# Quantum Computing and 

Lattice Field Theory Program

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University of Maryland, College Park


USQCD All-hands Meeting
Hosted by USQCD/MIT
May 1, 2021
i) Dense matter EOS, phase diagram of QCD


Path integral formulation...

$$
e^{-S[U, q, \bar{q}]}
$$

with a complex action:

$$
\mathcal{L}_{\mathrm{QCD}} \rightarrow \mathcal{L}_{\mathrm{QCD}}-i \mu \sum_{f} \bar{q}_{f} \gamma^{0} q_{f}
$$

## OUR MOTIVATION FOR LEVERAGING QUANTUM TECHNOLOGIES

ii) Real-time dynamics of matter (heavy-ion collisions, early universe...)

...and a wealth of dynamical response functions, transport properties, hadron distribution functions, and non-equilibrium physics of QCD.

Path integral formulation...

$$
e^{i S[U, q \bar{q}]}
$$

## A RANGE OF QUANTUM SIMULATORS WITH VARING CAPACITY AND CAPABILITY IS AVAILABLE!



## rigetti




## QUANTUM CHEMISTRY, CM vs. QUANTUM FIELD THEORY: SOME SIMILARITIES BUT MAJOR DIFFERENCES



# QUANTUM CHEMISTRY, CM vs. QUANTUM FIELD THEORY: SOME SIMILARITIES BUT MAJOR DIFFERENCES 



QUANTUM SIMULATION OF QUANTUM FIELD THEORIES: A MULTI-PRONG EFFORT

Conventional lattice field theory program
$+$
Classical computation
Implementation and benchmark

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## QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

Hamiltonian formalism maybe more natural than the path integral formalism for quantum simulation/computation:

Kogut and Susskind formulation:

$$
\begin{array}{ccc}
H_{\mathrm{QCD}}=-t \sum_{\langle x y\rangle} s_{x y}\left(\psi_{x}^{\dagger} U_{x y} \psi_{y}+\psi_{y}^{\dagger} U_{x y}^{\dagger} \psi_{x}\right)+m \sum_{x} s_{x} \psi_{x}^{\dagger} \psi_{x}+\frac{g^{2}}{2} \sum_{\langle x y\rangle}\left(L_{x y}^{2}+R_{x y}^{2}\right)-\frac{1}{4 g^{2}} \sum_{\square} \operatorname{Tr}\left(U_{\square}+U_{\square}^{\dagger}\right) . \\
\text { Fermion hopping term } & \text { Fermion } & \text { Energy of color }
\end{array} \quad \text { Energy of color } \quad \text { electric field } \quad \text { mass } \quad \text { magnetic field } \quad .
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## Gauge-field truncation

$S U(2)$ pure gauge in $3+1 D$
in group element basis


Hackett et al, Phys. Rev.
A 99, 062341 (2019)
$S U(2)$ with matter in $1+1 \mathrm{D}$ in electric-field basis


ZD, Raychowdhury, and Shaw, arXiv:2009.11802 [hep-lat]
$\mathrm{SU}(3)$ pure gauge in $2+1 \mathrm{D}$ in local-irreps basis


Ciavarella, Klco, and Savage, arXiv:2101.10227 [quant-ph]

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Generator of infinitesimal gauge transformation

$$
G_{x}^{a}=\psi_{x}^{i \dagger} \lambda_{i j}^{a} \psi_{x}^{j}+\sum_{k}\left(L_{x, x+\hat{k}}^{a}+R_{x-k, x}^{a}\right) \quad G_{x}^{i}\left|\psi\left(\left\{q_{x}^{(i)}\right\}\right)\right\rangle=q_{x}^{(i)}\left|\psi\left(\left\{q_{x}^{(i)}\right\}\right)\right\rangle
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$$



|  | $q_{x}=0$ |  |
| :---: | :---: | :---: |
|  |  |  |
| $q_{x} \neq 0$ | $q_{x} \neq 0$ |  |
|  | $\ldots$ |  |

## IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION

Can restore symmetries lost by Trotter expansion of the evolution...


