

Theory overview of dense QCD matter

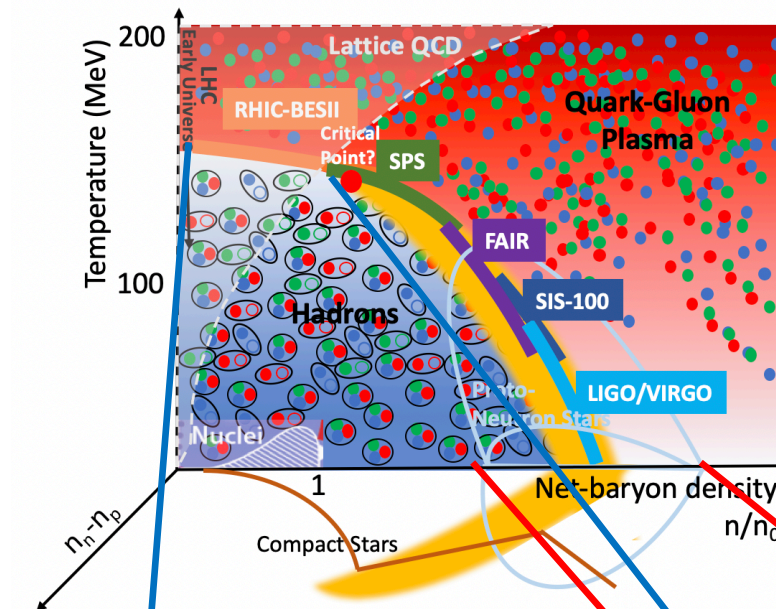
CLAUDIA RATTI

UNIVERSITY OF HOUSTON

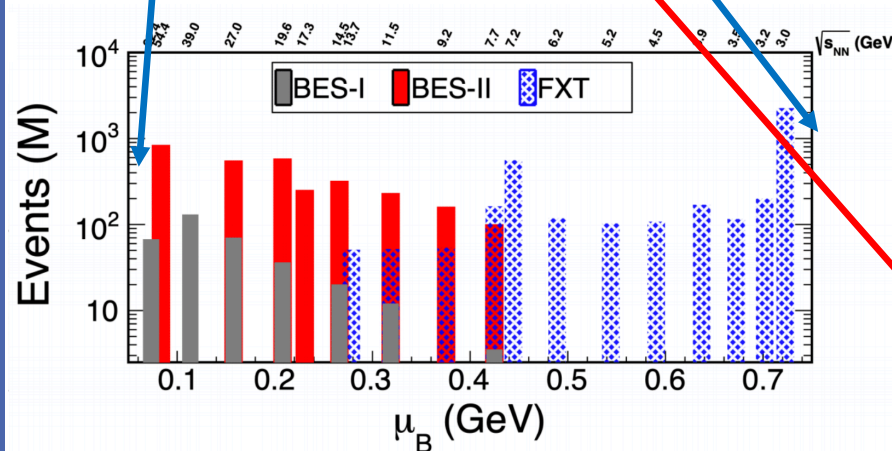


Motivating science goals

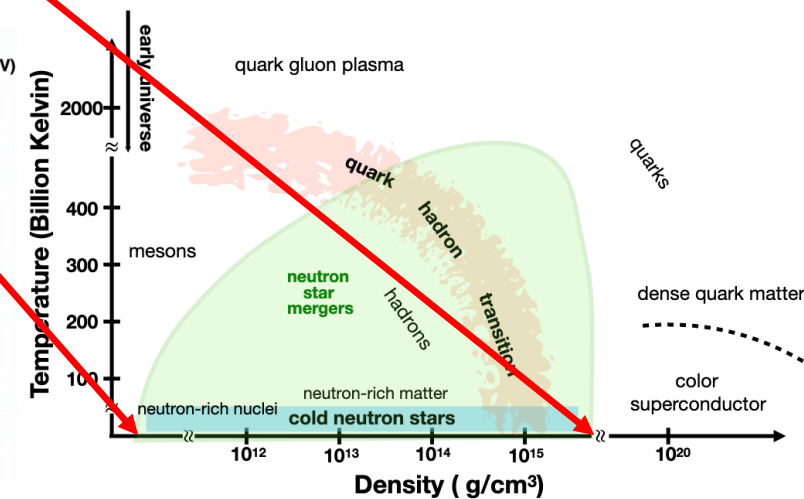
- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- Where is the transition line at high density?
- What are the phases of QCD at high density?
- Are we creating a thermal medium in experiments?



- Run 2019:
 - Collider: $v_{NN}=14.6, 19.6, 200$ GeV
 - Fixed target: $v_{NN}=3.2$ GeV
- Run 2020:
 - Collider: $v_{NN}=9.2, 11.5$ GeV
 - Fixed target: $v_{NN}=3.5, 3.9, 4.5, 5.2, 6.2, 7.2, 7.7$ GeV
- Run 2021:
 - Collider: $v_{NN}=7.7, 17.3$ GeV
 - Fixed target: $v_{NN}=3.0, 9.2, 11.5, 13.7$ GeV



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Comparison of the facilities

Compilation by D. Cebra

CP=Critical Point

OD= Onset of
Deconfinement

DHM=Dense
Hadronic Matter

Facility	RHIC BESII	SPS	SIS-100 SIS-300	J-PARC HI
Exp.:	STAR +FXT	NA61	CBM	JHITS
Start:	2019-2021	2009	2025	2025
Energy:	7.7– 19.6	4.9-17.3	2.7-8.2	2.0-6.2
$\sqrt{s_{NN}}$ (GeV)	2.5-7.7			
Rate:	100 HZ	100 HZ	<10 MHZ	100 MHZ
At 8 GeV	2000 Hz			
Physics:	CP&OD	CP&OD	OD&DHM	OD&DHM

Collider
Fixed target

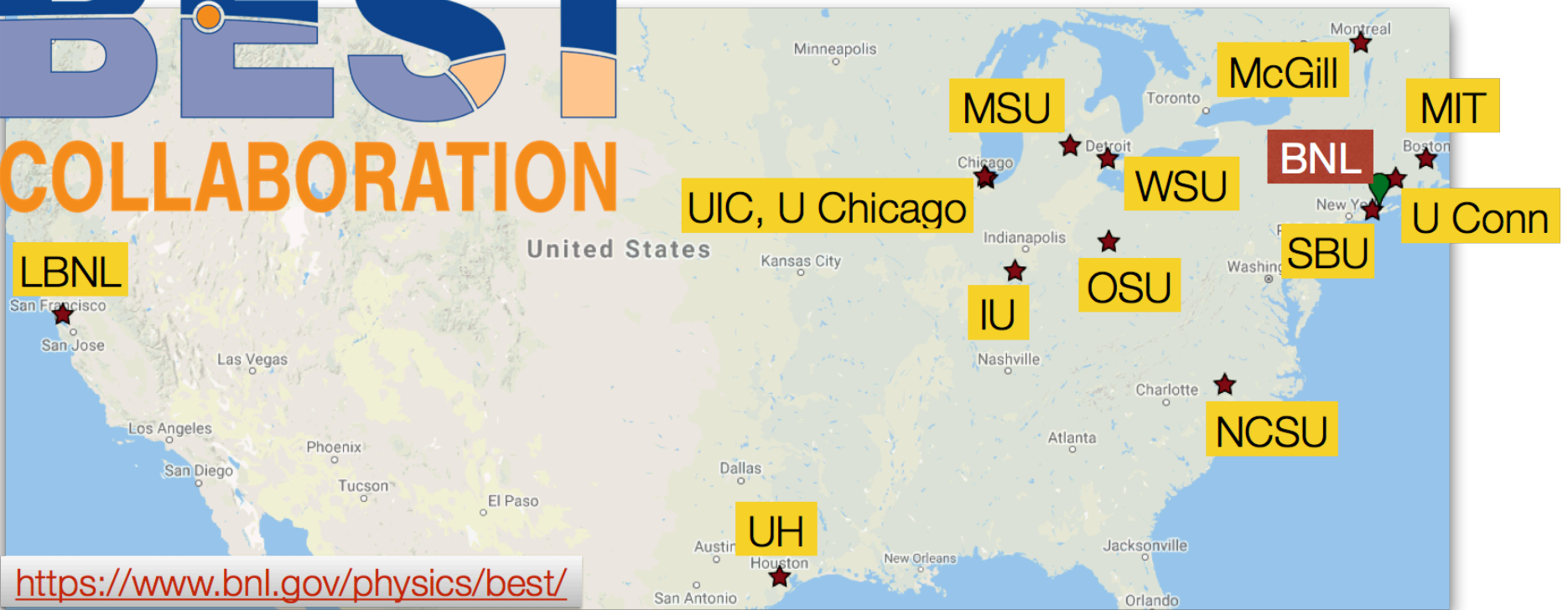
Fixed target
Lighter ion
collisions

Fixed target

Fixed target



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objectives:

- constraints on the existence of a critical point in the QCD phase diagram
- properties of baryon-rich QGP
- probe chiral symmetry restoration through chiral anomaly induced phenomena

path:

- construct a theoretical framework for interpreting the results from the BES @ RHIC



Hot and dense lattice QCD

BNL, UH

Major goals:

- QCD crossover temperature $T_c(\mu_B)$
 - switching temperature/energy density for fluid-dynamical modeling
- QCD equation of state (EoS) for $\mu_B > 0$
 - input for fluid-dynamical modeling & EoS with critical point
- skewness and kurtosis of conserved charge fluctuations for $\mu_B > 0$
 - equilibrium QCD baseline for the experimentally measured higher order cumulants of net proton, electric charge and kaon fluctuation

QCD transition line

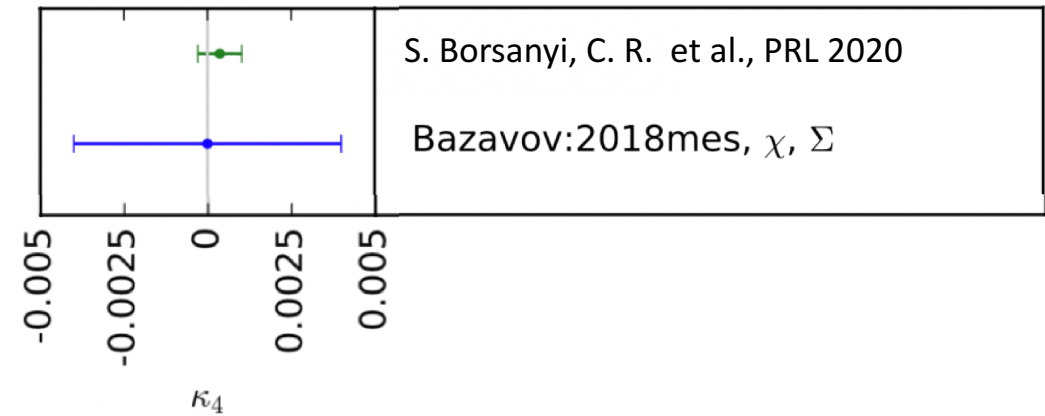
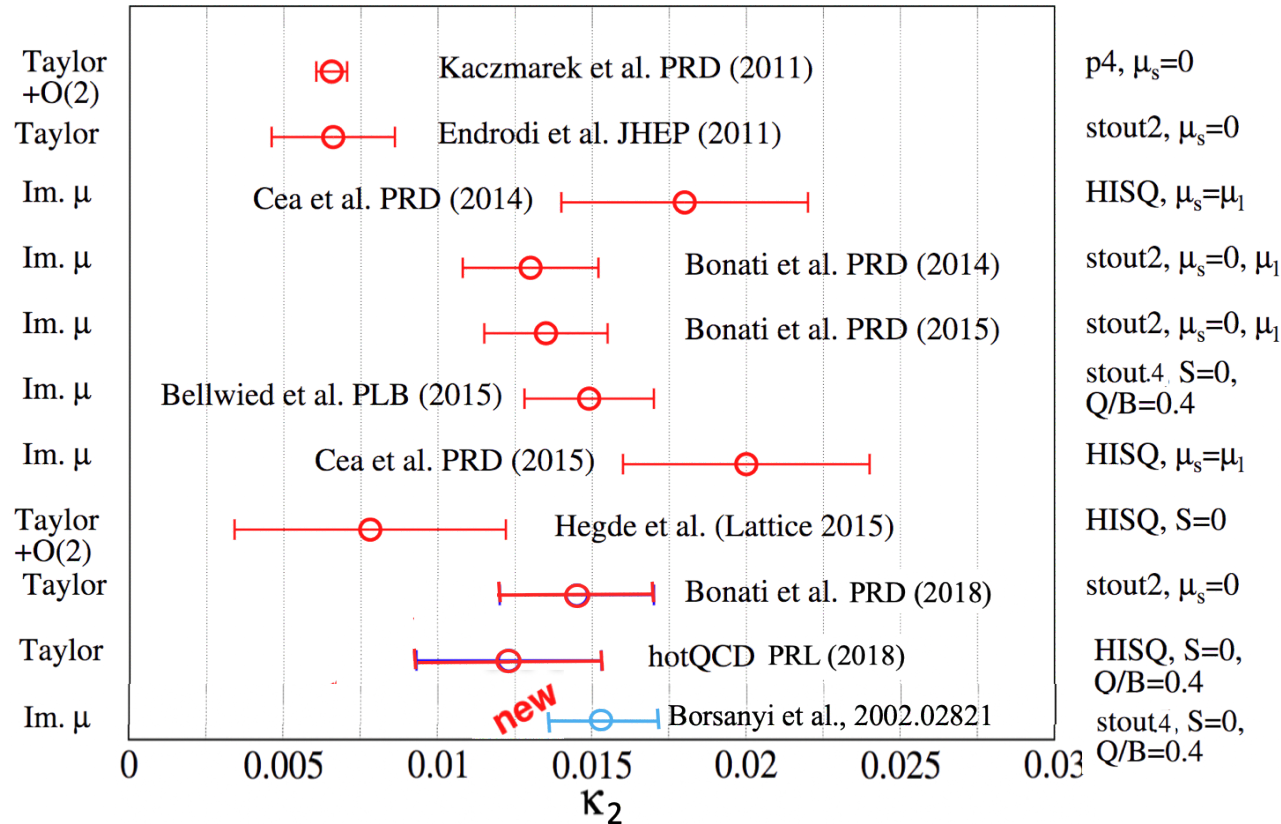
S. Borsanyi, C. R. et al., PRL (2020)

$$\frac{T_c(\mu_B)}{T_c(\mu_B = 0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)} \right)^4$$

$$T_c(LT = 4, \mu_B = 0) = 158.0 \pm 0.6 \text{ MeV}$$

$$\kappa_2 = 0.0153 \pm 0.0018,$$

$$\kappa_4 = 0.00032 \pm 0.00067$$



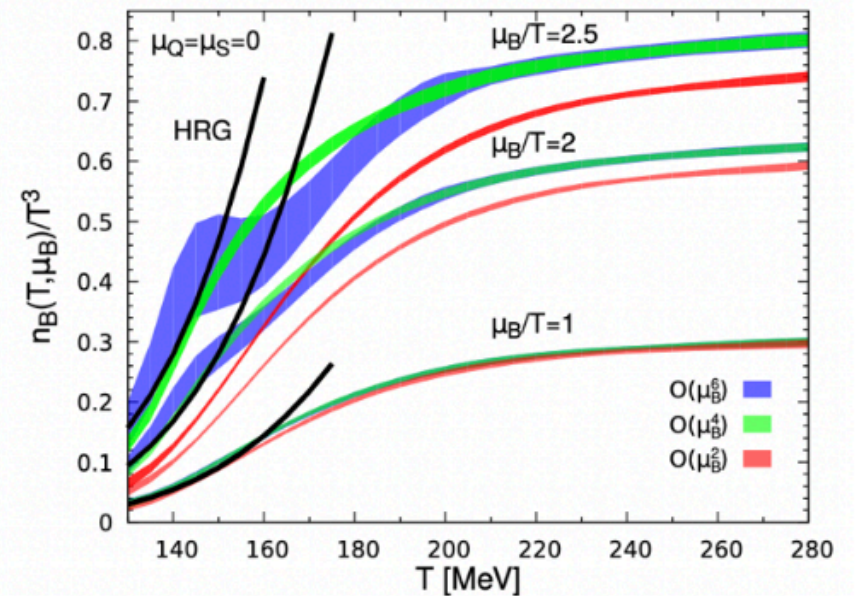
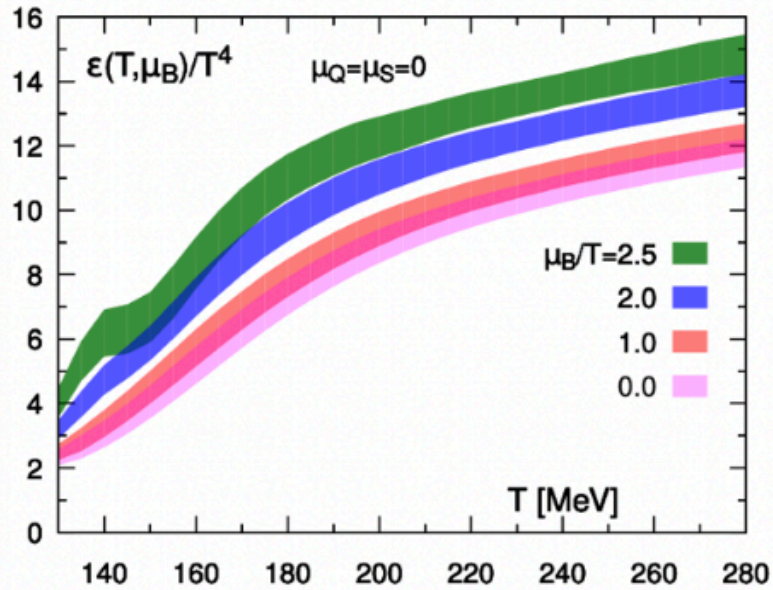
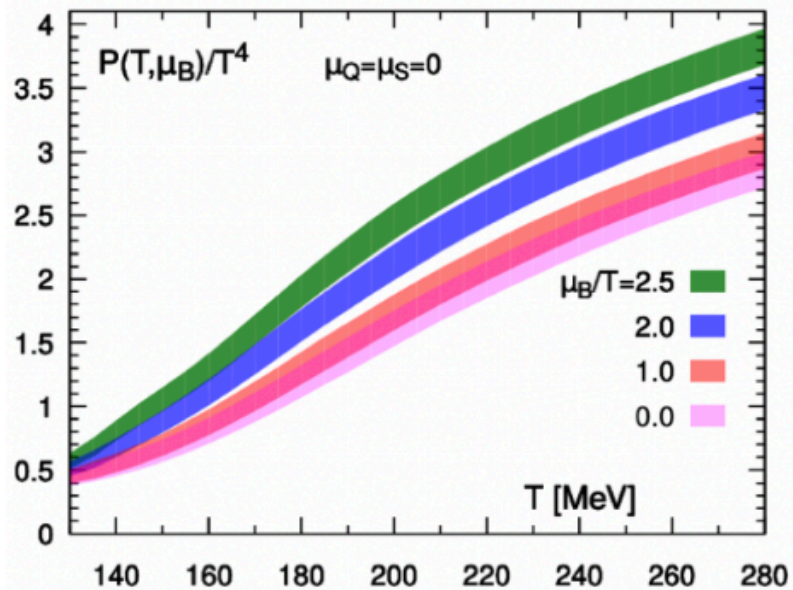
- Currently covers up to $\mu_B < 350$ MeV
- It will be expanded to larger μ_B
- No sign of criticality in the explored range

QCD Equation of state for $\mu_B > 0$

- Taylor expansion of the pressure:

$$\frac{p(T, \mu_B)}{T^4} = \frac{p(T, 0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left. \frac{d^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}} \right|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

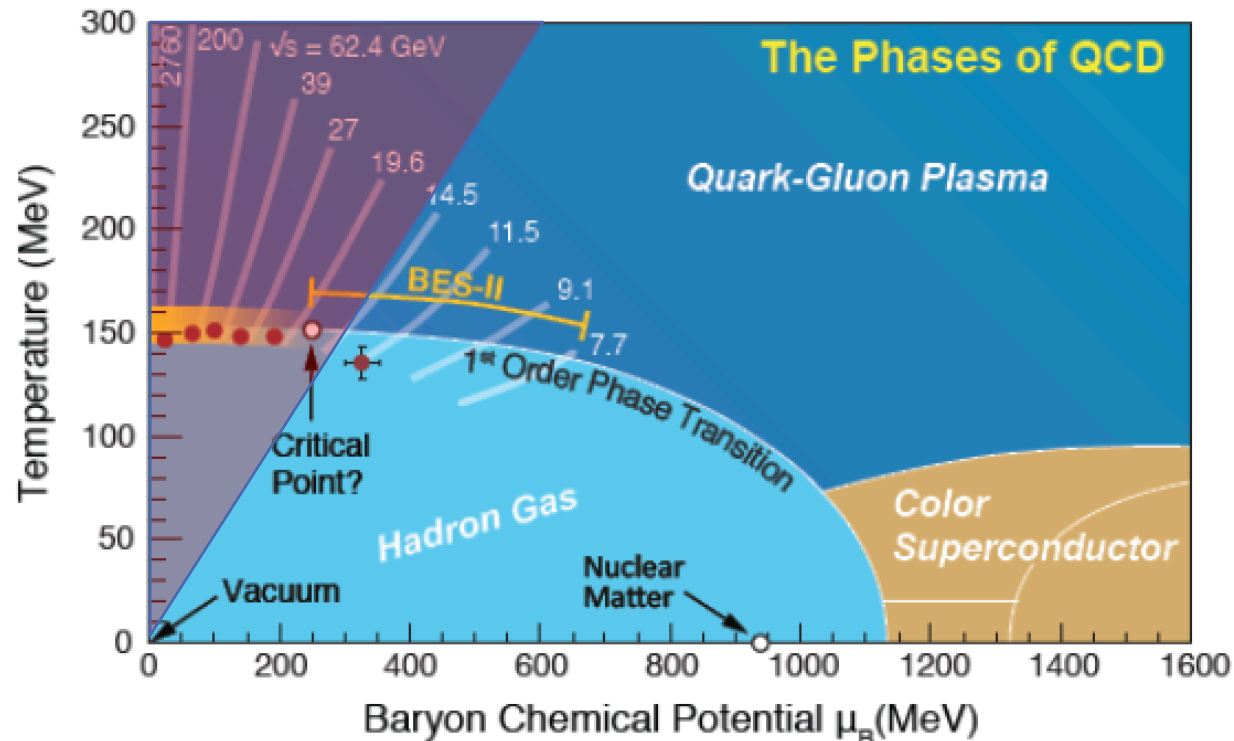
HotQCD: PRD (2017)



Range of validity of Taylor-expanded equation of state

- Taylor-expanded equation of state covers the range $\mu_B/T \leq 2$ or in terms of the RHIC energy scan:

$$\sqrt{s} = 200, 62.4, 39, 27, 19.6, 14.5 \text{ GeV}$$



Novel expansion method

WB: S. Borsanyi, C. R. et al, PRL (2021), PRD (2022)

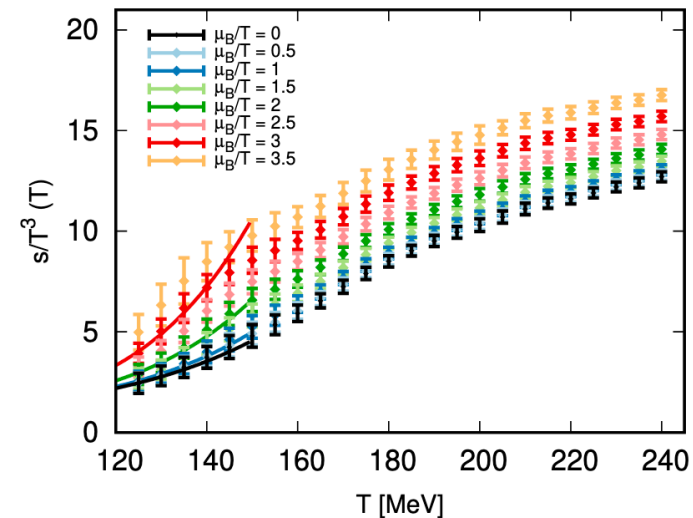
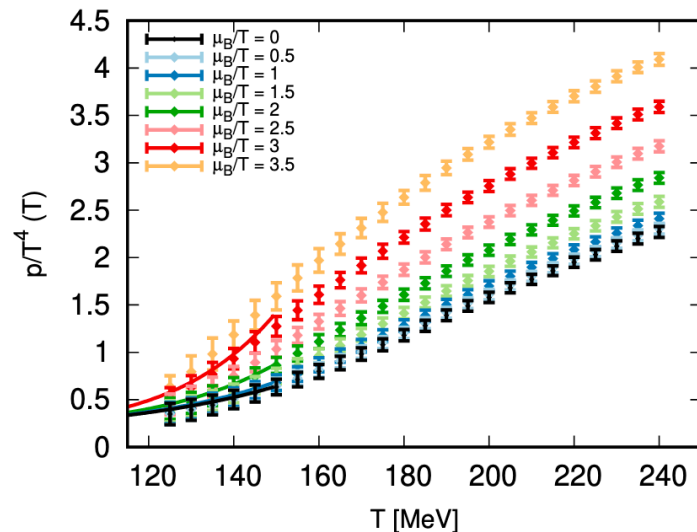
Observation: the temperature-dependence of baryonic density

$$n_B(T)/\bar{\hat{\mu}}_B = \chi_1^B(T, \hat{\mu}_B)/\bar{\hat{\mu}}_B$$

at finite imaginary chemical potential is just a shift in temperature from the $\mu_B = 0$ results for χ_2^B :

