SPC Report: Partonic Structure

Martha Constantinou

for the USQCD Scientific Program Committee

USQCD All-Hands Meeting 2023

April 20 - 21, 2023

Physics Investigations

★ Form Factors and Generalized Form Factors

★ Parton Distribution Functions (PDFs)

★ Parton Distribution Amplitudes (DA)

★ Generalized Parton Distributions (GPDs)

★ Transverse-Momentum Dependent PDFs (TMD PDFs)

★ Hadronic Tensor

Physics Investigations

★ Form Factors and Generalized Form Factors

★ Parton Distribution Functions (PDFs)

★ Parton Distribution Amplitudes (DA)

★ Generalized Parton Distributions (GPDs)

★ Transverse-Momentum Dependent PDFs (TMD PDFs)

★ Hadronic Tensor

★ Mellin moments (local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

local operators

Physics Investigations

★ Form Factors and Generalized Form Factors

★ Parton Distribution Functions (PDFs)

★ Parton Distribution Amplitudes (DA)

★ Generalized Parton Distributions (GPDs)

★ Transverse-Momentum Dependent PDFs (TMD PDFs)

★ Hadronic Tensor

★ Mellin moments (local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]}_{\text{local operators}}$$

local operators

★ Non-local operators, boosted states

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\underbrace{\mathscr{W}(z,0)}_{\text{Wilson line}}\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

Physics Investigations

★ Form Factors and Generalized Form Factors

★ Parton Distribution Functions (PDFs)

★ Parton Distribution Amplitudes (DA)

★ Generalized Parton Distributions (GPDs)

★ Transverse-Momentum Dependent PDFs (TMD PDFs)

★ Hadronic Tensor

★ Mellin moments (local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n} q\right]$$

local operators

★ Non-local operators, boosted states

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

★ 4pt-functions, auxiliary fields, …

$$W_{\mu\nu}(p,q) \propto \int d^4x e^{iqx}\,\langle\pi,\boldsymbol{p}|\,J_\mu^{\rm EM}(x)J_\nu^{\rm EM}(0)\,|\pi,\boldsymbol{p}\rangle$$

Motivation - Impact

★ **Complement the JLab 12 GeV, EIC, and LHC and program**
Physics investigations align with the experimental scientific program

★ **Provide constraints in global analysis**
PDFs: input in kinematic regions that lack data
GPDS: guide Q^2 parameterizations
TMD PDFs: provide Collins-Sopper Kernel to disentangle from TMDs

★ **Input on mesonic structure**
t-dependence of pion and kaon structure

★ **Gluonic structure from first principles**
complements picture for hadrons
Understand the mass decomposition with various sum rules

2022-2023 Type A Proposals

All continuation proposals

PI	Institution	Project Title
Constantinou	Temple	Twist-3 GPDs from Lattice QCD
Engelhardt	NMSU	Nucleon Quark-Gluon Structure with Clover-Wilson Fermions
Gao	ANL	Computing the large-Q^2 Kaon Form Factors with Physical Quark Masses
Jay	MIT	A Lattice Calculation of the Hadron Tensor of the Pion
Lin	MSU	Constraining the Bjorken-x Dependence of the Strange Distribution of the Proton Using Lattice Inputs
Richards	JLab	Parton Distribution Functions and Amplitudes of the PseudoscalarMesons and the Nucleon from Lattice QCD
Wagman	FNAL	Computing the Collins-Soper Kernel for TMD Evolution at the Physical Quark Masses

2022-2023 Type A Proposals

All continuation proposals

PI	Institution	Project Title
Constantinou	Temple	Twist-3 GPDs from Lattice QCD
Engelhardt	NMSU	Nucleon Quark-Gluon Structure with Clover-Wilson Fermions
Gao	ANL	Computing the large-Q^2 Kaon Form Factors with Physical Quark Masses
Jay	MIT	A Lattice Calculation of the Hadron Tensor of the Pion
Lin	MSU	Constraining the Bjorken-x Dependence of the Strange Distribution of the Proton Using Lattice Inputs
Richards	JLab	Parton Distribution Functions and Amplitudes of the PseudoscalarMesons and the Nucleon from Lattice QCD
Wagman	FNAL	Computing the Collins-Soper Kernel for TMD Evolution at the Physical Quark Masses

Dedicated talks in this session

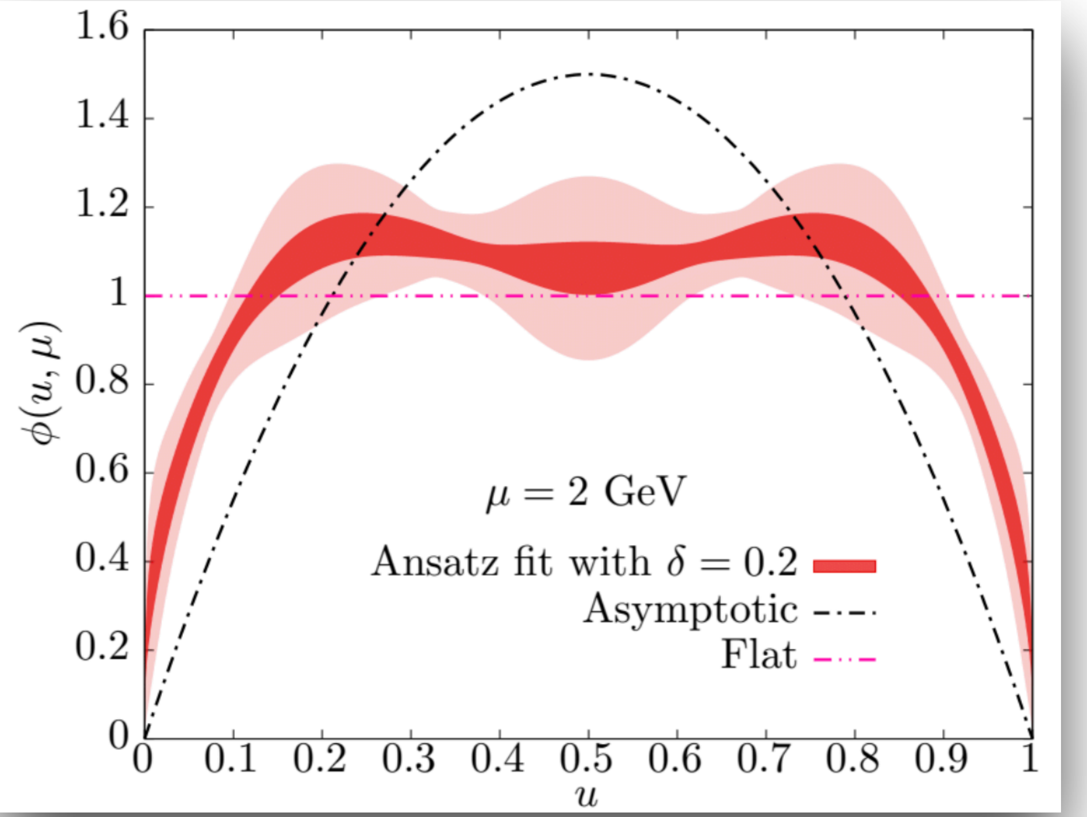
Pion and kaon structure

PI: X. Gao
Junior Investigator

Talk @ 3:25 pm

Progress with current allocation

★ DA of the pion and its Mellin moments within leading-twist framework (HISQ, $m_\pi = 140\,\mathrm{MeV},\ a = 0.076\,\mathrm{fm}$)



$\langle x^2 \rangle$= 0.287(6)(6)

[X. Gao et al., PRD 106 (2022) 7, 074505, arXiv:2206.04084]

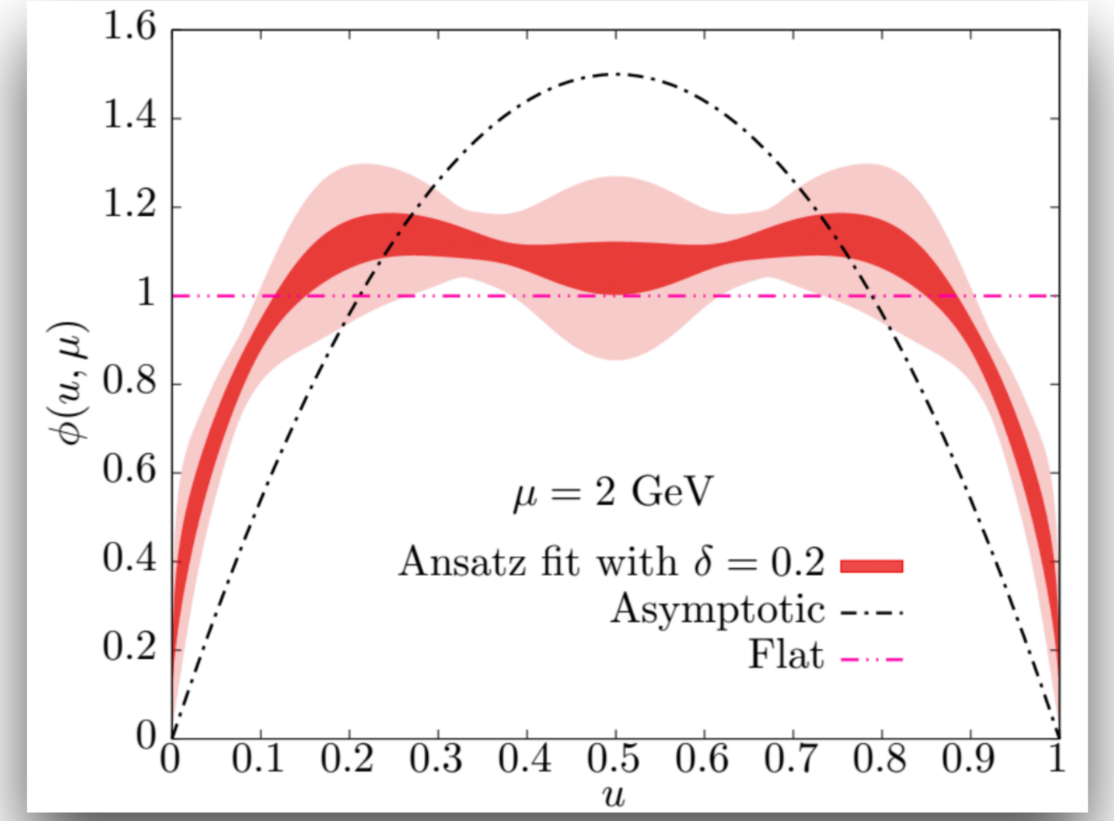
Pion and kaon structure

PI: X. Gao
Junior Investigator

Talk @ 3:25 pm

Progress with current allocation

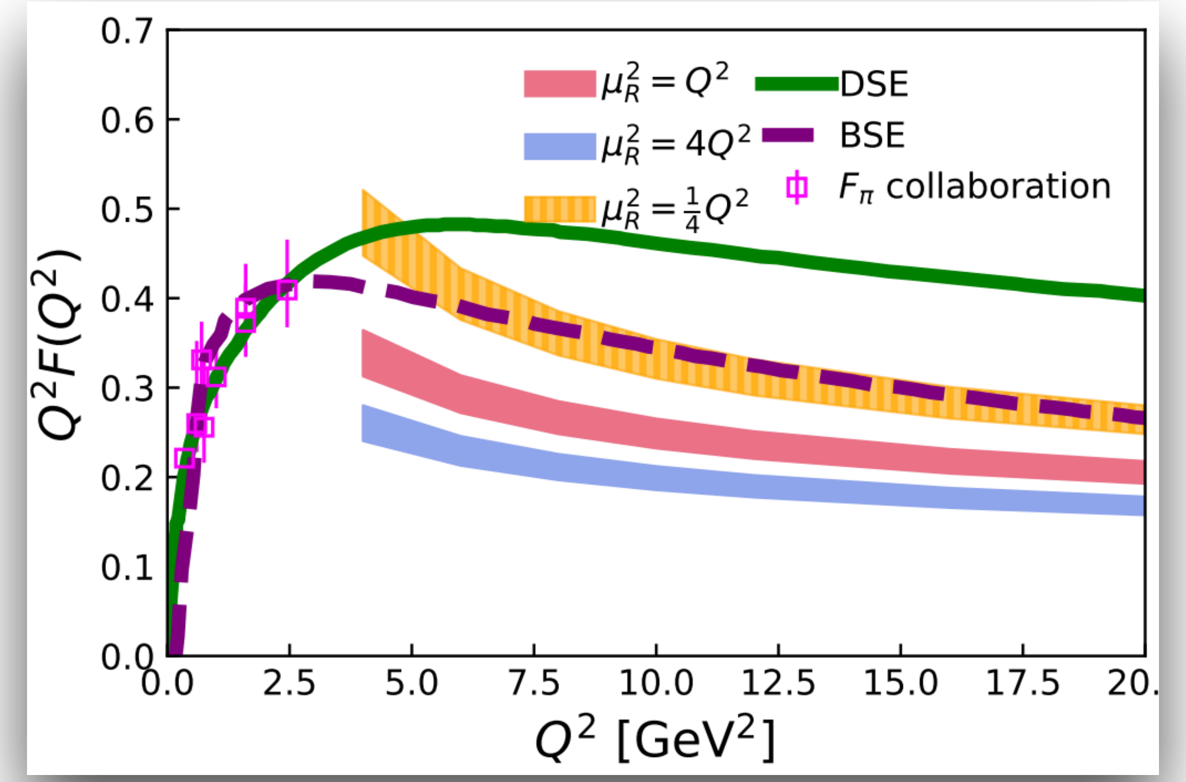
★ DA of the pion and its Mellin moments within leading-twist framework (HISQ, $m_\pi = 140\,\mathrm{MeV},\ a = 0.076\,\mathrm{fm}$)



$\langle x^2 \rangle$= 0.287(6)(6)

[X. Gao et al., PRD 106 (2022) 7, 074505, arXiv:2206.04084]

★ Extract the perturbative leading-twist for pion electric and gravitational FFs at large Q^2



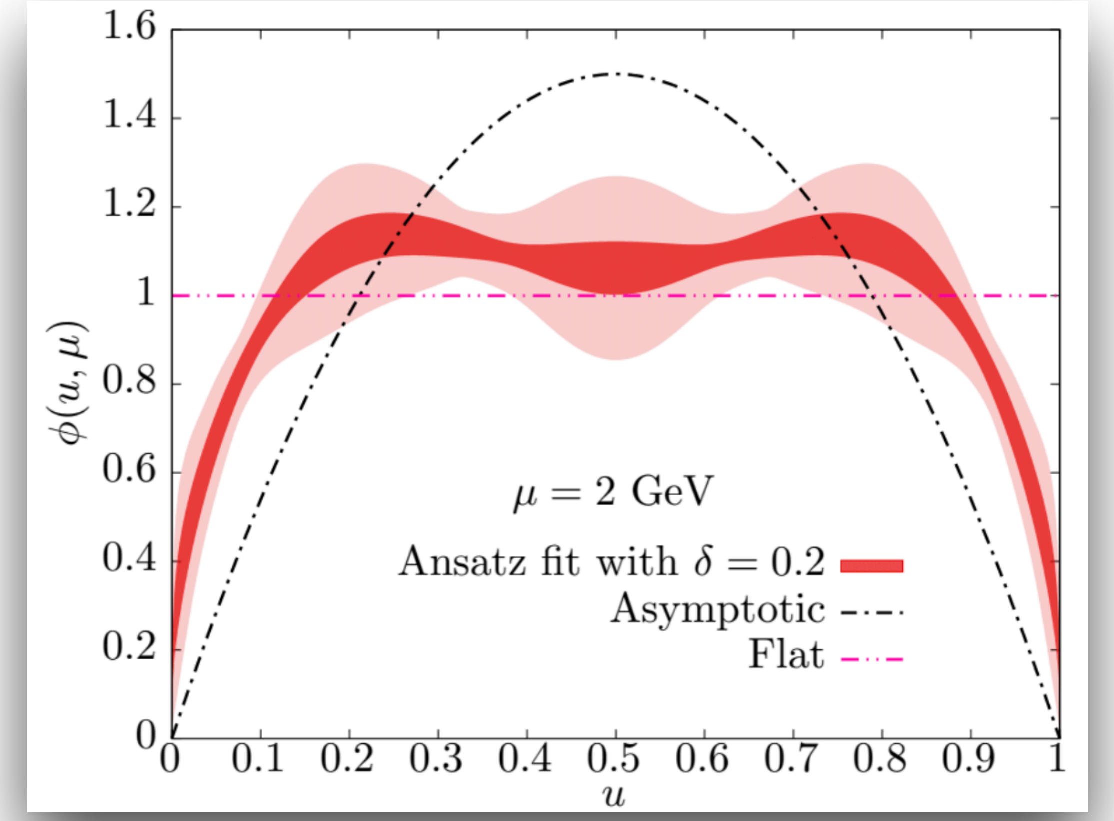
Pion and kaon structure

PI: X. Gao
Junior Investigator

Talk @ 3:25 pm

Progress with current allocation

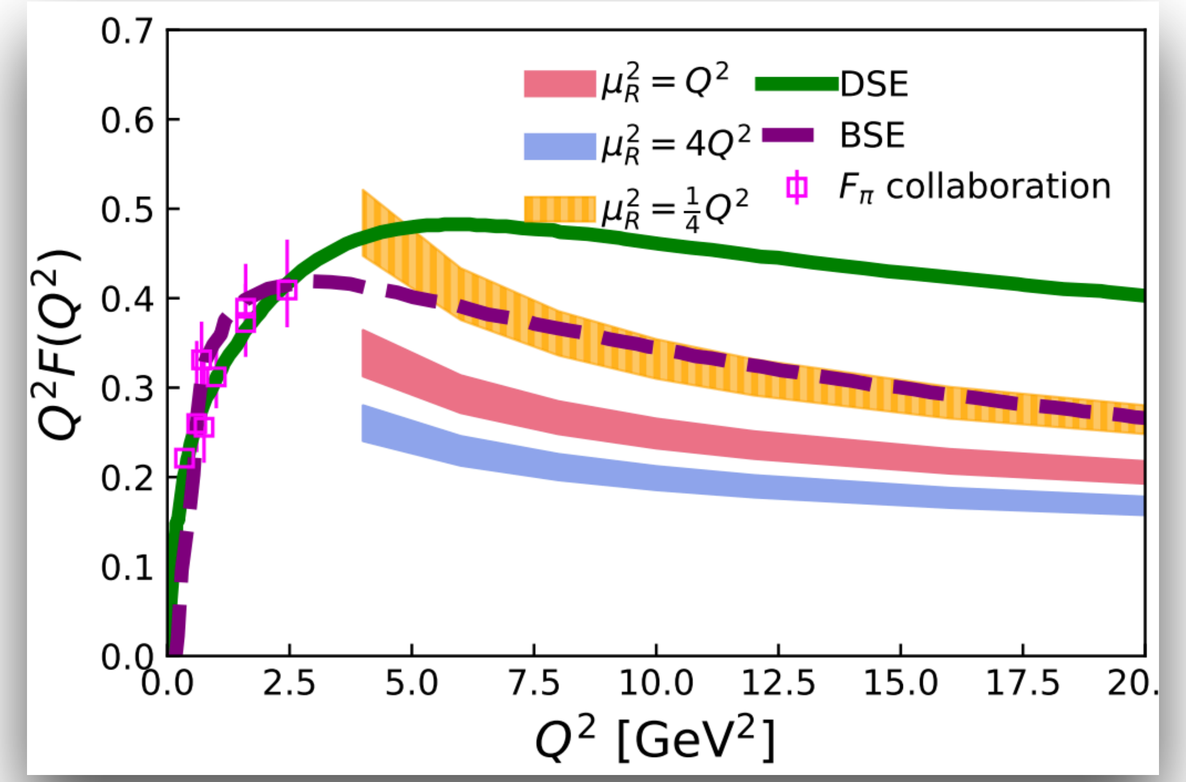
★ DA of the pion and its Mellin moments within leading-twist framework (HISQ, $m_\pi = 140\,\mathrm{MeV},\ a = 0.076\,\mathrm{fm}$)



$\langle x^2 \rangle$= 0.287(6)(6)

[X. Gao et al., PRD 106 (2022) 7, 074505, arXiv:2206.04084]

★ Extract the perturbative leading-twist for pion electric and gravitational FFs at large Q^2



★ 2023-2024: kaon form factors (HISQ confs with HYP-smeared clover fermions, $m_\pi = 140\,\mathrm{MeV},\ a = 0.076\,\mathrm{fm}$)

