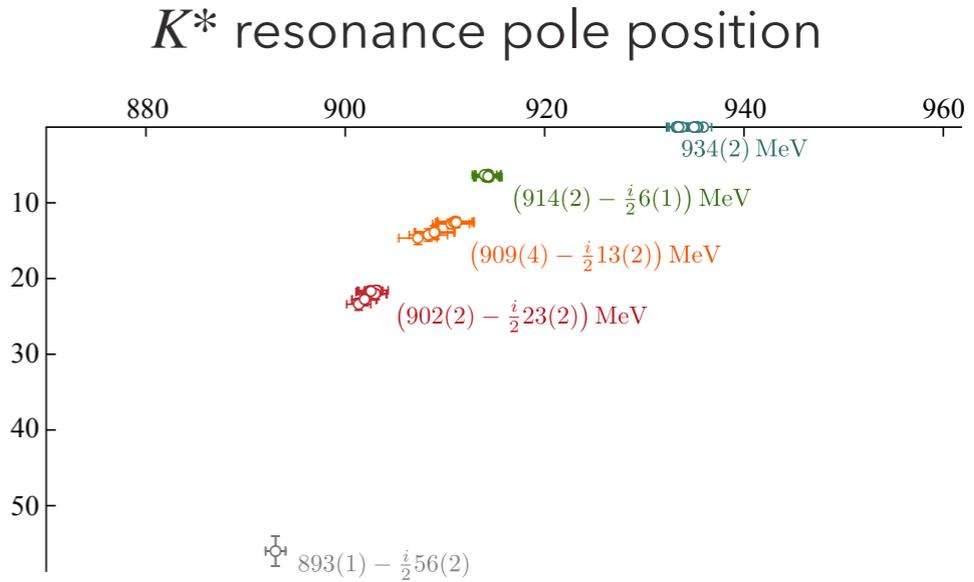
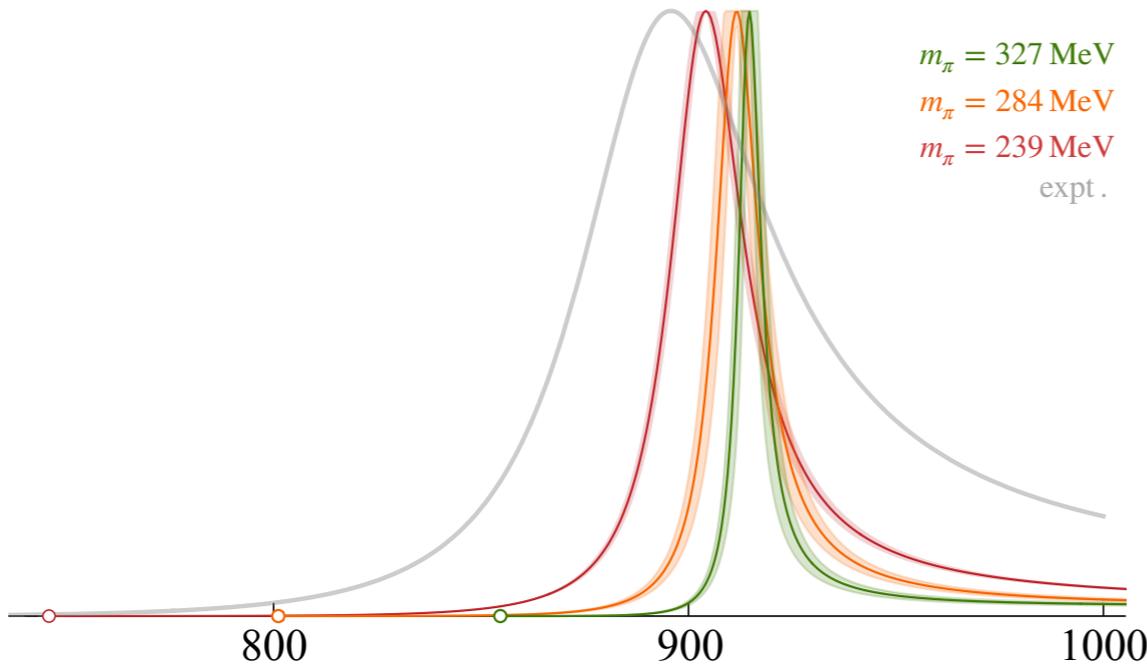
Lüscher & many others

rigorous relation between discrete spectrum in a finite-volume, $E_n(L)$ and coupled-channel scattering amplitudes, $\mathbf{t}(E)$

$$0 = \det [\mathbf{t}(E) + \mathbf{F}(E, L)]$$

e.g. $\pi K \rightarrow \pi K \quad J^P = 1^-$ elastic scattering

PRL 123 042002 (2019)

