3N SRCs, irreducible 3N forces and their implication on neutron star EOS

Misak Sargsian
Florida International University, Miami

4th Intl Workshop on Quantitative Challenges in SRC and EMC Effect Research
January 30-February 3, 2023 CEA Paris-Saclay

Nuclear Dynamics at Short Distances

Probing NN and NNN Interactions at < 1fm and their influence on the nuclear structure at short distances.

For NN interactions

-Identification of NN interaction in the nuclear dynamics

- Intermediate – short distance tensor forces

- Isospin dependence of the tensor forces

- NN repulsive core

- Hadron-quark transition in the core

- non-nucleonic components, hidden color, gluons

For NNN interactions

-Identification of NNN interaction in the nuclear dynamics

- Evaluation of irreducible 3N forces

- Evaluation of non-nucleonic component in NNN interactions

Modern NN Potentials

$$V^{2N} = V^{2N}_{EM} + V^{2N}_{\pi} + V^{2N}_{R}$$

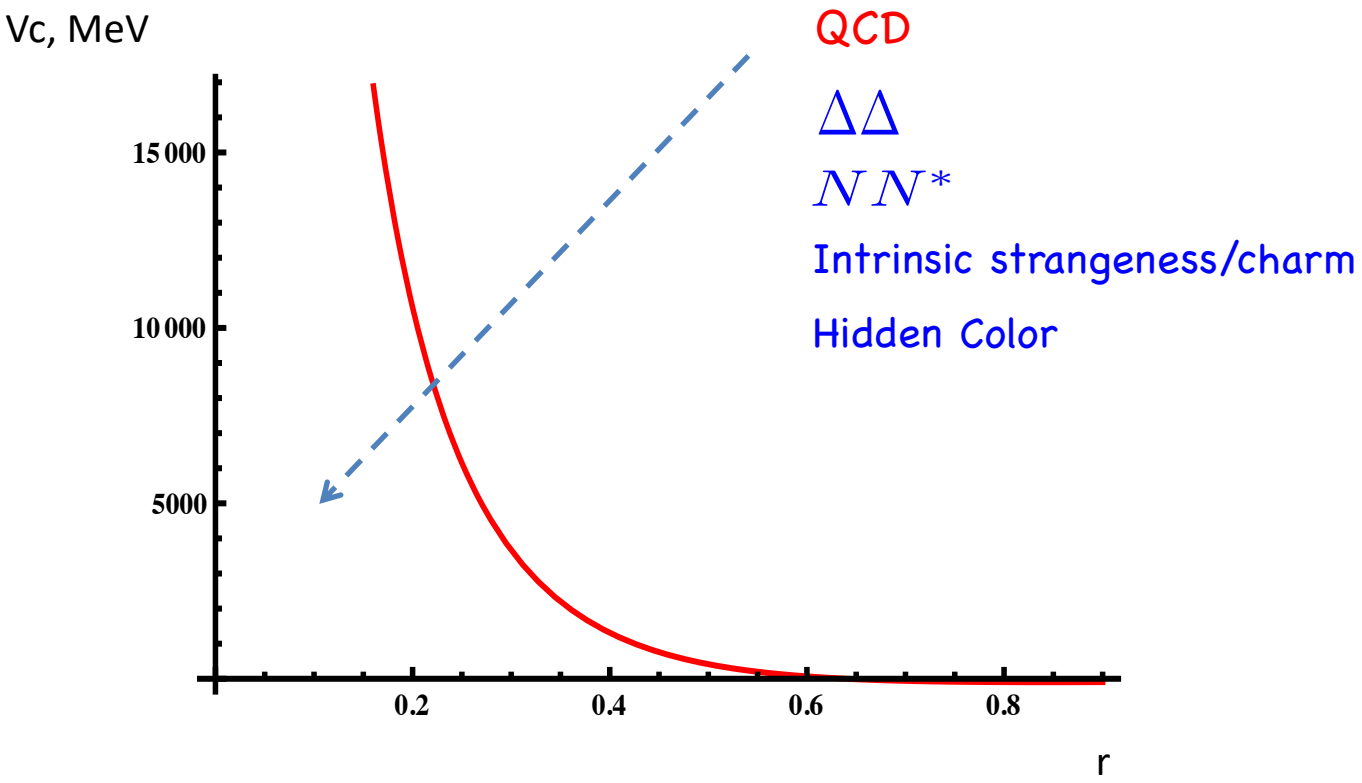
$$V^{2N}_{R} = V^{c} + V^{l2}L^2 + V^{t}S_{12} + V^{ls}L\cdot S + v^{ls2}(L\cdot S)^2$$

$$V^{i} = V_{int,R} + V_{core}$$

$$V_{core} = \left[1 + e^{\frac{r-r_0}{a}}\right]^{-1}$$

60's

Hans Bethe 1961: More intellectual energy is spent on nuclear physics than on all other issues taken together during entire human history.



~80% hidden color

Brodsky,Ji, Lepage, PRL 83

$$\Psi^{6q}_{T=0,S=1} = \sqrt{\tfrac{1}{9}}\Psi_{NN} + \sqrt{\tfrac{4}{45}}\Psi_{\Delta\Delta} + \sqrt{\tfrac{4}{5}}\Psi_{CC}$$

$$\Psi_{CC} \equiv \Psi_{N_C N_C}$$

The NN core can be due to the orthogonality of

$$\langle \Psi_{N_c,N_c} \mid \Psi_{N,N} \rangle = 0$$

From NN Interaction to Nuclear & Nuclear Matter Structure

"Standard" Nuclear Physics Approach

A-body Schroedinger equation interacting through NN -potential

$$\left[-\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2}\sum_{ij} V(x_i - x_j) + \sum_{ijk} V(x_i, x_j, x_k)\cdots\right]\psi(x_1, \cdots, x_A) = E\psi(x_1, \cdots, x_A)$$

Mean Field Approximation

$$\left[-\frac{\nabla_N^2}{2m} + V_{HF}(x)\right]\psi_N(x) = E_N\psi_N(x)$$

Hartree-Fock potential will smear out main properties NN potential 1990s

From NN Interaction to Nuclear & Nuclear Matter Structure

"Standard" Nuclear Physics Approach

A-body Schroedinger equation interacting through NN -potential

$$\left[-\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2}\sum_{ij} V(x_i - x_j) + \sum_{ijk} V(x_i, x_j, x_k)\cdots\right]\psi(x_1, \cdots, x_A) = E\psi(x_1, \cdots, x_A)$$

Ab Initio Calculations

NonRelativistic

Little Predictive Power

Conceptually: How to probe nuclei at short nucleon separations

- Probe bound nucleons at large internal momenta
- Need high energy probes to resolve such nucleons in nuclei
  in high energy nuclear processes

Theory of High Energy eA Scattering:

I. High-Energy approximations – small parameter

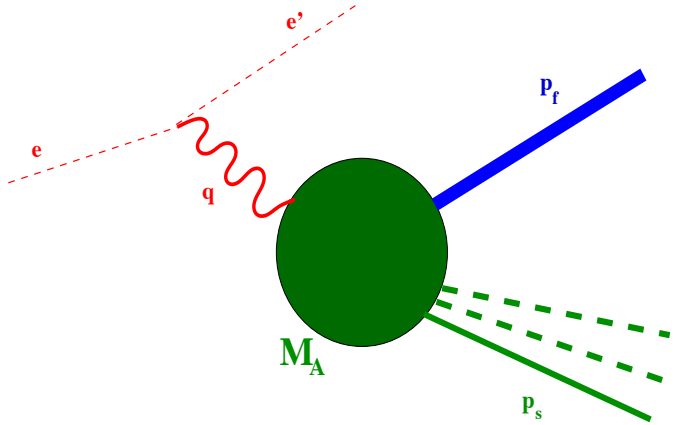
II. Emergence of Effective Theory – diagrammatic method

III. Light-Front Wave Function of Nucleus – relativism

IV. From Schroedinger Equation to LF Diagrams – LF-wavefunction + scattering

V. Emergence of small distance nuclear dynamics – predictions

I. High Energy Approximations:



$$|\vec{q}| = q_3 \sim p_{f3} \gg p \sim M_N$$

$$Q^2 \geq few \ \ \mathrm{GeV}^2$$

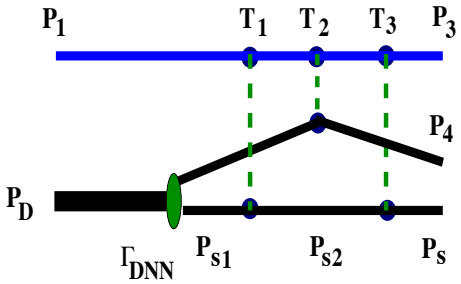
Both for QE/DIS

- Emergence of the small parameter

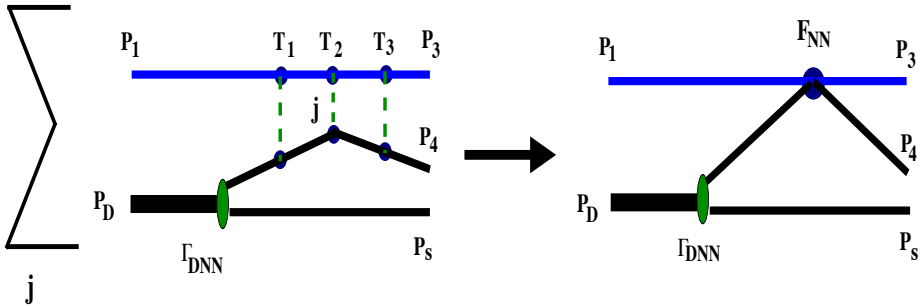
$$\frac{q_-}{q_+} = \frac{q_0 - q_3}{q_0 + q_3} \ll 1 \qquad \mathcal{O}\left(\frac{q_-}{q_+}\right)$$

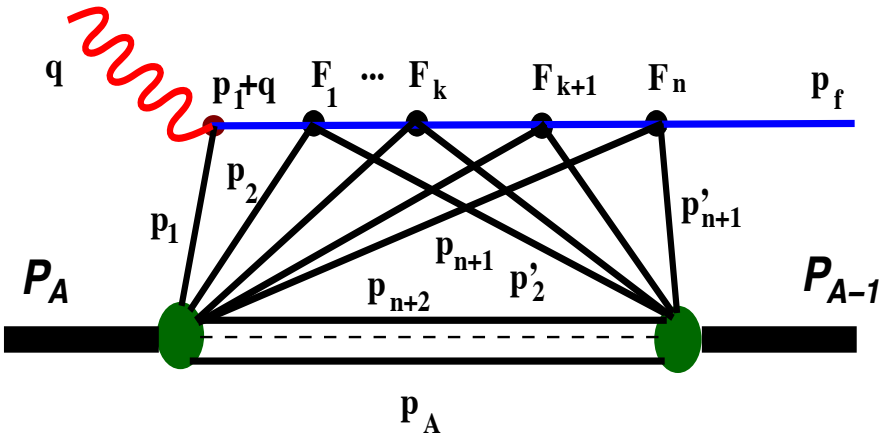
$$\frac{p_{f-}}{p_{f+}} = \frac{E_f - p_{f3}}{E_f + p_{f3}} \ll 1 \qquad \mathcal{O}\left(\frac{p_{f-}}{p_{f+}}\right)$$

II. Emergence of "effective" theory



$$\mathcal{O}\left(\frac{p_{1-}}{p_{1+}}\right)$$





Effective Feynman Diagrammatic Rules

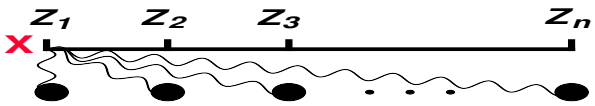
M.S. IJMS 2001

Wave function?

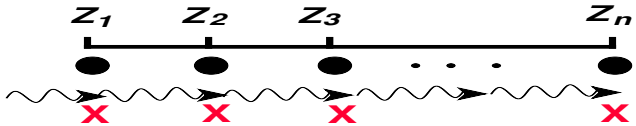
III. Light-Front Wave Function of the Nucleus

- Emergence of the light- front dynamics

$$\tau = t - z \sim \frac{1}{q_+} \longrightarrow 0$$



(a)



(b)

- non relativistic case: due to Galilean relativity observer X can probe all n-nucleons at the same time

$$\Psi(z_1, z_2, z_3, \cdots, z_n, t)$$

- relativistic case: observer X probes all n-nucleons at different n times

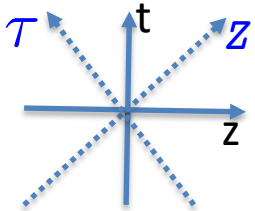
$$\Psi(z_1, t_1; z_2, t_2; z_3, t_3 \cdots; z_n, t_n)$$

- observer riding the light-front X probes all n-nucleons at same light-cone time:

$$\Psi_{LF}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \cdots, \mathcal{Z}_n, \tau)$$

$$\tau = t_1 - z_1 = t_2 - z_2 = \cdots = t_n - z_n$$

$$\mathcal{Z}_i = t_i + z_i$$



$$\Psi_{LF}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \cdots, \mathcal{Z}_n, \tau)$$

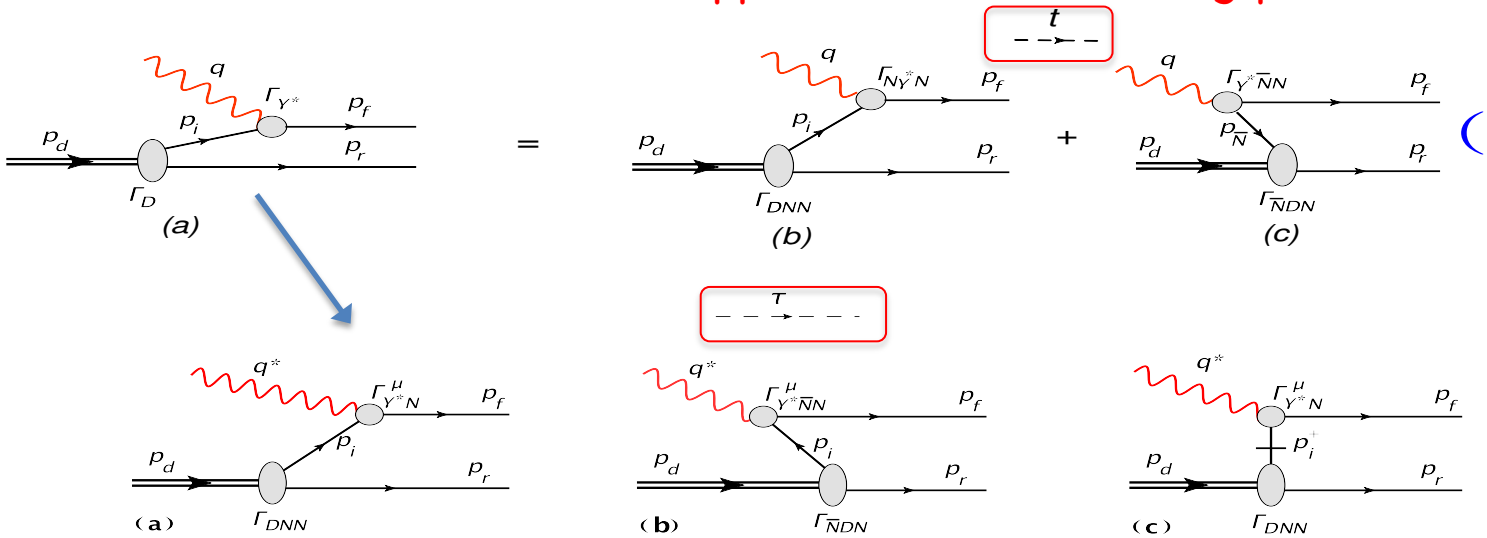
- in the momentum space

Variables

$$\Psi_{LF}(\alpha_1, p_{1\perp}; \alpha_2, p_{2\perp}; \alpha_3, p_{3\perp}; \cdots, \alpha_n, p_{n\perp})$$

$$\alpha_i = \frac{p_{i-}}{p_{A-}/A}, \; p_{i,\perp}$$

- How the LF wave function appears in the scattering process



$$(b) \sim (c)$$

$$(b) = 0$$

Frank Vera, M.S. PRC 2018

IV From Schroedinger Equation -> Light-Front Wave Function

Schroedinger eq. → Lipmann-Schwinger Eq.