★ Breit frame with boost up to ~1.5 GeV

★ multiple Tsink up to ~1fm

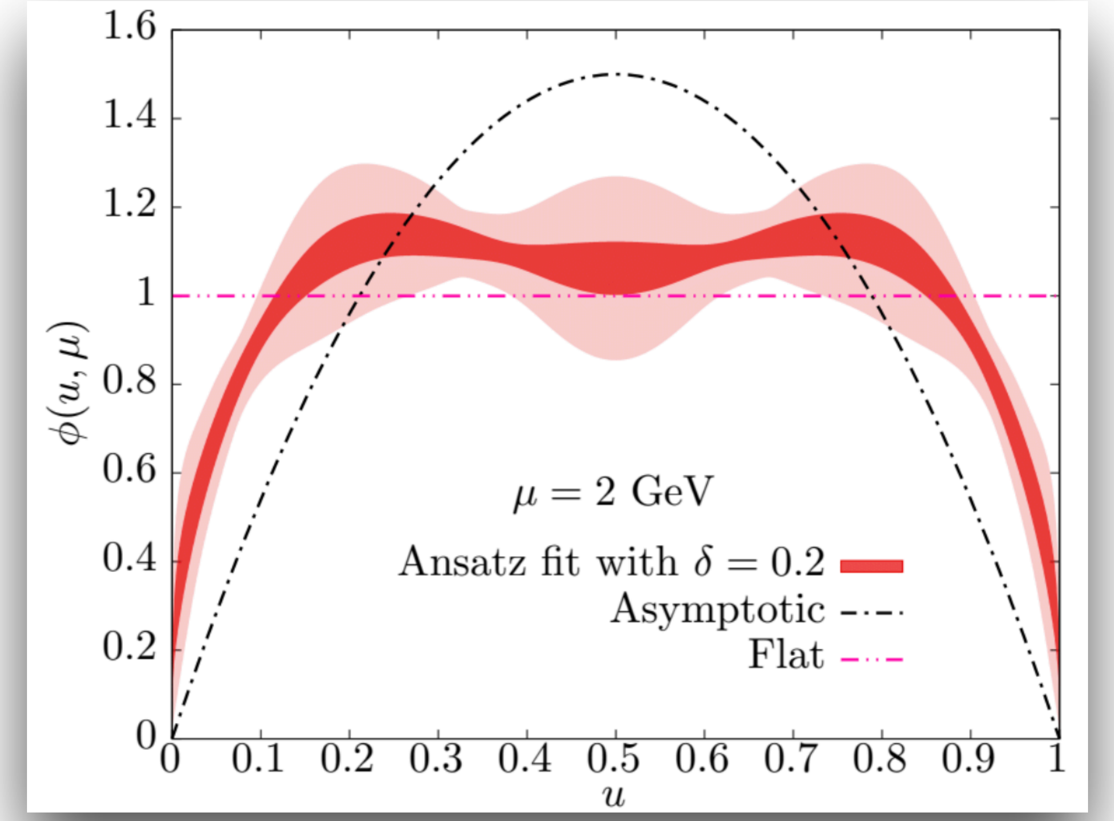
Pion and kaon structure

PI: X. Gao
Junior Investigator

Talk @ 3:25 pm

Progress with current allocation

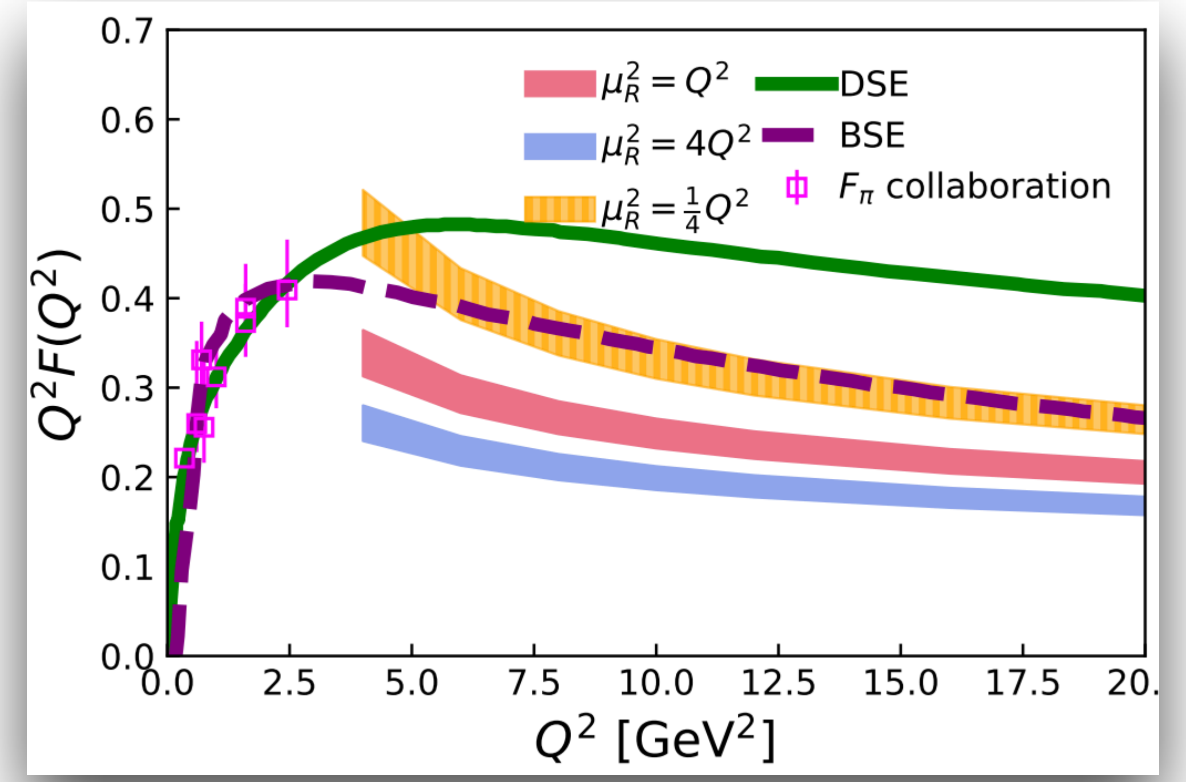
★ DA of the pion and its Mellin moments within leading-twist framework (HISQ, $m_\pi = 140\,\mathrm{MeV},\ a = 0.076\,\mathrm{fm}$)



$\langle x^2 \rangle$= 0.287(6)(6)

[X. Gao et al., PRD 106 (2022) 7, 074505, arXiv:2206.04084]

★ Extract the perturbative leading-twist for pion electric and gravitational FFs at large Q^2

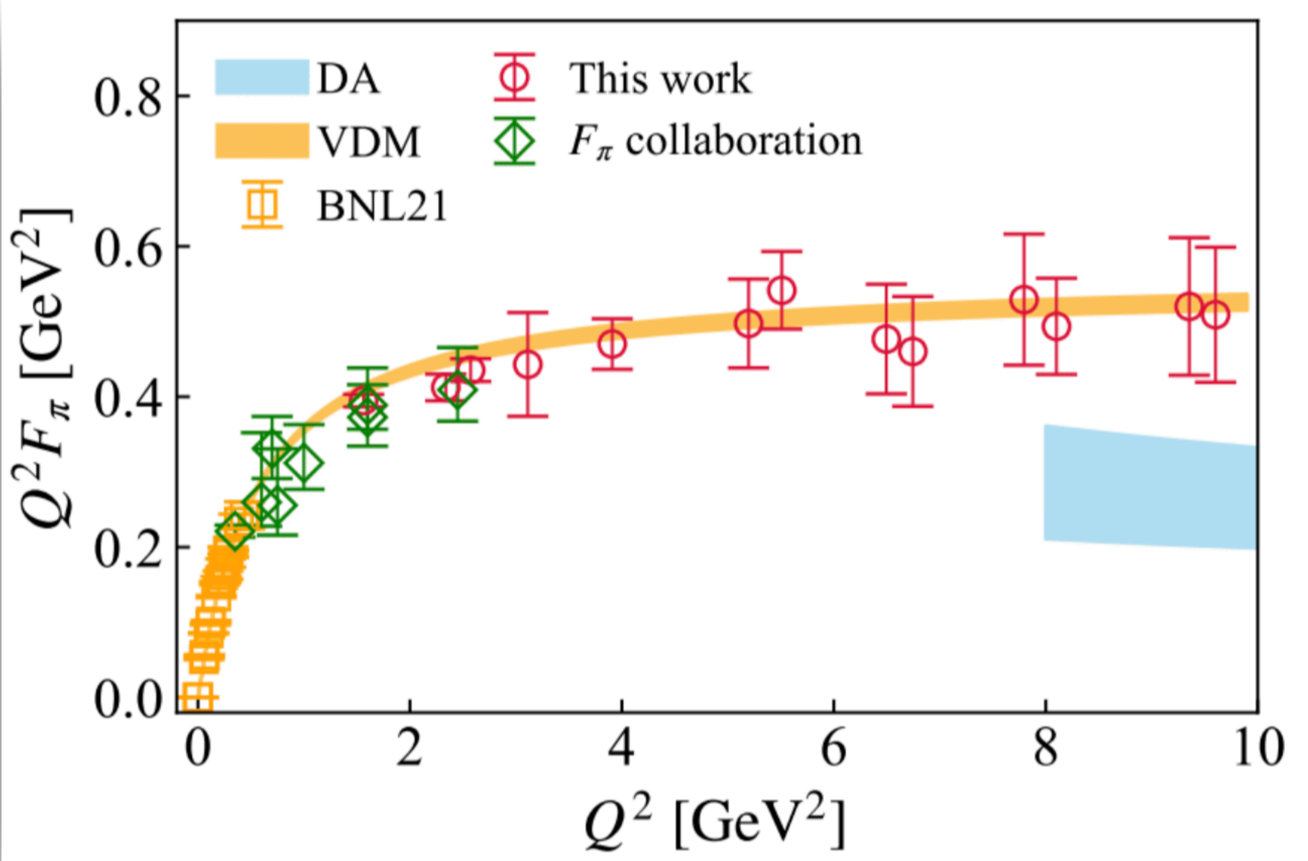


★ 2023-2024: kaon form factors (HISQ confs with HYP-smeared clover fermions, $m_\pi = 140\,\mathrm{MeV},\ a = 0.076\,\mathrm{fm}$)

★ Breit frame with boost up to ~1.5 GeV

★ multiple Tsink up to ~1fm

Preparatory work



★ Pion FF up to 9.3 GeV^2

★ Data from different methods in agreement

Nucleon Quark-Gluon structure

PI: M. Engelhardt

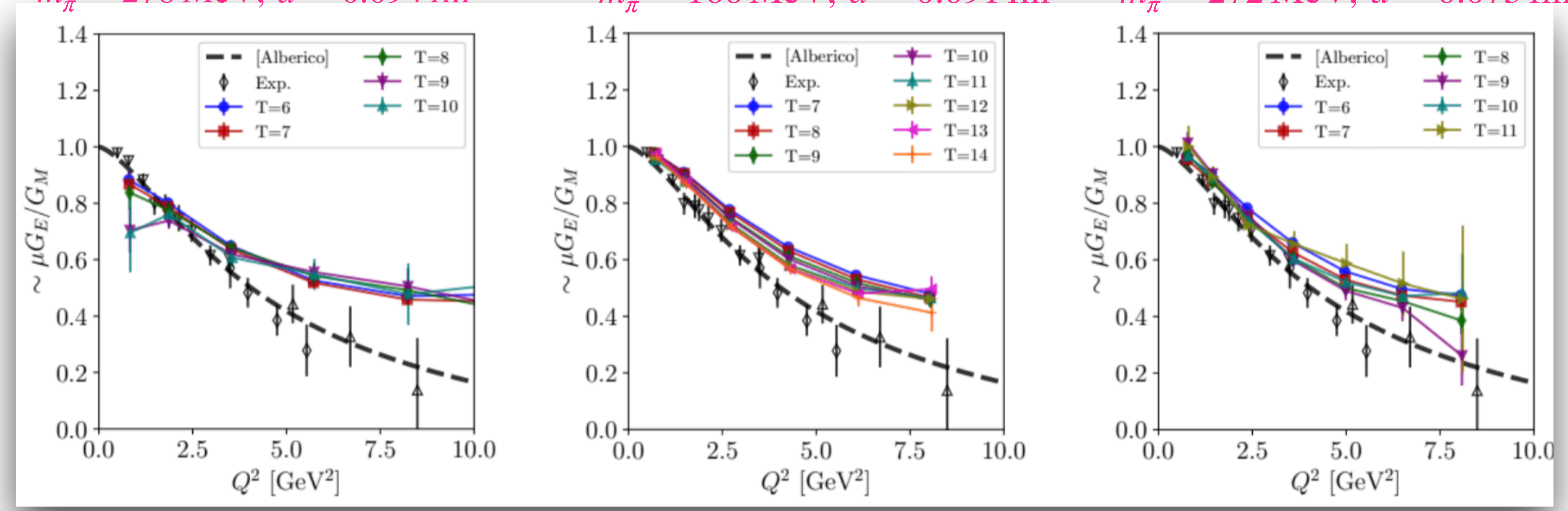
- ★ Vector, Axial FFs at high-Q^2 , TMDs, connections with quark OAM
- ★ Nf=2+1 dynamical Wilson-clover fermions, domain wall fermions (role of chiral symmetry)

Nucleon Quark-Gluon structure

PI: M. Engelhardt

- ★ Vector, Axial FFs at high-Q^2 , TMDs, connections with quark OAM
- ★ Nf=2+1 dynamical Wilson-clover fermions, domain wall fermions (role of chiral symmetry)

$m_\pi = 278\,\mathrm{MeV},\ a = 0.094\,\mathrm{fm}$ $m_\pi = 166\,\mathrm{MeV},\ a = 0.091\,\mathrm{fm}$ $m_\pi = 272\,\mathrm{MeV},\ a = 0.073\,\mathrm{fm}$



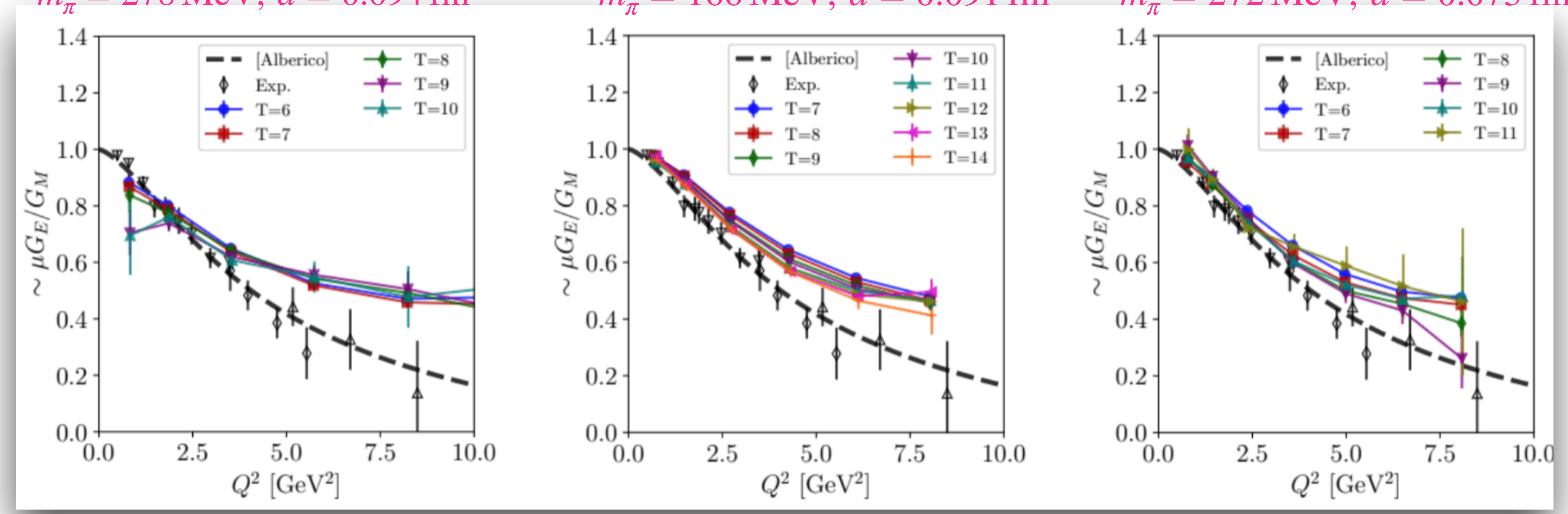
Tension with JLab12 GeV data — excited states investigation

Nucleon Quark-Gluon structure

PI: M. Engelhardt

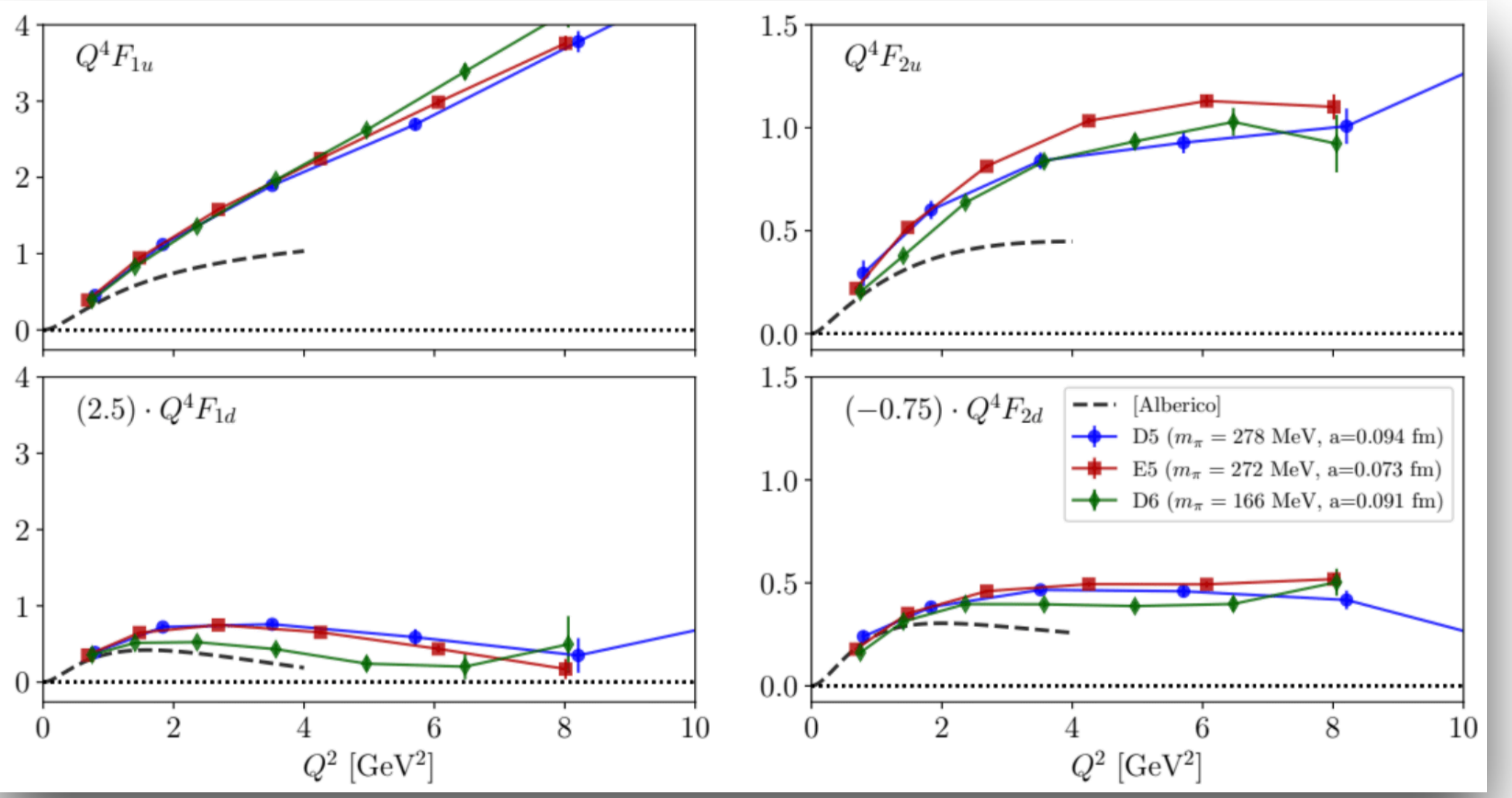
- ★ Vector, Axial FFs at high-Q^2 , TMDs, connections with quark OAM
- ★ Nf=2+1 dynamical Wilson-clover fermions, domain wall fermions (role of chiral symmetry)

$m_\pi = 278$ MeV, $a = 0.094$ fm $m_\pi = 166$ MeV, $a = 0.091$ fm $m_\pi = 272$ MeV, $a = 0.073$ fm



Tension with JLab12 GeV data — excited states investigation

- ★ u, d flavor decomposition of nucleon form factors



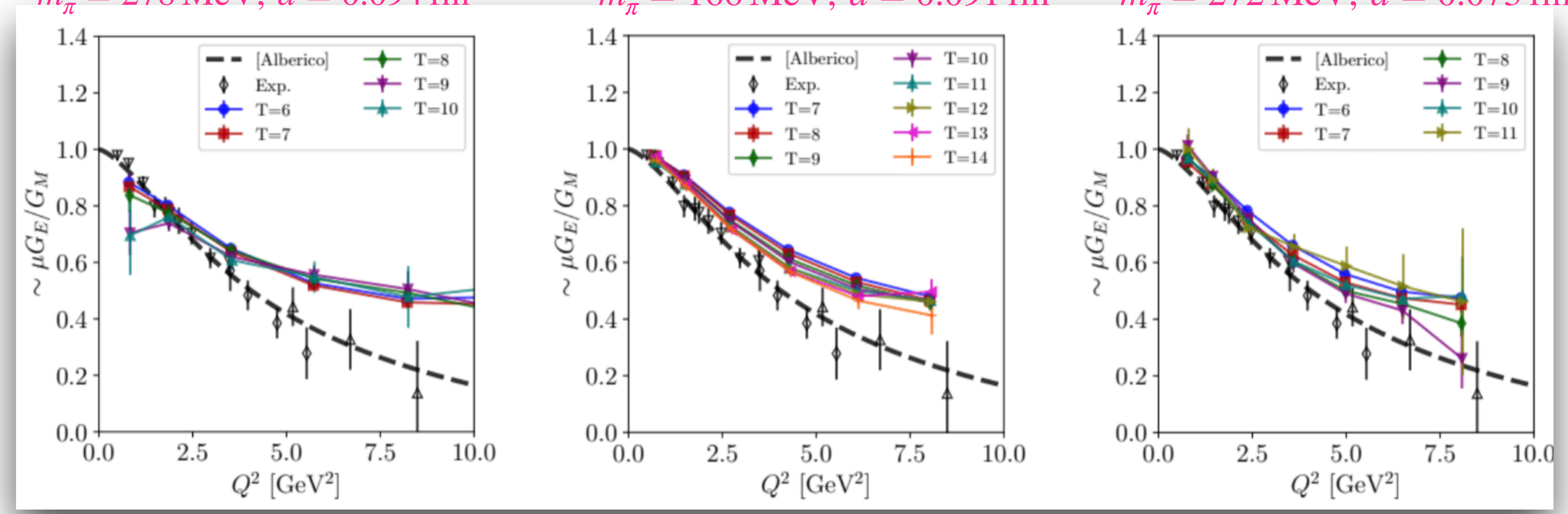
Inclusion of disconnected
will be considered
(near physical point)

Nucleon Quark-Gluon structure

PI: M. Engelhardt

- ★ Vector, Axial FFs at high-Q^2 , TMDs, connections with quark OAM
- ★ Nf=2+1 dynamical Wilson-clover fermions, domain wall fermions (role of chiral symmetry)

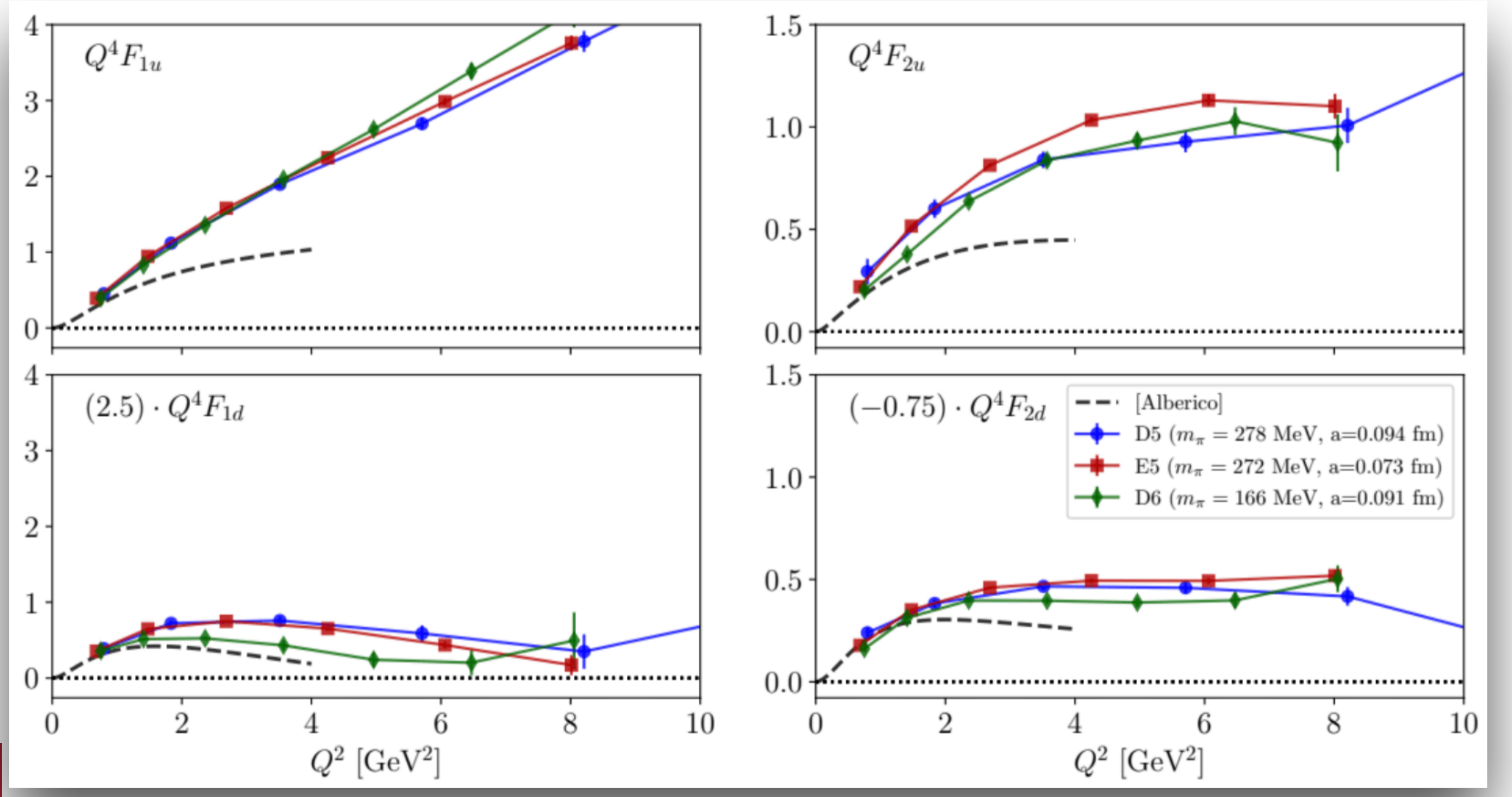
$m_\pi = 278\,\mathrm{MeV},\ a = 0.094\,\mathrm{fm}$ $m_\pi = 166\,\mathrm{MeV},\ a = 0.091\,\mathrm{fm}$ $m_\pi = 272\,\mathrm{MeV},\ a = 0.073\,\mathrm{fm}$



