Nucleon Spin Structure at Low x

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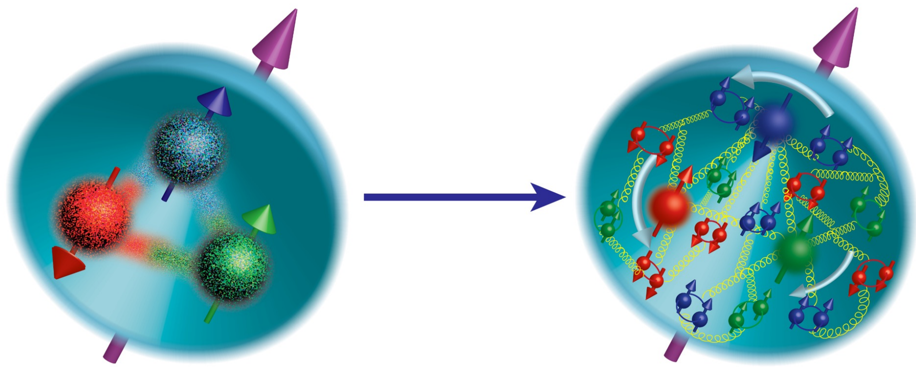
Outline

- Longitudinal spin:
  - proton spin puzzle;
  - gluon spin at RHIC;
  - spin at small-x: EIC and a possible path toward spin puzzle resolution.

- Transverse spin:
  - small-x asymptotics of the Sivers function;
  - transversity at small x.

Longitudinal Spin

Proton Spin



Our understanding of nucleon spin structure has evolved:

- In the 1980's the proton spin was thought of as a sum of constituent quark spins (left panel)
- Currently we believe that the proton spin is a sum of the spins of valence and sea quarks and of gluons, along with the orbital angular momenta of quarks and gluons (right panel)

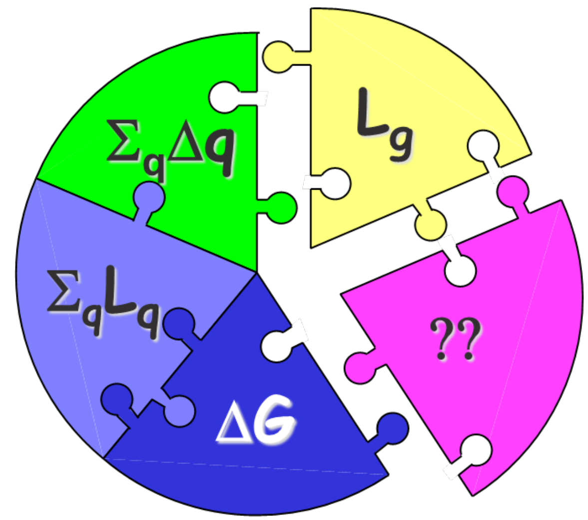
Proton Helicity Sum Rule

- Helicity sum rule (Jaffe&Manohar, 1989; cf. Ji, 1997):

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

with the net quark and gluon spin

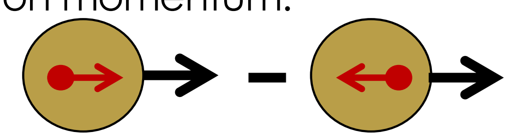
$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$



L_q and L_g are the quark and gluon orbital angular momenta (OAM)

- To quantify the contributions of quarks and gluons to the proton spin one defines helicity distribution functions: number of quarks/gluons with spin parallel to the proton momentum minus the number of quarks/gluons with the spin opposite to the proton momentum:

$$\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$



- $\Delta G(x, Q^2)$ is the gluon helicity distribution while the flavor–singlet quark helicity PDF is $\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$

Proton Spin Puzzle

- The spin puzzle began when the EMC collaboration measured the proton g_1 structure function ca 1988. Their data resulted in

$$S_q \approx 0.03$$

- It appeared (constituent) quarks do not carry all of the proton spin (which would have naively corresponded to $S_q = 1/2$).

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

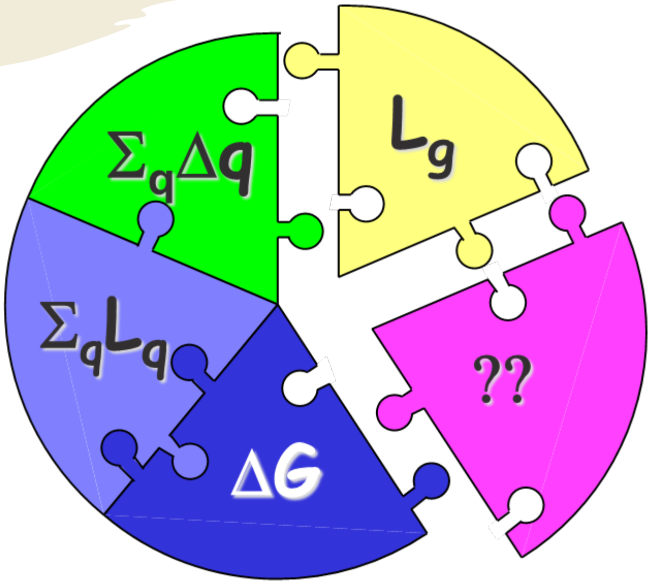
- Missing spin can be
  - Carried by gluons
  - In the orbital angular momenta of quarks and gluons
  - At small x:

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

Can't integrate down to zero, use x_{min} instead!

  - Or all of the above!

Current Knowledge of Proton Spin



- The proton spin carried by the quarks is estimated to be (for $0.001 < x < 1$)

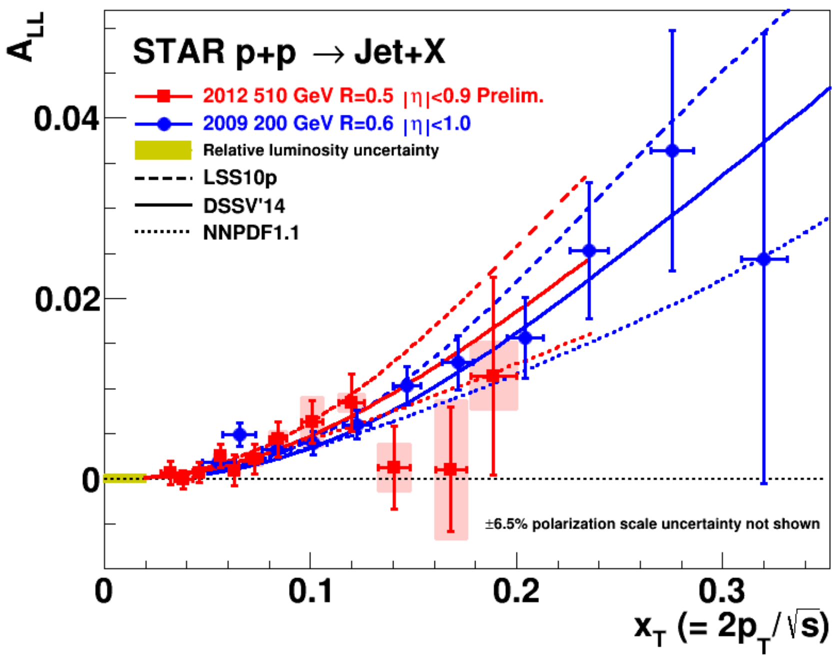
$$S_q(Q^2 = 10 \text{ GeV}^2) \in [0.15, 0.2]$$

- The proton spin carried by the gluons is (for $0.01 < x < 1$, STAR+PHENIX+COMPASS +HERMES+… , analyzed by DSSV, JAM, NNPDF…)

$$S_G(Q^2 = 10 \text{ GeV}^2) \in [0.13, 0.26]$$

- Negative S_G is also possible (JAM).
- Unfortunately, the uncertainties are large. Note also that the x-ranges are limited, with more spin (positive or negative) possible at small x.

A_{LL} Measurements



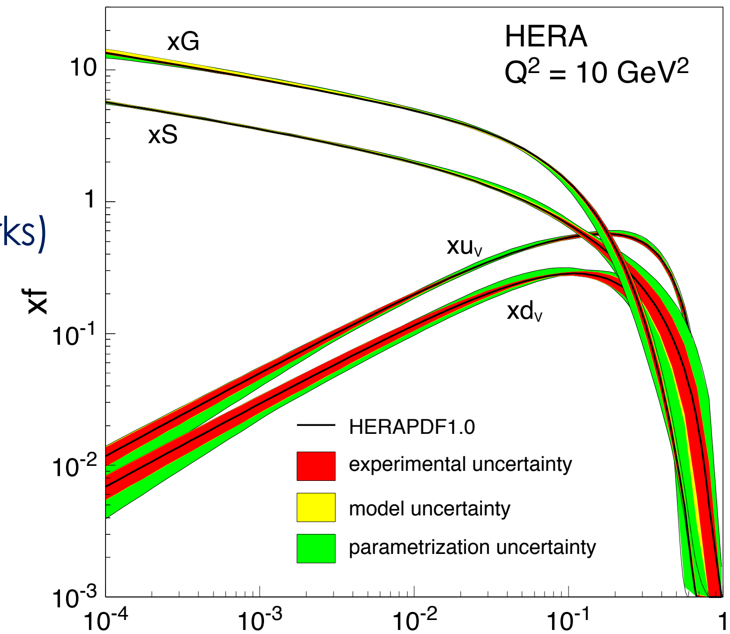
ΔG was extracted from A_{LL} for jets and neutral pions reported by STAR and PHENIX (see also talk by Elke Aschenauer on Saturday).

