physics@mit

MEMORANDUM

To: Yi, MIT AMS group

From: Peter

Subject: Memo #93 - Energy Loss by a bare monopole

Date: September 7, 2022

Assume monopoles are *bare*, $q_e = 0$ and $q_m \neq 0$. q_e has units A-s and q_m has units A-m. From $\vec{\nabla} \cdot \vec{B} = \mu_o \rho_m$, Figure 1,

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q_m}{r^2} \hat{r}$$

gives the static magnetic field for a monopole. Equating the forces between two electrons and two monopoles the same distance apart gives an arbitrary quantization of q_m,

$$\frac{1}{4\pi\epsilon_o} \frac{q_e^2}{r^2} = \frac{\mu_o}{4\pi} \frac{q_m^2}{r^2} \rightarrow q_m = c q_e.$$

Figure 2 shows the scattering of a magnetic monopole from a stationary atomic electron. The moving monopole creates an azimuthal electric field,

$$\vec{E}_\phi = \frac{v\mu_o}{4\pi} \frac{q_m}{r^2} \hat{\phi}$$

Maxwell's equations and Lorentz force equation with magnetic monopoles: SI units

Name	Without magnetic monopoles	With magnetic monopoles	
		Weber convention	Ampere-meter convention
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon_0}$		
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = \rho_m$	$\nabla \cdot \mathbf{B} = \mu_0 \rho_m$
Faraday's law of induction	$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$	$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{j}_m$	$-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mu_0 \mathbf{j}_m$
Ampère's law (with Maxwell's extension)	$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j}_e$		
Lorentz force equation	$\mathbf{F} = q_e (\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\mathbf{F} = q_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{q_m}{\mu_0} \left(\mathbf{B} - \mathbf{v} \times \frac{\mathbf{E}}{c^2} \right)$	$\mathbf{F} = q_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m \left(\mathbf{B} - \mathbf{v} \times \frac{\mathbf{E}}{c^2} \right)$

Figure 1: Maxwell's equations with magnetic monopoles in MKS units[**?**]

resulting in a force $F\phi = eE_\phi$ on the electron. $r = \sqrt{v^2t^2 + b^2}$ and

$$\Delta p = \int_{-\infty}^{+\infty} \frac{v\mu_o e q_m}{v^2t^2 + b^2} \quad (1)$$

$$= Q_m \frac{\pi \mu_o e^2}{4b} = Q_m \frac{\alpha \pi \hbar}{b} \quad (2)$$

where $Q_m = q_m/e$ and $\alpha = e^2/4\pi\epsilon_o\hbar c$. $K = \Delta p^2/2m_e$ gives the kinetic energy transferred to the electron from the monopole, assuming the electron does not move much during the collision, see below.

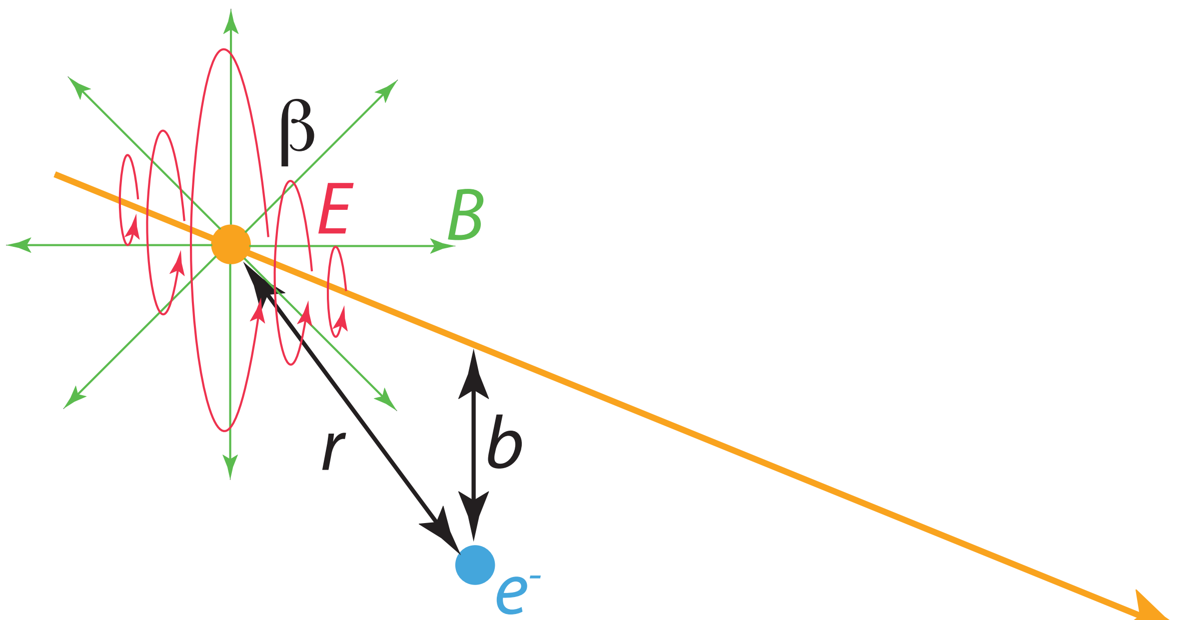

Figure 2: Layout of scattering. The orange disk show the position of the monopole that follows the orange vector with a speed $\beta = v/c$. The blue disk shows the electron, initially at reast. The monopole has a radial magnetic field shown in green and the red vectors shows the the monopole's circumferential electric arising from its motion.