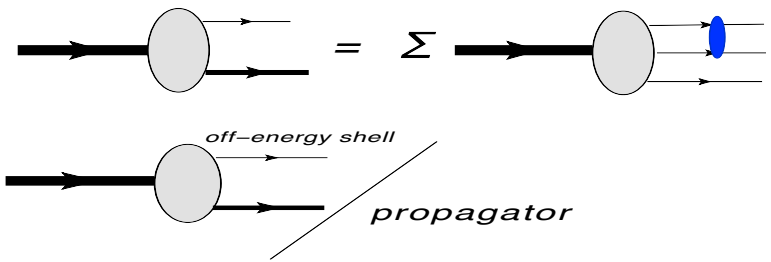
$$\left[-\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2}\sum_{i,j} V(x_i - x_j)\right]\psi(x_1,\cdots,x_A) = E\psi(x_1,\cdots,x_A)$$

$$\left(\sum_i \frac{k_i^2}{2m} - E_b\right)\Phi(k_1,\cdots,k_A) = -\frac{1}{2}\sum_{i,j}\int U(q)\,\Phi(k_1,\cdots,k_i-q,\cdots k_j+q,\cdots,k_A)d^3q$$

Lipmann-Schwinger Eq → t- ordered diagrammatic method

$$\left(\sum_i \frac{k_i^2}{2m} - E_b\right)\Phi(k_1,\cdots,k_A) = -\frac{1}{2}\sum_{i,j}\int U(q)\,\Phi(k_1,\cdots,k_i-q,\cdots k_j+q,\cdots,k_A)d^3q$$

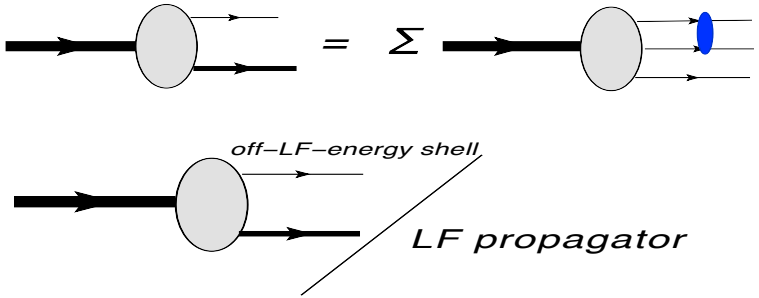
$$\Phi(k_1,\cdots k_A) = \frac{1}{\sum \frac{k_i^2}{2m} - E_b}\Gamma_{A\to N,A-1}$$



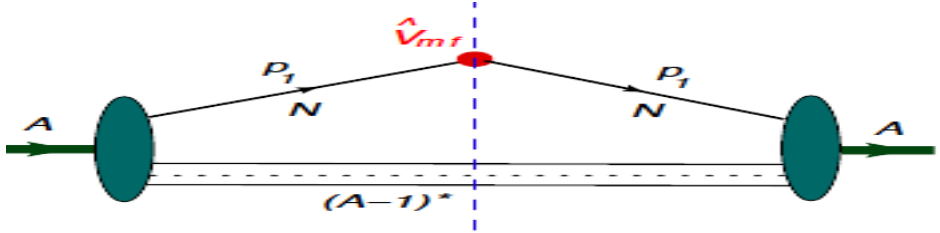
Weinberg Eq → $\mathcal{T}$ - ordered diagrammatic method

$$\left(\sum \frac{k_{i\perp}^2+m^2}{\alpha_i} - M_A^2)\Phi_{LF}(k_1,\cdots k_A)\right) = \frac{1}{2}\sum_{i,j}\int U_{LF}(q)\Phi_{LF}(k_1,\cdots k_A)\prod\frac{d\alpha_i}{\alpha_i}d^2k_{i\perp}$$

$$\Phi_{LF}(k_1,\cdots k_A) = \frac{1}{\sum \frac{k_{i\perp}^2+m^2}{\alpha_i} - M_A^2}\Gamma_{A\to N,A-1}$$



Spectral Function Calculations



$$S_A^{MF} = -Im \int \chi_A^\dagger \Gamma^\dagger_{A,N,A-1} \frac{\not{p}_1 + m}{p_1^2 - m^2 + i\varepsilon} \hat{V}^{MF} \frac{\not{p}_1 + m}{p_1^2 - m^2 + i \times \varepsilon} \left[\frac{G_{A-1}(p_{A-1})}{p_{A-1}^2 - M_{A-1}^2 + i\varepsilon} \right]^{on} \Gamma_{A,N,A-1} \chi_A \frac{d^4 p_{A-1}}{i(2\pi)^4}$$

$$\hat{V}^{MF} = i a^\dagger(p_1, s_1) \delta^3(p_1 + p_{A-1}) \delta(E_m - E_\alpha) a(p_1, s_1)$$

$$\psi_{N/A}(p_1, s_1, s_A, E_\alpha) = \frac{\bar{u}(p_1, s_1) \Psi^\dagger_{A-1}(p_{A-1}, s_{A-1}, E_\alpha) \Gamma_{A,N,A-1} \chi_A}{(M_{A-1}^2 - p_{A-1}^2) \sqrt{(2\pi)^3 2 E_{A-1}}}$$

$$S_A^{MF}(p_1, E_m) = \sum_\alpha \sum_{s_1, s_{A-1}} \mid \psi_{N/A}(p_1, s_1, s_A, E_\alpha) \mid^2 \delta(E_m - E_\alpha)$$

V. Emergence of small distance nuclear dynamics

-start with A-body Lipman Schwinger/Weinberg type equation for nuclear wave function interacting through NN -potential

$$\phi_A(k_1,\cdots k_n,\cdots k_A) = \frac{-\frac{1}{2}\int \sum_{i\neq j} V_{ij}(q)\phi_A(k_1,\cdots,k_i-q,\cdots,k_j+q,\cdots,k_A)\frac{d^3q}{(2\pi)^3}}{\sum_{i=1}^{A}\frac{k_i^2}{2m_N}-E_B}$$

$k=p,\ \frac{p^2}{2m}\gg E_B,\ k_i-q=p-q\approx 0\rightarrow q\approx p,\ k_j+q\approx k_j+p\approx 0\rightarrow \boxed{k_i\approx -k_j\approx p}$

$$\boxed{\phi_A^{(1)}(k_1,\cdots,k_i=p,\cdots,k_j\approx -p,\cdots,k_A)\sim \frac{V_{NN}(p)}{p^2}f(k_1,\cdots'\cdots'\cdots)}$$

$$\phi_A^{(2)}(\cdots p,\cdots)\sim \frac{1}{p^2}\int \frac{V_{NN}(q)V_{NN}(p)}{(p-q)^2}d^3q$$

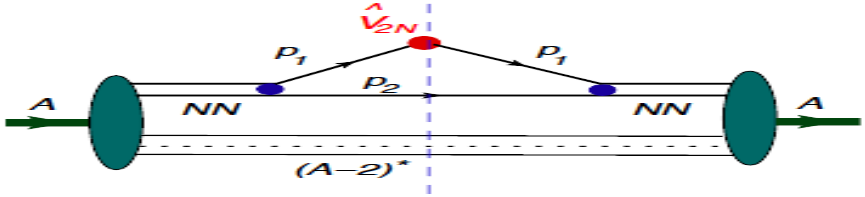
- IF: $V_{NN}=q^{-n}$ with $n>1$ and is finite range q_{min}

$$\boxed{\phi_A^{(2)}(\cdots p,\cdots)\sim \frac{V(p)}{p^2}\int_{q_{min}}^{\infty}\frac{dq}{q^n}}$$

$$\phi_A^{(2)}\ll\phi_A^{(1)}$$

Frankfurt, Strikman 1988
Frankfurt, MS, Strikman 2008

2N SRC model



$$P^N_{A,2N}(\alpha_1, p_{1,\perp}, \tilde{M}^2_N) = \sum_{s_2, s_{NN}, s_{A-2}} \int \chi^\dagger_A \Gamma^\dagger_{A\to NN,A-2} \chi_{A-2}(p_{A-2}, s_{A-2})$$
$$\times \frac{\chi_{NN}(p_{NN}, s_{NN})\chi^\dagger_{NN}(p_{NN}, s_{NN})}{p^2_{NN} - M^2_{NN}} \Gamma^\dagger_{NN\to NN} \frac{u(p_1, s_1)u(p_2, s_2)}{p^2_1 - M^2_N}$$
$$\times \left[2\alpha^2_1 \delta(\alpha_1 + \alpha_2 + \alpha_{A-2} - A)\delta^2(p_{1,\perp} + p_{2,\perp} + p_{A-2,\perp})\delta(\tilde{M}^2_N - \tilde{M}^{(2N),2}_N)\right] \frac{\bar{u}(p_1, s_1)\bar{u}(p_2, s_2)}{p^2_1 - M^2_N}$$
$$\times \Gamma_{NN\to NN} \frac{\chi_{NN}(p_{NN}, s_{NN})\chi^\dagger_{NN}(p_{NN}, s_{NN})}{p^2_{NN} - M^2_{NN}} \chi^\dagger_{A-2}(p_{A-2}, s_{A-2})\Gamma_{A,NN,A-2}\chi_A$$
$$\times \frac{d\alpha_2}{\alpha_2} \frac{d^2p_{2,\perp}}{2(2\pi)^3} \frac{d\alpha_{A-2}}{\alpha_{A-2}} \frac{d^2p_{A-2,\perp}}{2(2\pi)^3}. \qquad (1)$$

O. Artiles & M.S. Phys. Rev. C 2016

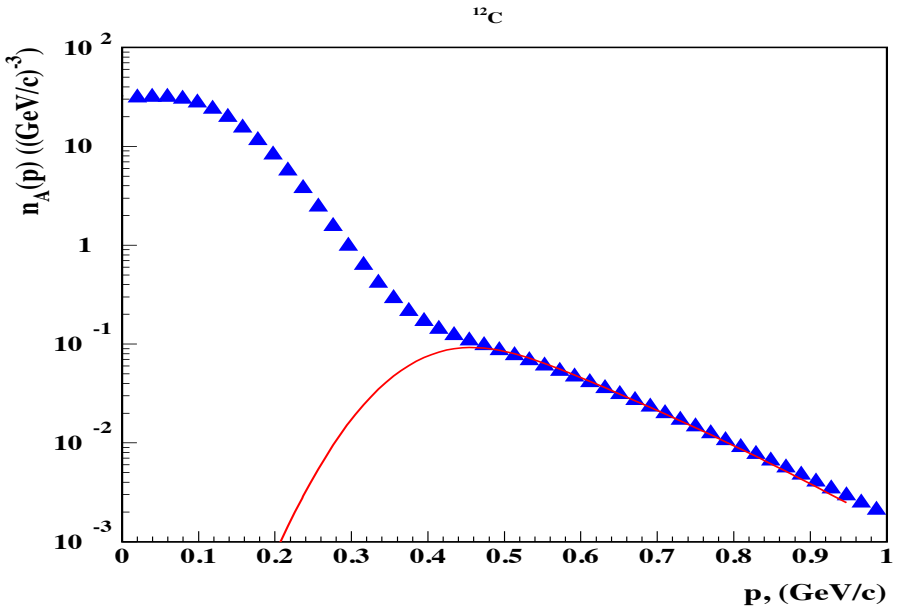
$$\rho_A(\alpha_N, p_{N,\perp}) = \int P_A(\alpha_N, p_{N,\perp}, \tilde{M}^2_N)\tfrac{1}{2} d\tilde{M}^2_N$$

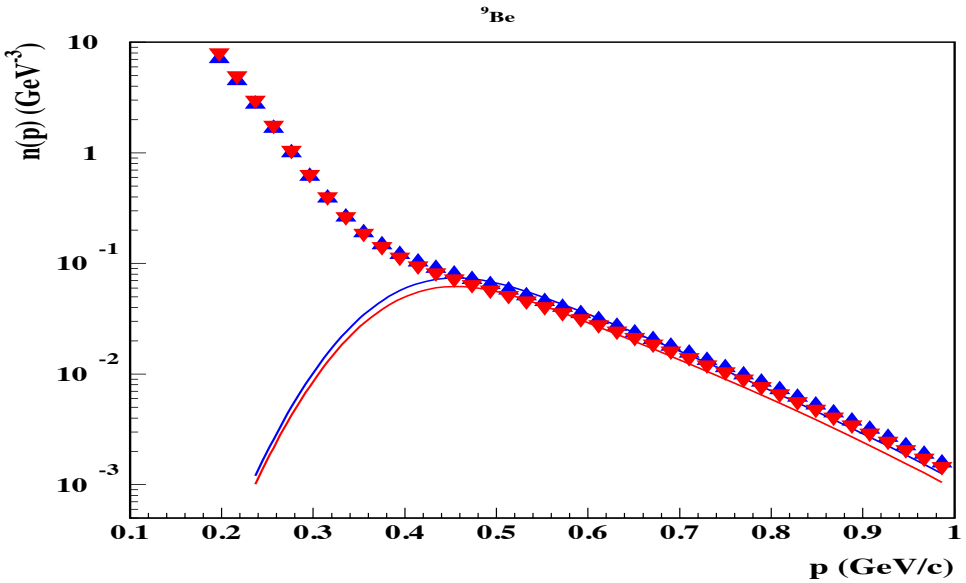
$$\psi^{LF}_{pn}(\alpha, p_\perp) \approx C\psi^{LF}_d(\alpha, p_\perp)$$

$$\psi^{s_{NN}}_{2N}(\beta_1, k_{1,\perp}, s_1, s_2) = -\frac{1}{\sqrt{2(2\pi)^3}} \frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}\cdot\chi_{NN}(p_{NN},s_{NN})}{\frac{1}{2}[M^2_{NN} - 4(M^2_N + k^2_1)]}$$

$$\psi_{CM}(\alpha_{NN}, k_{NN,\perp}) = -\frac{1}{\sqrt{\frac{A-2}{2}}} \frac{1}{\sqrt{2(2\pi)^3}} \frac{\chi^\dagger_{NN}(p_{NN},s_{NN})\chi^\dagger_{A-2}(p_{A-2},s_{A-2})\Gamma_{A\to NN,A-2}\chi^{s_A}_A}{\frac{2}{A}[M^2_A - s_{NN,A-2}(k_{CM})]}$$

2N SRC model Non Relativistic Approximation





Predictions made within the described approach

- Different location for maximum of FSI compared to Glauber model Frankfurt, MS., Strikman, PRC 1997, MS PRC2010

- Nuclear scaling due to 2N SRCs at: Frankfurt, Strikman Phys Rep.88
Frankfurt, Day, Strikman, MS PRC1993

- pn dominance in 2N SRCs: Piasetzky, MS, Frankfurt, Strikman, Watson PRL2006

- High momentum sharing in asymmetric nuclei MS PRC2014, ArXiv2012

- New Structure in the deuteron and non-nucleonic components MS &F.Vera ArXiV2022

- Nuclear scaling for 3N SRCs

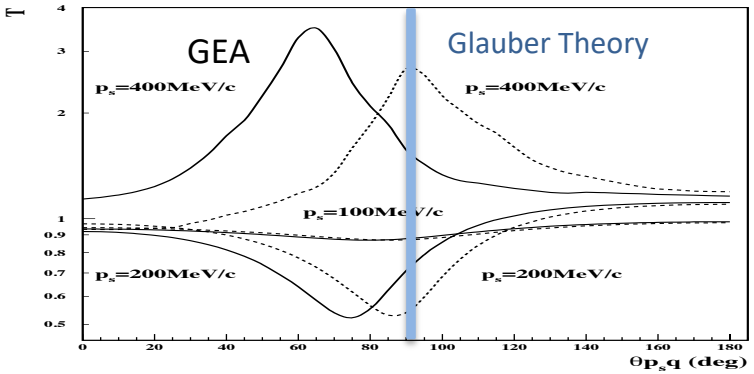
- Relation between 3N and 2N SRCs

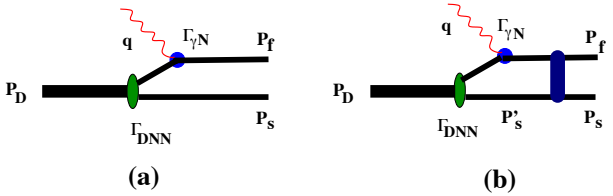
MS, Day,Frankfurt, Strikman PRC2019
Frankfurt, Day, MS,Strikman PRC2023