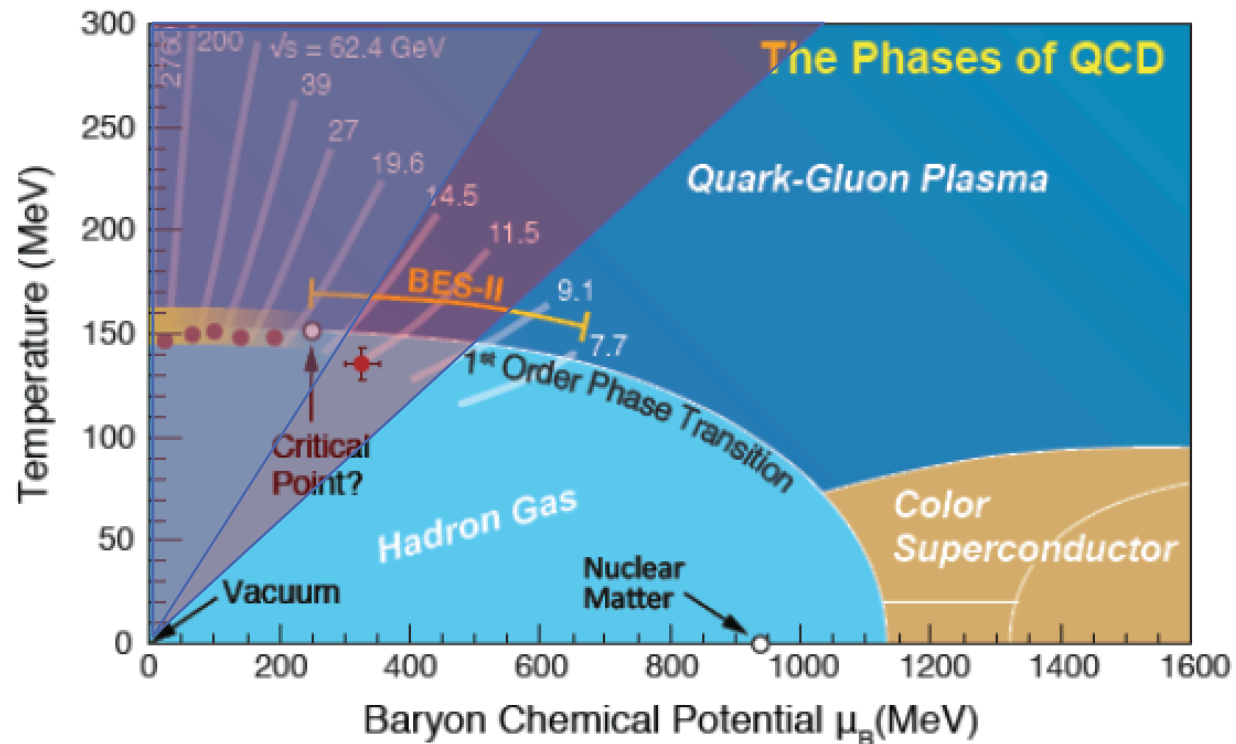
$$\frac{\chi_1^B(T, \hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T', 0),$$

$$T'(T, \hat{\mu}_B) = T \left(1 + \kappa_2^{BB}(T)\hat{\mu}_B^2 + \kappa_4^{BB}(T)\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6) \right)$$



New range of validity of equation of state

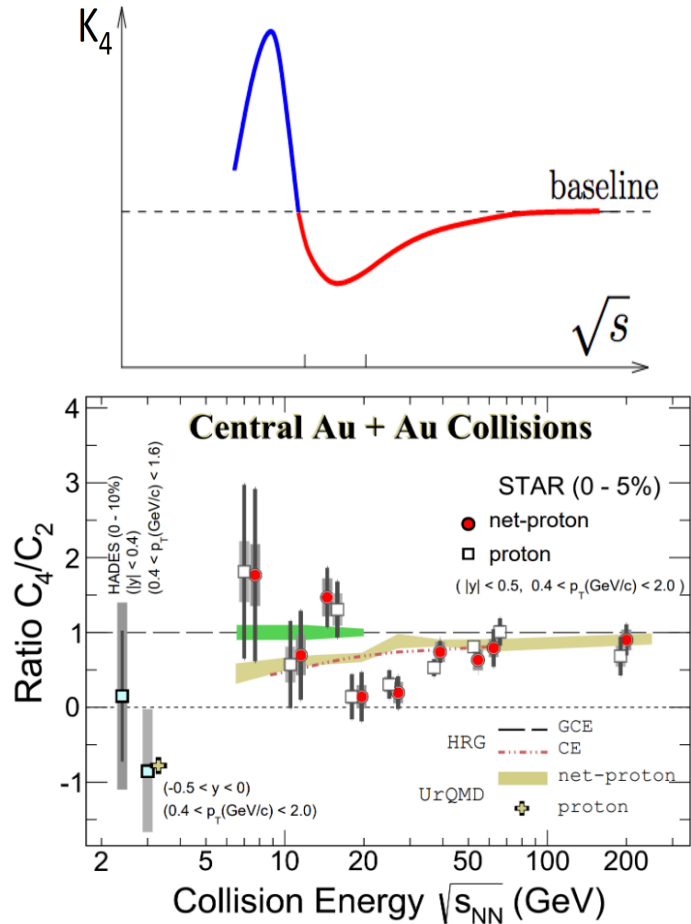
- New expansion scheme provides the equation of state for $\mu_B/T \leq 3.5$



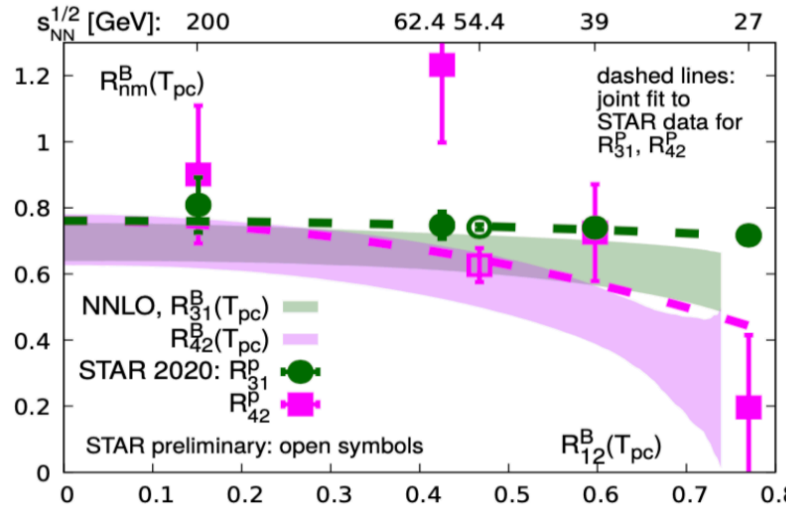
Net-proton fluctuations at the critical point

M. Stephanov, PRL (2009).

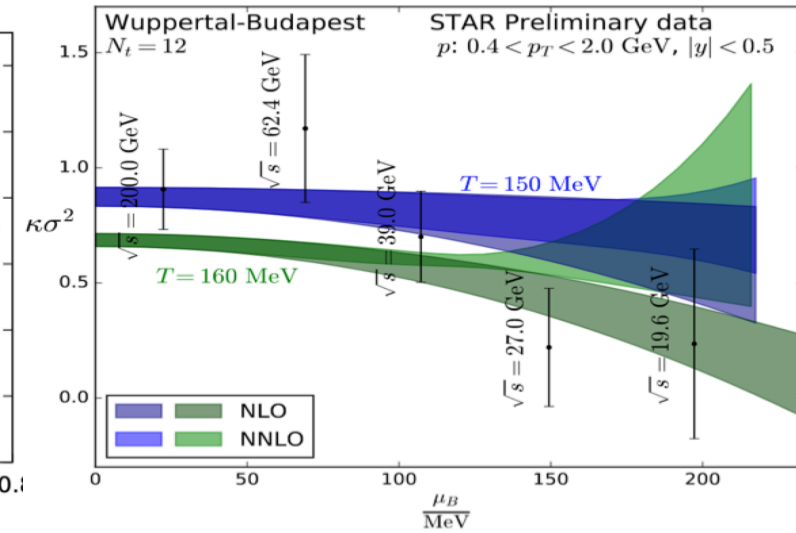
- Fluctuations are expected to diverge at the critical point



HotQCD, PRD (2020)



WB, JHEP (2018)



- Lattice QCD results reproduce the downward trend towards high μ_B



Moving forward

- Push simulations of all observables to larger μ_B
- First simulations at finite (real) μ_B on small lattices
M. Giordano et al., JHEP 05, 088; Borsanyi et al., PRD (2022)
- Alternative ways to resum the Taylor series
S. Mondal et al., PRL (2022); S. Mukherjee et al., PRD (2022); S. Mitra et al., PRD (2022)
- Progress in alternative methods (Langevin, Lefschetz Thimbles)
Aarts et al, 2010; Cristoforetti et al, 2012; Alexandru et al, 2016

Computational Nuclear Physics Workshop Resolution

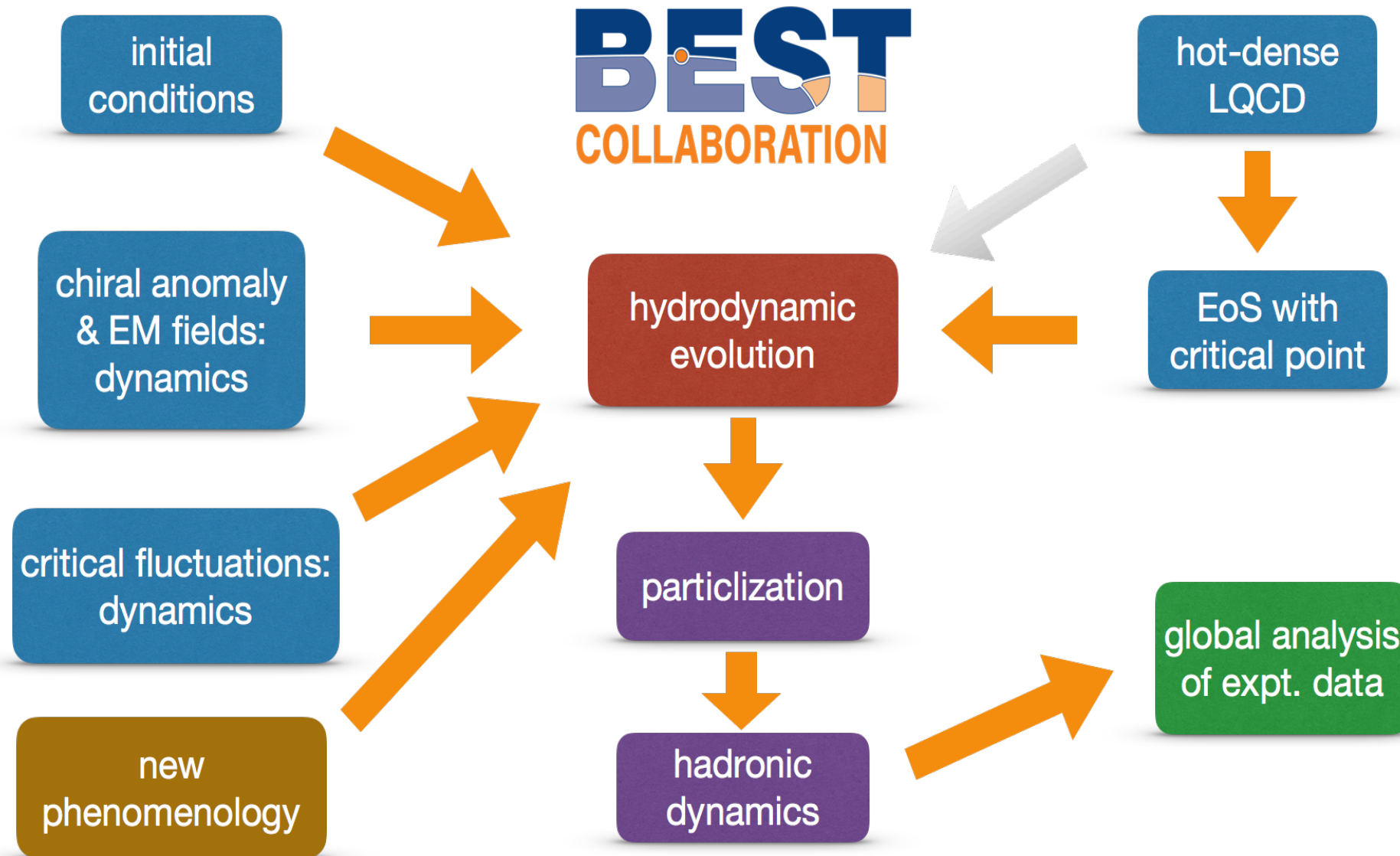
6-7 September 2022 at SURA in Washington, DC

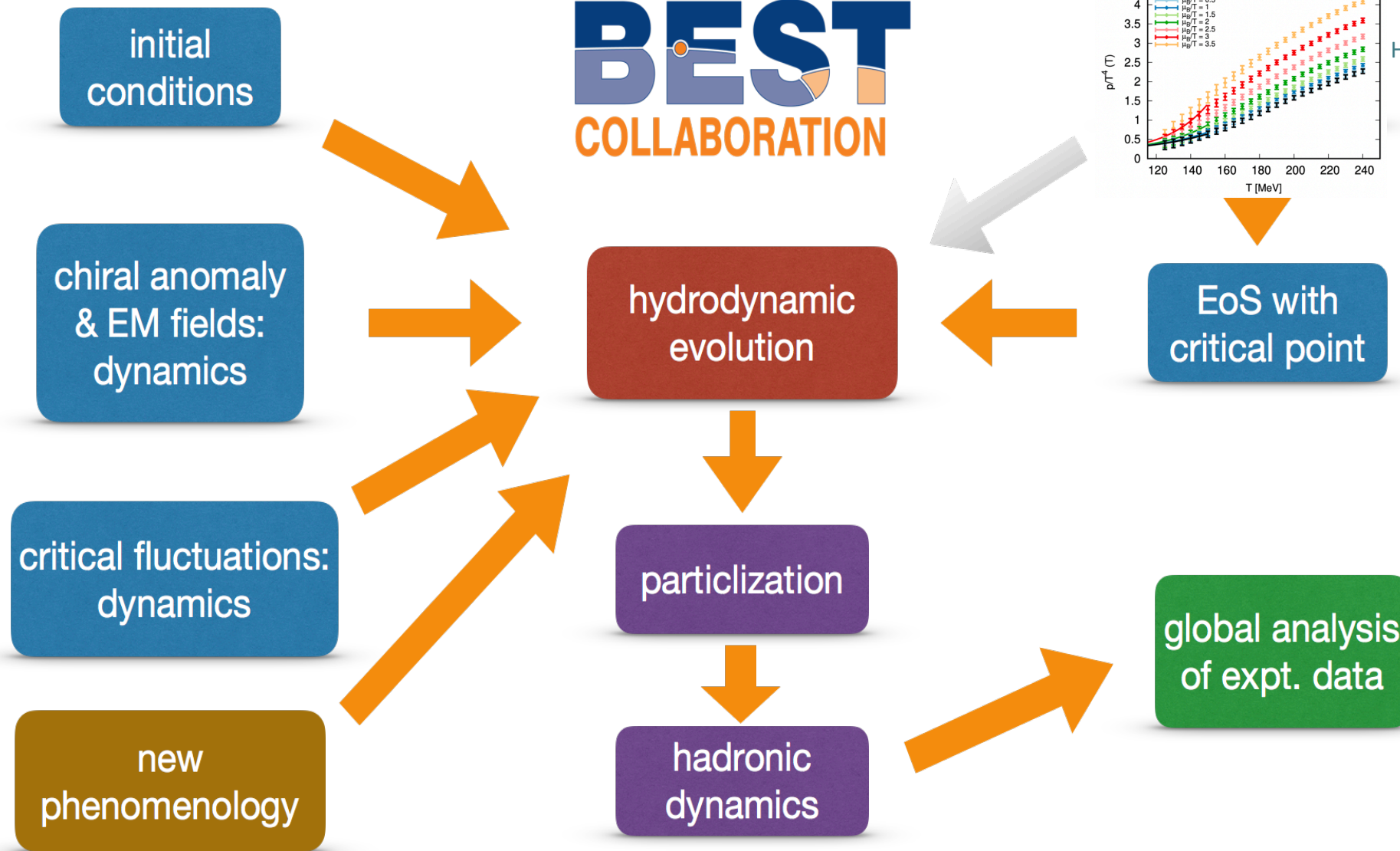
60 registered participants (40 in person, 20 on line), including DOE representation

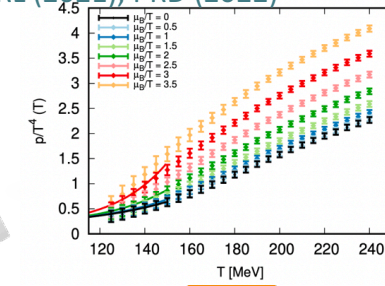
High-performance computing is essential to advance nuclear physics on the experimental and theory frontiers. Increased investments in computational nuclear physics will facilitate discoveries and capitalize on previous progress. Thus, we recommend a targeted program to ensure the utilization of ever-evolving HPC hardware via software and algorithmic development, which includes taking advantage of novel capabilities offered by AI/ML.

The key elements of this program are to:

- 1) Strengthen and expand programs and partnerships to support immediate needs in HPC and AI/ML, and also to target development of emerging technologies, such as quantum computing, and other opportunities.
- 2) Take full advantage of exciting possibilities offered by new hardware and software and AI/ML within the nuclear physics community through educational and training activities.
- 3) Establish programs to support cutting-edge developments of a multi-disciplinary workforce and cross-disciplinary collaborations in high-performance computing and AI/ML.
- 4) Expand access to computational hardware through dedicated and high-performance computing resources.







HotQCD: PRD (2017)

initial conditions

chiral anomaly & EM fields: dynamics

critical fluctuations: dynamics

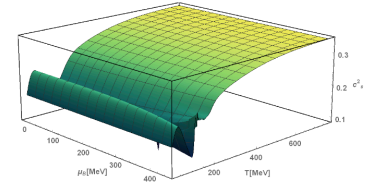
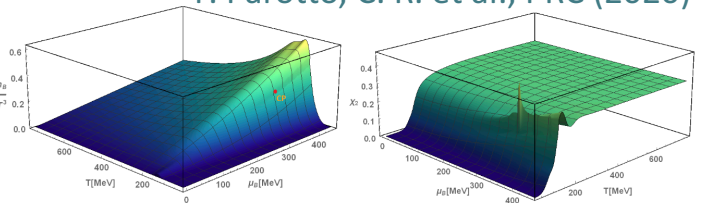
new phenomenology

hydrodynamic evolution

particlization

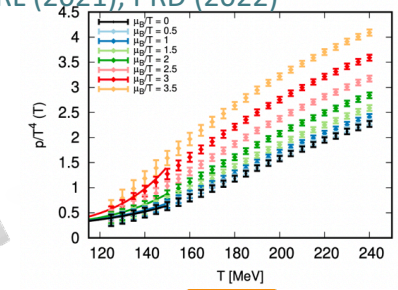
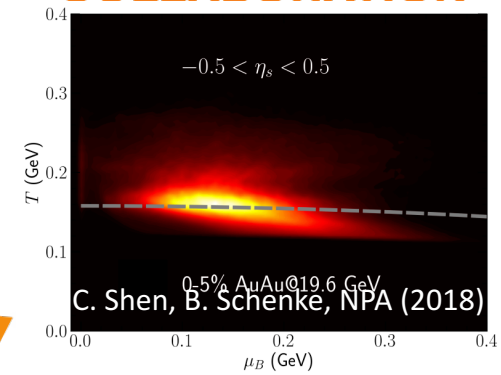
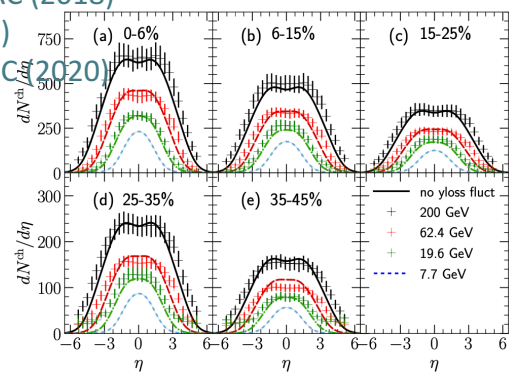
hadronic dynamics

P. Parotto, C. R. et al., PRC (2020)



global analysis of expt. data





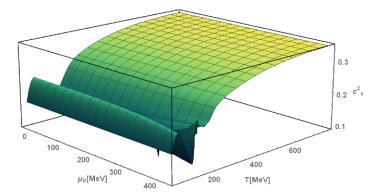
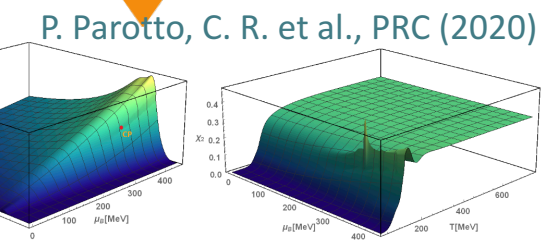
chiral anomaly
 & EM fields:
 dynamics

critical fluctuations:
 dynamics

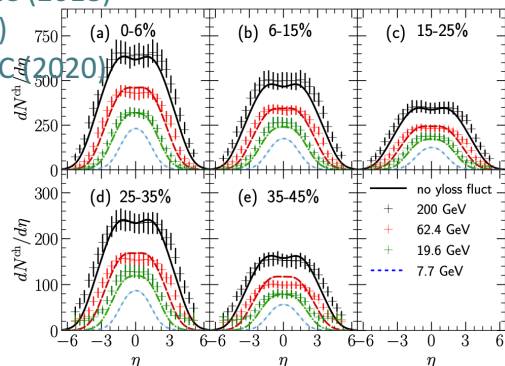
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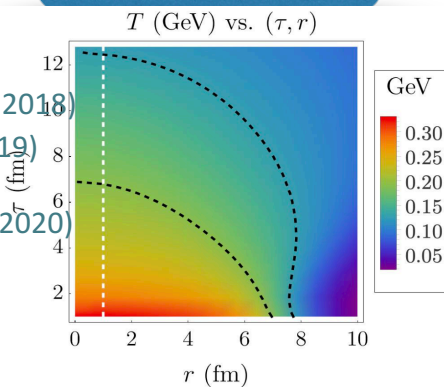
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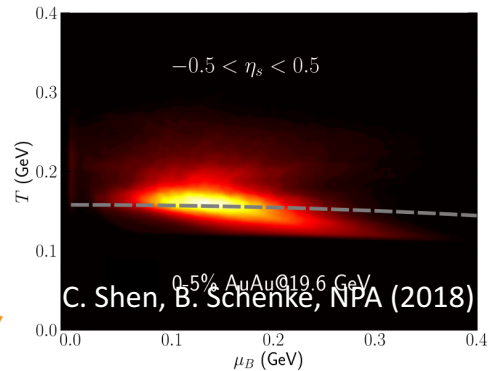


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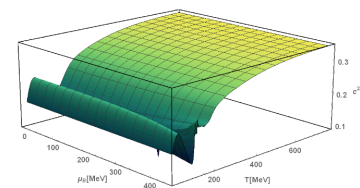
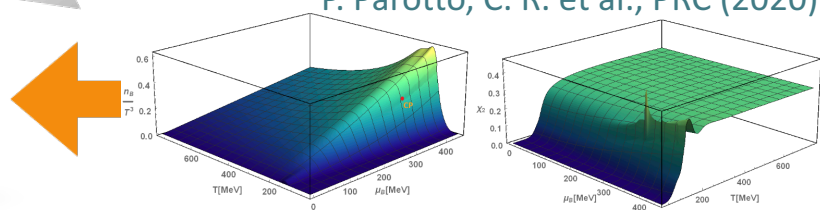
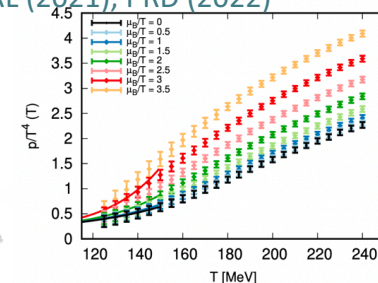
new
phenomenology

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particlization

hadronic
dynamics



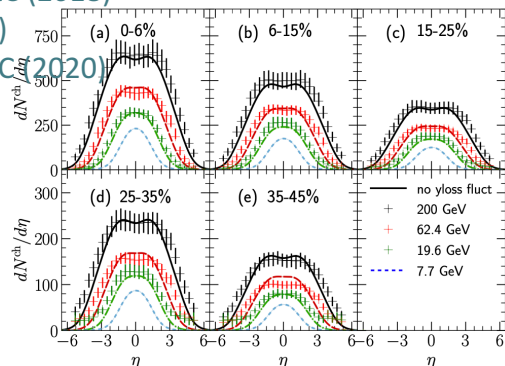
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C. Shen, B. Schenke, PRC (2018)

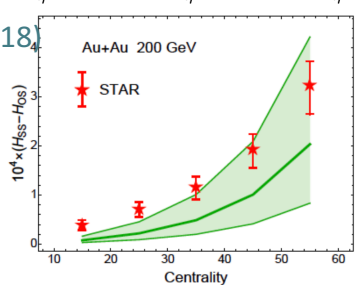
L. Du et al., NPA (2019)

C. Shen, S. Alzhrani, PRC (2020)



Shi et al., Annals of Physics (2018)

Shi, et al, PRL (2020)



Stephanov, Yin, PRD (2018)

Nahrgang et al., PRD (2019)

Akamatsu et al, PRC (2017, 2018)

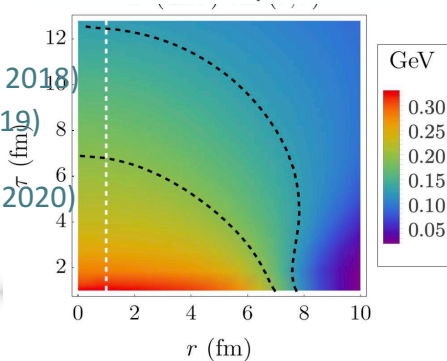
Martinez, Schaefer, PRC (2019)

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Pratt, Plumberg, PRC (2019,2020)

Rajagopal et al., PRD (2020)

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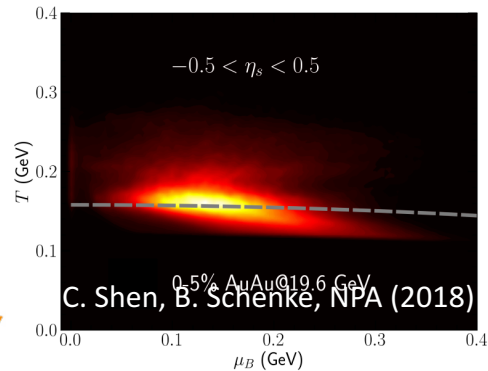


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WB: S. Borsanyi, C. R. et al, PRL (2021), PRD (2022)

BEST Report: NPA (2022)