Simulation circuits

$$
U_{t} \approx \prod_{k=1}^{r} C_{k}^{\dagger} S_{\delta t} C_{k} . \quad \text { with } \quad[C, H]=0 \quad \forall C \in \mathcal{S} .
$$

Example: a 4-site Schwinger model


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Example: a 4-site Schwinger model

...which mimics a large-angle single-qubit rotations procedure:

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\begin{array}{lr} 
& \text { Gauss's law operator } \\
\text { Add to the Hamiltonian: } & V H_{G}=V \sum c_{j} G_{j} .
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## IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION

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Symmetry protection


Simulation circuits

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Hauke, arXiv:2007.00668 [quant-ph]

See also Stannigel, et al, Phys. Rev. Lett. 112, 120406, Lamm, Lawrence, Yamauchi, arXiv:2005.12688 [quant-ph], and Kasper et al, arXiv:2012.08620 [quant-ph] for similar symmetry-protection ideas.

The time complexity of classical Hamiltonian-simulation algorithms for various formulations


ZD, Raychowdhury, and Shaw, arXiv:2009.11802 [hep-lat]

## IN FACT MANY MANY MORE FORMULATIONS EXIST, EACH WITH ITS OWN PROS AND CONS:

Gauge-field theories (Abelian and non-Abelian):

Group-element representation Zohar et al; Lamm et al

Spin-dual representation and hydrogen atom basis Mathur et al

## Prepotential formulation

Mathur, Raychowdhury et al

Fermionic basis
Hamer et al; Martinez et al; Banuls et al

Loop-String-Hadron basis
Raychowdhury and Stryker
Bosonic basis
Cirac and Zohar

Maximal tree and coupled-cluster basis Cirac and Zohar

Link models and qubit regularization Brower, Chandrasekharan, Wiese et al

Dual plaquette (magnetic) basis Bender, Zohar et al; Kaplan and Styker; Unmuth-Yockey; Hasse et al

Local irreducible representations Byrnes and Yamamoto; Ciavarella, Klco, and Savage
(Effective) models and light-front quantization Ortega at al; Kreshchuk, Love et al.

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Local irreducible representations Byrnes and Yamamoto; Ciavarella, Klco, and Savage

Manifold lattices
Buser et al
(Effective) models and light-front quantization Ortega at al; Kreshchuk, Love et al.

Scalar field theory

Field basis
Jordan, Lee, and Preskill

Continuous-variable basis
Pooser, Siopsis et al

Harmonic-oscillator basis Klco and Savage

Single-particle basis Barata, Mueller, Tarasov, and Venugopalan.

## OBSERVABLES TOO REQUIRE DEDICATED STUDIES TO BE CAST IN HAMILTONIAN LANGUAGE.

Structure functions and PDFs
Mueller, Tarasov, Venugopalan;
Lamm, Lawrence, Yamauchi

Scattering and decay amplitudes
Jordan, Lee, Preskill; Ciavarella;
Surace, Lerose; Gustafson, Meurice, et al

Viscosity and transport coefficients
Cohen, Lamm, Lawrence, Yamauchi
Thermalization and many-body localization Brenes, Dalmonte, et al

Dynamical phase transition and topological order Zache, Mueller, Berges, et al

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NORMALIZATION AND CONTINUUM LIMIT, TRUNCATION ERRORS, FINITE-VOLUME EFFECTS, etc. ALL MUST BE UNDERSTOOD.

Renormalization and continuum limit Klco, Savage; Mueller et al

Finite-volume effects in Minkowski amplitudes
Briceno, Hansen, et al; ZD, Kadam

Truncation effects in scalar and gauge theories
Hackett, et al; Klco, Savage; ZD, Raychowdhury, Shaw

Conventional lattice field theory program $+$
Classical computation
Implementation and benchmark

## DIFFERENT APPROACHES TO QUANTUM SIMULATION



## DIFFERENT APPROACHES TO QUANTUM SIMULATION

Analog


Analog-Digital

LET US PICK A PLATFORM (TRAPPED IONS) AND A MODEL (SCHWINGER MODEL) TO DEMONSTRATE FEATURES OF THESE APPROACHES:


## Lattice Schwinger model



Gauge DOF are eliminated in 1D by Gauss's law and gauge transformation

## Ions in a linear Paul trap





ZD, Hafezi, Monroe, Pagano, Seif and Shaw, Phys. Rev. R 2, 023015 (2020).