- Different location for maximum of FSI compared to Glauber model

$$e + d \rightarrow e' + p + n$$

Frankfurt, MS, Strikman, PRC 1997

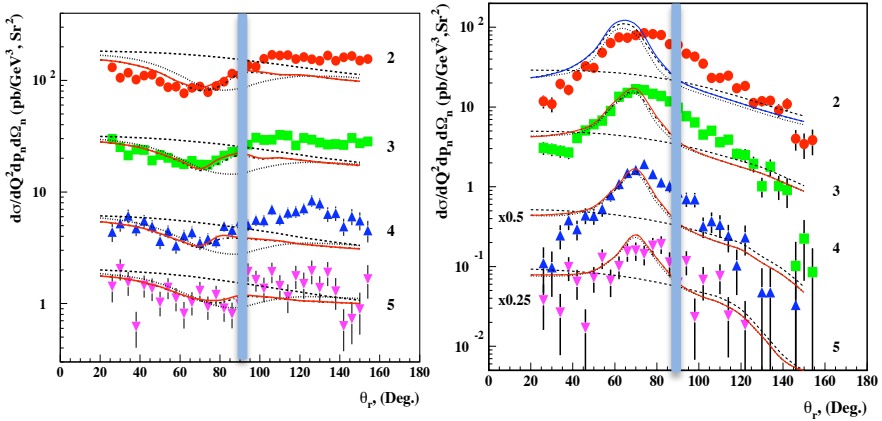


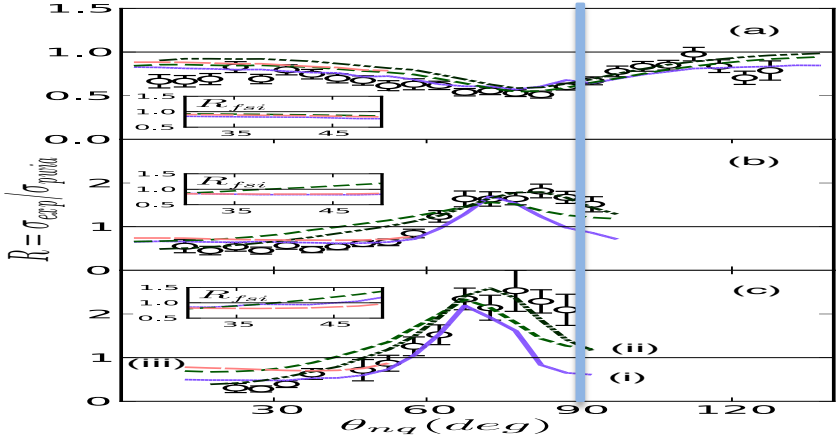


K. Egiyan et al PRL 2008

Unfactorized calculations MS PRC 2020

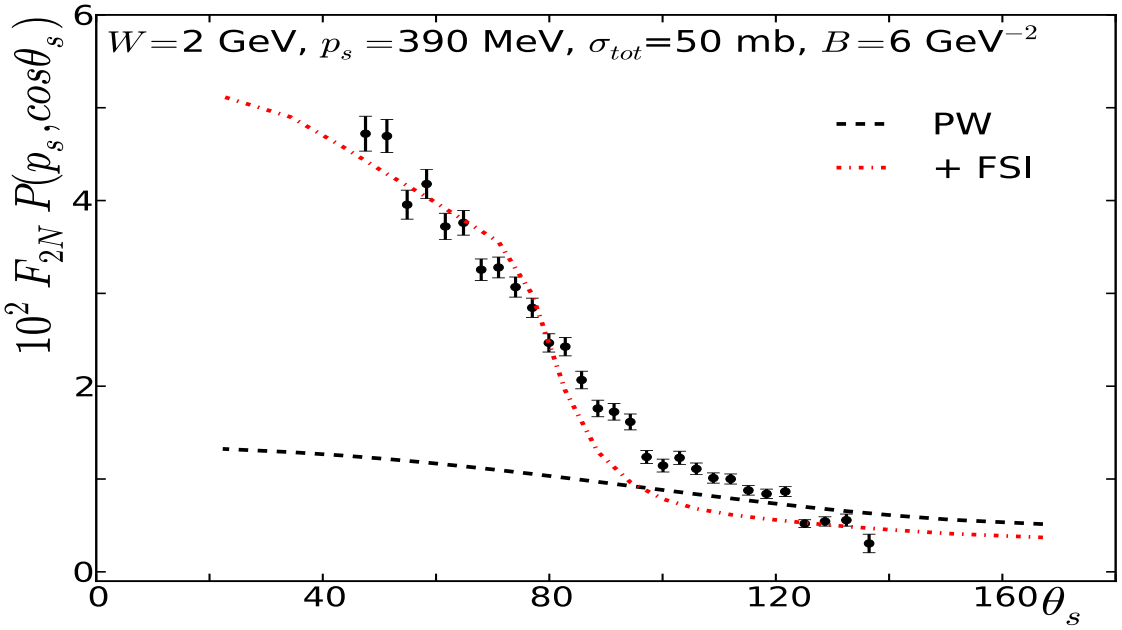
W. Boeglin et al PRL 2011





Extension to DIS: $e + d \to e' + p_s + X$

W.Cosyn & M.Sargsian, PRC 2011



- Nuclear scaling due to 2N SRCs at:

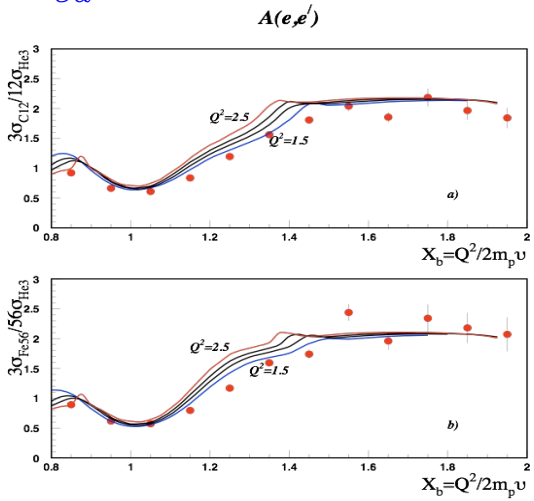
$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

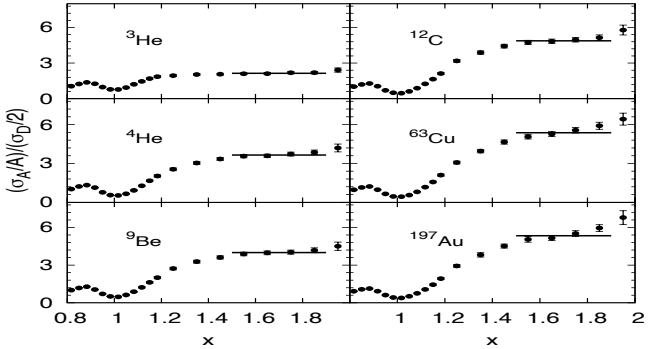
$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N}\left(1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}}\right) \quad 1.3 \leq \alpha_{2N} \leq 1.5$$

Frankfurt, Strikman Phys. Rep, 1988
Day,Frankfurt, Strikman, MS, Phys. Rev. C 1993

$$\boxed{\rho_A(\alpha) \approx a_2(A, z)\rho_{NN}(\alpha)}$$

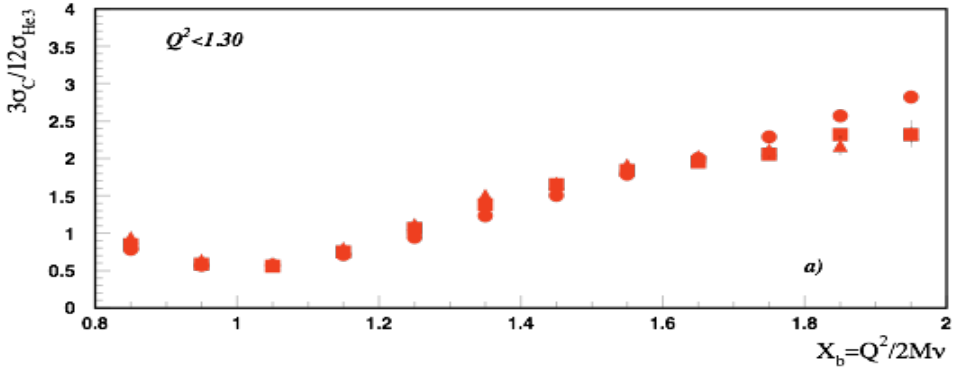
$$\frac{2\sigma_{eA}}{A\sigma_{ed}} \approx a_2(A, Z) \quad 1.3 \leq \alpha_{2N} \leq 1.5$$



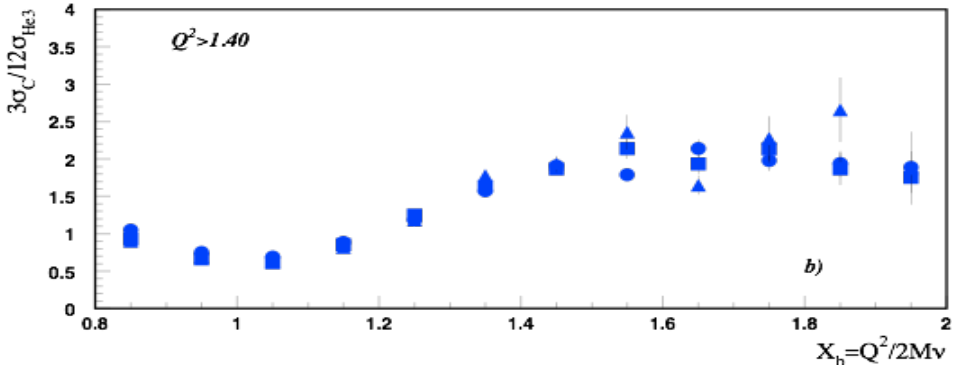


Egiyan et al, 2002,2006
Fomin et al, 2011

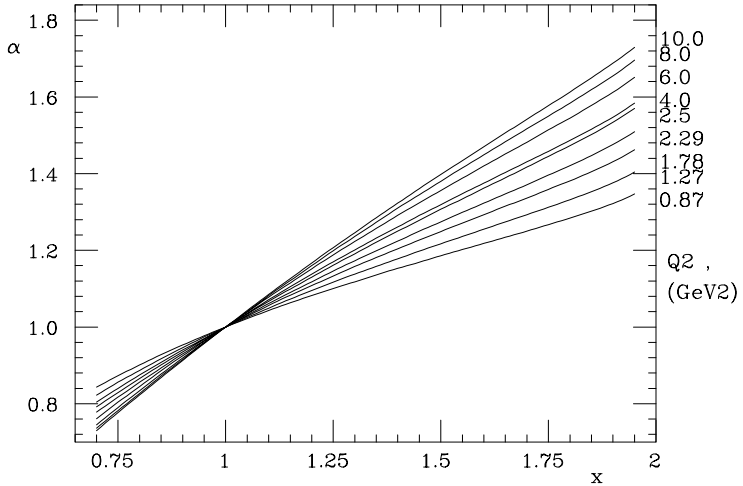
A(e,e')



$\alpha_{2N} < 1.3$



$\alpha_{2N} > 1.3$



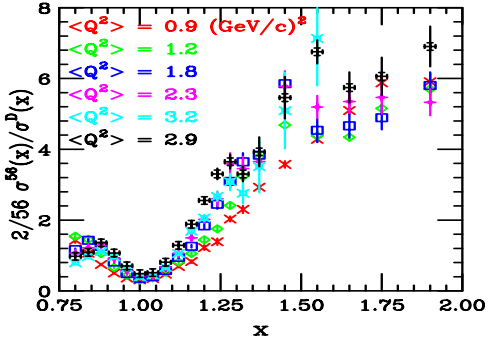
X vs α_{2N} Dependences

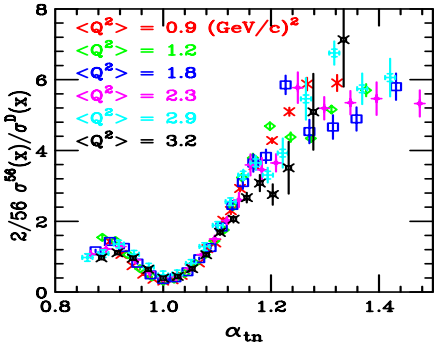
$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

$$1.3 \leq \alpha_{2N} \leq 1.5$$

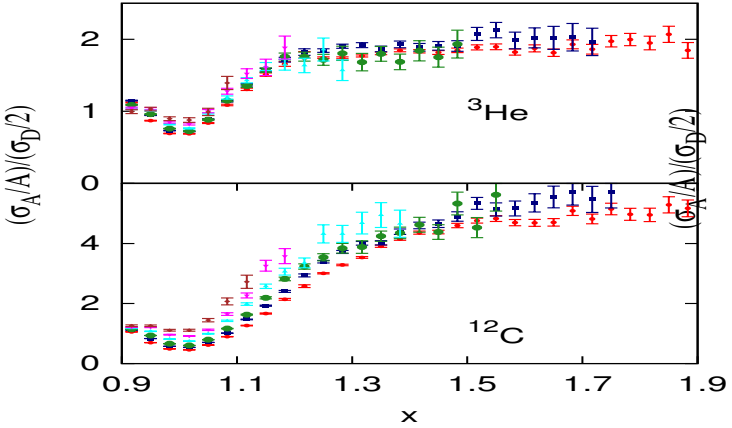
$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N}\left(1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}}\right)$$

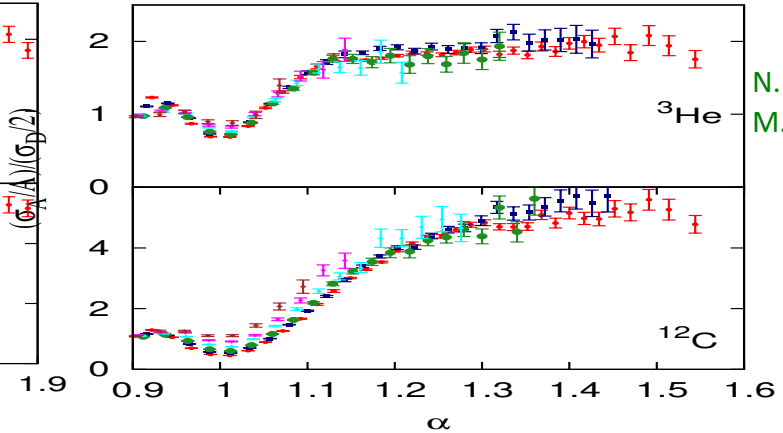
$$\alpha \,\Big|_{Q^2 \to \infty} \to x$$
$$\alpha \,\Big|_{x \to 1} \to 1$$





J.Arrington, D.Higinbotham
G.Rosner, M.S. Prog. PNP 2012





N.Fomin, D.Higinbotham
M.S., P.Sovignon ARNPS, 2017

a_2's as relative probability of 2N SRCs

Table 1: The results for $a_2(A, y)$

A	y	This Work	Frankfurt et al	Egiyan et al	Famin et al
^{3}He	0.33	2.07±0.08	1.7±0.3		2.13±0.04
^{4}He	0	3.51±0.03	3.3±0.5	3.38±0.2	3.60±0.10
^{9}Be	0.11	3.92±0.03			3.91±0.12
^{12}C	0	4.19±0.02	5.0±0.5	4.32±0.4	4.75±0.16
^{27}Al	0.037	4.50±0.12	5.3±0.6		
^{56}Fe	0.071	4.95±0.07	5.6±0.9	4.99±0.5	
^{64}Cu	0.094	5.02±0.04			5.21±0.20
^{197}Au	0.198	4.56±0.03	4.8±0.7		5.16±0.22

- pn dominance in 2N SRCs:

for large $k > k_{Fermi}$

$$P_{pn/pX} = 0.92^{+0.08}_{-0.18}$$

$$\frac{P_{pp}}{P_{pn}} \leq \frac{1}{2}(1 - P_{pn/pX}) = 0.04^{+0.09}_{-0.04}.$$

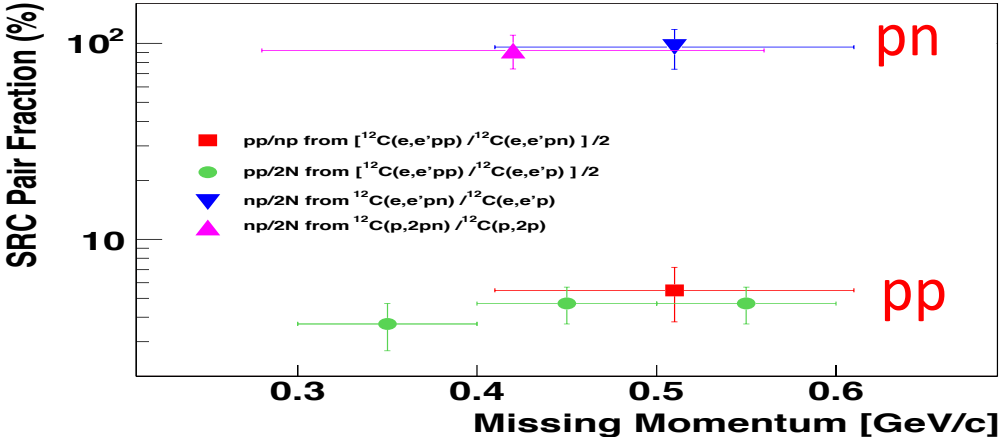
Theoretical analysis of BNL Data $A(p,2p)X$ reaction

E. Piasetzky, MS, L. Frankfurt,
M. Strikman,J.Watson PRL , 2006

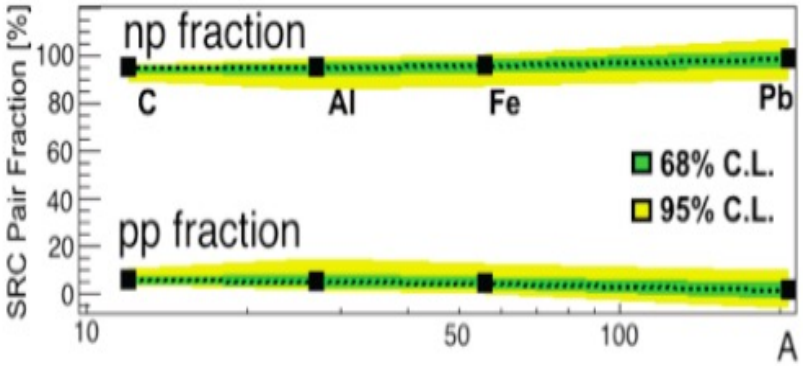
Factor of 20
Expected 4
(Wigner counting)

Direct Measurement at JLab R.Subdei, et al Science , 2008

$$P_{pp/pn} = 0.056 \pm 0.018$$



O. Hen, MS, L, Weinstein et.al. Science, 2014



- High momentum sharing in asymmetric nuclei

- Dominance of pn Correlations
(neglecting pp and nn SRCs)

Two properties were predicted

MS,arXiv:1210.3280. 2012
Phys. Rev. C 2014

First Property: Approximate Scaling Relation

$$x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p) = \frac{a_{pn}}{2}(A,Z)n_d(p)$$

where $x_p = \frac{Z}{A}$ and $x_n = \frac{A-Z}{A}$.