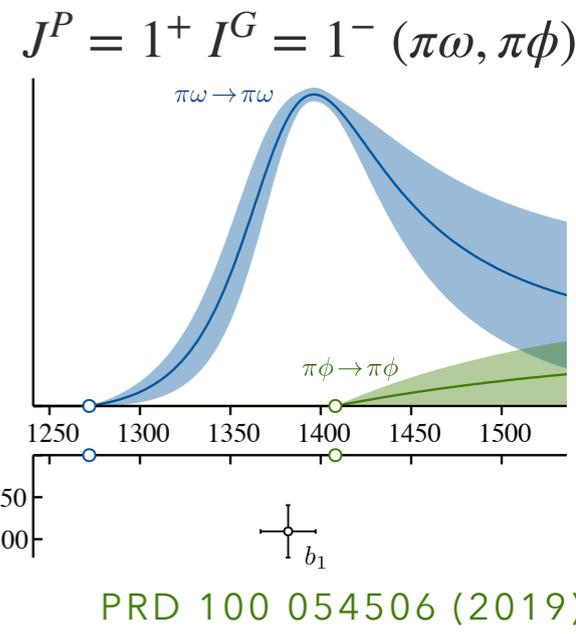
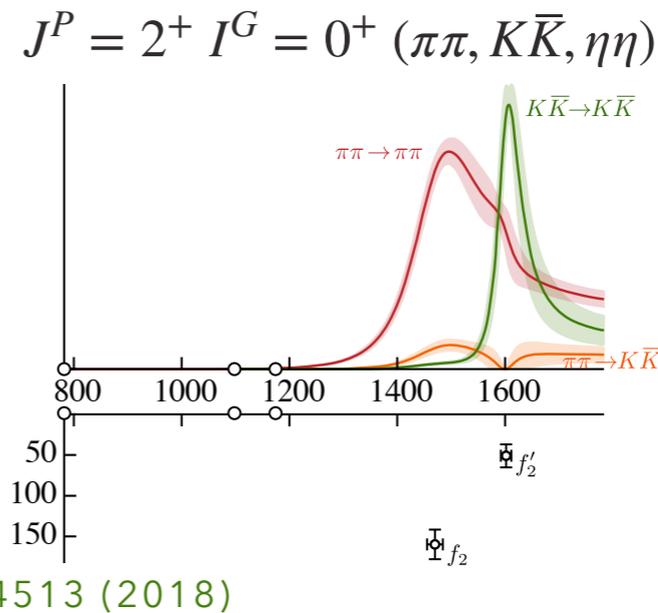
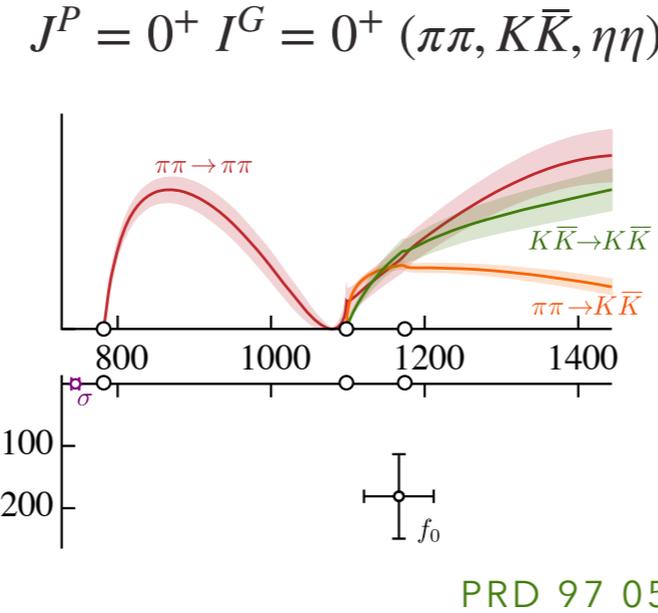
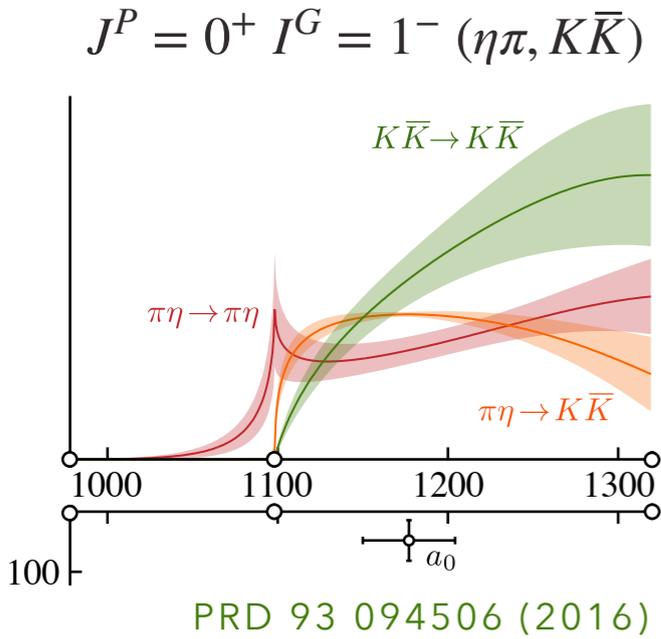


evolution of the K^* resonance as a function of varying light quark mass

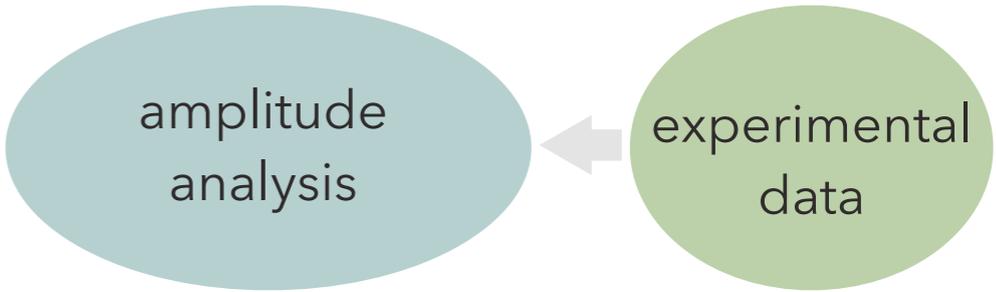
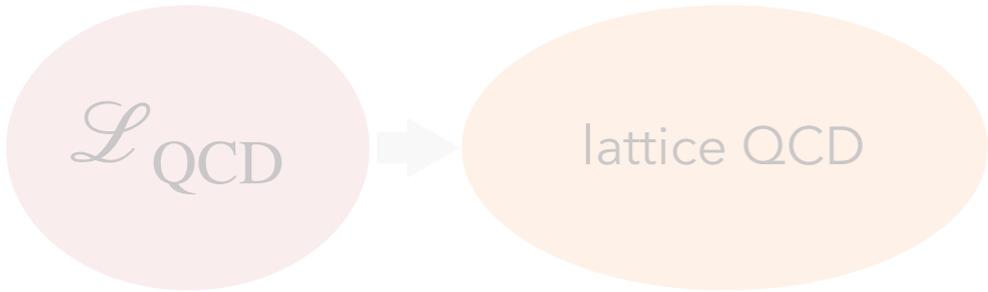
rigorous relation between discrete spectrum in a finite-volume, $E_n(L)$ and **coupled-channel** scattering amplitudes, $\mathbf{t}(E)$

