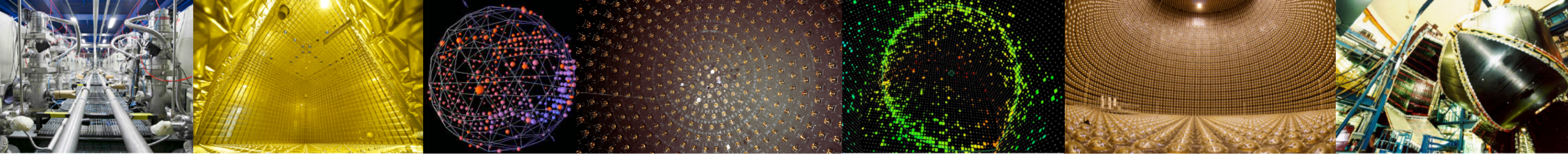
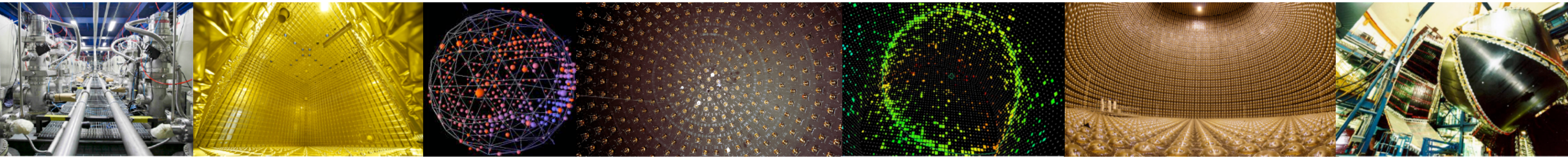


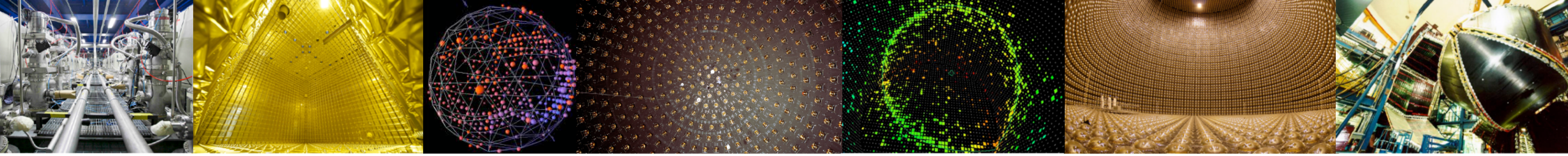
Lattice Calculation of the
Hadron Tensor of the Pion

William I. Jay - MIT

21 April 2023 — USQCD All-Hands Meeting

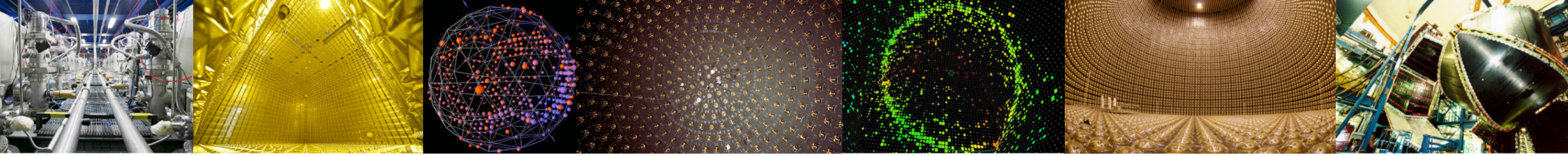
Work with:
Tom Blum (UConn), Luchang Jin (UConn),
Andreas Kronfeld (FNAL), Doug Stewart (UConn)





Outline

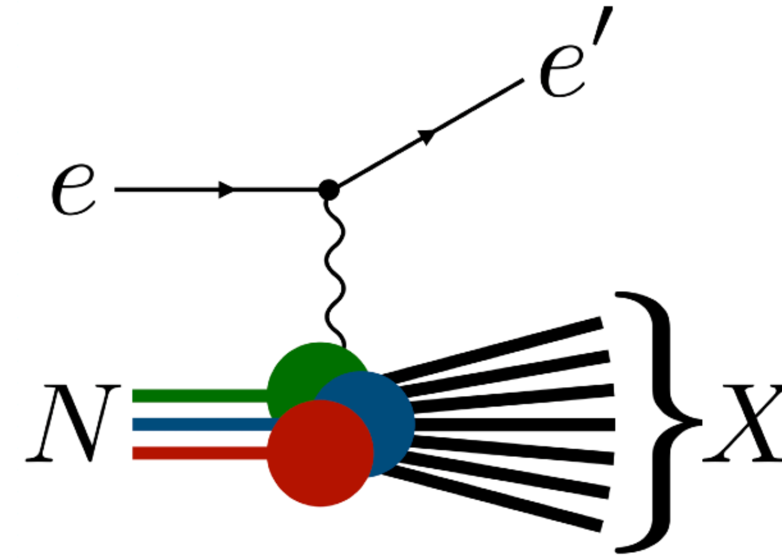
- Motivation
- Challenges
  - Computing a Euclidean 4pt function
  - Solving an inverse problem
- A few preliminary spectral reconstructions
- Outlook



Motivation

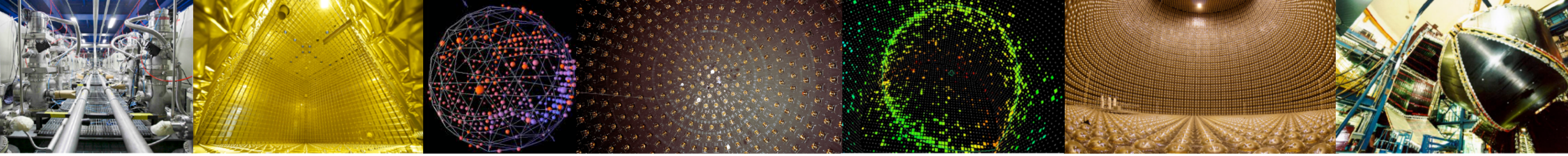
Inclusive eN scattering

Defined experimentally in terms of outgoing electron’s energy and scattering angle



$\Longleftrightarrow$ Q^2 and $x=Q^2/2m\nu$ (“momentum fraction”)

- Deep inelastic limit: $Q^2 \to \infty$ with fixed x
- Scaling of structure functions gave classic test of pQCD



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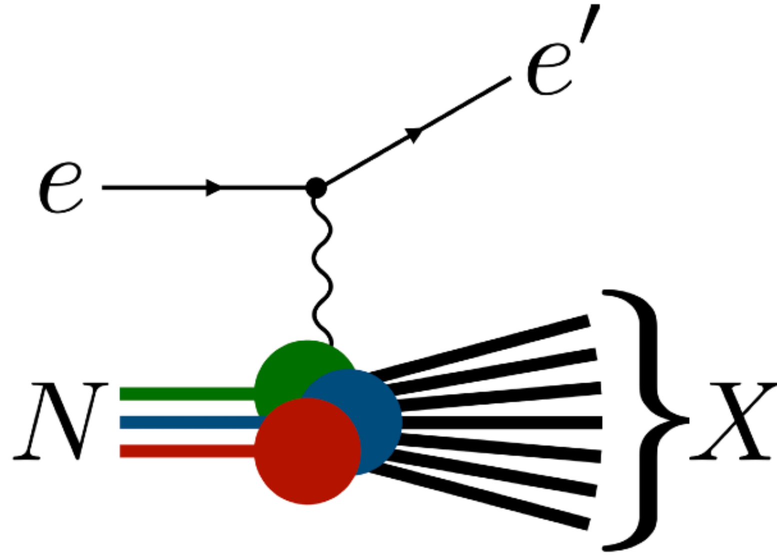
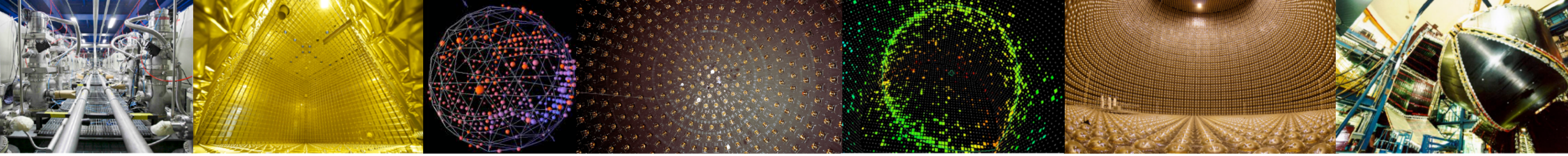


