Generalized Parton Distributions from Lattice QCD

USQCD All Hands' Meeting
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MAY 1, 2021

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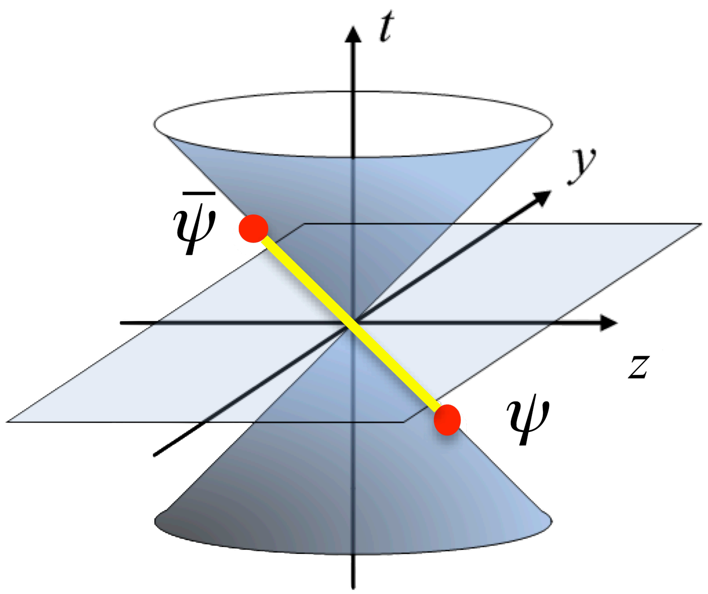
GPDs and 3D Imaging of the Proton

- Definition:

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle \bar{P} + \frac{\Delta}{2}, S' | O_{\gamma^+}(\lambda n) | \bar{P} - \frac{\Delta}{2}, S \rangle = \bar{u}(P', S') \left[H(x, \xi, t)\gamma^+ + E(x, \xi, t) \frac{i\sigma^{+\mu}\Delta_\mu}{2M} \right] u(P, S)$$



$$\tilde{O}_{\gamma^+}(z^-) = \bar{\psi}(\frac{z^-}{2})\gamma^+ W(\frac{z^-}{2}, -\frac{z^-}{2})\psi(-\frac{z^-}{2}),$$

$$\lambda = \bar{P} \cdot z = \bar{P}^+ z^- , \quad t \equiv \Delta^2, \quad \xi \equiv -\frac{\Delta^+}{2\bar{P}^+}$$

Skewness parameter

Breit frame:

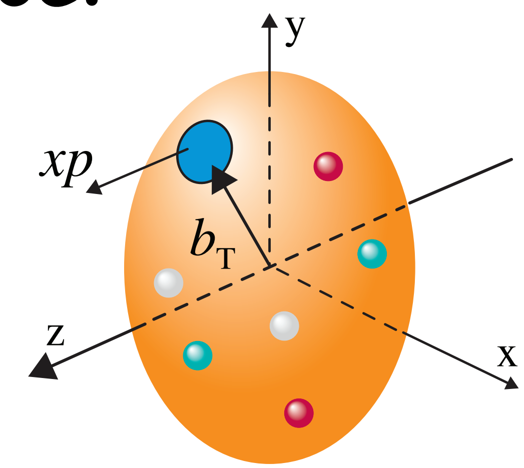
$$\bar{P}^\mu = \bar{P}^+ n^\mu = (\bar{P}^+, 0, 0, \bar{P}^+)/\sqrt{2}$$

- X. Ji, PRL 78 (1997) 610-613;
- X. Ji, PRD 55 (1997) 7114-7125;
- A. Radyushkin, PRD 56 (1997) 5524-5557.

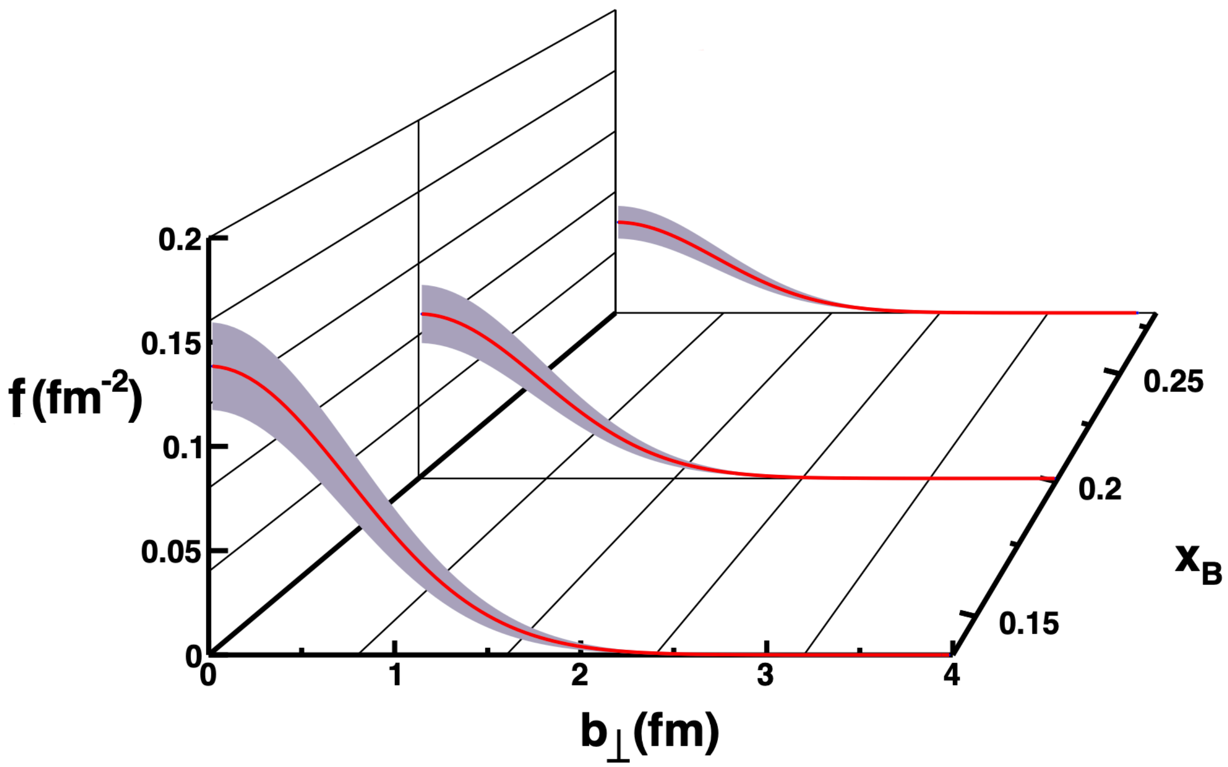
GPDs and 3D Imaging of the Proton

- GPDs at zero skewness are number densities of color charges in the longitudinal x- and transverse position space:

$$f(x, \overrightarrow{b}_T) = \int d^2\overrightarrow{\Delta}_T \, e^{i\overrightarrow{\Delta}_T \cdot \overrightarrow{b}_T} H(x, \xi = 0, t = -\Delta_T^2)$$



- M. Burkadrt, PRD 62 (2000) 071503;
- M. Burkardt, IJMPA 18 (2003) 173-208.



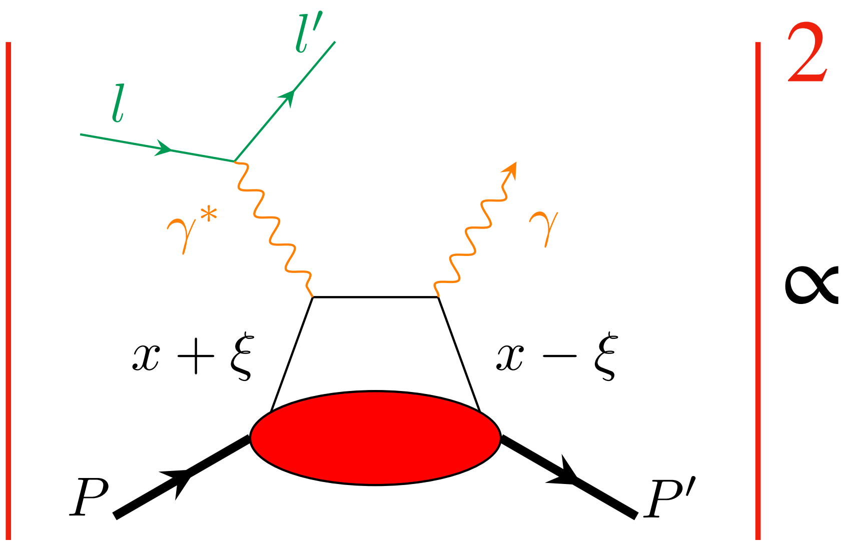
Goal: dissecting the proton in the transverse plane