Near term cost

|  | $\delta_{g}=10^{-3}$ |  | $\delta_{g}=10^{-4}$ |  | $\delta_{g}=10^{-5}$ |  | $\delta_{g}=10^{-6}$ |  | $\delta_{g}=10^{-7}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\tilde{\epsilon}^{2}$ | CNOT | $\tilde{\epsilon}^{2}$ | CNOT | $\tilde{\epsilon}^{2}$ | CNOT | $\tilde{\epsilon}^{2}$ | CNOT | $\tilde{\epsilon}^{2}$ | CNOT |
| $x=10^{-2}$ | - | 7.3 e 4 | - | 1.6 e 5 | - | 3.4 e 5 | - | 7.3 e 5 | $5.6 \mathrm{e}-2$ | 1.6 e 6 |
| $x=10^{-1}$ | - | 1.6 e 4 | - | 3.5 e 4 | - | 7.5 e 4 | $5.9 \mathrm{e}-2$ | 1.6 e 5 | $2.7 \mathrm{e}-3$ | 3.5 e 5 |
| $x=1$ | - | 4.6 e 3 | - | 9.9 e 3 | $1.0 \mathrm{e}-1$ | 2.1 e 4 | $4.7 \mathrm{e}-3$ | 4.6 e 4 | $2.2 \mathrm{e}-4$ | 9.9 e 4 |
| $x=10^{2}$ | - | 2.8 e 3 | $8.3 \mathrm{e}-1$ | 6.1 e 3 | $3.8 \mathrm{e}-2$ | 1.3 e 4 | $1.8 \mathrm{e}-3$ | 2.8 e 4 | $8.2 \mathrm{e}-5$ | 6.0 e 4 |



$$
\left\{E_{j}, U_{j}\right\} \quad\left\{E_{j+1}, U_{j+1}\right\}
$$



Analog-Digital


Fermion mass term

## Lattice Schwinger model

Ions in a linear Paul trap

Collective normal modes used to perform two-ion entangling gates.


Schwinger model
Fermion-gauge interaction Fermion mass Electric-field term

| Analog-digital | $\mathcal{O}(N)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ |
| :---: | :---: | :---: | :---: |
| Digital | $\mathcal{O}\left(N(\log \Lambda)^{2}\right)$ | $\mathcal{O}(1)$ | $\mathcal{O}\left(N(\log \Lambda)^{2}\right)$ |

Conventional lattice field theory program $+$
Classical computation

## Implementation and benchmark



Martinez, Muschik, Schindler, Nigg, Erhard, Heyl, Hauke,
Dalmonte, Monz, Zoller, Blatt, Nature 534, 516-519 (2016)


Nguyen, Shaw, Zhu, Huerta Alderete, ZD, Linke (2020)


Klco, Dumitrescu, McCaskey, Morris, Pooser, Sanz, Solano, Lougovski, Savage, Phys. Rev. A 98, 032331 (2018)


Lu, Klco, Lukens, Morris, Bansal, Ekström, Hagen, Papenbrock, Weiner, Savage, Lougovski, Phys. Rev. A 100, 012320 (2019)


Real-time dynamic of pure $\operatorname{SU}(3)$ with global irrupts on IBM


```
Ciavarella, Klco, and Savage,
``` arXiv:2101. 10227 [quant-ph]

Real-time dynamic of pure \(\operatorname{SU}(2)\) with global irreps on IBM


Low-lying spectrum of \(\operatorname{SU}(2)\) with matter in 1+1 D on IBM

```

Atas et al,
arXiv:2102.08920 [quant-ph]

```

See also another SU(2) study on D-wave by Rahman et al, arXiv:2103.08661 [hep-lat]

\section*{ANALOG EXAMPLES FOR SCHWINGER MODEL}

MWWW

A realization of lattice Schwinger model within QLM with cold atoms in a trapping potential


B


Mil, Zache, Hegde, Xia, Bhatt, Oberthaler, Hauke, Berges, Jendrzejewski, Science 367, 1128-1130 (2020)


\section*{ANALOG EXAMPLES FOR SCHWINGER MODEL}

WMM Meurice, Tsai, Unmuth-Yockey, Zhang, Phys. Rev. D 92, 076003 (2015). Luo, El-Khadra, et al, Phys. Rev. A 102, 032617 (2020).