The non-zero ΔG measurement is one of the most important results of RHIC Spin program.

Gluons and Quarks at Small-x

- There is a large number of small-x gluons (and sea quarks) in a proton:

x (# gluons or quarks)

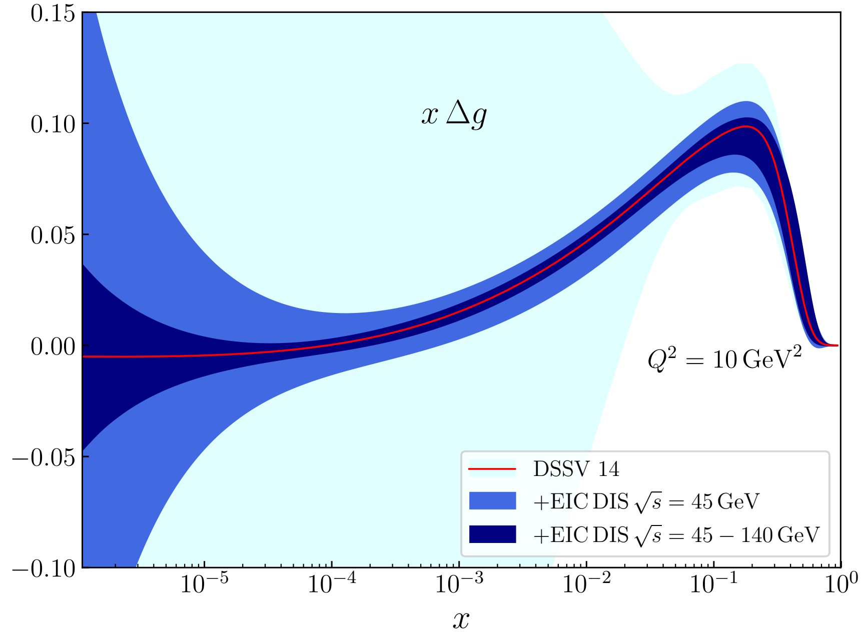


x =fraction of the proton momentum they carry

Can the small-x gluons and quarks carry a lot of the proton spin?

- $G(x, Q^2)$, $q(x, Q^2)$ = gluon and quark number densities / parton distribution functions (q=u,d, or S for sea).

How much spin is there at small x?



- E. Aschenauer et al, 2020 (DGLAP-based helicity PDF extraction from data)

- Uncertainties are very large at small x. Note that this is xΔG, the uncertainties for ΔG are 1/x = 100-10000 times larger! EIC will reduce them, but only where there will be data.

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

Small-x Spin Challenge

- Can we constrain theoretically the amount of proton spin and OAM coming from small x?

- Any existing and future experiment probes the helicity distributions and OAM down to some x_{min} .

$$S_q(Q^2) = \frac{1}{2} \int\limits_0^1 dx \, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int\limits_0^1 dx \, \Delta G(x, Q^2)$$

$$L_{q+\bar{q}}(Q^2) = \int\limits_0^1 dx \, L_{q+\bar{q}}(x, Q^2) \qquad L_G(Q^2) = \int\limits_0^1 dx \, L_G(x, Q^2)$$

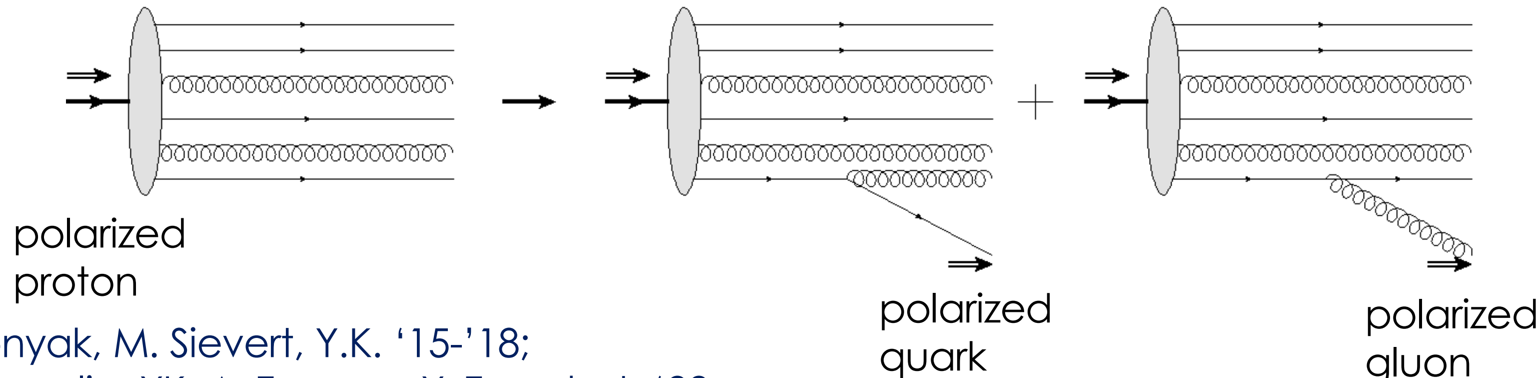
- At very small x (for the proton), saturation sets in: that region likely carries a negligible amount of proton spin. But what happens at larger (but still small) x?

Philosophy of small-x approach to spin

- DGLAP equation evolves in Q^2, it does not evolve in x.
- Hence, DGLAP-based analyses (DSSV, NNPDF, standard JAM) cannot predict the x-dependence of PDFs.
- If we want to predict helicity PDFs at small x, we need a different evolution equation evolving in x.
- Such equations were constructed by D. Pitonyak, M. Sievert, and YK, 2015-2018 along with a recent important correction by F. Cougoulic, YK, A. Tarasov, Y. Tawabutr in 2022; both works use an approach similar to the BK/JIMWLK evolution.
- Other important work on spin at small x: J. Bartels, B. Ermolaev, M. Ryskin '95-'96; Y. Hatta et al '16; R. Boussarie, Y. Hatta, F. Yuan '19; G. Chirilli '21. Significant related works by R. Kirschner and L. Lipatov '83; T. Altinoluk, G. Beuf et al '20, '21.
- Axial anomaly: important for helicity, even at small x. See recent work by A. Tarasov and R. Venugopalan, '19-'21, building on earlier work on the anomaly.

Helicity Evolution at Small x

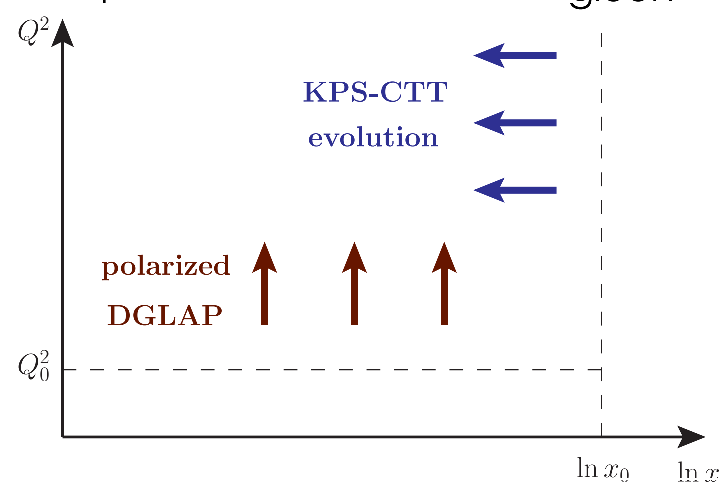
- To understand how much of the proton's spin is at small x one can construct a helicity analogue of the BFKL equation:



D. Pitonyak, M. Sievert, Y.K. '15-'18;
F. Cougoulic, YK, A. Tarasov, Y. Tawabutr '22

- This new helicity evolution equations give the following small-x asymptotics of helicity PDFs and the g_1 structure function at large N_c (cf. Bartels et al, '96):

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$



(Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK, 2102.06159 [hep-ph] = JAMsmallx)