Tension with JLab12 GeV data — excited states investigation

- ★ 2023-2024:
  FFs at high-Q^2 (physical point, Tsink > 1fm, Q^2 up to 11.6 GeV^2
- ★ Longitudinally polarized TMD/GTMD

- ★ u, d flavor decomposition of nucleon form factors



Inclusion of disconnected will be considered (near physical point)

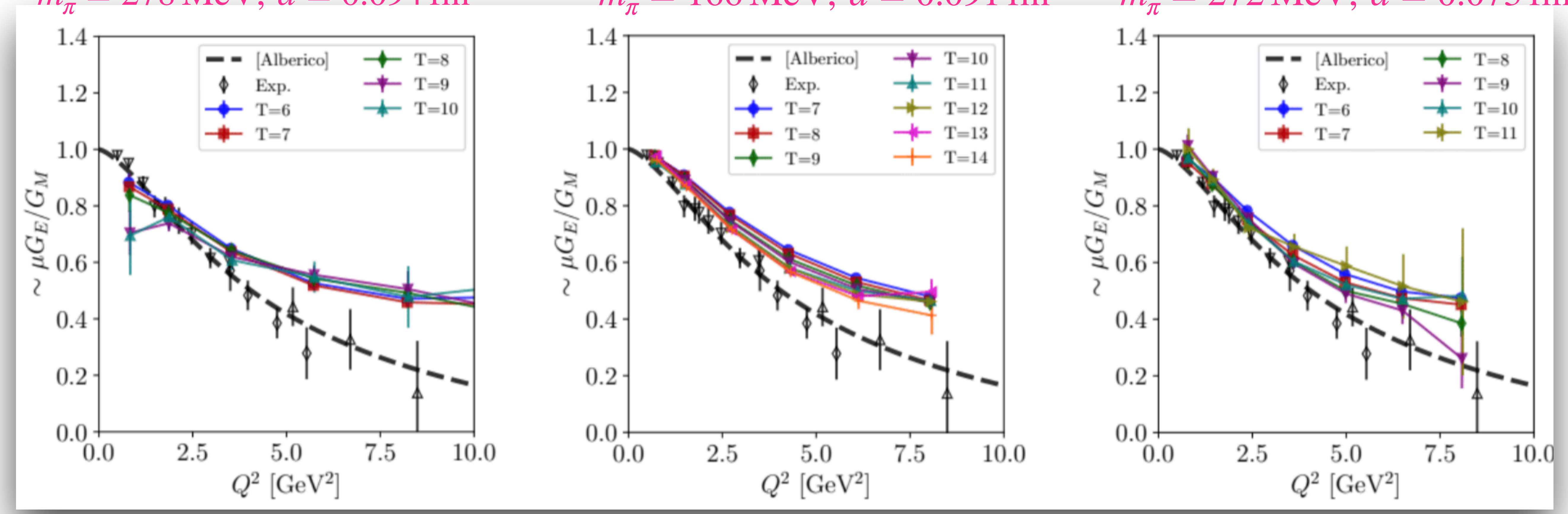
Nucleon Quark-Gluon structure

PI: M. Engelhardt

★ Vector, Axial FFs at high-Q^2, TMDs, connections with quark OAM

★ Nf=2+1 dynamical Wilson-clover fermions, domain wall fermions (role of chiral symmetry)

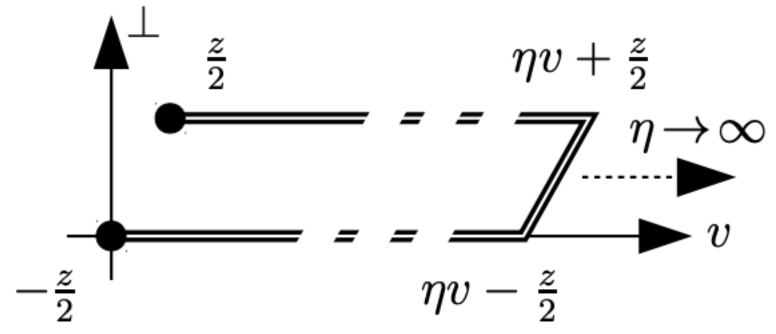
$m_\pi = 278$ MeV, $a = 0.094$ fm $\quad$ $m_\pi = 166$ MeV, $a = 0.091$ fm $\quad$ $m_\pi = 272$ MeV, $a = 0.073$ fm



Tension with JLab12 GeV data — excited states investigation

★ 2023-2024:
FFs at high-Q^2 (physical point, Tsink > 1fm, Q^2 up to 11.6 GeV^2

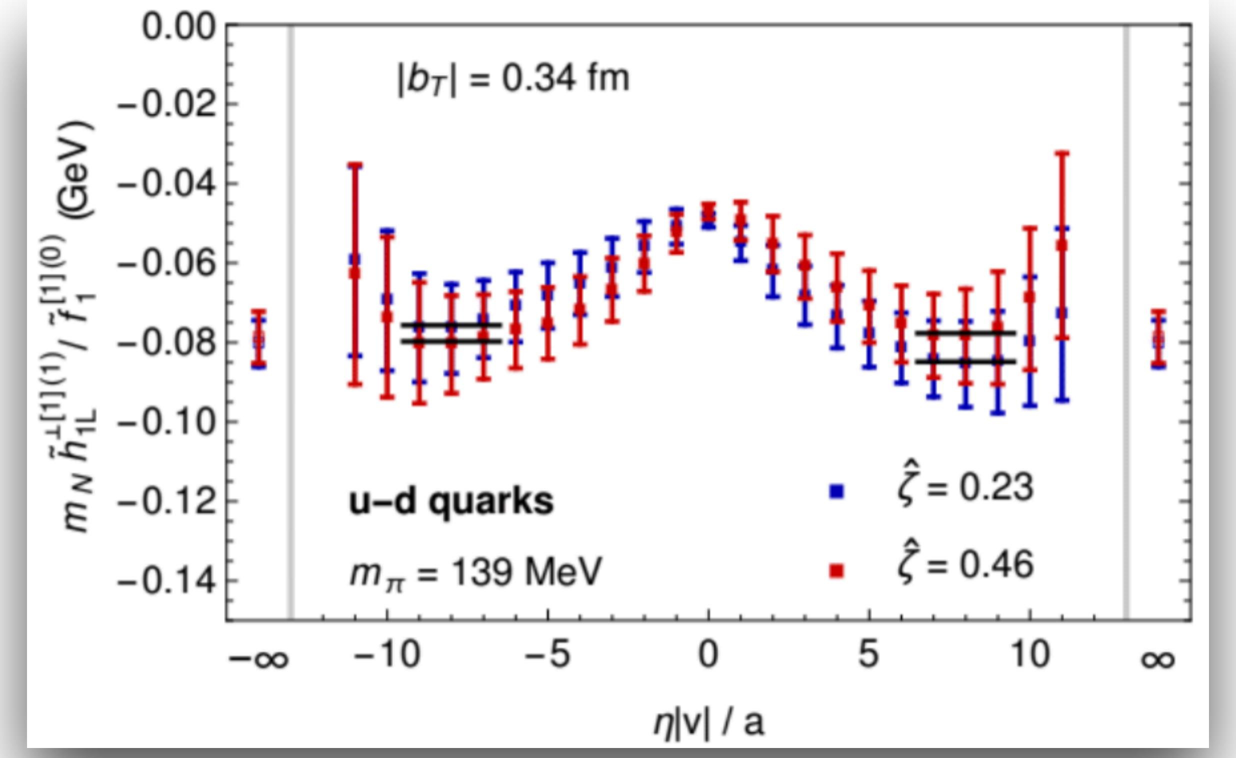
★ Longitudinally polarized TMD/GTMD



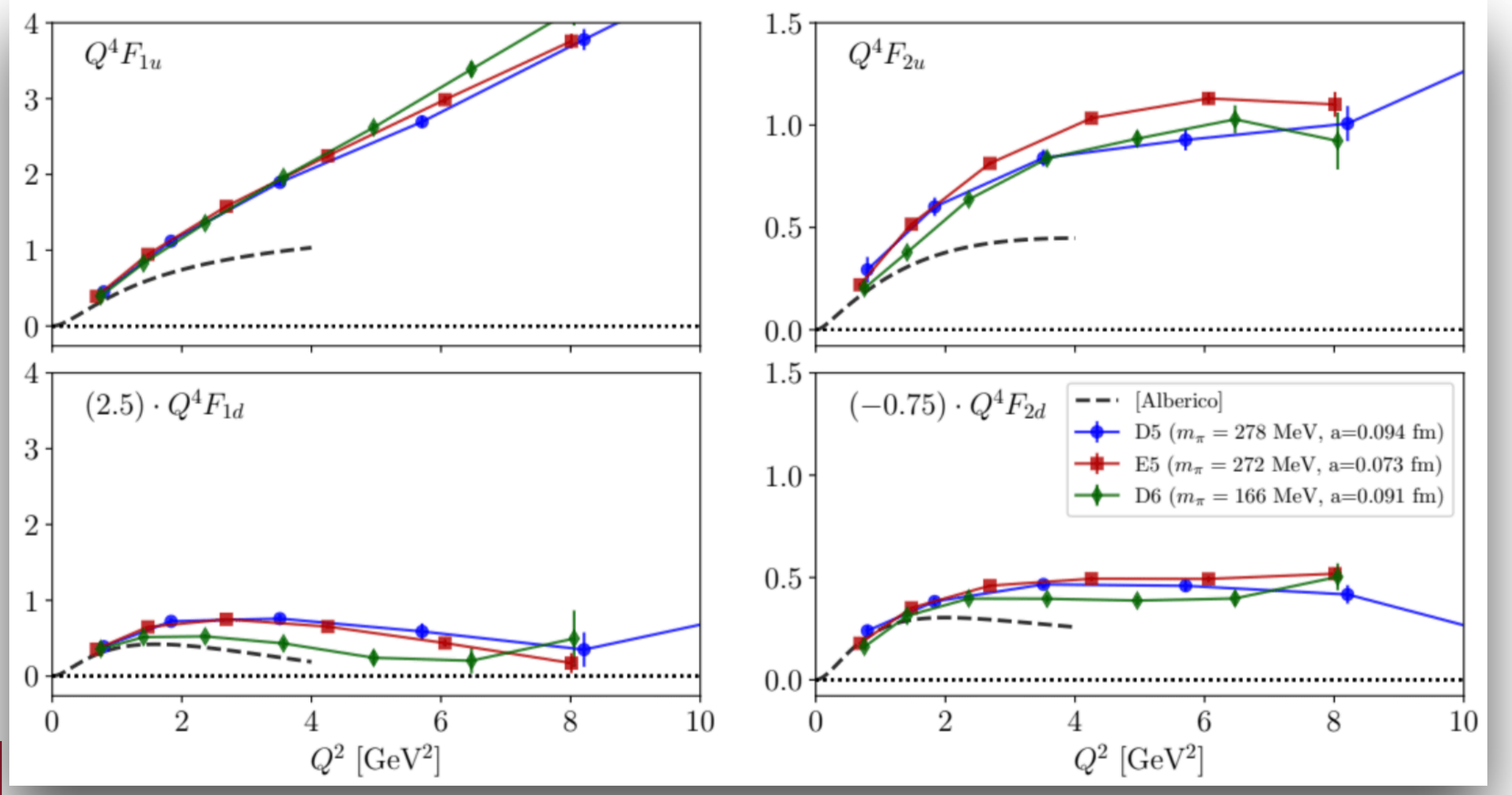
Preparatory work

★ transverse nucleon spin quantities
(e.g., generalized worm-gear shift)

★ OAM from Ji, Jaffe-Manohar sum rules



★ u, d flavor decomposition of nucleon form factors



Inclusion of disconnected will be considered (near physical point)

Strange-quark PDFs for nucleon

PI: H.-W. Lin

2023-2024 proposal

- ★ Extend work on strange PDFs using Nf=2+1+1 clover on HISQ fermions ($m_\pi = 310\,\mathrm{MeV}, a = 0.06\,\mathrm{fm}$)
- ★ Analysis improvement on strange asymmetry - input for CT18 global fits

Strange-quark PDFs for nucleon

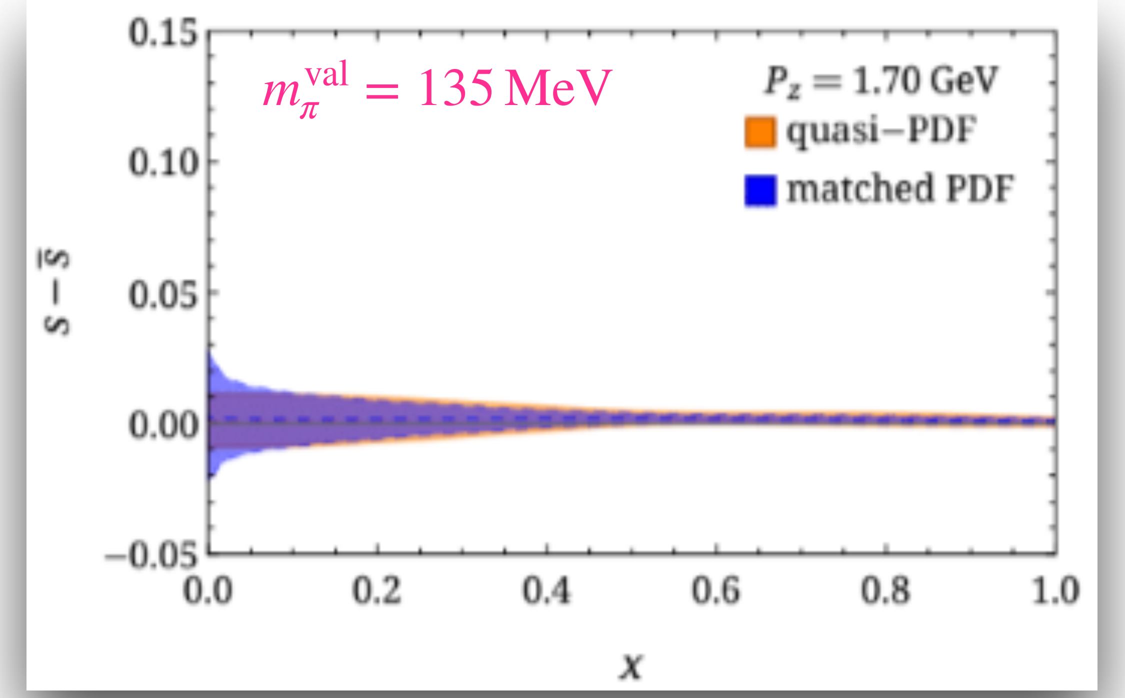
2023-2024 proposal

★ Extend work on strange PDFs using Nf=2+1+1 clover on HISQ fermions ($m_\pi = 310\,\text{MeV}, a = 0.06\,\text{fm}$)

★ Analysis improvement on strange asymmetry - input for CT18 global fits

Preparatory work

★ Nf=2+1+1 clover on HISQ fermions ($m_\pi^{\text{sea}} = 310\,\text{MeV},\ a = 0.12\,\text{fm},\ m_\pi^{\text{val}} = 310, 690\,\text{MeV}$)



[Zhang et al., PRD 104, (2021) 9, 094511]

Strange-quark asymmetry from lattice data compatible with zero

Strange-quark PDFs for nucleon

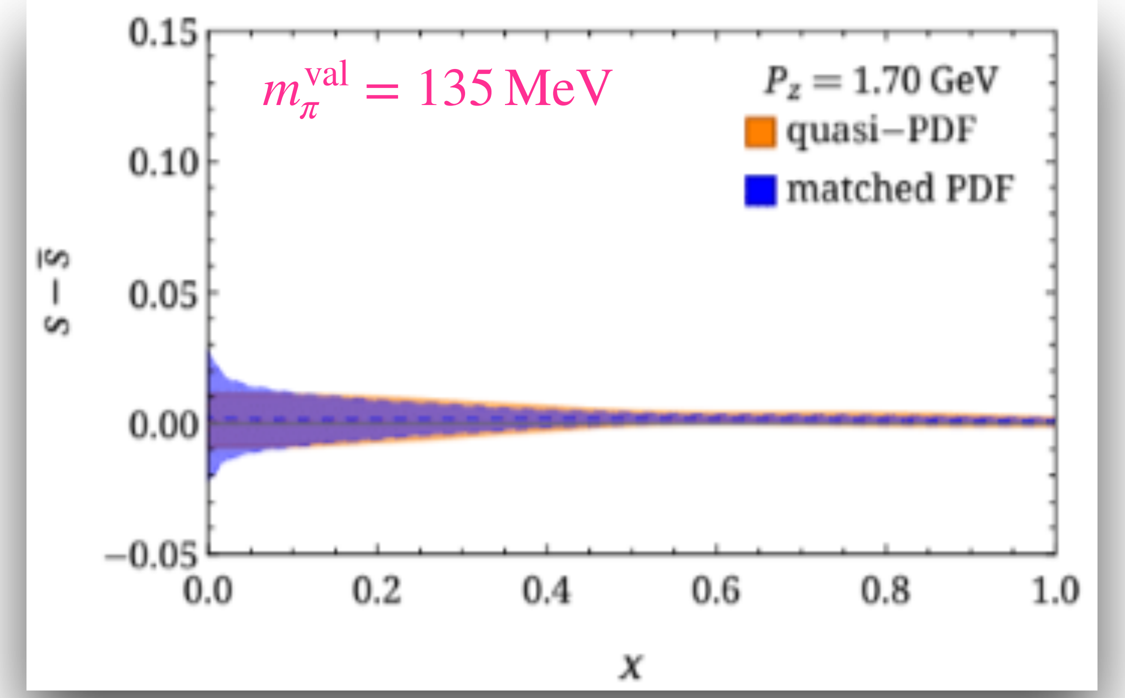
PI: H.-W. Lin

2023-2024 proposal

- ★ Extend work on strange PDFs using Nf=2+1+1 clover on HISQ fermions ($m_\pi = 310\,\text{MeV}, a = 0.06\,\text{fm}$)
- ★ Analysis improvement on strange asymmetry - input for CT18 global fits

Preparatory work

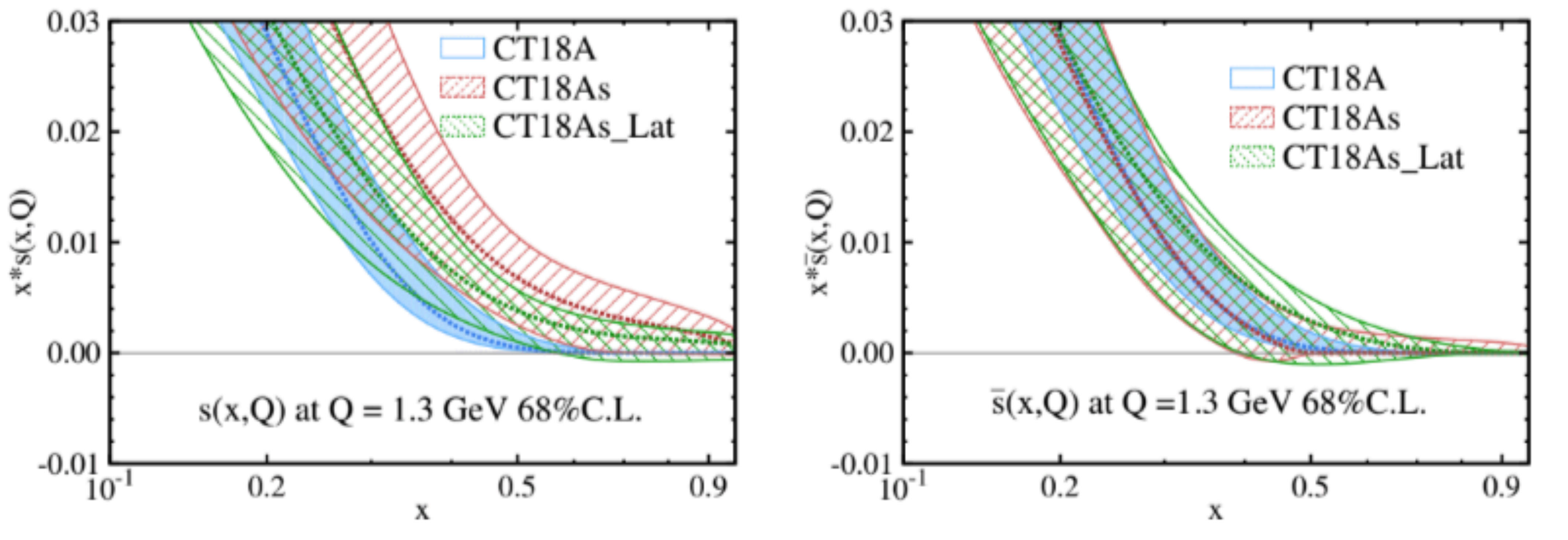
- ★ Nf=2+1+1 clover on HISQ fermions ($m_\pi^{\text{sea}} = 310\,\text{MeV},\ a = 0.12\,\text{fm},\ m_\pi^{\text{val}} = 310, 690\,\text{MeV}$)



[Zhang et al., PRD 104, (2021) 9, 094511]

Strange-quark asymmetry from lattice data compatible with zero

Lattice data incorporated in CT18A (includes ATLAS √s= 7 TeV W,Z combined cross-section measurements) with no assumption of strange-quark asymmetry