Next,

$$\frac{d\sigma}{dK} = b\left|\frac{db}{dK}\right| = Q_m\alpha^2\frac{m_e b^4}{\pi^2\hbar^2} = Q_m\alpha^2\frac{\pi^2 hbar^2}{4m_e K^2}$$

gives the differential cross section and $d^2P/dKdx = n\frac{d\sigma}{dK}$ gives the probability for the monopole to transfer kinetic energy K to an atomic electron while cross a medium of electron number density n and thickness dx. Finally,

$$\left\langle \frac{dK}{dx} \right\rangle = \int_{K_{min}}^{K_{max}} Kn\frac{d^P}{dxdK}dK \quad (3)$$

$$= Q_m\alpha^2\frac{\pi^2\hbar^2 n}{4m_e}\ln\frac{K_{max}}{K_{min}} \quad (4)$$

gives the energy energy loss per unit length. The electron binding energy fixes K_{min} ~10 eV, depending on the material, and, the maximum energy transfer to the electron,

$$K_{max} = \frac{1+\beta^2}{1-\beta^2} m_e$$

fixes $K_{max} \sim 2\gamma m_e$ if $\beta \sim 1$.

For silicon, $n = 7 \times 10^2 9/\mathrm{m}^3$. For a monopole moving with $\beta = 0.5$. $M_{max} = 4$ MeV, giving $b_{min} = 303$ fm. $Kmin = 8$ eV, giving $b_{max} = 216$ pm. $\langle dK/dx \rangle = 0.36 MeV/\mathrm{g/cm}^2$, compared with $\sim 2MeV/g/\mathrm{cm}^2$ for minimum ionizing particles.

Does the electron move during the collision? Equation 2 remains valid if the electron move by $d << b$ during the collision. The collision takes place over a time $\tau \sim 2v/b$, which translates to a condition,

$$\begin{aligned}
\beta \quad &>> \quad \frac{\Delta p}{m_e} && (5)\\
&>> \quad \sqrt{\frac{2K}{m_e}} && (6)\\
\frac{1}{2} m_e \beta^2 \quad &= \quad 2.5\mathrm{keV} >> K, && (7)
\end{aligned}$$

implying the approximation remains valid for recoil energies up to $K_{valid} = m_e\beta^2/2$=2.5 keV. Higher energy recoild contribute very little to the cross section: the differential cross section $d\sigma/dK$ gives the probability of a recoil energy K and,

$$\frac{\int_{K_{min}}^{K_{valid}} \frac{d\sigma}{dK} dK}{\int_{K_{min}}^{K_{max}} \frac{d\sigma}{dK} dK} = 0.997.$$

In experiments, base monopole will show up in at least two ways: anomalous energy loss and parabolic trajectory in a magnetic spectrometer. Figure 3 shows the average energy loss in silicon for a variety of energies compared with the light isotopes.

For Q_m, monopoles traversing a magneitc spectrometer will show large deflections. For example, in a 0.15 T field and a 1 m measuring distance, a $\beta = 0.3$ 1 GeV monopole will deflect by 25 cm.

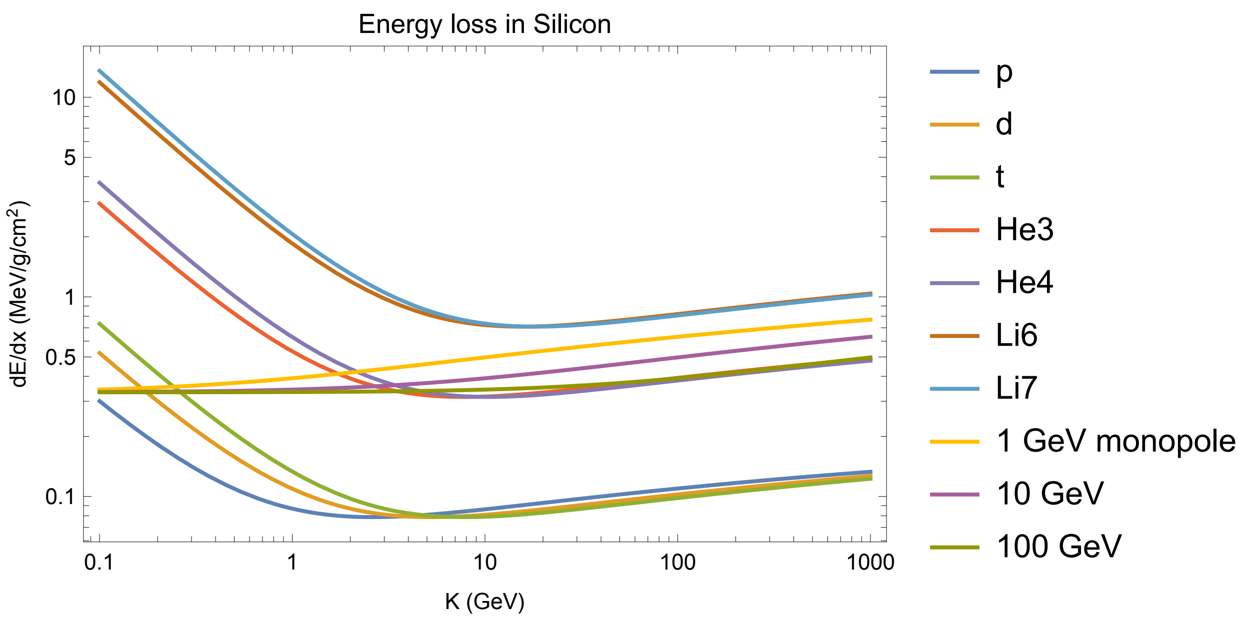

Figure 3: Bare monopole energy loss in silicon for $Q_M = 1$.