W. Armstrong et al., 1708.00888.

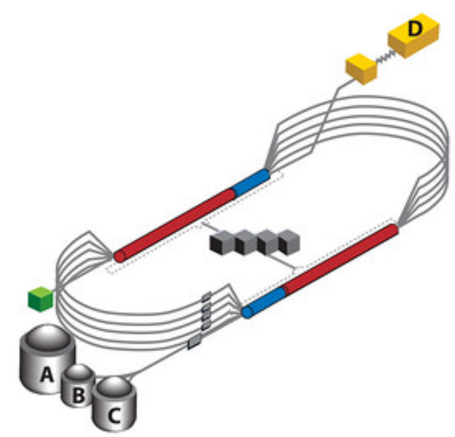
GPDs and 3D Imaging of the Proton

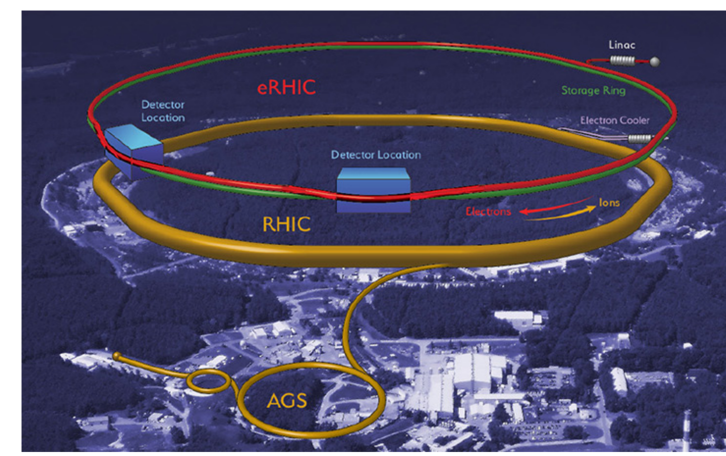
- GPDs can be measured from hard exclusive processes such as deeply virtual Compton scattering. X. Ji, PRD 55 (1997) 7114-7125.

$$\mathscr{H}(\xi,t) = \int_{-1}^{1} dx \left[\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0}\right] H(x,\xi,t)$$



$$\propto |\mathscr{H}(\xi,t)|^2$$



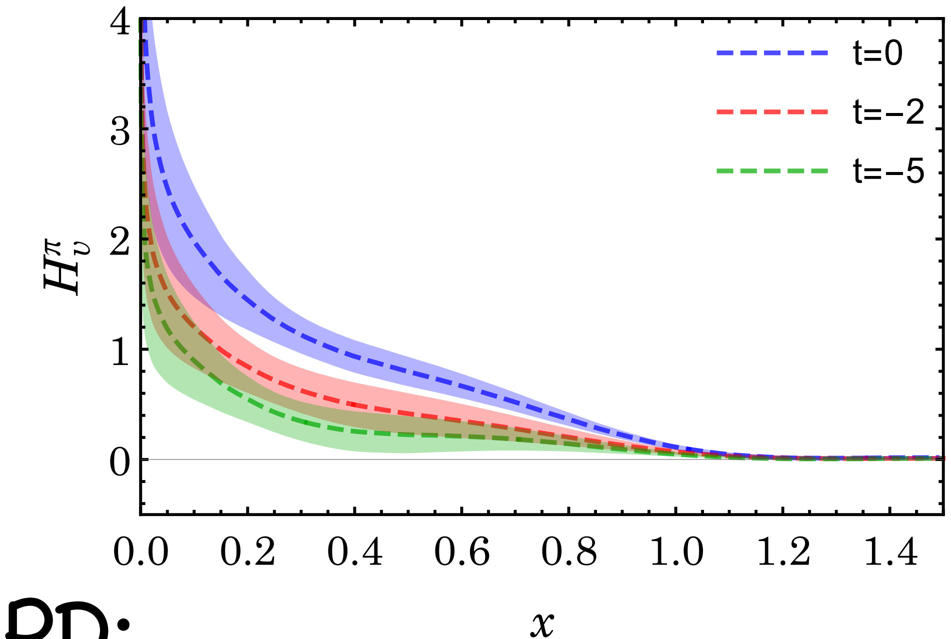


- However, it is the Compton form factors that are extracted from the experiments:
  - Inversion problem to obtain the GPD;
  - Modeling the GPD is inevitable without nonperturbative input, but it is hard to satisfy the many constraints such as forward limit as PDF, polynomiality, etc.

First Lattice Calculations

- Pion valence GPD:

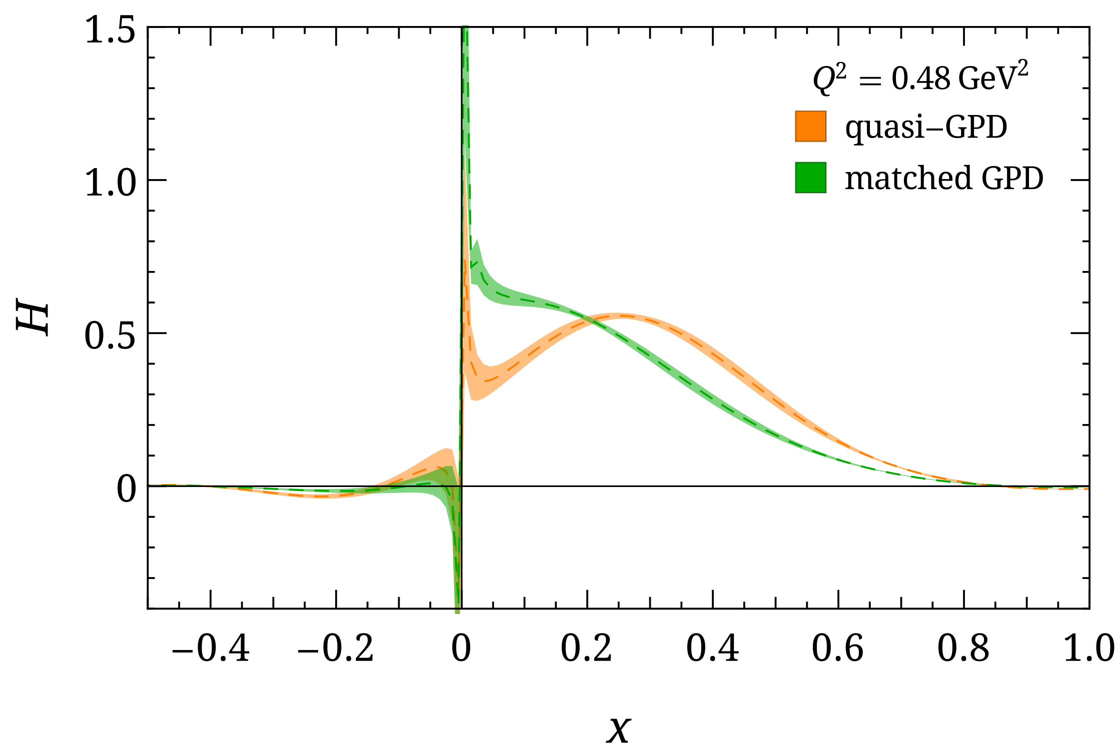
J.W. Chen et al., NPB 952 (2020) 114940.



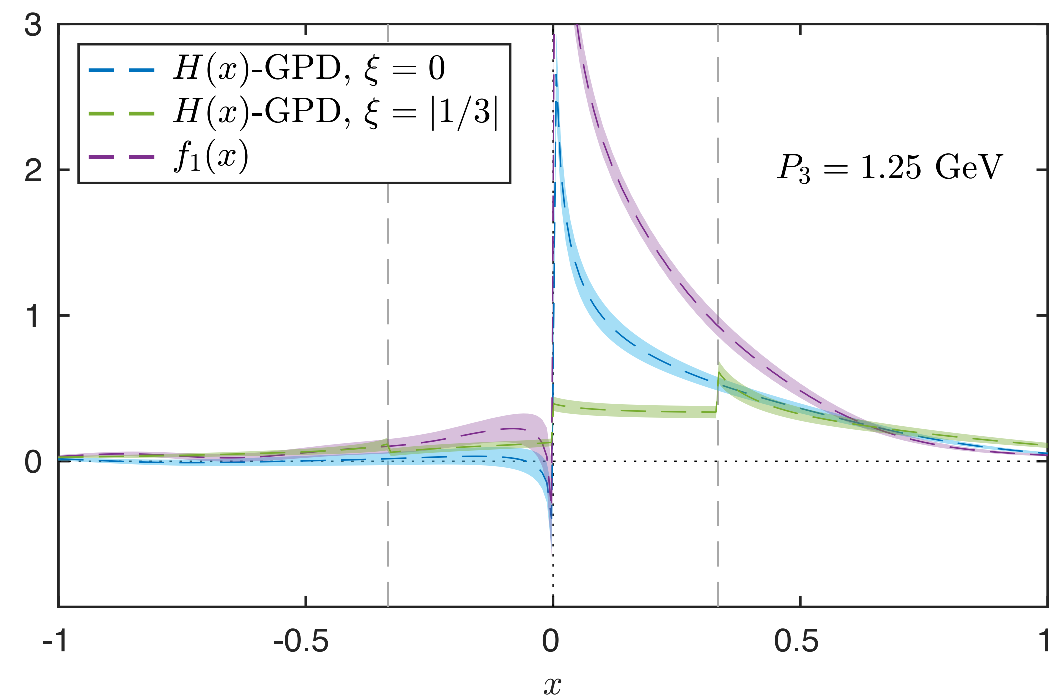
Method: large-momentum effective theory (LaMET) or quasi-GPD

X. Ji, PRL 110 (2013) 262002

- Proton isovector GPD:



H. W. Lin, 2008.12474.



C. Alexandrou et al. (ETM), PRL 125 (2020) 26, 262001. Unpolarized and Helicity cases.

Pion GPD calculation at BNL

Why do we study the pion/kaon structure?

- Pseudo-Nambu-Goldstone boson of spontaneous chiral symmetry breaking in QCD, key to understanding hadron mass and its relation to hadron structure;

- Lattice computation is cheaper than the nucleon. Can achieve large Lorentz boost with small momenta due to light mass;

- Systematic improvements in the current pion calculations will set the stage for computing the proton GPDs.