Defined experimentally in terms of outgoing electron's energy and scattering angle

$\Longleftrightarrow$ Q^2 and W (final hadronic invariant mass)

- Shallow inelastic region: W $\gtrsim$ Δ(1232), $Q^2 \approx$ few GeV
- Many open channels, rich resonant structure
- Essentially no first-principles understanding from QCD in this region
- Experimentally relevant for accelerator-based neutrino program (e.g., DUNE). Knowledge of axial-current resonant structure very poorly known. Important input for neutrino event generators (like the theory-driven generator Achilles)
- Connections exist to the EIC program



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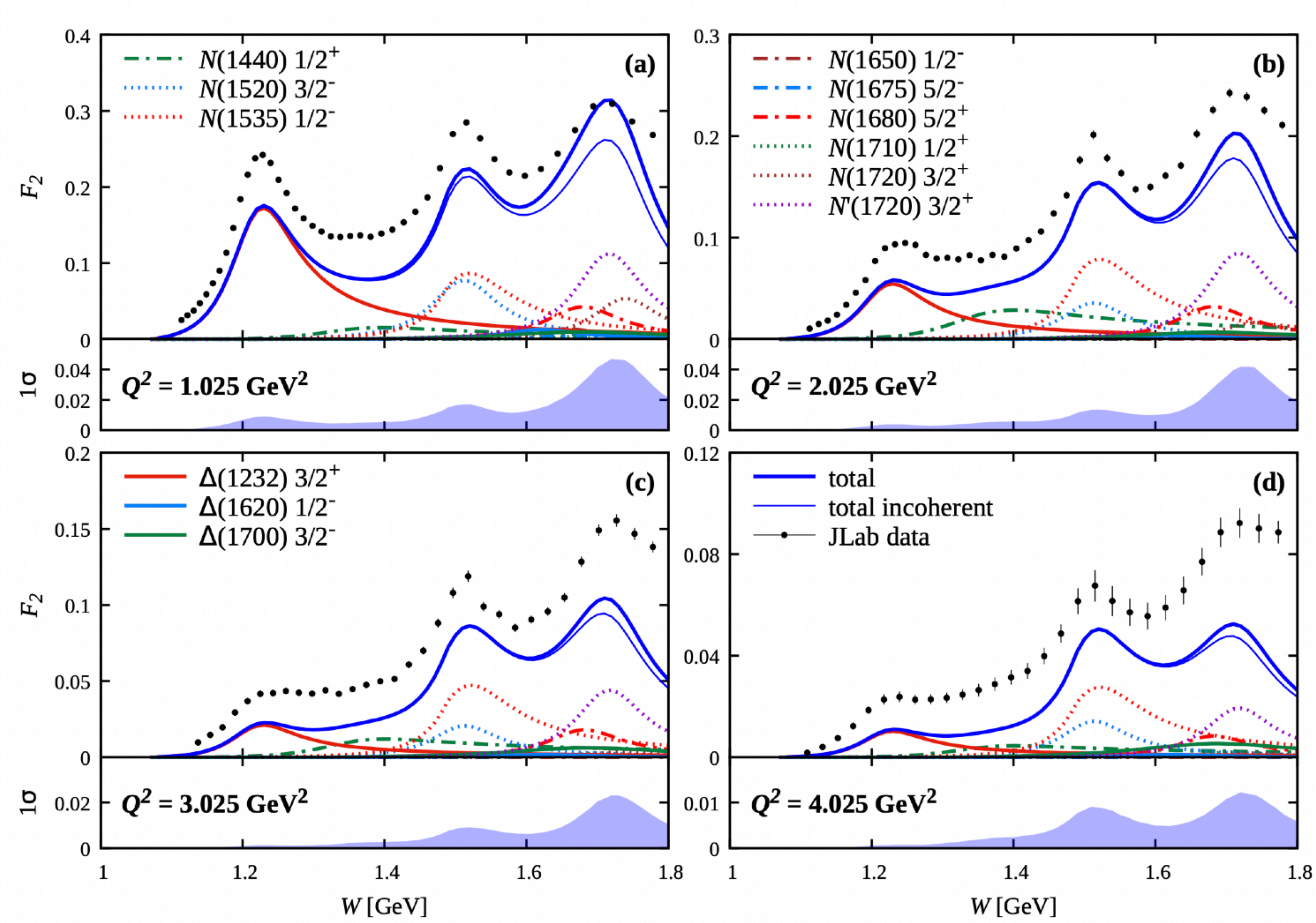
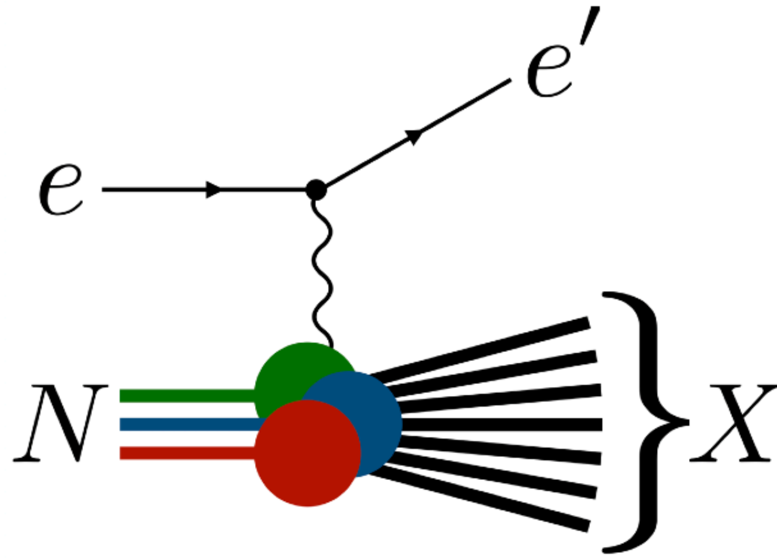
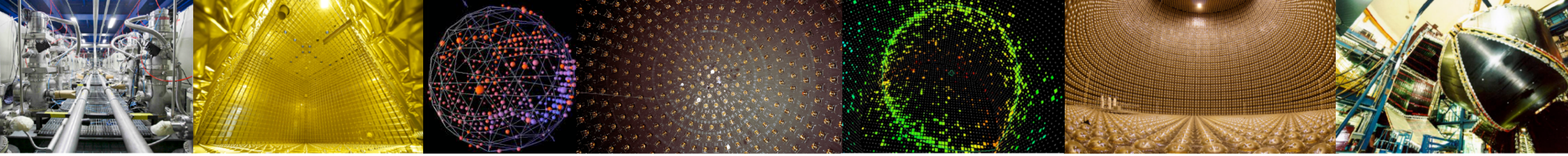




A.N. Hiller Blin et al
arXiv:2105.05834

CLAS
hep-ph/0301204



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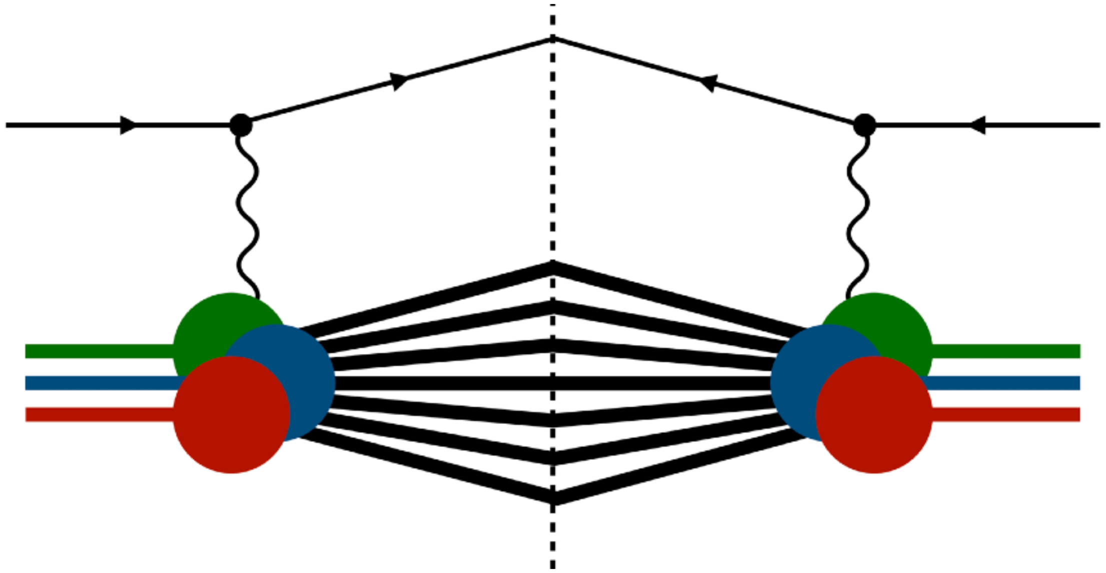