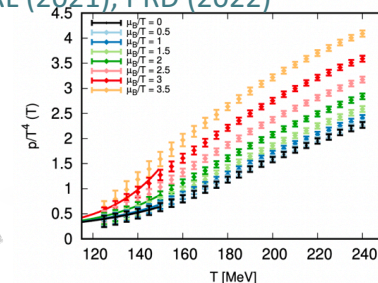
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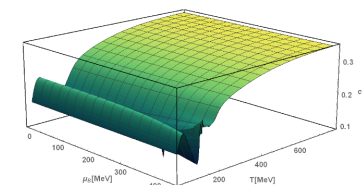
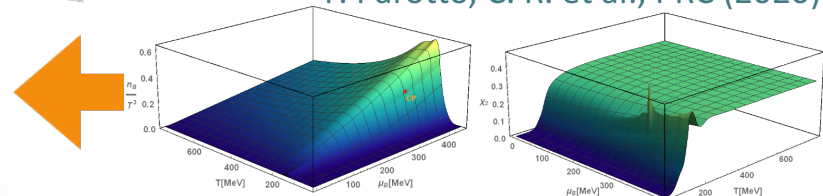
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HotQCD: PRD (2017)

P. Parotto, C. R. et al., PRC (2020)



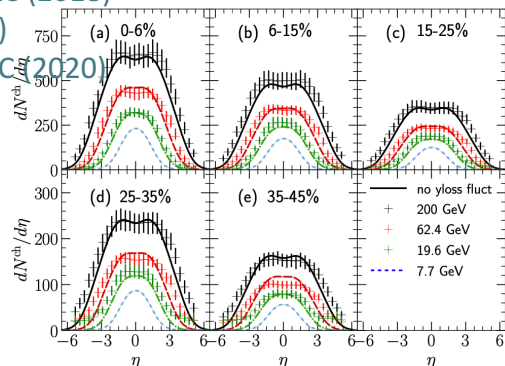
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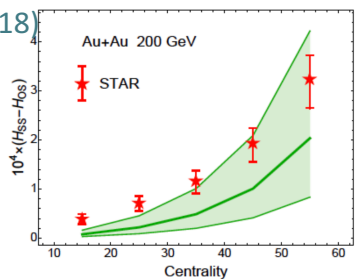
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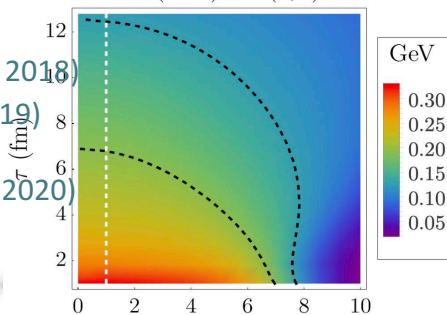
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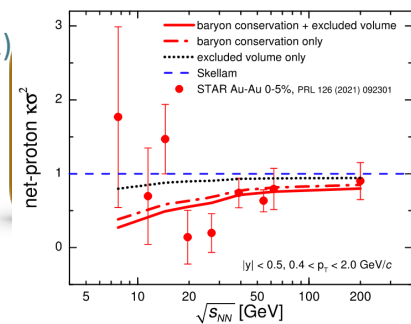
Pratt, Plumberg, PRC (2019,2020)

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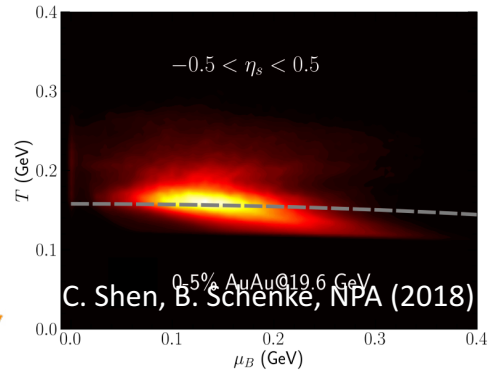


Vovchenko et al., PRC (2022)



WB: S. Borsanyi, C. R. et al, PRL (2021), PRD (2022)

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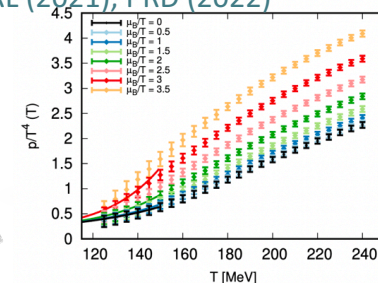


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particlization

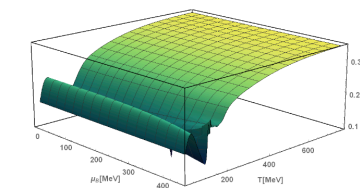
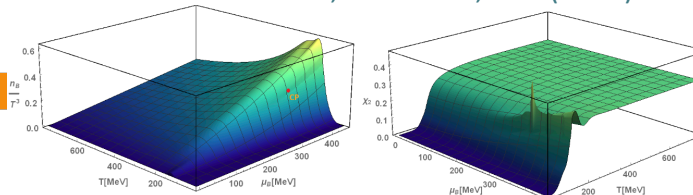
hadronic dynamics

BEST Report: NPA (2022)



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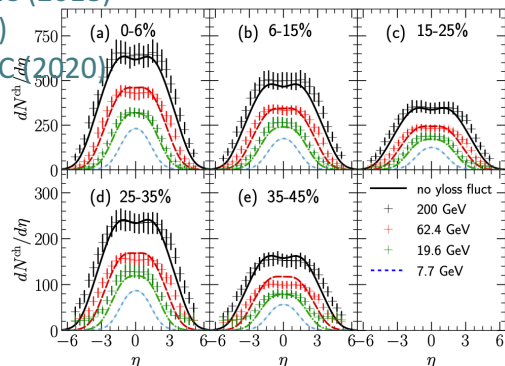
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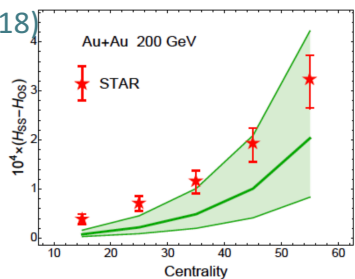
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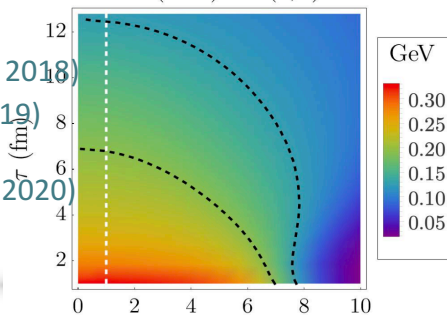
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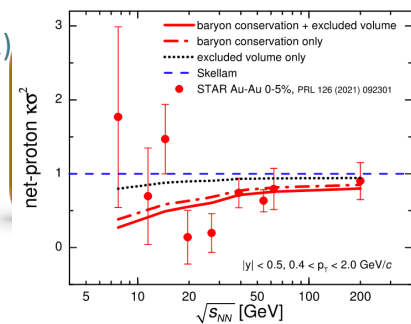
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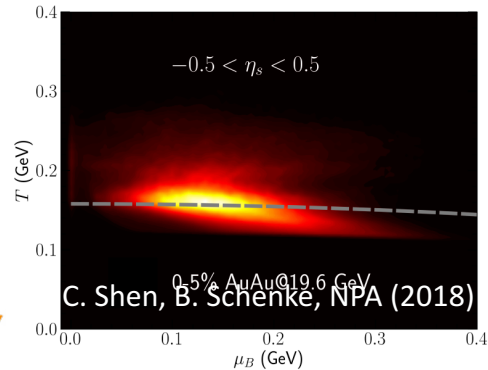


Vovchenko et al., PRC (2022)



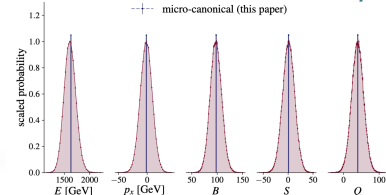
WB: S. Borsanyi, C. R. et al, PRL (2021), PRD (2022)

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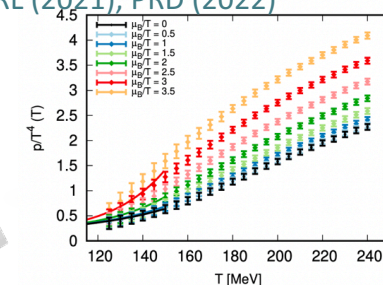
C. Shen, B. Schenke, NPA (2018)

Oliinychenko et al., PRC (2020)



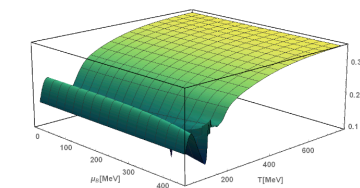
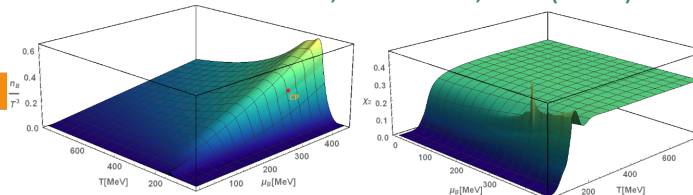
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BEST Report: NPA (2022)



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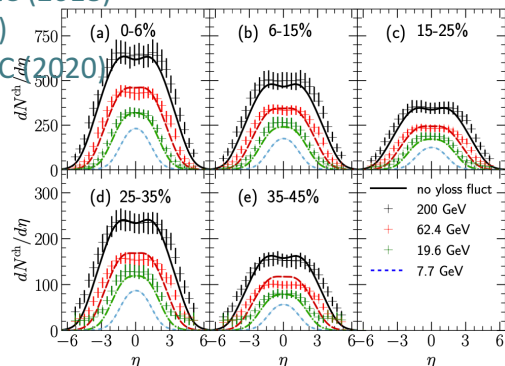
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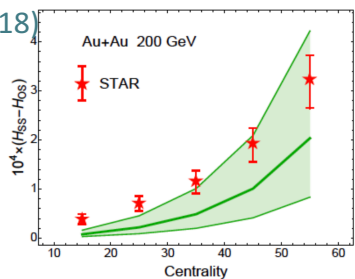
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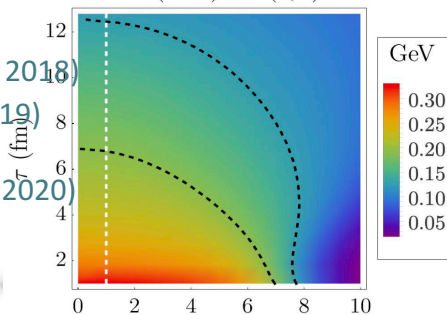
Martinez, Schaefer, PRC (2019)

An et al., PRC (2020)

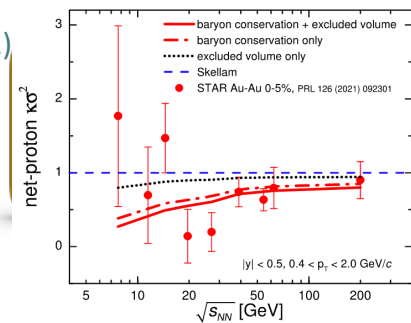
Pratt, Plumberg, PRC (2019,2020)

Rajagopal et al., PRD (2020)

Du et al., PRC (2020)

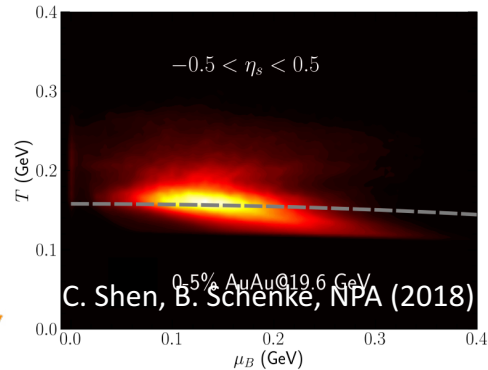


Vovchenko et al., PRC (2022)



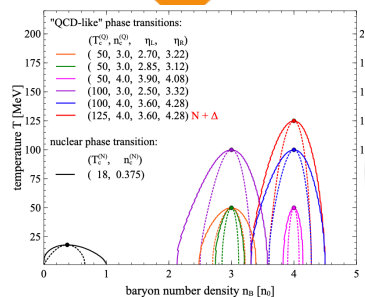
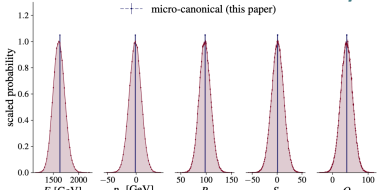
WB: S. Borsanyi, C. R. et al, PRL (2021), PRD (2022)

BEST COLLABORATION



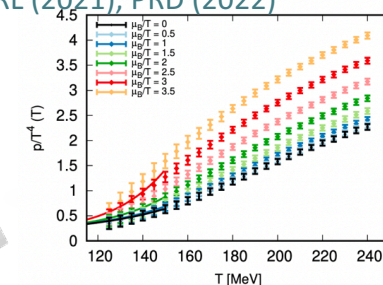
C. Shen, B. Schenke, NPA (2018)

Oliinychenko et al., PRC (2020)



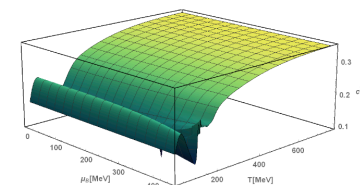
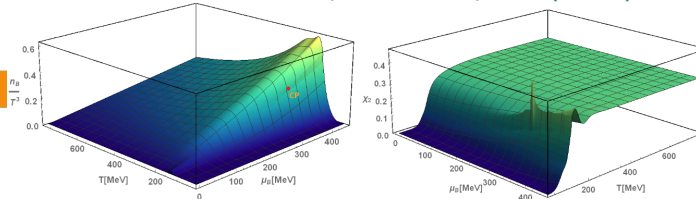
Sorensen, Koch, PRC (2021)

BEST Report: NPA (2022)



HotQCD: PRD (2017)

P. Parotto, C. R. et al., PRC (2020)



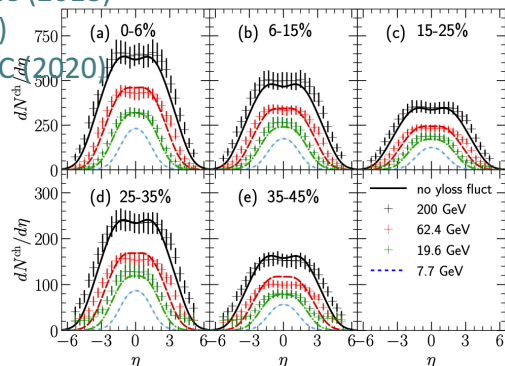
global analysis of expt. data



C. Shen, B. Schenke, PRC (2018)

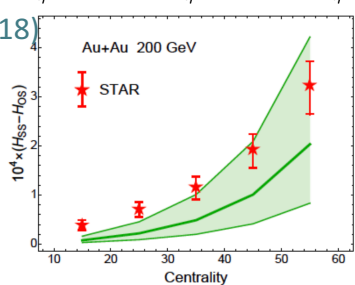
L. Du et al., NPA (2019)

C. Shen, S. Alzhirani, PRC (2020)



Shi et al., Annals of Physics (2018)

Shi, et al, PRL (2020)



Stephanov, Yin, PRD (2018)

Nahgang et al., PRD (2019)

Akamatsu et al, PRC (2017, 2018)

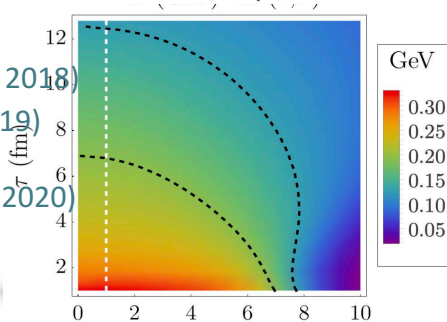
Martinez, Schaefer, PRC (2019)

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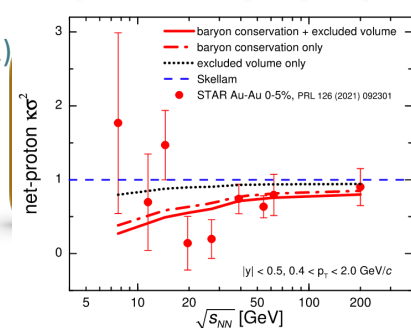
Pratt, Plumberg, PRC (2019,2020)

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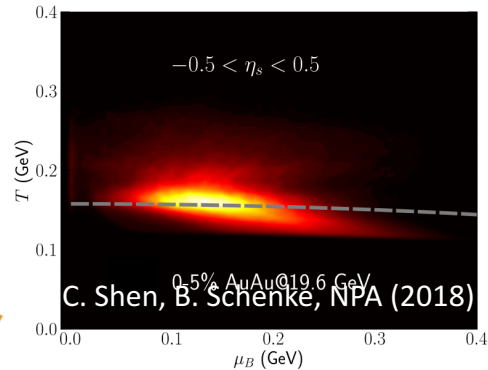


Vovchenko et al., PRC (2022)

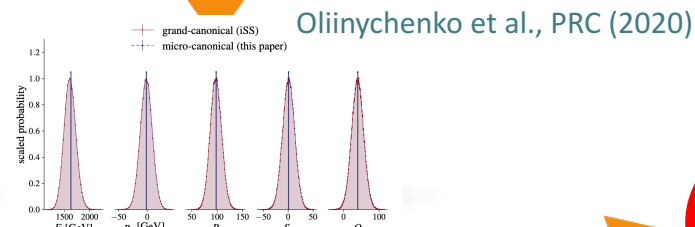


WB: S. Borsanyi, C. R. et al, PRL (2021), PRD (2022)

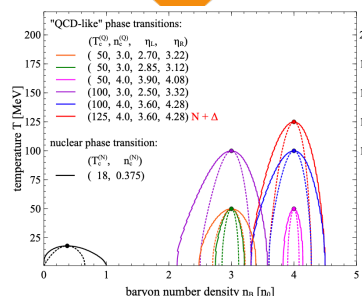
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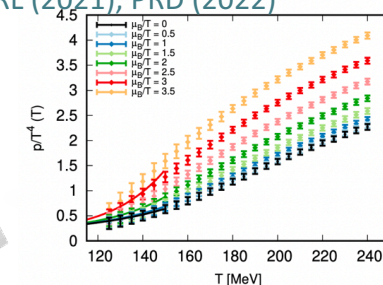


Oliinychenko et al., PRC (2020)



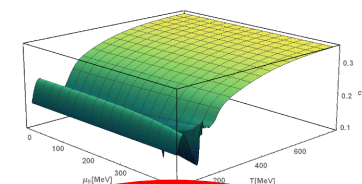
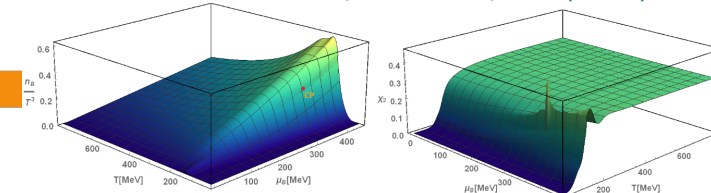
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HotQCD: PRD (2017)

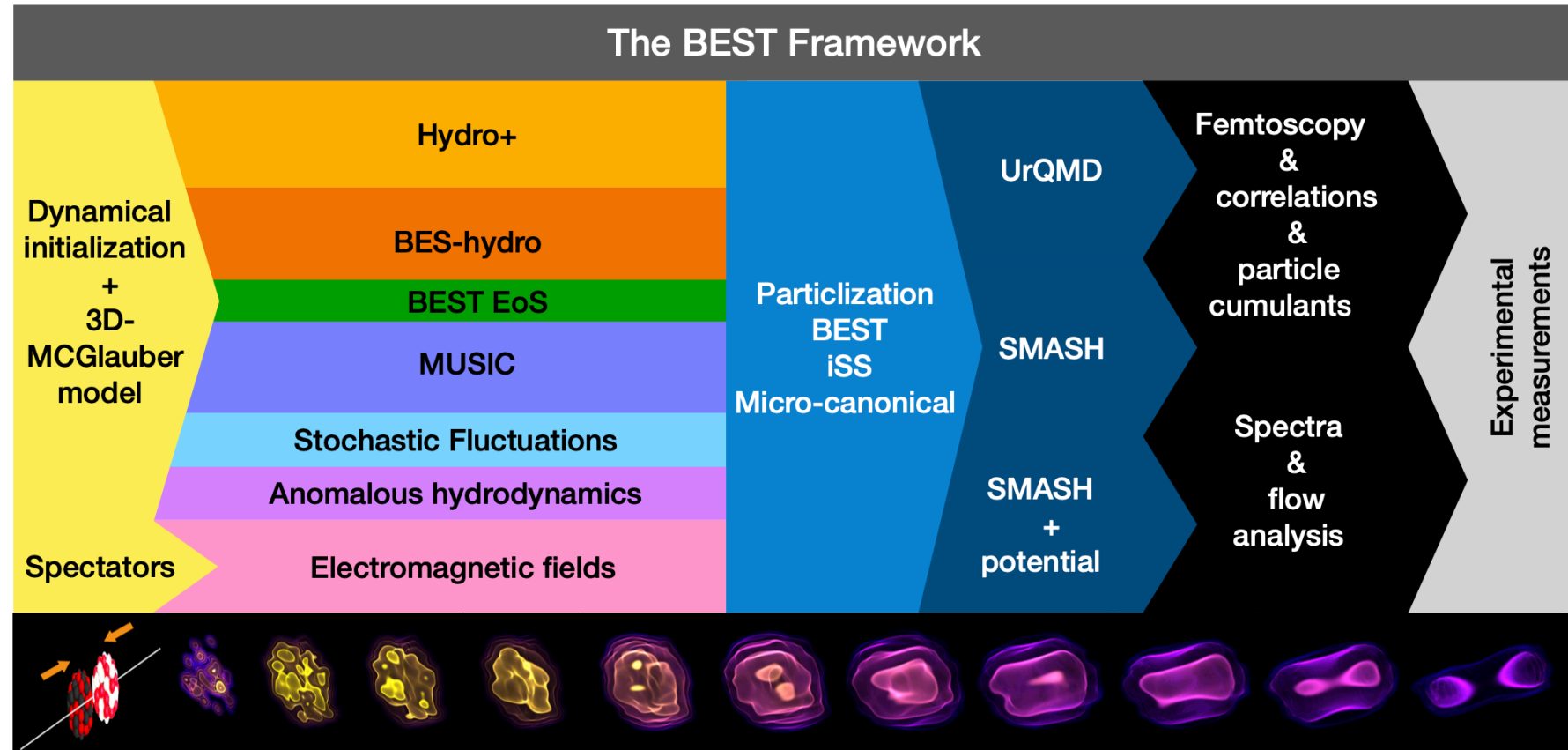
P. Parotto, C. R. et al., PRC (2020)



global analysis of expt. data



Future opportunities



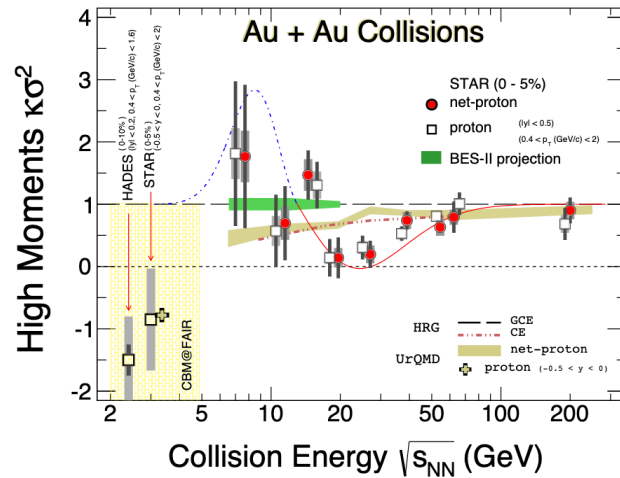
- BEST has made tremendous strides towards a dynamical framework for a **quantitative** description of low-energy HICs
- The design will accommodate a global Bayesian analysis of BESII data
- This framework can be applied to data from future low-energy HIC experiments

- Some BEST members are still working together on these topics, joining forces with low-energy nuclear theorists (FRIB physics) → **proposed topical collaboration** dense Quantum Chromo-Dynamics from the lab to neutron stars and back (**d-QCD**)
- Jaki Noronha-Hostler (PI), M. Alford, V. Dexheimer, S. Gandolfi, G. Hagen, A. Lovato, J. Noronha, T. Papenbrock, S. Pastore, M. Piarulli, K. Rajagopal, C. Ratti, T. Schaefer, S. Sen, C. Shen, M. Stephanov, I. Thews, V. Vovchenko

Science possibilities with CBM

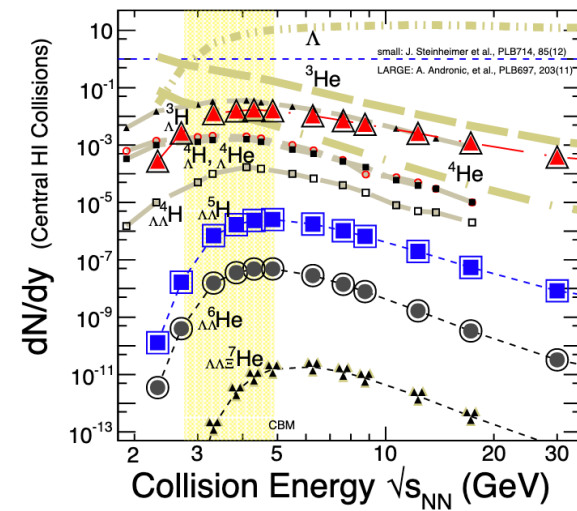
1. Is there a critical point on the QCD phase diagram? 2. Hyperon-hyperon interactions

- Net-proton fluctuations: explore the range $2.9 \text{ GeV} \leq \sqrt{s} \leq 4.9 \text{ GeV}$