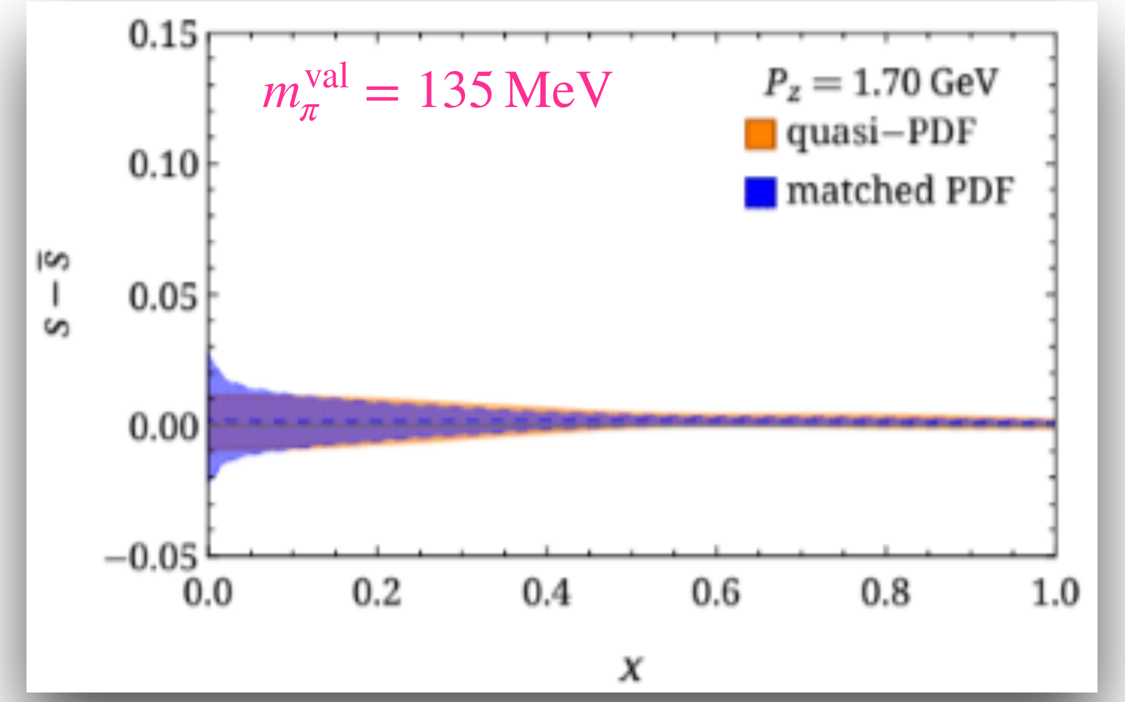
Strange-quark PDFs for nucleon

2023-2024 proposal

- ★ Extend work on strange PDFs using Nf=2+1+1 clover on HISQ fermions ($m_\pi = 310\,\text{MeV}, a = 0.06\,\text{fm}$)
- ★ Analysis improvement on strange asymmetry - input for CT18 global fits

Preparatory work

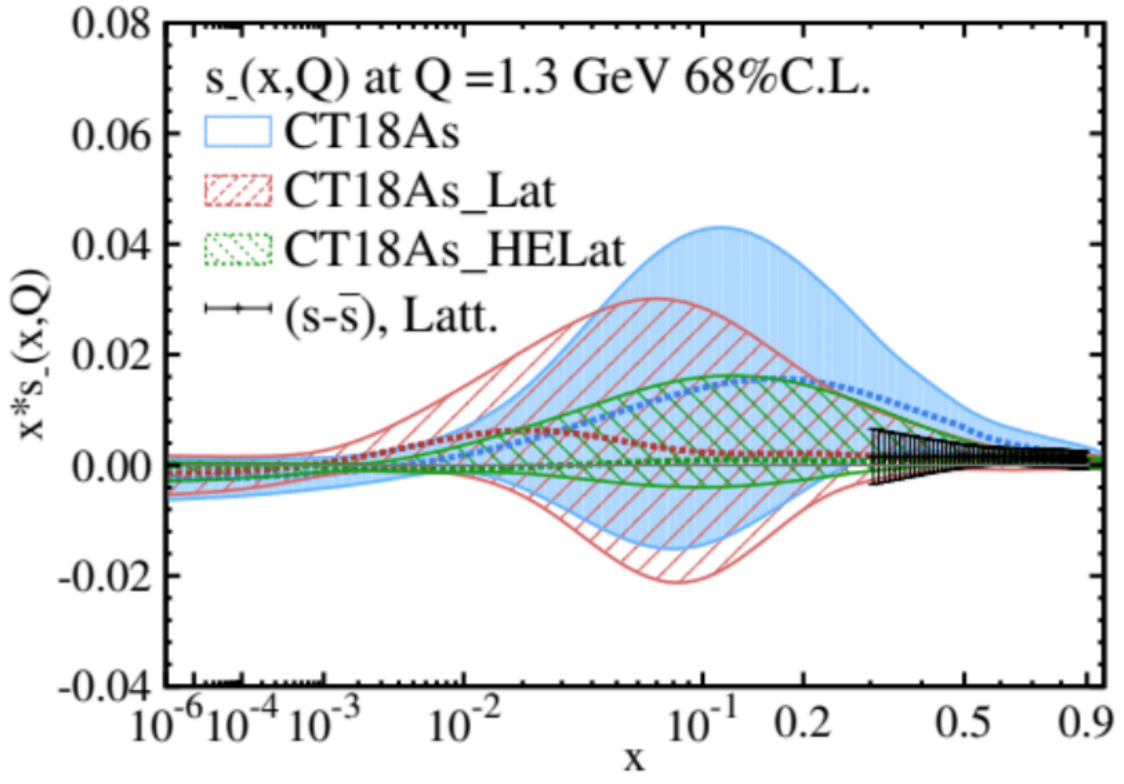
- ★ Nf=2+1+1 clover on HISQ fermions ($m_\pi^{\text{sea}} = 310\,\text{MeV},\ a = 0.12\,\text{fm},\ m_\pi^{\text{val}} = 310, 690\,\text{MeV}$)

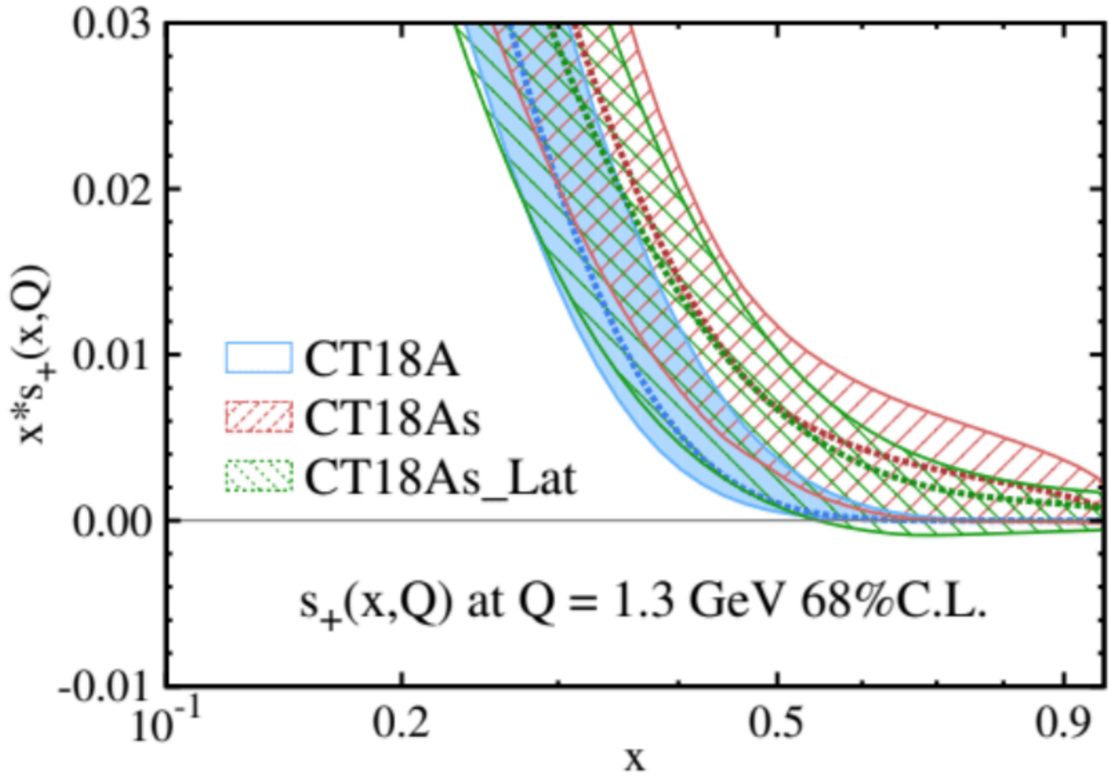


[Zhang et al., PRD 104, (2021) 9, 094511]

Strange-quark asymmetry from lattice data compatible with zero

Lattice data incorporated in CT18A (includes ATLAS √s= 7 TeV W,Z combined cross-section measurements) with no assumption of strange-quark asymmetry





Strange-quark PDFs for nucleon

PI: H.-W. Lin

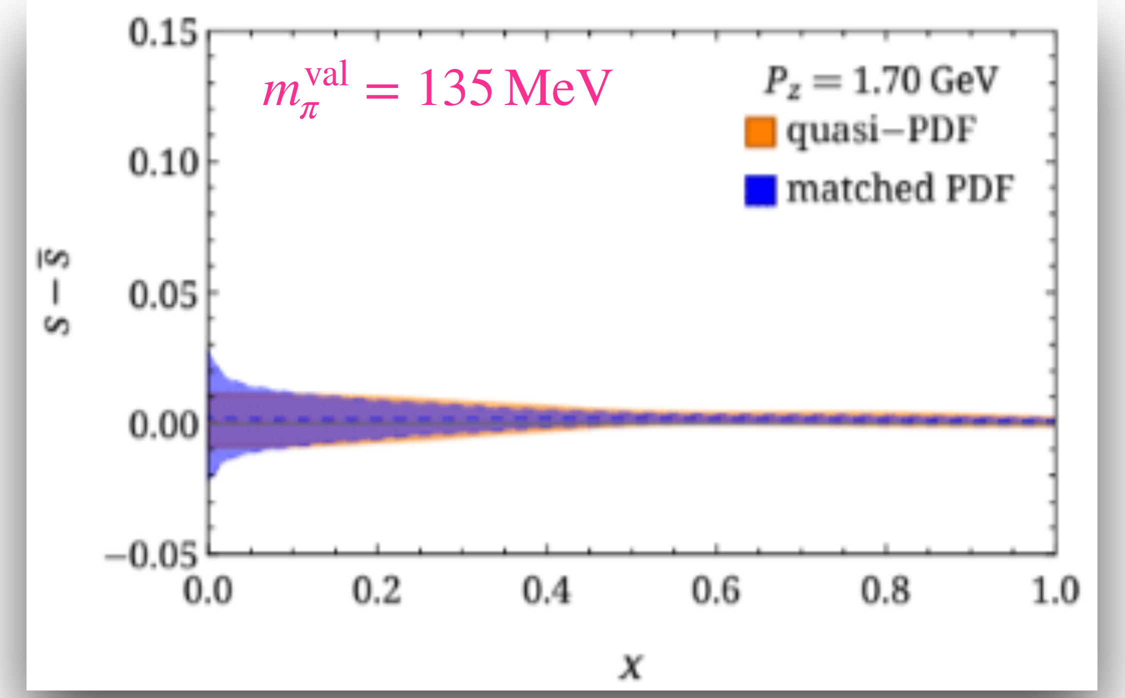
2023-2024 proposal

★ Extend work on strange PDFs using Nf=2+1+1 clover on HISQ fermions ($m_\pi = 310\,\mathrm{MeV}, a = 0.06\,\mathrm{fm}$)

★ Analysis improvement on strange asymmetry - input for CT18 global fits

Preparatory work

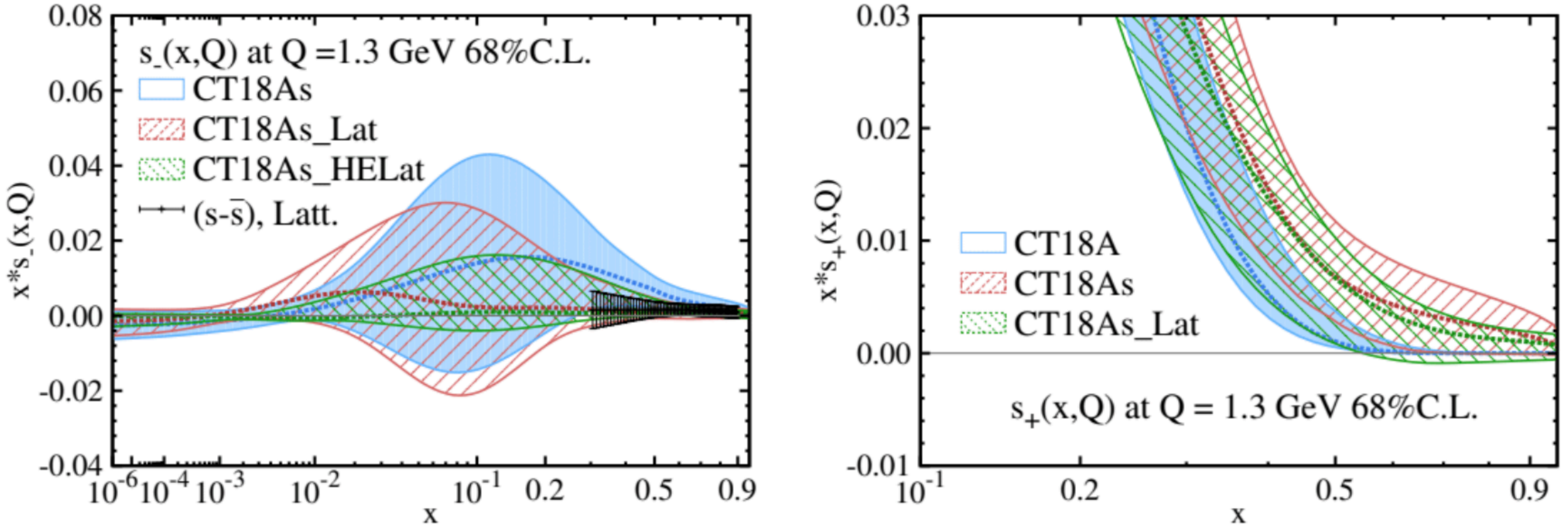
★ Nf=2+1+1 clover on HISQ fermions ($m_\pi^{\mathrm{sea}} = 310\,\mathrm{MeV},\ a = 0.12\,\mathrm{fm},\ m_\pi^{\mathrm{val}} = 310, 690\,\mathrm{MeV}$)



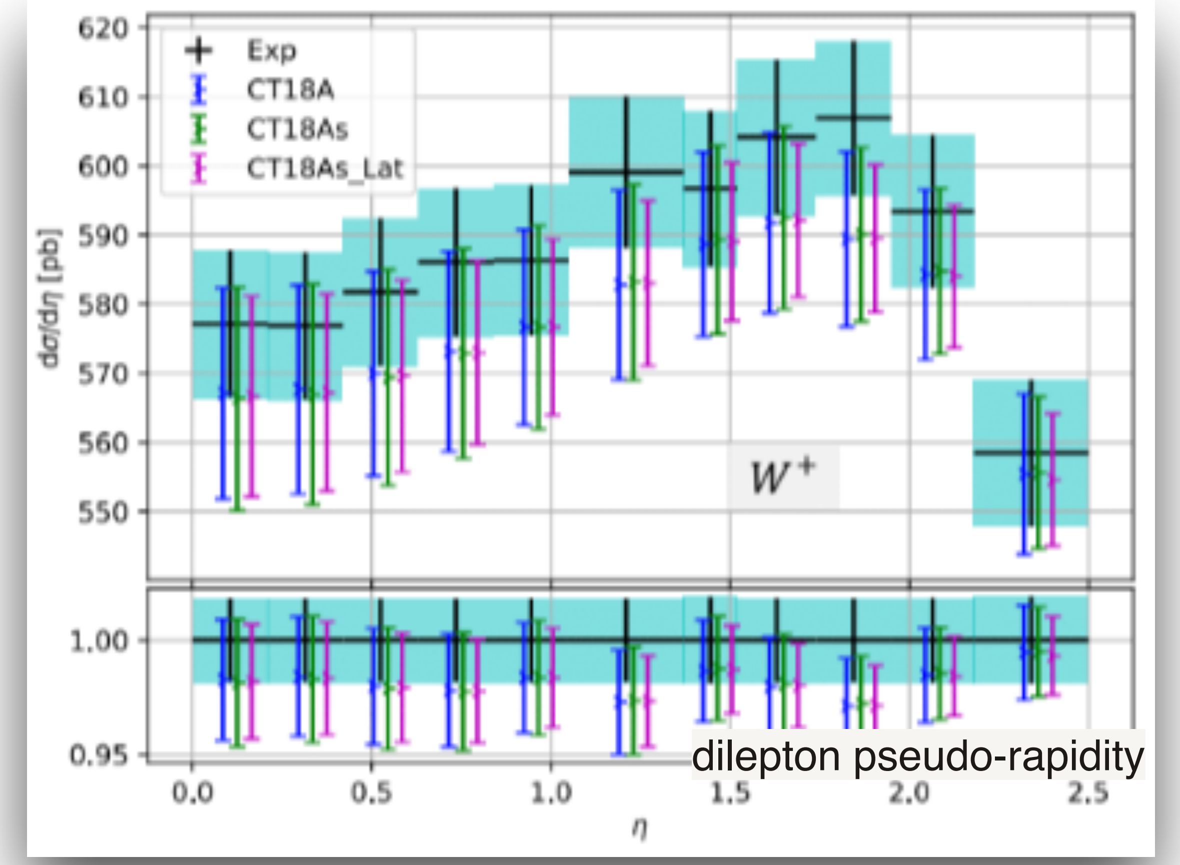
[Zhang et al., PRD 104, (2021) 9, 094511]

Strange-quark asymmetry from lattice data compatible with zero

Lattice data incorporated in CT18A (includes ATLAS √s= 7 TeV W,Z combined cross-section measurements) with no assumption of strange-quark asymmetry



Phenomenology Impact on ATLAS 7 TeV W and Z production at the LHC



CT18A analysis vs exper. values of differential cross-sections for W+

Proton and meson structure

PI: D. Richards

- Methodology: pseudo-PDFs, distillation / syst. effects: higher-twist effects, z^2 contaminating amplitude
- 2022: results with 2+1 clover fermions ($m_\pi = 360\,\mathrm{MeV}, a = 0.093\,\mathrm{fm}$)
  quark helicity & transversity PDF, gluon unpolarized & helicity PDF

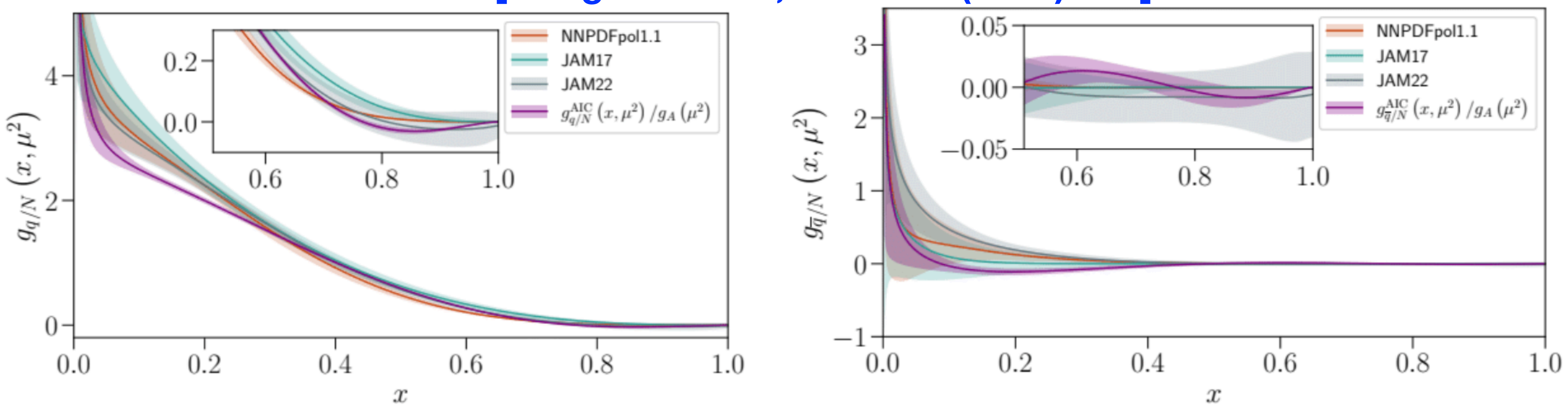
Proton and meson structure

PI: D. Richards

★ Methodology: pseudo-PDFs, distillation / syst. effects: higher-twist effects, z^2 contaminating amplitude

★ 2022: results with 2+1 clover fermions ($m_\pi = 360\,\mathrm{MeV}, a = 0.093\,\mathrm{fm}$)
quark helicity & transversity PDF, gluon unpolarized & helicity PDF

[C. Egerer et al., *JHEP* 03 (2023) 086]



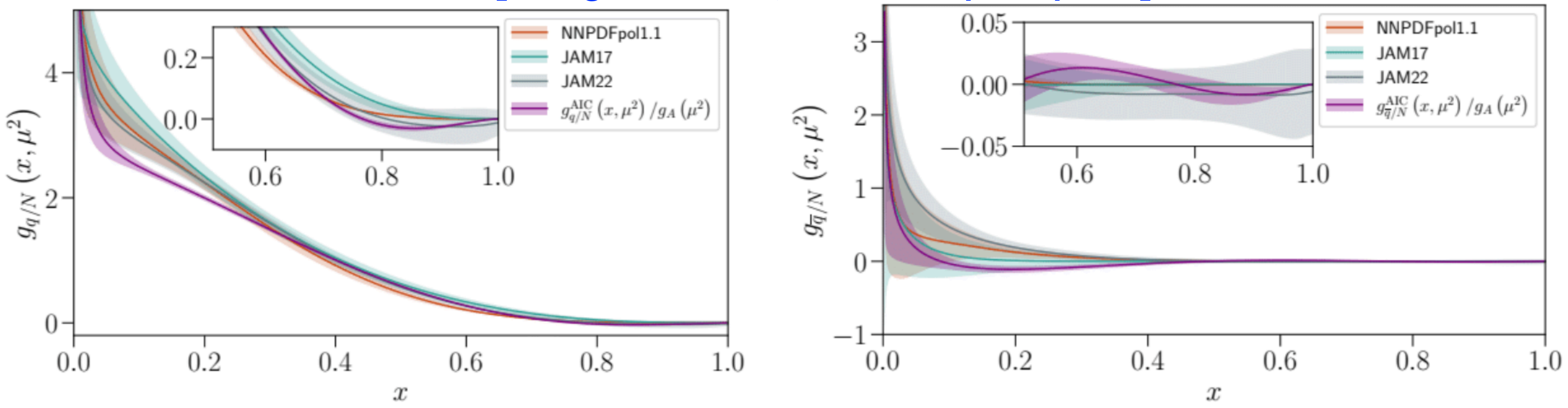
Akaike Information Criterion used to average over model functions and cuts in range of lattice data.