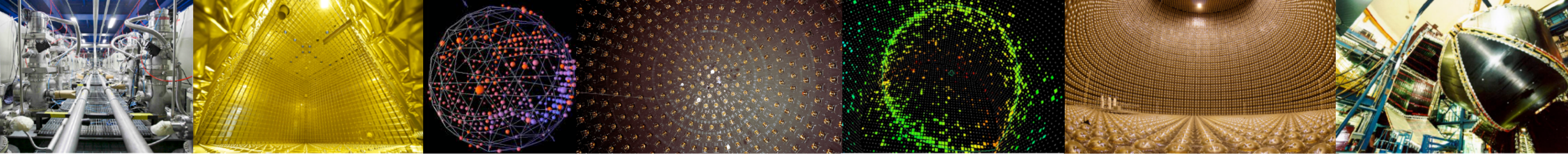
Inclusive eN scattering

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}$$

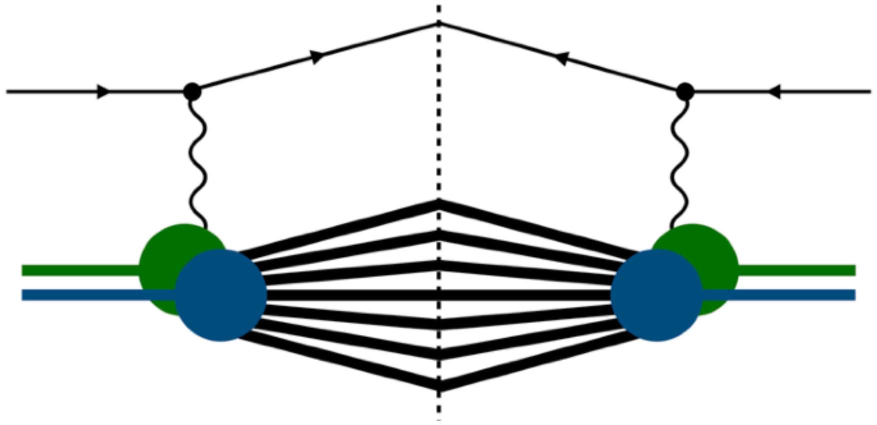
- Differential cross section specified completely by two objects
  - Leptonic tensor $L_{\mu\nu}$ (perturbatively calculable in QED, known)
  - Hadronic tensor $W_{\mu\nu}$ (generically nonperturbative in QCD)

$$W_{\mu\nu}(p,q) = \int \frac{d^4x}{4\pi} e^{iq\cdot x} \langle p|[j_\mu^{\text{EM}}(x), j_\nu^{\text{EM}}(0)]|p\rangle$$

- Our project **hadtensor** is developing techniques to access observables like these using lattice QCD.



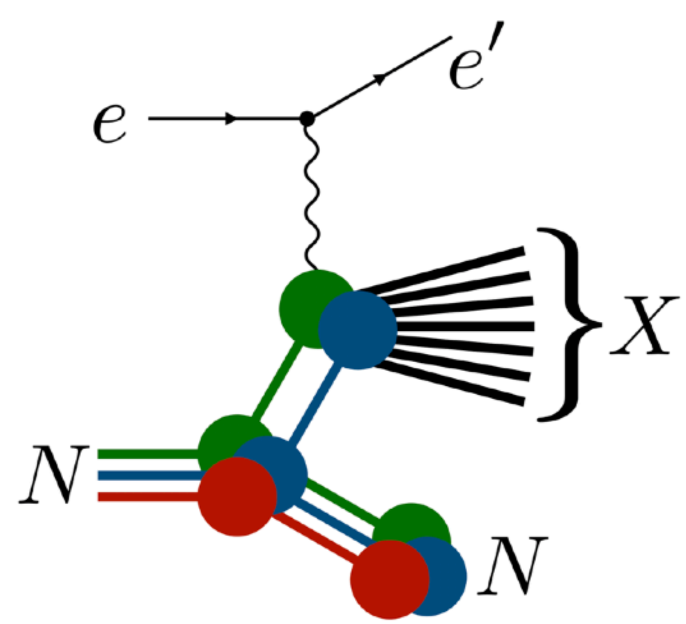
Hadron tensor of the pion

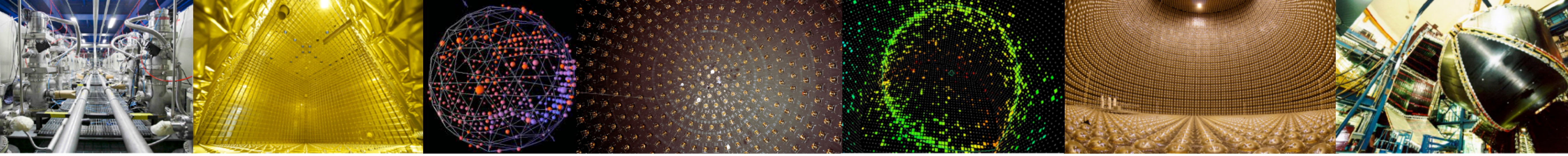


Inclusive eπ scattering

$$W_{\mu\nu}(p,q) = \int \frac{d^4x}{4\pi} e^{iq\cdot x} \langle \pi | [j_\mu^{\mathrm{EM}}(x), j_\nu^{\mathrm{EM}}(0)] | \pi \rangle$$

- Nucleons are difficult in LQCD due to signal-to-noise problems
- Pions have advantageous signal-to-noise properties and are a good proving ground for new methods
- Little is known experimentally about the hadronic (resonant) structure of mesons
- There are prospects to measure F_1^π, F_2^π at the EIC via the DIS Sullivan Process





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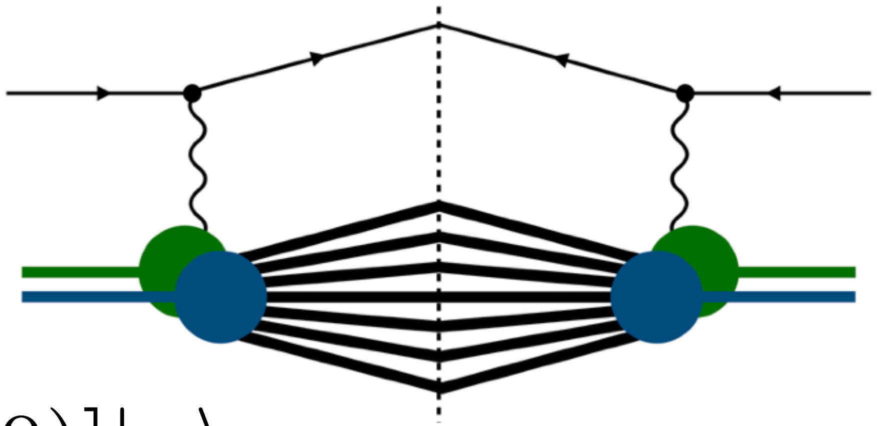


Inclusive eπ scattering

$$W_{\mu\nu}(p,q) = \int \frac{d^4x}{4\pi} e^{iq\cdot x} \langle \pi | [j_\mu^{\rm EM}(x), j_\nu^{\rm EM}(0)] | \pi \rangle$$

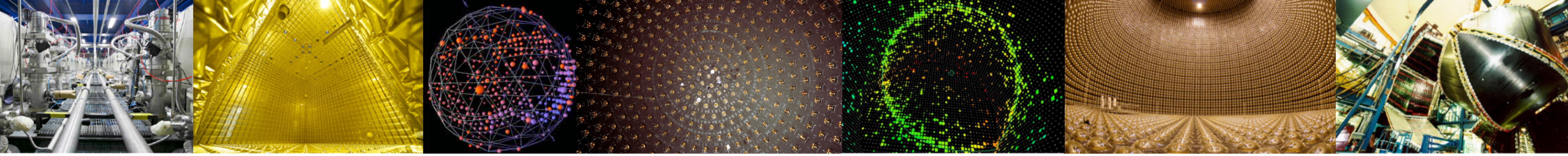
- LQCD calculations of the hadron tensor face two challenges:

