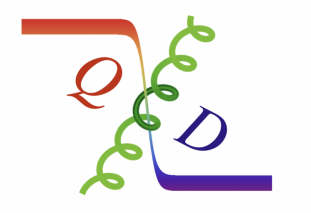
Gluon trace anomaly form factors of the energy-momentum tensor

Bigeng Wang

(χQCD collaboration)
Department of Physics and Astronomy
University of Kentucky

USQCD All-hands Meeting
April 20–21, 2023



Pion Mass Puzzle

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \tag{1}$$

- From the forward matrix element of the EMT $\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$, pion mass can be obtained from the trace of the EMT:

$$m_\pi = \underbrace{\frac{\langle\pi|\int d^3\vec{x}\gamma[\frac{\beta(g)}{2g}G^2 + \sum_f \gamma_m(g)m_f\bar{\psi}_f\psi_f]|\pi\rangle}{\langle\pi|\pi\rangle}}_{\text{conformal symmetry breaking}\leftrightarrow\text{trace anomaly ME}} + \underbrace{\frac{\langle\pi|\int d^3\vec{x}\gamma\sum_f m_f\bar{\psi}_f\psi_f|\pi\rangle}{\langle\pi|\pi\rangle}}_{\sigma\,term,\,\frac{1}{2}m_\pi\propto\sqrt{m_q}\text{[1]}} \tag{2}$$

- 1st order in the chiral symmetry breaking:

$$m_\pi \propto \sqrt{m_q}, \quad for \quad m_q = m_u = m_d \tag{3}$$

[1] Based on the Gellmann-Oakes-Renner relation and the Feynman-Hellman theorem.

Pion Mass Puzzle

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \tag{1}$$

- From the forward matrix element of the EMT $\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$, pion mass can be obtained from the trace of the EMT:

$$m_\pi = \underbrace{\frac{\langle\pi|\int d^3\vec{x}\gamma[\frac{\beta(g)}{2g}G^2 + \sum_f \gamma_m(g)m_f\bar{\psi}_f\psi_f]|\pi\rangle}{\langle\pi|\pi\rangle}}_{\text{conformal symmetry breaking}\leftrightarrow\text{trace anomaly ME}} + \underbrace{\frac{\langle\pi|\int d^3\vec{x}\gamma\sum_f m_f\bar{\psi}_f\psi_f|\pi\rangle}{\langle\pi|\pi\rangle}}_{\sigma\,term,\,\frac{1}{2}m_\pi\propto\sqrt{m_q}\text{[1]}} \tag{2}$$

- 1st order in the chiral symmetry breaking:

$$m_\pi \propto \sqrt{m_q}, \quad for \quad m_q = m_u = m_d \tag{3}$$

- **How** does the **conformal** symmetry breaking correlate with the **chiral** symmetry breaking?

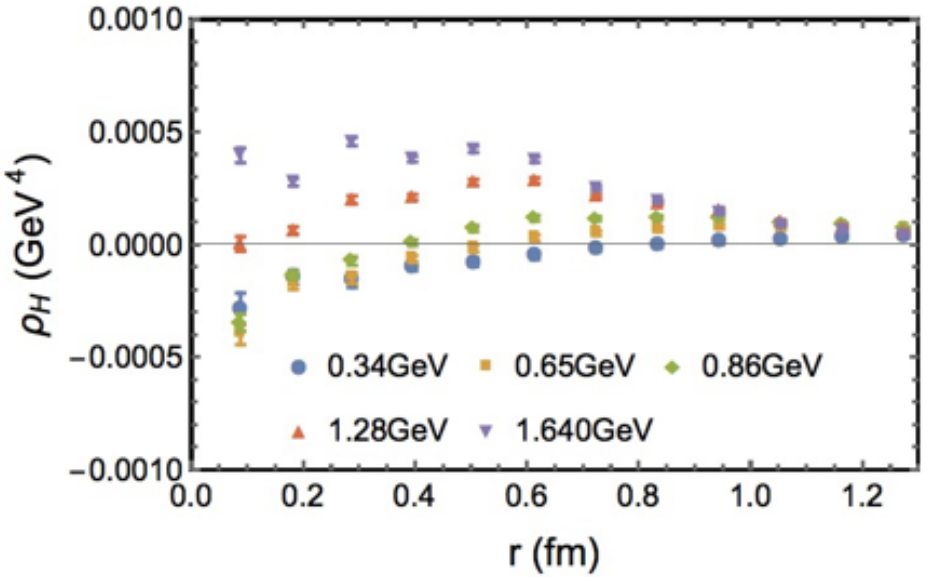
[1] Based on the Gellmann-Oakes-Renner relation and the Feynman-Hellman theorem.