Second Property: Inverse Fractional Dependence of High Momentum Component

$$n_{p/n}^A(p) \approx \frac{1}{2x_{p/n}} a_2(A,y) \cdot n_d(p)$$

$$n_{p/n}^A(p) \approx \frac{1}{2(x_{p/n})^\gamma} a_2(A,y) \cdot n_d,\ \gamma \leq 1$$

$\gamma \approx 0.85$ for ^3He

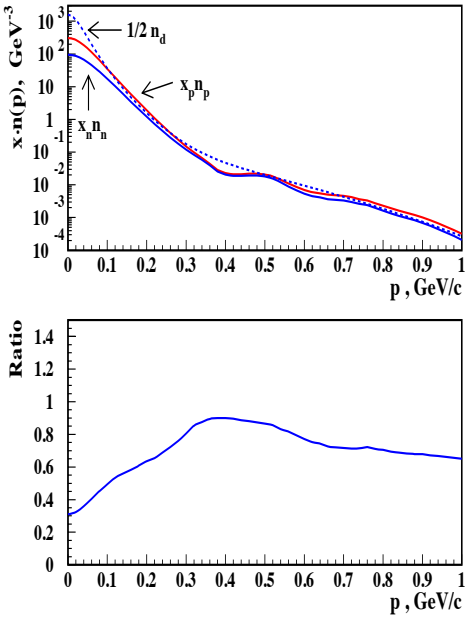
$\gamma \approx 1.00$ for $^9Be, ^{10}B$

Minority component has more high momentum fraction

$$P_{p/n}(A,y) = \frac{1}{2x_{p/n}} a_2(A,y) \int_{k_F}^{\infty} n_d(p) d^3p$$

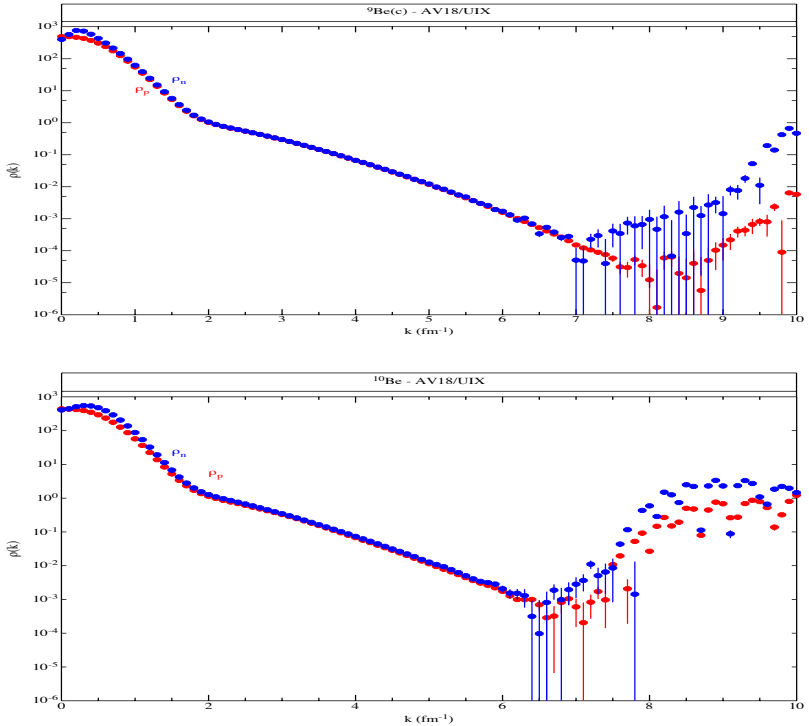
First Property: $x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p)$

^{3}He



Be9,B10 Variational Monte Carlo Calculation: Robert Wiringa 2013

http://www.phy.anl.gov/theory/research/momenta/



Predictions: High Momentum Fractions

MS,arXiv:1210.3280,2012
Phys. Rev. C 2014

$$P_{p/n}(A,y) = \frac{1}{2x_{p/n}} a_2(A,y) \int_{k_F}^{\infty} n_d(p) d^3p$$

$$y = |x_p - x_n|$$

A	P_p(%)	P_n(%)
12	20	20
27	23	22
56	27	23
197	31	20

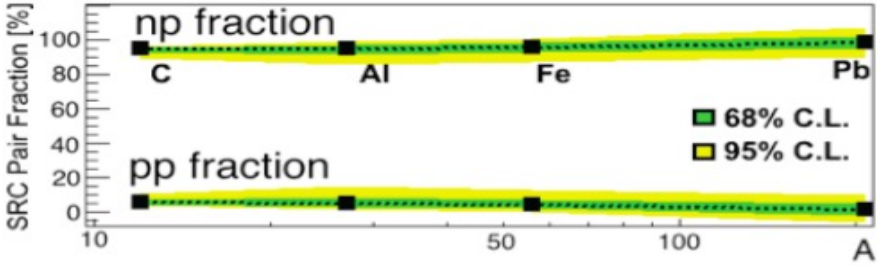
Table 1: Kinetic energies (in MeV) of proton and neutron

A	y	E^p_{kin}	E^n_{kin}	$E^p_{kin} - E^n_{kin}$
^{8}He	0.50	30.13	18.60	11.53
^{6}He	0.33	27.66	19.06	8.60
^{9}Li	0.33	31.39	24.91	6.48
^{3}He	0.33	14.71	19.35	-4.64
^{3}H	0.33	19.61	14.96	4.65
^{8}Li	0.25	28.95	23.98	4.97
^{10}Be	0.2	30.20	25.95	4.25
^{7}Li	0.14	26.88	24.54	2.34
^{9}Be	0.11	29.82	27.09	2.73
^{11}B	0.09	33.40	31.75	1.65

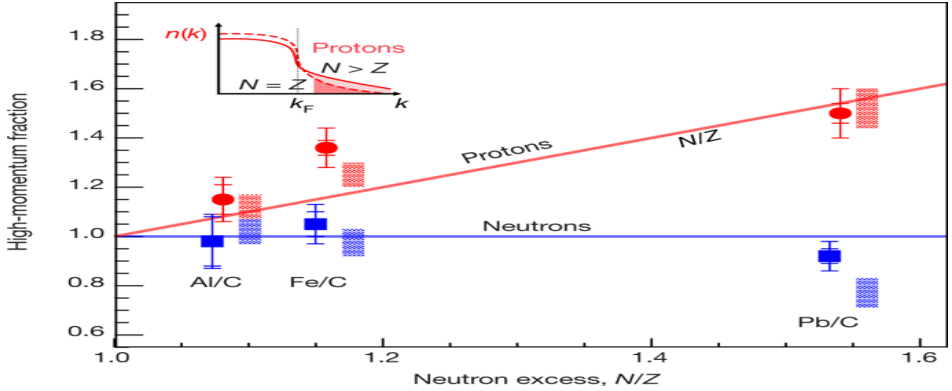
- Experimental Verification of Momentum Sharing Effects

- pn dominance persist for heavy nuclei

O. Hen, MS, L, Weinstein et.al. Science, 2014

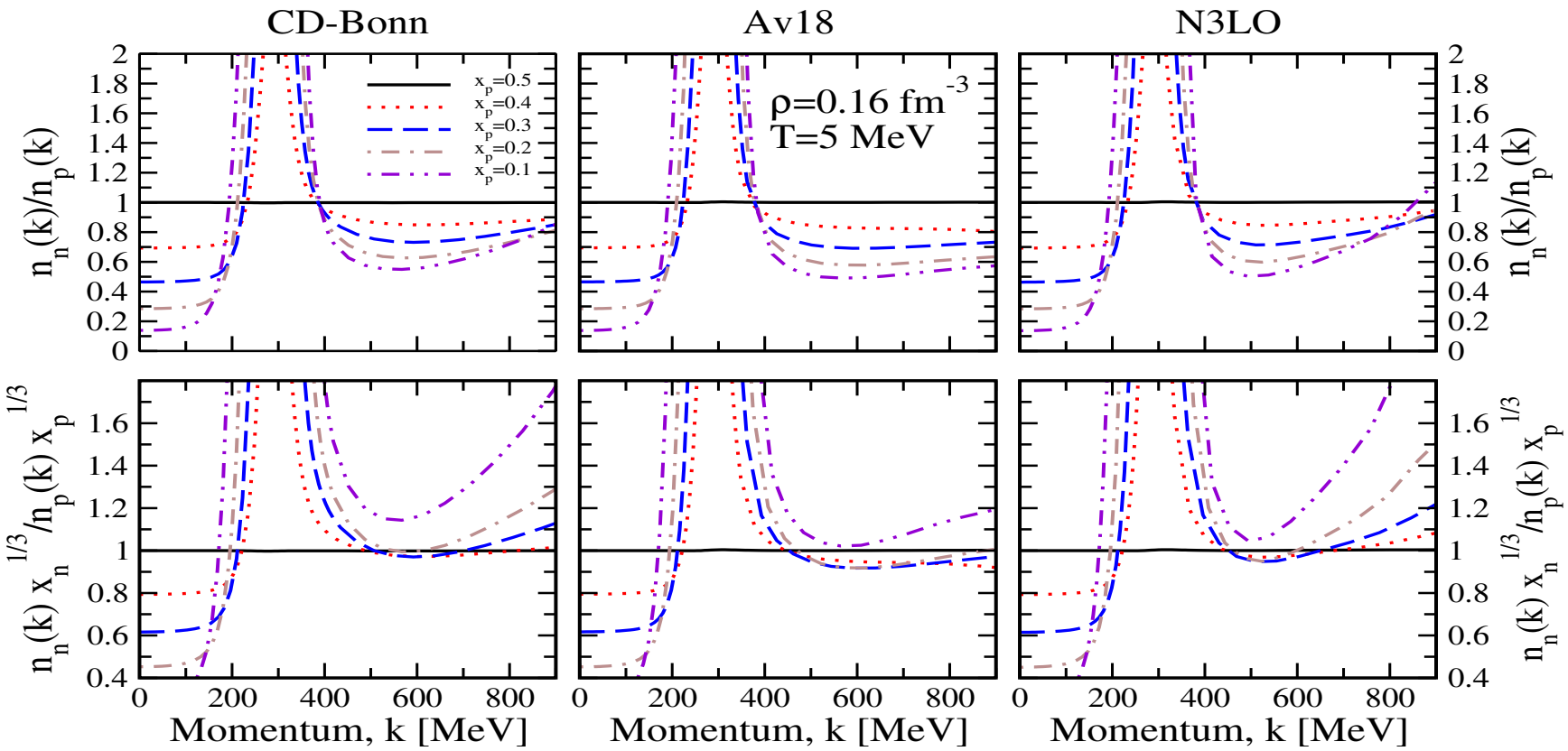


Duer et al, Nature 2018



Asymmetric Nuclear Matter Calculations

A.Rios, A. Polls and W. H. Dickhoff, Phys. Rev. C 2014



- New Structure in the deuteron and non-nucleonic components

MS &F.Vera ArXiV2022

Monday's talk

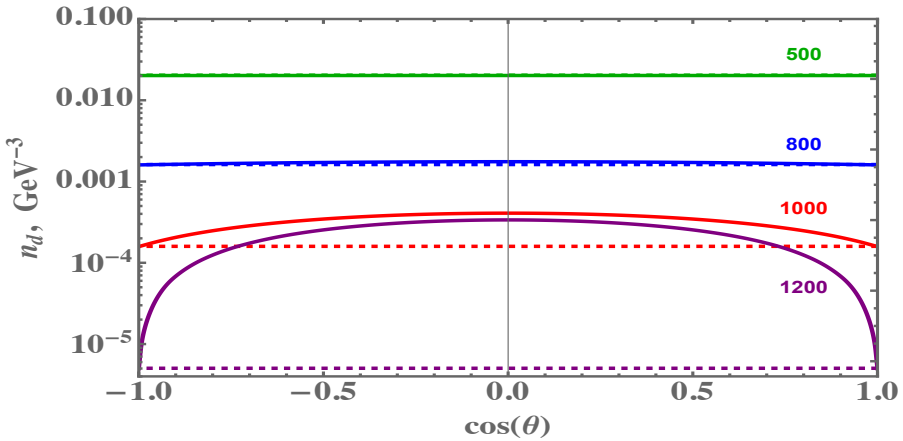
$$\psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = \sum_{\lambda_1'} \phi^\dagger_{\lambda_2} \sqrt{E_k} \left[\frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_\mathbf{d}^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi}\sqrt{2}} \left(\frac{3(\sigma\mathbf{k})(\mathbf{k}\mathbf{s}_\mathbf{d}^\lambda)}{k^2} - \sigma \mathbf{s}_\mathbf{d}^\lambda \right) + (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1,|\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda_1'}}{\sqrt{2}} \phi_{\lambda_1'}$$

$$U(k) = \frac{2\sqrt{4\pi}\sqrt{E_k}}{3} \left[\Gamma_1 (2 + \frac{m_N}{E_k}) + \Gamma_2 \frac{k^2}{m_N E_k} \right]$$

$$W(k) = \frac{2\sqrt{4\pi}\sqrt{2E_k}}{3} \left[\Gamma_1 (1 - \frac{m_N}{E_k}) - \Gamma_2 \frac{k^2}{m_N E_k} \right]$$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k)\sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

$$n_d(k, k_\perp) = \frac{1}{3} \sum_{\lambda_d=-1}^{1} |\psi_d^{\lambda_d}(\alpha, k_\perp)|^2 = \frac{1}{4\pi} \left(U(k)^2 + W(k)^2 + \frac{k_\perp^2}{k^2} P^2(k) \right)$$



- Nuclear scaling for 3N SRCs & - Relation between 3N and 2N SRCs

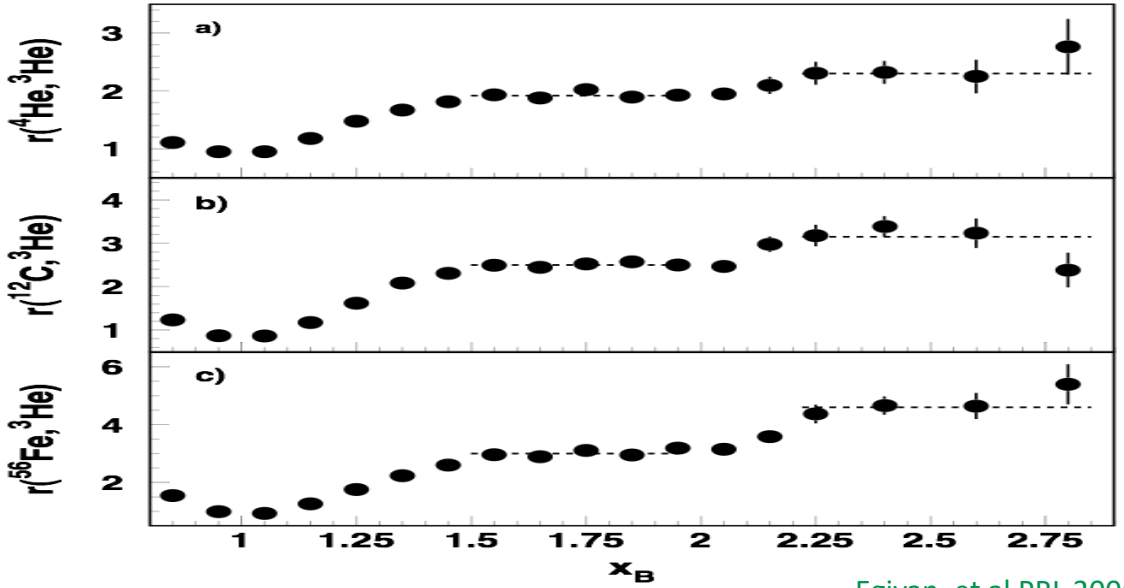
Looking for the Plateau in Inclusive Cross Section Ratios x>2