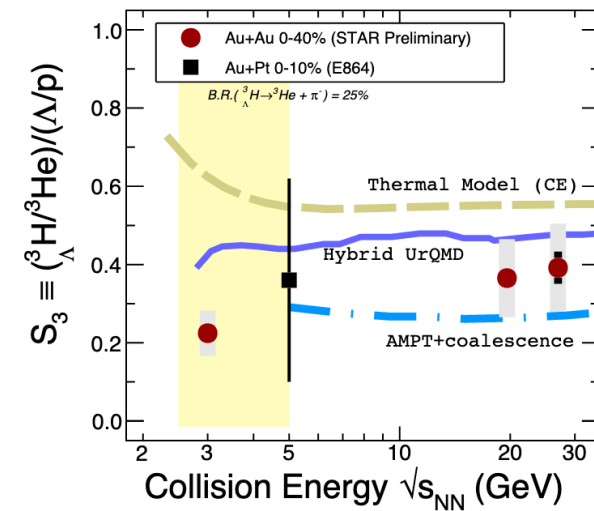
Theory:

- Propagate critical fluctuations beyond second order
- Understand and model non-thermal effects
- Bayesian analysis to narrow down critical point location and strength



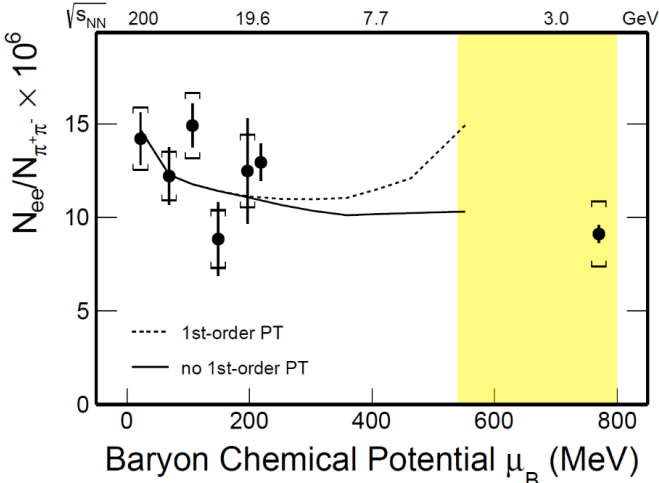
Theory:

- Link measurements to models
- Hyperon-nucleon and hyperon-hyperon interactions
- Can constrain the equation of state of compact stars



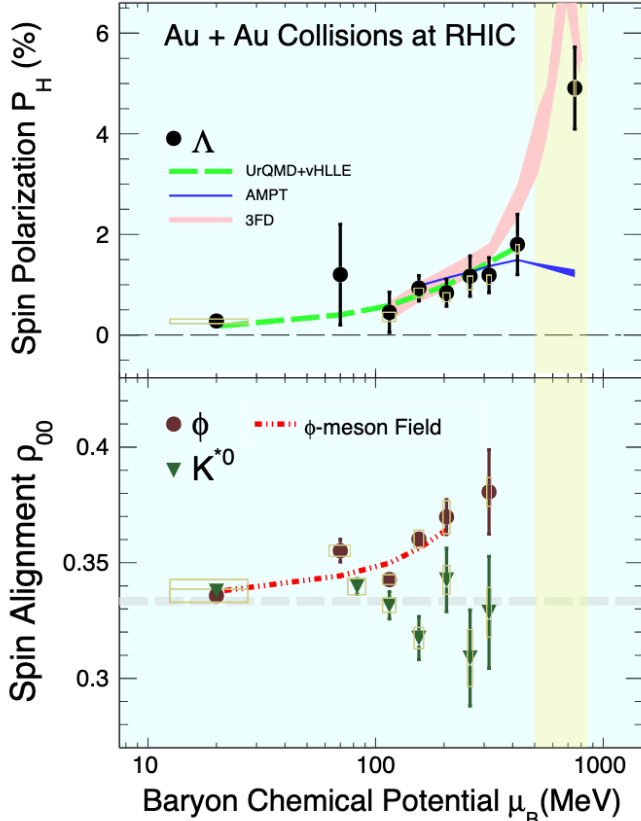
Science possibilities with CBM

3. Dilepton spectra and collective flow



- Theory:
- Interplay between chiral symmetry restoration and deconfinement
 - Can investigate the quarkyonic phase
 - First order phase transition can manifest itself in an increase of the low-mass di-lepton yield

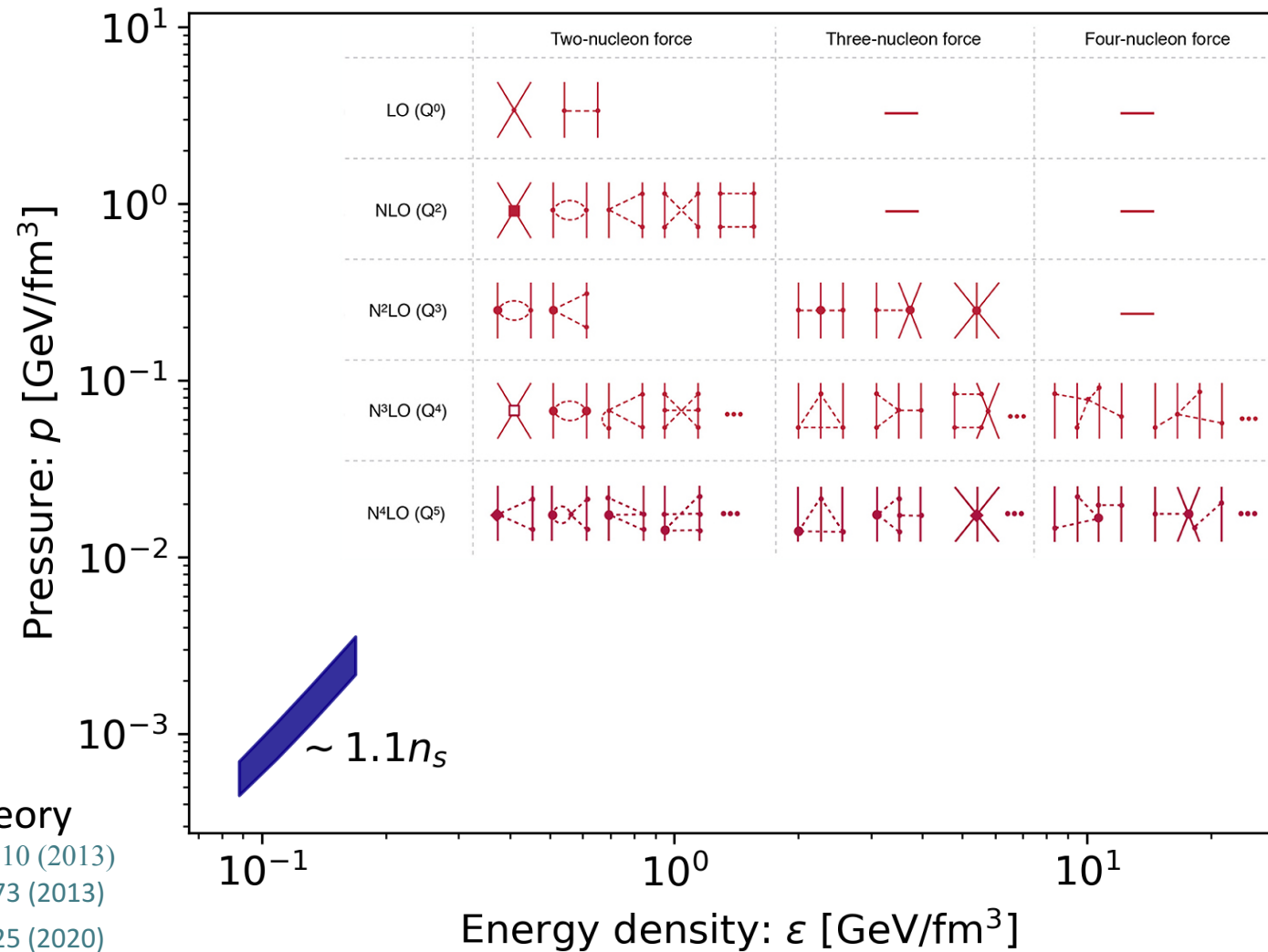
4. Global polarization



- Theory:
- Study of strong vorticity field
 - Baryon stopping effects?
 - Connections between fields and vorticity at high density



Neutron stars and mergers



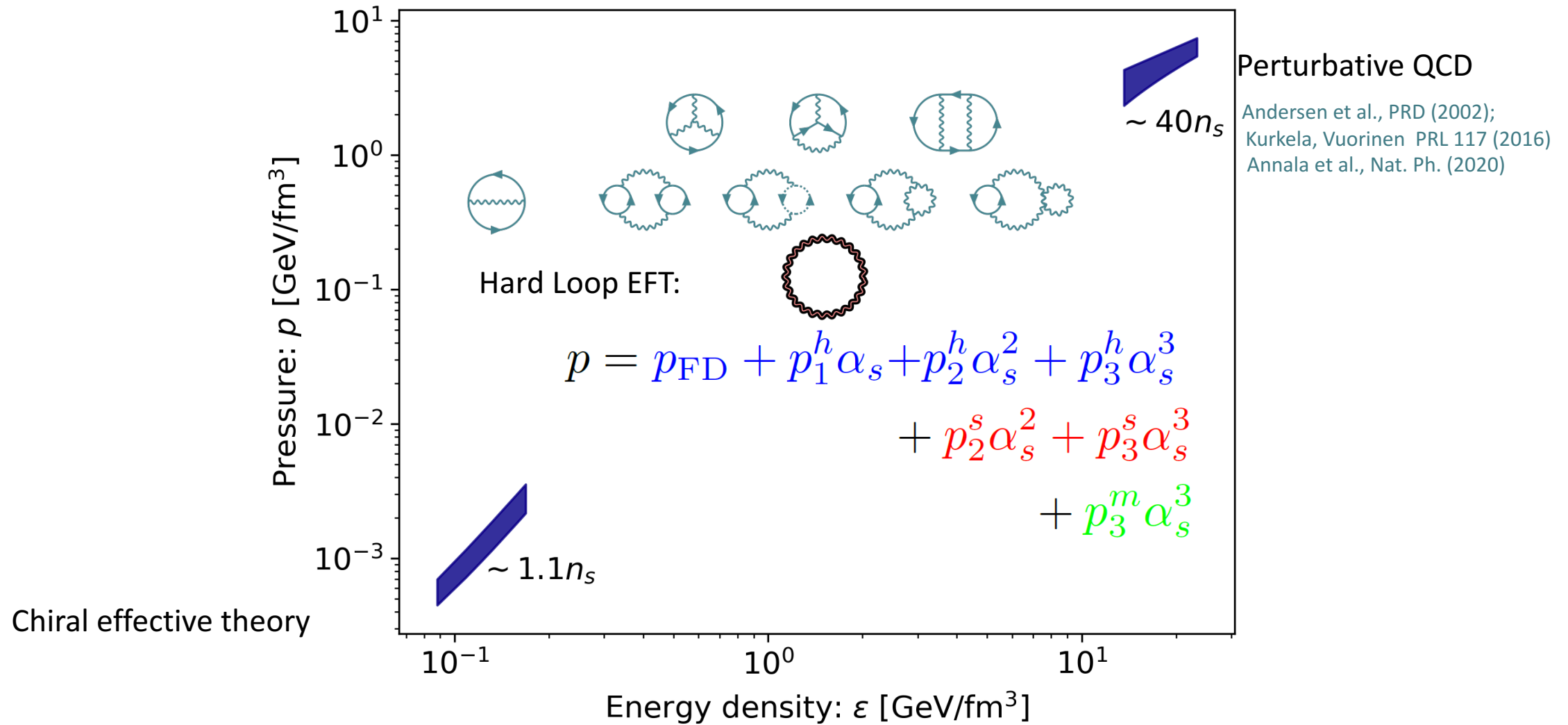
Chiral effective theory

Tews et al. PRL 110 (2013)

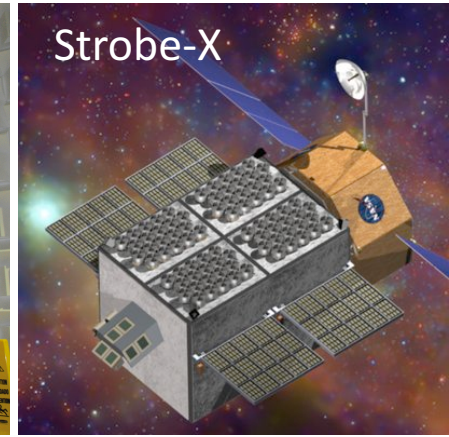
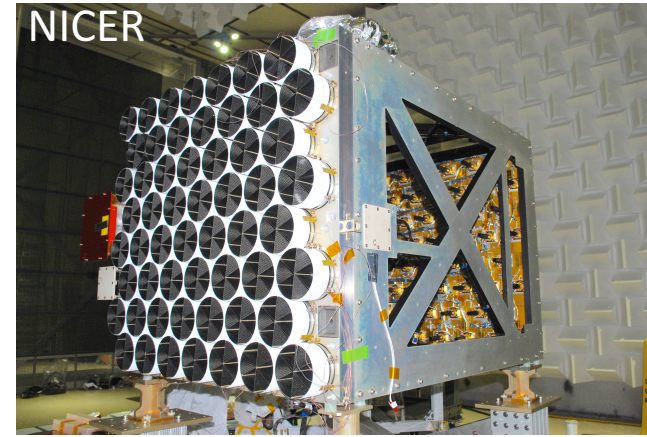
Hebeler, Lattimer et.al. APJ 773 (2013)

Drischler, Furnstahl et.al. PRL 125 (2020)

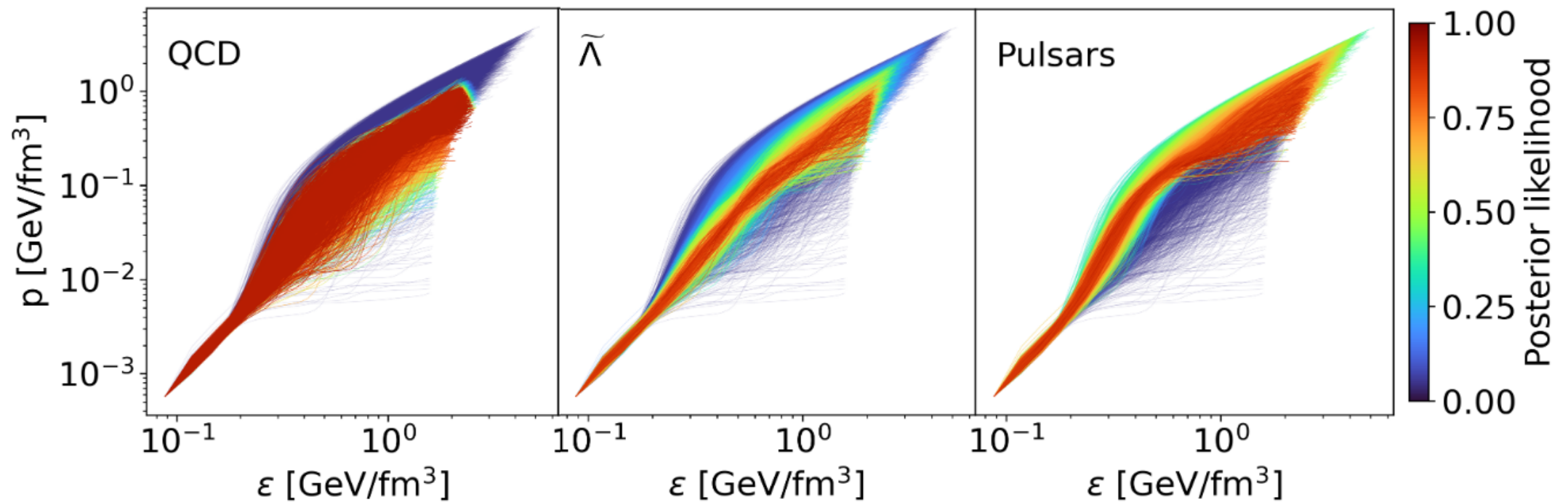
Neutron stars and mergers



Neutron stars and mergers



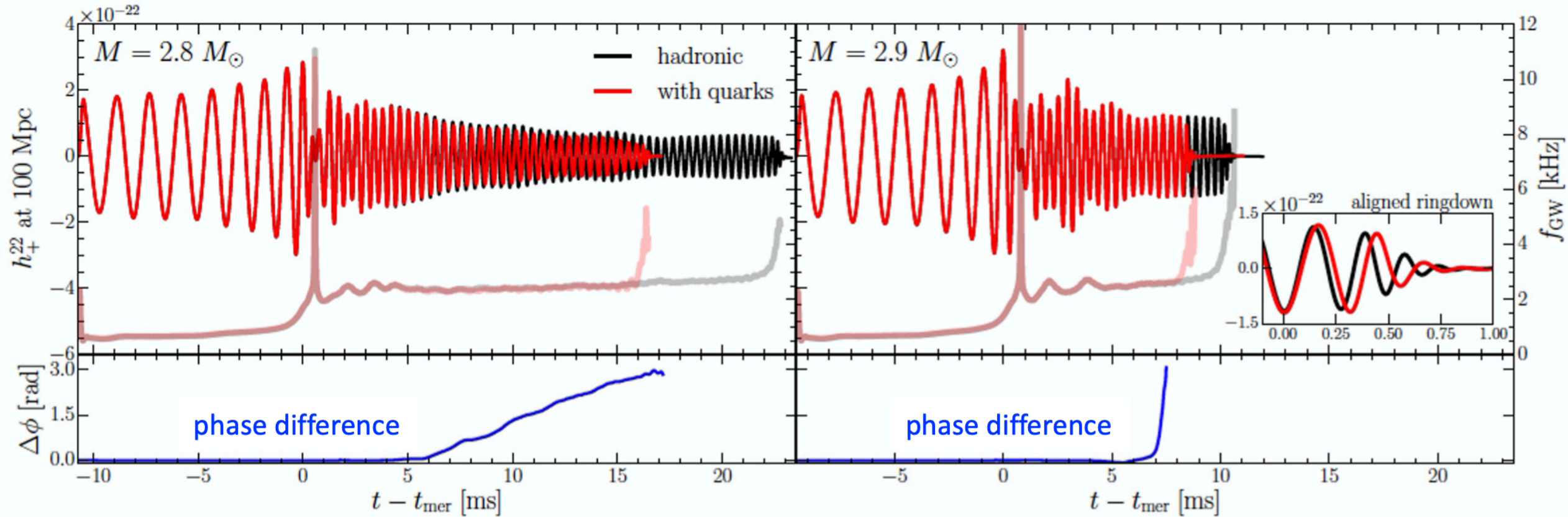
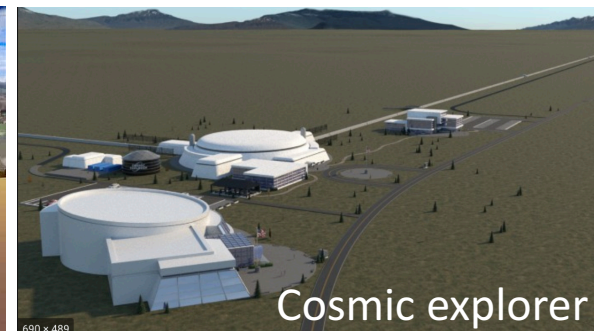
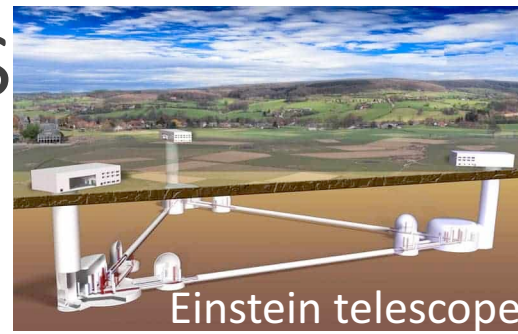
QCD input and NS observations can constrain the interpolation



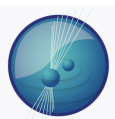
Slide adapted from talk by A. Kurkela

Neutron stars and mergers

- Post-merger signal sensitive to order of the phase transition
- Next generation observatories will be able to detect it!
- Need to combine the nuclear physics input and simulations

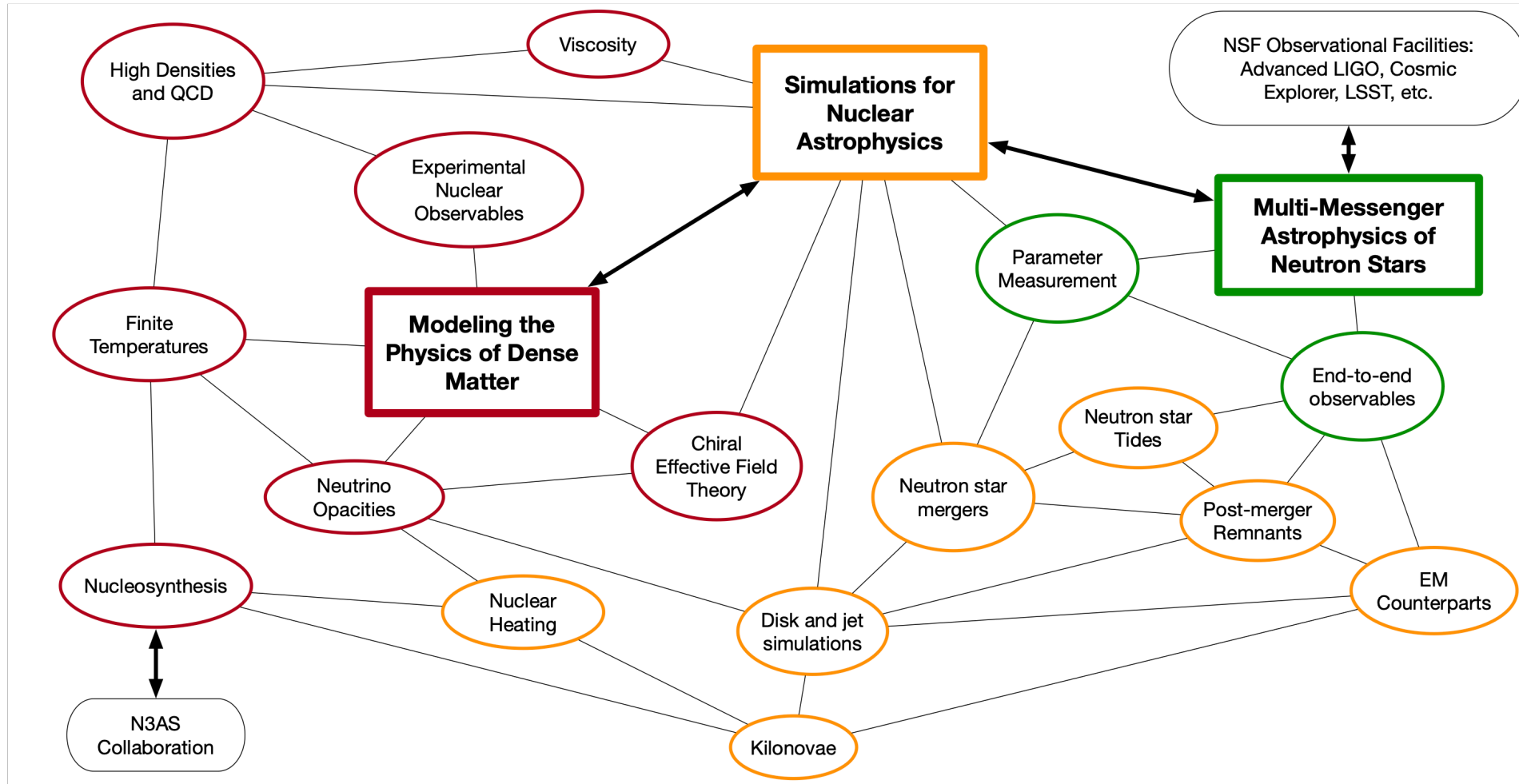


E. Most et al., PRL (2019)



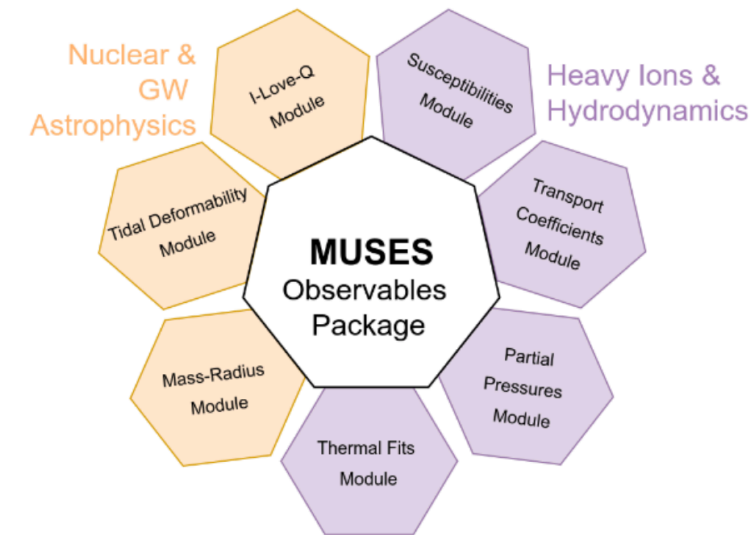
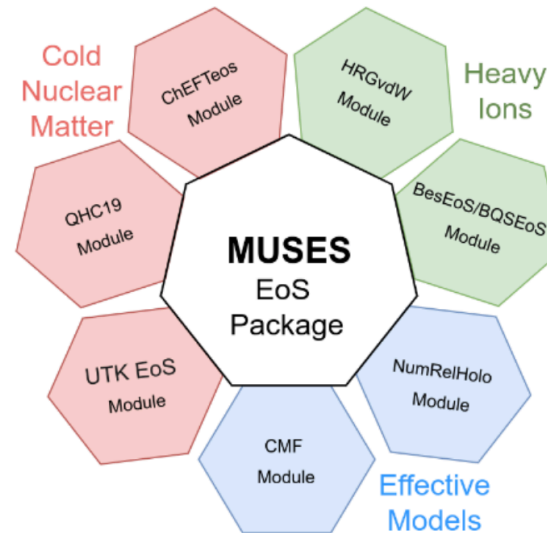
The Problem is Too Big For One Group

Progress needs a **close, coordinated, and sustained** collaboration across different research groups



Modular Unified Solver of the Equation of State collaboration

- Funded by NSF through CSSI program
- **Developers** and **Users** are working together to create a sustainable software to generate equations of state in the whole phase space
- **Modular**: Different models (“modules”) to describe the EoS in different regimes of phase space
- **Unified**: Modules smoothly integrated to (i) ensure maximal coverage of phase space, and (ii) respects constraints