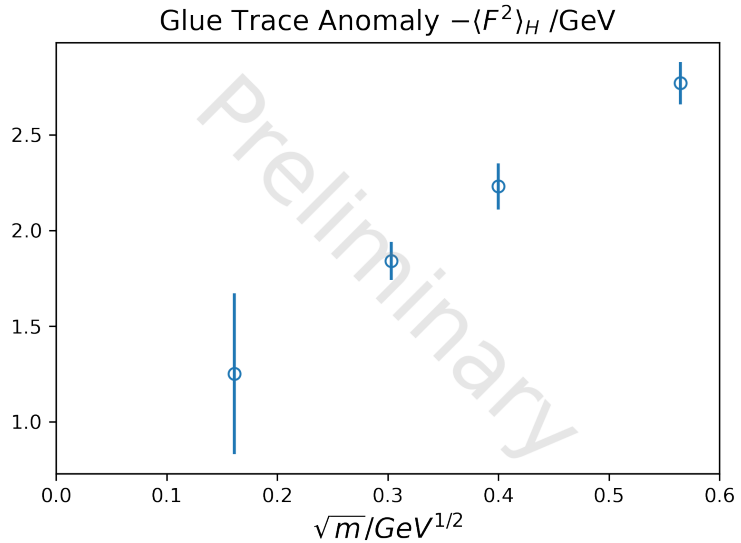
Pion Mass Puzzle

- **How** does the **conformal** symmetry breaking correlate with the **chiral** symmetry breaking?
- **How** does the gluon trace anomaly matrix element keep itself proportional to $\sqrt{m_q}$ as $m_q \to 0$?

  F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

  As $m_q \to 0$, the **density function** **changes sign** and the sum approximately is proportional to $\sqrt{m_q}$