Pion GPD calculation at BNL

Computational details: Wilson-clover fermion on HotQCD HISQ ensembles

n_z	P_z (GeV)		ζ
	$a = 0.06$ fm	$a = 0.04$ fm	
0	0	0	0
1	0.43	0.48	0
2	0.86	0.97	1
3	1.29	1.45	2/3
4	1.72	1.93	3/4
5	2.15	2.42	3/5
	$48^3 \times 64$	$64^3 \times 64$	

$m_\pi = 300$ MeV

$a = 0.076$ fm

$P_z = 0$ GeV, 0.25 GeV, 0.51 GeV, 0.76 GeV, 1.02 GeV, 1.27 GeV, 1.53 GeV, 1.78 GeV, 2.04 GeV, 2.29 GeV

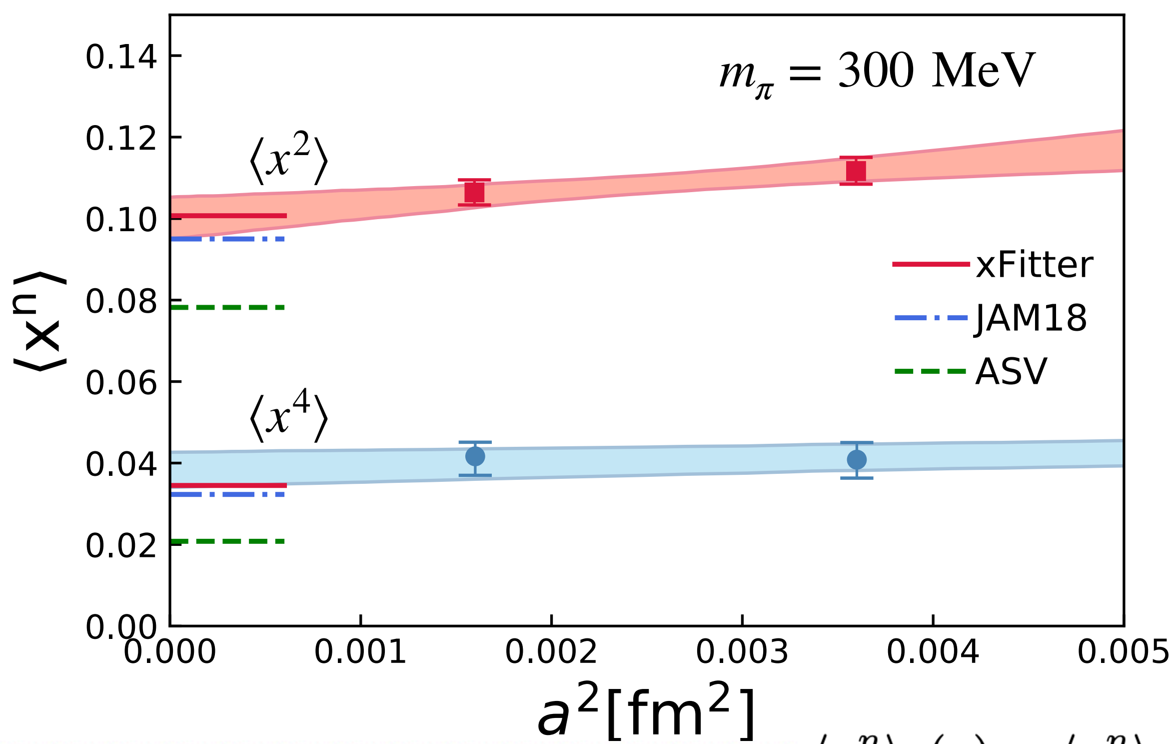
$64^3 \times 64$

$m_\pi = 140$ MeV

Pion valence PDF

With fine lattice spacings at unphysical pion mass:

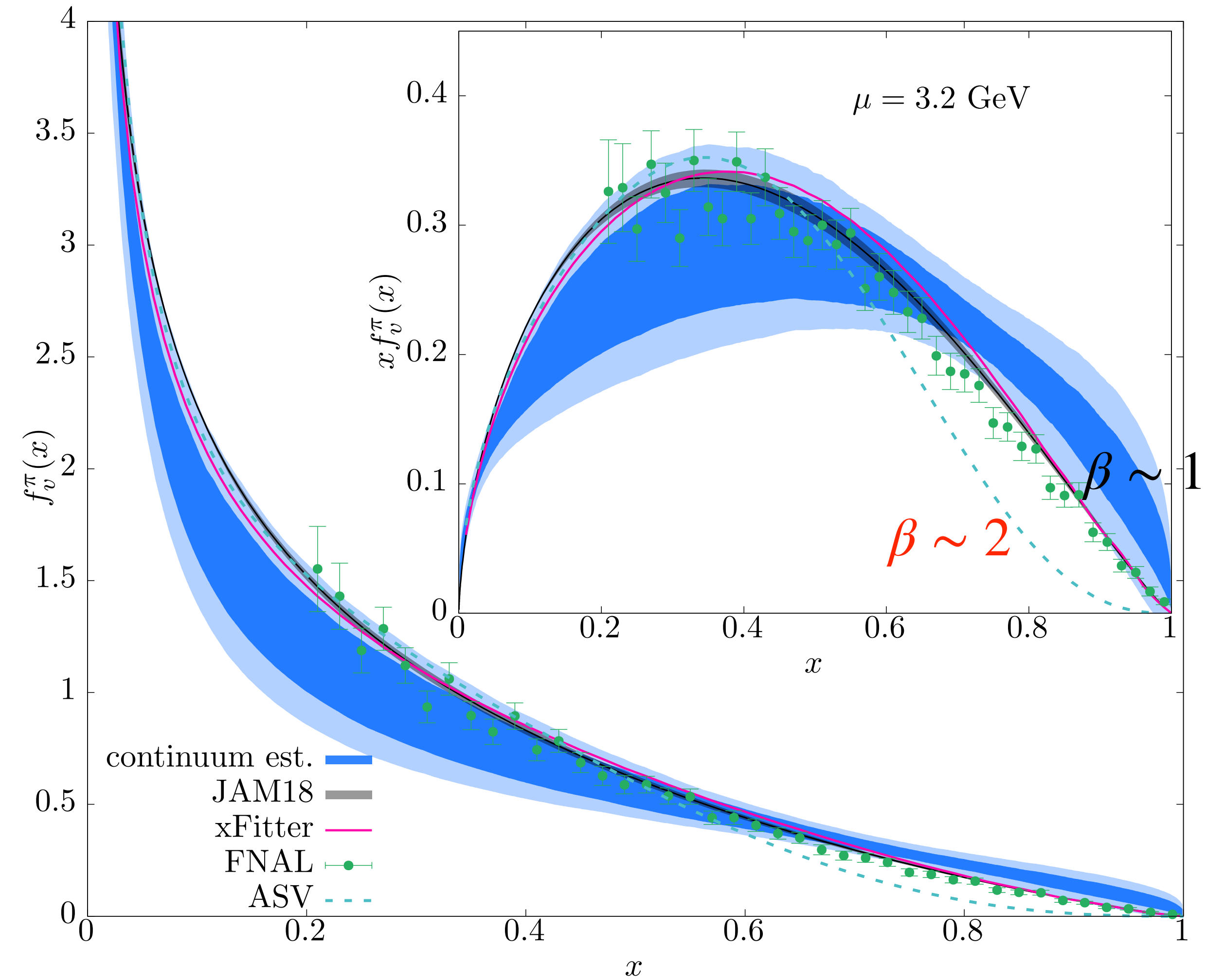
Model-independent extraction of Mellin moments from twist-2 OPE with NLO Wilson coefficients:



$$\langle x^n \rangle_v(a) = \langle x^n \rangle_v + d_n a^2$$

Model fitting of the PDF:

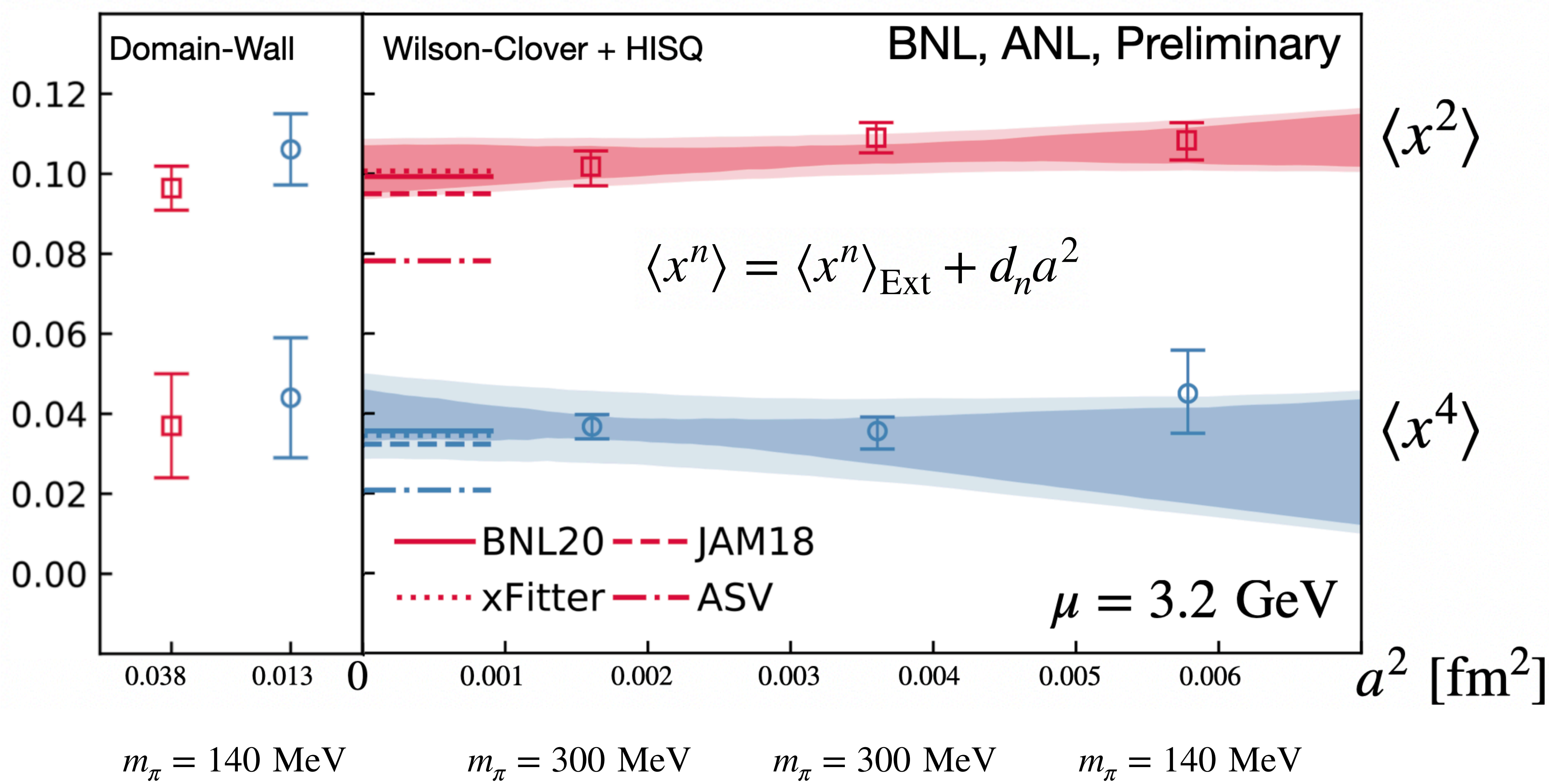
$$f(x,\mu) \sim x^\alpha (1-x)^\beta (1 + s\sqrt{x} + tx)$$