Conclusions

- Need for quantitative results at finite-density to support the experimental programs
- Lattice needs computational support to provide a larger coverage for
 - Equation of state
 - Phase transition line
 - Fluctuations of conserved charges
- BEST framework needs to be extended and applied to analysis of data from low-energy heavy-ion collision
- Connections to astrophysics and future terrestrial facilities will allow to map out the phase diagram

Backup slides

Computational Nuclear Physics and AI/ML Workshop

Organized by:

- Alessandro Lovato (ANL)
- Joe Carlson (LANL)
- Phiala Shanahan (MIT)
- Bronson Messer (ORNL)
- Witold Nazarewicz (FRIB/MSU)
- Amber Boehnlein (JLab)
- Peter Petreczky (BNL)
- Robert Edwards (JLab)
- David Dean (JLab)

6-7 September 2022 at SURA in Washington, DC

60 registered participants (40 in person, 20 on line), including DOE representation

<https://indico.jlab.org/event/581/>

- All talks archived
- Short white paper being prepared for the LRP

**Computational
Nuclear Physics
and AI/ML
Workshop**



6-7 September, 2022 / SURA headquarters

Organized by:

Alessandro Lovato – Joe Carlson (LANL), Phiala Shanahan (MIT), Bronson Messer (ORNL)
Witold Nazarewicz (FRIB/MSU), Amber Boehnlein (JLab), Peter Petreczky (BNL)
Robert Edwards (JLab), David Dean (JLab)

Admin support: Jae Cho jcho@jlab.org Tea Jojua tjojua@sura.org Sherry Thomas sthas@jlab.org

Schedule

Registration, schedule, and other information can be found at: <https://indico.jlab.org/event/581/>

Tuesday, 6 September

1:00 – 1:05 Welcome, David Dean and Sean Hearne
1:05 – 1:20 DOE remarks, Tim Hallman
1:20 – 2:00 QCD, William Detmold (JLab) and Swagato Mukherjee (BNL)
2:00 – 2:40 Quantum many-body problems, Thomas Papenbrock (UT/ORNL)
2:40 – 3:00 BREAK
3:00 – 3:40 Fundamental Symmetries, Emanuele Mereghetti (LANL)
3:40 – 4:20 Astrophysics, George Fuller (UCSD)
4:20 – 5:00 AI/ML, Amber Boehnlein (JLab)
5:00 – 5:40 Preliminary list of recommendations discussion (Peter Petreczky, lead)
5:40 – 7:30 Reception

Wednesday, 7 September

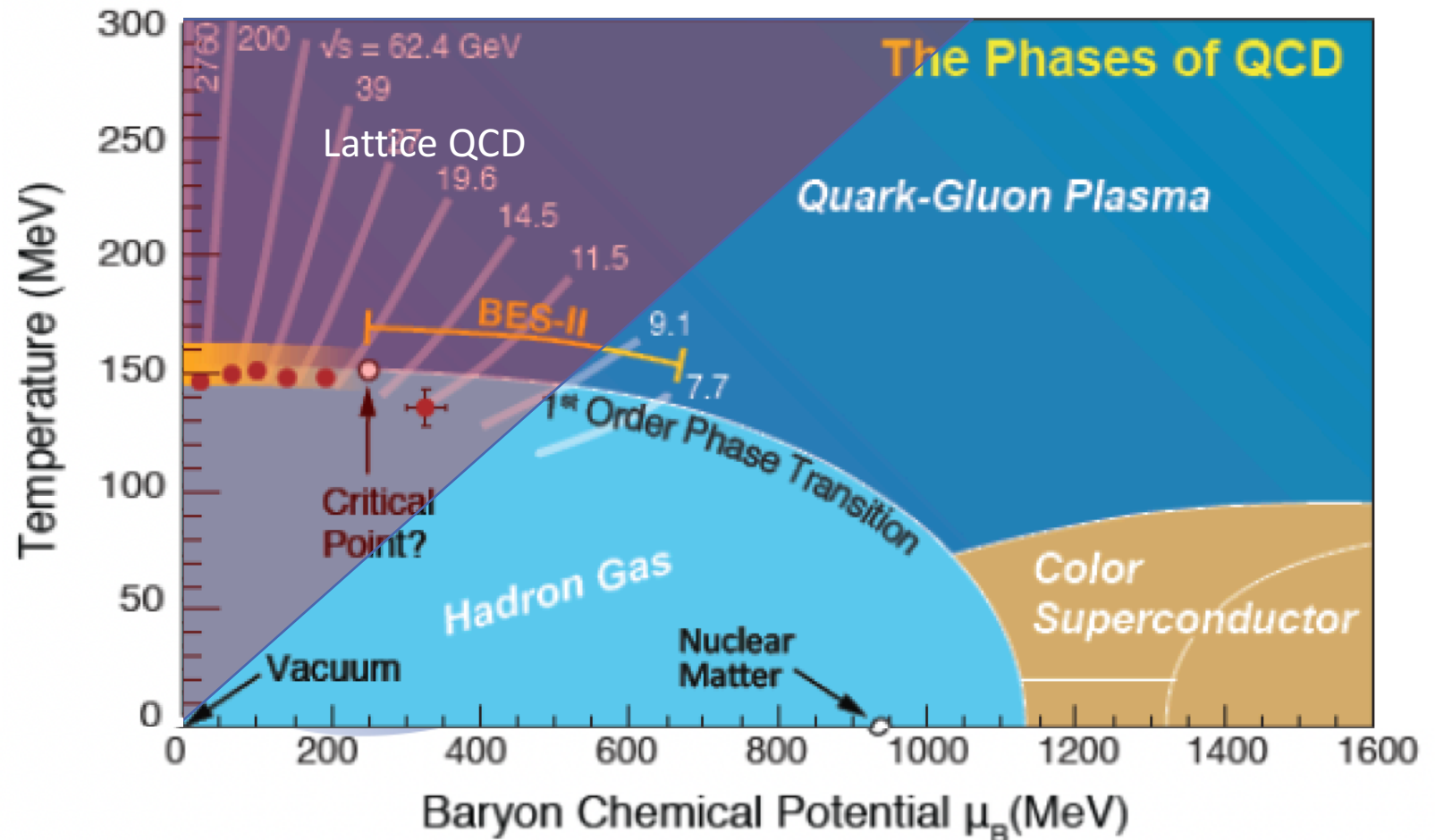
7:45 – 8:30 Continental Breakfast
8:30 – 10:00 Breakout Sessions
1. QCD (Phiala Shanahan, lead)
2. Nuclear Structure and fundamental symmetries (Alessandro Lovato, lead)
3. Astrophysics (Bronson Messer, lead)
10:00 – 10:30 Break
10:30 – 12:00 Breakout reports
12:00 – 1:00 Lunch
1:00 – 2:30 Recommendations discussion and next steps

Jefferson Lab
Thomas Jefferson National Accelerator Facility

SURA

What happens at large densities?

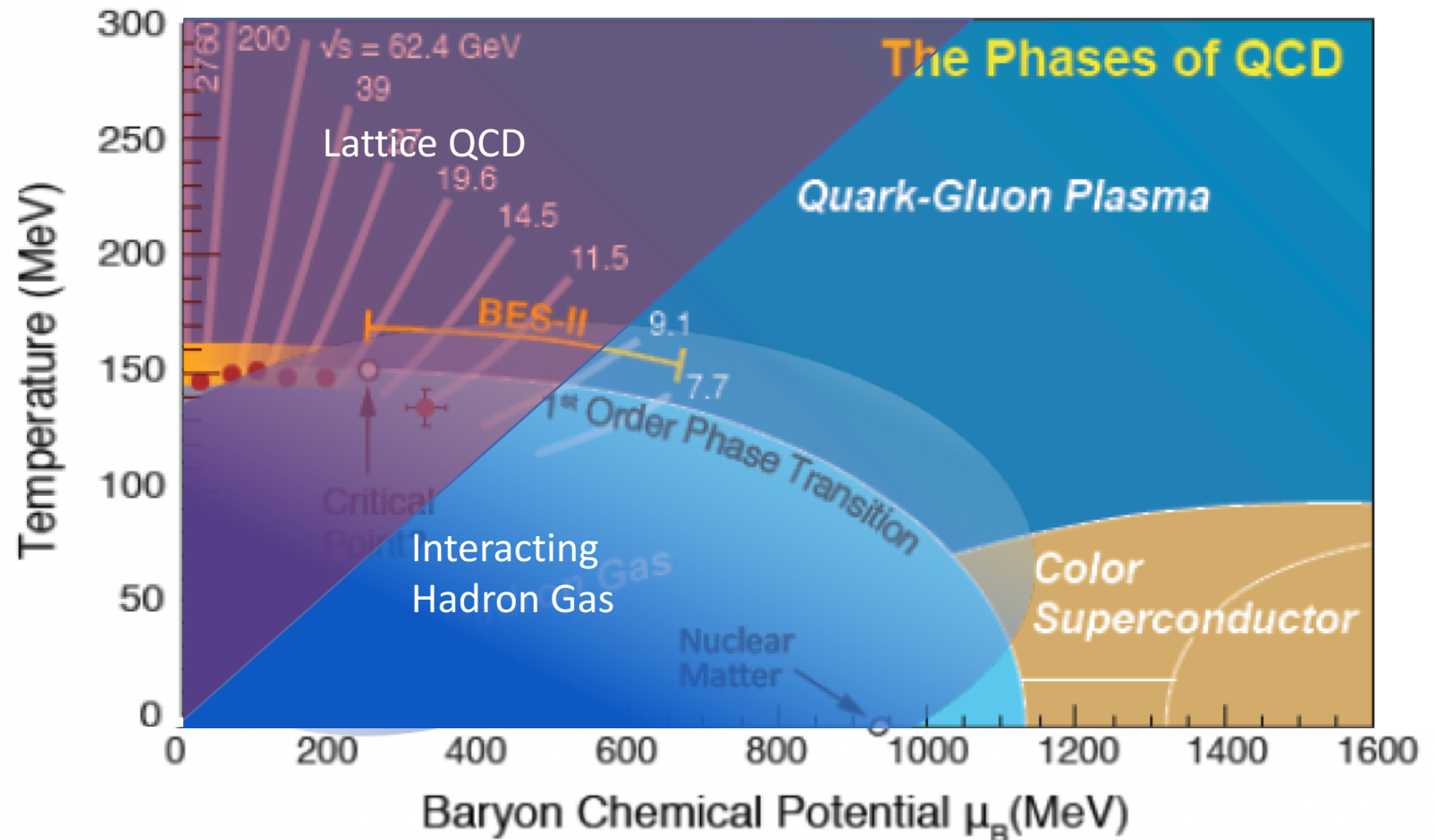
- We need to merge the lattice QCD equation of state with other effective theories
- Careful study of their respective range of validity
- Constrain the parameters to reproduce known limits
- Test different possibilities and validate/exclude them



Lattice QCD: S. Borsanyi, C. R. et al, PRL (2021)

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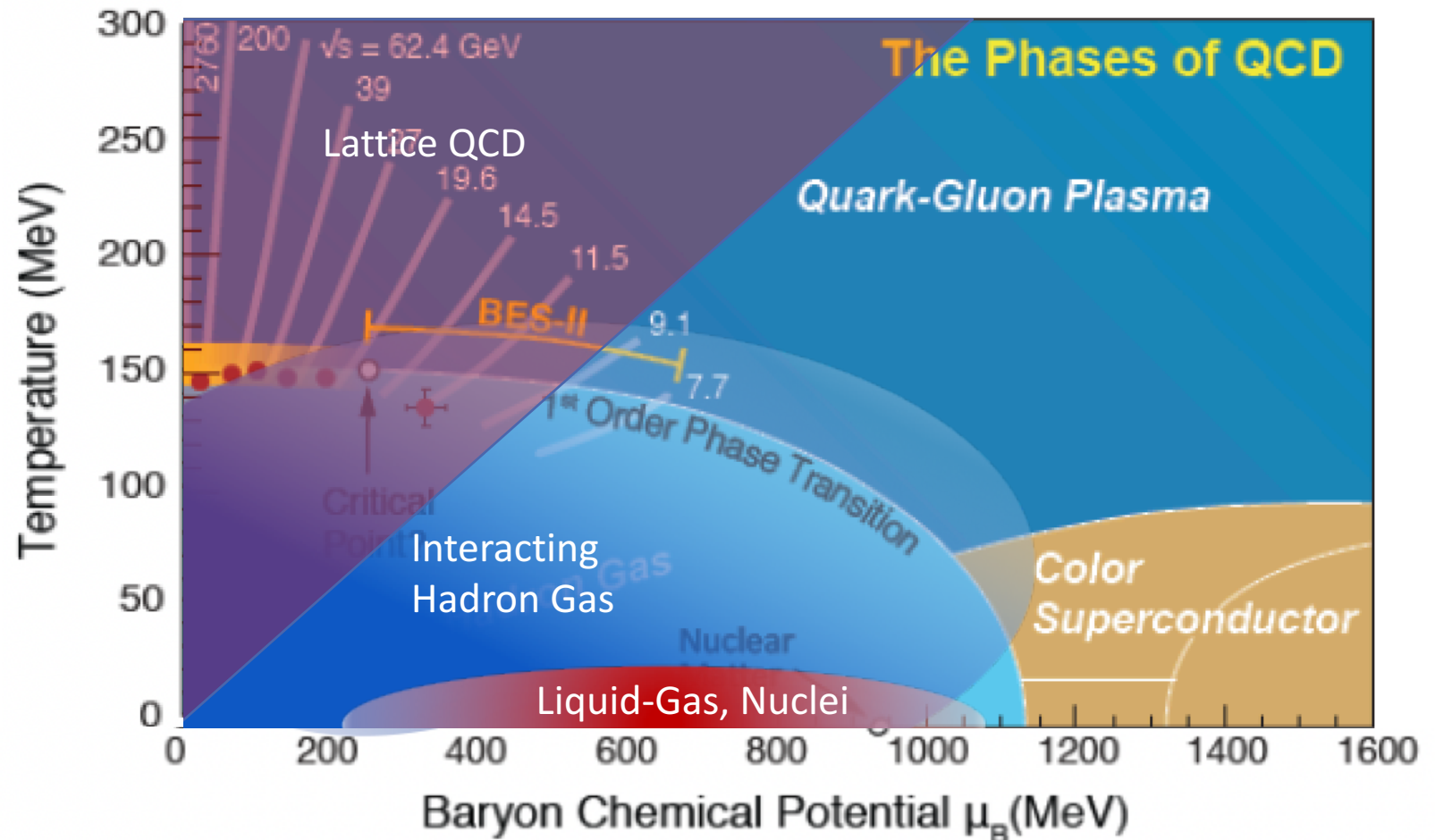


Lattice QCD: S. Borsanyi, C. R. et al, PRL (2021)

Interacting HRG: J. Karthein, C. R. et al., PRD (2022)

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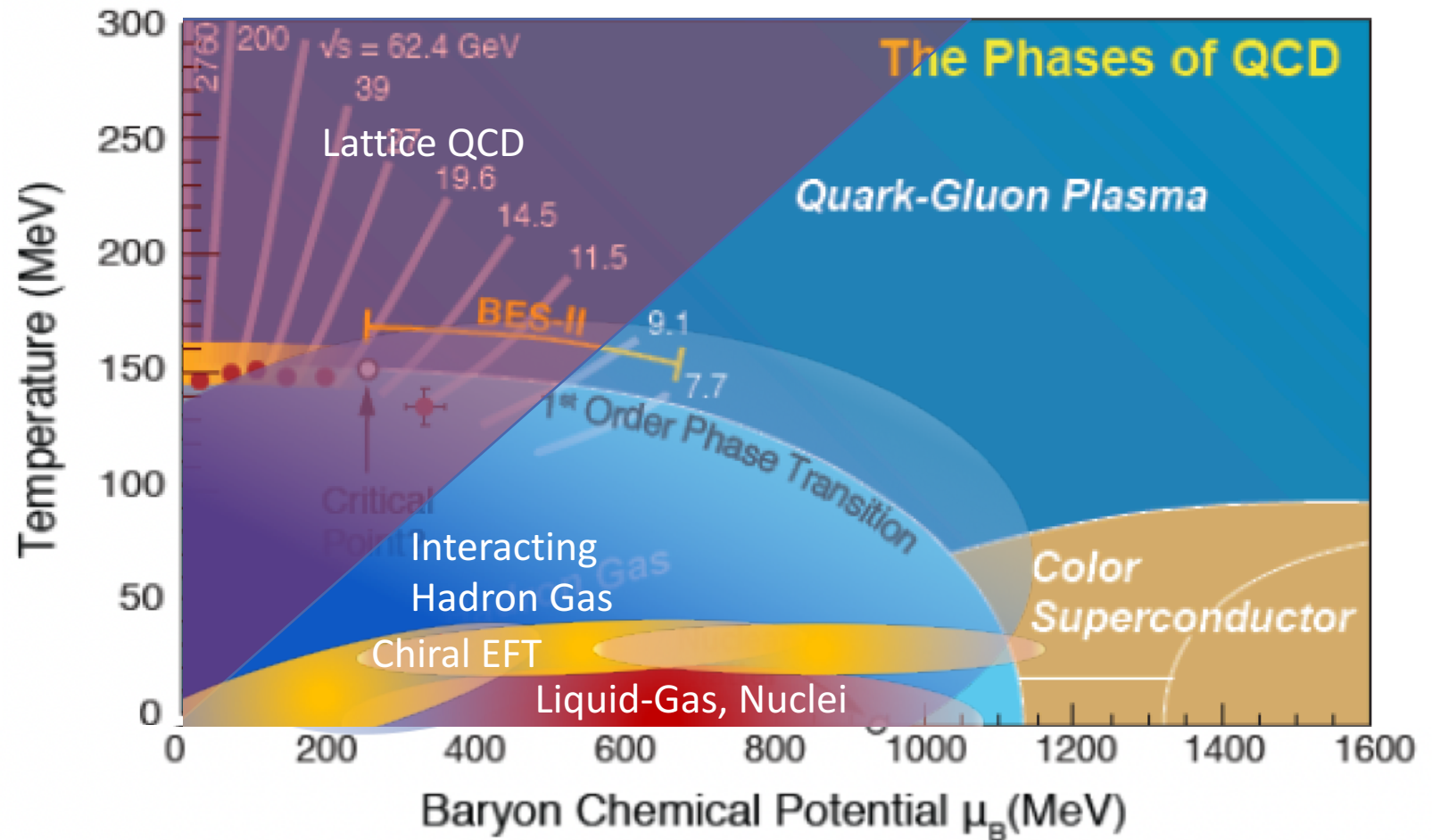
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Interacting HRG: J. Karthein, C. R. et al., PRD (2022)

Liquid-gas, Nuclei: see e.g. Du et al. PRC (2019)

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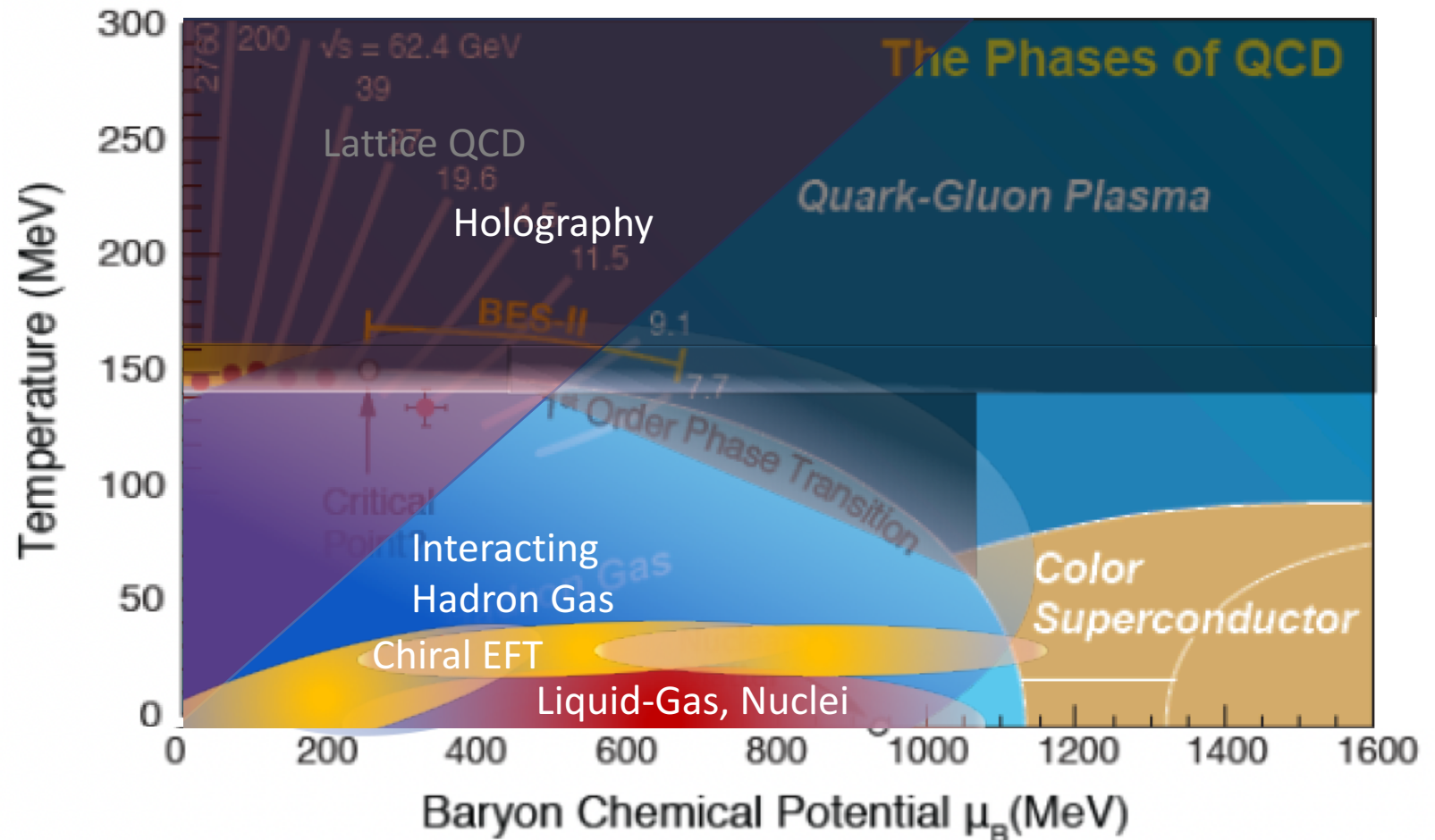
Interacting HRG: J. Karthein, C. R. et al., PRD (2022)

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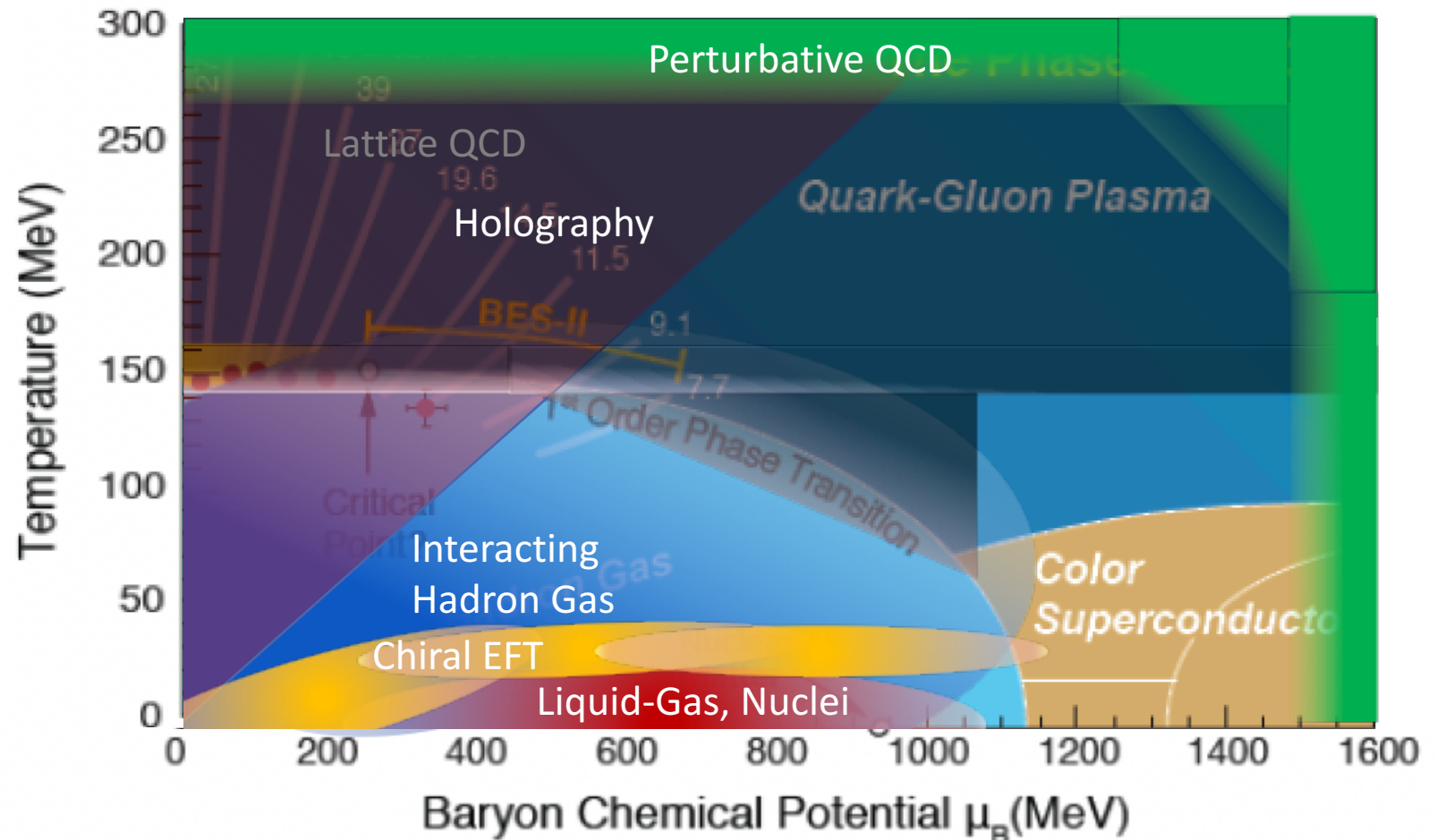
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Holography: R. Critelli, C. R. et al., PRD (2017)