For $1 < x < 2$ $R \approx \frac{a_2(A_1)}{a_2(A_2)}$

2N SRCs

For $2 < x < 3$ $R \approx \frac{a_3(A_1)}{a_3(A_2)}$

3N SRCs



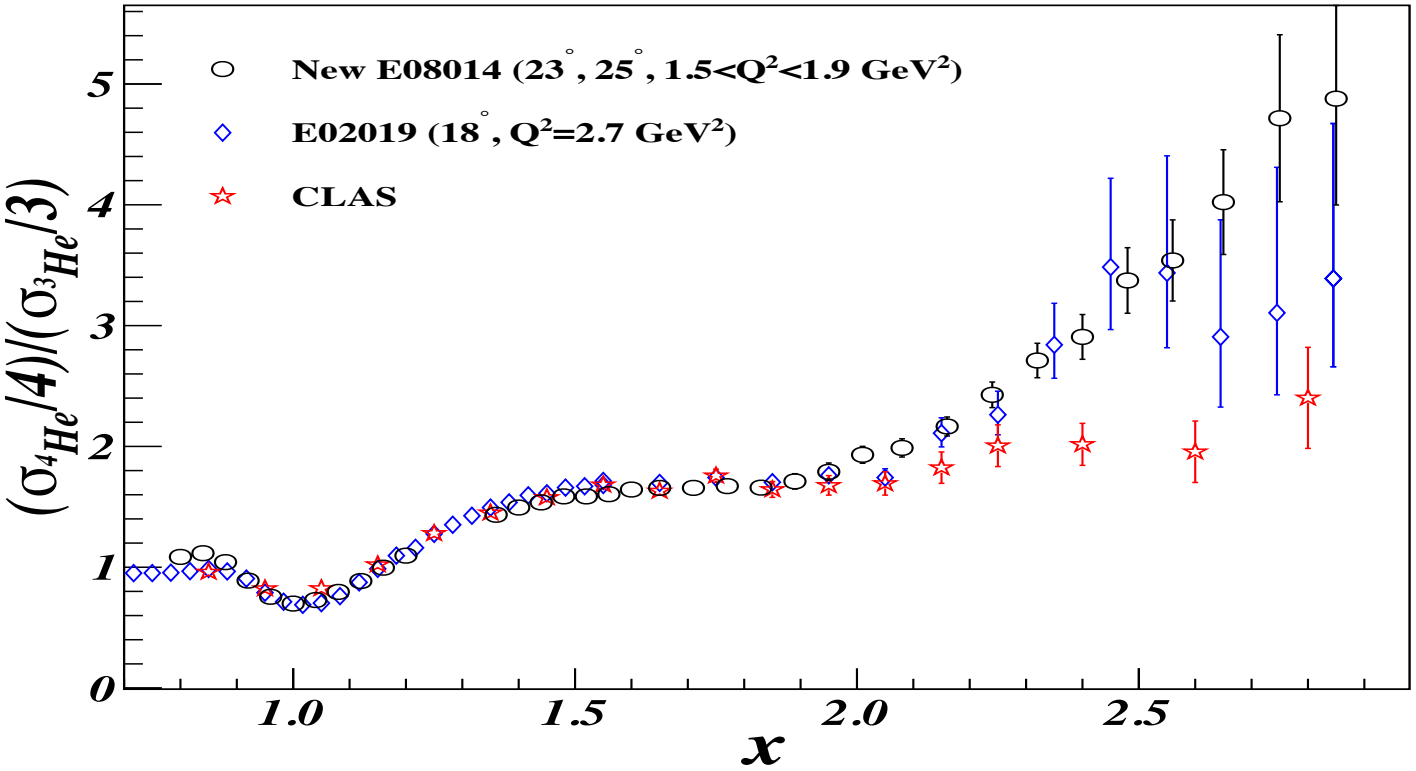
$Q^2 = 1.4\ GeV^2$

Egiyan, et al PRL 2006

Three Nucleon Short Range Correlations

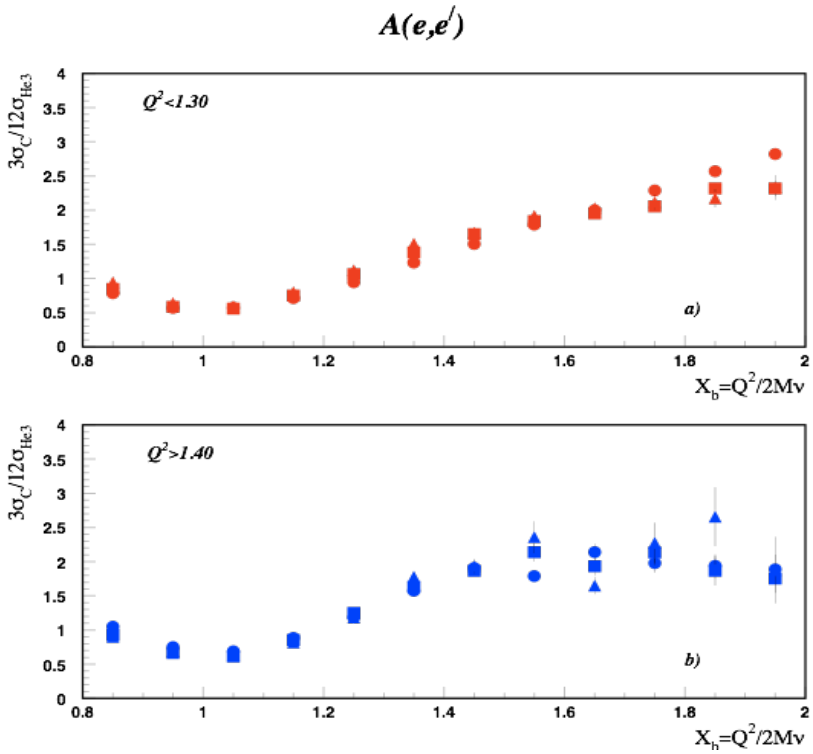
Z. Ye, at al, 2017

Looking for the Plateau in Inclusive Cross Section Ratios x>2



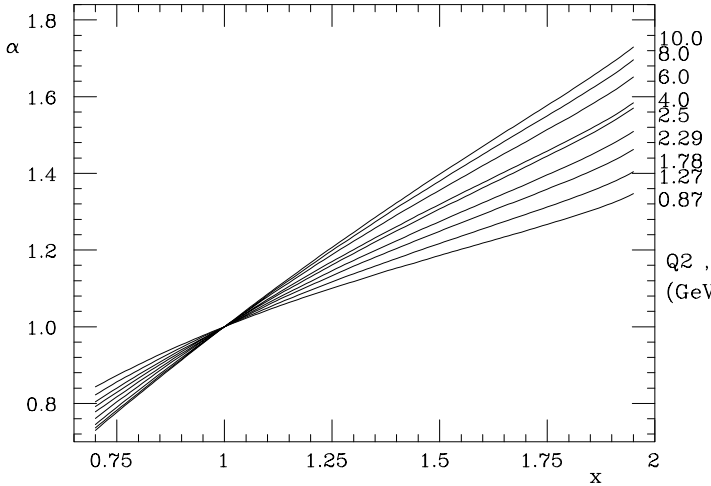
Do we really measure momentum fractions: α relevant for 3N SRCs?

Remember for 2N SRCs the onset of scaling was related to α



$\alpha_{2N} < 1.3$

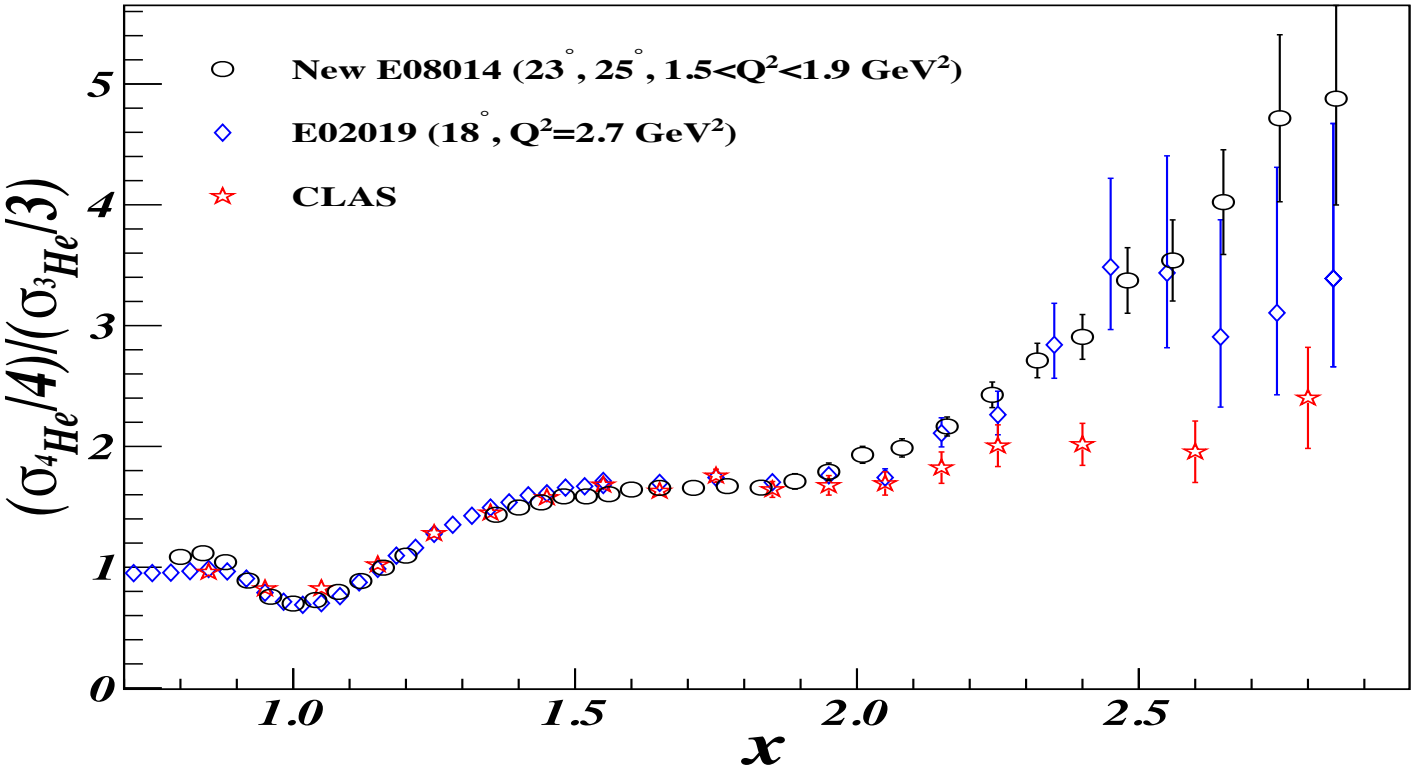
$\alpha_{2N} > 1.3$



Three Nucleon Short Range Correlations

Z. Ye, at al, 2017

Looking for the Plateau in Inclusive Cross Section Ratios x>2



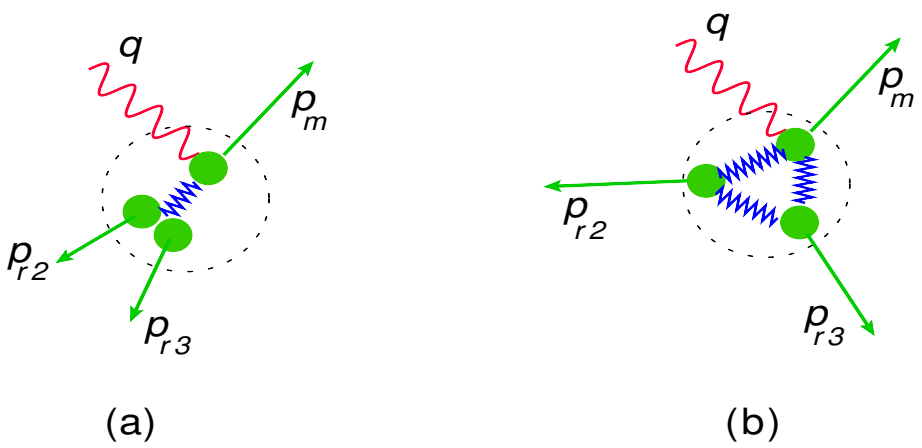
α_{3N} ?

Do we really measure momentum fractions: α relevant for 3N SRCs?

3N SRCs:

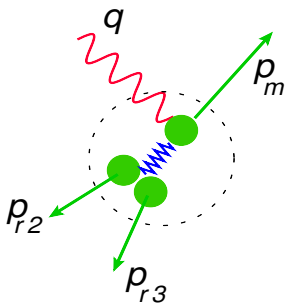
Proper Variables of 3N SRC are

- the Light Front Momentum Fraction: $\alpha = \frac{p_N^+}{p_{3N}^+}$
- transverse momentum: $p_\perp$

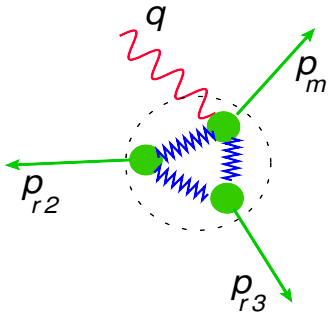


3N SRCs in Inclusive A(e,e′)X Reactions

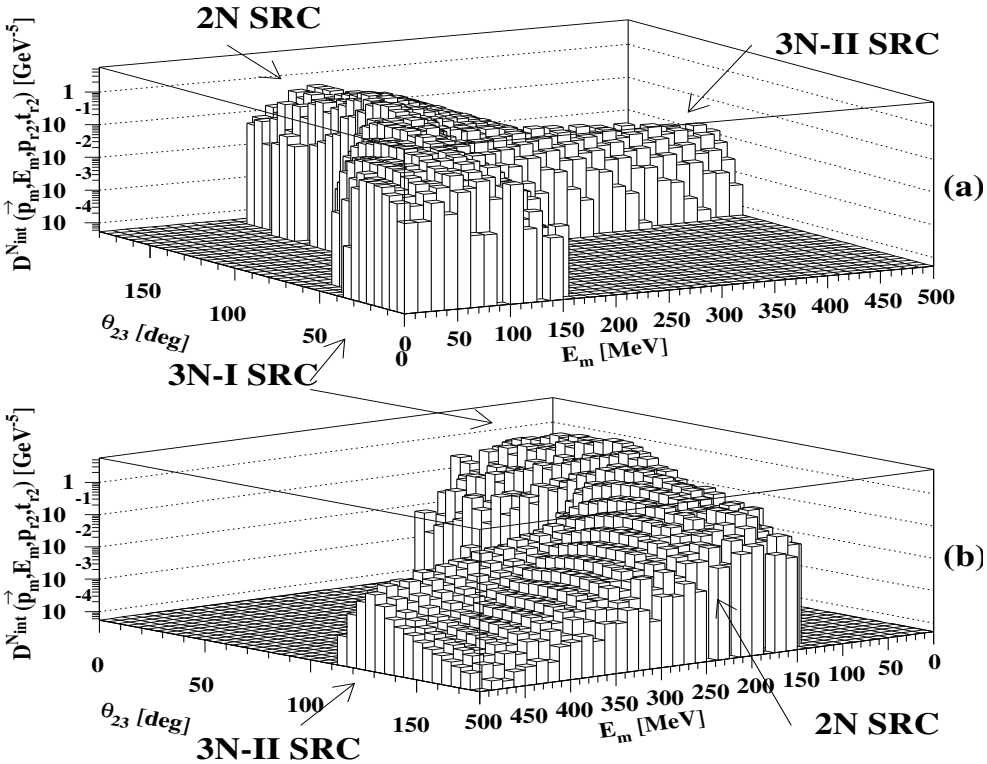
$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$



(a)



(b)