X. Gao et al. (BNL-SBU), PRD 102 (2020), 074504.

Pion valence PDF

Preliminary results: at physical pion mass from HISQ and Domain Wall fermion ensembles, with NNLO Wilson coefficients.



- Pion mass dependence is mild;

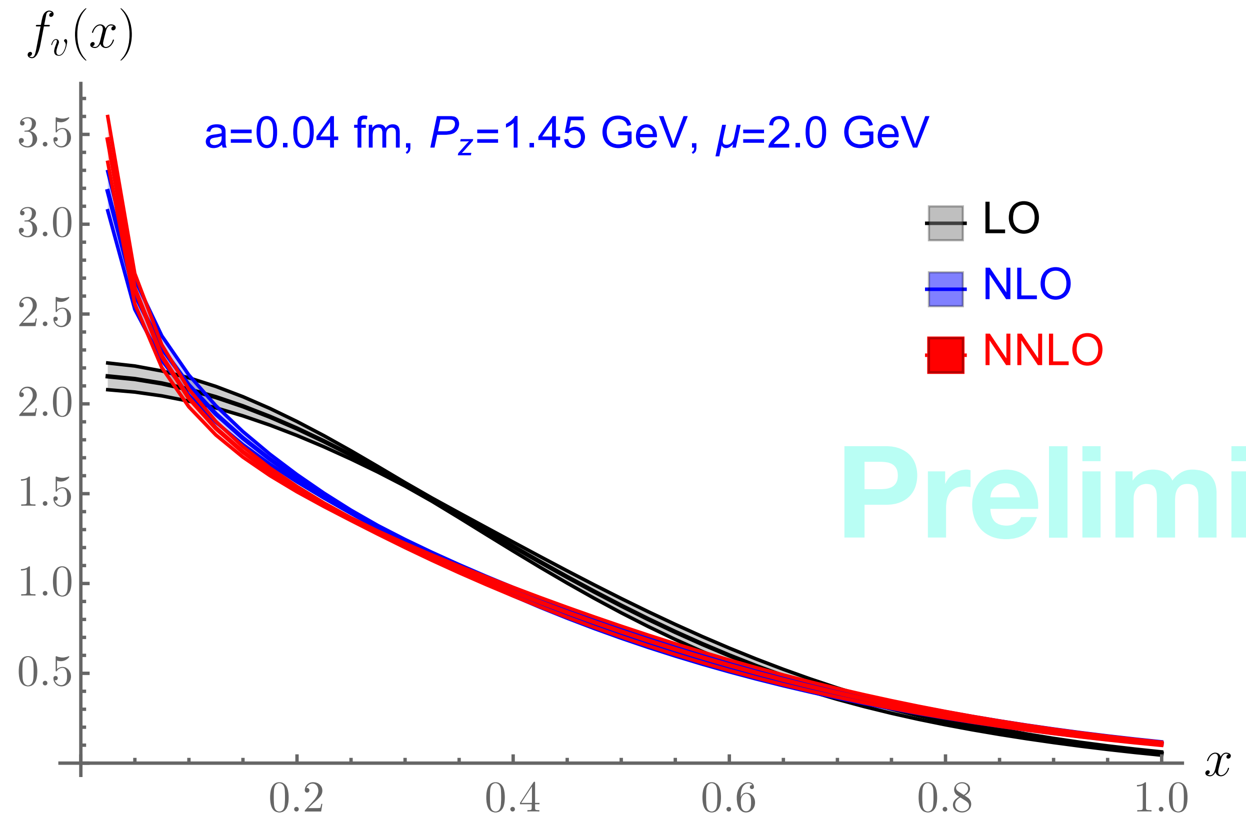
See Xiang Gao's talk at DIS2021.

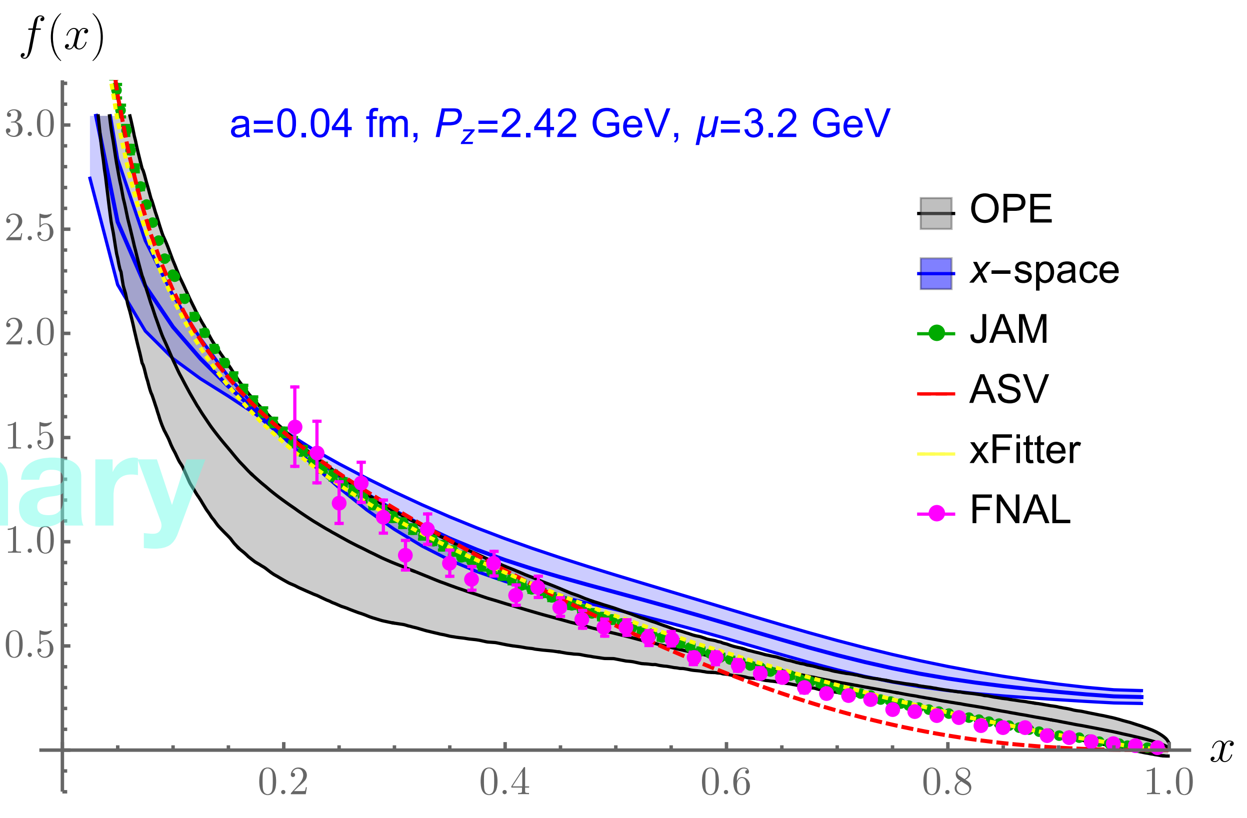
- Chiral fermion results shows good agreement with Wilson-Clover + HISQ fermion ones.

Pion valence PDF

Preliminary results: direct x-space calculation (quasi-PDF) with NNLO matching.

See Yong Zhao's talk at the 2021 GHP workshop.