1. Calculating the Euclidean four-point function

$$\sum_{\boldsymbol{x}_i,\boldsymbol{x}_f} \sum_{\boldsymbol{x}_1,\boldsymbol{x}_2} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \langle \emptyset | \chi_\pi(x_f) J_\mu(x_2) J_\nu(x_1) \bar{\chi}_\pi(x_i) | \emptyset \rangle$$

2. Connecting the Euclidean object to the “real-time” $W_{\mu\nu}$

$$W_{\mu\nu}^{\rm Euc.}(\boldsymbol{q}^2,\tau) \equiv \int d\omega\, e^{-\omega\tau} W_{\mu\nu}(\boldsymbol{q}^2,\omega).$$

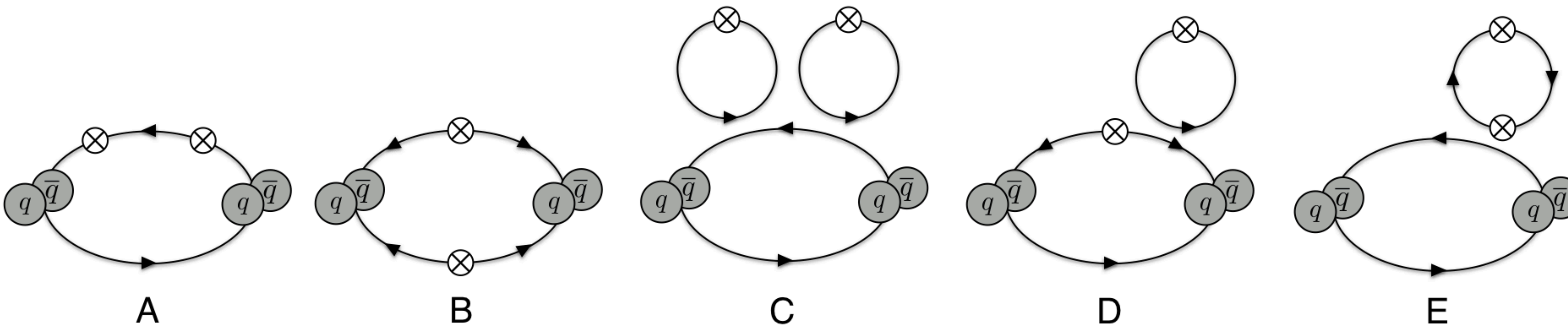


Calculating the Euclidean 4pt function

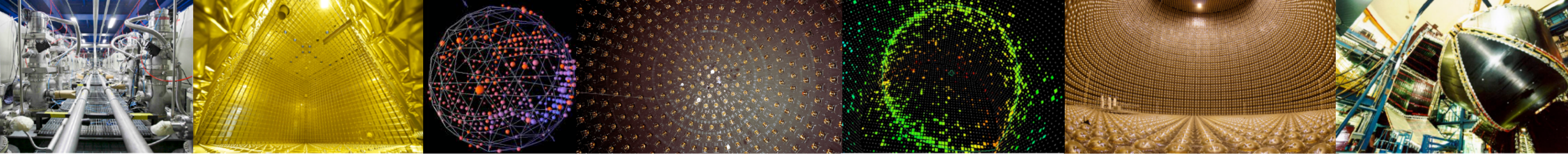
Diagrams and topologies

$$\sum_{\boldsymbol{x}_i,\boldsymbol{x}_f}\sum_{\boldsymbol{x}_1,\boldsymbol{x}_2} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)}\langle\emptyset|\chi_\pi(x_f)J_\mu(x_2)J_\nu(x_1)\bar{\chi}_\pi(x_i)|\emptyset\rangle$$

- Evaluating the Wick contractions yields five basic topologies, including disconnected diagrams:



- Computing all five classes of diagrams naturally leads to all-to-all methods for quark propagators



All-to-All Methods

Meson fields

- Consider the usual decomposition of the propagator:

$$D^{-1}_{\mathrm{A2A}}(x,y) = \sum_{l=1}^{N_{\mathrm{low}}} v_l(x) w_l^\dagger(y) + \sum_{h=N_{\mathrm{low}}+1}^{N_{\mathrm{total}}} v_h(x) w_h^\dagger(y)$$

$$v_l(x) = \phi_l(x) \quad w_l(x) = \phi_l(x)/\lambda_l$$

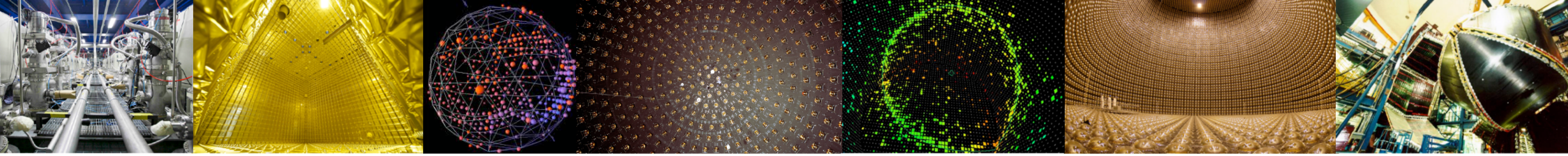
- Compute low modes exactly (Lanczos), high modes stochastically

- Define meson fields:

$$\Pi_{ij}(t_x, \boldsymbol{p}; \Gamma) \equiv \sum_{\mathbf{x}} w_i^\dagger \Gamma v_j(x) e^{i\boldsymbol{x}\cdot\boldsymbol{p}}$$

- Evaluate correlation functions via traces of products, e.g.,

$$C(t_f - t_i) = \sum_{jk} \Pi_{jk}(t_f; \gamma_5 \otimes \gamma_5) \Pi_{kj}(t_i; \gamma_5 \otimes \gamma_5)$$

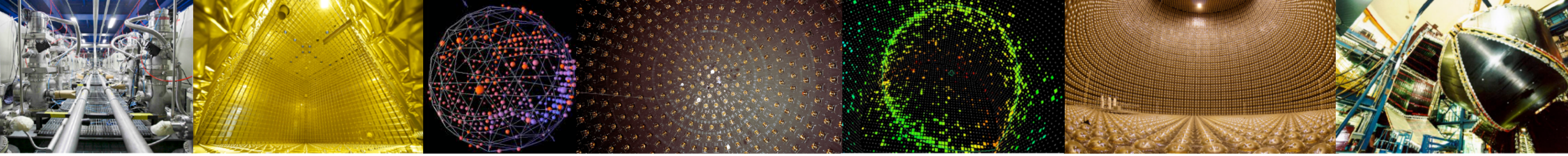


Spectral Reconstruction

Inverse Laplace transform ⟺ Extract the spectral density ρ(ω)

$$C(\tau) = \int d\omega\, e^{-\omega\tau} \rho(\omega) \to C_i(\tau_i) = K_{ij}\rho(\omega_j)$$

- Conceptual difficulty #1:
  - Finite volume: $$\rho(\omega) = \sum_n \langle\emptyset|\mathcal{O}|n\rangle \delta(\omega - E_n)$$
  - Infinite volume:
    - ρ ∍ δ-functions from single-particle states
    - ρ ∍ continuous functions from multi-particle states
- Conceptual difficulty #2:
  - Ambiguity in definition of “the” solution ⟺ K_{ij} has a large null space



Spectral Reconstruction

Connecting finite and infinite volumes

- Smeared spectral densities bridge the gap between finite-volume and infinite-volume observables [Hansen, Meyer, and Robaina, arXiv:1704.08993]:

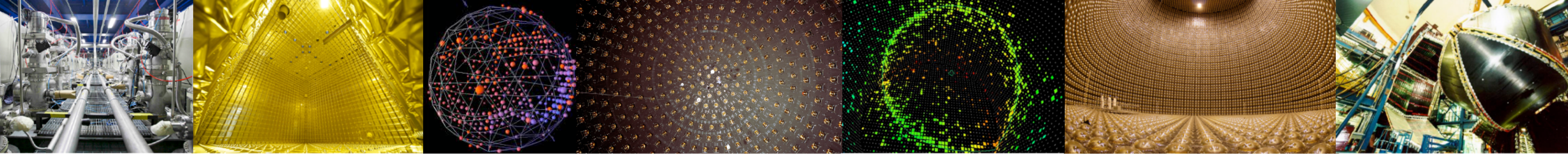
$$\rho_\sigma(\omega; L) = \int d\omega' \delta_\sigma(\omega - \omega') \rho(\omega; L)$$

Smearing function, e.g., Gaussian of width σ

$$\rho(\omega) = \lim_{\sigma \to 0} \lim_{L \to \infty} \rho_\sigma(\omega; L)$$

Ordered limit
- $L \to \infty$ first
- $\sigma \to 0$ second

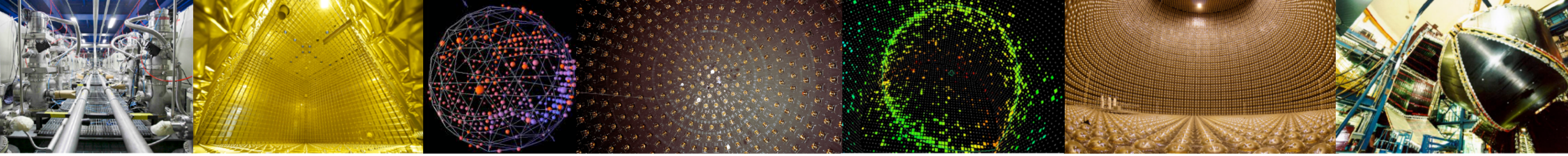
- A nice algorithm to compute $\rho_\sigma(\omega)$ was proposed by Martin Hansen, Lupo, and Tantalo [arXiv:1903.06476]. The HLT method generalizes and improves the "Backus-Gilbert" method.