Small-x Polarized DIS Data

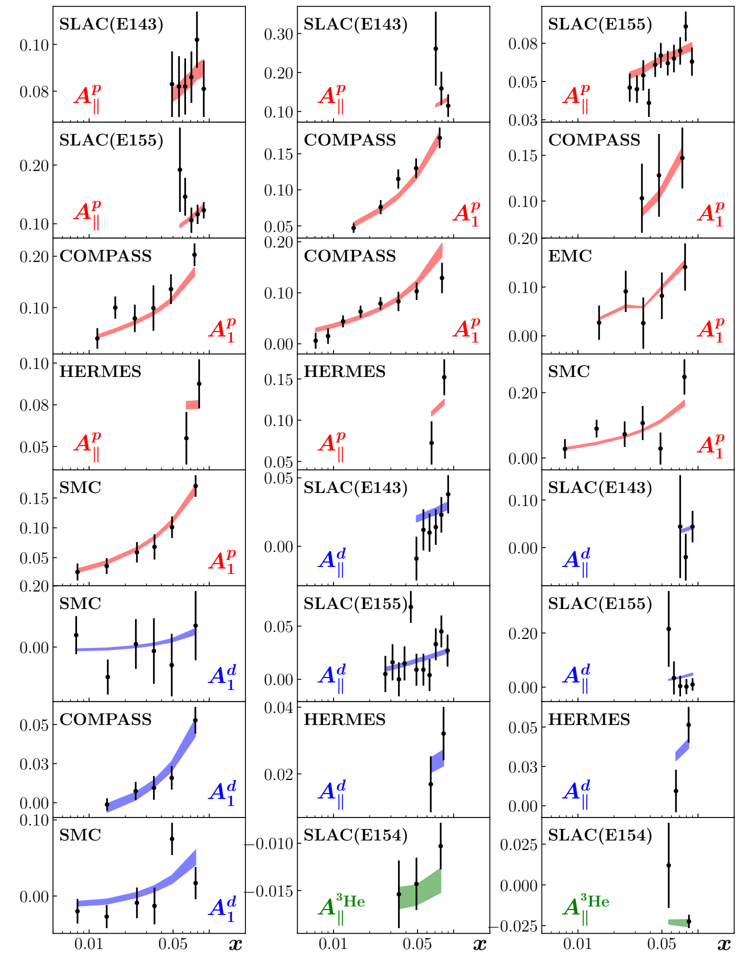
$$g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 \left[\Delta q_f + \Delta q_{\bar{f}} \right]$$

$$A_1 \sim A_\parallel = \frac{\sigma_{+-} - \sigma_{++}}{\sigma_{+-} + \sigma_{++}} \sim \frac{g_1}{F_1}$$

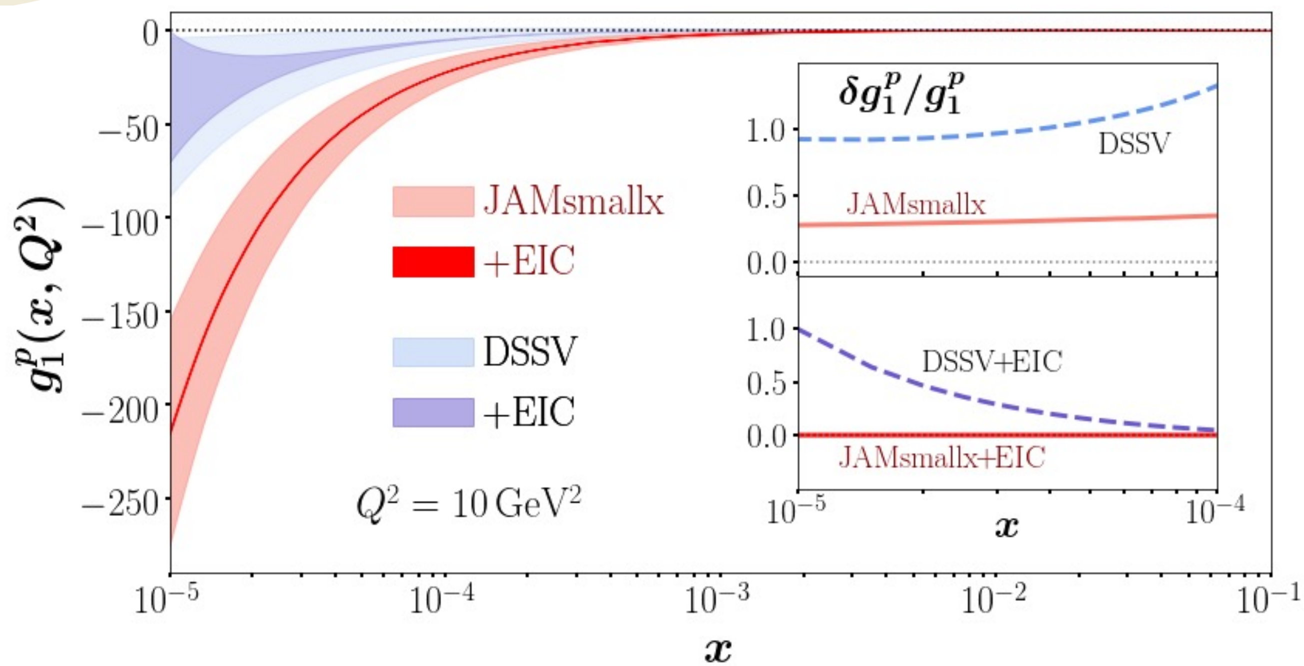
- Analysis of the world polarized DIS data with x<0.1=x$_0$, $Q^2 > m_c^2$ (122 data points) using the large-N$_C$ KPS evolution with the Born-inspired initial conditions (8 parameters for 2 flavors, 11 parameters for 3 flavors).

- χ^2/N$_{pts}$ =1.01 (cf. JAM16: χ^2/N$_{pts}$ =1.07)

- Small-x evolution starts at x$_0$=0.1 ! (cf. x$_0$=0.01 for unpolarized BK/JIMWLK evolution)



JAMsmallx prediction for g_1 structure function



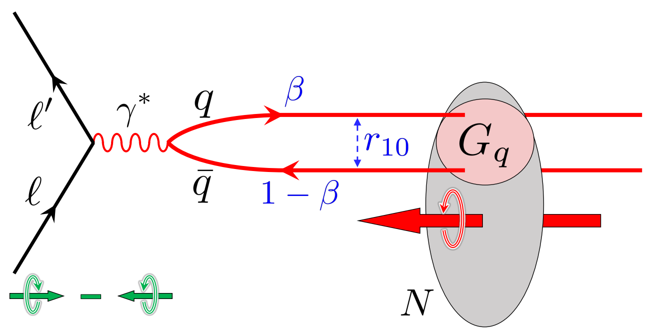
g_1 structure function ≈ spin-dependent part of σ^{e+p} $\quad g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 \left[\Delta q_f + \Delta q_{\bar{f}} \right]$

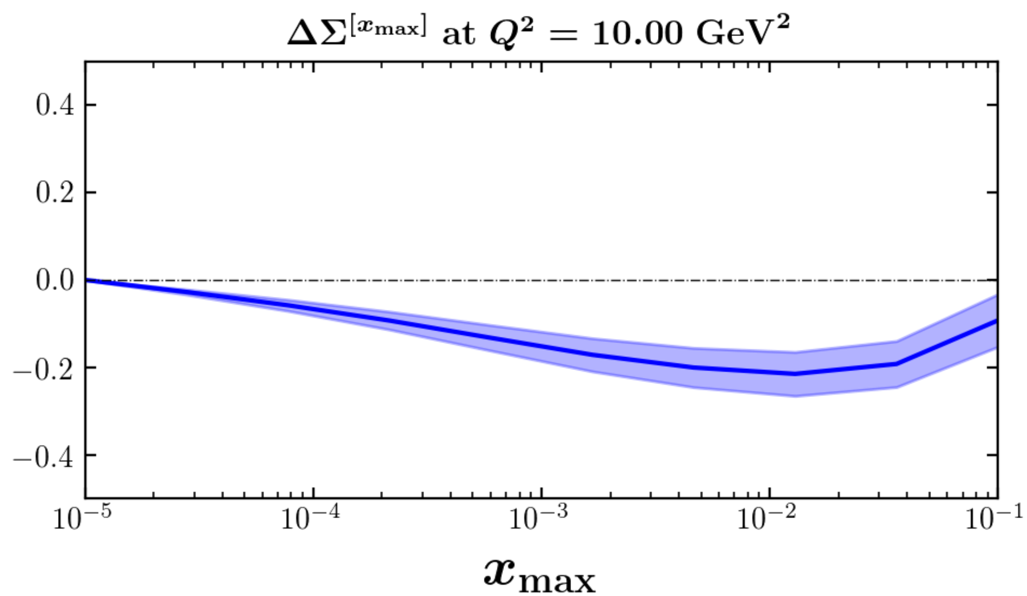
Thick band: 1σ CL; **thin band: impact of EIC data**. With the EIC pseudo-data we have 1096 data points. Despite tremendous progress we still do not know the fate of g_1 at small *x*: EIC with small-*x* evolution can answer this question.