Proton and meson structure

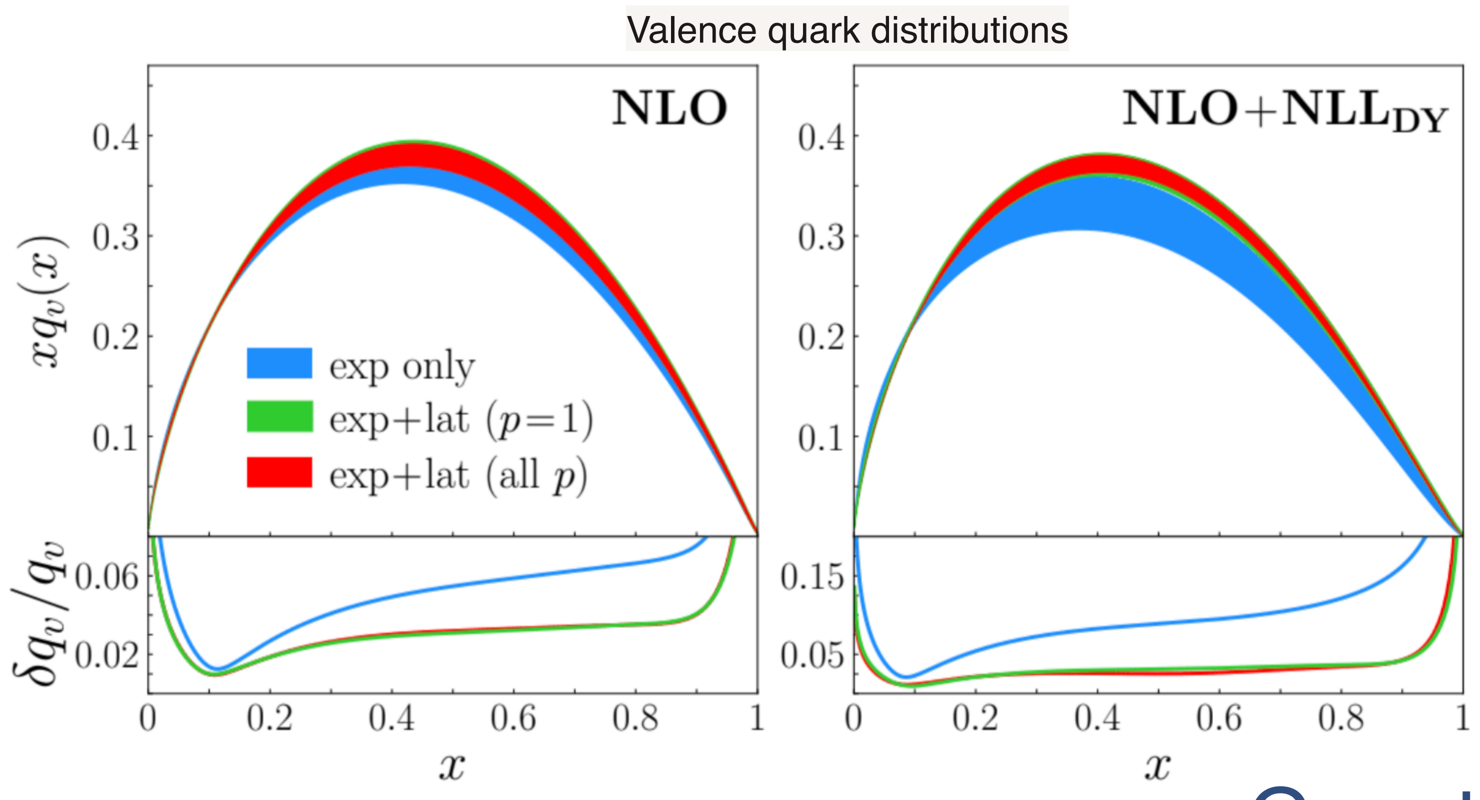
PI: D. Richards

- ★ Methodology: pseudo-PDFs, distillation / syst. effects: higher-twist effects, z^2 contaminating amplitude
- ★ 2022: results with 2+1 clover fermions ($m_\pi = 360\,\mathrm{MeV}, a = 0.093\,\mathrm{fm}$)
quark helicity & transversity PDF, gluon unpolarized & helicity PDF

[C. Egerer et al., *JHEP* 03 (2023) 086]



Akaike Information Criterion used to average over model functions and cuts in range of lattice data.



[P. Barry et al. PRD 105 114051]

Constraining power of LQCD results on pion PDF

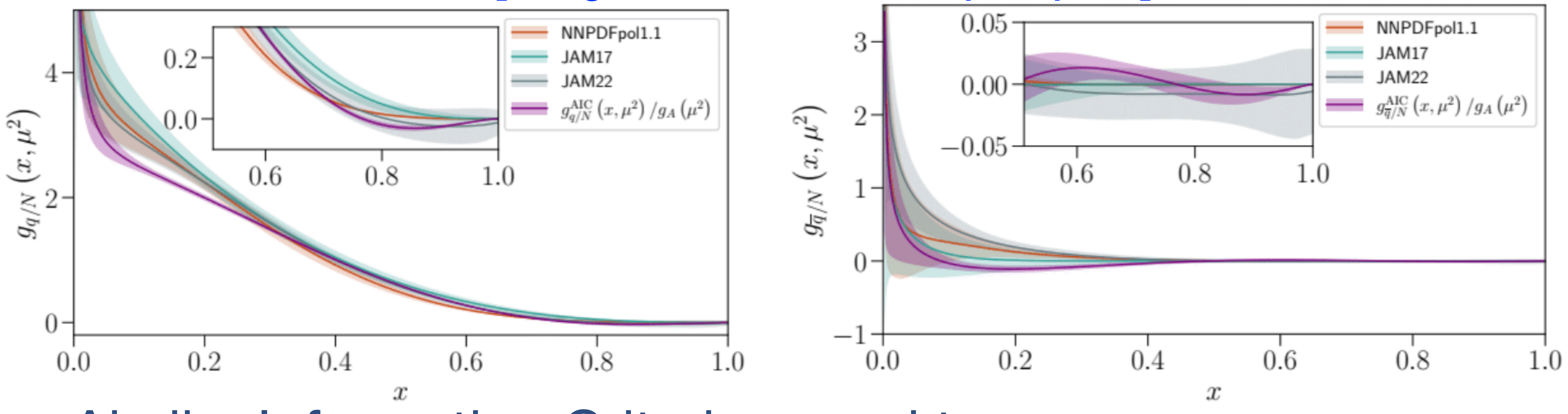
Proton and meson structure

PI: D. Richards

★ Methodology: pseudo-PDFs, distillation / syst. effects: higher-twist effects, z^2 contaminating amplitude

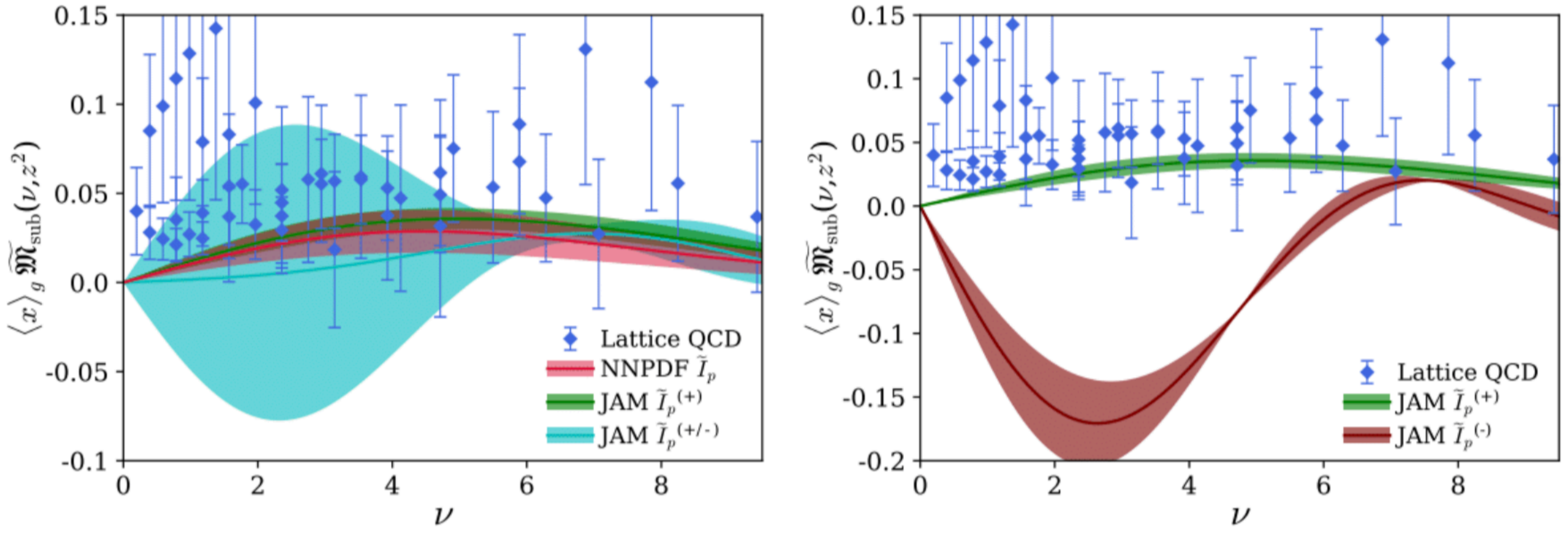
★ 2022: results with 2+1 clover fermions ($m_\pi = 360\,\mathrm{MeV}, a = 0.093\,\mathrm{fm}$)
quark helicity & transversity PDF, gluon unpolarized & helicity PDF

[C. Egerer et al., *JHEP* 03 (2023) 086]

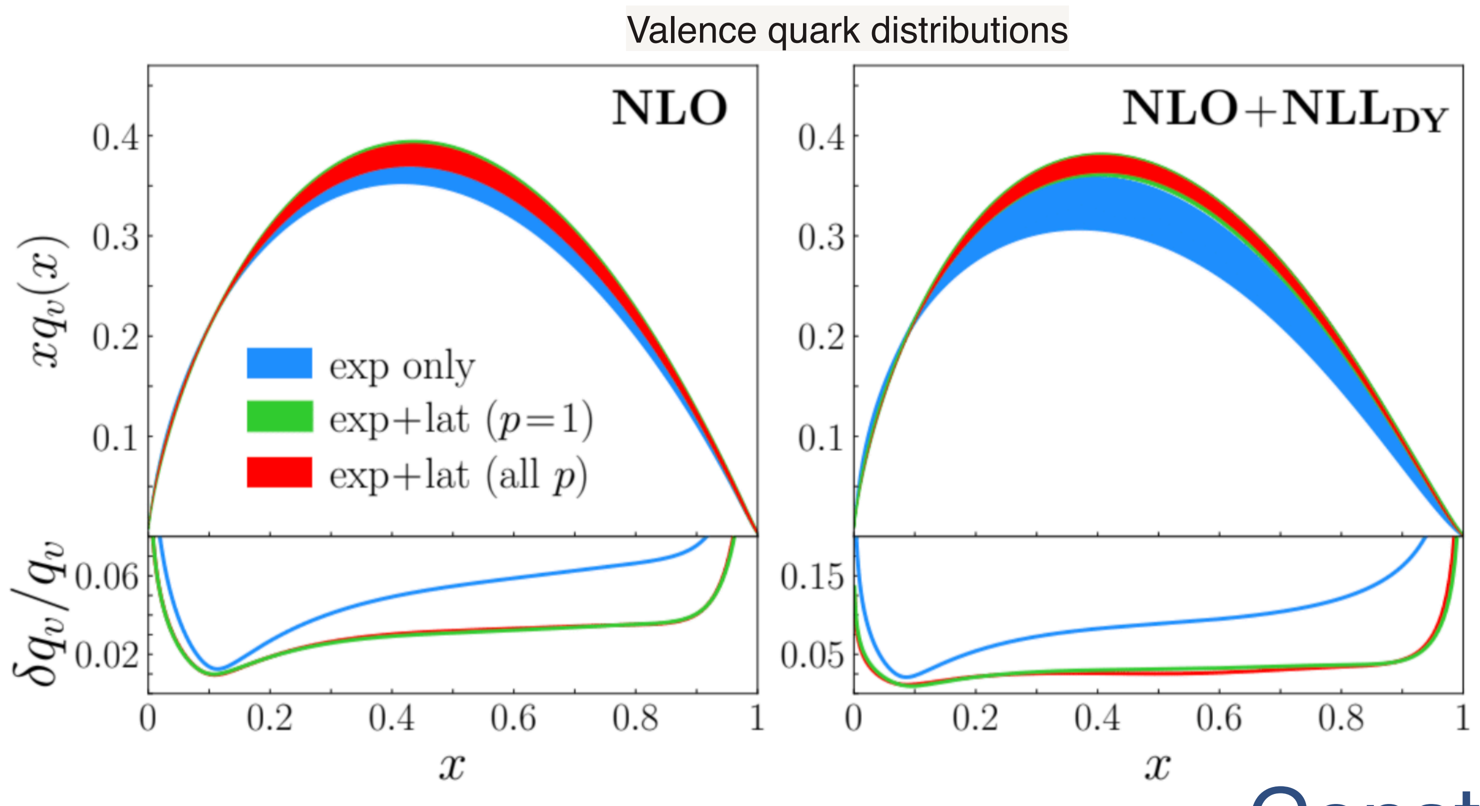


Akaike Information Criterion used to average over model functions and cuts in range of lattice data.

[C. Egerer et al., PRD 105, 034507]



Hint for a nonzero gluon spin contribution to proton spin
No positivity constraint: magnitude/sign of gluon helicity undetermined



[P. Barry et al. PRD 105 114051]

Constraining power of LQCD results on pion PDF

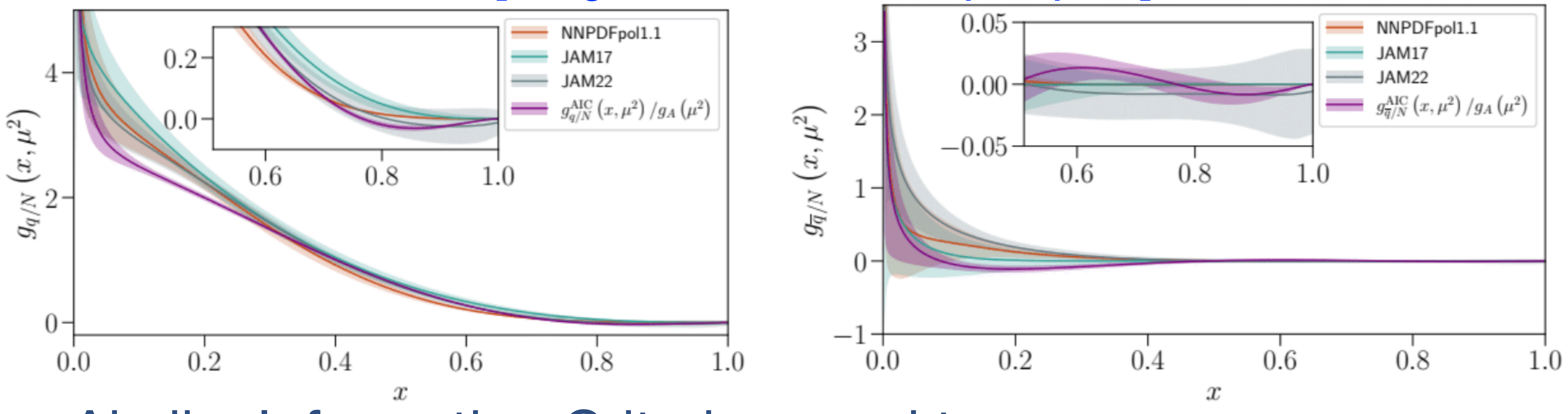
Proton and meson structure

PI: D. Richards

★ Methodology: pseudo-PDFs, distillation / syst. effects: higher-twist effects, z^2 contaminating amplitude

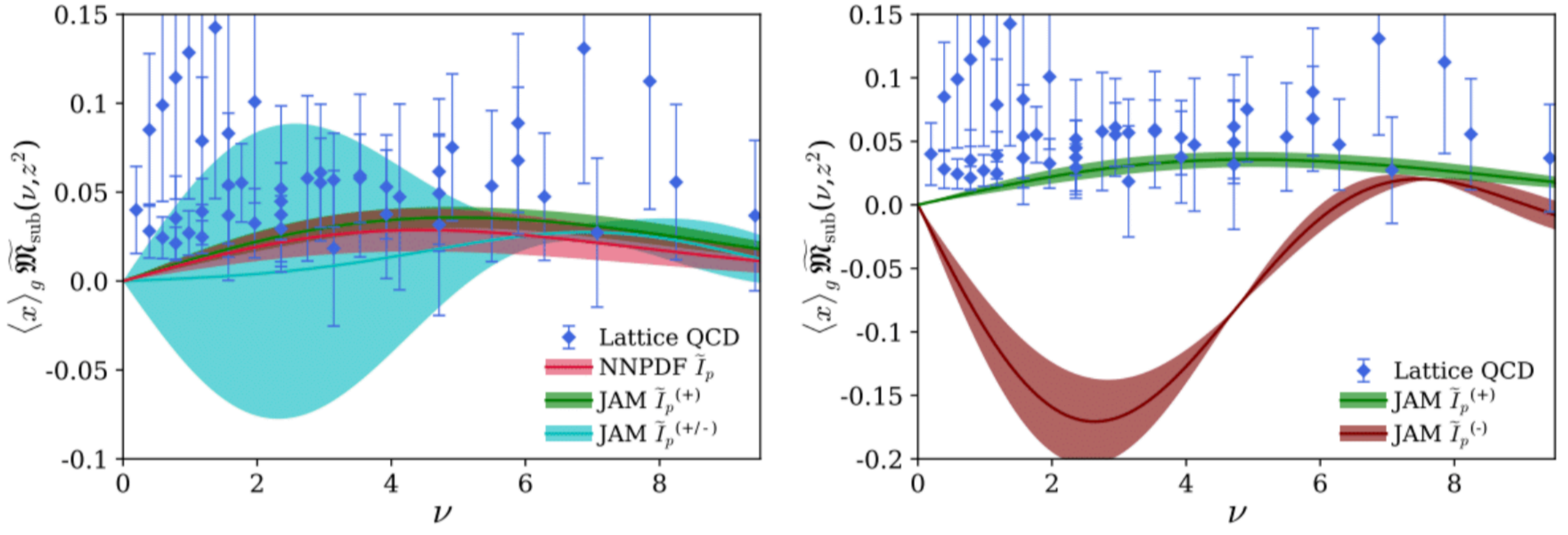
★ 2022: results with 2+1 clover fermions ($m_\pi = 360\,\text{MeV}, a = 0.093\,\text{fm}$)
quark helicity & transversity PDF, gluon unpolarized & helicity PDF

[C. Egerer et al., *JHEP* 03 (2023) 086]



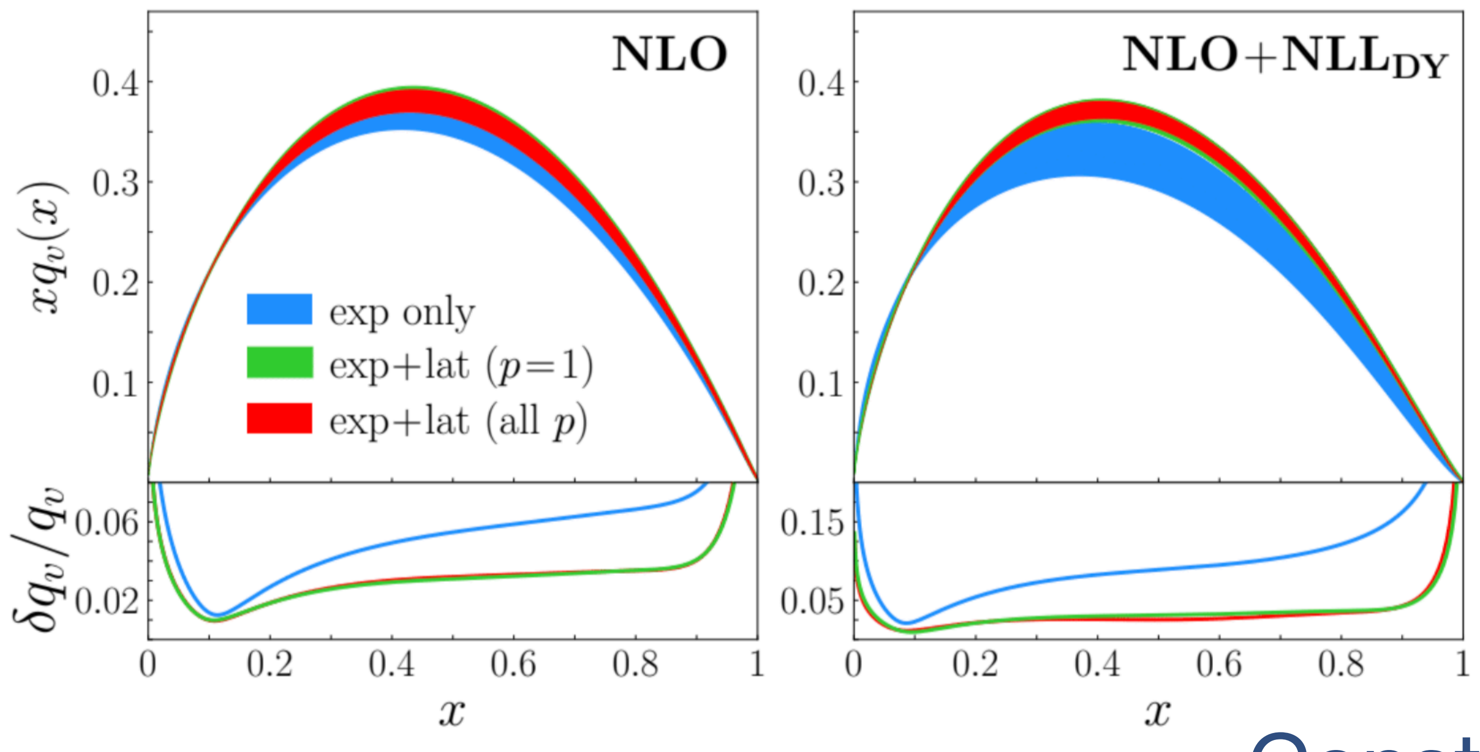
Akaike Information Criterion used to average over model functions and cuts in range of lattice data.