Spectral Reconstruction

Resolving the ambiguity in analytic continuation

- Some of us (WJ + MIT students) have developed a new reconstruction method:
  - Imposes known analytic structure for Green functions in the complex plane.
  - Reduces to HLT method for a certain choice of smearing kernel
  - Furnishes a robust quantification of uncertainties
    - Total error = (Systematic) $\oplus$ (Statistical)
    - Systematic = “Compute the null space explicitly”
    - “Null space” = Space of functions consistently with imposed analytic structure and given Euclidean data
- Dedicated methods paper is nearly complete, arXiv:2205.XXXX

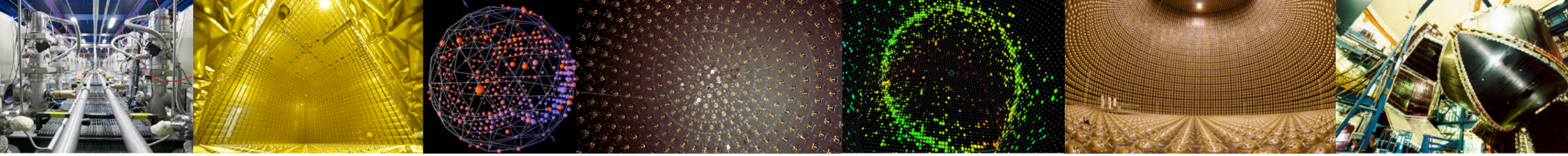


Our calculation

- N_f=2+1+1 HISQ ensembles generated by the MILC collaboration

$\approx a[\,\mathrm{fm}]$	$N_s^3 \times N_t$	$\approx L[\,\mathrm{fm}]$	Configurations (analyzed to date)	(proposed 2023)	Total
0.15	$32^3 \times 48$	4.8	30	-	30
0.15	$48^3 \times 64$	7.2	0	**175**	175
0.12	$48^3 \times 64$	5.8	50	**125**	175

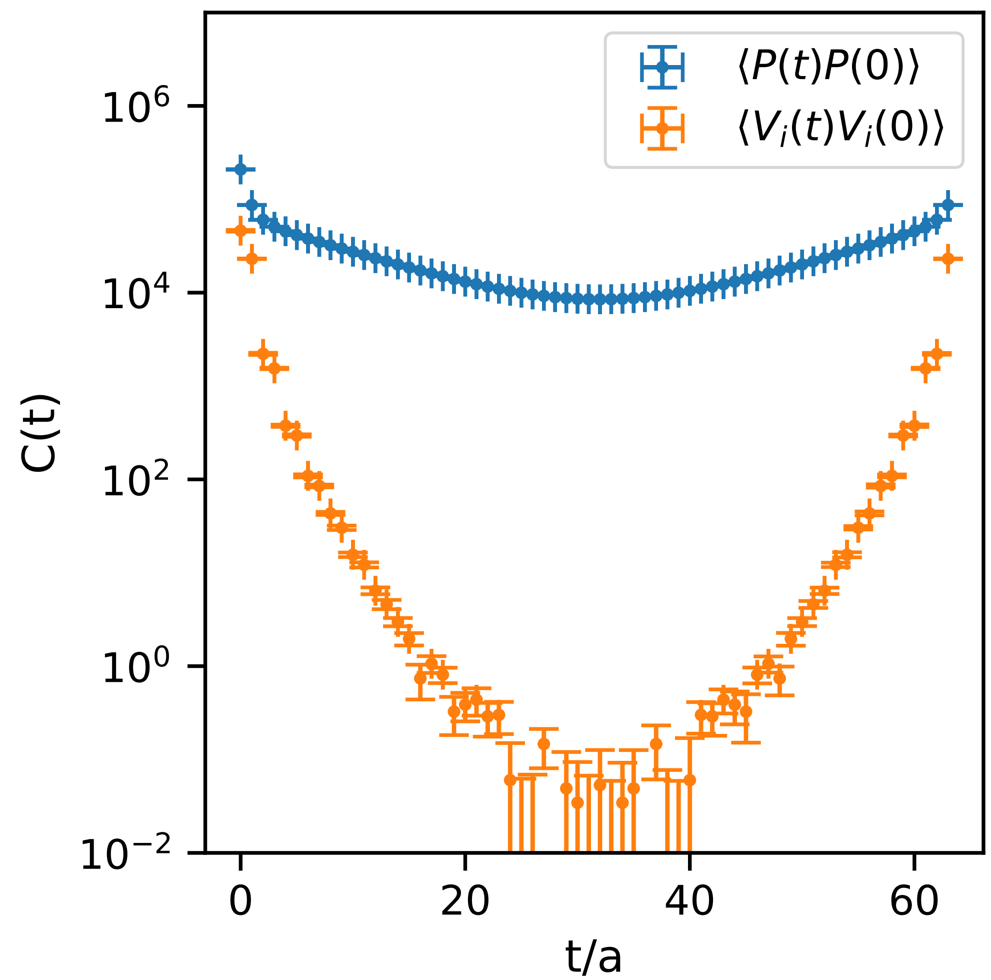
- Code: `Grid` + `Hadrons`
  - Native support for improved staggered fermions
  - Required local staggered currents implemented by us, tested against MILC
  - Smeared links computed externally