Small-x quarks impact on the proton spin

- Potentially negative 10-20% of the proton spin may be carried by small-x quarks helicity (JAMsmallx, preliminary):

$$\Delta\Sigma^{[x_{max}]}(Q^2) = \int_{10^{-5}}^{x_{max}} dx\, \Delta\Sigma(x, Q^2)$$





Speculation on a path to resolving the spin puzzle

- Above we discussed quark helicity at small x. Let's add the orbital angular momentum (OAM) (Hatta & Yang, '18; YK '19):

$$\frac{1}{2}\,\Delta\Sigma(x,Q^2) + L_{q+\bar{q}}(x,Q^2) = -\frac{1}{2}\,\Delta\Sigma(x,Q^2)$$

- So, the net quark (1/2) helicity+OAM = (-1/2) helicity.
- For x<0.001 we thus expect (preliminary!)

$$\left[\tfrac{1}{2}\Delta\Sigma + L_{q+\bar{q}}\right]_{Q^2=10\,\text{GeV}^2,\ x<0.001} \approx -\frac{1}{2}\,(-0.2) = 0.1$$

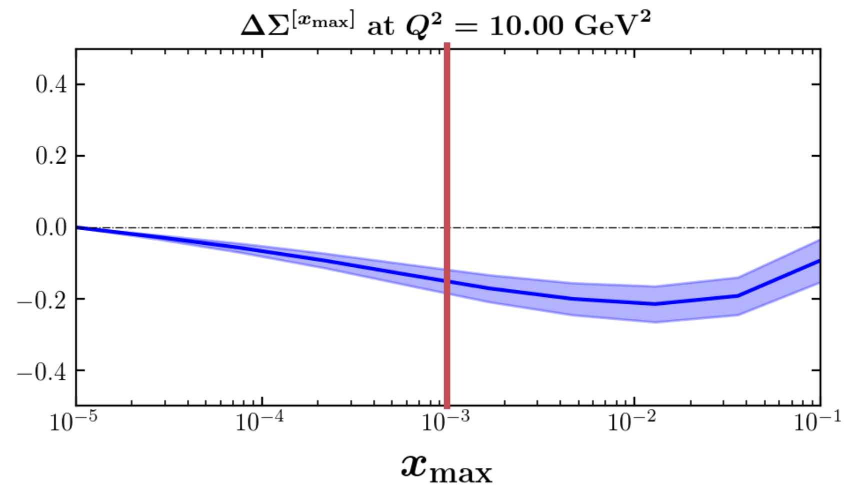
- Add to this the larger-x numbers

$$S_q(Q^2 = 10\,\text{GeV}^2, x > 0.001) \approx 0.18$$
$$S_G(Q^2 = 10\,\text{GeV}^2, x > 0.05) \approx 0.2$$

- We get

$$0.18 + 0.2 + 0.1 = 0.48$$

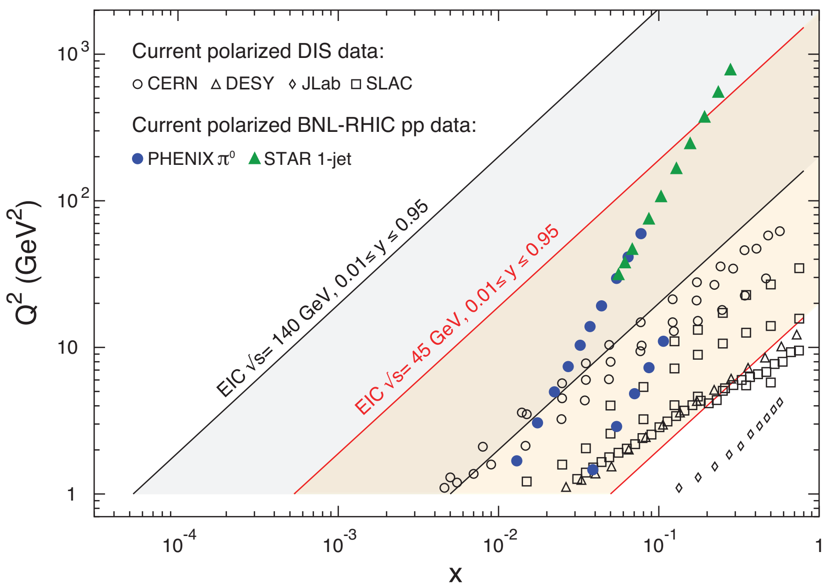
JAMsmallx, preliminary, Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK

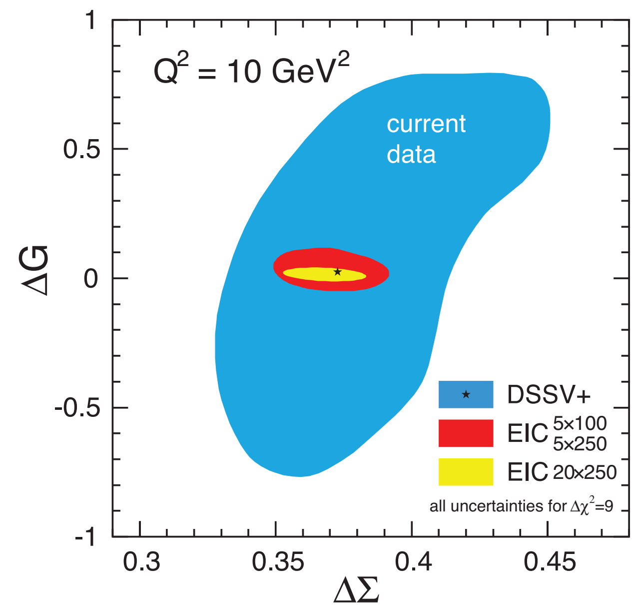


EIC & Spin Puzzle

Still, even with the EIC data we need to extrapolate quark and gluon spin down to smaller x: small-x evolution can do that.

- Parton helicity distributions are sensitive to low-x physics.
- EIC would have an unprecedented low-x reach for a polarized DIS experiment, allowing to pinpoint the values of quark and gluon contributions to proton's spin:





- ΔG and $\Delta\Sigma$ are integrated over x in the $0.001 < x < 1$ interval.

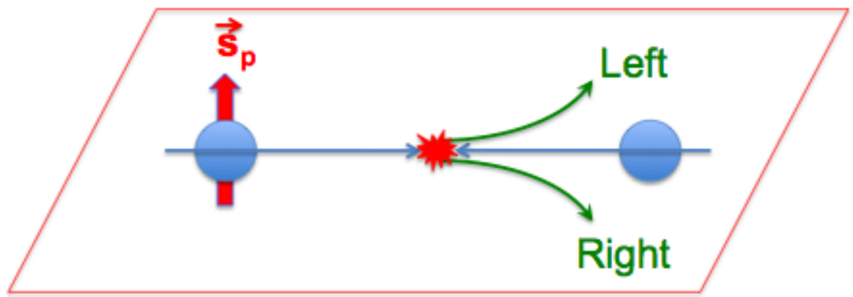
Transverse Spin

Single Transverse Spin Asymmetry

Single Transverse Spin Asymmetry

- Consider transversely polarized proton scattering on an unpolarized proton or nucleus.

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$



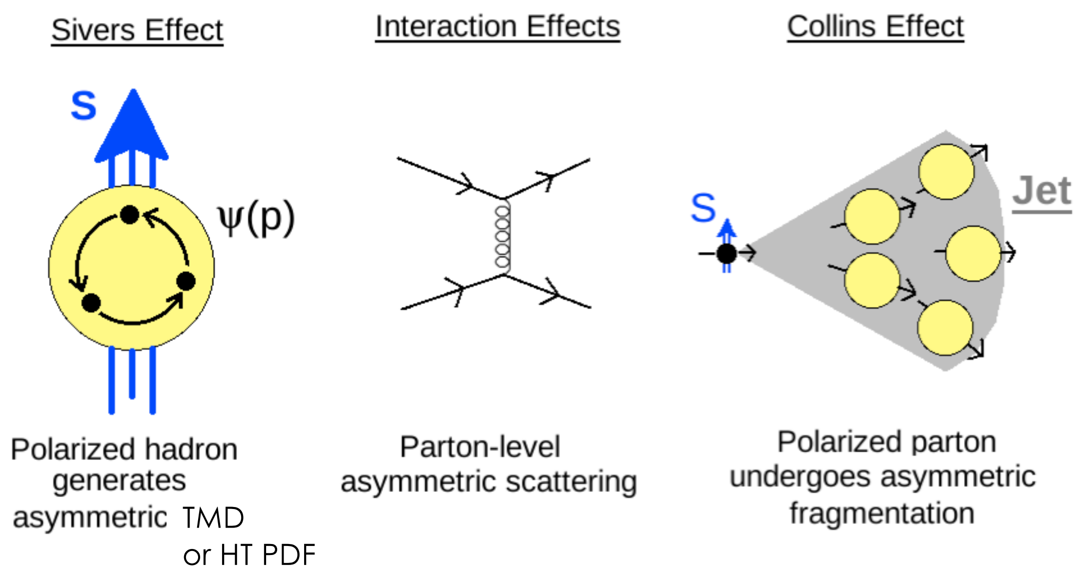
- Single Transverse Spin Asymmetry (STSA) is defined by

$$A_N(\mathbf{k}) \equiv \frac{\frac{d\sigma^\uparrow}{d^2k\,dy} - \frac{d\sigma^\downarrow}{d^2k\,dy}}{\frac{d\sigma^\uparrow}{d^2k\,dy} + \frac{d\sigma^\downarrow}{d^2k\,dy}} = \frac{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) - \frac{\mathbf{d\sigma^\uparrow}}{\mathbf{d^2k\,dy}}(-\mathbf{k})}{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) + \frac{\mathbf{d\sigma^\uparrow}}{\mathbf{d^2k\,dy}}(-\mathbf{k})} \equiv \frac{d(\Delta\sigma)}{2\,d\sigma_{unp}}$$