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A realization of lattice Schwinger model within OLM with cold atoms in a trapping potential


Mil, Zache, Hegde, Xia, Bhatt, Oberthaler, Hauke, Berges, Jendrzejewski, Science 367, 1128-1130 (2020)


Schwinger model within quantum link model formulation...

...mapped to a 71-site Bose-Hubbard quantum simulator:


Yang et al, Nature 587 (2020) 7834, 392-396.

Gauss's law violating effects are suppressed:



Schwinger model within quantum link model formulation...

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Some non-Abelian gauge theory analog proposals:
Zohar, Cirac, Reznik, Phys. Rev. A 88023617 (2013).
Zohar, Cirac, Reznik, Phys. Rev. Lett. 110, 125304
(2013), Phys. Rev. A 88023617 (2013), Rep. Prog. Phys.

79, 014401 (2016). González Cuadra, Zohar, Cirac, New
J. Phys. 19063038 (2017). Dasgupta and Raychowdhury, arXiv:2009. 13969 [hep-lat].

Yang et al, Nature 587 (2020) 7834, 392-396.

Gauss's law violating effects are suppressed:




\section*{EXAMPLE I: STATE PREPARATION ROUTINE FOR LATTICE GAUGE THEORIES}

State preparation can be done using Monte Carlo methods if no sign or signal-to-noise problems occur, and time evolution can be ported to quantum hardware.




Harmalkar, Lamm, Lawrence1, arXiv:2001.11490 [hep-lat] Gustafson and Lamm and Phys. Rev. D 103, 054507 (2021)

Kokail et al, Nature 569, 355 (2019).
Hamiltonian under which the system evolves respects some symmetries of the original theory and is implemented in an analog fashion.


See also Atas et al, arXiv:2102.08920 [quant-ph] for a VQE study of SU(2) hadrons.


\section*{EXAMPLE III: TENSOR NETWORKS FORM CLASSICAL TO QUANTUM COMPUTING}

SU(2) gauge theory coupled to matter in 1+1D with an MPS ansatz

\(\left.\left.\mid j_{3} l_{3}\right\}_{3}^{\prime}\right\rangle\left|n_{4}^{1}, n_{4}^{2}\right\rangle\left|j_{4} \ell_{4}\right\rangle\)
\(Z(3)\) gauge theory in \(2+1 \mathrm{D}\) with a PEPS ansatz


Emonts, Bañuls, Cirac, Zohar, Phys. Rev. D 102, 074501 (2020)

Quantum computing holds the promise of enabling access to quantities which are intractable with our current techniques due to sign and signal-to-noise problem.

Even if scalable noise-resilient quantum computers were available today, we are still not ready to express our LGT simulations in their language.

Theory, algorithm, and implementation and benchmark on hardware define the pillars of the program now and in upcoming years. Hardware co-design and interactions with other disciplines will be crucial.

The rate of progress and magnitude of developments are significant and lattice gauge theorists, including USOCD physicists, are making impactful contributions.

\section*{OUTLOOK FOR USQCD}

USQCD has formed a sub-committee on OIS+OC (chair: Martin Savage, members: Bazavov, ZD, Hasenfratz, Kronfeld, Meurice, Osborn, Petreczky, Simone, and El-Khadra), but concrete activities are yet to be planned.

USQCD members, acting as conveners in the Snowmass process and representing OIS+OC in LGT, are: Catterall, ZD, Izubuchi, Neil, and Savage, and of course El-Khadra and Gottlieb as co-leaders of Theory and Comp. Frontiers.

A Snowmass whitepaper in Quantum Simulation for HEP is commissioned by the Comp. Frontier and with Theory Frontier representatives (editors: Brauer and ZD). We will try to define better the role of USQCD there.

We begin with a cross country trip in 1967, from Brooklyn NY to Palo Alto CA.


The George Washington Bridge


My GTO


The Golden Gate Bridge


So much for history. Now we are in a new era. Hopefully the efforts of the past will inspire great progress in the next era of Quantum Computing!

This is a new field, full of workers with "fire in the belly". That's great! You remind of the characters in this story.

Taken from a nice recent presentation by J. Kogut.

InQubator for
Quantum Simulation

Quantum Simulation of Strong Interactions (QuaSI) Workshop 1 : Theoretical Strategies for Gauge Theories

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