M.S. Abrahamyan, Frankfurt,
Strikman ,Phys. Rev. C 2005

$p_m > 700 MeV/c$

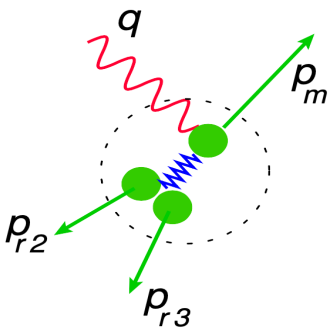
3N SRC model

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

$$1.6 \leq \alpha_{3N} < 3$$

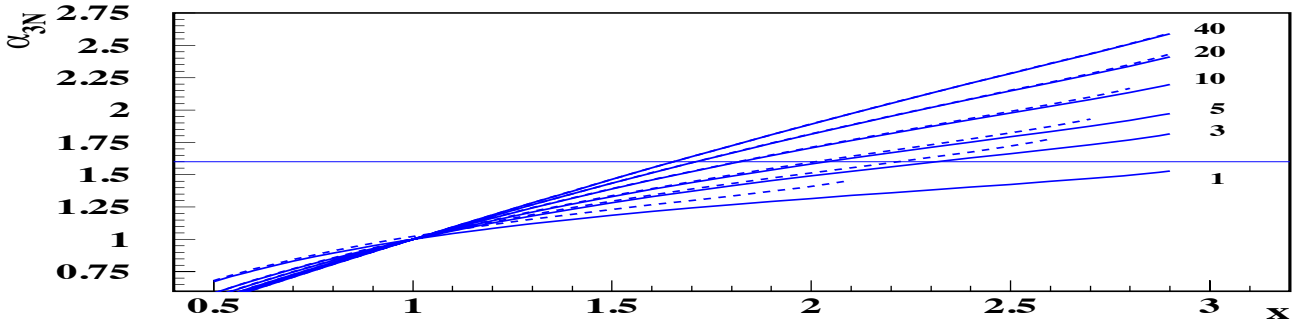
$$q + 3m_N = p_f + p_s$$

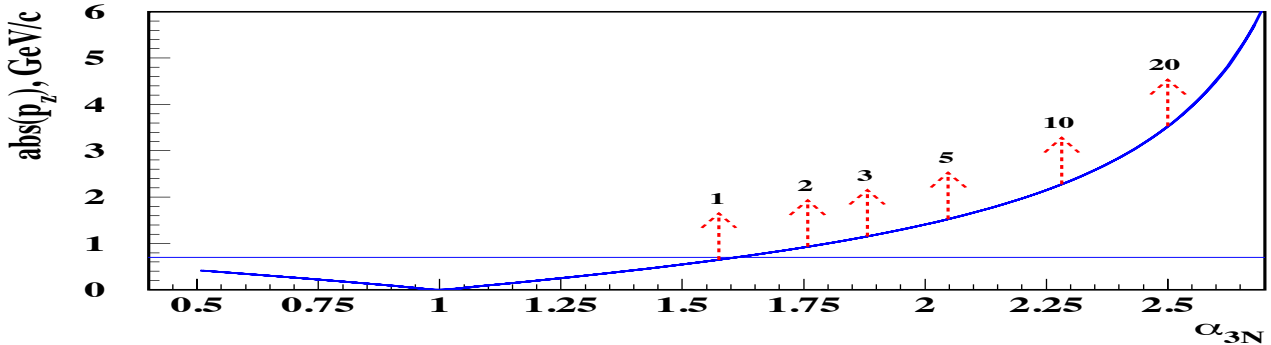
$$\alpha_{3N} = 3 - \frac{q_- + 3m_N}{2m_N}\left[1 + \frac{m_S^2 - m_N^2}{W_{3N}^2} + \sqrt{\left(1 - \frac{(m_S + m_n)^2}{W_{3N}^2}\right)\left(1 - \frac{(m_S - m_n)^2}{W_{3N}^2}\right)}\right]$$



$$\alpha_{3N} > 1.6$$

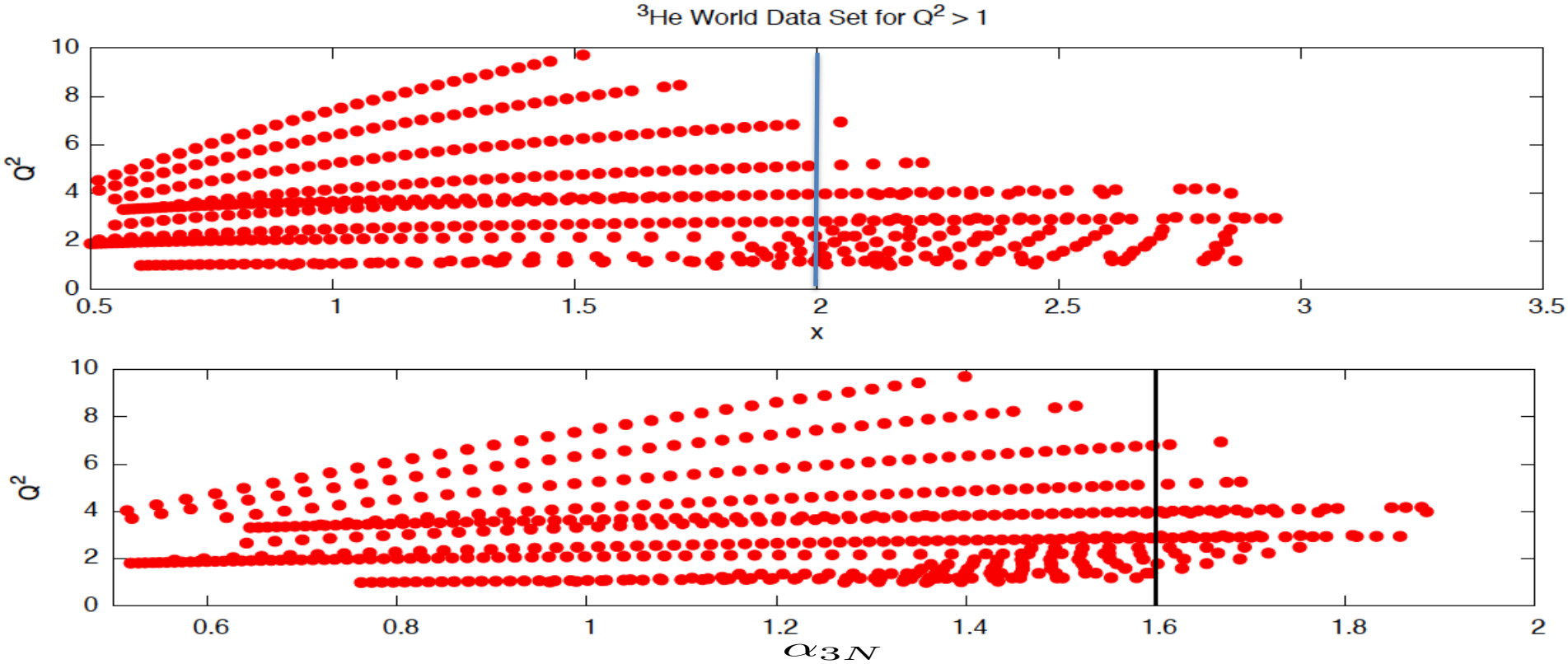
$$p_i^{min} \geq 700 MeV/c$$





3N SRC model $\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha)$ where $\rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$

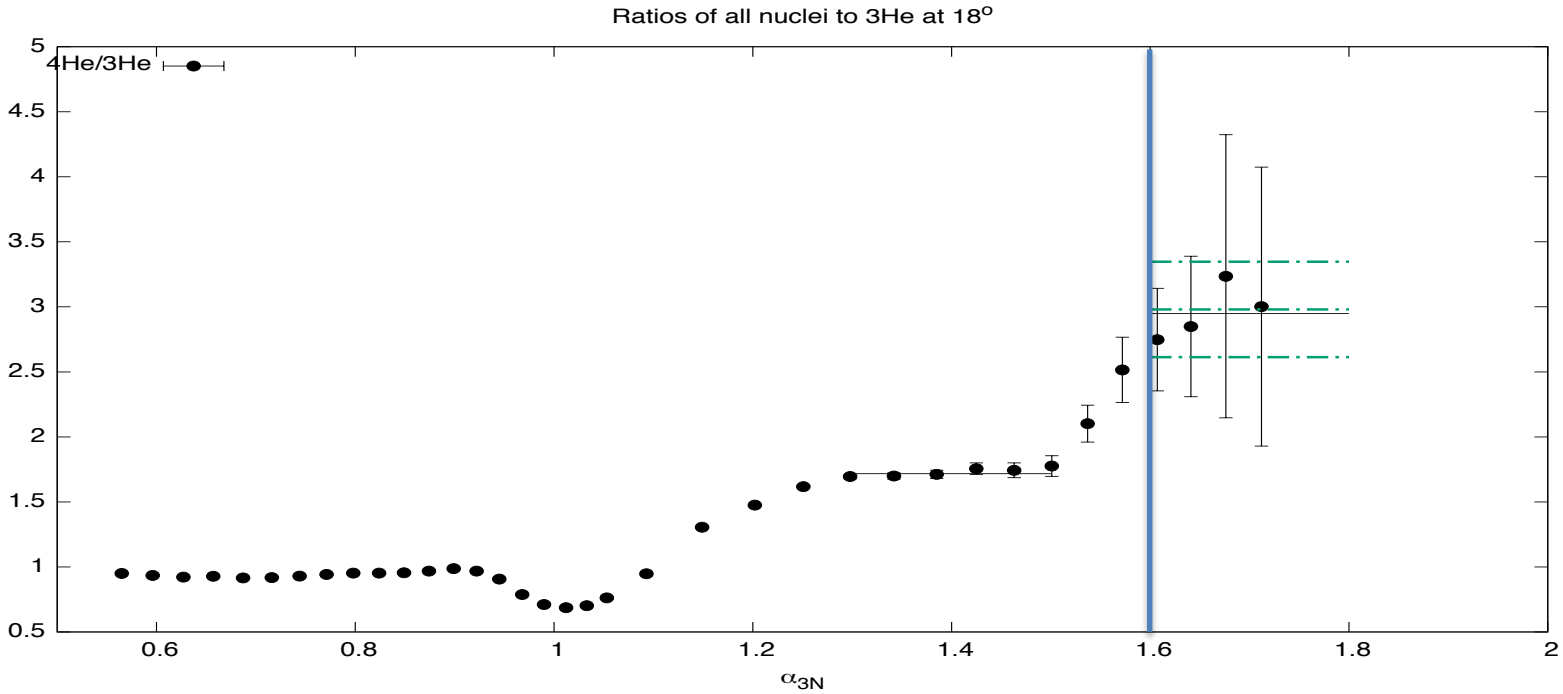
$$1.6 \leq \alpha_{3N} < 3$$



3N SRCs

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

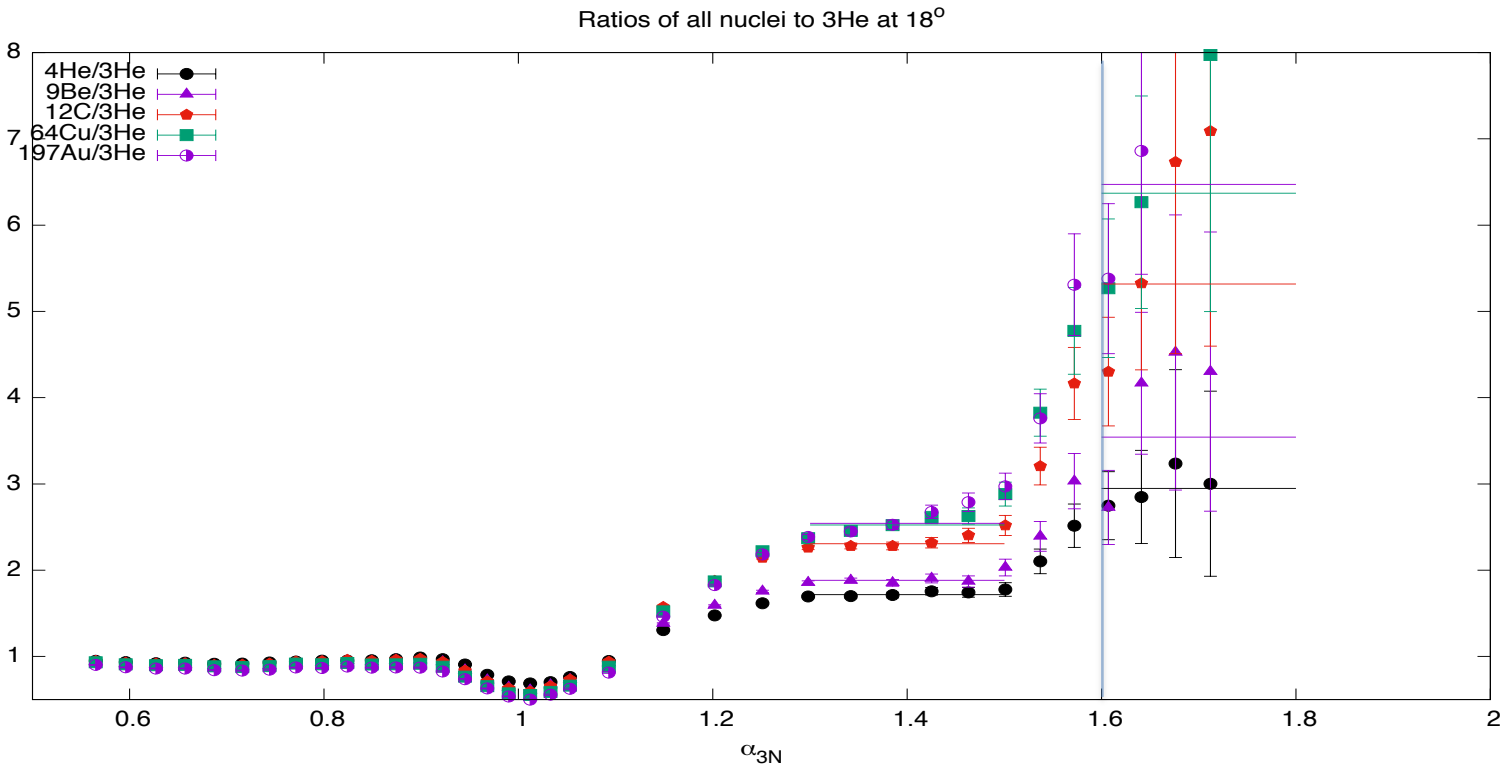
$$1.6 \leq \alpha_{3N} < 3$$



JLab - E02019 - Data

3N SRC model $\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha)$ where $\rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$

$$1.6 \leq \alpha_{3N} < 3$$

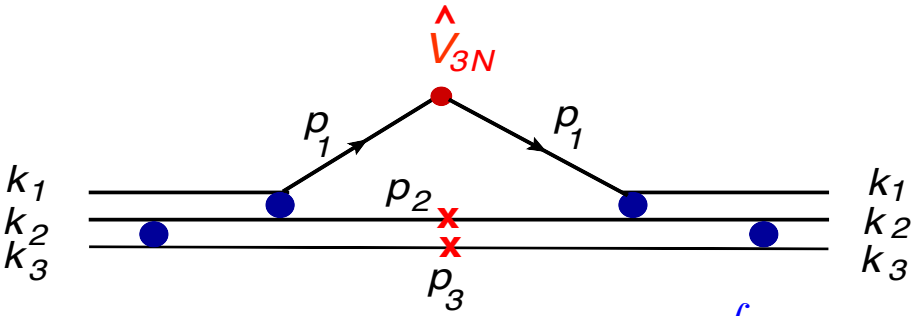


JLab - E02019 - Data

3N SRC: Light-Cone Momentum Fraction Distribution

A.Freese, M.S., M.Strikman, Eur. Phys. J 2015

O. Artiles M.S. Phys. Rev. C 2016



$$P^N_{A,3N}(\alpha_1, p_{1,\perp}, s_1, \tilde{M}_N) = \sum_{s_2,s_3,s_{2'},\tilde{s}_{2'}} \int \bar{u}(k_1)\bar{u}(k_2)\bar{u}(k_3)\Gamma^\dagger_{NN\to NN} \frac{u(p_{2'},\tilde{s}_{2'})\bar{u}(p_{2'},\tilde{s}_{2'})}{p^2_{2'} - M^2_N}$$