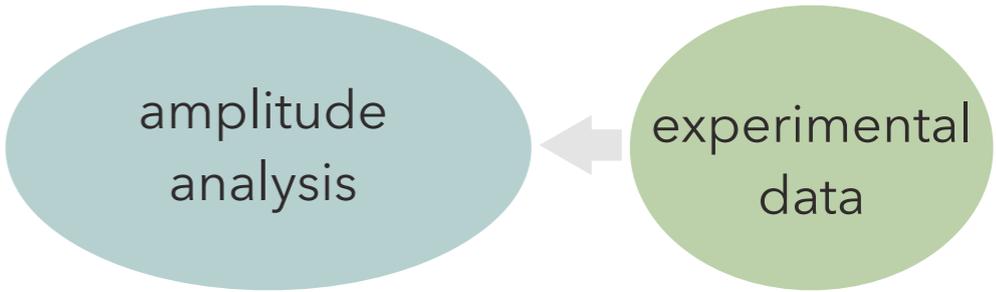
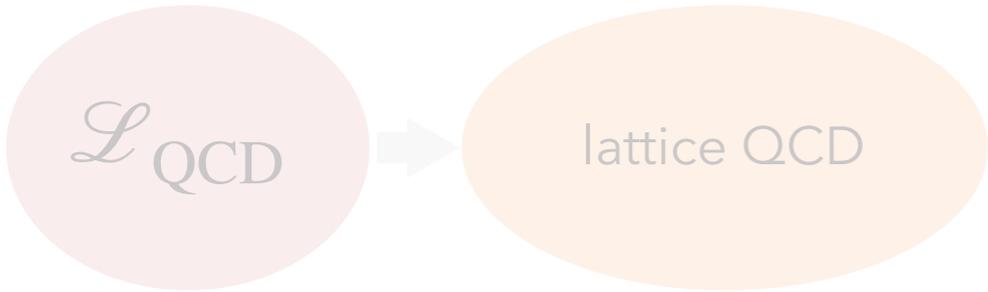
$$0 = \det [\mathbf{t}(E) + \mathbf{F}(E, L)]$$

parameterize **coupled-channel** $\mathbf{t}(E)$ and describe lattice spectra



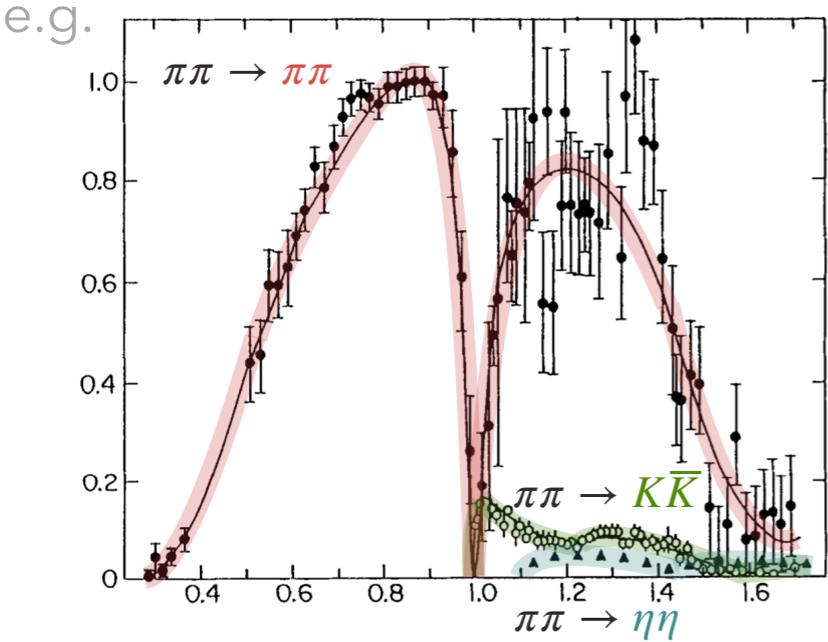
...





hadron scattering amplitudes

should satisfy generally true constraints: **unitarity**, **analyticity/causality**, **crossing symmetry** ...
 build relevant constraints into amplitudes used in analysis



just summing Breit-Wigners will lead to incorrect results

needs coupled-channels, unitarity, correct threshold behavior

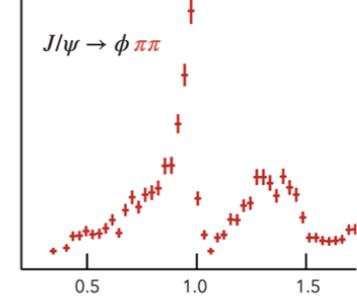
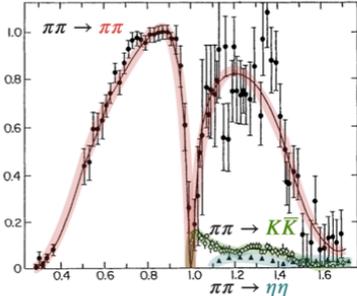
hadron scattering amplitudes

should satisfy generally true constraints: **unitarity, analyticity/causality, crossing symmetry ...**
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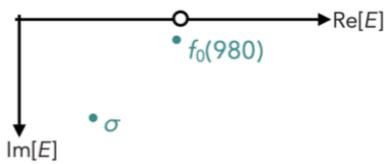
enhancements determined by **singularities in the complex energy plane**

rigorous determination of **resonances as pole singularities**

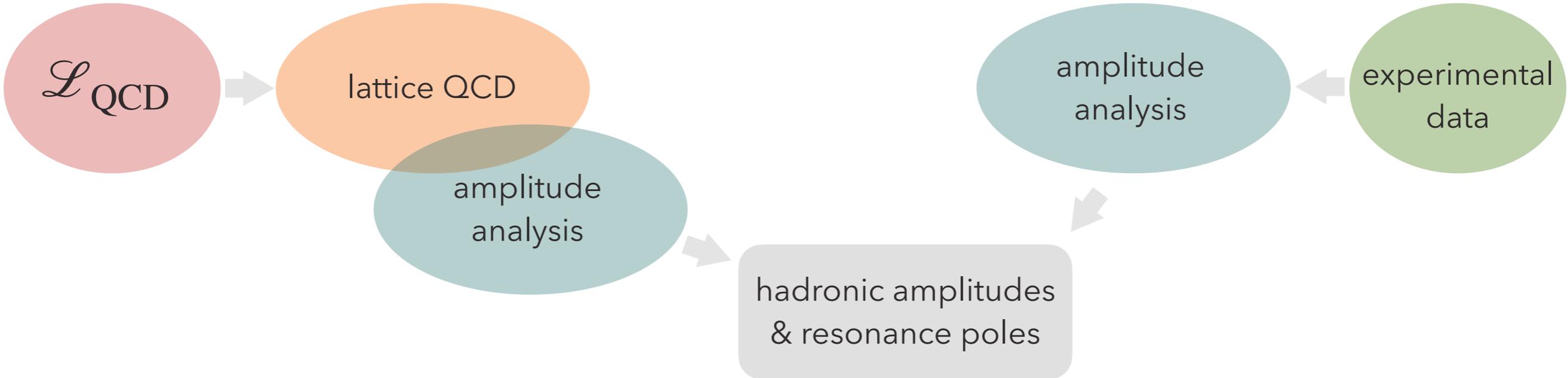
must have same location
in every process