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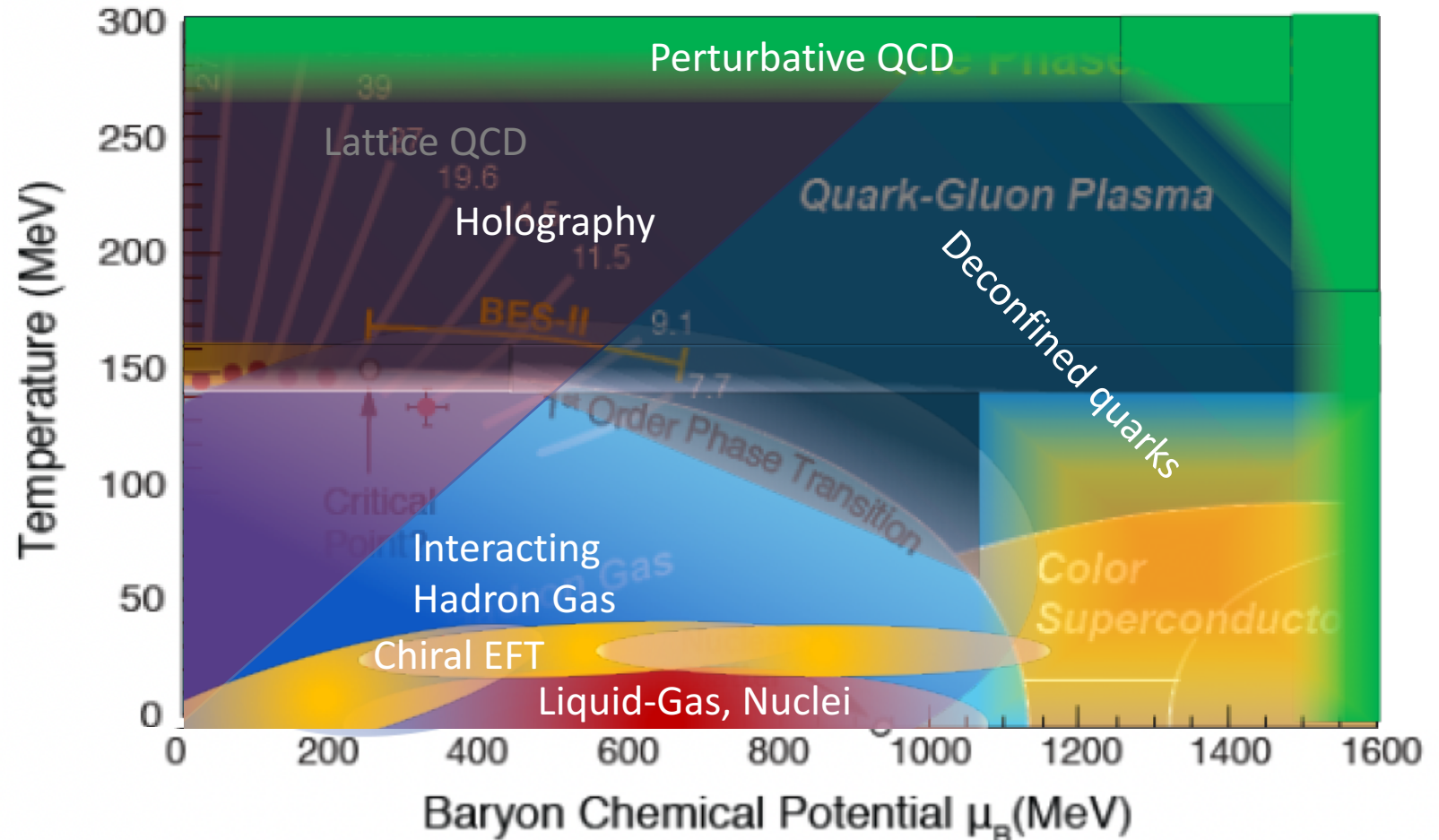
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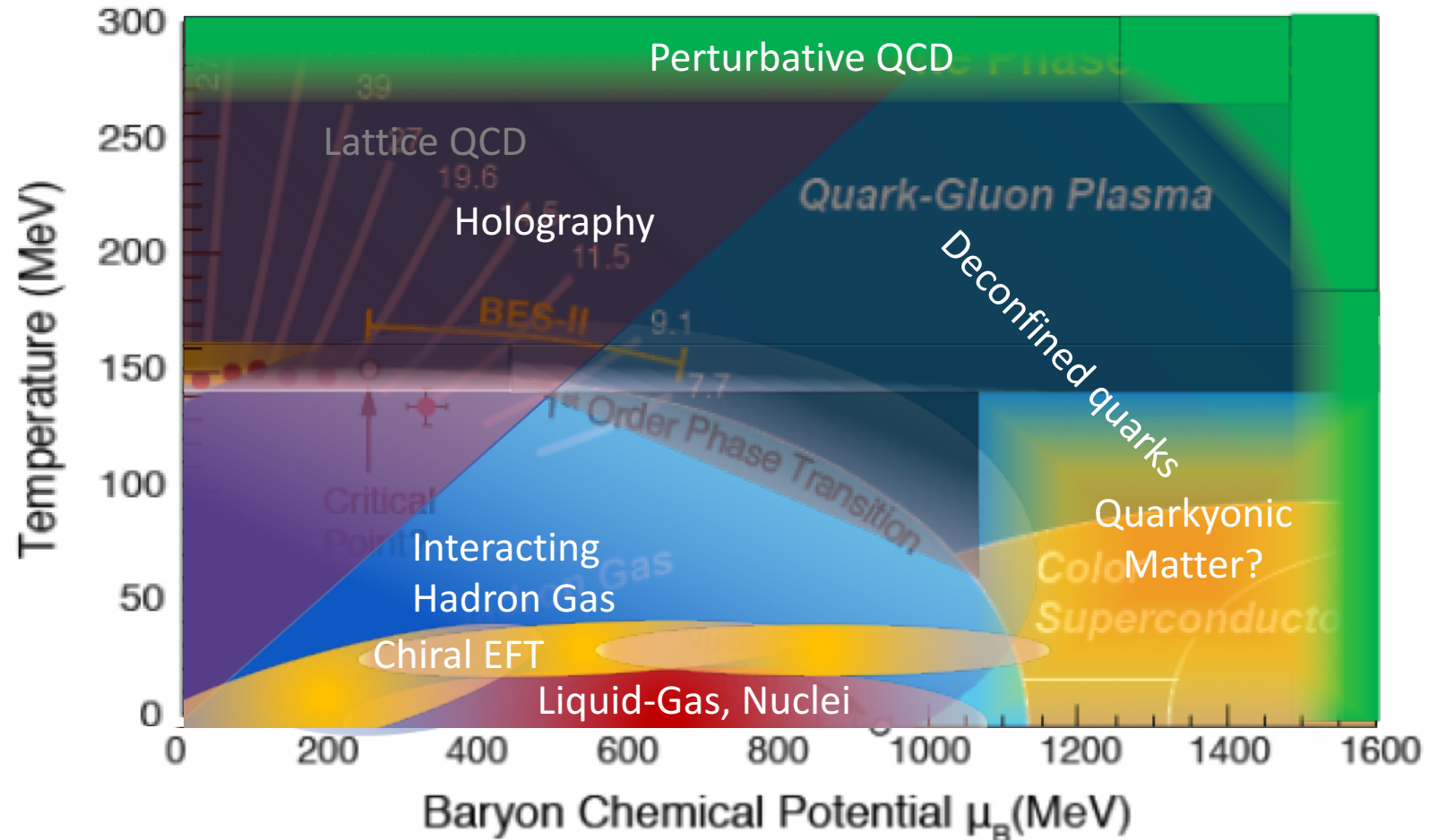
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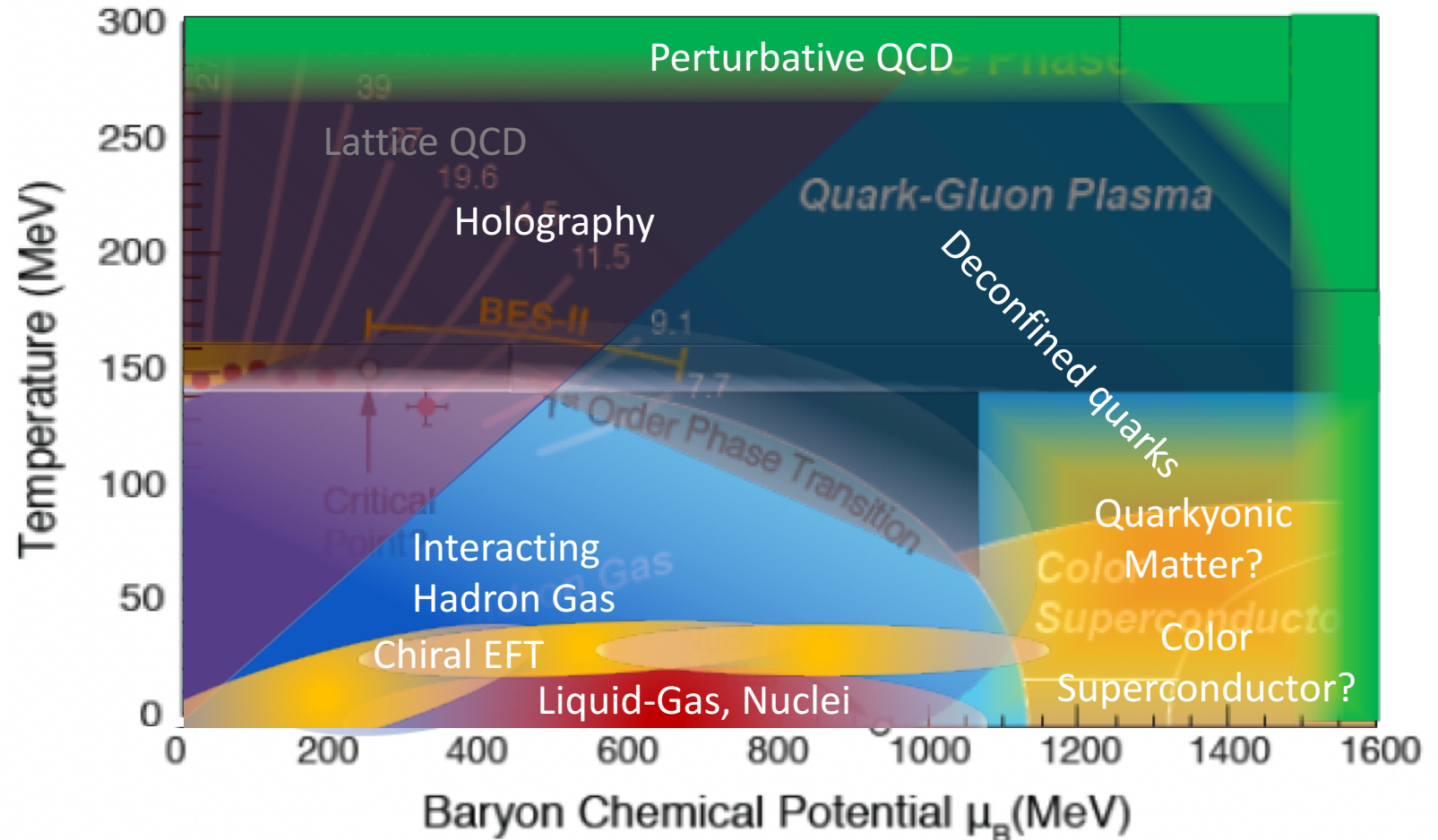
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QCD matter under extreme conditions

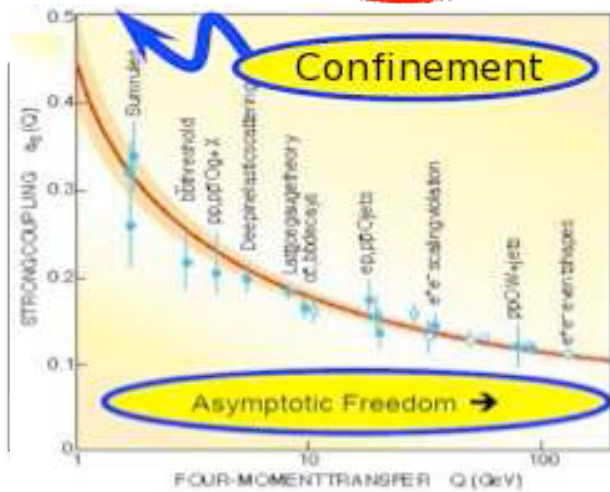
To address these questions, we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- ▶ QCD is the fundamental theory of strong interactions
- ▶ It describes interactions among quarks and gluons

$$L_{QCD} = \sum_{i=1}^{n_f} \bar{\psi}_i \gamma_\mu \left(i \partial^\mu - g A_a^\mu \frac{\lambda_a}{2} \right) \psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + i f_{abc} A_b^\mu A_c^\nu$$



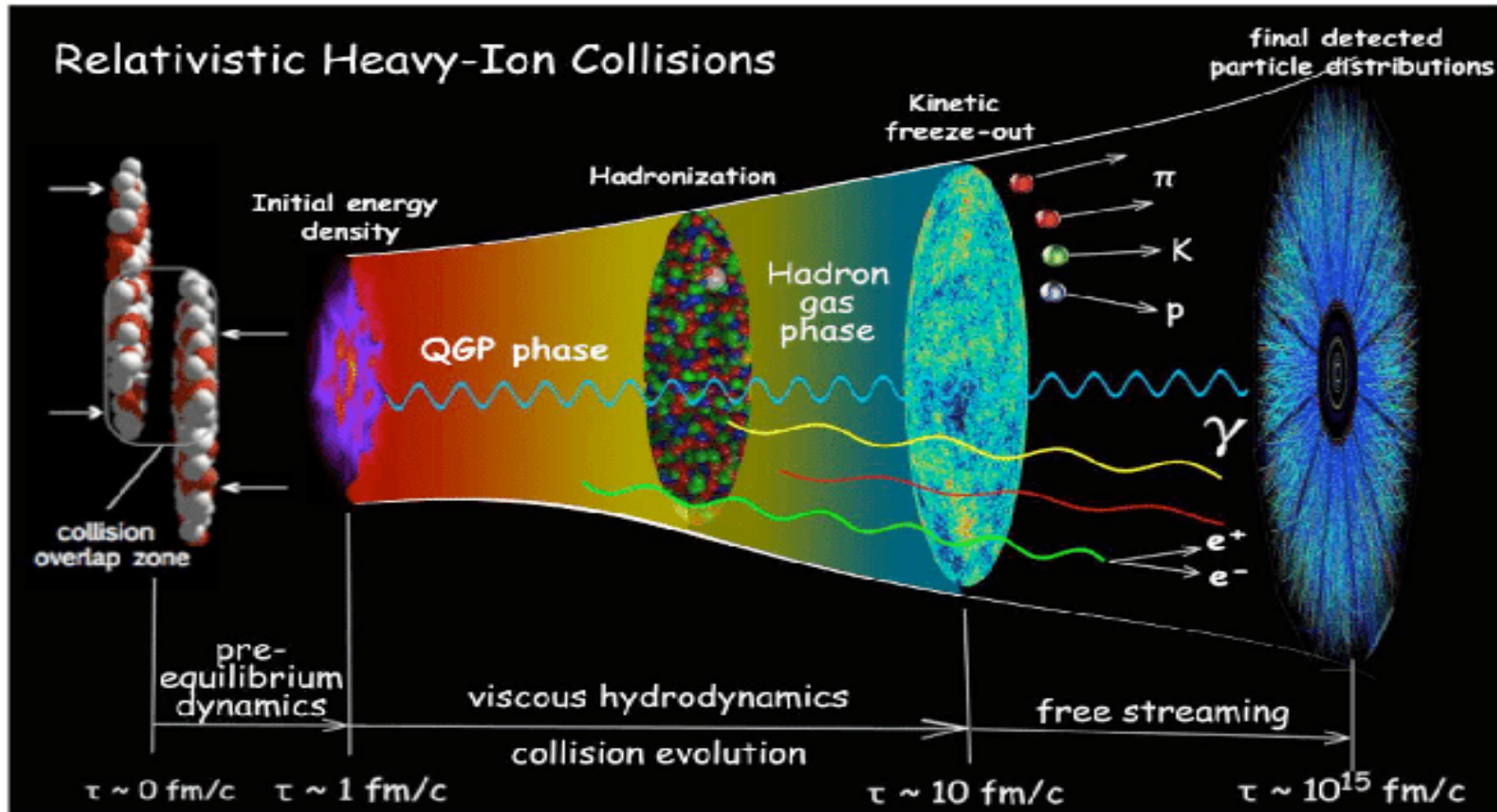
Experiment: heavy-ion collisions



- ▶ Quark-Gluon Plasma (QGP) discovery at RHIC and LHC:
- ▶ SURPRISE!!! QGP is a **PERFECT FLUID**
- ▶ Changes our idea of QGP (no weak coupling)
- ▶ Microscopic origin still unknown



Anatomy of a heavy-ion collision



How can lattice QCD support the experiments?

Equation of state

- Needed for **hydrodynamic** description of the QGP

QCD phase diagram

- Transition line at finite density
- Constraints on the location of the critical point

Fluctuations of conserved charges

- Can be **simulated** on the lattice and **measured** in experiments
- Can give information on the **evolution** of heavy-ion collisions
- Can give information on the **critical point**



Formulation

S. Borsanyi, C. R. et al., PRL (2021)

- We have observed the $\hat{\mu}_B$ -dependence seems to amount to a simple T - rescaling
- A simplistic scenario with a single T - independent parameter κ does not provide a systematic treatment which can serve as an alternative expansion scheme
- We allow for more than $\mathcal{O}(\hat{\mu}^2)$ expansion of T' and let the coefficients be T -dependent:

$$\frac{\chi_1^B(T, \hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T', 0) , \quad T' = T (1 + \kappa_2(T) \hat{\mu}_B^2 + \kappa_4(T) \hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6))$$

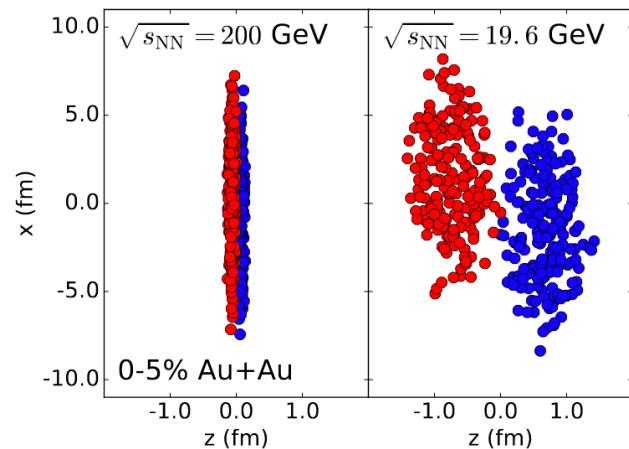
- **Important:** we are simply re-organizing the Taylor expansion via an expansion in the shift

$$\Delta T = T - T' = (\kappa_2(T) \hat{\mu}_B^2 + \kappa_4(T) \hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6))$$

- Comparing the (Taylor) expansion in $\hat{\mu}_B$ and our expansion in ΔT order by order, we can relate $\chi_n^B(T)$ and $\kappa_n(T)$

Scientific goals

- Model the fluctuating initial conditions for the baryon-asymmetric matter for baryon, electric charge, and strangeness



C. Shen, B. Schenke, PRC (2018)
C. Shen, B. Schenke, NPA (2019)

- Develop **(3+1)D** viscous hydrodynamic code which includes all conserved currents and connect it to model for initial conditions
- Extract transport properties of nuclear matter at finite baryon density

G. Denicol et al., PRC (2018)
L. Du et al., NPA (2019)

M. Li, C. Shen, PRC (2018)
C. Gale et al., NPA (2019)

Hydrodynamics evolution

- The sequential collisions between nucleons contribute as energy-momentum and net-baryon density sources to the hydrodynamic fields

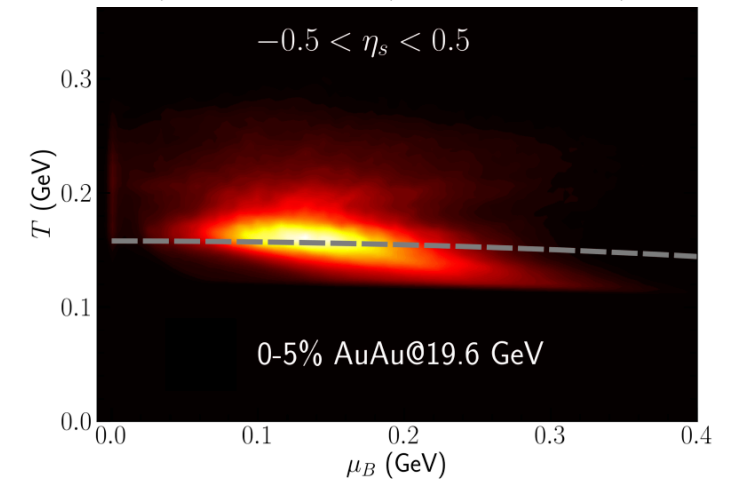
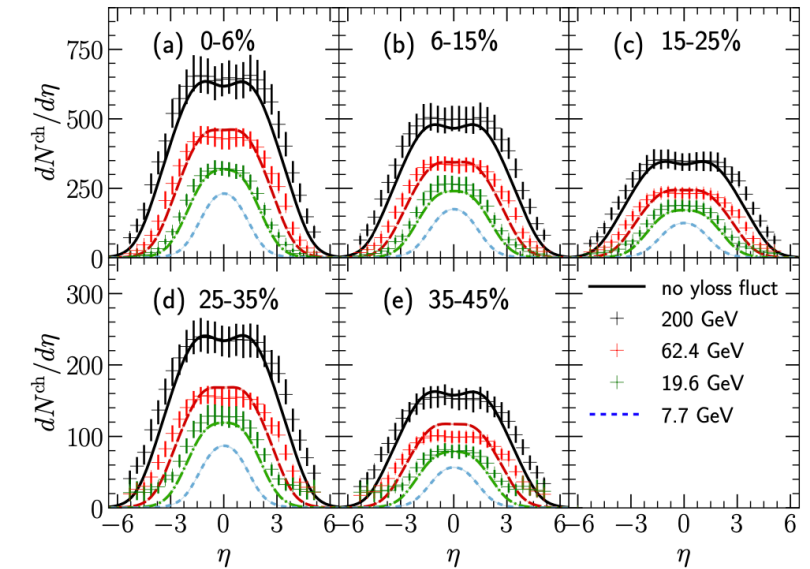
C. Shen, B. Schenke, PRC (2018)

L. Du et al., NPA (2019)

- For recent developments and an alternative method based on a minimal extension of the Glauber model see C. Shen, S. Alzhrani, PRC (2020)

- Relativistic viscous hydrodynamic simulations extended to include the propagation of net baryon current including its dissipative diffusion

C. Shen, B. Schenke, NPA (2018)



Hydrodynamics evolution

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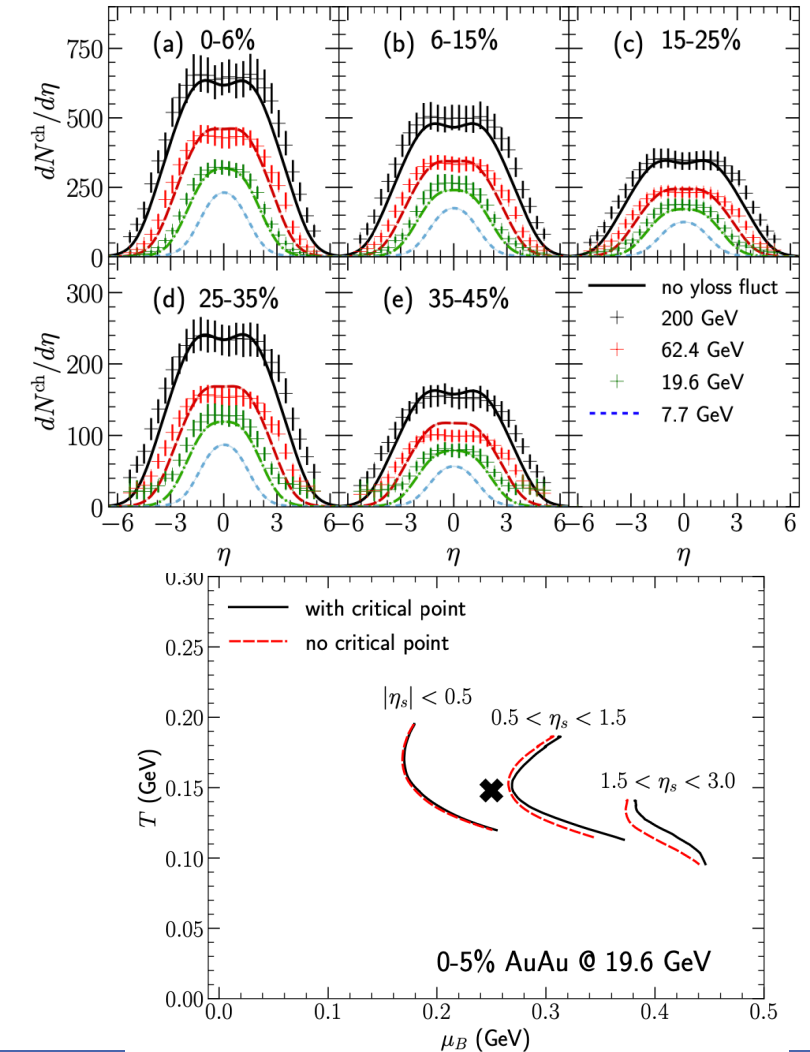
C. Shen, B. Schenke, PRC (2018);

L. Du et al., NPA (2019)

- For recent developments and an alternative method based on a minimal extension of the Glauber model see C. Shen, S. Alzhrani, PRC (2020);

- Relativistic viscous hydrodynamic simulations extended to include the propagation of net baryon current including its dissipative diffusion

C. Shen, B. Schenke, NPA (2018)



Approaches

One of the central goals of the BEST collaboration is to develop quantitative understanding of fluctuations near the CP

- Stochastic approach with noise

M. Nahrgang et al., PRD (2019)

- Deterministic approach in which correlation functions are treated as additional variables with the hydrodynamics ones (Hydro+)

M. Stephanov and Yi Ying, PRD (2018)

- So far only applicable to crossover side of phase boundary
- So far limited to two-point functions