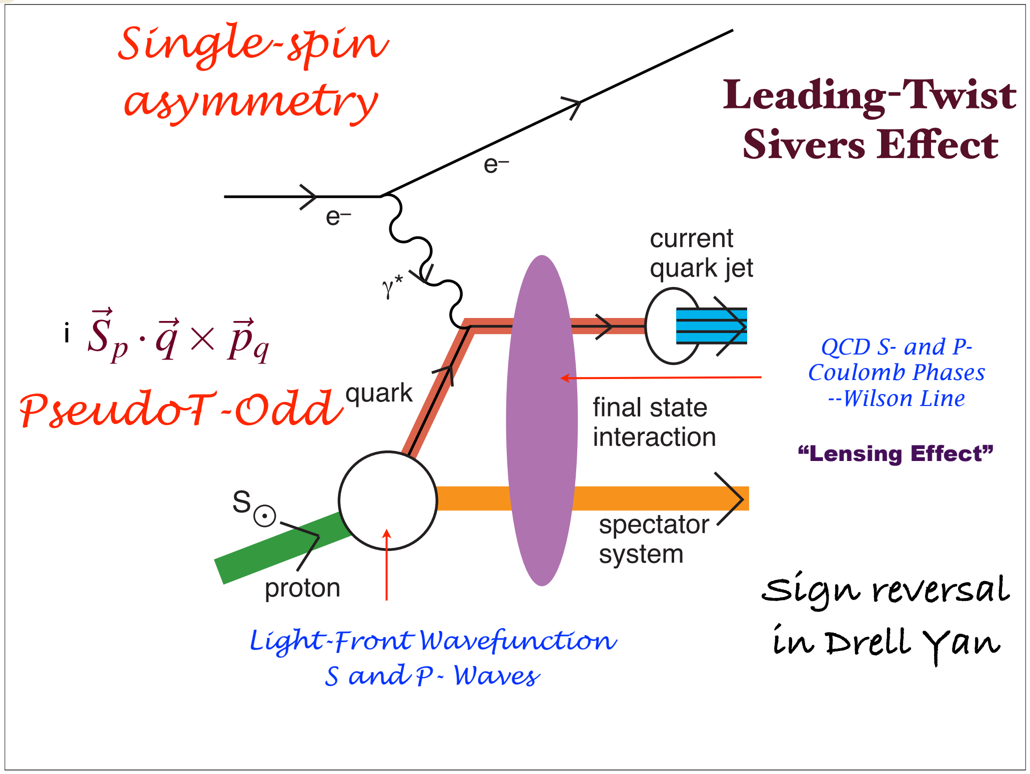
Theoretical Explanations
(TMD of HT collinear factorization framework)

The origin of STSA is in

- Transverse spin-dependent TMD (Sivers function) or higher-twist PDF (ETQS function)
- Transverse spin-dependent fragmentation (Collins effect)
- Hard scattering



STSA IN SIDIS



Brodsky, Hwang, Schmidt '02; Collins '02

- To generate STSA need a final state interaction (the blob above) -- lensing.
- In TMD factorization this is usually absorbed into the polarized proton TMD and is referred to as the initial-state effect, and hence identified with **the Sivers function**.

x-Dependence of A_N

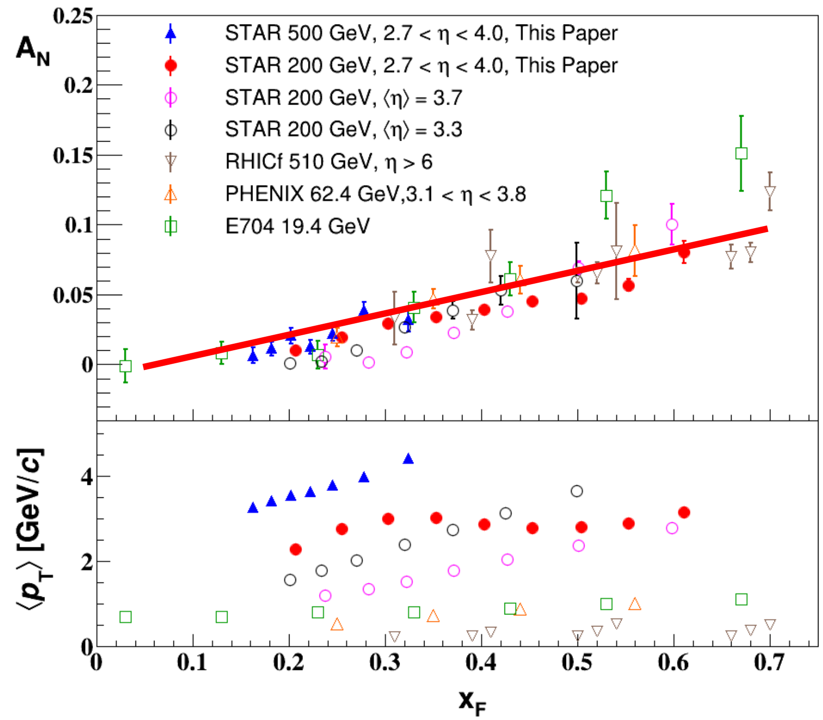
$$A_N \sim x\, f_{1T}^{\perp}$$

Sivers function at small x scales as

$$f_{1T}^{\perp NS}(x \ll 1, k_T^2) = C_O(x, k_T^2)\,\frac{1}{x} + C_1(x, k_T^2)\left(\frac{1}{x}\right)^{3.4\sqrt{\frac{\alpha_s N_c}{4\pi}}}$$

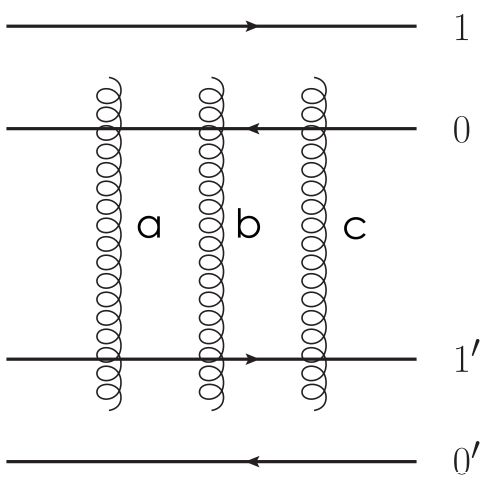
- The spin-dependent odderon (D. Boer, M. G. Echevarria, P. Mulders and J. Zhou '15) gives the first term above. It predicts A_N=const(x).
- The second term is due to sub-eikonal small-x evolution (YK, M.G. Santiago, '22), with the power of x also close to 1.
- The data indicates more like A_N~x, albeit at not very small x.
- **Can the EIC help find the spin-dependent odderon? test the sub-eikonal x-dependence?**

π^0's, STAR, arXiv:2012.11428 [hep-ex]



Odderon as a 3-gluon exchange

- In perturbation theory, the *C*-odd exchange can be due to
  - 1-gluon exchange: yes, it is *C*-odd, but not color-singlet, cannot give an elastic amplitude.
  - 2-gluon exchange: can be color-singlet, but not *C*-odd. (Each gluon has *C*=-1.)
  - 3 gluon exchange: can be both color-singlet and *C*-odd! That's the Odderon at the lowest order.