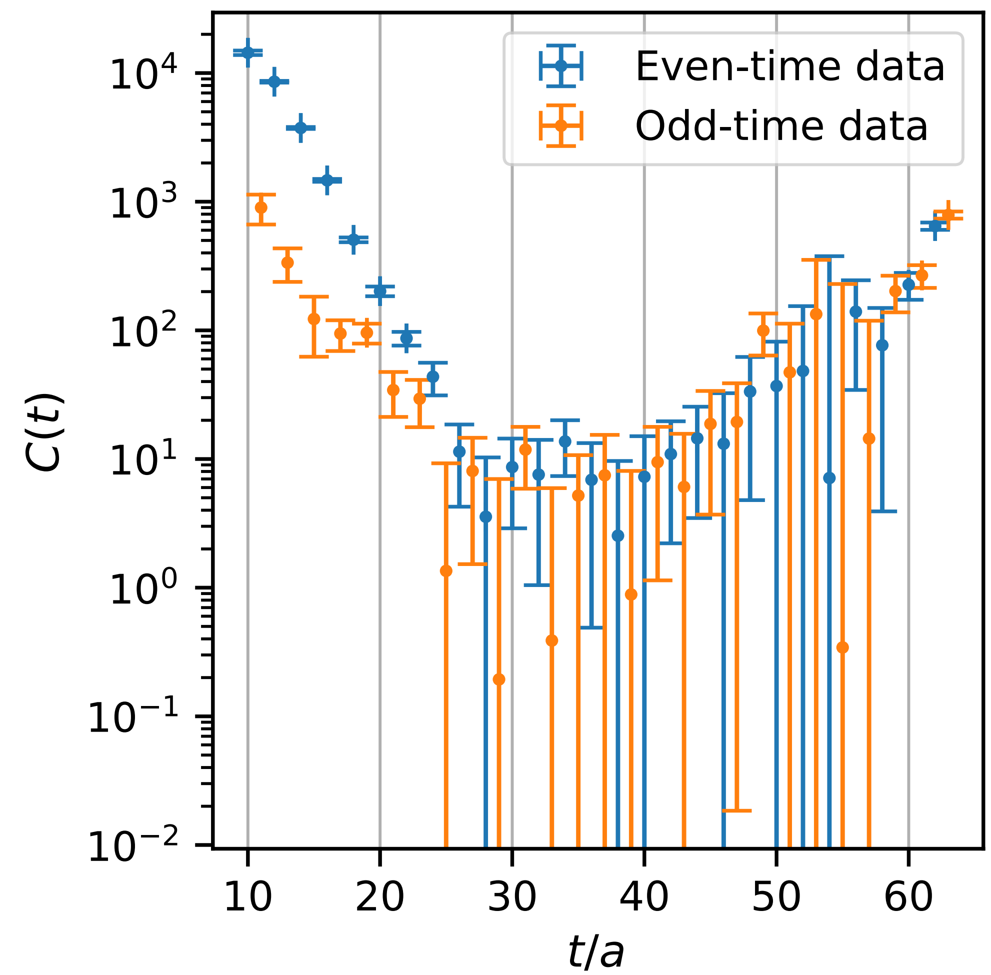
Spectral Reconstruction

Some preliminary results: a≈0.12 fm, V=48³×64

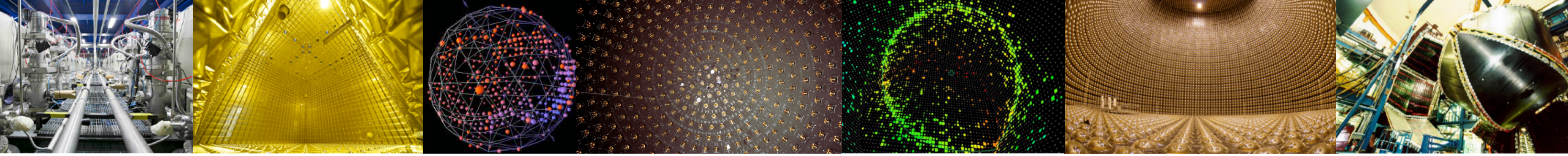
Two-point functions



Four-point function



$$\langle P(t+2\Delta t) V_i(t+\Delta t) V_i(\Delta t) P(0) \rangle$$



Spectral Reconstruction

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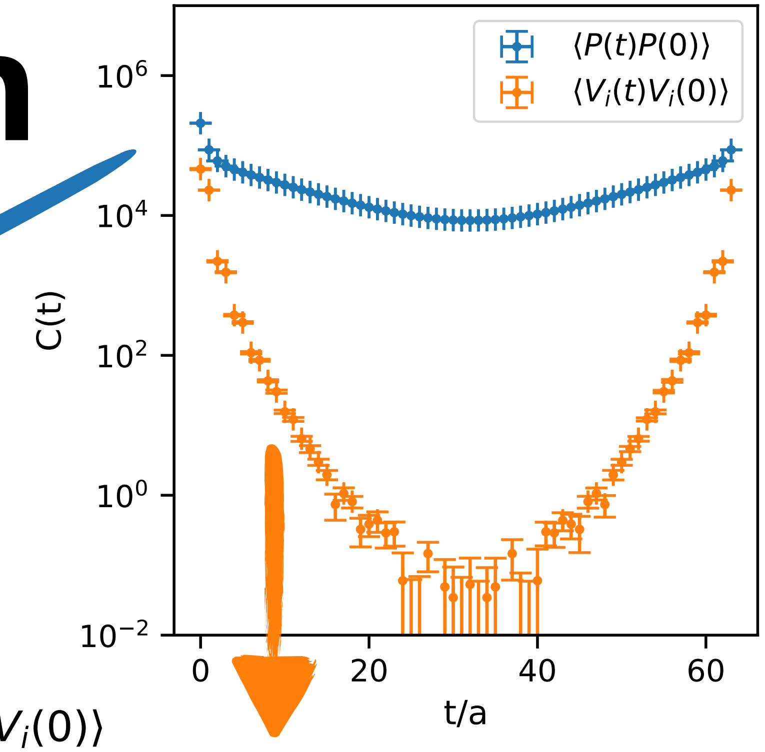
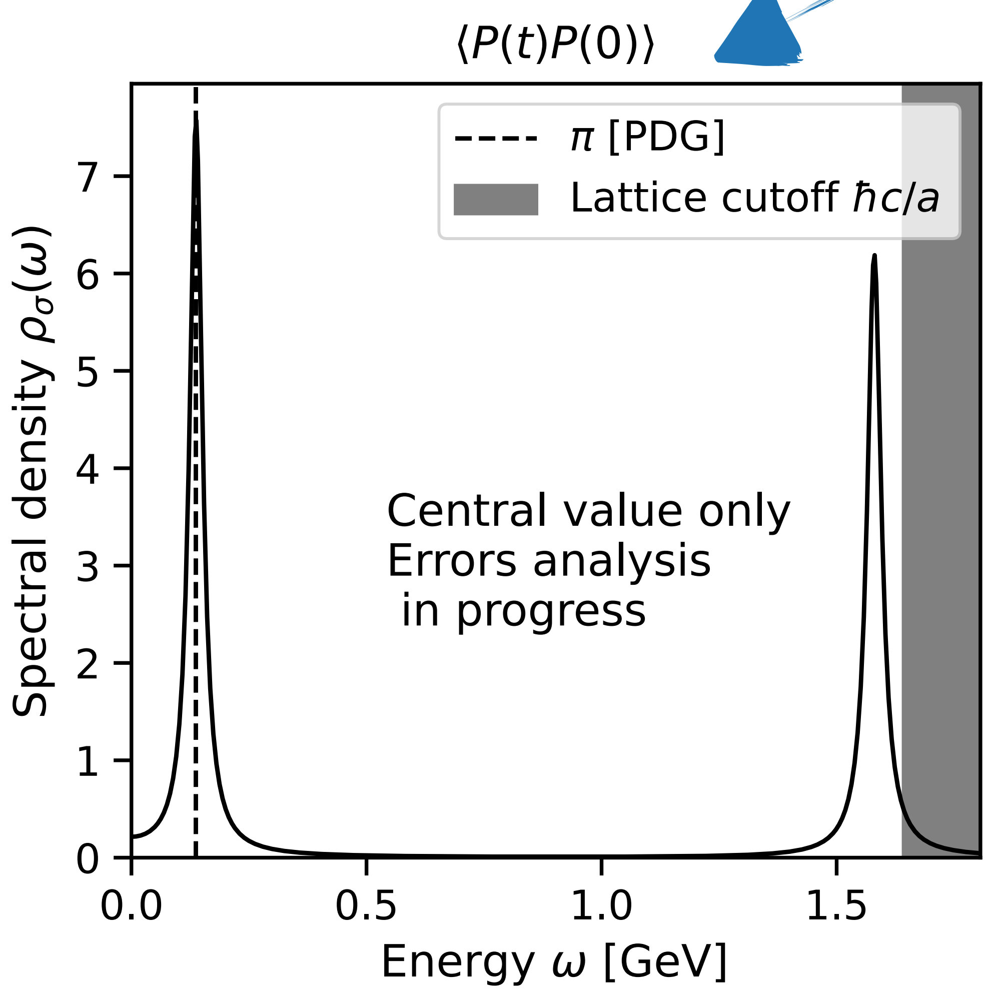
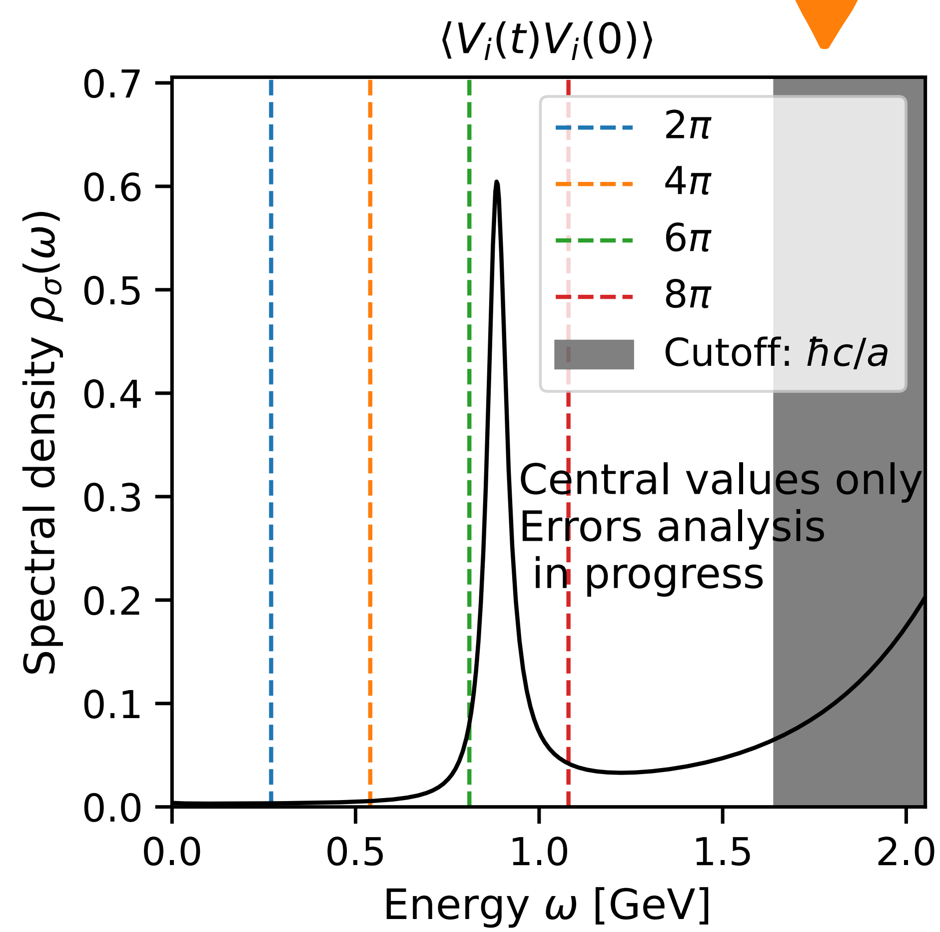
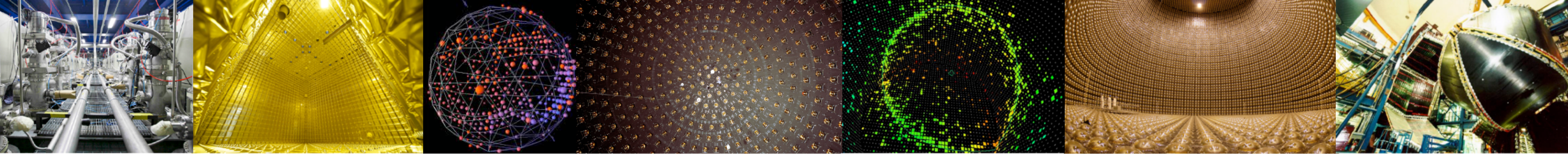