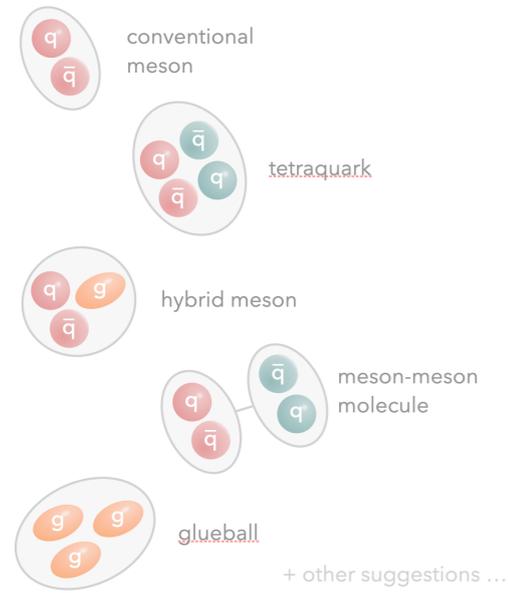
same poles $\sigma, f_0(980)$
different couplings



kinematical singularities, e.g. triangles in three-body
 are process dependent
 (may explain some XYZ inconsistencies)



can "picture" assignments be made robust within QCD ?



there are ideas in this direction ...

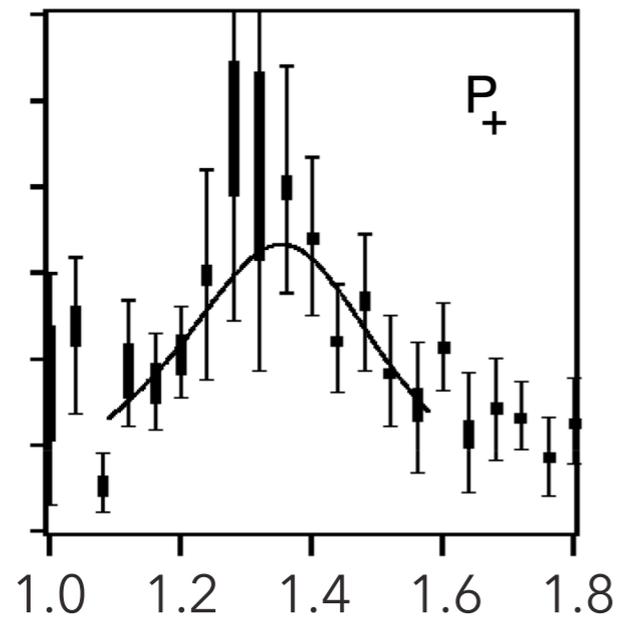
robust connection made between real experiment & QCD

until recently, a confused situation:

$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^-+)$

Mass $m = 1354 \pm 25$ MeV
Full width $\Gamma = 330 \pm 35$ MeV

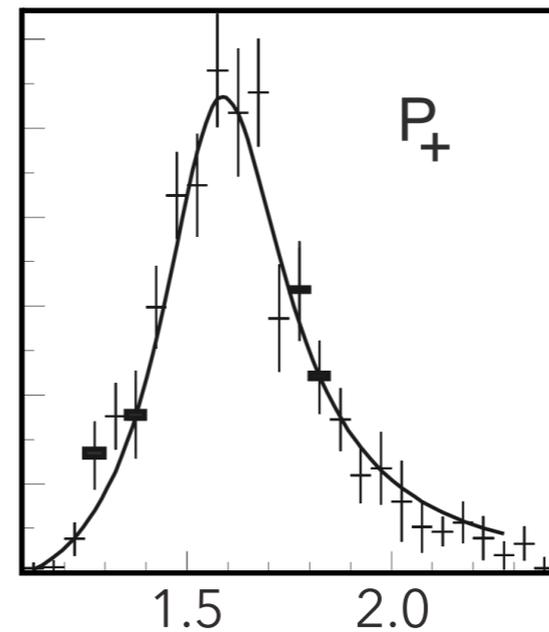
BNL e852 (1997)



$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^-+)$

Mass $m = 1661^{+15}_{-11}$ MeV
Full width $\Gamma = 240 \pm 50$ MeV

BNL e852 (2001)