[C. Egerer et al., PRD 105, 034507]



Hint for a nonzero gluon spin contribution to proton spin
No positivity constraint: magnitude/sign of gluon helicity undetermined

Valence quark distributions



[P. Barry et al. PRD 105 114051]

Constraining power of LQCD results on pion PDF

2023 work plan:

★ Focus on gluon and sea-quark contributions, GPDs

★ Two additional ensembles ($m_\pi = 270\,\text{MeV},\ a = 0.072, 0.093\,\text{fm}$)

Proton GPDs

PI: M. Constantinou

★ Lack density interpretation, but can be sizable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

★ Kinematically suppressed
Difficult to isolate experimentally

★ Understanding importance of q-g-q:
WW approximation

Proton GPDs

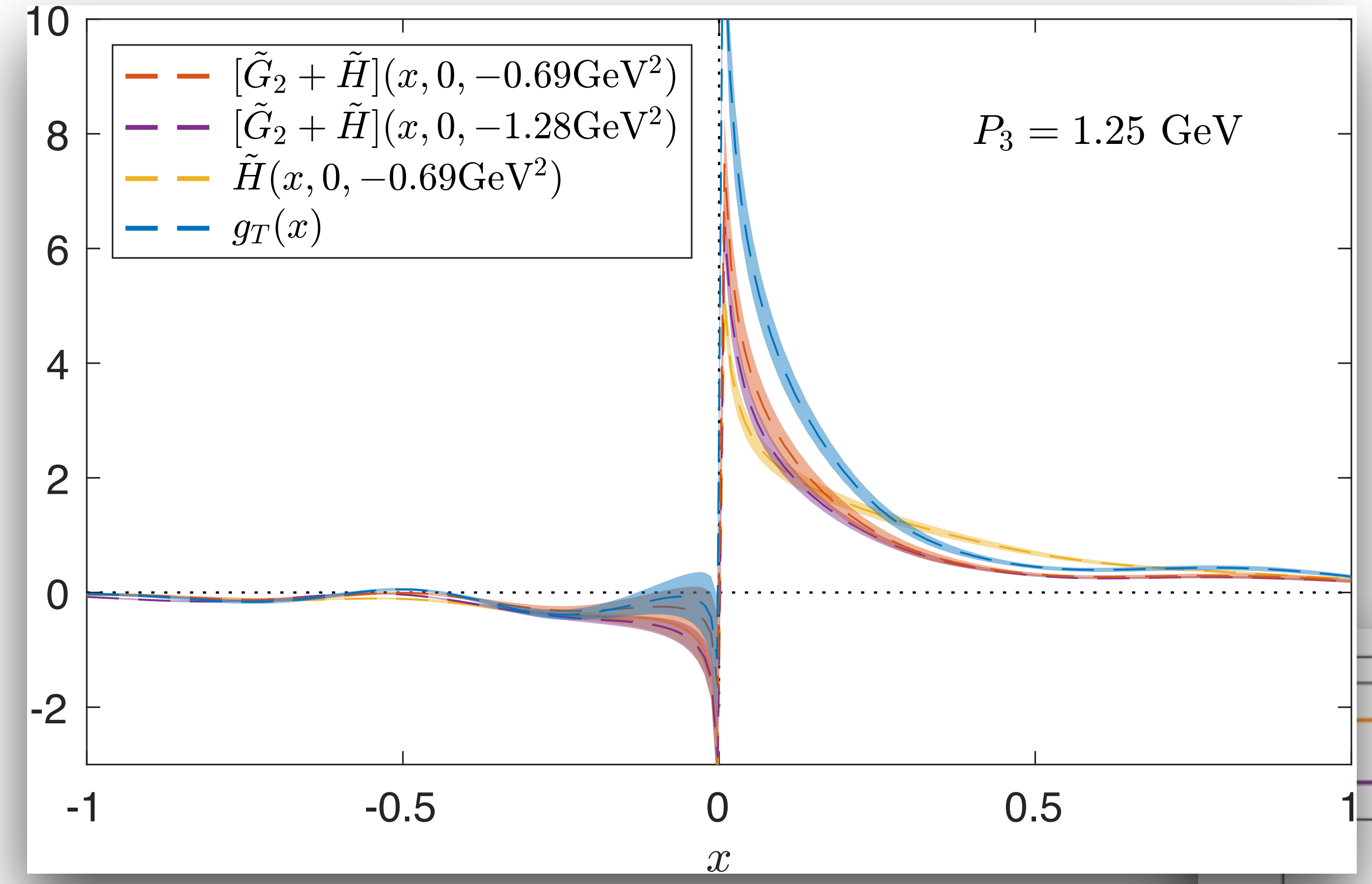
PI: M. Constantinou

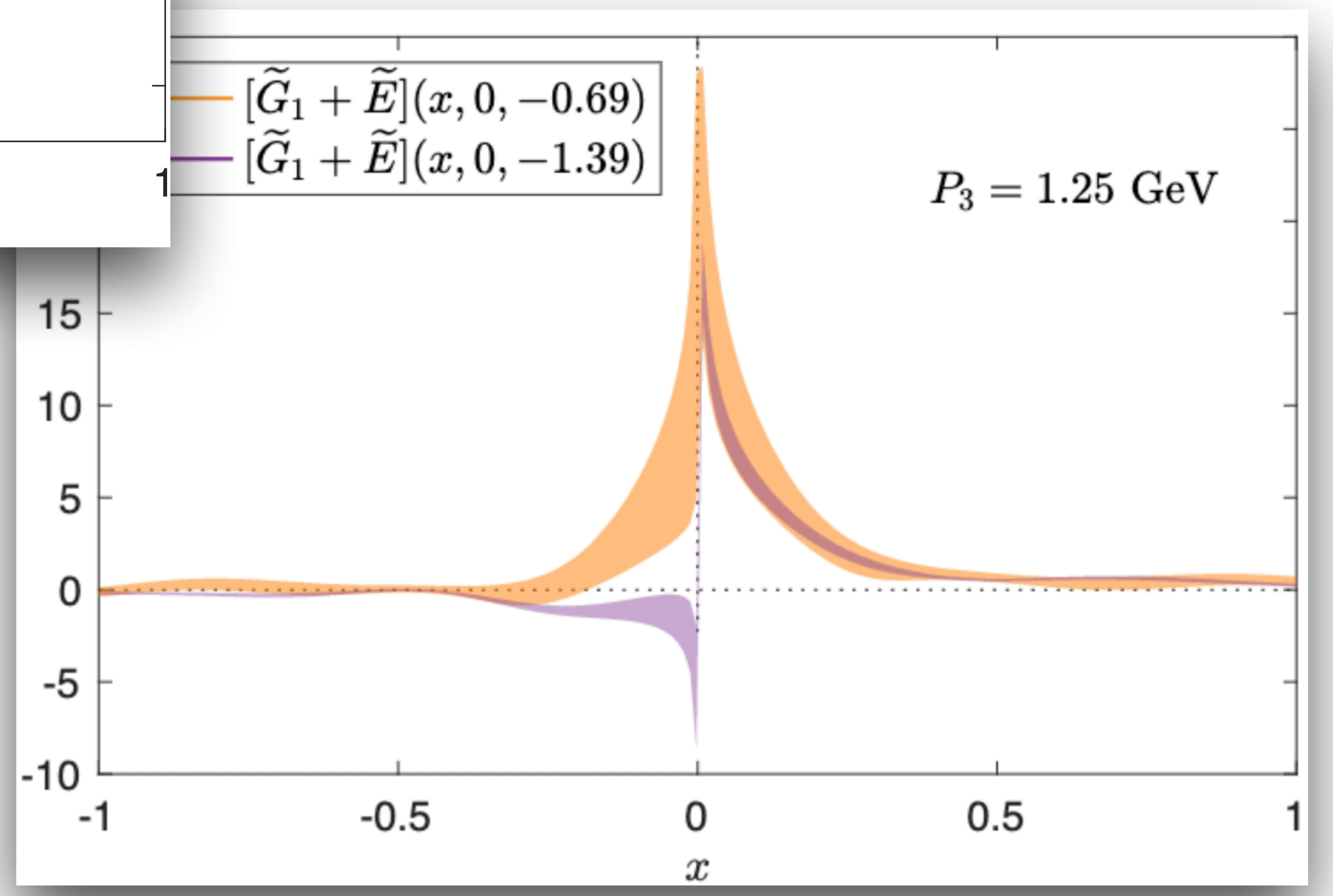
- ★ Lack density interpretation, but can be sizable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

- ★ Kinematically suppressed
  Difficult to isolate experimentally

- ★ Understanding importance of q-g-q:
  WW approximation





[S. Bhattacharya et al.,
PoS LATTICE2021 (2022) 054
arXiv:2112.05538]

Proton GPDs

PI: M. Constantinou

★ Lack density interpretation, but can be sizable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2}\cdots$$

★ Kinematically suppressed
Difficult to isolate experimentally

★ Understanding importance of q-g-q: WW approximation

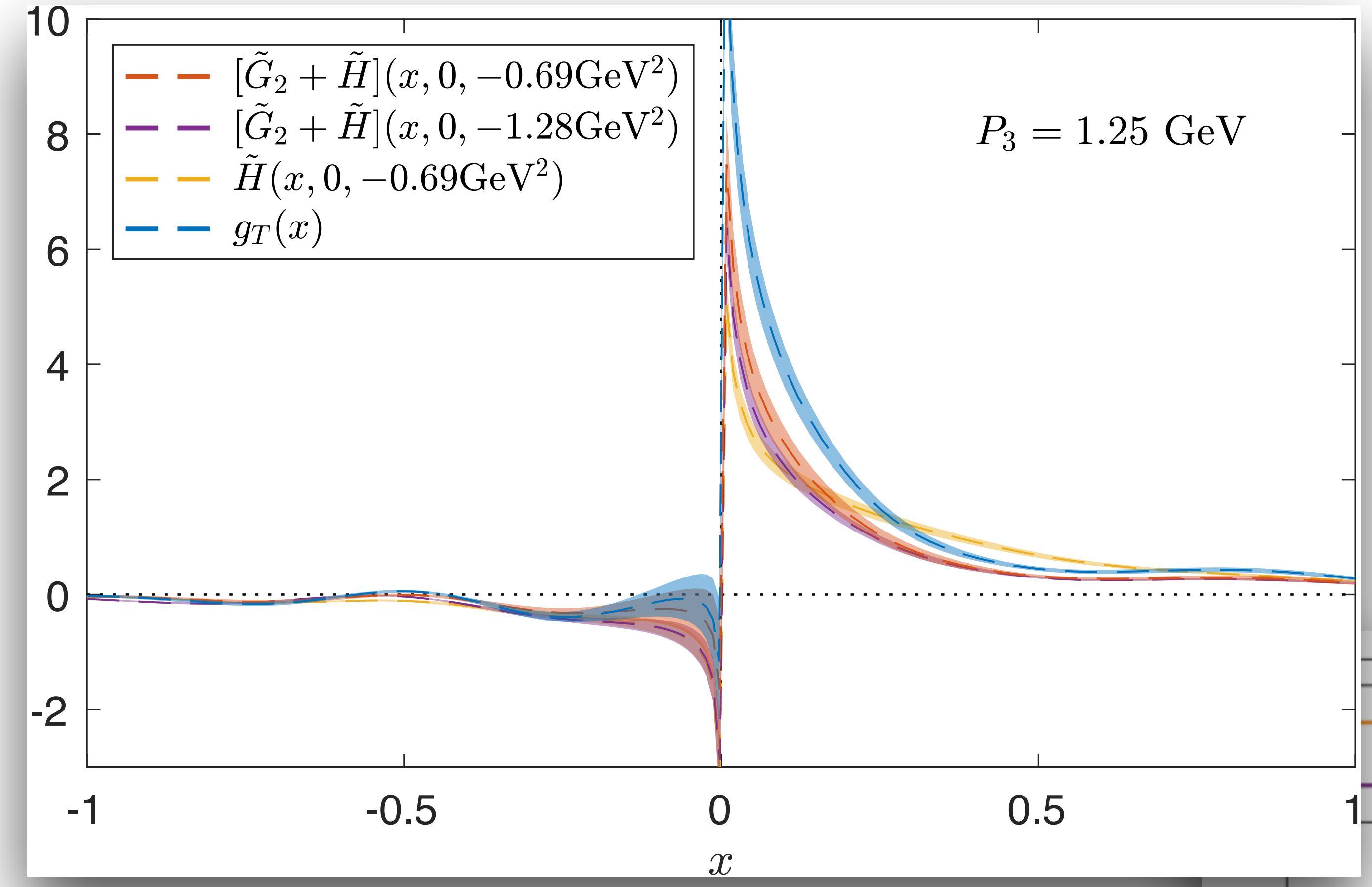
★ Proposal: GPDs, Nf=2+1+1 twisted mass fermions
$\{m_\pi, a\} = \{250\,\mathrm{MeV}, 0.08\,\mathrm{fm}\}$

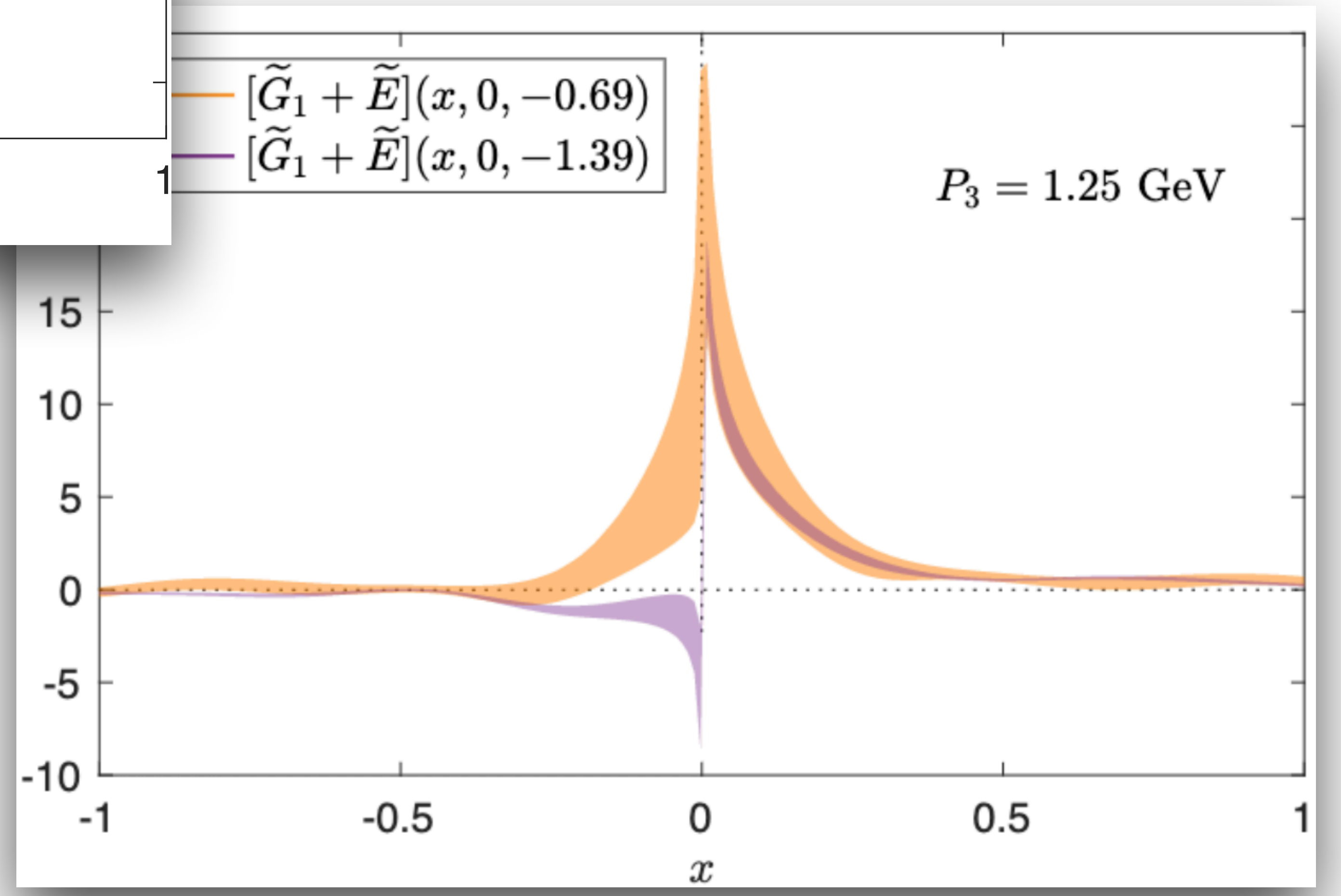
★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^\mu_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Main advantage

- Applicable to any kinematic frame





[S. Bhattacharya et al., PoS LATTICE2021 (2022) 054 arXiv:2112.05538]

Proton GPDs

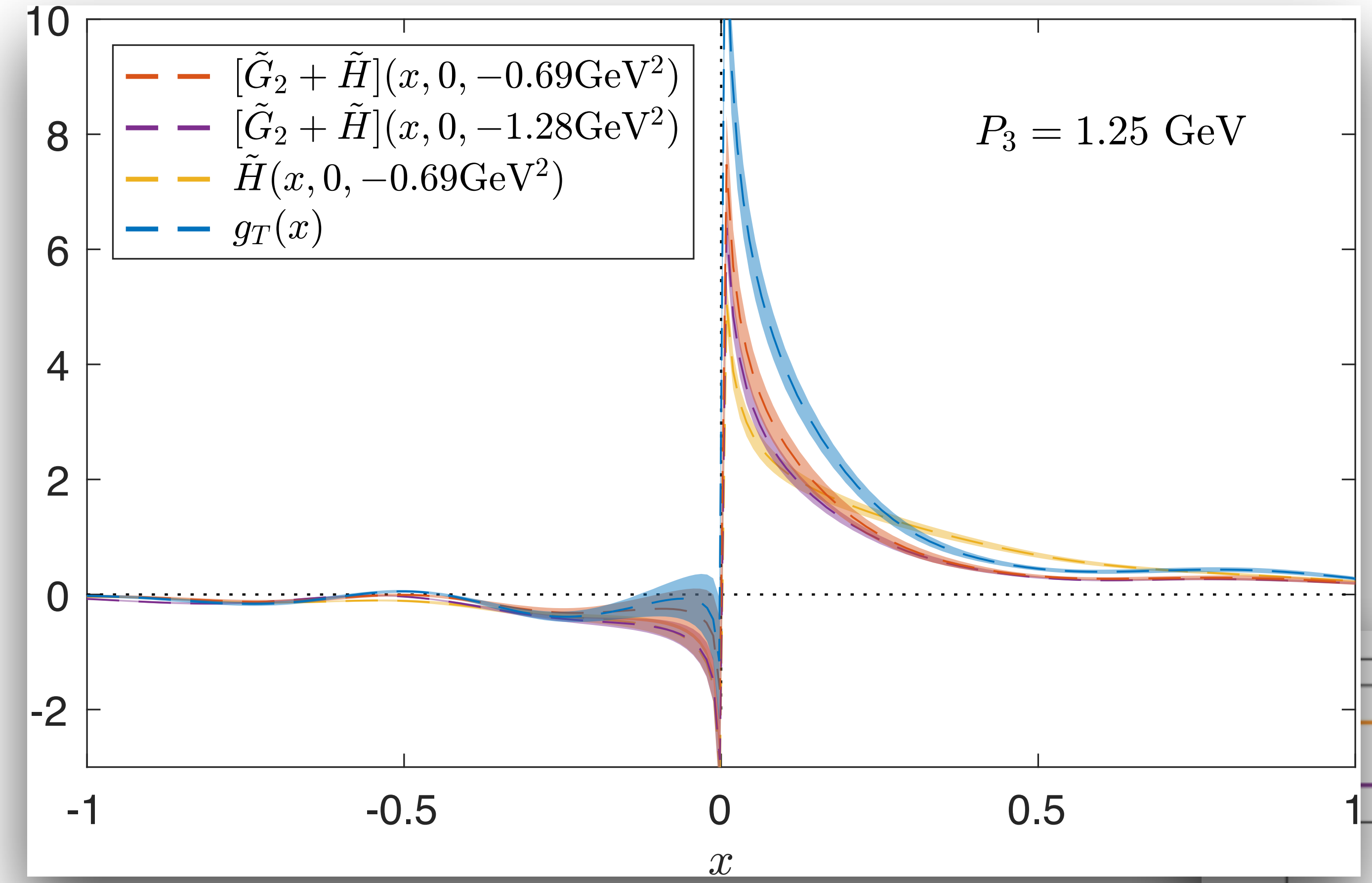
PI: M. Constantinou

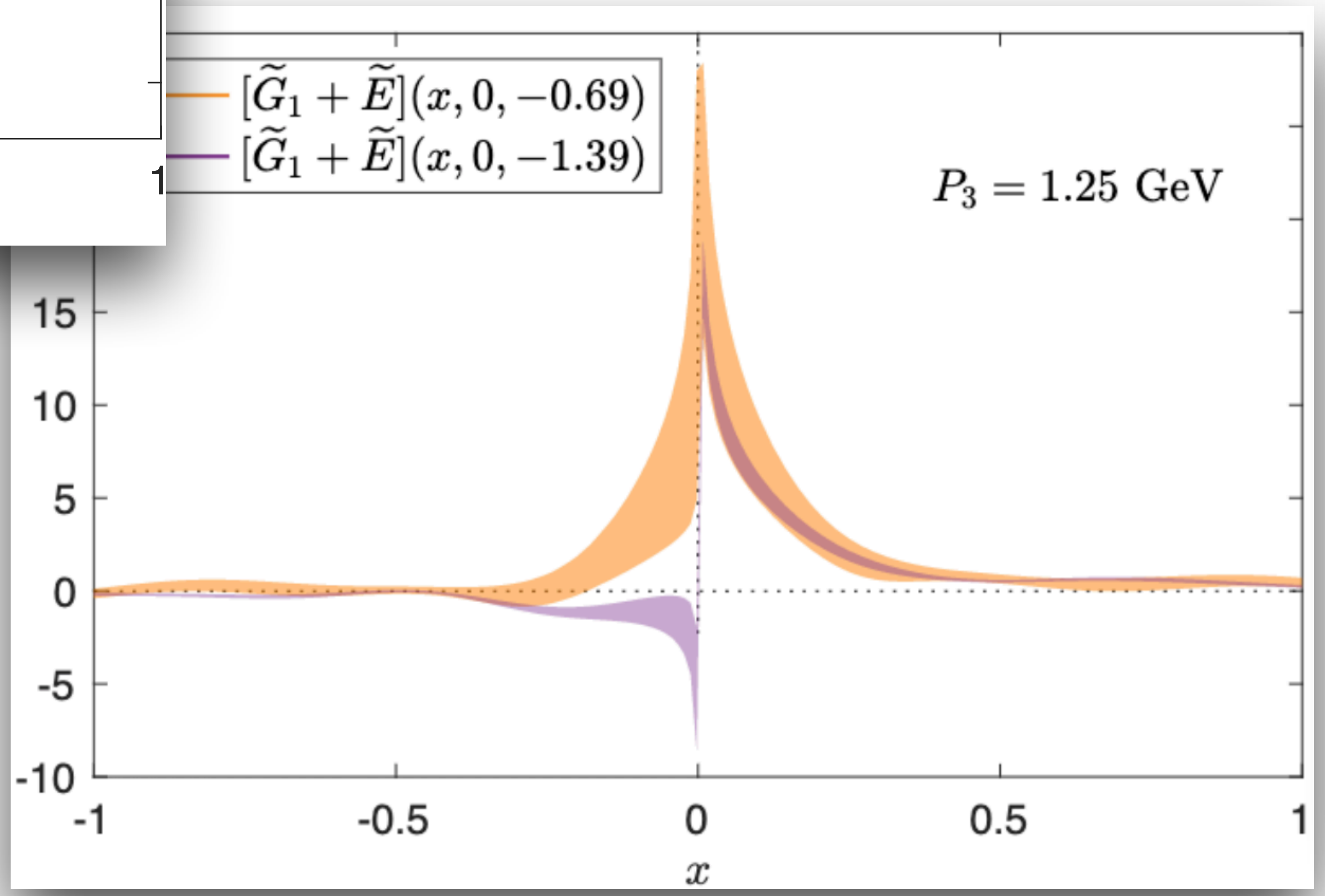
- ★ Lack density interpretation, but can be sizable $f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Understanding importance of q-g-q: WW approximation