See also Y. Akamatsu et al, PRC (2017 and 2018); M. Martinez and T. Schaefer, PRC (2019); X. An et al., PRC (2020)
S. Pratt and C. Plumberg, PRC (2019 and 2020)

Implementation

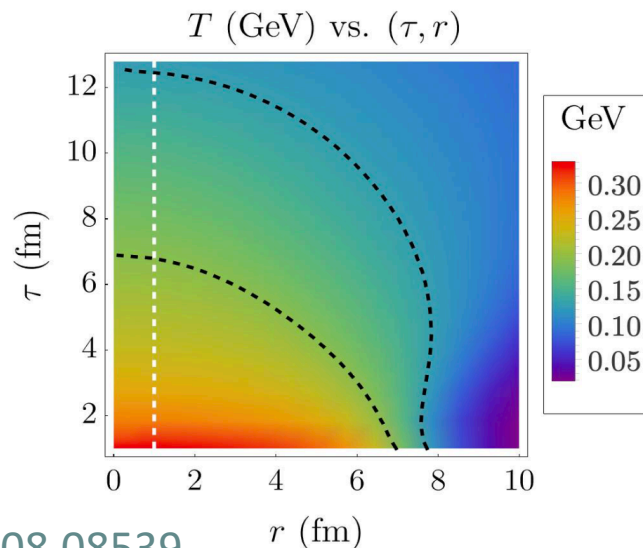
- Solution of stochastic hydro equations using a momentum filter by which fluctuating modes above a cutoff given by a microscopic scale are removed

M. Singh et al., QM2018 proceedings

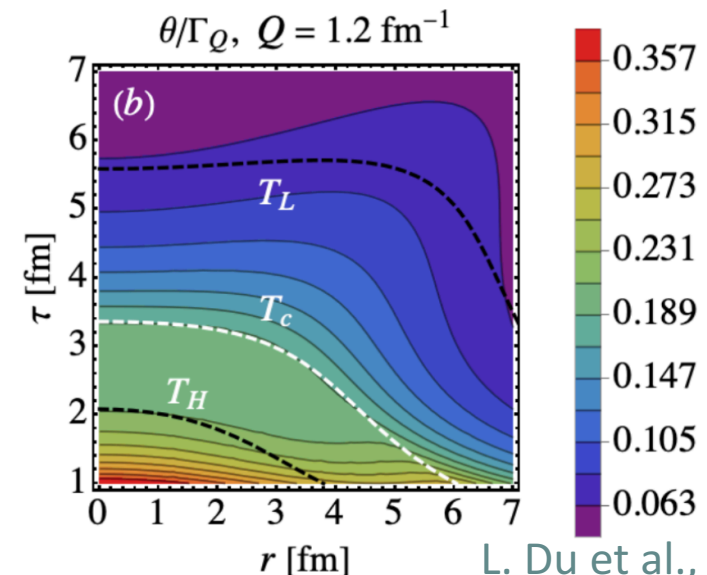
- Solution of full stochastic diffusive equation in a finite-size system with Gaussian white noise: critical slowing down is observed

M. Nahrgang et al., 1804.05728

- Hydro+ implemented in two main simulations



K. Rajagopal et al., 1908.08539



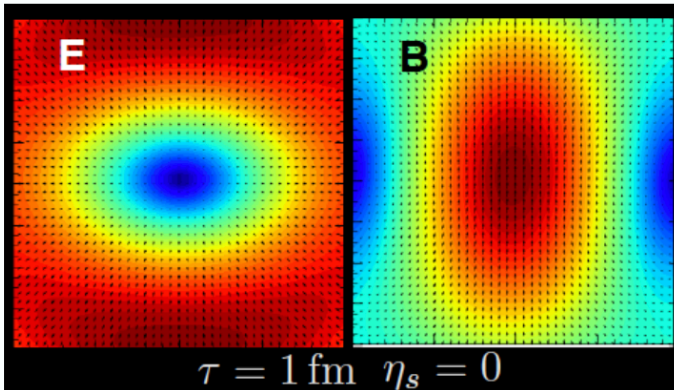
L. Du et al., 2004.02719

Scientific goals and achievements

- Model fluctuating initial conditions for axial charges

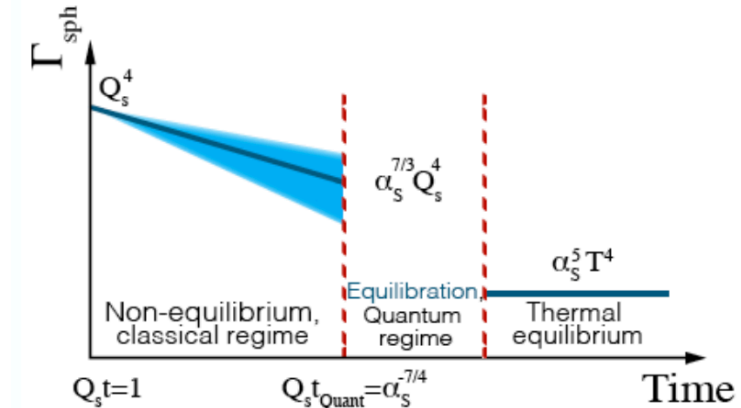
Mace et al., PRD (2016)

Shi et al., PRL (2020)



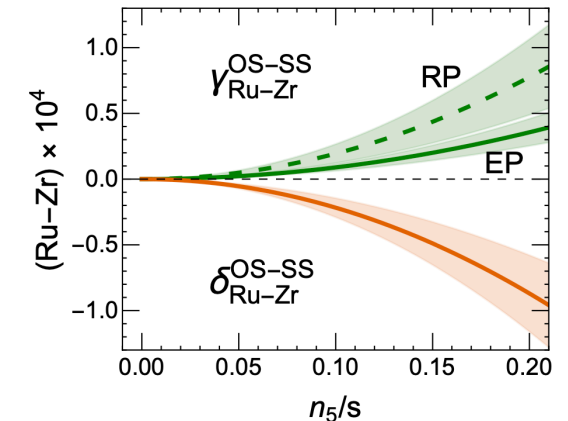
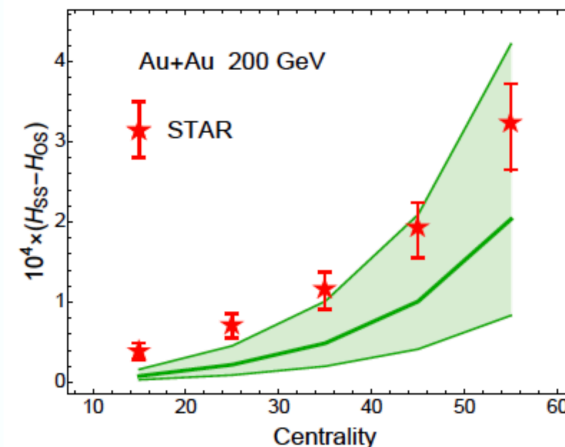
- Quantitatively characterize the experimental signals of CME

Shi et al., Annals of Physics (2018)



- Develop magneto-hydro code and incorporate anomalous hydro terms, studying the co-evolution of the dynamical magnetic field with the medium

U. Gursoy et al., PRC (2018)



Particlization

- Develop the interface between the hydrodynamic evolution and hadronic transport, such that it preserves fluctuations

- micro-canonical Metropolis sampling algorithm: conserves all the charges as well as energy and momentum as given by hydrodynamics

D. Oliinychenko, V. Koch, PRL (2019)

- Particlization of hydro+: projects fluctuations from hydro+ onto the represented hadrons

Pradeep et al., 2109.1318

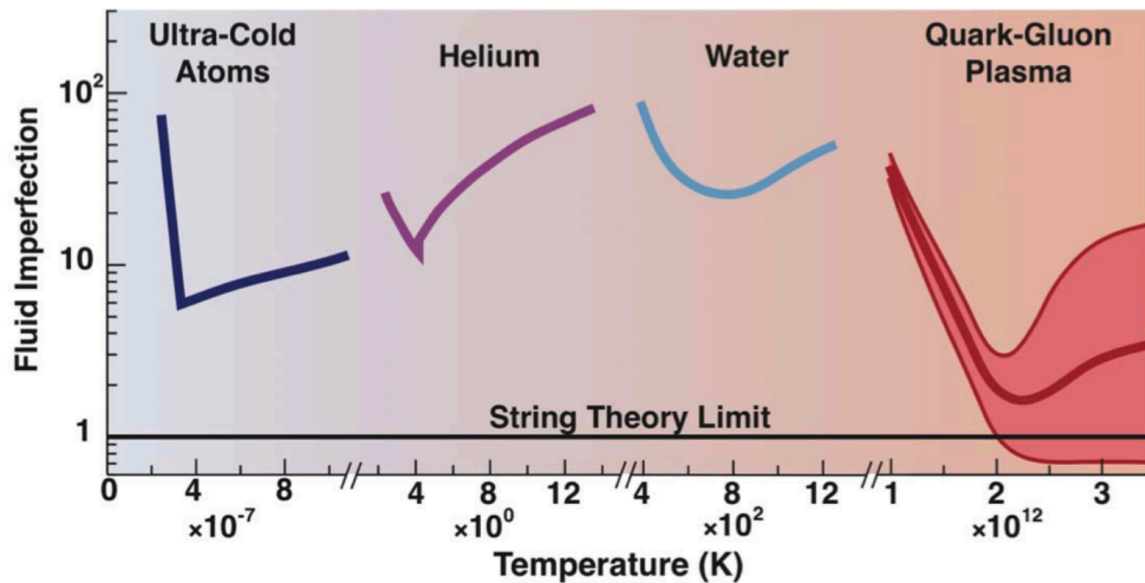
- Hadronic transport with tunable potentials

A. Sorensen, V. Koch, 2011.06635

Things to keep in mind

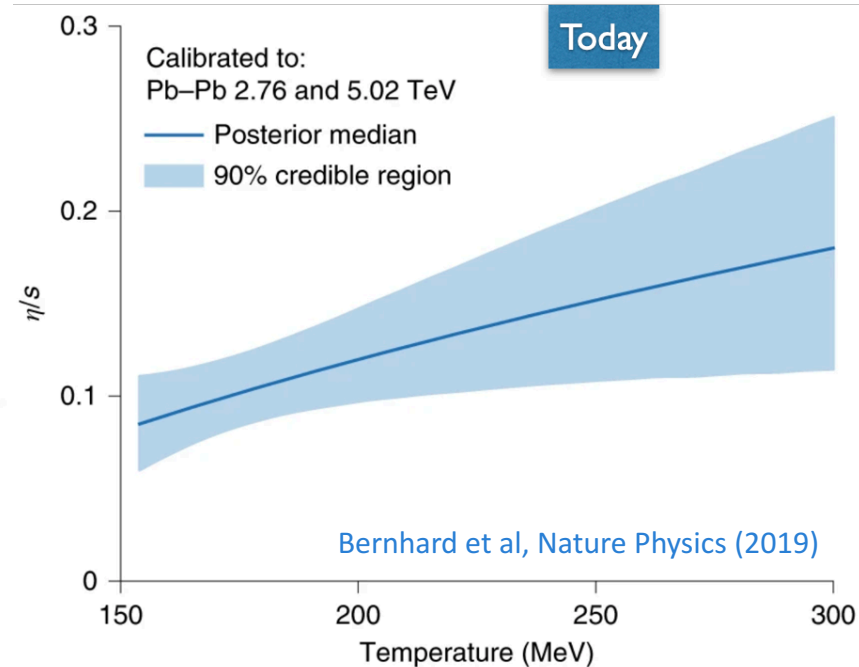
- Effects due to volume variation because of finite centrality bin width
 - Experimentally corrected by centrality-bin-width correction method
V. Skokov et al., PRC (2013), P. Braun-Munzinger et al., NPA (2017),
V. Begun and M. Mackowiak-Pawlowska (2017)
- Finite reconstruction efficiency
 - Experimentally corrected based on binomial distribution
A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
 - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
 - Experimental cuts in the kinematics and acceptance
V. Koch, S. Jeon, PRL (2000)
P. Braun-Munzinger et al., NPA (2017)
- Baryon number conservation
 - Experimental data need to be corrected for this effect
- Proton multiplicity distributions vs baryon number fluctuations
 - Recipes for treating proton fluctuations
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238
- Final-state interactions in the hadronic phase
 - Consistency between different charges = fundamental test
J.Steinheimer et al., PRL (2013)

Nearly perfect fluidity



Kovtun, Son, Starinets,
PRL (2005)

$$\eta/s = 1/4\pi$$



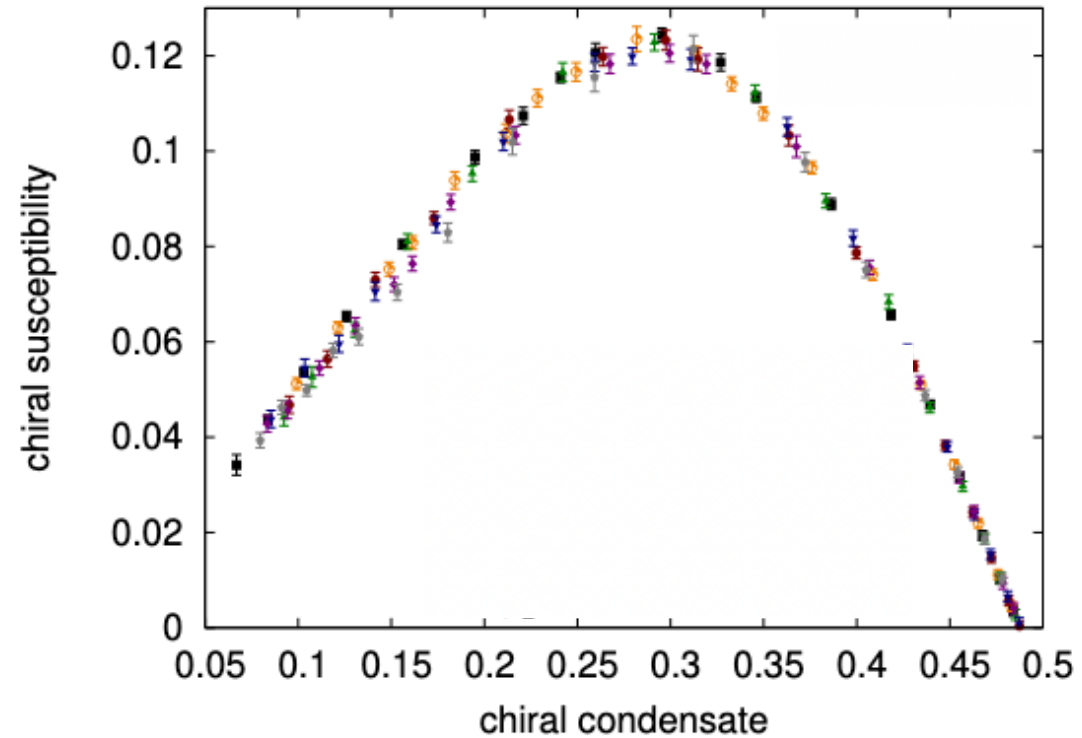
Bernhard et al, Nature Physics (2019)

- Hydrodynamic description of the system created in heavy-ion collisions works extremely well
- It needs an equation of state as input

Phase Diagram from Lattice QCD

The transition at $\mu_B=0$ is a smooth crossover

Aoki et al., Nature (2006)
Borsanyi et al., JHEP (2010)
Bazavov et al., PRD (2012)



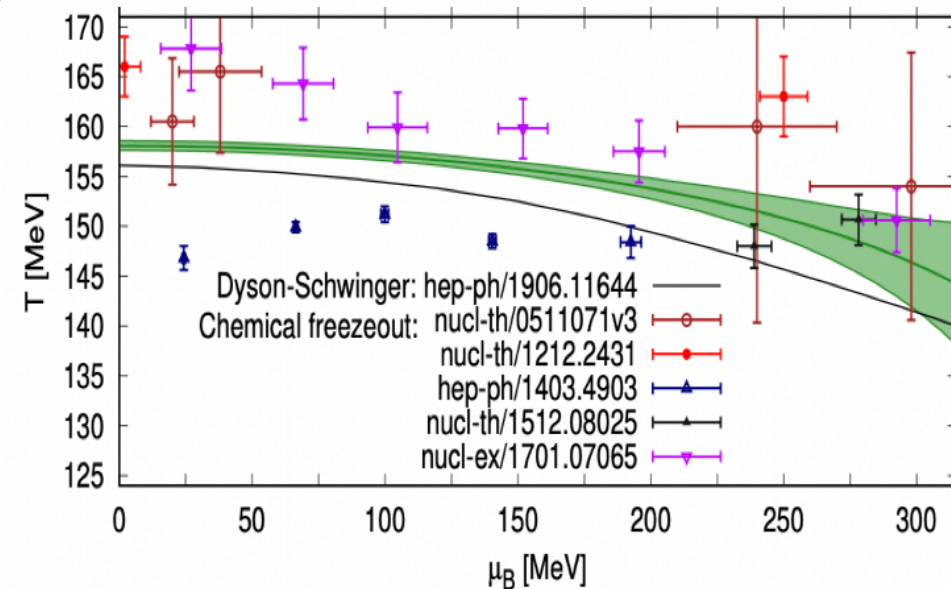
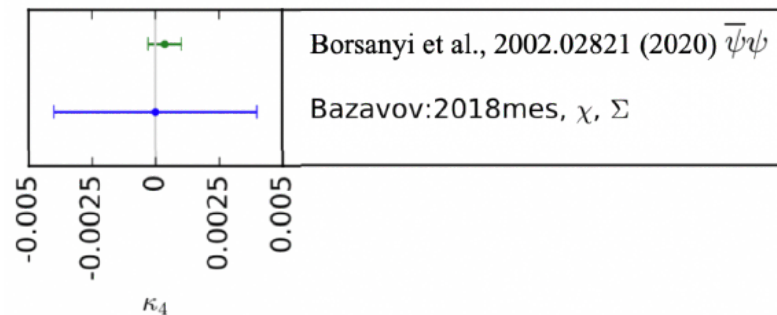
QCD transition temperature and curvature

$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + O(\mu_B^6)$$

Borsanyi, C. R. et al. PRL (2020)

- Latest results on T_0 from WB collaboration based on subtracted chiral condensate and chiral susceptibility

$T_0 = 158.0 \pm 0.6$ MeV



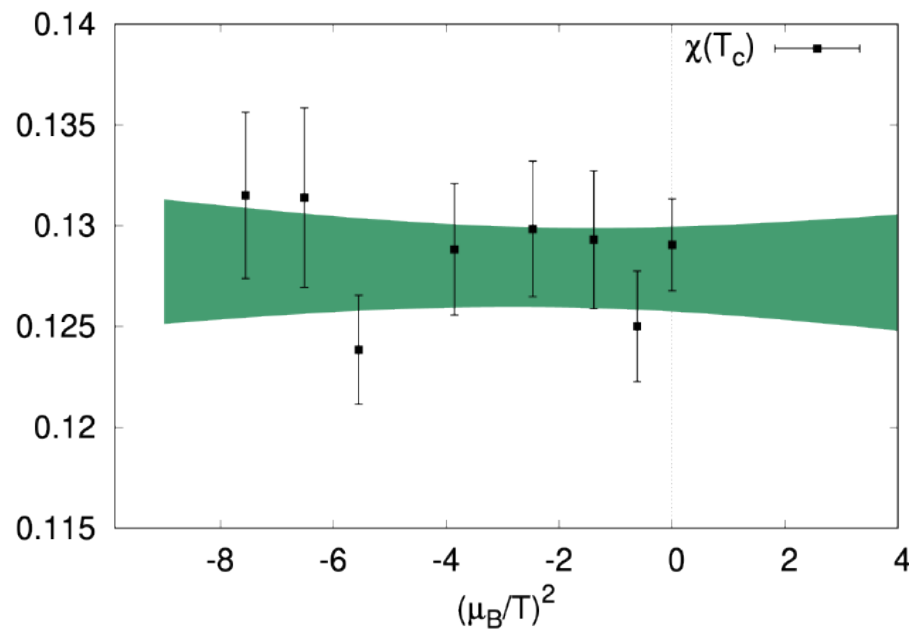
$$\kappa_2 = 0.0153 \pm 0.0018$$

$$\kappa_4 = 0.00032 \pm 0.00067$$

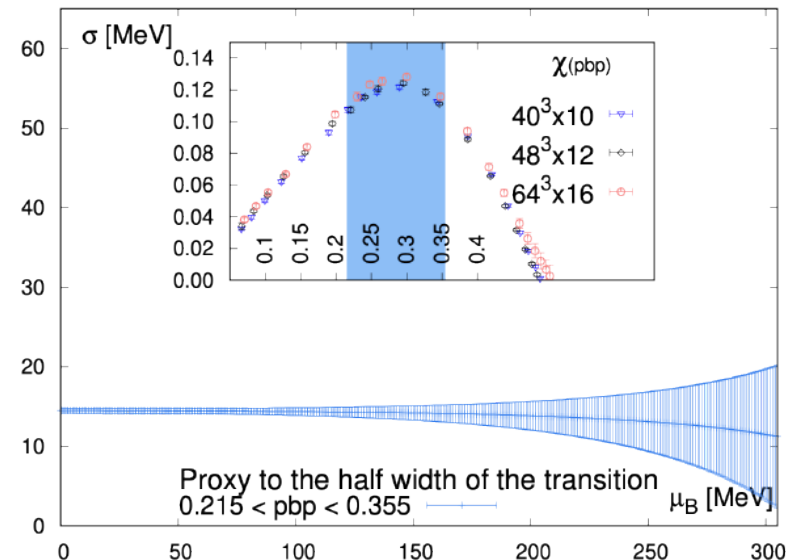
Limit on the location of the critical point

For a genuine phase transition, the height of the peak of the chiral susceptibility diverges and the width shrinks to zero

Height of chiral susceptibility peak



Width of chiral susceptibility peak



No sign of criticality for $\mu_B < 300$ MeV

Borsanyi, C. R. et al. PRL (2020)

Fluctuations of conserved charges

Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p / T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

Relationship between chemical potentials:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q;$$

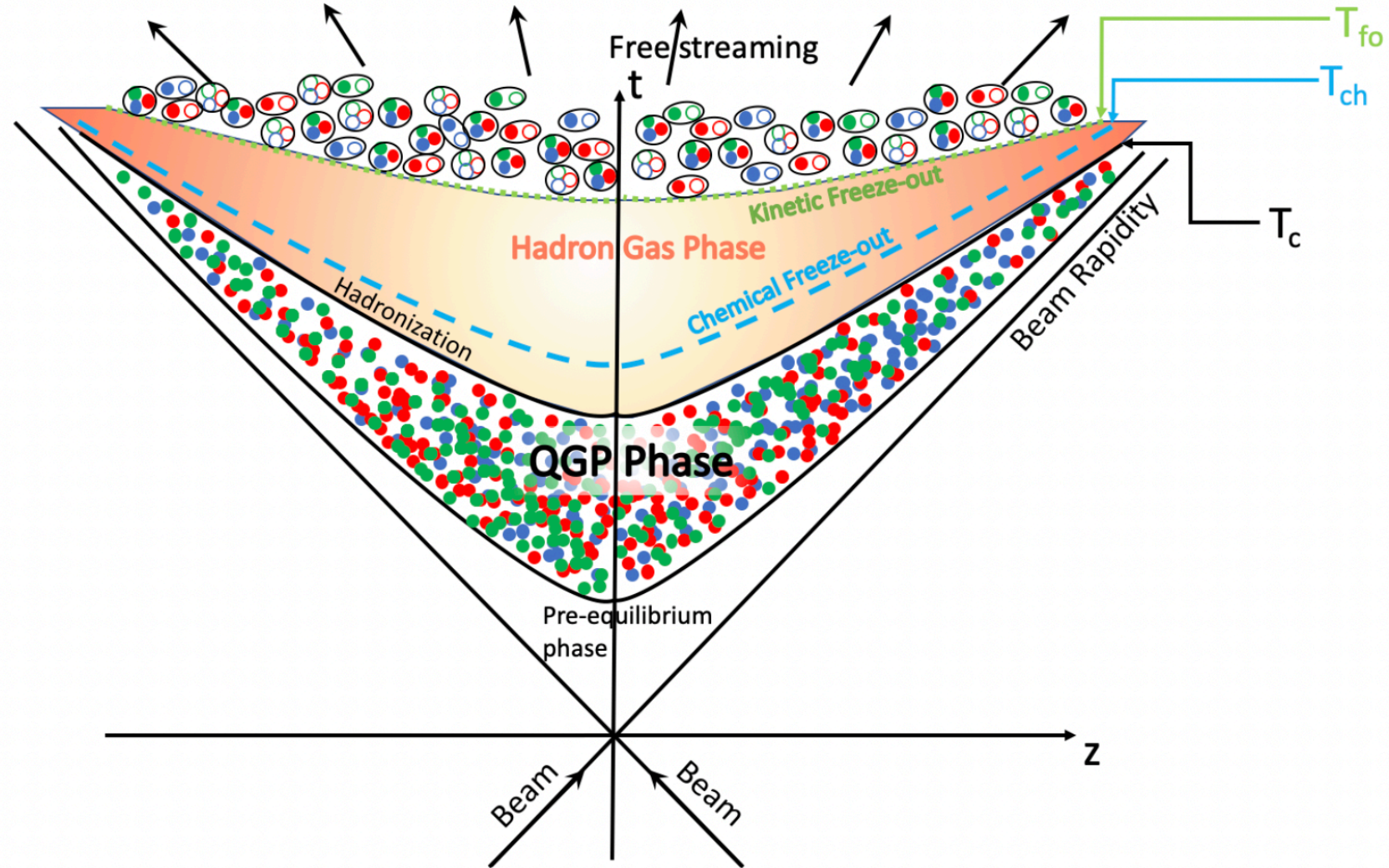
$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q;$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.$$