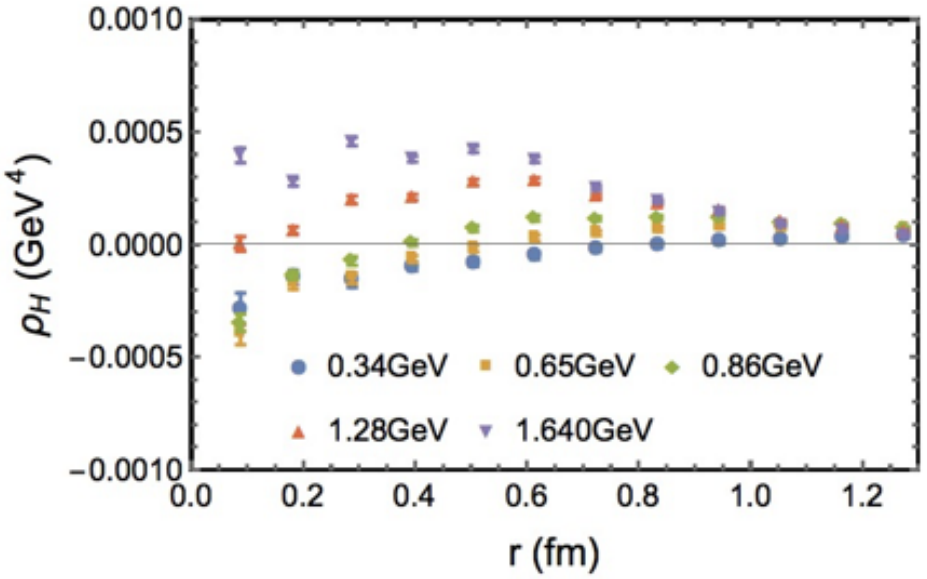


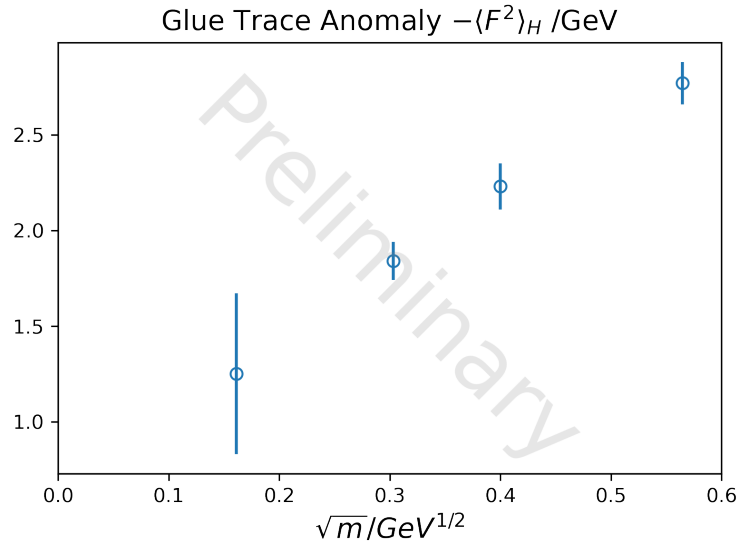
Pion Mass Puzzle

- **How** does the **conformal** symmetry breaking correlate with the **chiral** symmetry breaking?
- **How** does the gluon trace anomaly matrix element keep itself proportional to $\sqrt{m_q}$ as $m_q \to 0$?

  F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

  As $m_q \to 0$, the **density function** **changes sign** and the sum approximately is proportional to $\sqrt{m_q}$





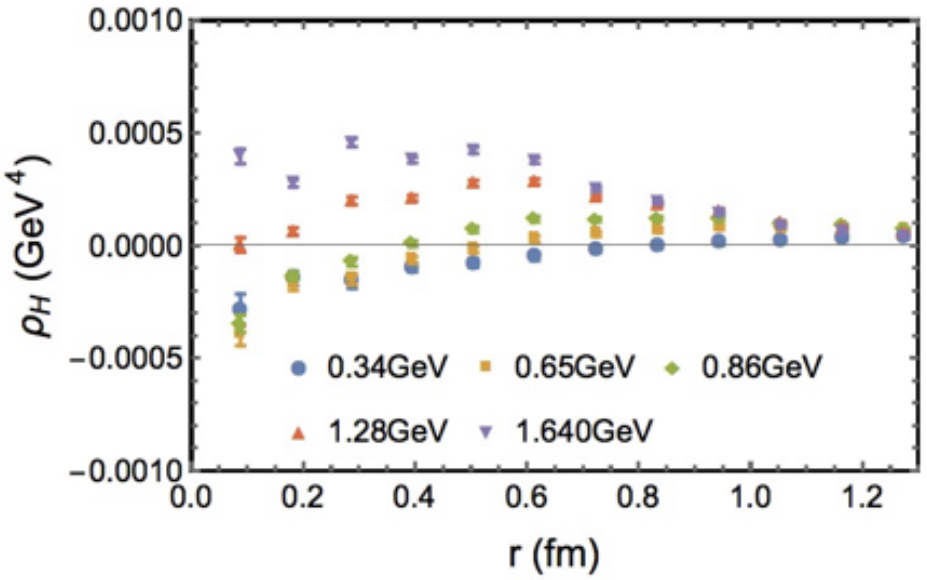
- This is a clear direct evidence that the **conformal** (scale) symmetry breaking in the **pion** is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600

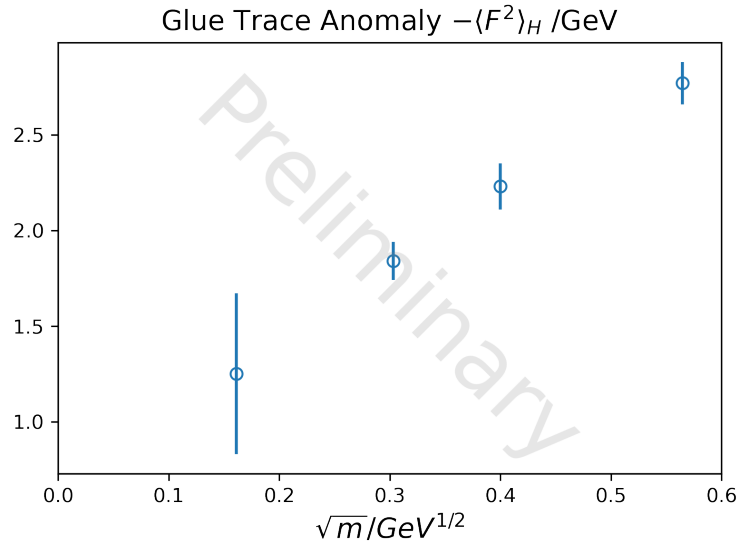
Pion Mass Puzzle

- **How** does the **conformal** symmetry breaking correlate with the **chiral** symmetry breaking?
- **How** does the gluon trace anomaly matrix element keep itself proportional to $\sqrt{m_q}$ as $m_q \to 0$?