DGLAP evolution and threshold resummation

- OPE:

$$\tilde{h}_{\gamma^t}(\lambda, z^2\mu^2) = \sum_{N=0}^{\infty} \frac{(-i\lambda)^N}{N!} C_N(\alpha_s(\mu), z_0^2\mu^2) a_N(\mu) + \ldots$$

$$z_0^2 = z^2 e^{2\gamma_E}/4$$

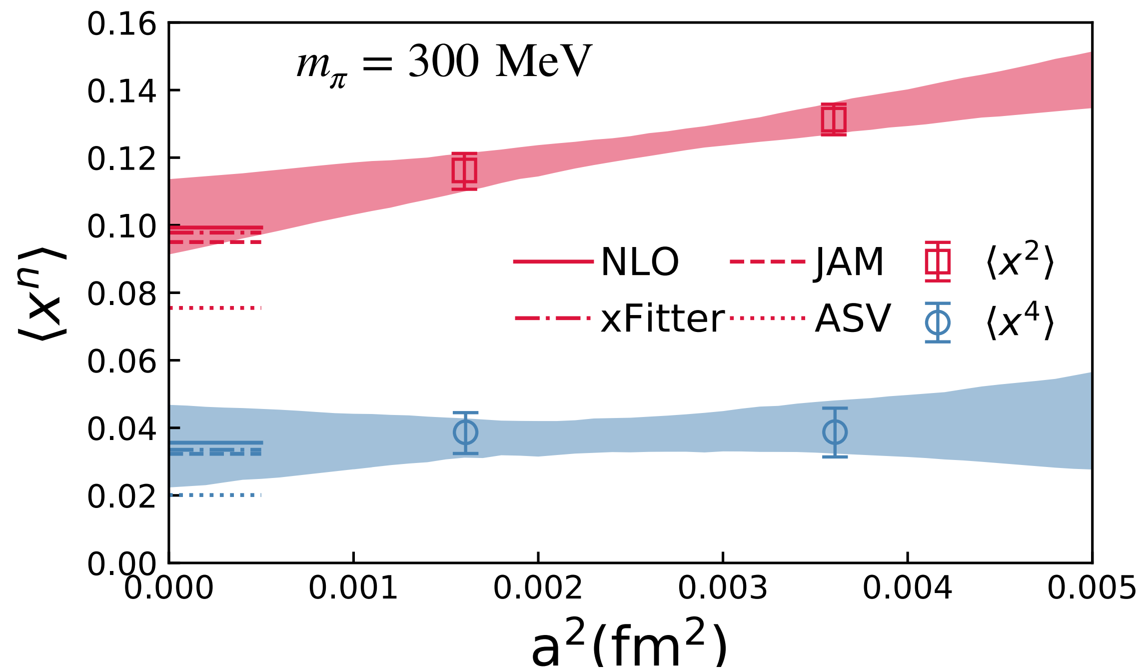
ln(z²μ²) resummed by solving the DGLAP evolution equation.

$$\lim_{N\to\infty} C_N^{\rm NLO} = \frac{\alpha_s(\mu)C_F}{2\pi}\left[2\ln N' \ln(z_0^2\mu^2) - 2\ln^2 N' + 2\ln N' - \frac{\pi^2}{3}\right],$$

$$N' = N e^{\gamma_E}$$

Leading and subheading threshold logs of N' need be resummed.

- LO DGLAP evolution and NLL threshold resummation.

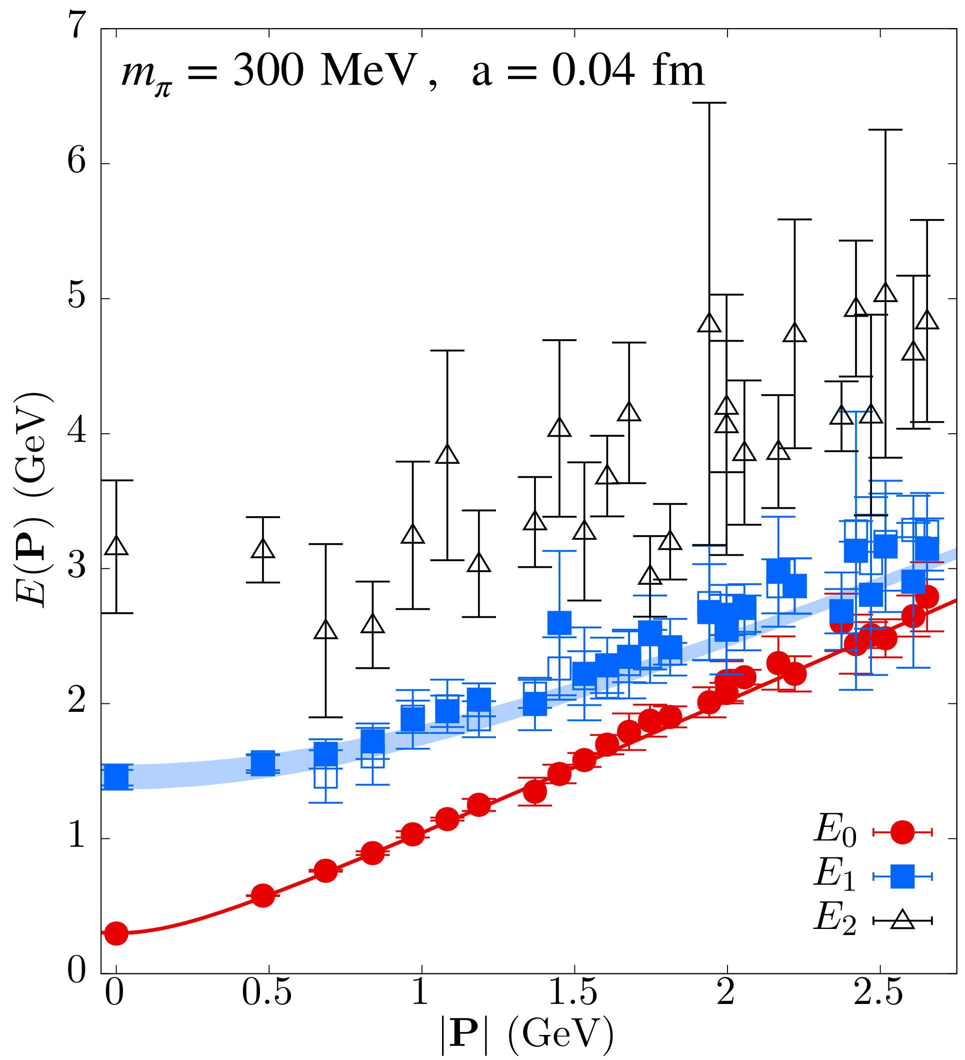


- **Threshold resummation can lead to larger exponent of (1-x);**
- **Current lattice data are only sensitive to the lowest moments or moderate x, where the threshold resummation effect is negligible.**

X. Gao et al. (BNL-SBU-ANL), 2102.01101, accepted by PRD.

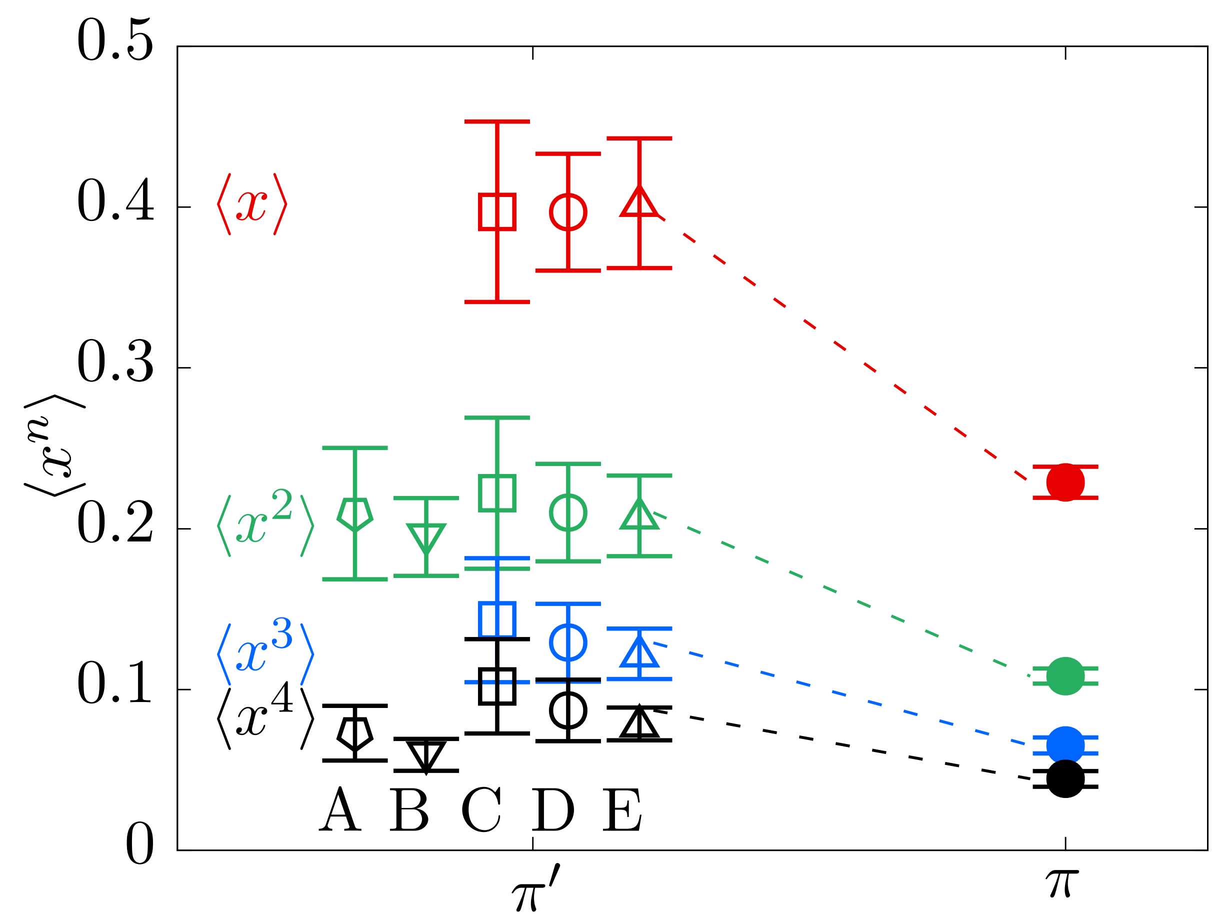
Structure of pion radial excitation π'(1300)

Evidence of π'(1300):



X. Gao et al. (BNL-SBU), 2101.11632.

Valence PDF of π'(1300):



Valence quarks carry about twice the momentum fraction of π'(1300) to that carried by them in the ground-state pion.