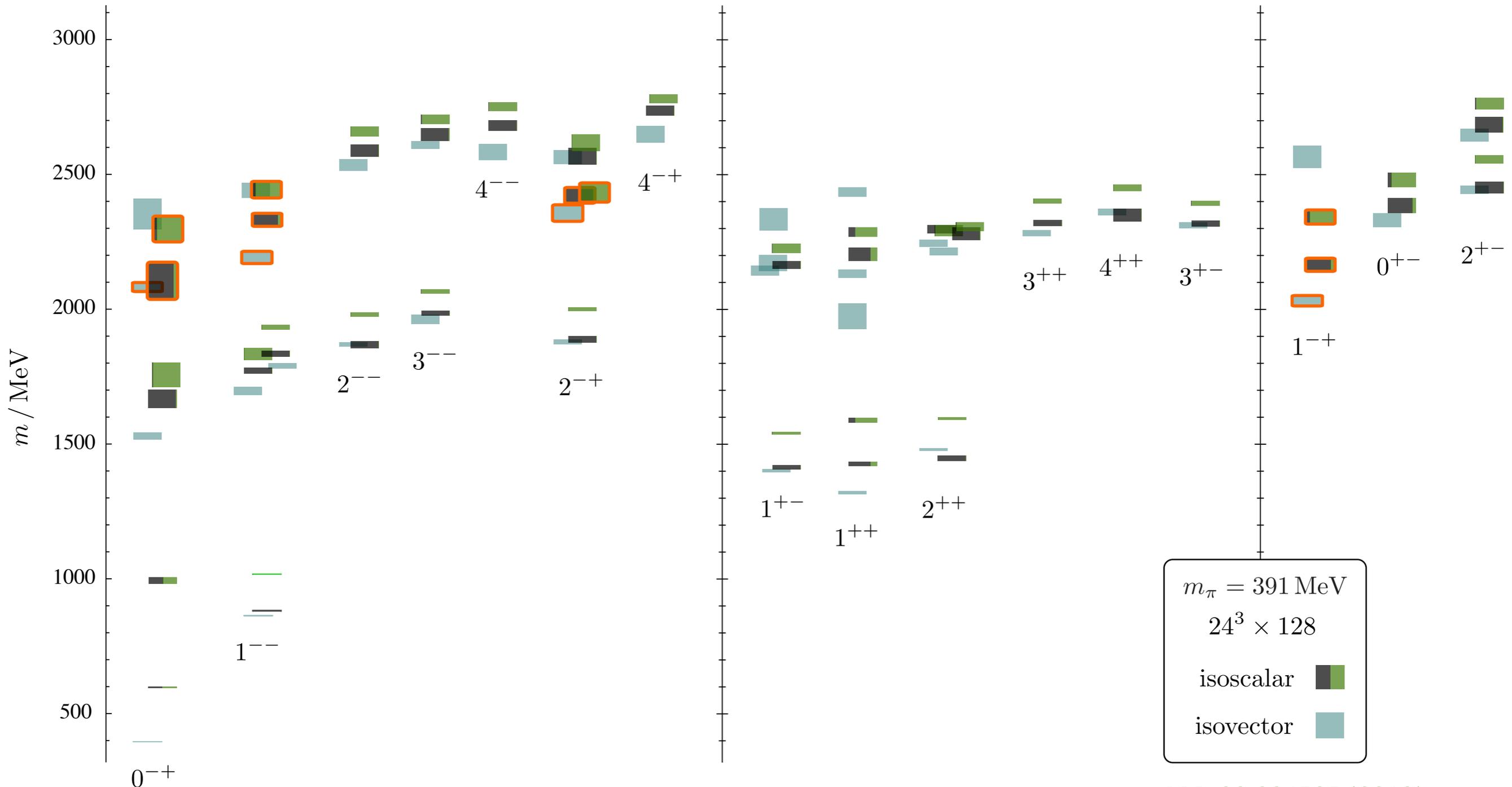
also seen in $\pi\rho$

two light states, where models predict only one ?

but models could be wrong ...

the lightest exotic hybrid meson – lattice QCD

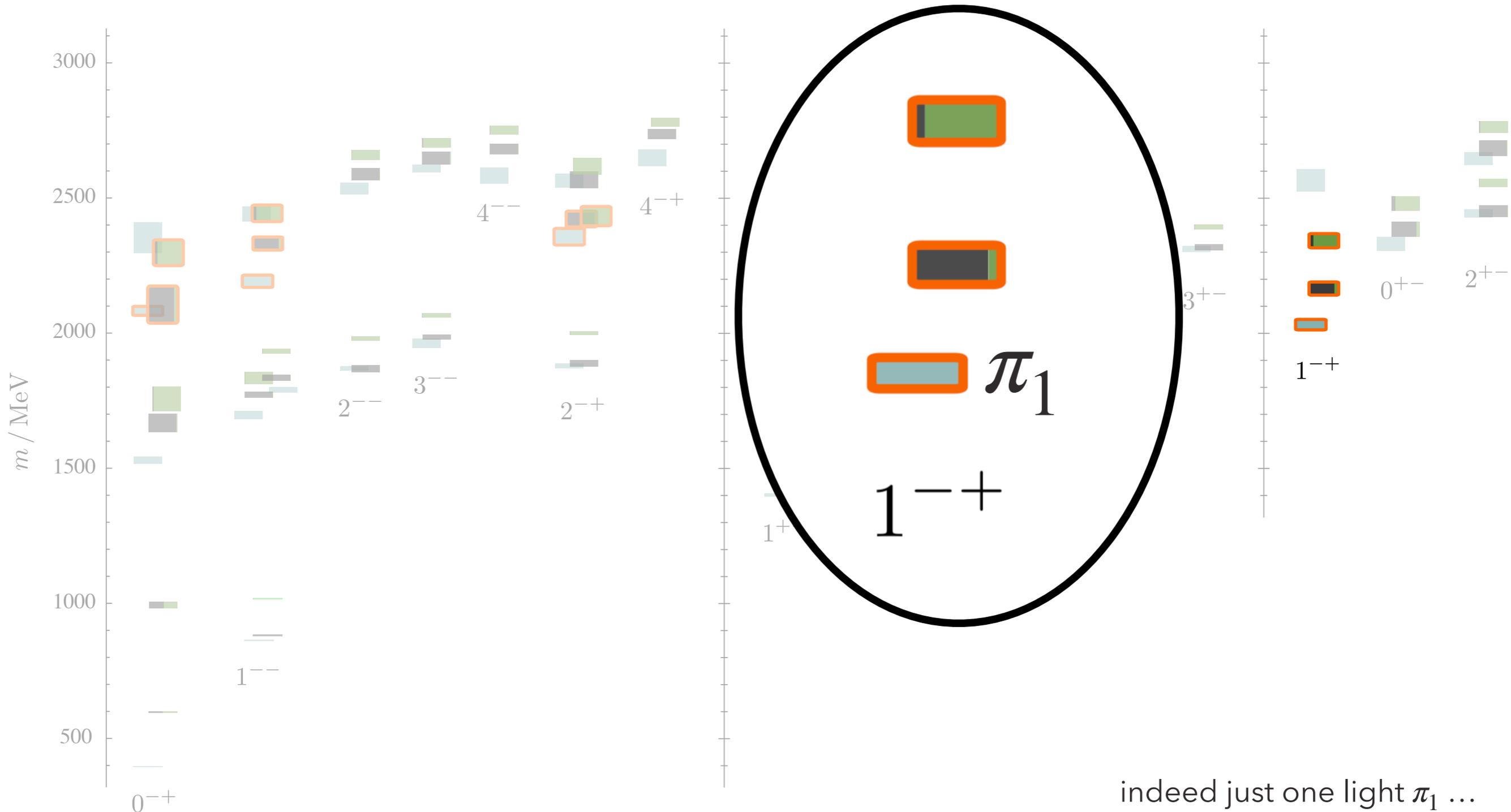
in 2013 *hadspec* calculates the “stable” light meson spectrum