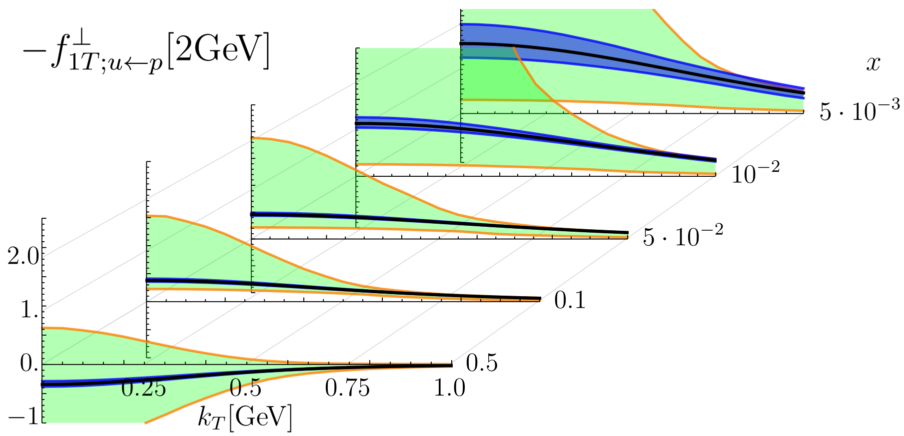
Note that the gluons must be in a symmetric d^{abc} color state ($d^{abc} = 2\,\mathrm{tr}[t^a\,\{t^b, t^c\}]$). If the color group was SU(2), there would be no Odderon.

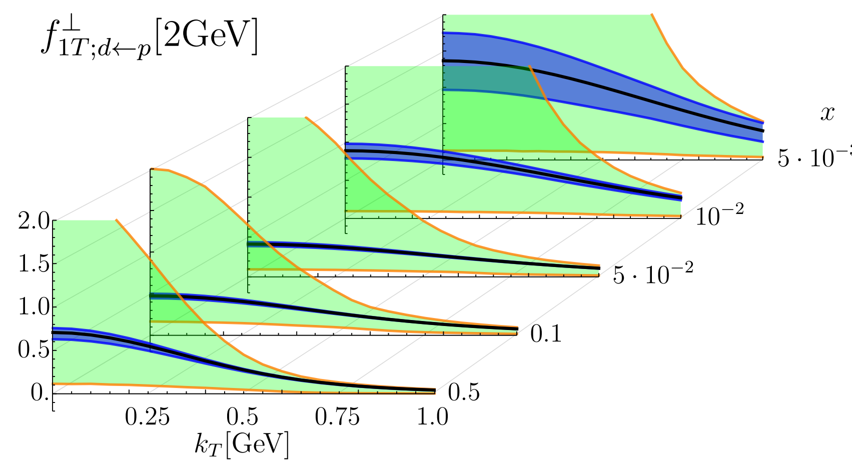
Disconnected gluon lines imply sum over all possible gluon connections to the quark and anti-quark lines.

TOTEM and D0 Collaborations announced the odderon discovery in pp vs. ppbar elastic scattering in late 2020. Results from other experiments are needed to seal the discovery.

Sivers function at EIC

- The Sivers function can be measured at EIC in the SIDIS process (YR plots below by A. Vladimirov):





- It would help understand the small-x asymptotics of this function.
- Spin-dependent odderon search: a dedicated study is needed.

Transversity

Transversity

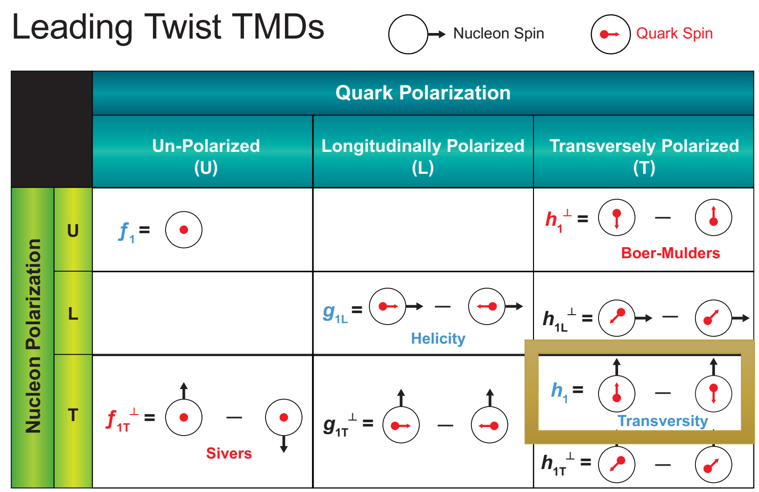
- Another fundamental object, but C-odd, hence hard to measure.
- Related to the proton's tensor charge.

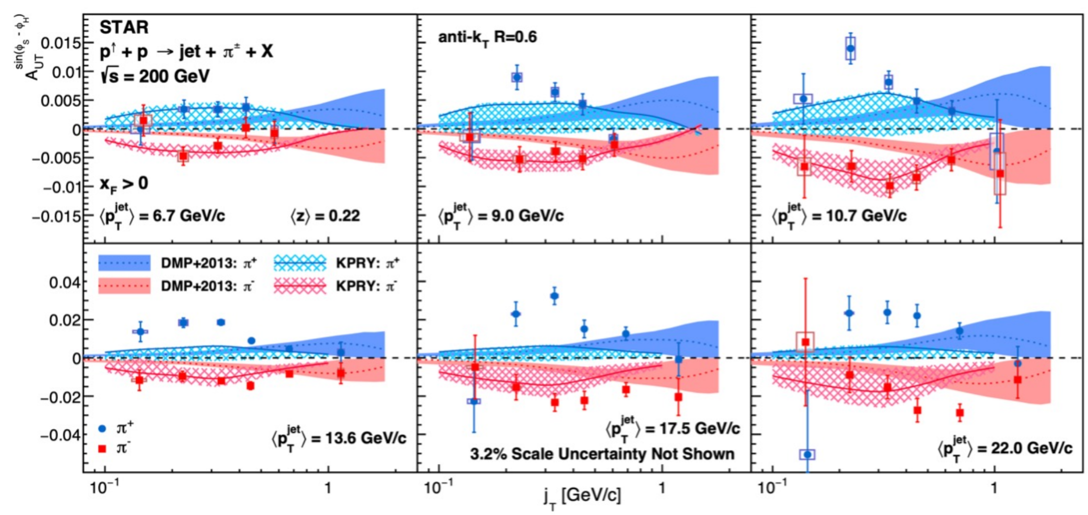
$$\delta q(Q^2) = \int_0^1 dx\, h_1(x, Q^2)$$

- Transversity can be extracted from A_{UT} at RHIC due to the Collins effect,

  $A_{UT} \sim$ transversity $\times$ interference fragmentation function.

- See pioneering work by M. Radici and A. Bacchetta, '18.
- EIC would certainly help narrow down the error bars.





STAR, 2205.11800 [hep-ex]

Small-x Asymptotics of Quark Transversity

- The small-x asymptotics of transversity is given by (R. Kirschner et al, '96; YK, M. Sievert, '18)

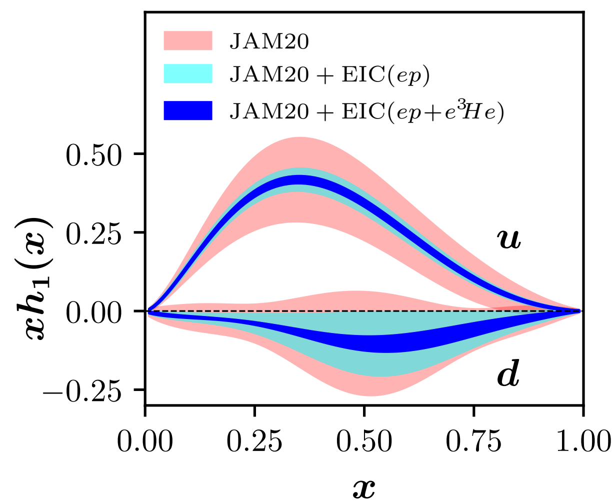
$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s C_F}{\pi}}$$

- For $\alpha_s = 0.3$ we get

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp NS}(x, k_T^2) \sim x^{0.243}$$

This result agrees with extractions from data (see J. Benel, A. Courtoy, and R. Ferro-Hernandez, '19).

It can be further tested at EIC (YR plot by JAM Collaboration):



Conclusions

- RHIC Spin produced important measurements of A_{LL} leading to the first-ever non-zero ΔG extraction. This is a major step towards solving the proton spin puzzle.
- **A large portion of the missing spin of the proton may be at small x**, with theory and phenomenology being developed. May solve the spin puzzle. **EIC will help resolve the issue!**
- **x-Dependence of transversity and Sivers function** has been investigated theoretically and **can be tested experimentally at EIC**.

EIC UG SC Recommendation: We recommend expeditious completion of the EIC as the highest priority for facility construction.