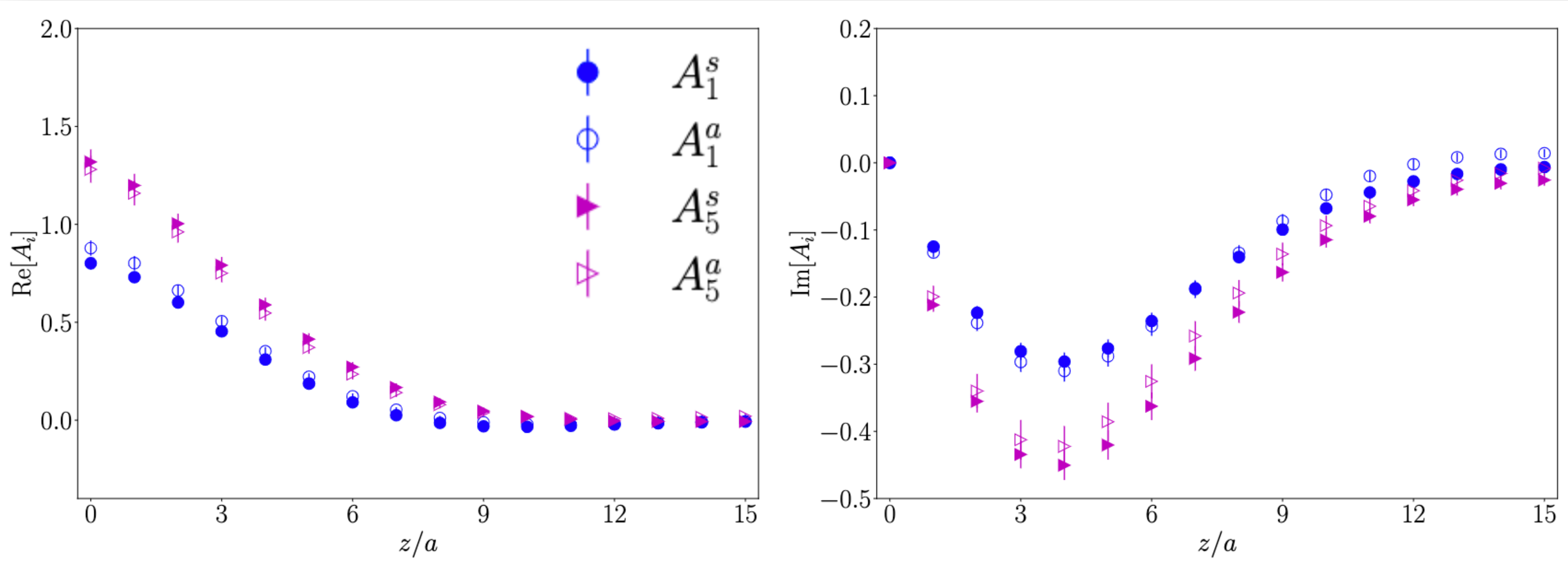
- ★ Proposal: GPDs,
  Nf=2+1+1 twisted mass fermions
  $\{m_\pi, a\} = \{250\,\text{MeV}, 0.08\,\text{fm}\}$
- ★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^\mu_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Main advantage • Applicable to any kinematic frame







[S. Bhattacharya et al., PoS LATTICE2021 (2022) 054 arXiv:2112.05538]

TMDs

PI: M. Wagman

2023-2024 proposal

★ Collins-Soper evolution kernel: (Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.09\,\mathrm{fm}$) relates transverse momentum-dependent parton distribution functions at different energy scales

Scale evolution

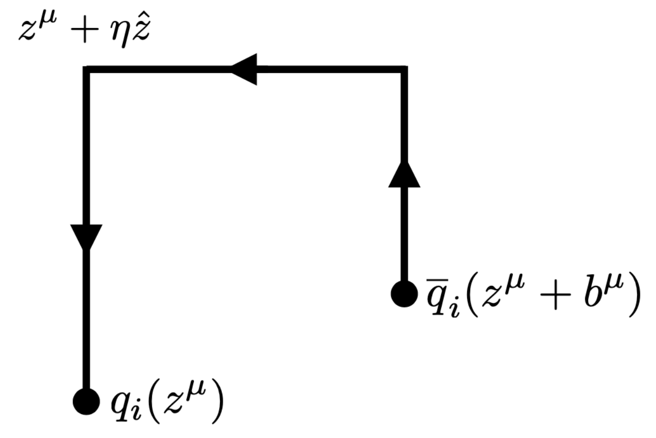
$$f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu, \zeta\right) = f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu_0, \zeta_0\right) \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i\left(\mu', \zeta_0\right)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i\left(\mu, b_T\right) \ln \frac{\zeta}{\zeta_0}\right]$$

b_T : transverse displacement

μ_0 : virtuality scale

ζ_0 : hadron mom. scale

Ratios of operators with staple-shape WL



TMDs

PI: M. Wagman

2023-2024 proposal

★ Collins-Soper evolution kernel: (Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.09\,\mathrm{fm}$) relates transverse momentum-dependent parton distribution functions at different energy scales

Scale evolution

$$f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu, \zeta\right) = f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu_0, \zeta_0\right) \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i\left(\mu', \zeta_0\right)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i\left(\mu, b_T\right) \ln\frac{\zeta}{\zeta_0}\right]$$

Perturbative

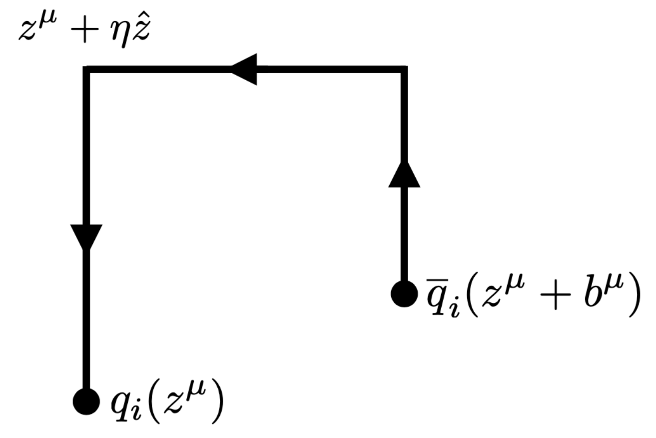
Non-pert (small parton transverse momentum): $1/b_T \sim \Lambda_{QCD}$

b_T : transverse displacement

μ_0 : virtuality scale

ζ_0 : hadron mom. scale

Ratios of operators with staple-shape WL



TMDs

PI: M. Wagman

2023-2024 proposal

★ **Collins-Soper evolution kernel: (Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.09\,\mathrm{fm}$)**
relates transverse momentum-dependent parton distribution functions at different energy scales

Scale evolution

$$f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu, \zeta\right) = f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu_0, \zeta_0\right) \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i\left(\mu', \zeta_0\right)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i\left(\mu, b_T\right) \ln\frac{\zeta}{\zeta_0}\right]$$

Perturbative

Non-pert (small parton transverse momentum): $1/b_T \sim \Lambda_{QCD}$

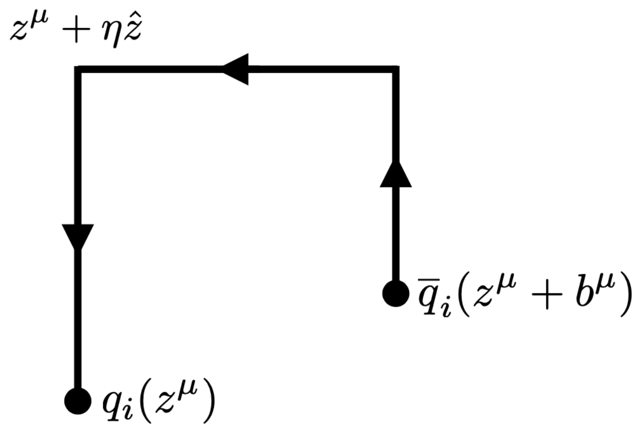
b_T : transverse displacement

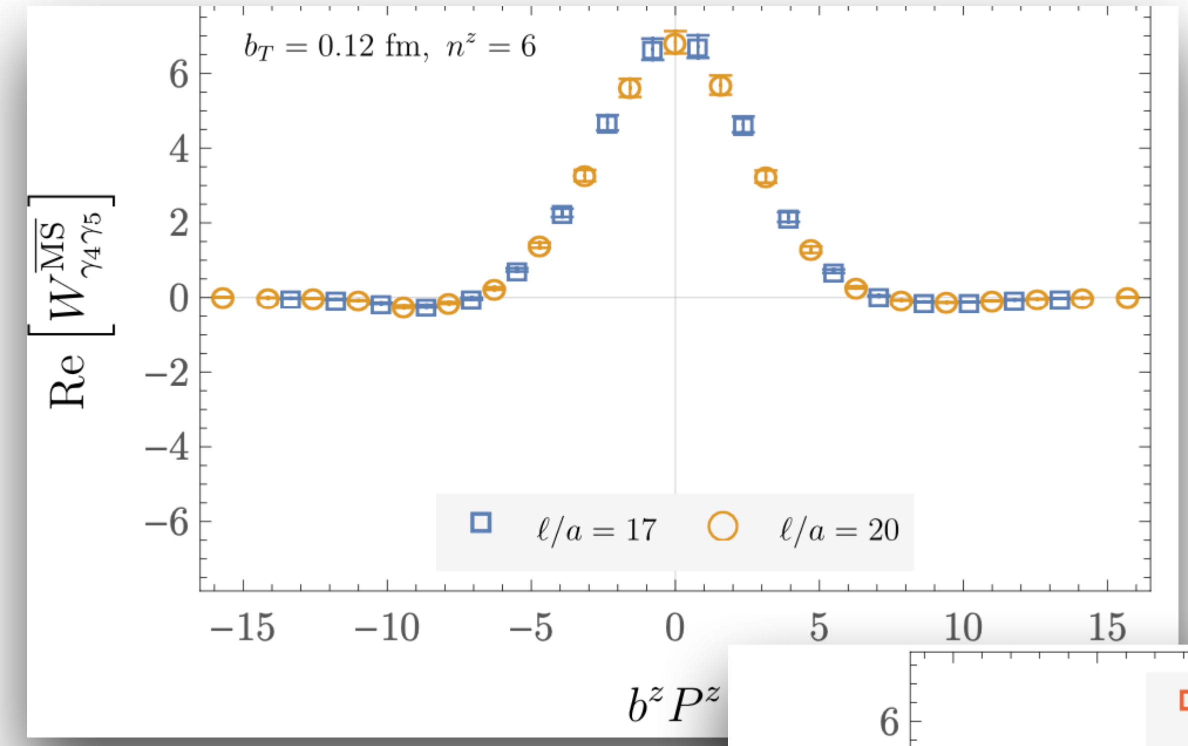
μ_0 : virtuality scale

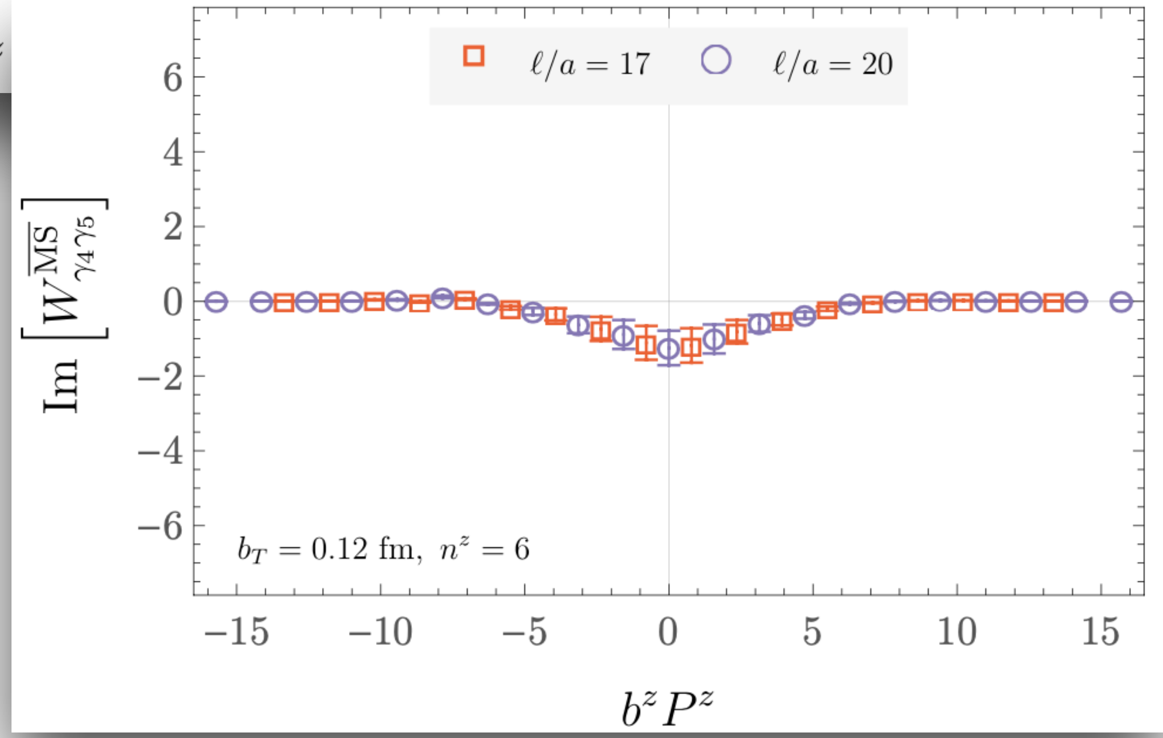
ζ_0 : hadron mom. scale

★ **2022: Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.12\,\mathrm{fm}$**
TMDWF via pion quasi TMDWFs with non-pert. renormalization and NLO matching ($b_T \lesssim 0.6$ fm)

Ratios of operators with staple-shape WL







quasi-wavefunction ratios

TMDs

PI: M. Wagman

2023-2024 proposal

★ **Collins-Soper evolution kernel: (Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.09\,\mathrm{fm}$)**
relates transverse momentum-dependent parton distribution functions at different energy scales

Scale evolution

$$f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu, \zeta\right) = f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu_0, \zeta_0\right) \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i\left(\mu', \zeta_0\right)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i\left(\mu, b_T\right) \ln\frac{\zeta}{\zeta_0}\right]$$

Perturbative

Non-pert (small parton transverse momentum): $1/b_T \sim \Lambda_{QCD}$

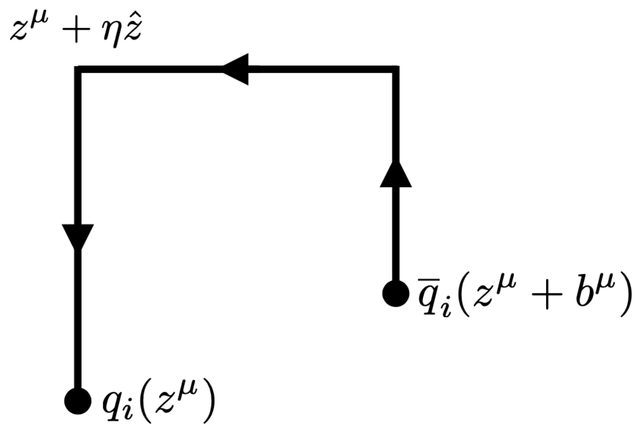
b_T : transverse displacement

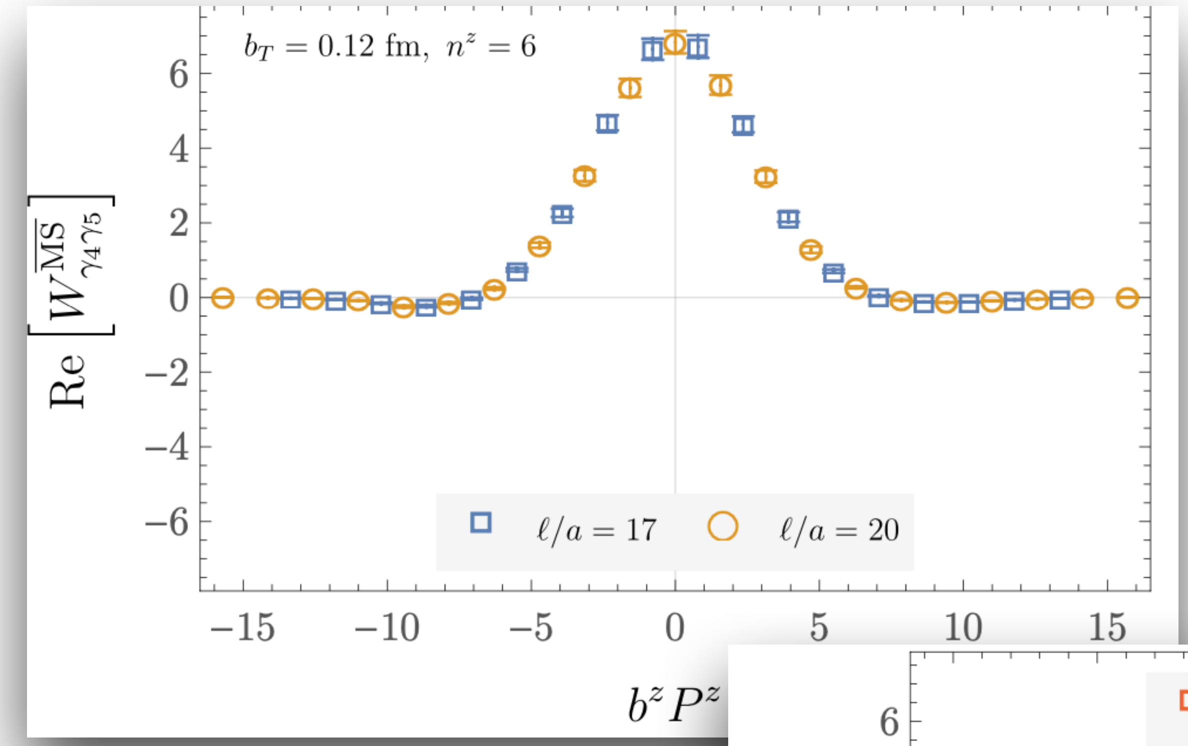
μ_0 : virtuality scale

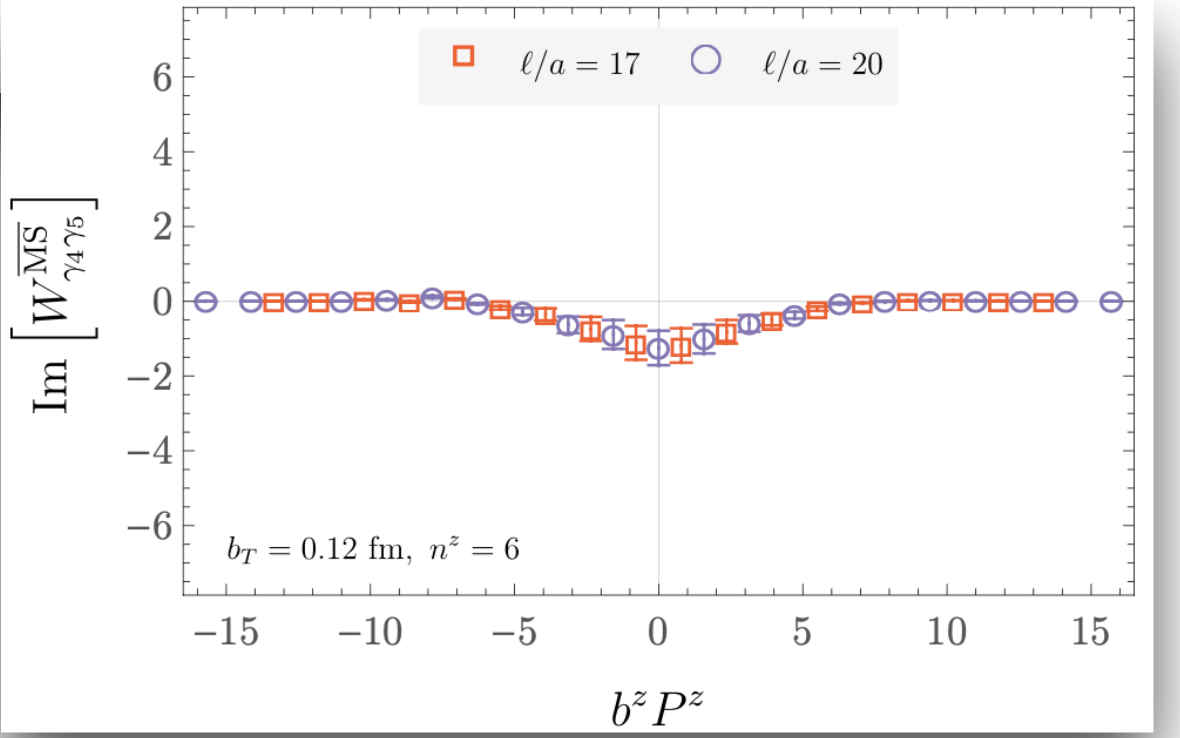
ζ_0 : hadron mom. scale

★ **2022: Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.12\,\mathrm{fm}$**
TMDWF via pion quasi TMDWFs with non-pert. renormalization and NLO matching ($b_T \lesssim 0.6$ fm)

Ratios of operators with staple-shape WL

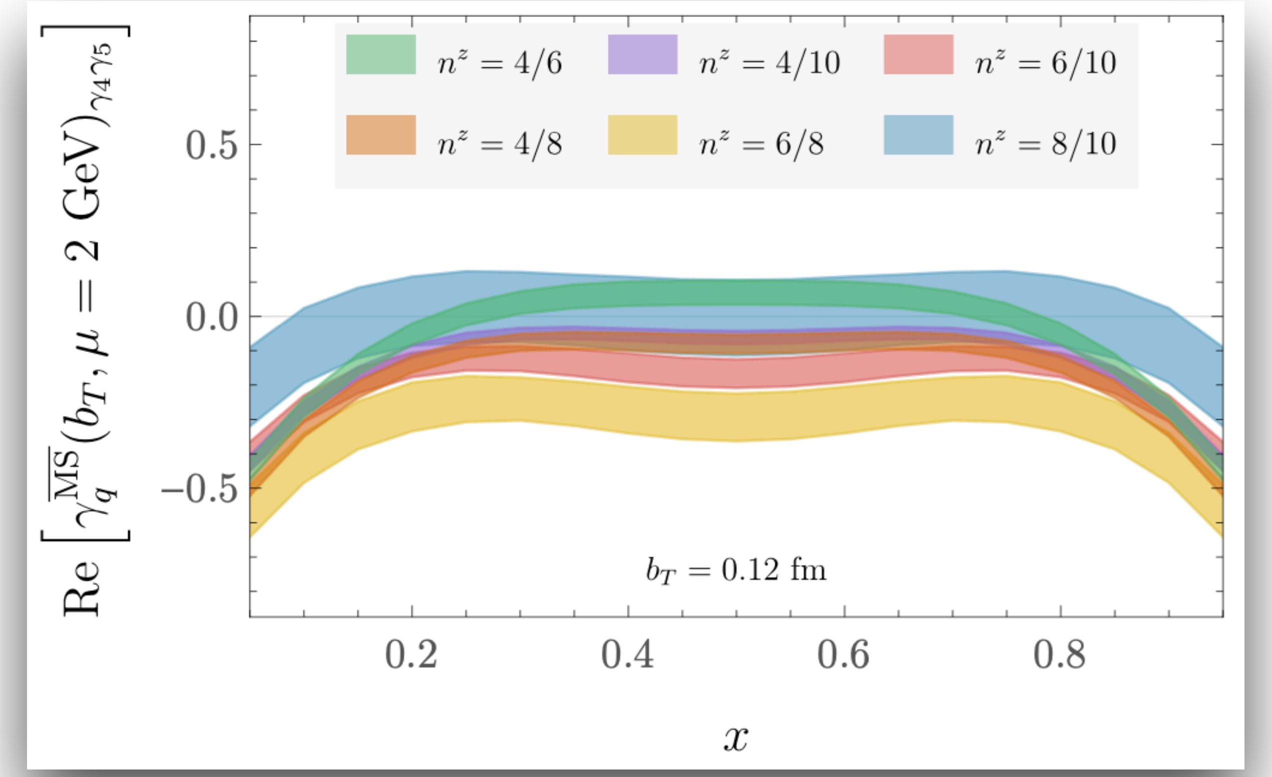






quasi-wavefunction ratios

matching in x space



TMDs

PI: M. Wagman

2023-2024 proposal

★ Collins-Soper evolution kernel: (Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.09\,\mathrm{fm}$) relates transverse momentum-dependent parton distribution functions at different energy scales