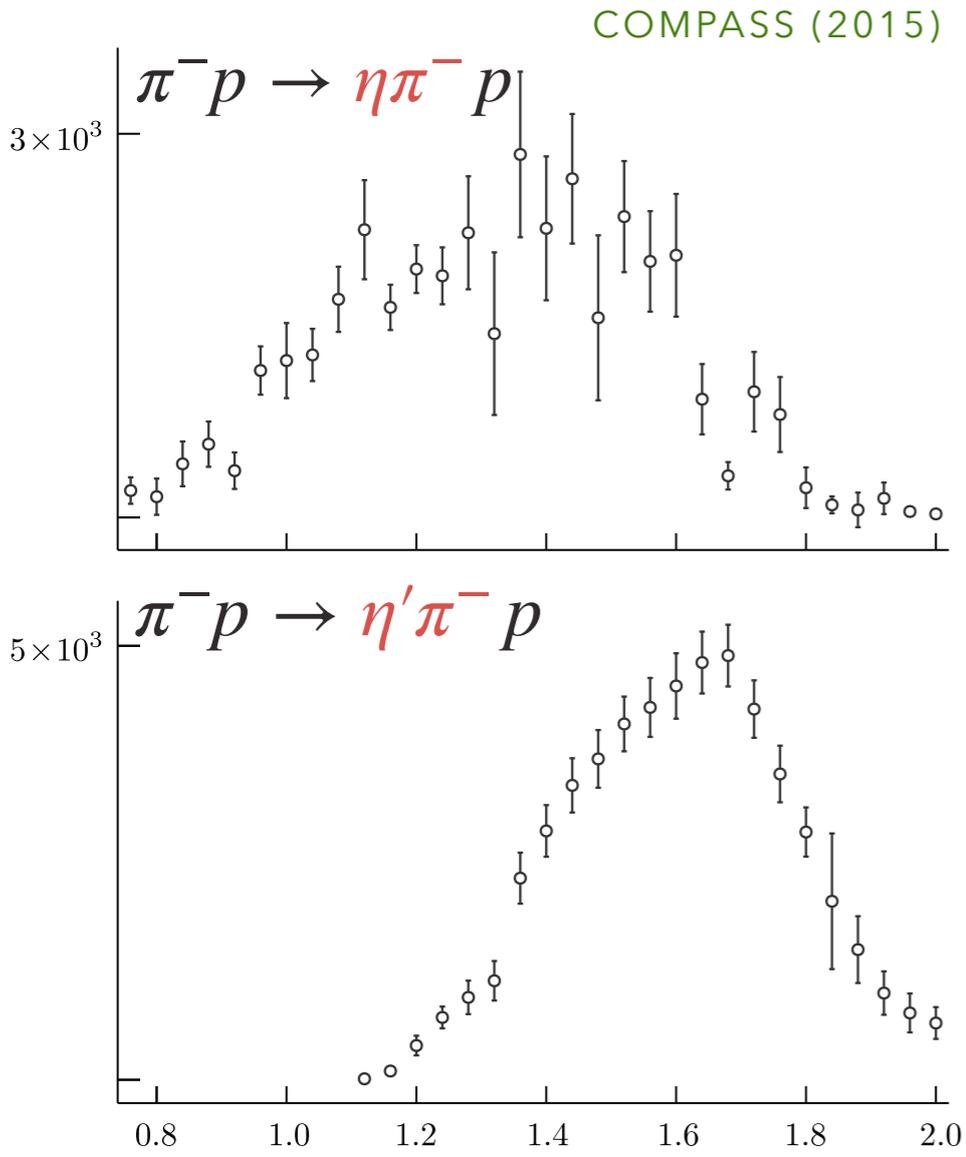


$m_\pi = 391 \text{ MeV}$
 $24^3 \times 128$
 isoscalar ■
 isovector ■

PRD 88 094505 (2013)

 (matrix element overlaps with chromomagnetic operators)