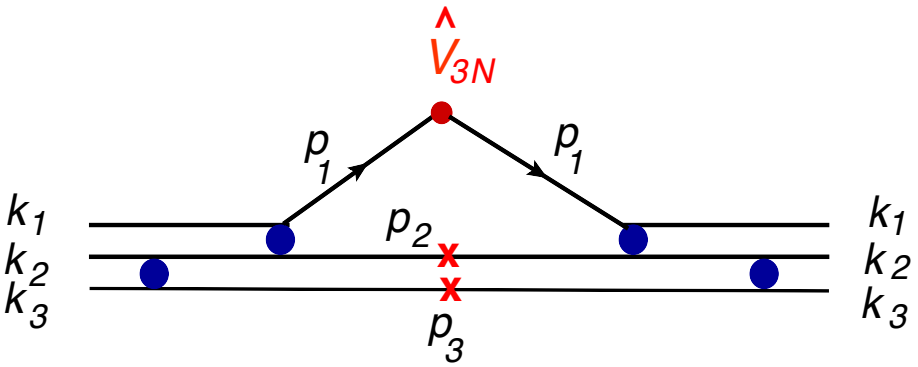
$$\times\ \Gamma^\dagger_{NN\to NN} \frac{u(p_1,s_1)}{p_1^2 - M_N^2} u(p_2,s_2) \left[2\alpha_1^2 \delta(\alpha_1+\alpha_2+\alpha_3-3)\delta^2(p_{1\perp}+p_{2\perp}+p_{3\perp})\delta(\tilde{M}_N^2 - M_N^{3N,2})\right]$$

$$\times\ \bar{u}(p_2,s_2)\frac{\bar{u}(p_1,s_1)}{p_1^2 - M_N^2}\Gamma_{NN\to NN}\frac{u(p_{2'},s_{2'})\bar{u}(p_{2'},s_{2'})}{p^2_{2'} - M^2_N} u(p_3,s_3)\bar{u}(p_3,s_3)\Gamma^\dagger_{NN\to NN} u(k_1)u(k_2)u(k_3)$$

$$\times\ \frac{d\alpha_2}{\alpha_2}\frac{d^2p_{2\perp}}{2(2\pi)^3}\frac{d\alpha_3}{\alpha_3}\frac{d^2p_{3\perp}}{2(2\pi)^3}, \qquad (1)$$

$$P^N_{A,3N}(\alpha_1, p_{1,\perp}, \tilde{M}_N) = \int \frac{3-\alpha_3}{2(2-\alpha_3)^2} \rho_{NN}(\beta_3, p_{3\perp})\rho_{NN}(\beta_1, \tilde{k}_{1\perp}) 2\delta(\alpha_1+\alpha_2+\alpha_3-3)$$
$$\delta^2(p_{1\perp}+p_{2\perp}+p_{3\perp})\delta(\tilde{M}_N^2 - M_N^{3N,2}) d\alpha_2 d^2p_{2\perp} d\alpha_3 d^2p_{3\perp}, \qquad (1)$$

3N SRC: Light-Cone Momentum Fraction Distribution



$$\rho_{3N}(\alpha_1) = \int \frac{1}{4}\left[\frac{3-\alpha_3}{(2-\alpha_3)^3}\boxed{\rho_{pn}\left(\alpha_3, p_{3\perp}\right)\rho_{pn}}\left(\frac{2\alpha_2}{3-\alpha_3}, p_{2\perp} + \frac{\alpha_1}{3-\alpha_3}p_{3\perp}\right) + \frac{3-\alpha_2}{(2-\alpha_2)^3}\boxed{\rho_{pn}\left(\alpha_2, p_{2\perp}\right)\rho_{pn}}\left(\frac{2\alpha_3}{3-\alpha_2}, p_{3\perp} + \frac{\alpha_1}{3-\alpha_2}p_{2\perp}\right)\right]\delta\left(\sum_{i=1}^{3}\alpha_i - 3\right) d\alpha_2 d^2p_{2\perp} d\alpha_3 d^2p_{3\perp}, \quad (1)$$

$$\boxed{\rho_{pn}(\alpha, p_\perp) \approx a_2(A)\rho_d(\alpha, p_\perp)}$$

$$-\rho_{3N} \sim a_2(A, z)^2$$

3N SRC Observables:

- For A(e,e') X reactions: $\sigma_{eA} = \sum_N \sigma_{eN} \rho_{3N}(\alpha_{3N})$

- Observable: $R_3(A,Z) = \frac{3\sigma_{eA}}{A\sigma_{e^3He}} \Big|_{\alpha_{3N} \geq \alpha^0_{3N}}$ = 1.6

- Define: $a_3(A,Z) = \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e^3He} + \sigma_{e^3H})/2}$

- Relate: $R_3(A,Z) = a_3(A,Z) \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3}$

- Using: $\rho^A_{3N}(\alpha) \sim a_2(A)^2$ and charge symmetry

$$a_3(A,Z) = \frac{a_2(A)^2}{a_2^p(^3He)a_2^n(^3He)},$$

3N SRC Observables:

- using $$a_3(A,Z) = \frac{a_2(A)^2}{a_2^p(^3He)a_2^n(^3He)},$$
- and $$n^A_{p/n}(p) \approx \frac{1}{2(x_{p/n})^\gamma} a_2(A,y)\cdot n_d,\ \gamma \le 1$$ form MS: PRC2014
- obtains: $$a_2^n(^3He) = \frac{a_2(^3He)}{2(1/3)^\gamma} \quad \text{and} \quad a_2^p(^3He) = \frac{a_2(^3He)}{2(2/3)^\gamma},$$
- Relate: $$R_3(A,Z) = a_3(A,Z)\frac{(\sigma_{ep}+\sigma_{en})/2}{(2\sigma_{ep}+\sigma_{en})/3} = 4\left(\frac{2}{9}\right)^\gamma \frac{(\sigma_{ep}+\sigma_{en})/2}{(2\sigma_{ep}+\sigma_{en})/3}\left(\frac{a_2(A,Z)}{a_2(^3He)}\right)^2 = 4\left(\frac{2}{9}\right)^\gamma \frac{(\sigma_{ep}+\sigma_{en})/2}{(2\sigma_{ep}+\sigma_{en})/3} R_2^2(A,Z)$$
- Where: $$R_2(A,Z) = \frac{3}{A}\frac{\sigma_{eA}}{\sigma_{e^3H}}\Big|_{1.3<\alpha_{3N}<1.5} = \frac{a_2(A)}{a_2(^3He)}$$

for $Q^2 \sim 3\ \mathrm{GeV}^2$, $\sigma_{ep} \approx 3\sigma_{en}$ and for $3He$ $\gamma \approx 0.85$.

$$\boxed{R_3(A,Z)|_{\alpha_{3N}>1.6} \approx 0.96 R_2(A,Z)^2 \approx R_2(A,Z)^2|_{1.3\le\alpha_{3N}\le1.5}.}$$

$$\alpha_{3N} \approx \alpha_{2N}$$

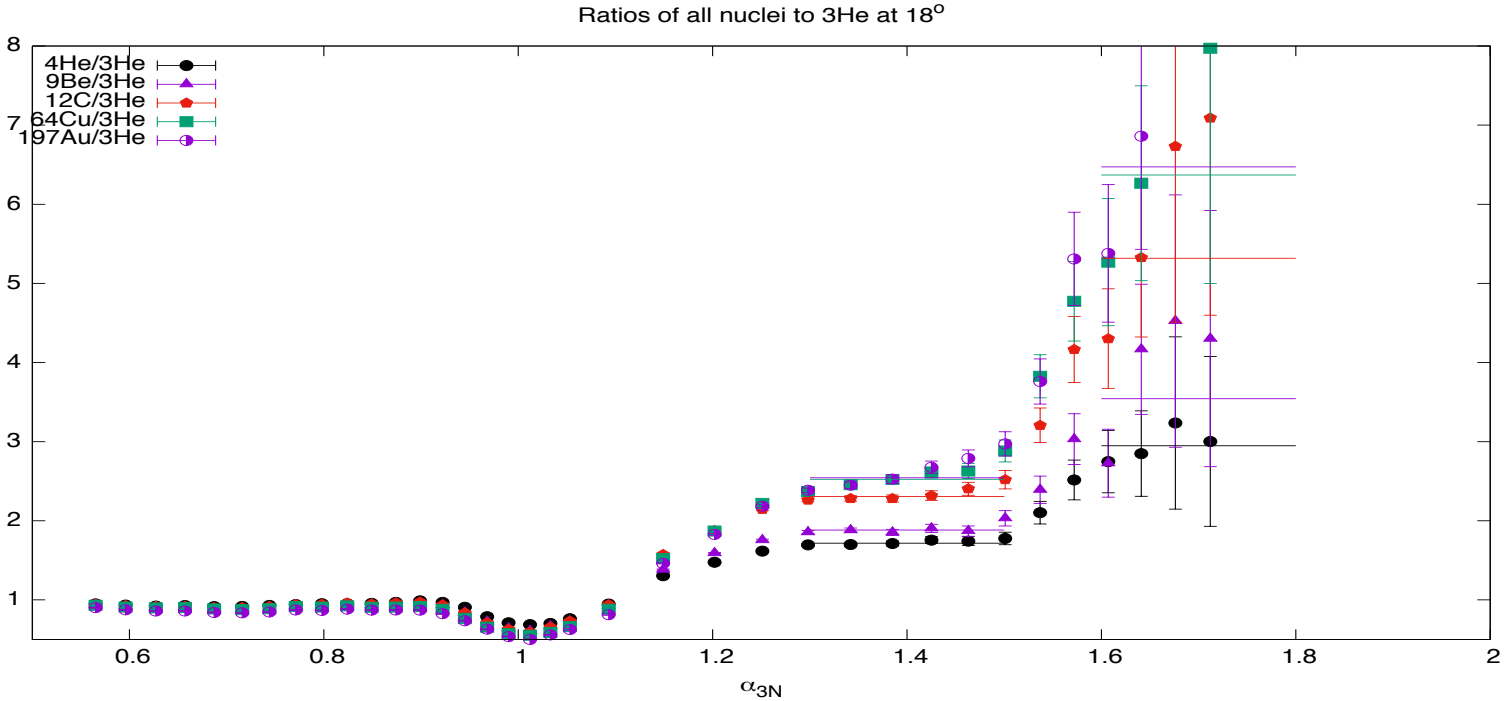
3N SRC model

$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5$$

$$1.6 \leq \alpha_{3N} < 3$$

$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8$$

$$R_3(A,Z) \approx R_2(A,Z)^2$$



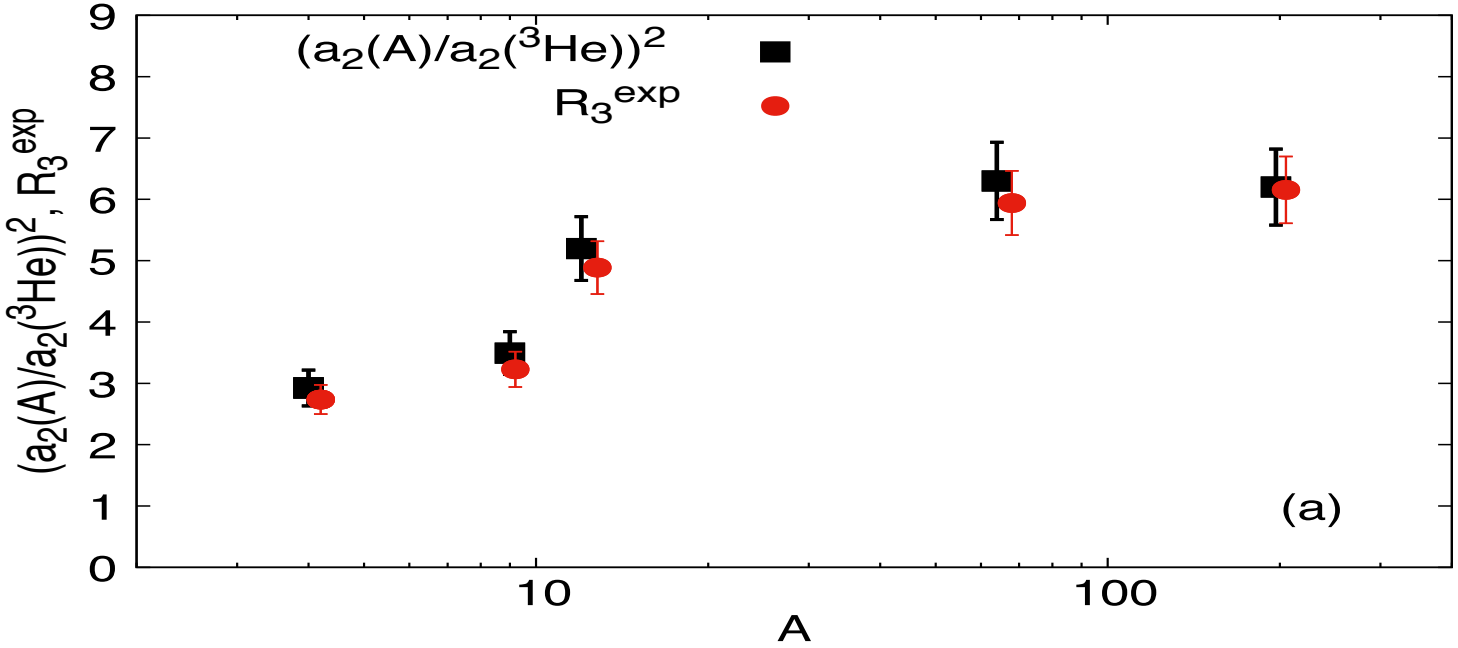
3N SRC model

$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5$$

$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8$$

$$1.6 \leq \alpha_{3N} < 3$$

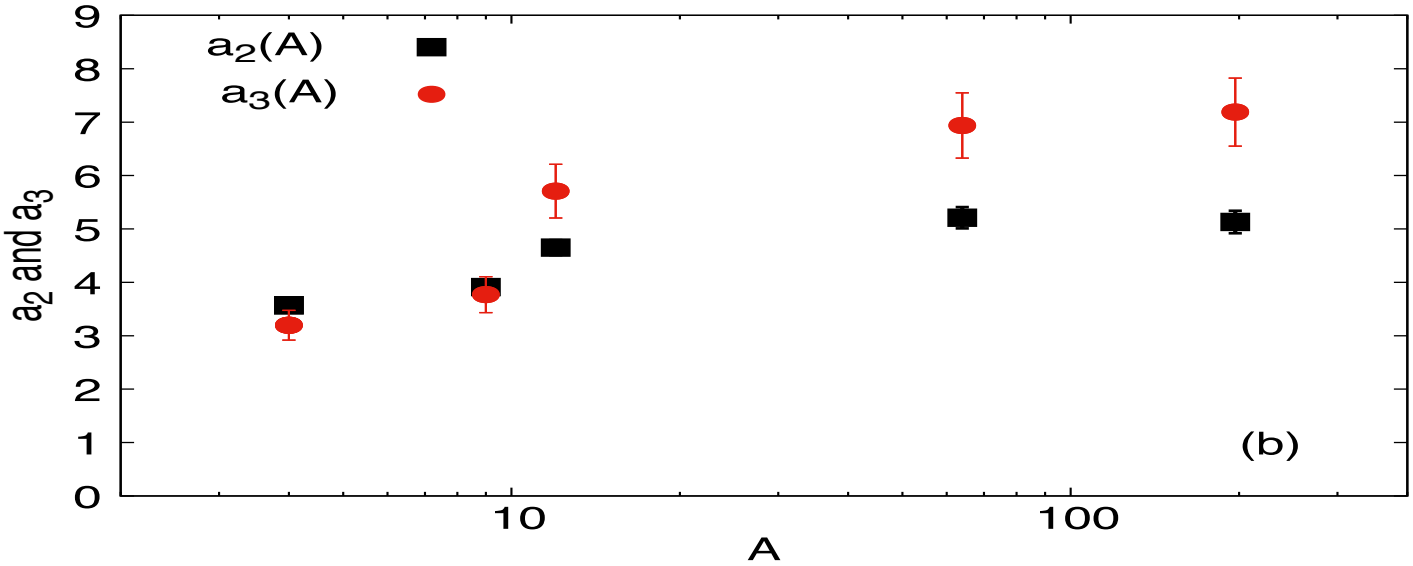
$$R_3(A) = R_2(A)^2$$



3N SRC model

Defining: $a_3(A,Z) = \frac{3}{A}\frac{\sigma_{eA}}{(\sigma_{e^3He}+\sigma_{e^3H})/2}$

One relates: $a_3(A,Z) = \frac{(2\sigma_{ep}+\sigma_{en})/3}{(\sigma_{ep}+\sigma_{en})/2} R_3(A,Z)$



3N SRC model

Defining: $a_3(A,Z) = \frac{3}{A}\frac{\sigma_{eA}}{(\sigma_{e^3He}+\sigma_{e^3H})/2}$

One relates: $a_3(A,Z) = \frac{(2\sigma_{ep}+\sigma_{en})/3}{(\sigma_{ep}+\sigma_{en})/2} R_3(A,Z)$

A	a_2	R_2	R_3^{exp}	a_3
3	2.13 $\pm$0.04	1	NA	NA
4	3.57 $\pm$0.09	1.68 $\pm$ 0.03	2.74 $\pm$0.24	3.20 $\pm$ 0.28
9	3.91 $\pm$0.12	1.84 $\pm$ 0.04	3.23 $\pm$0.29	3.77 $\pm$ 0.34
12	4.65 $\pm$0.14	2.18 $\pm$ 0.04	4.89 $\pm$0.43	5.71 $\pm$ 0, 50
64	5.21 $\pm$0.20	2.45 $\pm$ 0.04	5.94 $\pm$0.52	6.94 $\pm$ 0.77
197	5.13 $\pm$0.21	2.41 $\pm$ 0.05	6.15 $\pm$0.55	7.18 $\pm$ 0.64