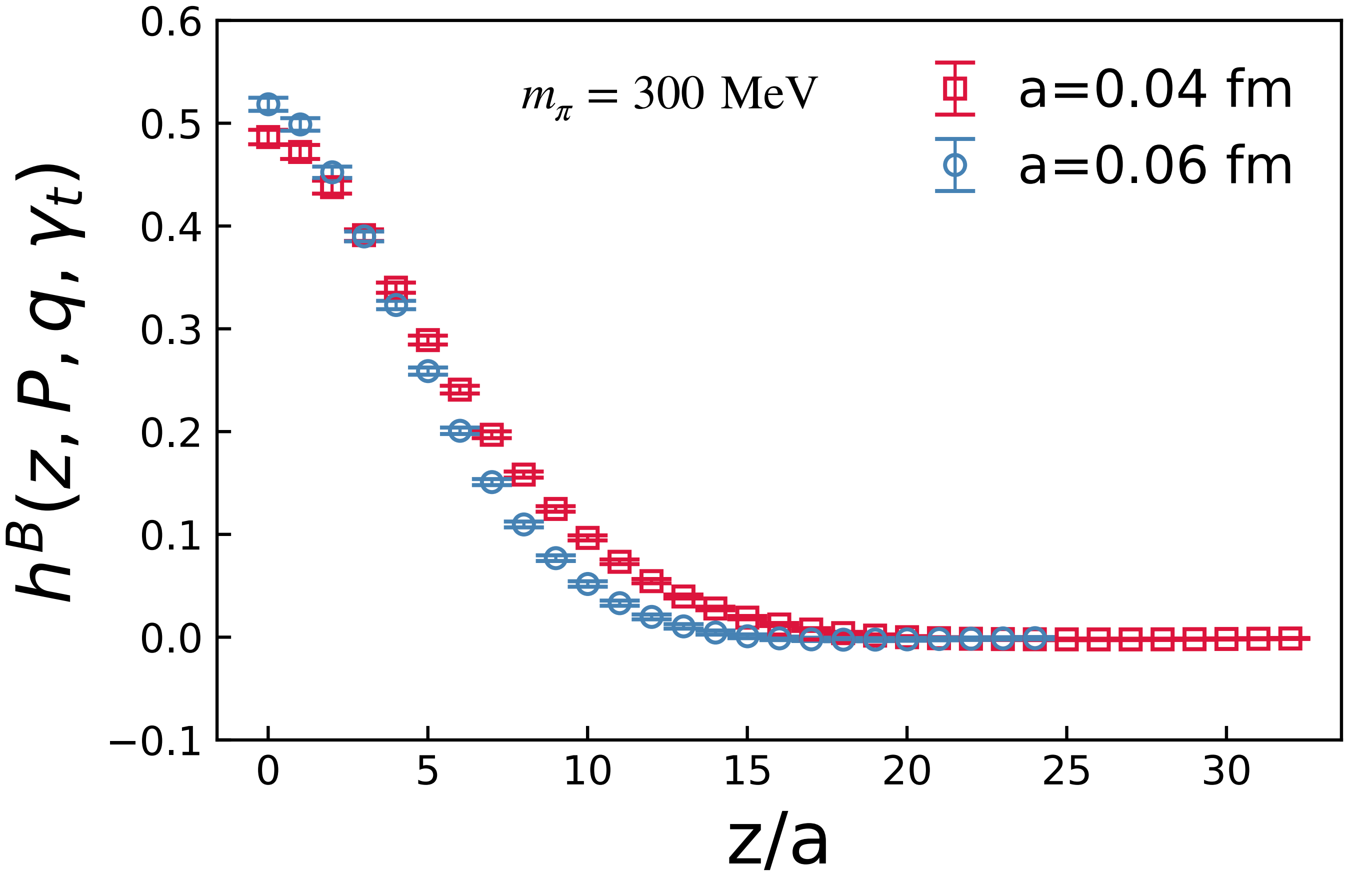
Matrix elements for pion quasi-GPD at fine lattice spacings

Preliminary results: bare matrix elements in the Breit frame.

$$\bar{P} = \frac{2\pi}{L}(0,0,n_z)\,, \qquad \frac{\Delta}{2} = \frac{2\pi}{L}(n_x, n_y, 0)$$

$$a = 0.04\ \text{fm}\,, \quad \bar{P} \pm \frac{\Delta}{2} = (\pm 0.97, 0, 0.48)\ \text{GeV}$$

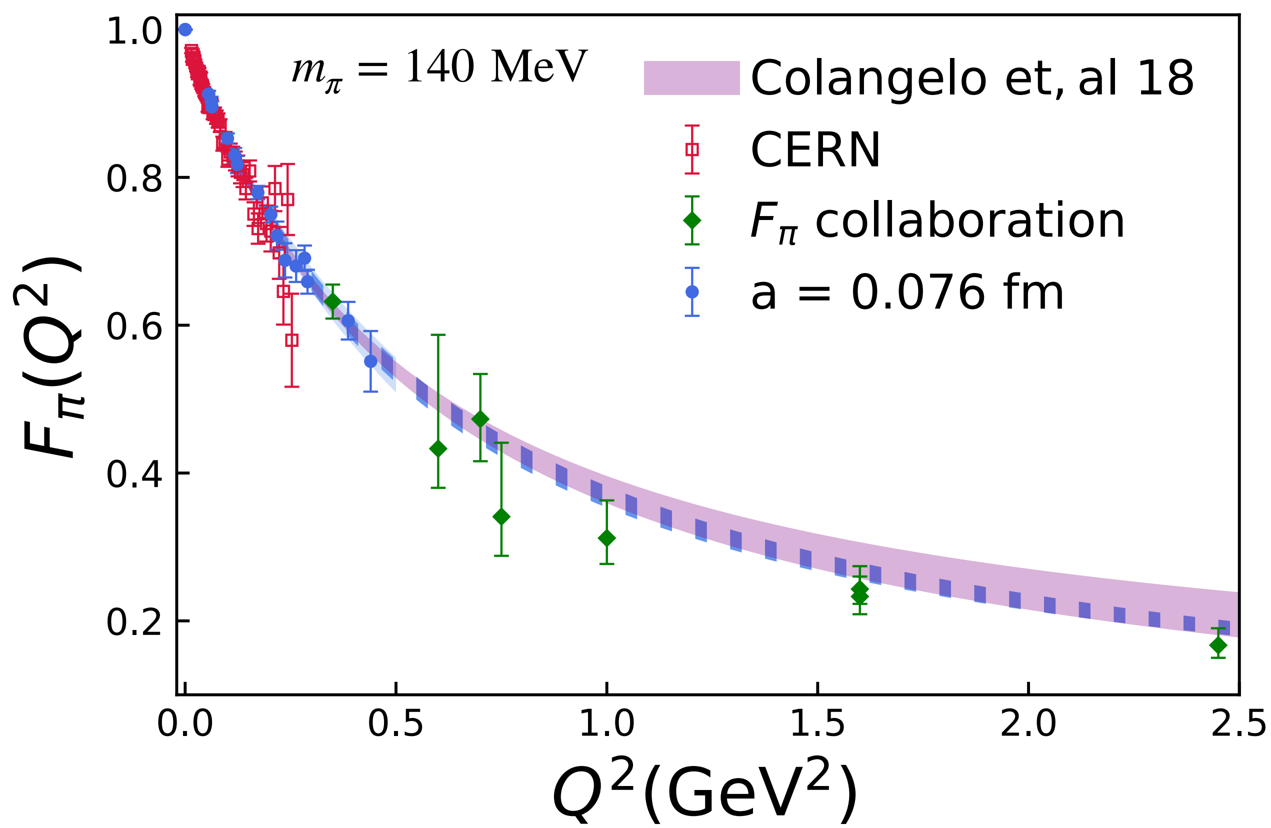
$$a = 0.06\ \text{fm}\,, \quad \bar{P} \pm \frac{\Delta}{2} = (\pm 0.86, 0, 0.43)\ \text{GeV}$$