  F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

  As $m_q \to 0$, the **density function** **changes sign** and the sum approximately is proportional to $\sqrt{m_q}$





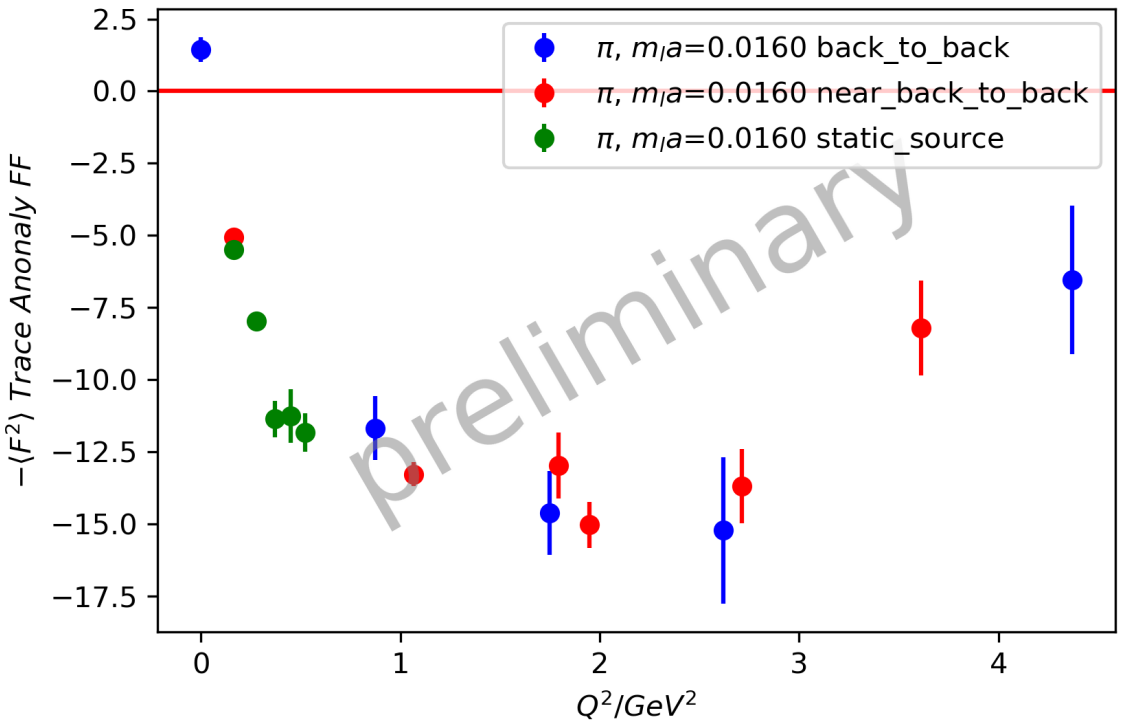
- This is a clear direct evidence that the **conformal** (scale) symmetry breaking in the **pion** is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600
- Will the **form factor** $G_{\rm H}(t) = \langle p_f|O_g(q)|p_i\rangle/m_{\rm H}$ **changes sign** as well?

Results using grid-source propagators and LMS preliminary

- Overlap fermions on DWF at near-physical pion mass:

	Ensemble	$L^3 \times T$	a (fm)	L(fm)	m_π (MeV)	N_{conf}	N_{src}
1	24I	$24^3 \times 64$	0.1105(3)	2.65	340	794	16