3N SRC Summary & Outlook

- Proper variable for studies of 2N and 3N SRS are Light-Cone momentum fractions: α_{2N}, α_{3N}

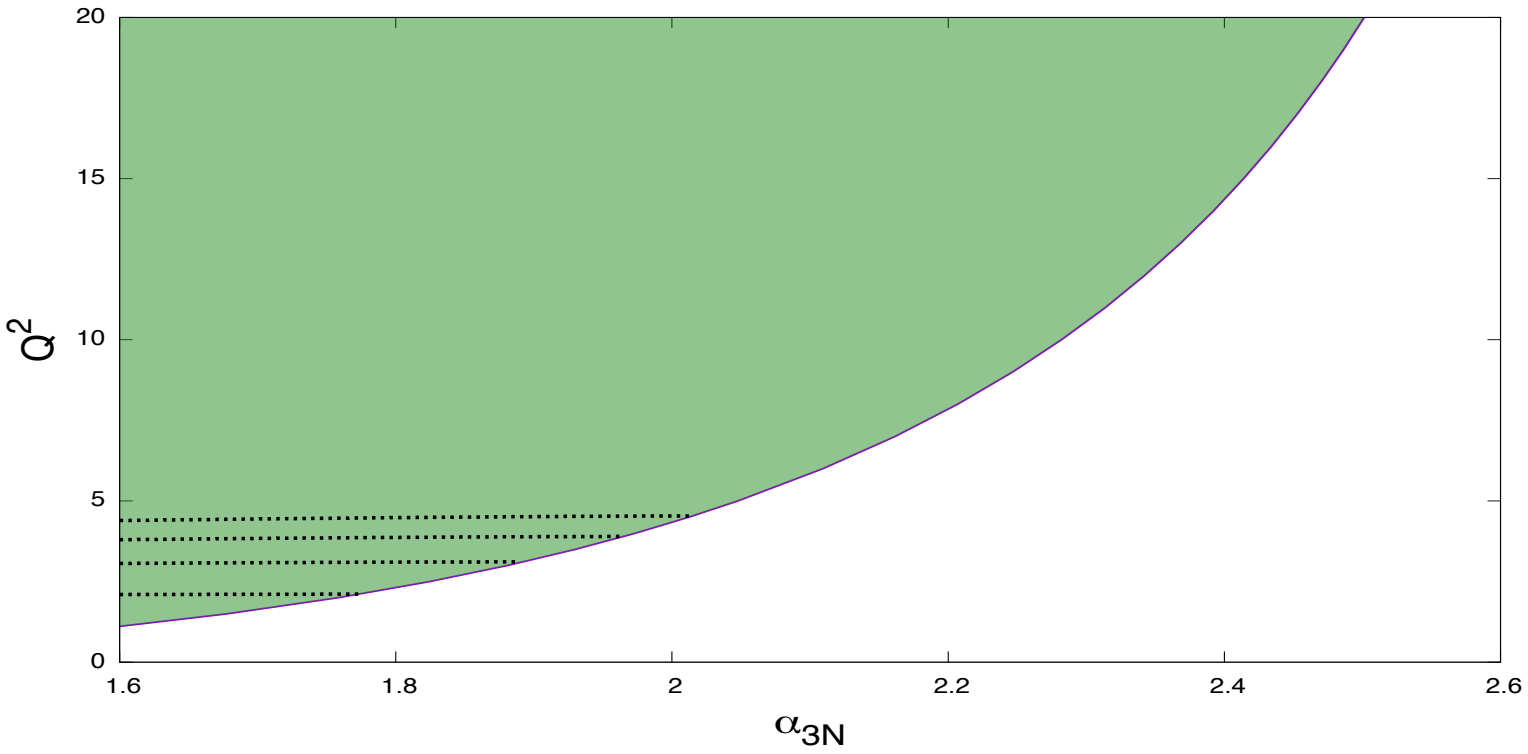
- It seems we observed first signatures of 3N SRCs in the form of the "scaling"

- Existing data in agreement with the prediction of: $R_3(A,Z) \approx R_2(A,Z)^2$

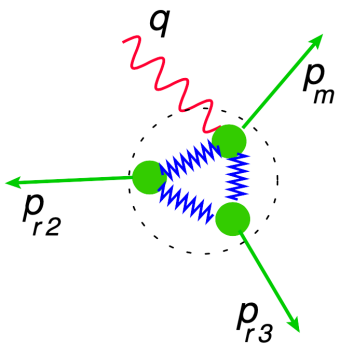
- Unambiguous verification will require larger Q2 data to cover larger α_{3N} region

- Reaching Q2 > 5 GeV2 will allow to reach: $\alpha_{3N} > 2$

3N SRC Outlook



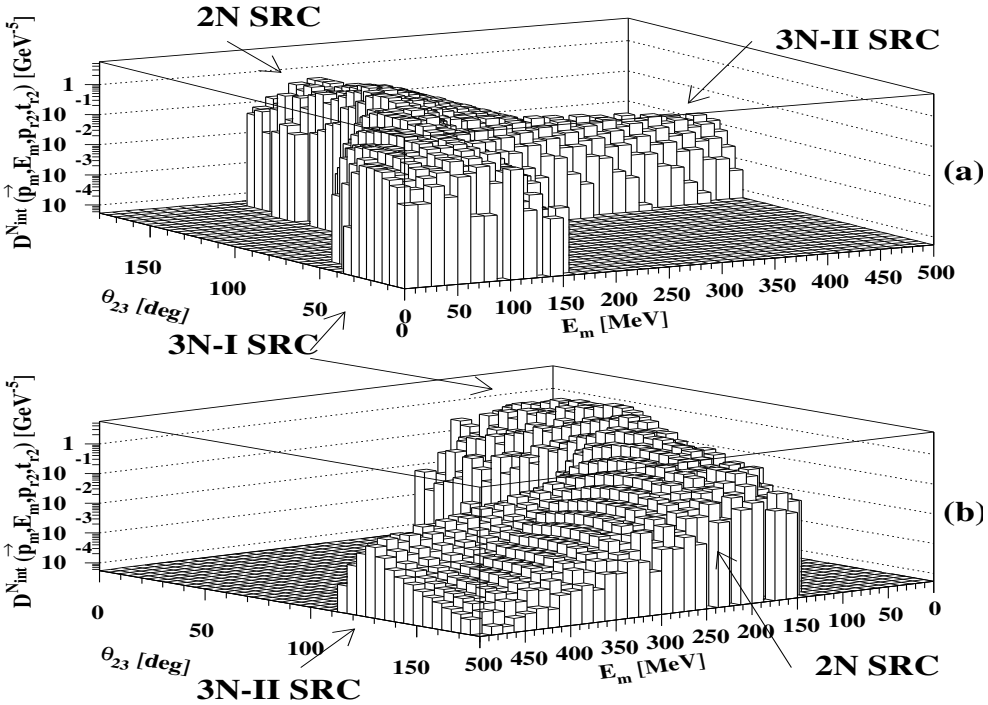
What about type II 2N SRCs



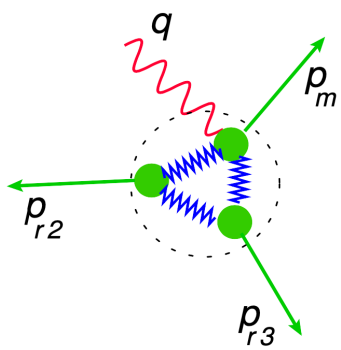
e+ ^{3}He → e' + p_f + p_{r1} + p_{r2}

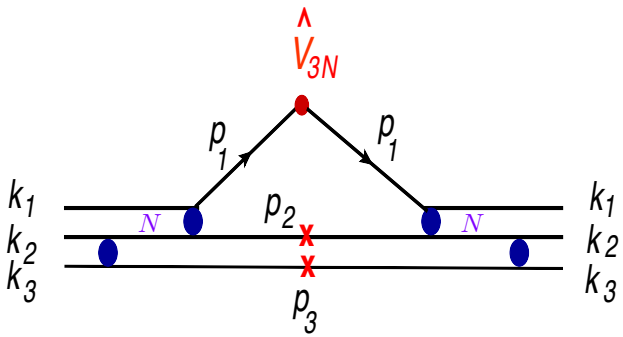
$p_m > 700 MeV/c$

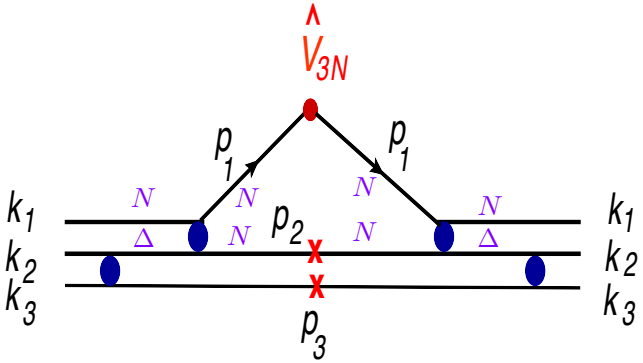
$E_m > 300 MeV$



What about type II 3N SRCs



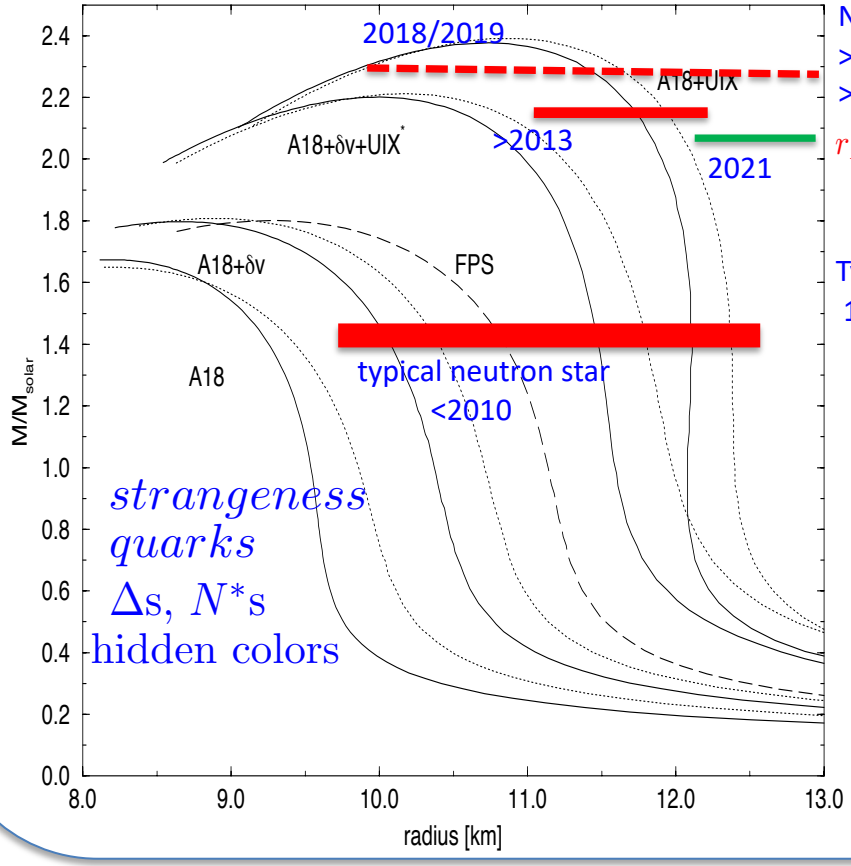




Irreducible 3N-Force

- Introduced first to describe binding energy of ^{3}H Wigner – Phys. Rev. 1933
- In ordinary cases accuracy of 1% needed to disentangle the 3N-Force effects in low energy phenomena Friar Nucl.Phys.A - 2001
- Significant contribute to the equation of state of dense nuclear matter. Heiselberg & Pandharipande ARNPS 2000

"Unreasonable" Persistence of Nucleons



Neutron star masses
>2 solar masses
>2010

$r_{NN} \sim 0.6 - 0.8 fm$

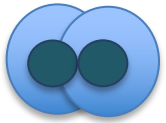
Typical neutron star
1.4 Solar mass

$r_{NN} \sim 0.7 - 1.0 fm$

H. Heiselberg,
V.Pandharipande
ARNPS 2000

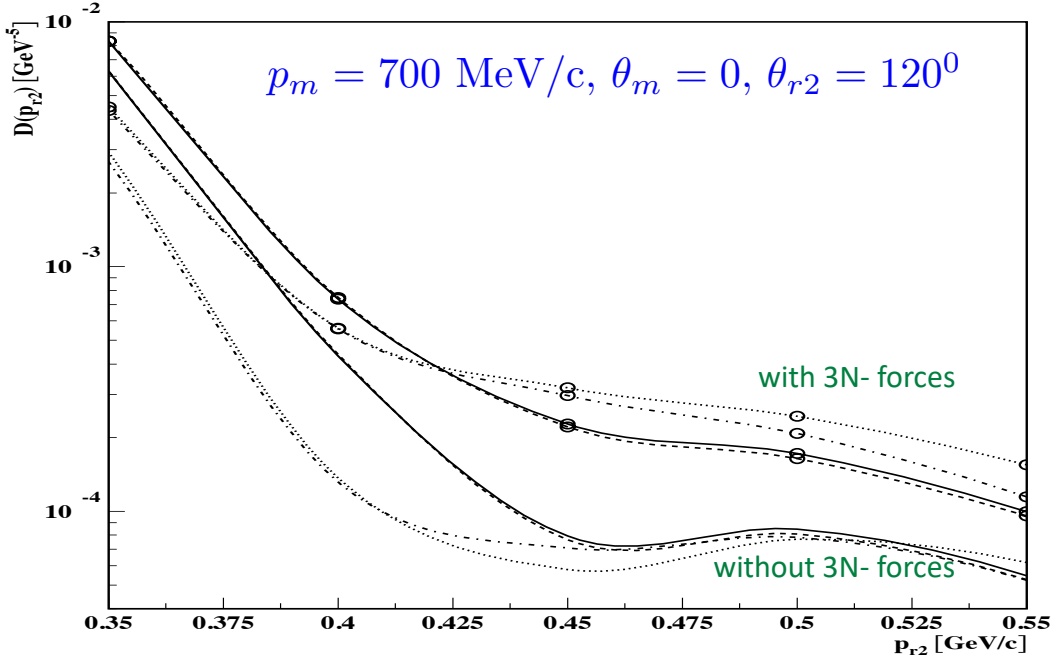
$r_c \sim 0.3 fm$

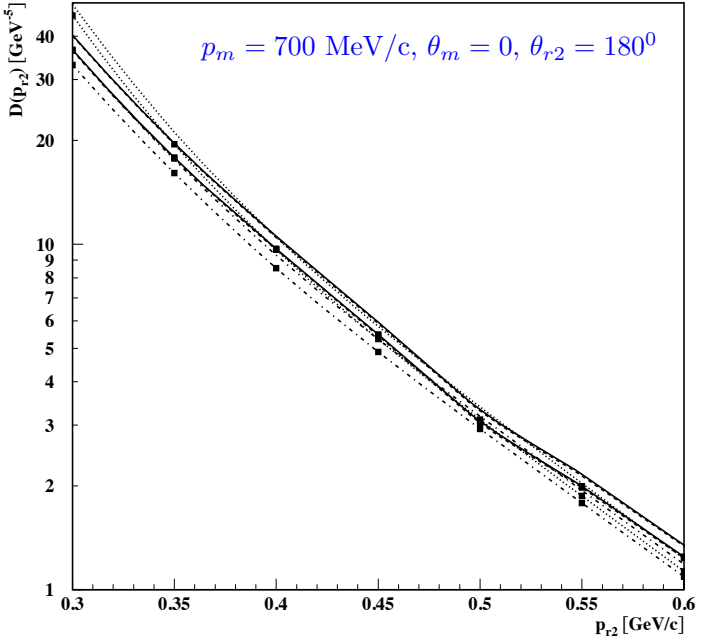
color singlet core



two color singlet
"nucleons"

Calculation of $e + 3He \rightarrow e' p_f + p_{r1} + p_{r2}$ In high Q2 kinematics





Wave functions

MS, Abrahamyan, Frankfurt Strikman, PRC 2005

Nogga, Kievsky, Kamada,Gloeckle et al, PRC2003

Outlook – on 3N-Forces

- Detailed exploration of triple-coincidence scattering on ^{3}He may allow to isolate and measure the strength of the 3N-Forces

3N SRC: Light-Cone Momentum Fraction Distribution

O. Artiles M.S. Phys. Rev. C 2016