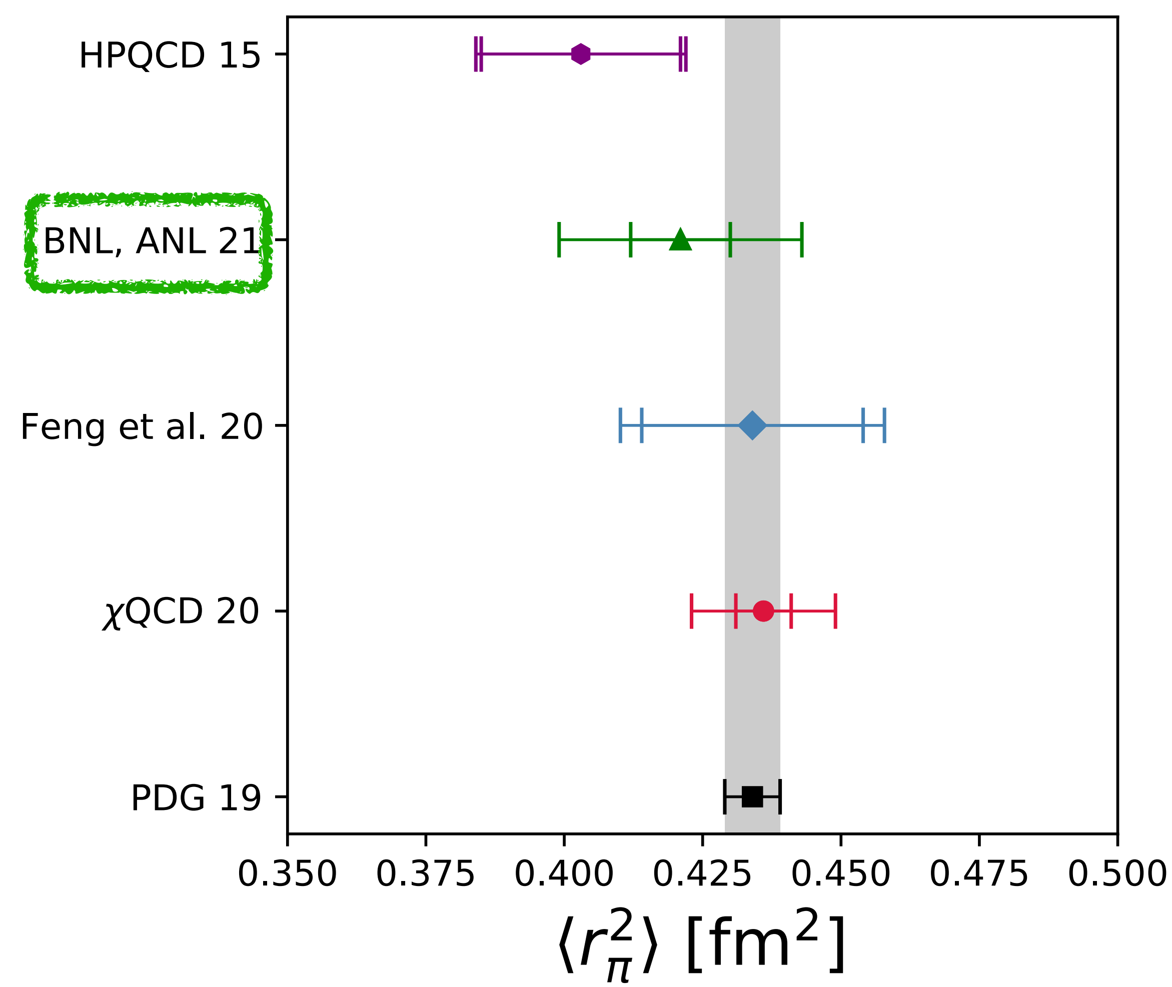
Pion form factor at physical pion mass

- We have computed the off-forward matrix elements of the vector current, the local limit of the quasi-GPD correlator.

Pion form factor:



Pion charge radius:



X. Gao et al. (BNL-SBU-ANL), 2102.06047.

$\langle r_\pi^2 \rangle = 0.42(2)$ fm

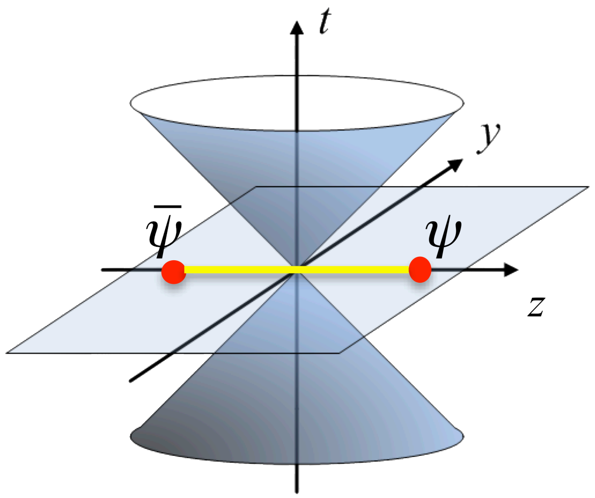
Pion GPD calculation from lattice QCD: method

- Moments as form factors of local-gauge invariant operators:

- M. Gockeler et al. (QCDSF), PRL 2004;
- P. Hagler et al. (LHPC), PRD 2008;
- C. Alexandrou et al. (ETM), PRD 2020.

Pion GPD calculation from lattice QCD: method

- Equal-time correlation in boosted hadron states:
  - x-space expansion: LaMET or Quasi-GPD



- X. Ji, PRL 110 (2013) 262002, SCPMA 57 (2014) 1407-1412;
- X. Ji, Y.S. Liu, Y.Z. Liu, J.H. Zhang and YZ, 2004.03543.

$$\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle \bar{P} + \frac{\Delta}{2}, S' | \tilde{O}_{\gamma^t}(z) | \bar{P} - \frac{\Delta}{2}, S \rangle = \bar{u} \left[\tilde{H}(x, \xi_z, t)\gamma^t + \tilde{E}(x, \xi_z, t) \frac{i\sigma^{t\mu}\Delta_\mu}{2M} \right] u$$

$$\tilde{O}_{\gamma^t}(z) = \bar{\psi}(\frac{z}{2})\gamma^0 W(\frac{z}{2}, -\frac{z}{2})\psi(-\frac{z}{2}), \qquad \lambda = zP^z, \qquad \xi_z = -\frac{\Delta^z}{2\bar{P}^z} \approx \xi$$

Large-momentum expansion (MSbar scheme):

$$(H/E)(x, \xi, t) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y\bar{P}^z}\right) (\tilde{H}/\tilde{E})(y, \xi, t, \bar{P}^z, \mu) + \ldots$$

- Ji et al., PRD 92 (2015) 014039;
- Xiong and Zhang, PRD 92 (2015) 5, 054037;
- Liu et al., PRD 100 (2019) 3, 034006.

Renormalization: same as the quasi-PDF operator.

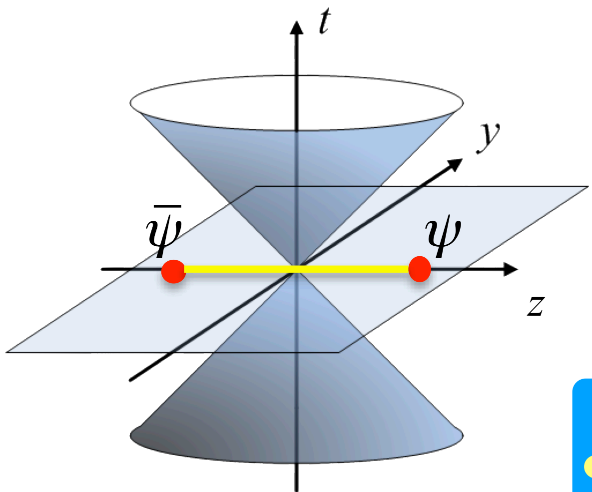
- RI'/MOM — Liu et al., PRD 100 (2019) 3, 034006.
- Hybrid renormalization. — Ji et al., NPB 964 (2021) 115311.

*Matching kernel C is the same as the PDF case when ξ=0.

Predicting a range of x where the power corrections are under control.

Pion GPD calculation from lattice QCD: method

- Equal-time correlation in boosted hadron states:
  - x-space expansion: LaMET or Quasi-GPD



- X. Ji, PRL 110 (2013) 262002, SCPMA 57 (2014) 1407-1412;
- X. Ji, Y.S. Liu, Y.Z. Liu, J.H. Zhang and YZ, 2004.03543.

$$\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle \bar{P} + \frac{\Delta}{2}, S' | \tilde{O}_{\gamma^t}(z) | \bar{P} - \frac{\Delta}{2}, S \rangle = \bar{u} \left[\tilde{H}(x, \xi_z, t) \gamma^t + \tilde{E}(x, \xi_z, t) \frac{i\sigma^{t\mu}\Delta_\mu}{2M} \right] u$$

$^z,\qquad \xi_z = -\frac{\Delta^z}{2\bar{P}^z} \approx \xi$

- RI'/MOM introduces nonperturbative effects at large z;
- Hybrid scheme features a Wilson line mass subtraction at large z and avoids this issue;
- We have made much progress in determining the Wilson line mass correction (YZ GHP2021).

Large-momentum

i et al., PRD 92 (2015) 014039;
iong and Zhang, PRD 92 (2015) 5, 54037;
iu et al., PRD 100 (2019) 3, 034006.

$(H/E)(x, \xi, t) = \int_{-\infty}^{\infty}$

Renormalization: same as the quasi-PDF operator.

- RI'/MOM Liu et al., PRD 100 (2019) 3, 034006.
- Hybrid renormalization. Ji et al., NPB 964 (2021) 115311.

kernel C is the same as the PDF case when ξ=0.

Predicting a range of x where the power corrections are under control.