The EIC is a powerful and versatile new accelerator facility, capable of colliding high-energy beams ranging from heavy ions to polarized light ions and protons with high-energy polarized electron beams. In the 2015 Long Range Plan the EIC was put forward as the highest priority for new facility construction and the expeditious completion remains a top priority for the nuclear physics community. The EIC, accompanied by a general-purpose large-acceptance detector, ePIC, will be a discovery machine that addresses fundamental questions such as the origin of mass and spin of the proton as well as probing dense gluon systems in nuclei. It will allow for the exploration of new landscapes in QCD, permitting the “tomography”, or high-resolution multidimensional mapping of the quark and gluon components inside of nucleons and nuclei. Realizing the EIC will keep the U.S. on the frontiers of nuclear physics and accelerator science and technology.

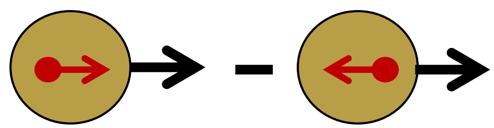
CFNS workshop: EIC Theory in the next decade, Sep. 20-22, 2022, MIT

- Organizers: Ian Cloët (ANL), Dmitri Kharzeev (Stony Brook University/BNL), Xiangdong Ji (University of Maryland), Peter Petreczky (BNL), Jianwei Qiu (JLab), Phiala Shanahan (MIT), Iain Stewart (MIT), Ivan Vitev (LANL), Feng Yuan (LBNL)
- #participants: 65, https://indico.bnl.gov/event/16740/
- Goal: review the needs and challenges for EIC theory including interdisciplinary aspects, 15 talks (5 by young faculty), 7 open mic discussion sessions
- Resolution: "**We recommend the establishment of a national EIC theory alliance to enhance and broaden the theory community needed to advance EIC physics goals and the experimental program. This theory alliance will develop a diverse workforce through a competitive national EIC theory fellow program and tenure-track bridge positions, including appointments at minority serving institutions.**"

Backup Slides

Helicity Distributions

- To quantify the contributions of quarks and gluons to the proton spin one defines helicity distribution functions: number of quarks/gluons with spin parallel to the proton momentum minus the number of quarks/gluons with the spin opposite to the proton momentum:



- The helicity parton distributions are

$$\Delta f(x,Q^2) \equiv f^+(x,Q^2) - f^-(x,Q^2)$$

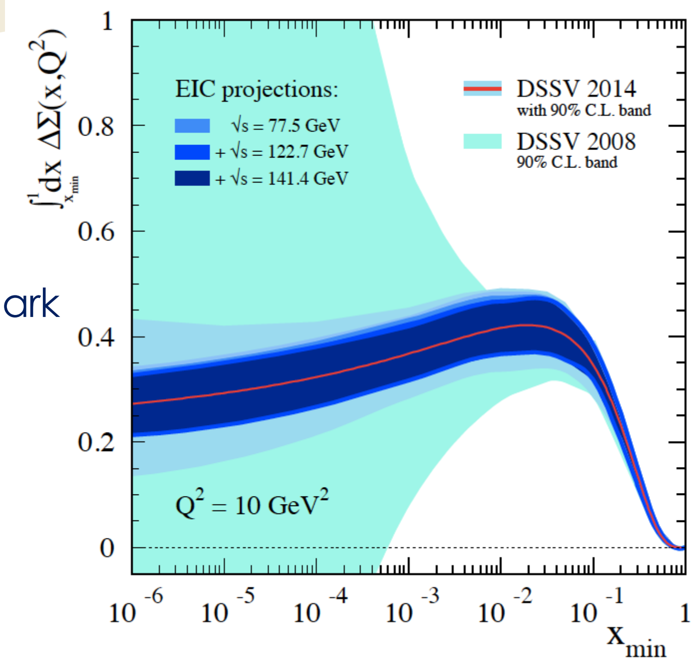
with the net quark helicity distribution

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

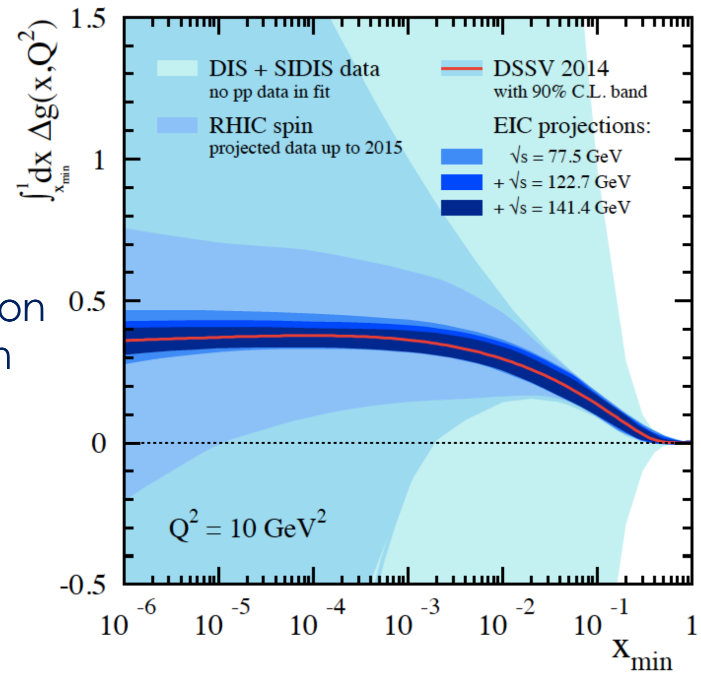
and $\Delta G(x,Q^2)$ the gluon helicity distribution.

How much spin is there at small x?

2 x (quark spin)



gluon spin



- E. Aschenauer et al, arXiv:1509.06489 **[hep-ph]**, (DSSV = de Florian, Sassot, Stratmann, Vogelsang, DGLAP-based helicity PDF extraction from data)
- Uncertainties are very large at small x! (EIC may reduce them.)

Large N_c

- KPS-CTT: At large-N_c the equations close (Q → G). Everything with 2 in the subscript (e.g., G_2 and Γ_2) is new compared to KPS papers.

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[\Gamma(x_{10}^2, x_{21}^2, z's) + 3\, G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z's) \Big],$$

KPS

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3\, G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z''s) \Big],$$

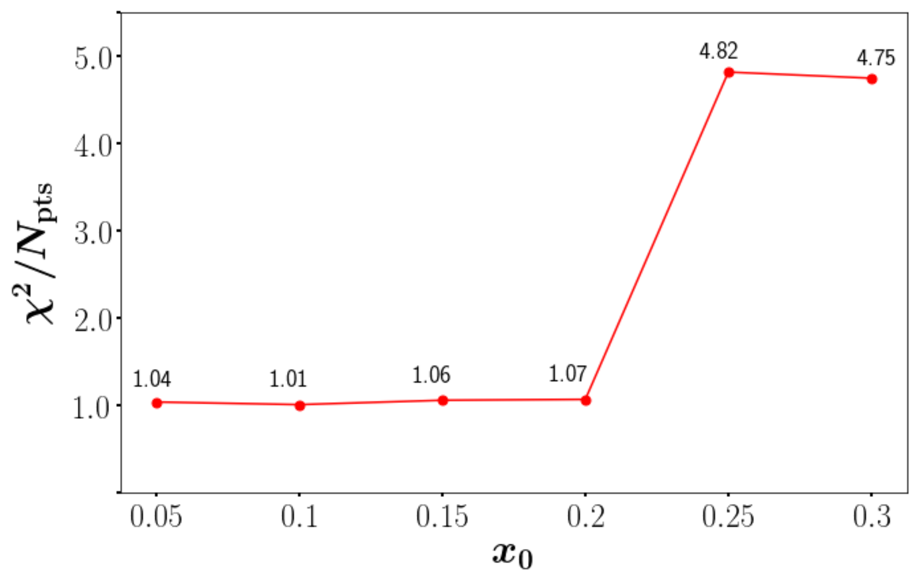
KPS

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) \right],$$

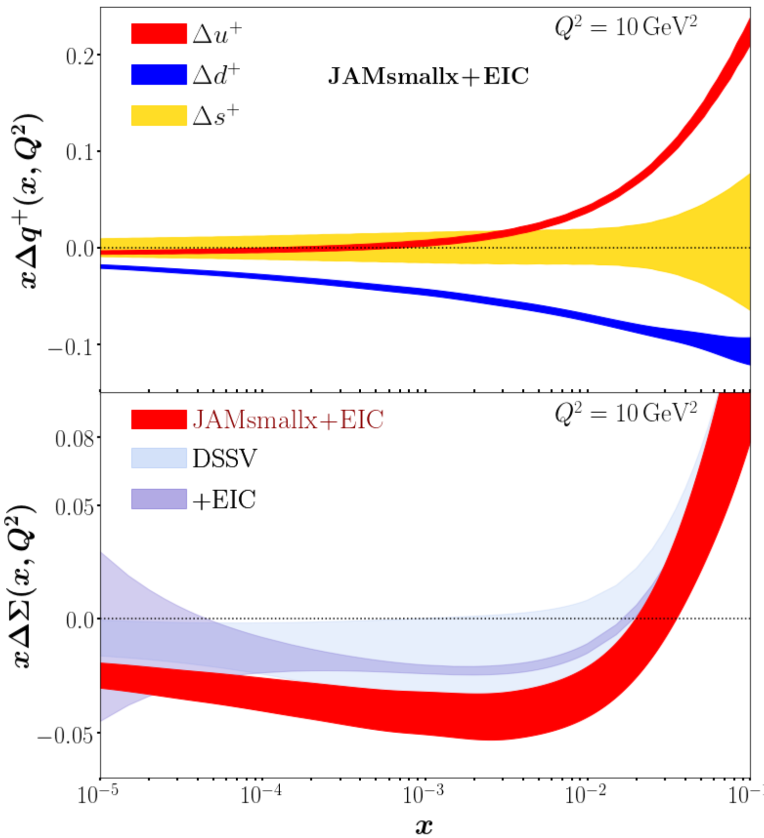
$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) \right]$$