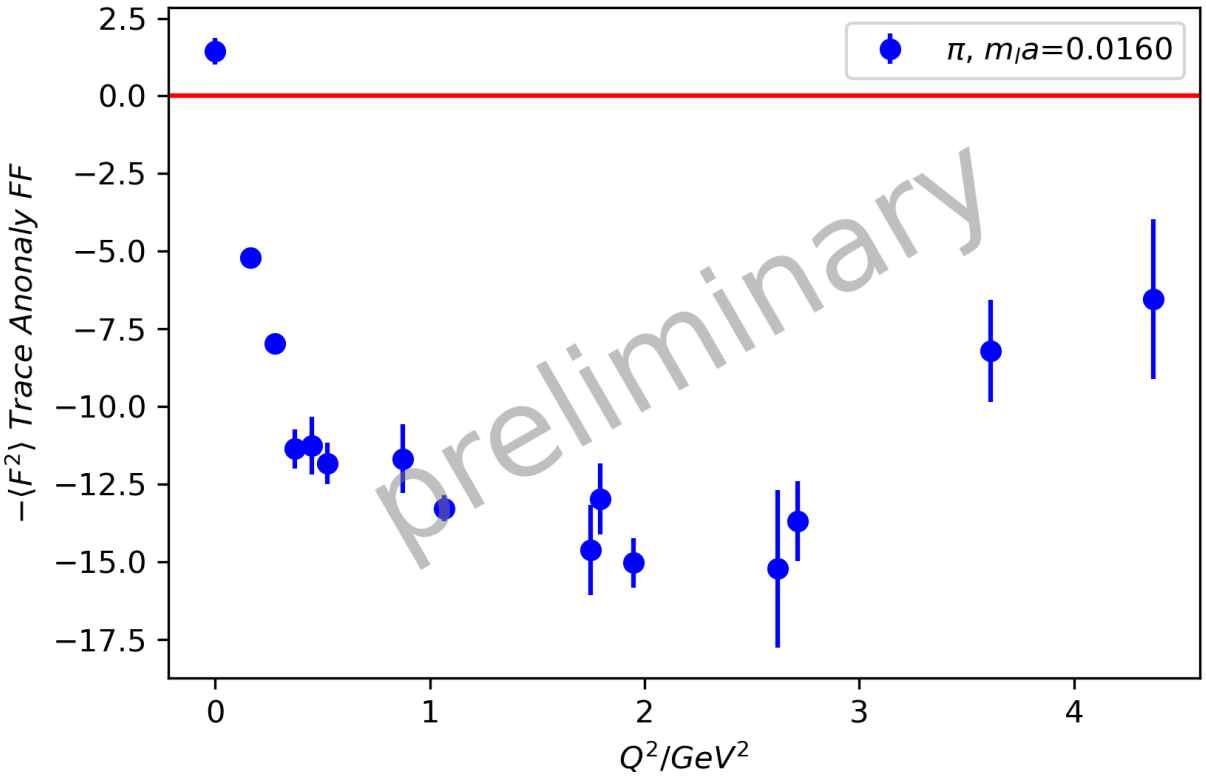
- Supplemented by three different momentum transfer scenarios, each has 225 source-sink momentum combinations (same Q^2 averaged)



Three cases of momentum transfer:

1. $|\vec{p}_{\text{i}}| = 0$ with $\vec{q} = \vec{p}_{\text{f}}$ or $|\vec{p}_{\text{f}}| = 0$ with $\vec{q} = -\vec{p}_{\text{i}}$
2. the back-to-back case: $\vec{p}_{\text{f}} = -\vec{p}_{\text{i}}$ with $\vec{q} = 2\vec{p}_{\text{f}}$
3. the near-back-to-back case: for a given $\vec{q}$, $\vec{p}_{\text{f}}$ and $-\vec{p}_{\text{i}}$ are close to $\vec{q}/2$.

Results using grid-source propagators and LMS preliminary



- **positive** at $Q^2 = 0\,\mathrm{GeV}^2$(contribution to the hadron mass)
- **sign change** of gluon trace anomaly form factors for **pion**, consistent with the density calculation results
- form factor calculated up to $Q^2 \sim 4\,\mathrm{GeV}^2$

Summary and Outlook

1. pion mass puzzle (motivation):
   - trace anomaly matrix element is proportional to $\sqrt{m_q}$ as $m_q \to 0$ and gluon trace anomaly has a **sign change** in the spatial distribution(density) to achieve this.
   - a clear direct evidence that the **conformal** (scale) symmetry breaking in the pion is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600

Summary and Outlook

1. pion mass puzzle (motivation):
   - trace anomaly matrix element is proportional to $\sqrt{m_q}$ as $m_q \to 0$ and gluon trace anomaly has a **sign change** in the spatial distribution(density) to achieve this.
   - a clear direct evidence that the **conformal** (scale) symmetry breaking in the pion is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600
2. gluon trace anomaly form factors of the EMT(**preliminary**):
   - consistent with hadron mass results at $Q^2 = 0\,\mathrm{GeV}^2$.
   - **sign change** of gluon trace anomaly form factors for **pion**
   - to be confirmed in experiments: J/Ψ production at the threshold of photoreaction, etc.

Summary and Outlook

1. pion mass puzzle (motivation):
   - trace anomaly matrix element is proportional to $\sqrt{m_q}$ as $m_q \to 0$ and gluon trace anomaly has a **sign change** in the spatial distribution(density) to achieve this.
   - a clear direct evidence that the **conformal** (scale) symmetry breaking in the pion is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600
2. gluon trace anomaly form factors of the EMT(**preliminary**):
   - consistent with hadron mass results at $Q^2 = 0\,\mathrm{GeV}^2$.
   - **sign change** of gluon trace anomaly form factors for **pion**
   - to be confirmed in experiments: J/Ψ production at the threshold of photoreaction, etc.

- We expect the calculation on the 48I ensemble will give a prediction of the gluon trace anomaly form factors at physical pion mass.

Thanks for your attention!