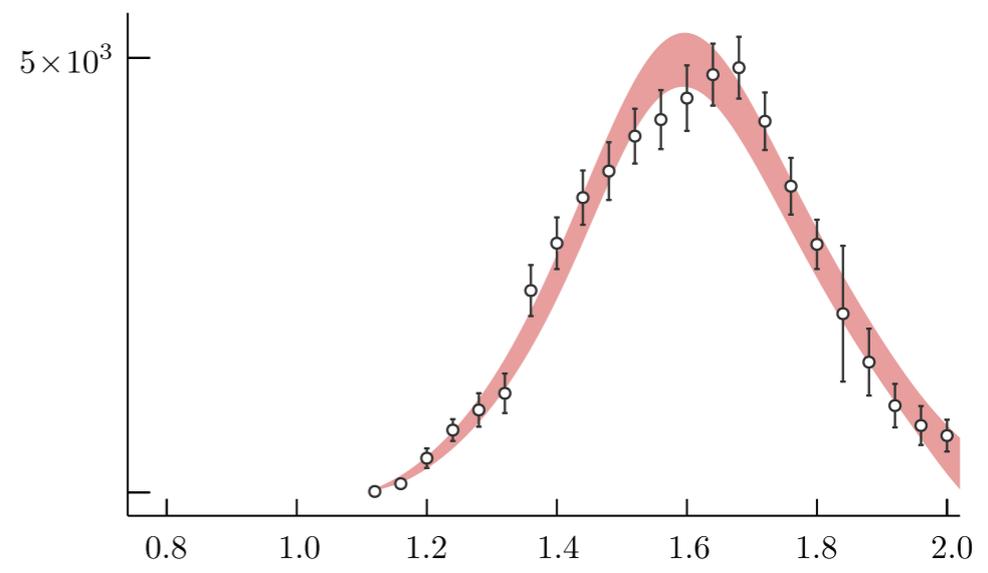
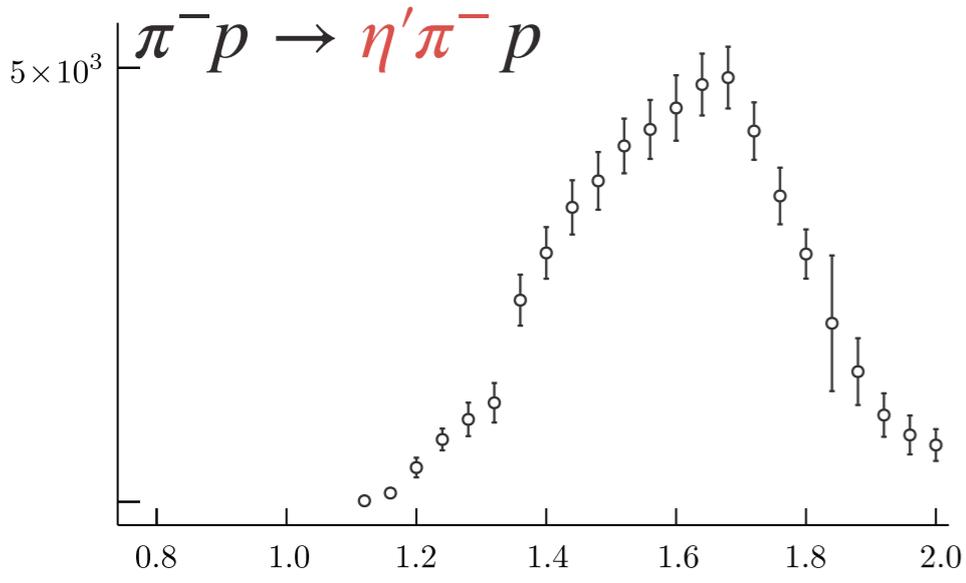
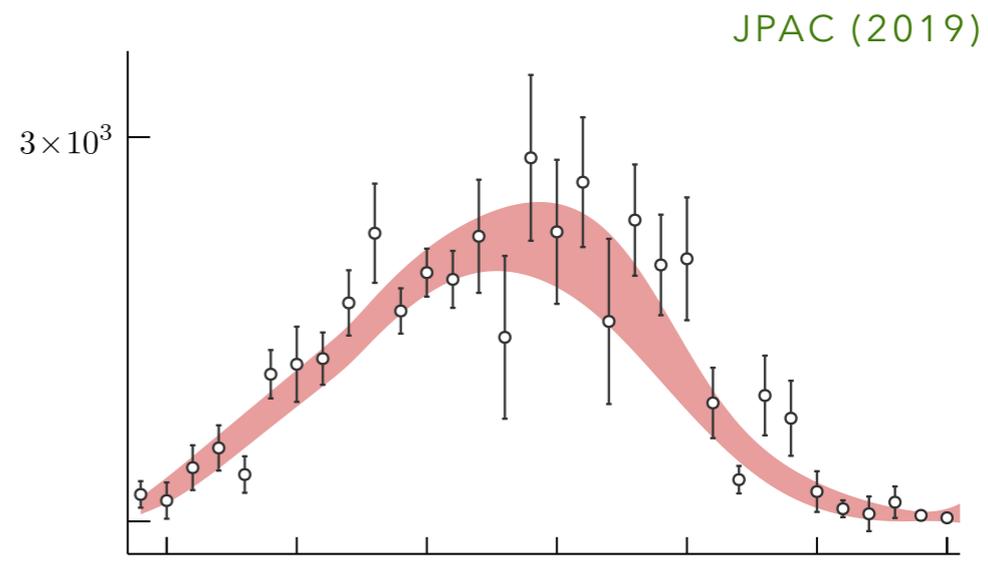
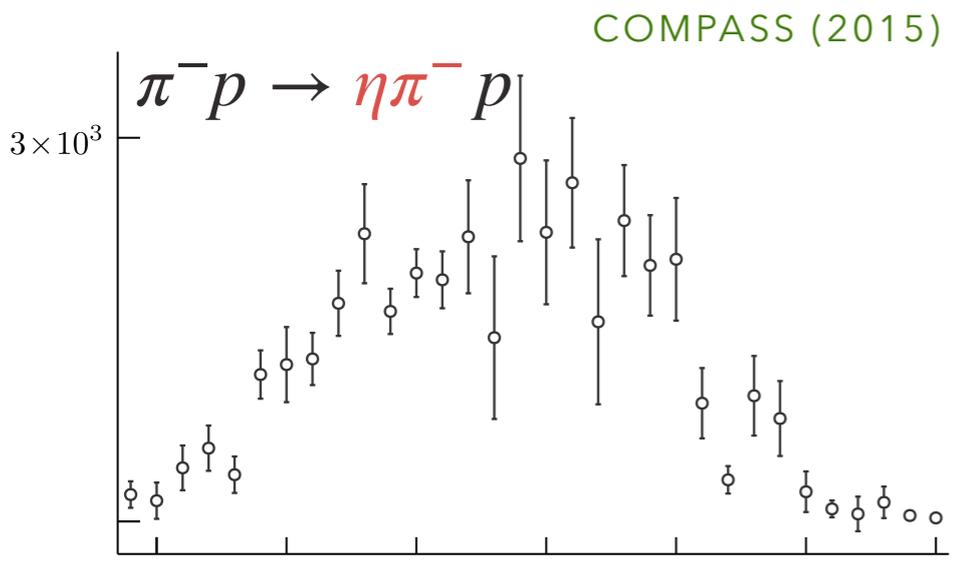




compatible with structures seen at BNL

the lightest exotic hybrid meson – constrained amplitude analysis

in 2019 *JPAC* analyses COMPASS data with **unitary coupled-channel amplitudes**



amplitudes have only **one pole singularity**

$$m = 1564(89) \text{ MeV}$$

$$\Gamma = 492(115) \text{ MeV}$$

a broad resonance ...

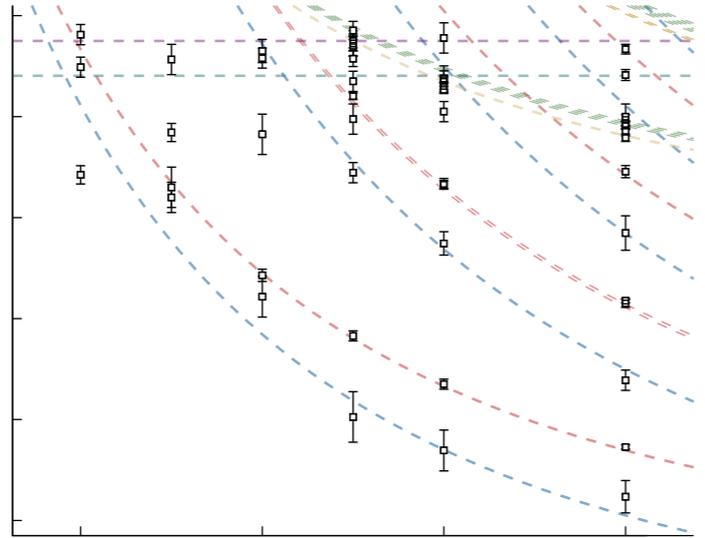
the lightest exotic hybrid meson – lattice QCD resonance ?