Pion GPD calculation from lattice QCD: method

- Equal-time correlation in boosted hadron states:
  - Coordinate-space factorization: pseudo-GPD

A. Radyushkin, PRD 96 (2017) 3, 034025;
A. Radyushkin, PRD 100 (2019) 11, 116011.

$$\langle \bar{P}+\frac{\Delta}{2}, S' | \tilde{O}_{\gamma^t}(\lambda n) | \bar{P}-\frac{\Delta}{2}, S \rangle = 2\bar{P}^0 \mathscr{M}(\lambda, \xi_z, t; z^2)$$

Renormalization done by forming the ratio:

$$\tilde{\mathscr{M}}(\lambda, \xi, t; z^2) = \frac{\mathscr{M}(\lambda, \xi, t; z^2)}{\mathscr{M}(0,0,0; z^2)}$$

Factorization at short distance (MSbar scheme):

Modeling the GPD and fit to the short-distance lattice data.

$$\mathscr{M}(\lambda, \xi, t; z^2) = \int_0^1 d\omega \mathscr{C}(\omega, \nu, \xi; z^2) \int_{-1}^1 dx\ e^{ix\nu} H(x, \xi, t, \mu) + O(z^2\Lambda_{\rm QCD}^2)$$

Conformal operator product expansion (OPE):

$$\mathscr{M}(\lambda, \xi, t; z^2) = \sum_{n=0}^{\infty} \mathscr{F}_n(\lambda)\ \xi^n \int_{-1}^1 dy\ C_n^{\frac{3}{2}}(\frac{y}{\xi}) H(x, \xi, t, \mu)\ +\ O(\alpha_s) +\ \ldots$$

Pion GPD calculation from lattice QCD: method

- Equal-time correlation

    - Coordinate-space factor

$$\langle \bar{P}+\frac{\Delta}{2}, S' | \tilde{O}_{\gamma^t}(\lambda n) | \bar{P}-\frac{\Delta}{2}, S\rangle$$

We will consider replacing $\mathscr{M}(0,0,0;z^2)$ with boosted pion valence quasi-PDF or quasi-GPD matrix elements to suppress the power corrections (X. Gao et al. (BNL/SBU), PRD 102 (2020), 074504.).

Renormalization done by forming the ratio:

$$\tilde{\mathscr{M}}(\lambda,\xi,t;z^2) = \frac{\mathscr{M}(\lambda,\xi,t;z^2)}{\mathscr{M}(0,0,0;z^2)}$$

Factorization at short distance (MSbar scheme):

Modeling the GPD and fit to the short-distance lattice data.

$$\mathscr{M}(\lambda,\xi,t;z^2) = \int_0^1 d\omega \mathscr{C}(\omega,\nu,\xi;z^2)\int_{-1}^1 dx\ e^{ix\nu}H(x,\xi,t,\mu) + O(z^2\Lambda^2_{\rm QCD})$$

Conformal operator product expansion (OPE):

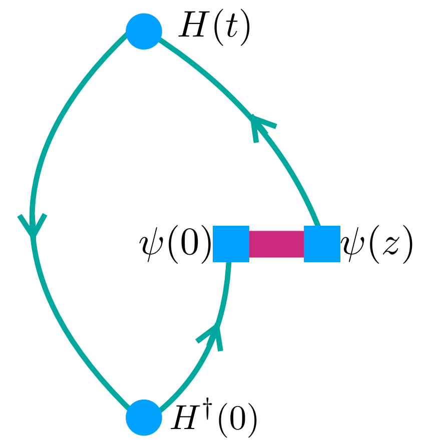
$$\mathscr{M}(\lambda,\xi,t;z^2) = \sum_{n=0}^{\infty}\mathscr{F}_n(\lambda)\ \xi^n\int_{-1}^1 dy\ C_n^{\frac{3}{2}}(\frac{y}{\xi})H(x,\xi,t,\mu)\ +\ O(\alpha_s) +\ \dots$$

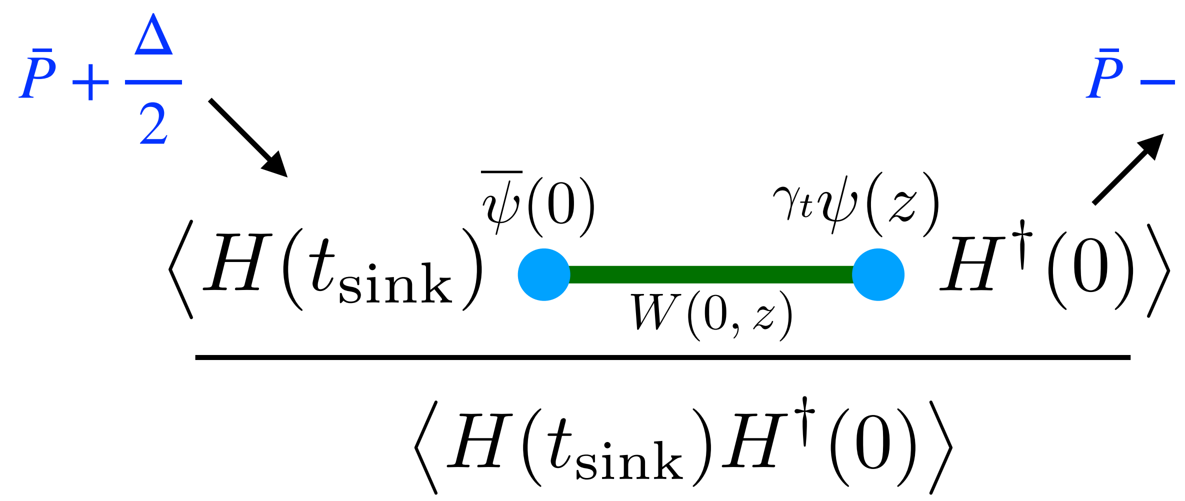
The current proposal:

- Lattice setup: Wilson-Clover + HISQ

$a = 0.076$ fm $\qquad m_\pi = 140$ MeV $\qquad 64^3 \times 64$

- 3pt function:





- Breit frame kinematics:

$$\bar{P} = \frac{2\pi}{L}(0,0,n_z)\,, \quad \frac{\Delta}{2} = \frac{2\pi}{L}(n_x, n_y, 0)$$

$n_z = 0,\ 2,\ 4, \qquad \bar{P}^z = 0,\ 0.51,\ 1.02$ GeV

$(n_x, n_y) = (1,0), (1,1), (2,0), \qquad t = 0.26,\ 0.52\ ,1.04$ GeV^2

- Calculate GPD moments and GPD within a range of x at discrete ξ and t;
- Differentiate GPD models and provide inputs for phenomenology;

The current proposal:

- Software:
  - Qlua for 3pt functions;
  - Multigrid (MG) algorithm solvers for quark propagators;
  - Ported with QUDA all the inversion, contraction and smearing functionality.
- Requested resources:
  - 140k K80 hours
  - 80TB disk storage
  - 150TB archival storage

extended source/sink creation	t_{ext}	8.5s
MG inversion sloppy	t_{inv}	28s
MG inversion exact	t_{inv}	52s
contraction for 2pt function	t_{2pt}	7.6s
contraction for 3pt function	t_{3pt}	11s

Timing breakdown for the physical pion mass ensemble using 16 nodes of Annie@BNL-IC cluster (32 K80 cards).

Conclusion

- Lattice QCD can calculate GPD moments and predict GPDs within a range of x at fixed ξ and t, providing useful inputs for the extraction of GPDs from experiments;
- The BNL-SBU-ANL group have made much progress in the pion valence PDF (with systematic improvements), form factor and generation of matrix elements for the GPD;
- Outlook: 1) Breit-frame matrix elements for pion valence GPD at physical pion mass; 2) analyze the lattice data with both the x- and coordinate-space approaches.

This year's proposal: Pion valence GPD at the physical point
Requested time: 140k K80 hours

Backup slides

ensemble $a, L_t \times L^3$	$m_q a$	$m_\pi L_t$	n_z	z range	#cfgs	(#ex,#sl)
$a = 0.06$ fm, 64×48^3	-0.0388	5.85	0,1	[0,15]	100	$(1, 32)$
			2,3,4,5	[0,8]	525	$(1, 32)$
				[9,15]	416	$(1, 32)$
				[16,24]	364	$(1, 32)$
$a = 0.04$ fm, 64×64^3	-0.033	3.90	0,1	[0,32]	314	$(3, 96)$
			2,3	[0,32]	314	$(4, 128)$
			4,5	[0,32]	564	$(4, 128)$

Backup slides

Ensemble:	m_π^{val} (GeV)	c_{sw}	t_s/a	r_G fm	n_z	n_i $(i = x, y)$	j_z	#cfgs	(#ex,#sl)
$a = 0.076$ fm, $m_\pi^{sea} = 0.14$ GeV, 64×64^3	0.14	1.0372	6, 8, 10	0.342	[0,3]	±1,±2	2	350	$(5, 100)$
					[4,7]	±1,±2	5	350	$(5, 100)$
$a = 0.06$ fm, $m_\pi^{sea} = 0.16$ GeV, 64×48^3	0.3	1.0336	8, 10, 12	0.312	[0,1]	±1,±2	0	100	$(1, 32)$
					[2,3]	±1,±2	2	525	$(1, 32)$
					[4,5]	±1,±2	3	525	$(1, 32)$
$a = 0.04$ fm, $m_\pi^{sea} = 0.16$ GeV 64×64^3	0.3	1.02868	9,12, 15,18	0.208	[0,1]	±1	0	314	$(3, 96)$
					[0,1]	±2	0	314	$(2, 64)$
					[2,3]	±1	2	564	$(4, 128)$
					[2,3]	±2	2	564	$(3, 96)$

Ensemble $a, L_t \times L_s^3$	m_π^{val} (GeV)	t_s/a	n_z^p	n_i^p $i = x, y$	n_i^q $i = x, y$	#cfgs	(#ex,#sl)
$a = 0.06$ fm, $m_\pi^{sea} = 0.16$, 64×48^3	0.3	8, 10	2	±1	∓2	120	$(1, 32)$
$a = 0.04$ fm, $m_\pi^{sea} = 0.16$, 64×64^3	0.3	9,12, 15,18	2	±1	∓2	120	$(1, 32)$