Scale evolution

$$f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu, \zeta\right) = f_i^{\mathrm{TMD}}\left(x, \vec{b}_T, \mu_0, \zeta_0\right) \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i\left(\mu', \zeta_0\right)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i\left(\mu, b_T\right) \ln\frac{\zeta}{\zeta_0}\right]$$

Perturbative (the first exponential)

Non-pert (small parton transverse momentum): $1/b_T \sim \Lambda_{QCD}$ (the second exponential)

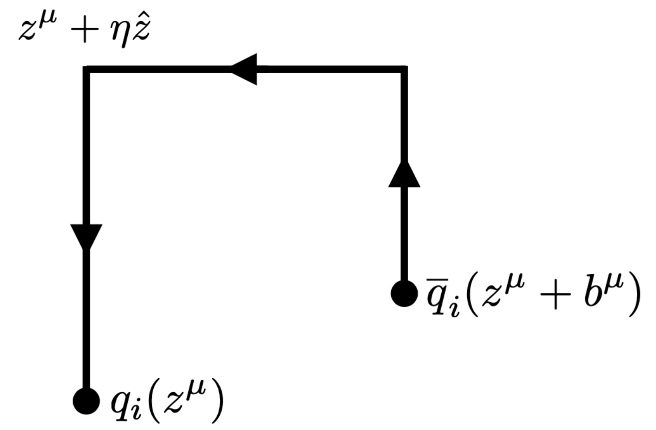
b_T : transverse displacement

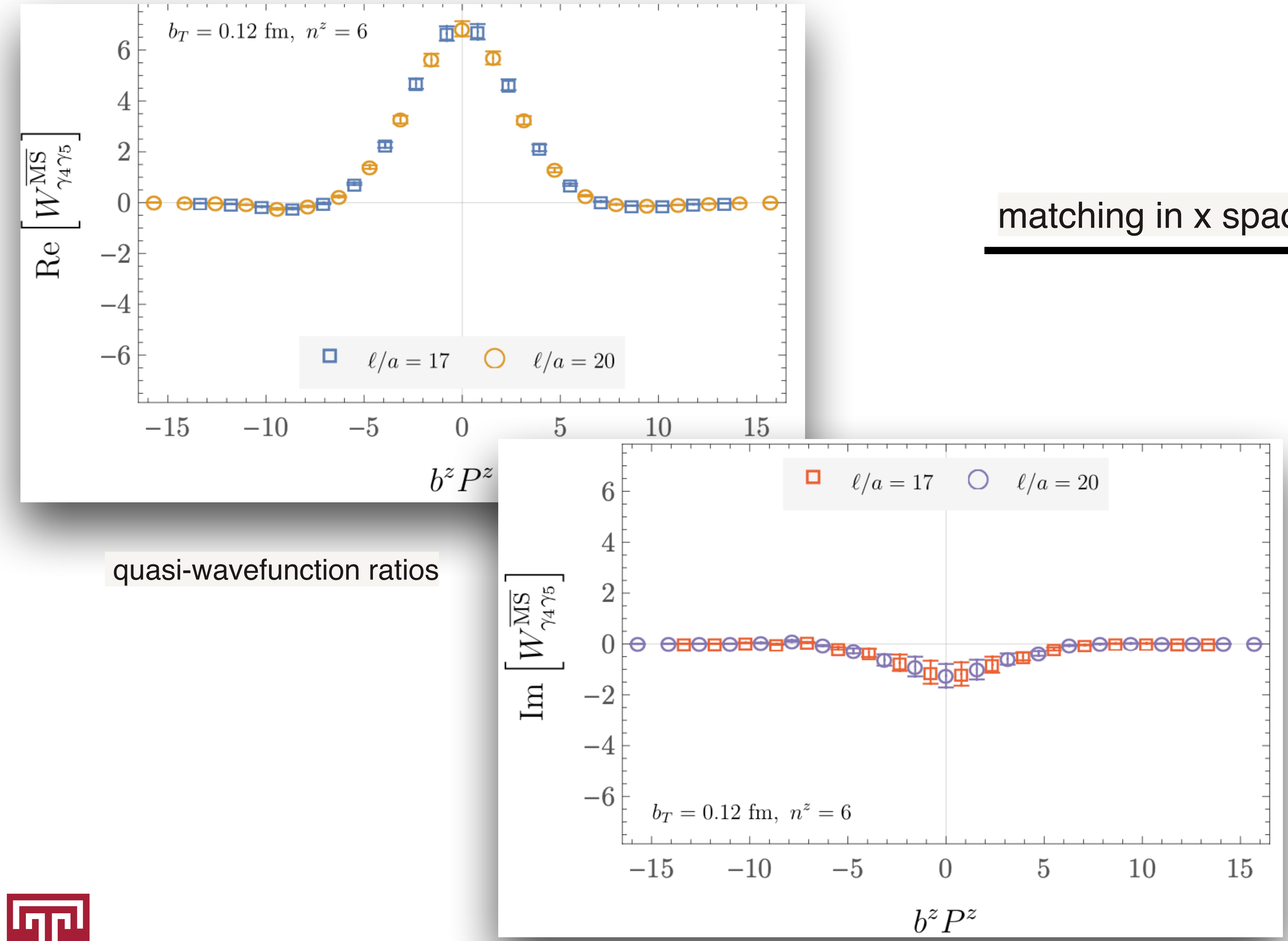
μ_0 : virtuality scale

ζ_0 : hadron mom. scale

★ 2022: Nf=2+1+1, HISQ, $m_\pi = 140\,\mathrm{MeV}, a = 0.12\,\mathrm{fm}$ TMDWF via pion quasi TMDWFs with non-pert. renormalization and NLO matching ($b_T \lesssim 0.6$ fm)

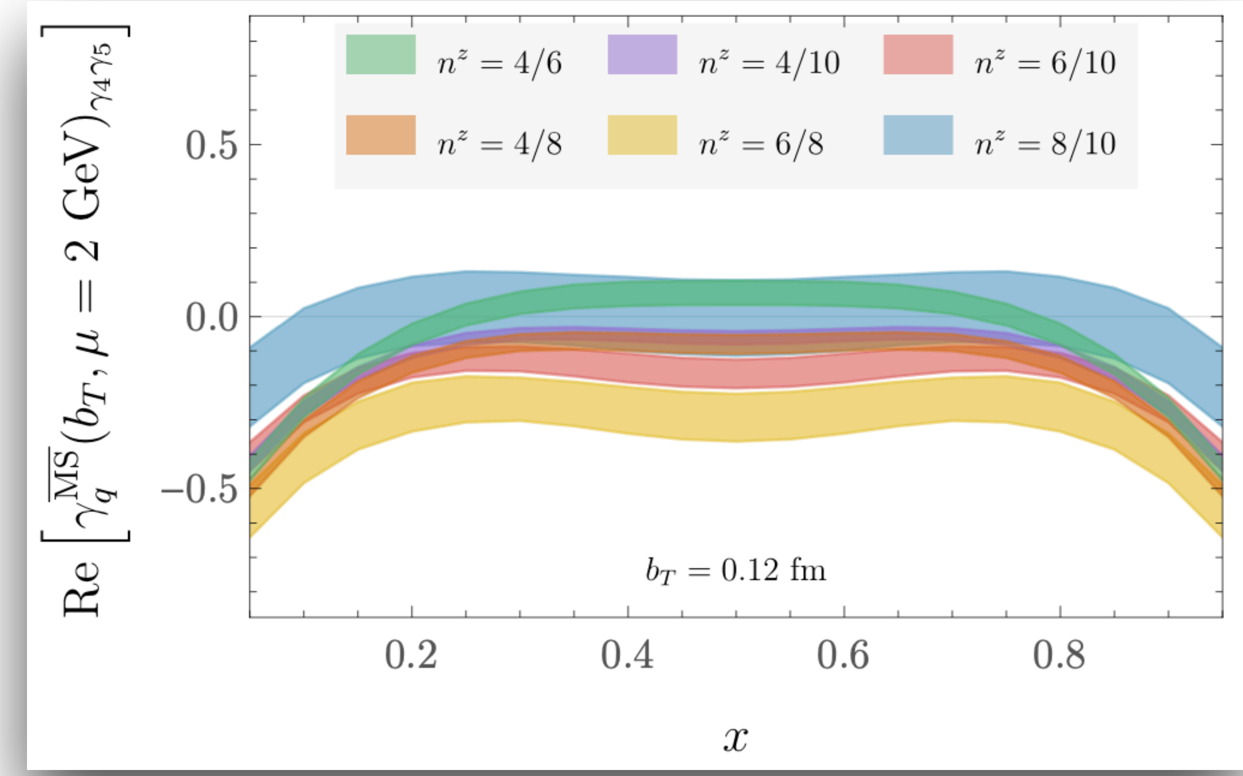
Ratios of operators with staple-shape WL



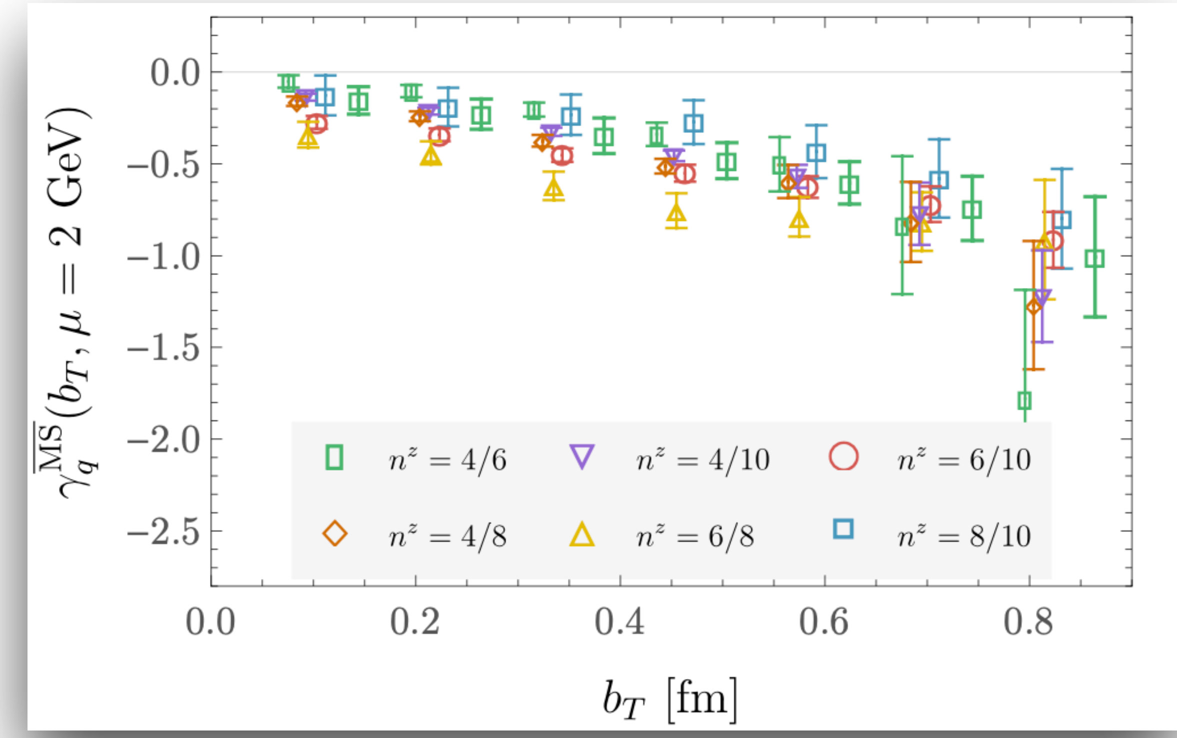


quasi-wavefunction ratios

matching in x space



NLO matching in b_T space



Hadron tensor of pion

PI: W. Jay

Junior investigator

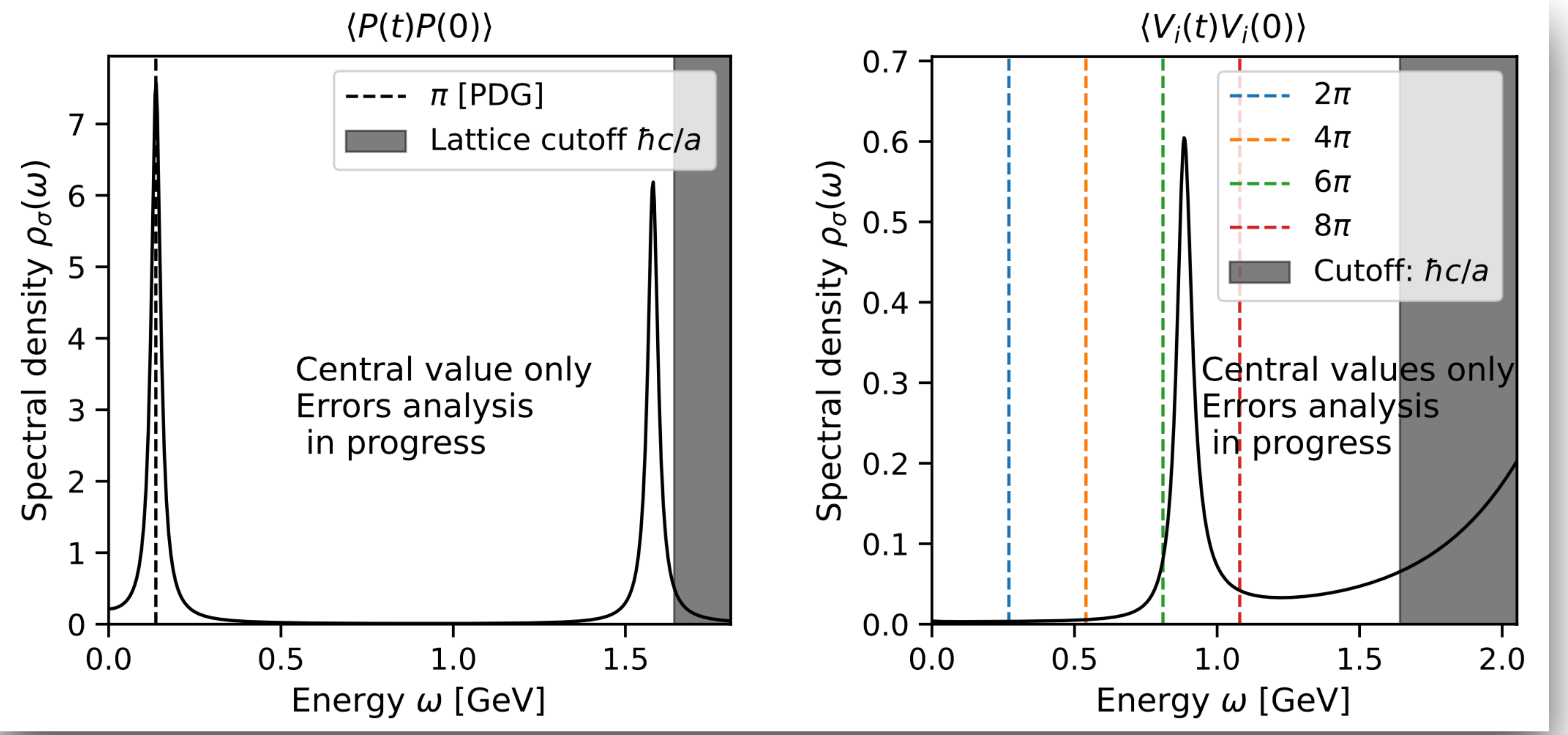
Talk @ 3:10 pm

- ★ 2023: Nf=2+1+1 HISQ fermions ($m_\pi = 135\,\mathrm{MeV}, a = 0.12, 0.15\,\mathrm{fm}$)
- ★ Calculation of 4-pt functions: computationally challenging, but no need for high momentum boost or matching formalism $W_{\mu\nu}(p,q) \propto \int d^4x e^{iqx} \langle \pi, \boldsymbol{p}| J_\mu^{\mathrm{EM}}(x) J_\nu^{\mathrm{EM}}(0) |\pi, \boldsymbol{p}\rangle$
- ★ Scalar and vector cases
- ★ Study of reconstruction methods of the physical HT from Euclidean HT: Laplace transform

Hadron tensor of pion

PI: W. Jay
Junior investigator
Talk @ 3:10 pm

- ★ 2023: Nf=2+1+1 HISQ fermions ($m_\pi = 135\,\mathrm{MeV}, a = 0.12, 0.15\,\mathrm{fm}$)
- ★ Calculation of 4-pt functions: computationally challenging, but no need for high momentum boost or matching formalism $W_{\mu\nu}(p,q) \propto \int d^4x e^{iqx} \langle \pi, \boldsymbol{p}| J^{\mathrm{EM}}_\mu(x) J^{\mathrm{EM}}_\nu(0) |\pi, \boldsymbol{p}\rangle$
- ★ Scalar and vector cases
- ★ Study of reconstruction methods of the physical HT from Euclidean HT: Laplace transform

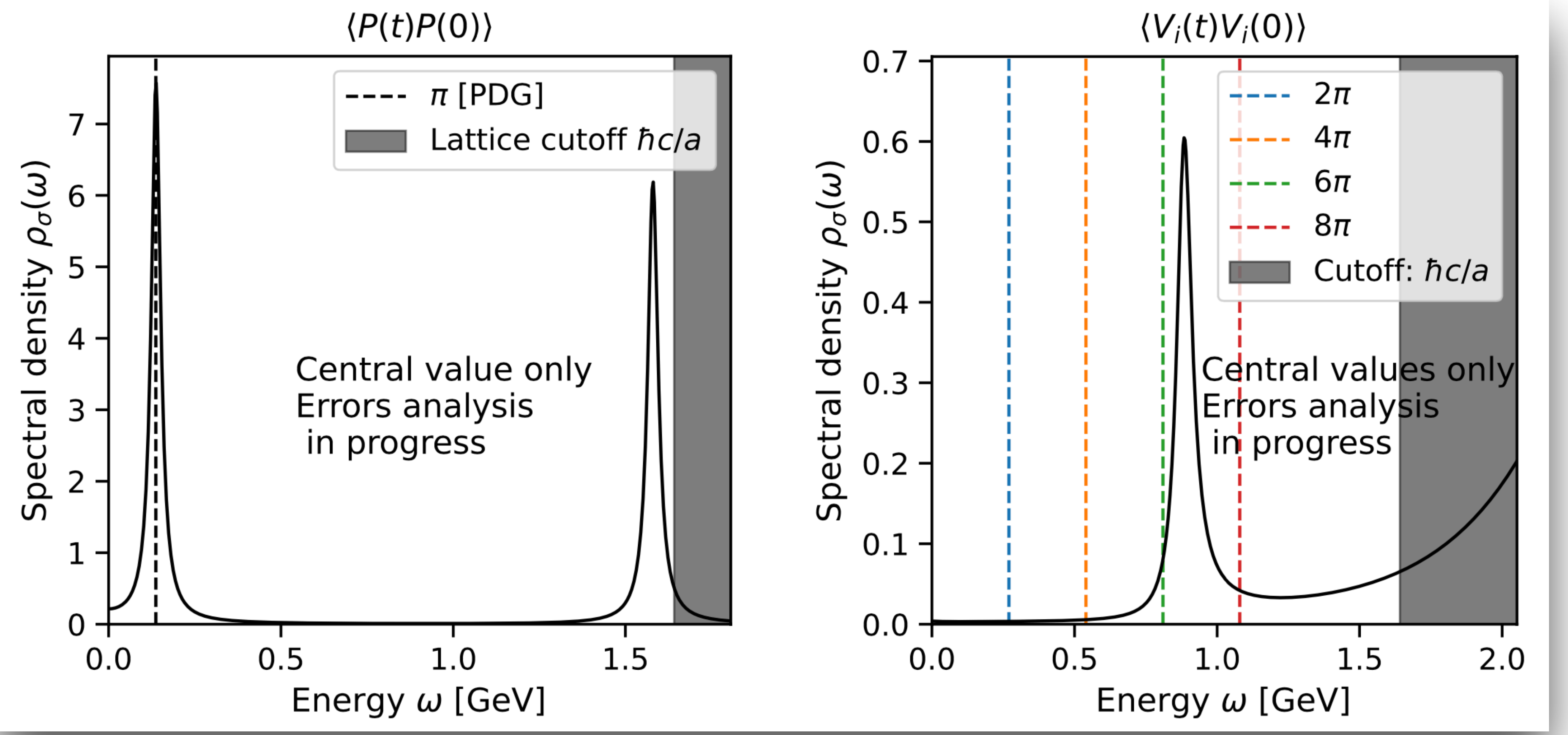


Finite-volume spectral densities ($a = 0.12\,\mathrm{fm}$) associated with scalar and vector 2pt functions. Reconstruction using a novel procedure developed by group. Peaks compatible to ϱ-meson around 770 MeV near the location of the lightest vector resonances.

Hadron tensor of pion

PI: W. Jay
Junior investigator
Talk @ 3:10 pm

- ★ 2023: Nf=2+1+1 HISQ fermions ($m_\pi = 135\,\mathrm{MeV}, a = 0.12, 0.15\,\mathrm{fm}$)
- ★ Calculation of 4-pt functions: computationally challenging, but no need for high momentum boost or matching formalism $W_{\mu\nu}(p,q) \propto \int d^4x e^{iqx} \langle \pi, \boldsymbol{p}| J_\mu^{\mathrm{EM}}(x) J_\nu^{\mathrm{EM}}(0) |\pi, \boldsymbol{p}\rangle$
- ★ Scalar and vector cases
- ★ Study of reconstruction methods of the physical HT from Euclidean HT: Laplace transform



Finite-volume spectral densities ($a = 0.12\,\mathrm{fm}$) associated with scalar and vector 2pt functions. Reconstruction using a novel procedure developed by group. Peaks compatible to ϱ-meson around 770 MeV near the location of the lightest vector resonances.

- ★ Focus: quantify systematics related to finite-volume that may be leading the uncertainties
- ★ Long-term plan: investigate other systems, e.g., proton

Session Schedule

Time	Talk	Speaker / Slot
	SPC Report	*Martha Constantinou* 14:40 - 14:55
15:00	**QCD trace anomaly form factors**	*Bigeng Wang* 14:55 - 15:10
	Hadronic Tensor of the pion	*William Jay* 15:10 - 15:25
	Kaon form factors	*Dr Xiang Gao* 15:25 - 15:40
	Discussion	*Martha Constantinou* 15:40 - 15:55