Where to start small-x evolution

- The evolution starts at $x=x_0$, and continues toward smaller x.
- The quality of our fit rapidly deteriorates for $x_0>0.2$, as expected from a small-x approach.
- In unpolarized BK/JIMWLK evolution, typically $x_0=0.01$, so the fact that our fit works up to such a high x_0 is quite remarkable.



Predictions for helicity PDFs



D. Adamiak, W. Melnitchouk,
D. Pitonyak, N. Sato, M. Sievert & YK,
2102.06159 [hep-ph], in the
JAM Collaboration framework.

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

$$\Delta\Sigma(x,Q^2) = \sum_f \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

Our (red) error band does not explode in the unmeasured region. We can **predict** spin at small x.

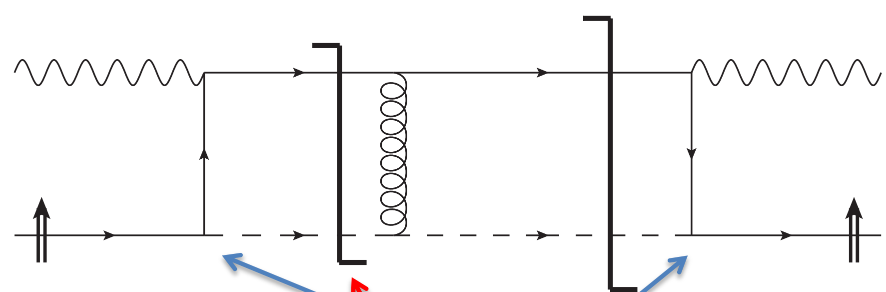
If we plug in $\alpha_S = 0.25$
we get $\alpha_h^q = 0.80$

old intercept!

$$\Delta q(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

STSA in SIDIS

- STSA arises from the interference diagrams between Born-level and the one-rescattering graphs:



- Spin-dependence comes from the vertex. This brings in an "i". Need another "i" to get a real cross-section. Hence, need a phase.

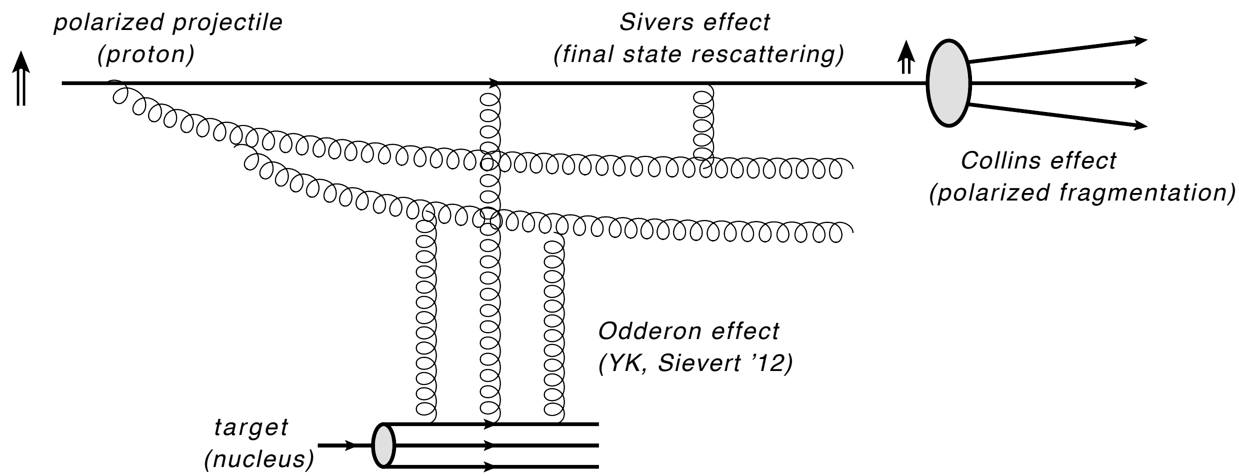
- The phase is generated by an extra rescattering, which gives the amplitude an Im part represented by the second "cut".

Brodsky, Hwang, Schmidt '02;
(see also Brodsky, Hwang, YK, Schmidt, Sievert '13)

Possible Origins of A_N

The single transverse spin asymmetry in p↑+p or p↑+A can be generated via the following mechanisms:

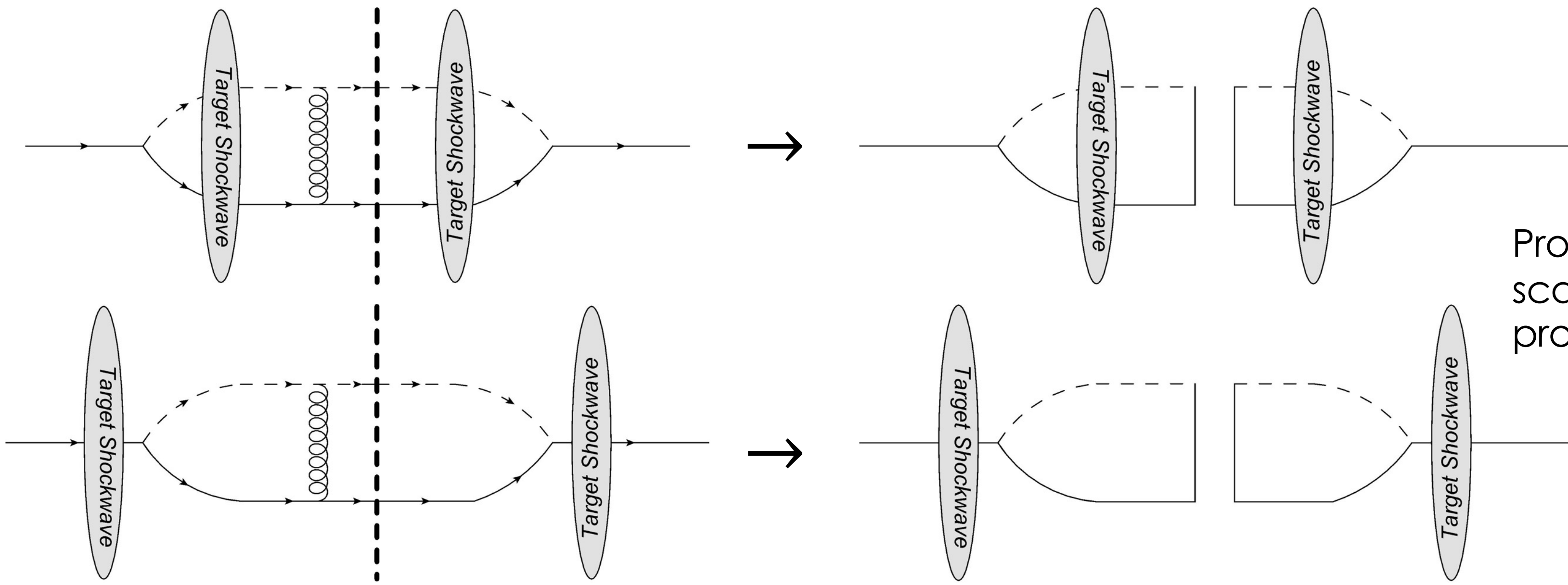
- Sivers effect – multiple rescattering in p↑+p – This work.
- Collins effect – fragmentation of a polarized quark
- Odderon – multiple rescattering in the unpolarized target



Large-N_c Limit → Elastic Dominance

M.G. Santiago, YK, '20

- At large-N_c the gluon line becomes a double quark line



Product of elastic scattering processes!

$$3 \otimes \bar{3} = 1 \oplus 8 \quad \Rightarrow \quad N_c \otimes \bar{N}_c = 1 \oplus (N_c^2 - 1) \approx N_c^2 - 1$$