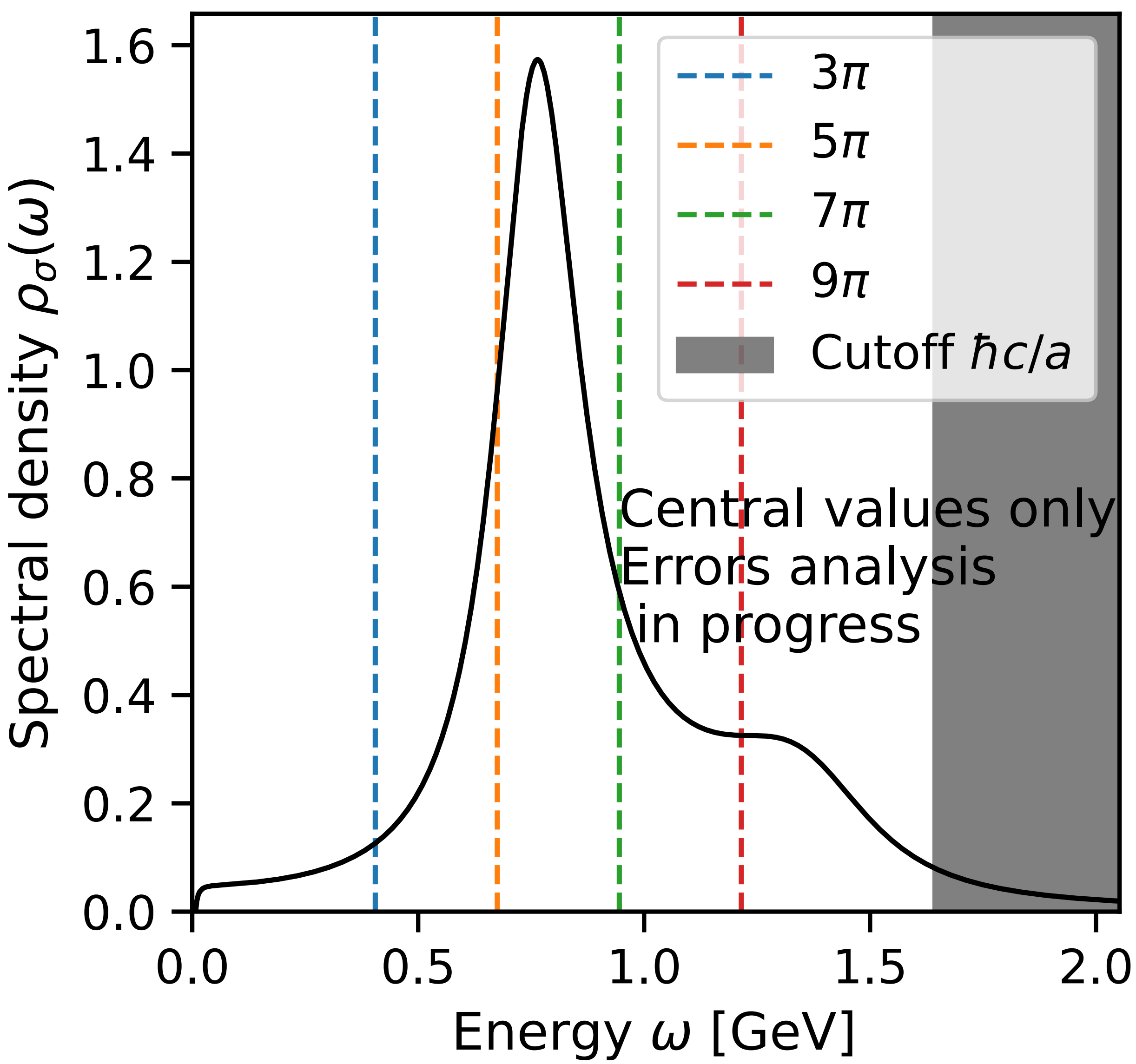


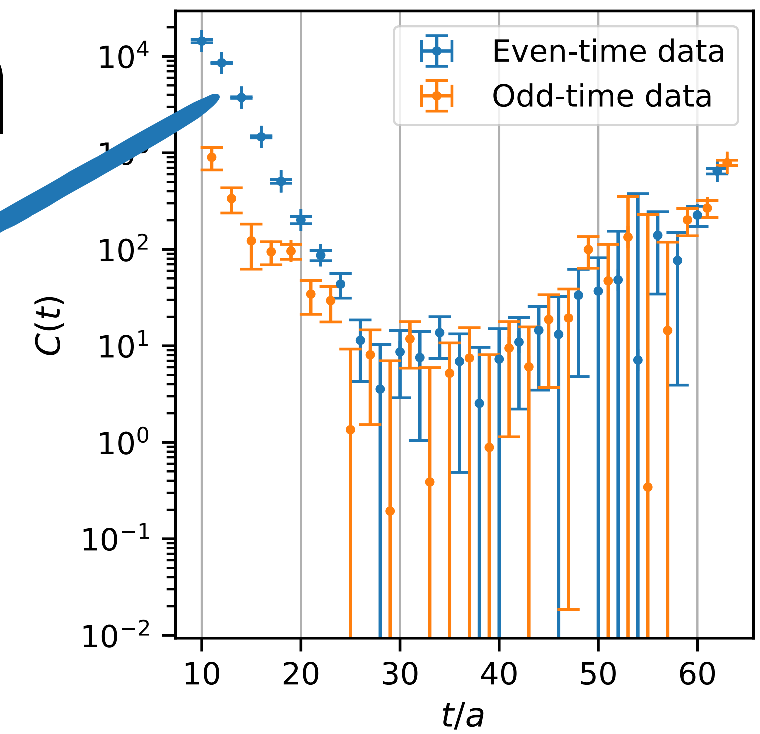
Spectral Reconstruction

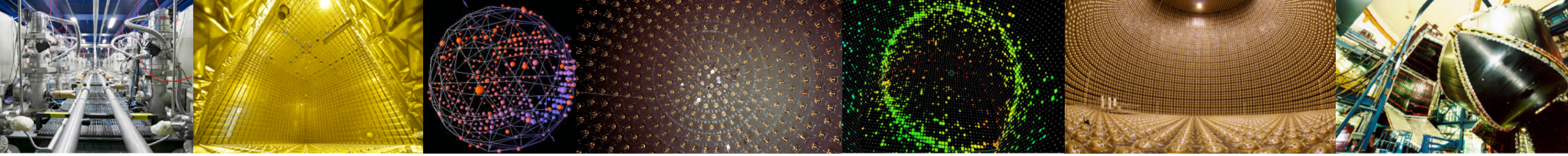
Some preliminary results

Four-point function

$\langle P(0)V_i(\Delta t)V_i(t+\Delta t)P(t+2\Delta t)\rangle$



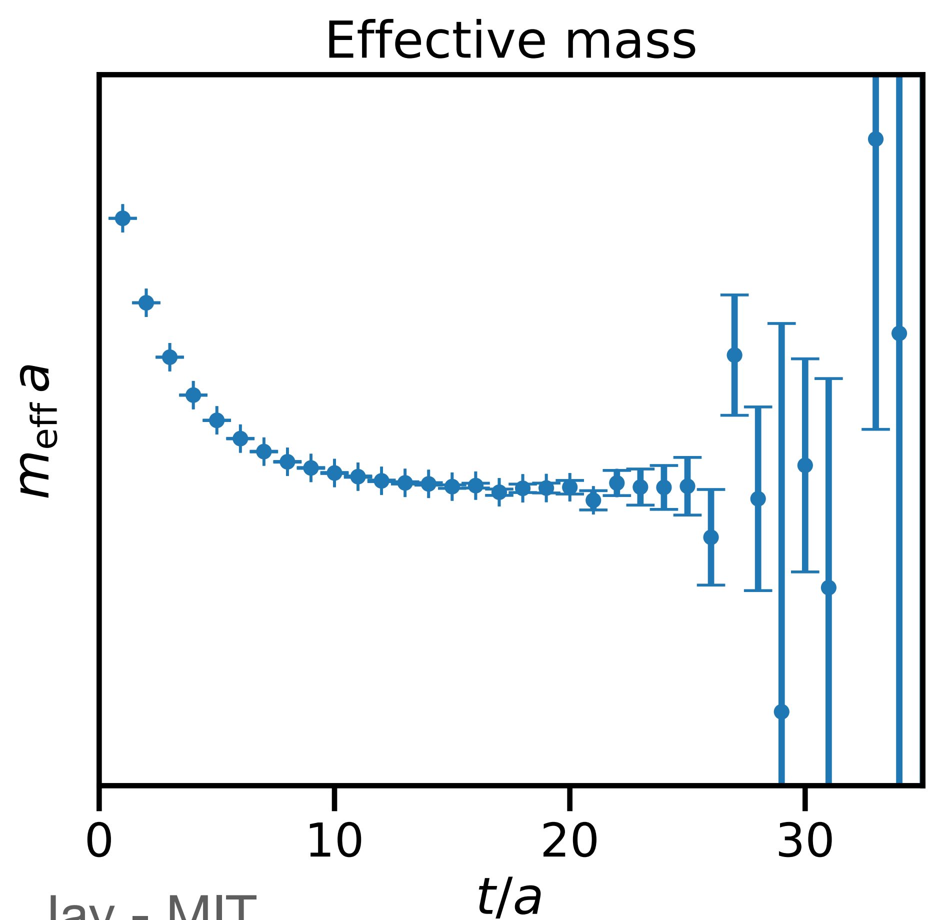


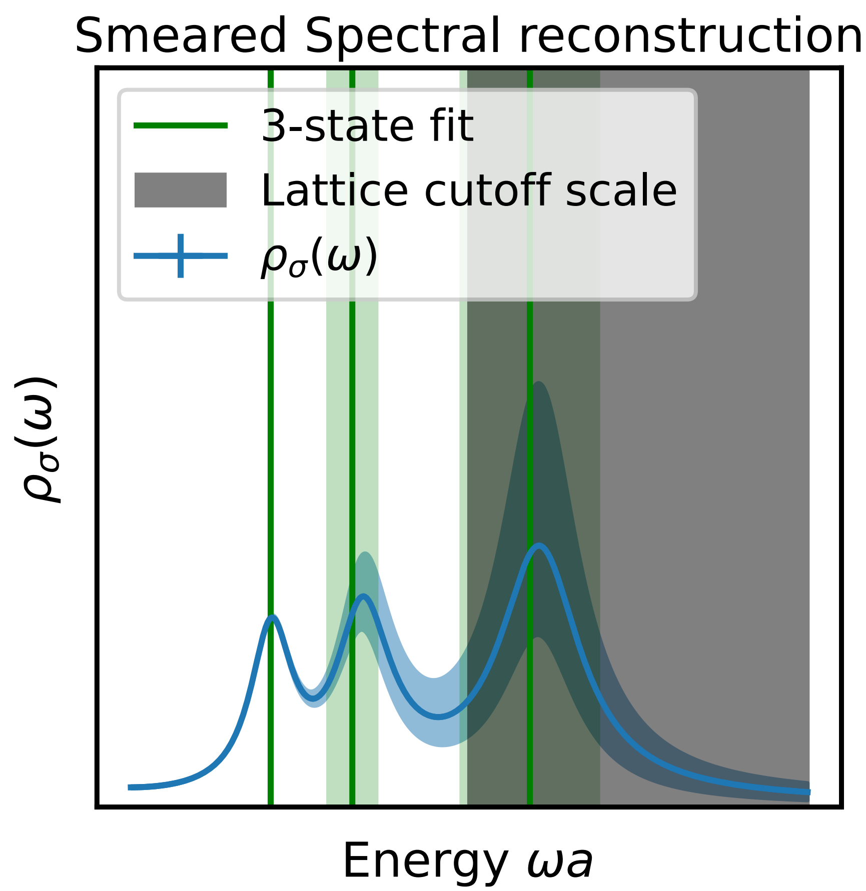


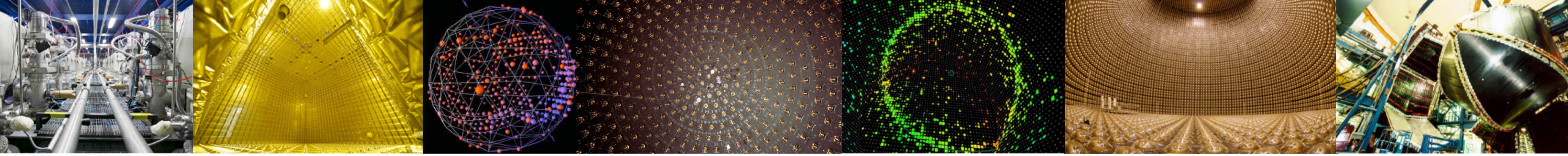
Spectral Reconstruction

Example from a different project

- Nucleon-nucleon 2pt function with high statistics
- Blue band = systematic = “compute null space analytically”
- Excellent compatibility with a 3-state Bayesian fit (wide priors)
- *Nota bene*: reconstruction is nonparametric: 3-peak structure is an <u>output of the method</u>

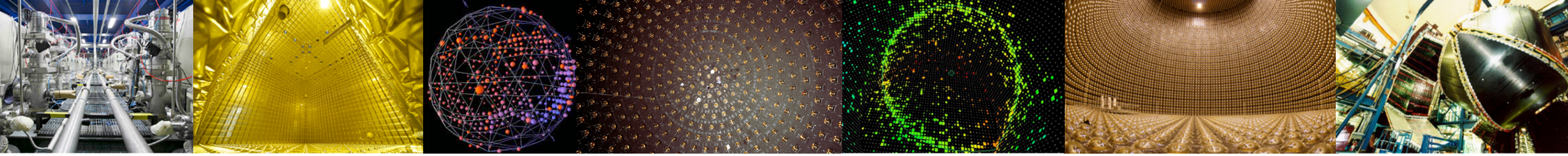






Next Steps

- We continue to run productively with our 2022-2023 allocation
- Our 2023-2024 proposal aims to:
  - Increase statistics at $a \approx 0.12$ fm, $V = 48^3 \times 64$
  - Calculate correlators on a larger physical volume — $a \approx 0.12$ fm, $V = 48^3 \times 64$
- We have developed a new method for spectral reconstruction
  - Dedicated paper describing the method 2205.XXXX
  - Analysis of data is preliminary, but we're excited about the performance of the method to date
- A key goal of our project is robust estimation of systematic uncertainty
  - New reconstruction method developed specifically with this goal in mind
  - 2023-2024 proposal targeted at measuring finite-volume dependence



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Questions?