They can be calculated on the lattice and compared to experiment

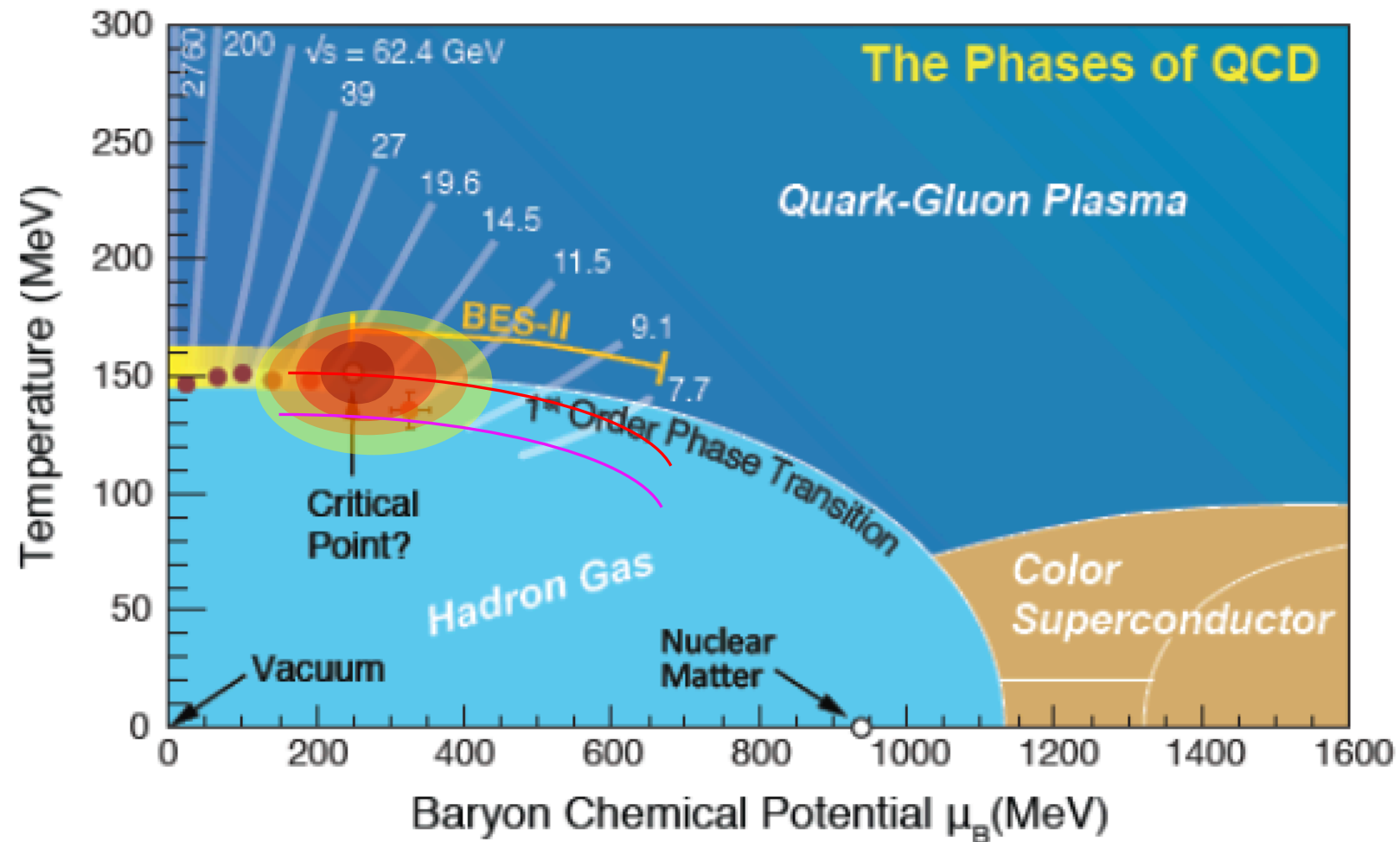
Evolution of a heavy-ion collision

- **Chemical freeze-out:** inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- **Kinetic freeze-out:** elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector



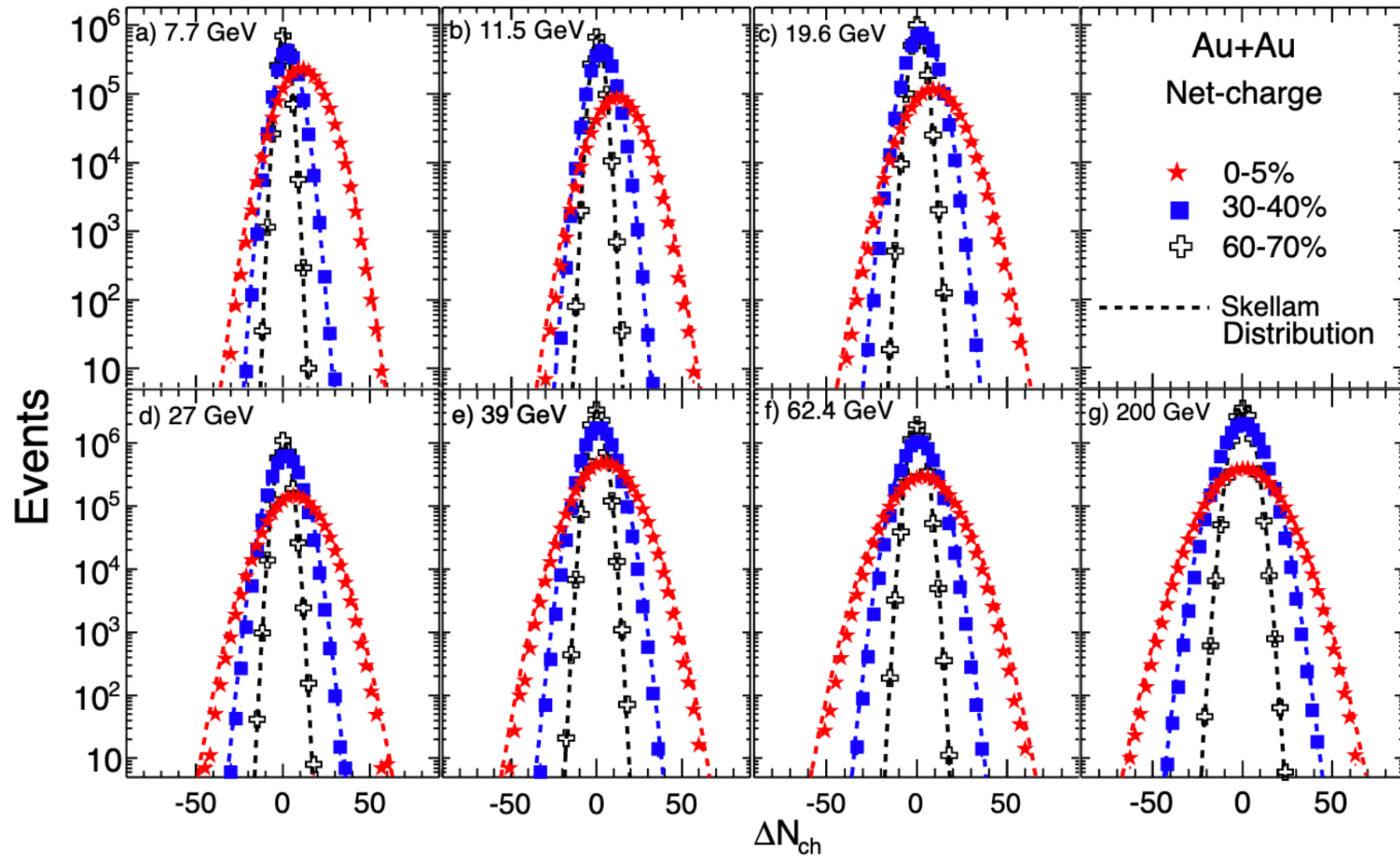
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- **Kinetic freeze-out:** elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector



Connection to experiment

- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to measure its **event-by-event distribution**



Connection to experiment

Fluctuations of conserved charges are the **cumulants** of their event-by-event distribution

$$\text{mean : } M = \chi_1$$

$$\text{variance : } \sigma^2 = \chi_2$$

$$\text{skewness : } S = \chi_3/\chi_2^{3/2}$$

$$\text{kurtosis : } \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2$$

$$\kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2$$

$$S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0$$

$$\langle n_Q \rangle = 0.4 \langle n_B \rangle$$

“Baryometer and Thermometer”

Let us look at the Taylor expansion of $R^{B_{31}}$

$$R_{31}^B(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = \frac{\chi_4^B(T, 0) + \chi_{31}^{BQ}(T, 0)q_1(T) + \chi_{31}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

- To order μ_B^2 it is independent of μ_B : it can be used as a **thermometer**
- Let us look at the Taylor expansion of $R^{B_{12}}$

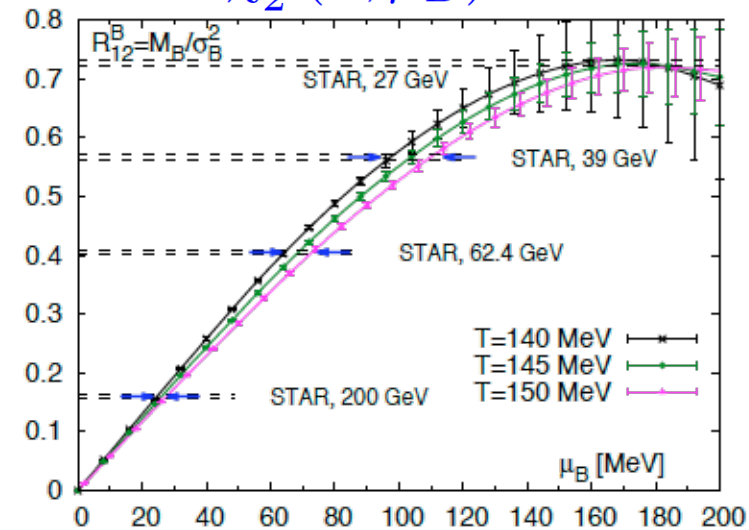
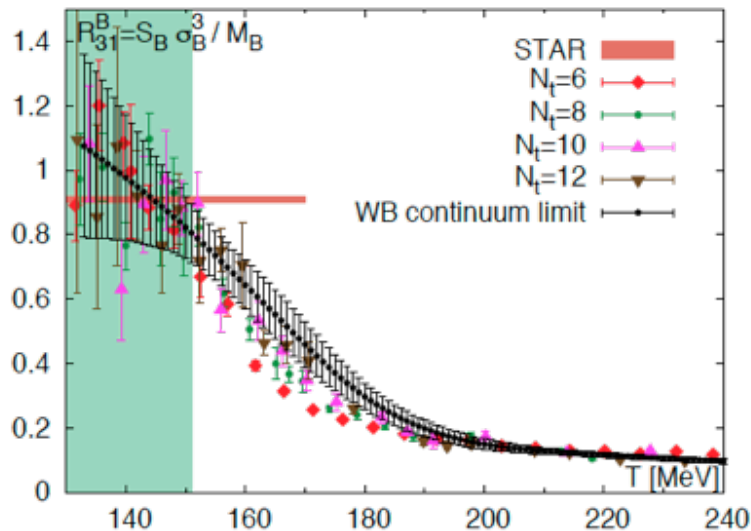
$$R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

- Once we extract T from $R^{B_{31}}$, we can use $R^{B_{12}}$ to extract μ_B

Freeze-out parameters from B fluctuations

➤ Thermometer: $\frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = S_B \sigma_B^3 / M_B$

Baryometer: $\frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \sigma_B^2 / M_B$



Upper limit: $T_f \leq 151 \pm 4$ MeV

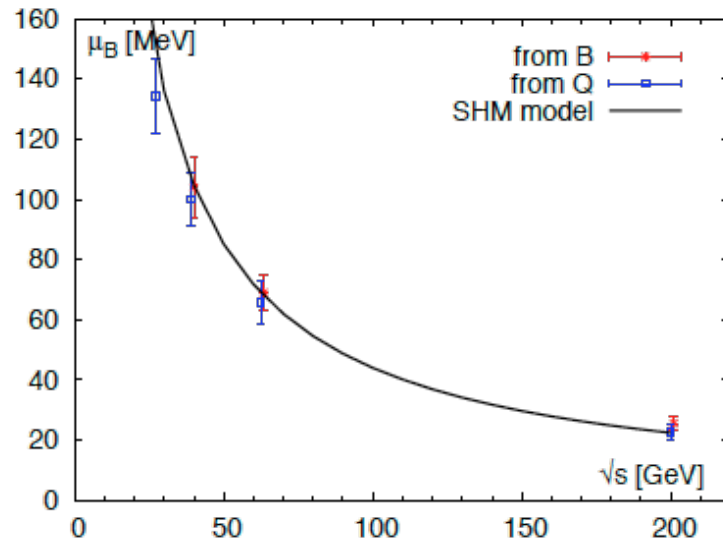
WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

Consistency between freeze-out chemical potential from electric charge and baryon number is found.

Freeze-out parameters from B fluctuations

➤ Thermometer: $\frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = S_B \sigma_B^3 / M_B$

Baryometer: $\frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \sigma_B^2 / M_B$



\sqrt{s} [GeV]	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8 ± 2.7	22.8 ± 2.6
62.4	69.7 ± 6.4	66.6 ± 7.9
39	105 ± 11	101 ± 10
27	-	136 ± 13.8

WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

Upper limit: $T_f \leq 151 \pm 4$ MeV

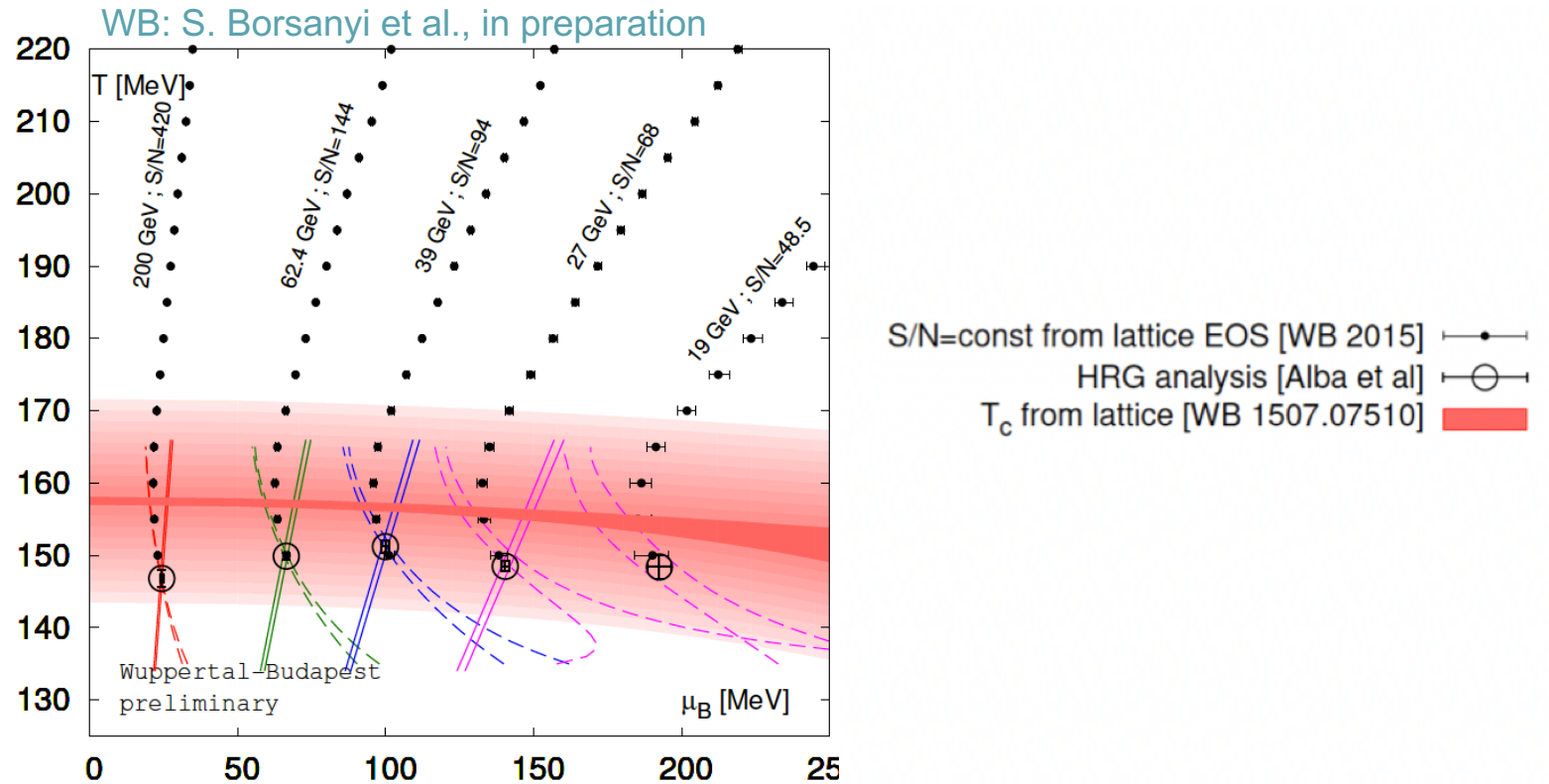
Consistency between freeze-out chemical potential from electric charge and baryon number is found.

Freeze-out line from first principles

Use T - and μ_B -dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T, \mu_B) = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} = \frac{\chi_{11}^{QB}(T, 0) + \chi_2^Q(T, 0)q_1(T) + \chi_{11}^{QS}(T, 0)s_1(T)}{\chi_2^Q(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

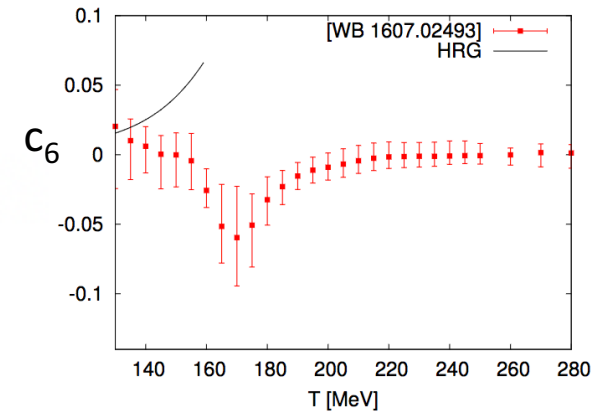
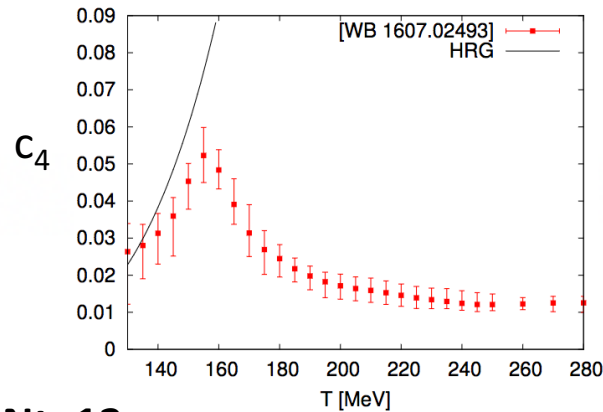
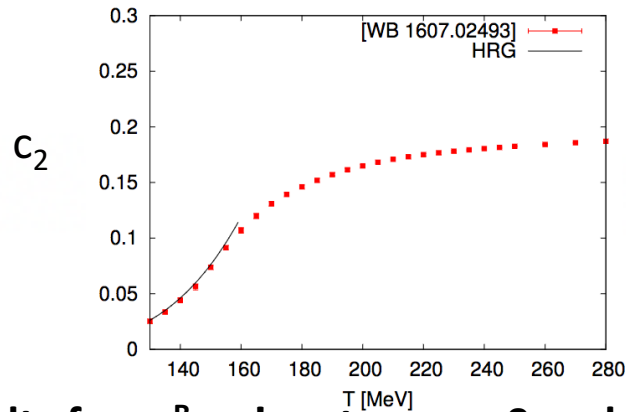
$$R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$



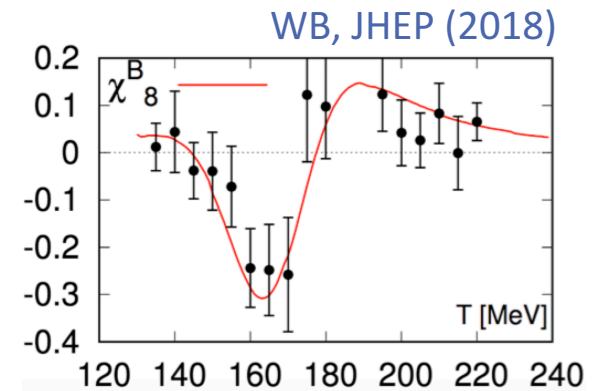
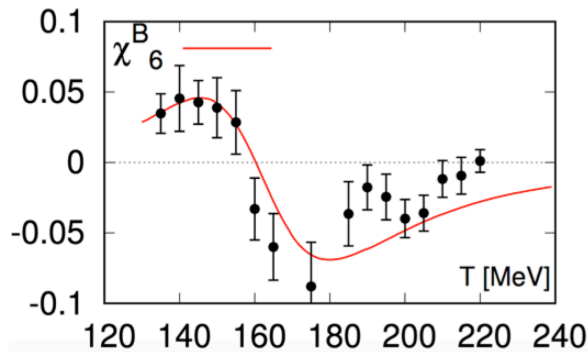
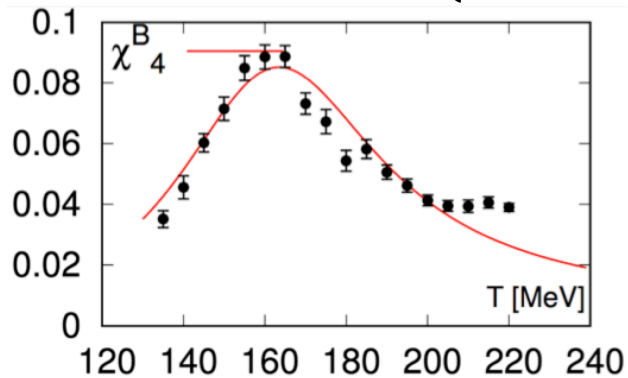
Pressure coefficients

Simulations at imaginary μ_B :

Continuum, $O(10^4)$ configurations, errors include systematics (WB: NPA (2017)) **Strangeness neutrality**

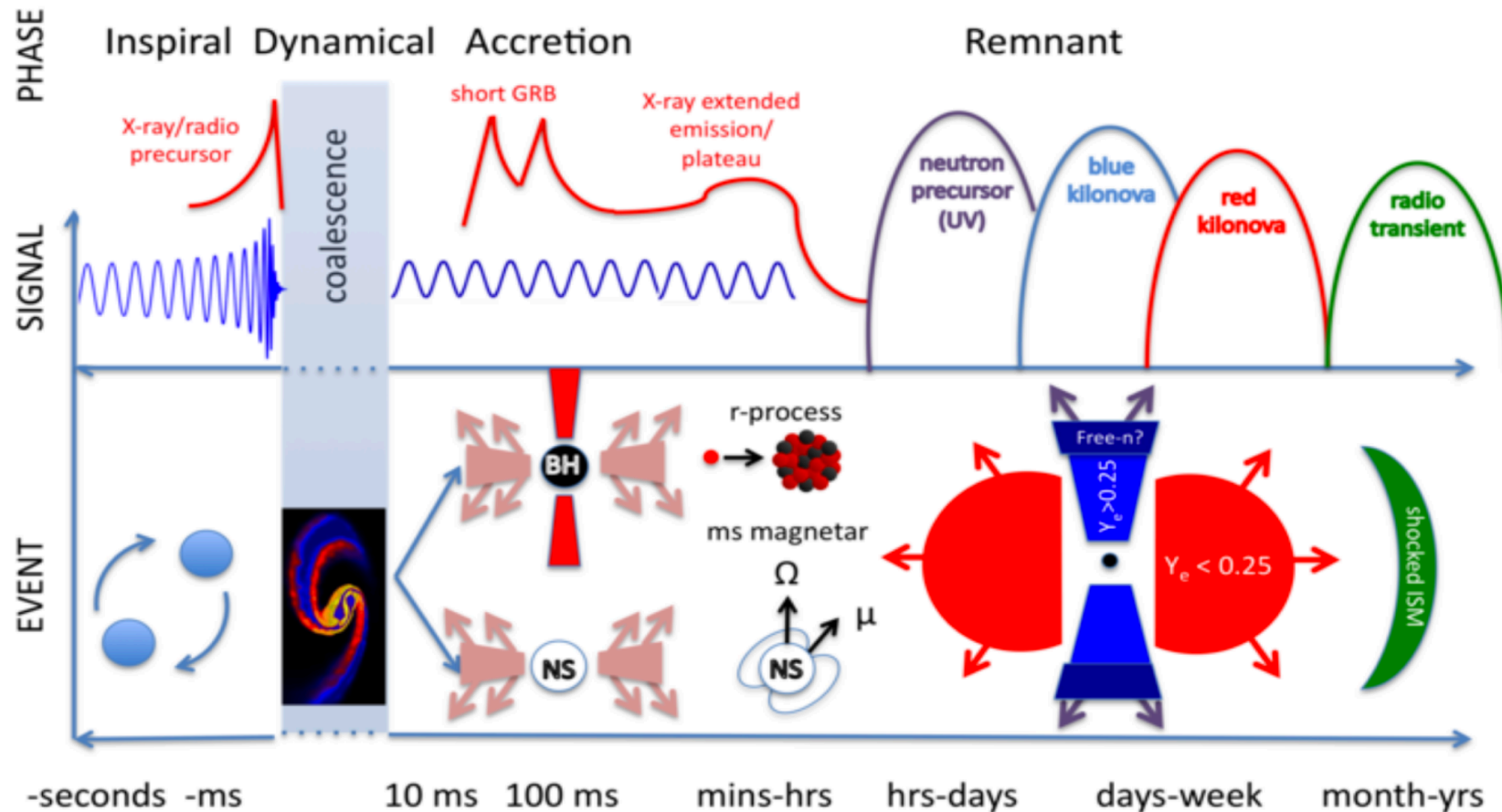


Results for $\chi_n^B = n!c_n$ at $\mu_S = \mu_Q = 0$ and $Nt=12$



WB, JHEP (2018)

Anatomy of a multi-messenger merger



Nuclear Physics is encoded in all phases!



A few Lessons learned

- Heavy ion collisions:
 - Phase transition at small μ_B is a smooth crossover
 - If a critical point exists, it is in the 3D-Ising model universality class
 - Equation of state and phase diagram are known from 1st principles at $\mu_B/T < 3.5$
 - Quark-Gluon Plasma is a strongly coupled fluid with very small viscosity/entropy
- Neutron star mergers:
 - GWs travel essentially at the speed of light
 - binary neutron star mergers are progenitors of short gamma ray bursts
 - they are prolific sites for the formation of heavy elements
 - constrained neutron-star radii to be between 9.5 and 13 km