in 2021 *hadspec* computes π_1 as an **unstable resonance**

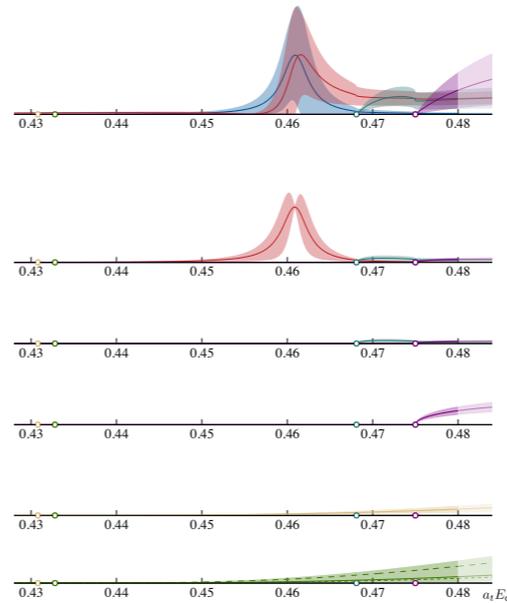
(not as tightly constrained as other cases shown earlier)

SU(3)_F point
 $m_u = m_d = m_s^{\text{phys}}$
 $m_\pi \sim 700 \text{ MeV}$

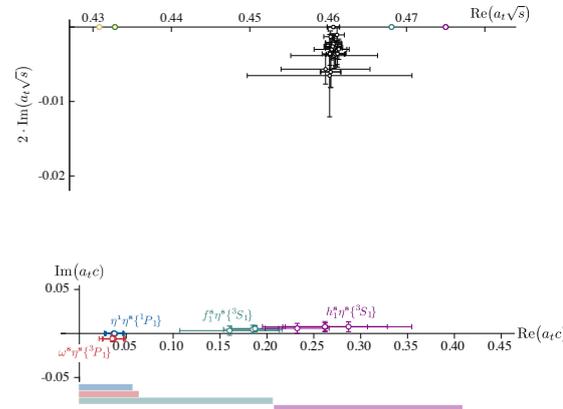
spectrum in six lattice volumes



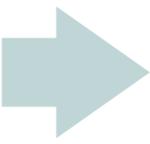
amplitudes in eight channels



decay couplings of one resonance pole



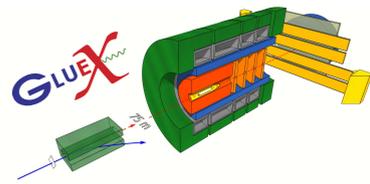
narrow resonance at this heavy quark mass



extrapolated to physical quark mass

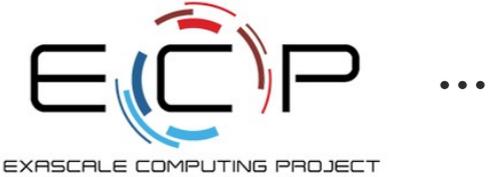
for a $\pi_1(1564)$
 $\Gamma \sim 140 - 600 \text{ MeV}$
 $\Gamma(\pi\eta) \lesssim 1 \text{ MeV}$
 $\Gamma(\pi\eta') \lesssim 12 \text{ MeV}$
 $\Gamma(\pi\rho) \lesssim 20 \text{ MeV}$
 $\Gamma(\pi b_1) \sim 139 - 529 \text{ MeV}$

compatible with being a **broad resonance**
 suggests dominant decay into $\pi b_1 \rightarrow \pi\pi\omega \rightarrow \pi\pi\pi\pi$



lattice QCD collaborations e.g. *hadspec*
www.hadspec.org

hardware & software development grants

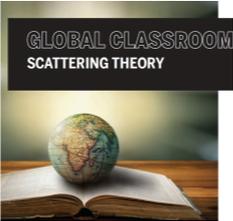


computing time awards



amplitude analysis collaborations e.g. *JPAC*
cgl.soic.indiana.edu/jpac

workforce development



relationships with experiments



broad collaborative efforts proposed *DoE topical collaboration* to co-ordinate, also with modeling, EFTs

novel & high statistics data

new structures

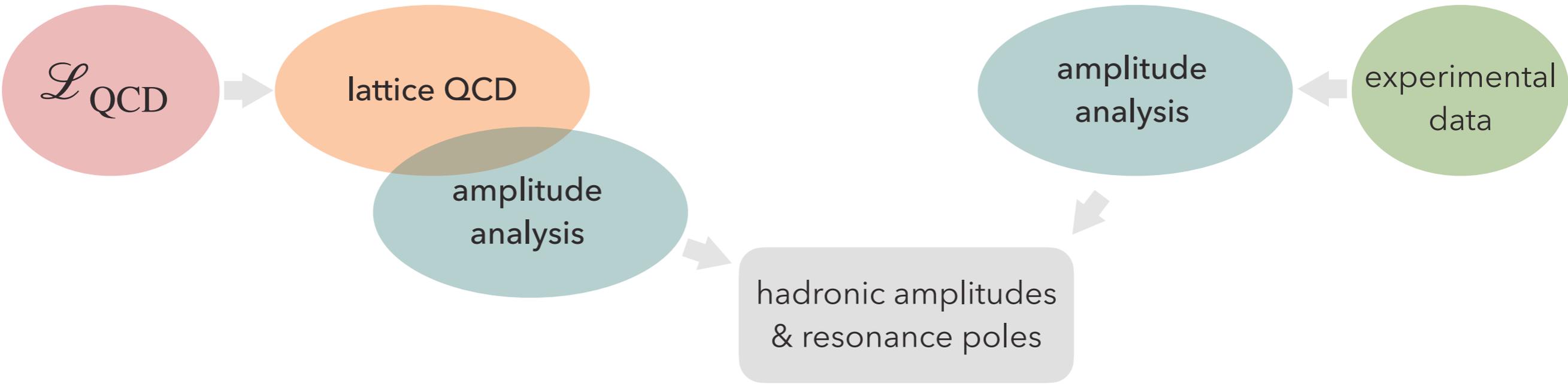
reliable description requires robust amplitudes

new tools & computational advances in lattice QCD

excited hadrons

faithfully, as resonances

tools are common to light quark and heavy quark systems



hadron spectroscopy has evolved into a rigorous probe of the emergent properties of strongly-coupled QCD