Computing the large-Q^2 Kaon Form Factors with Physical Quark Masses

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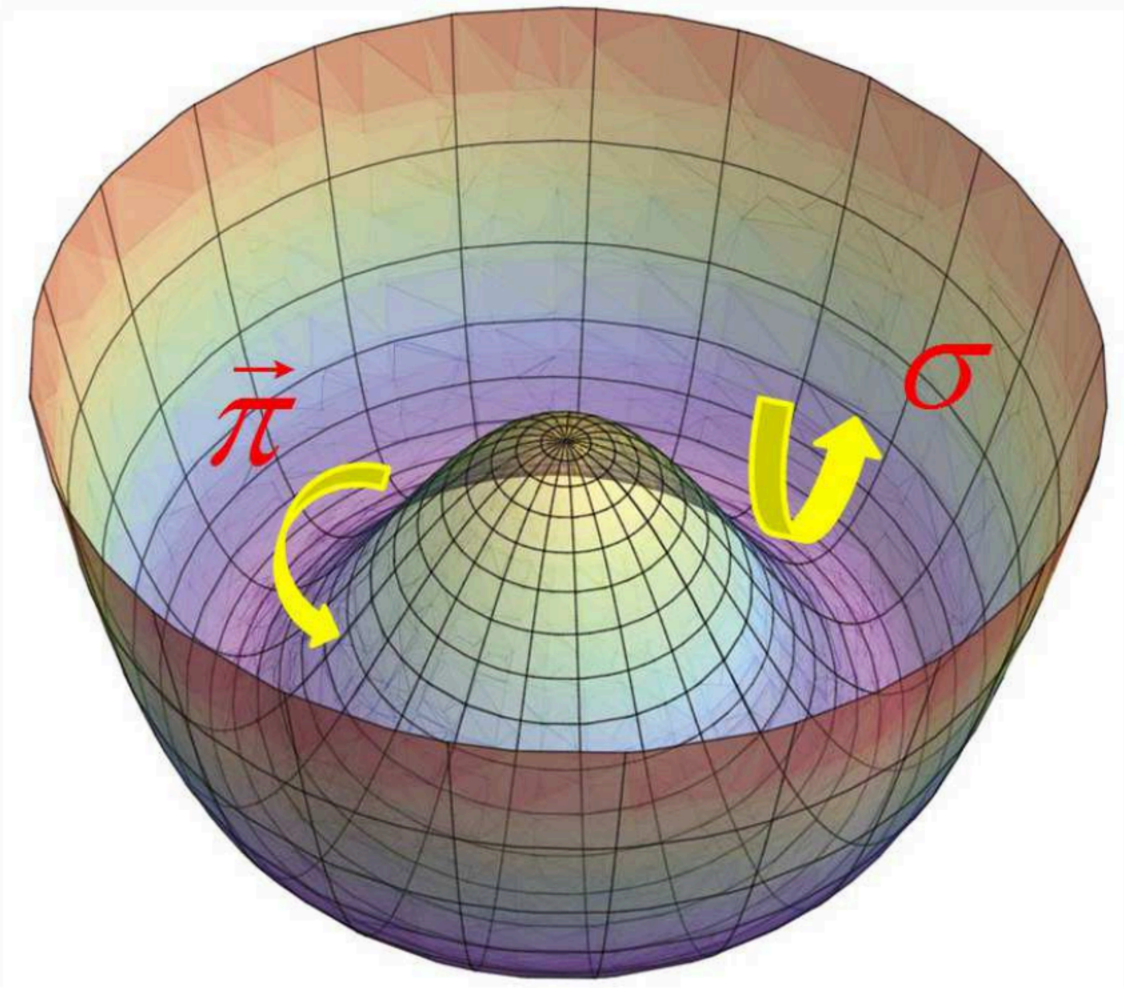
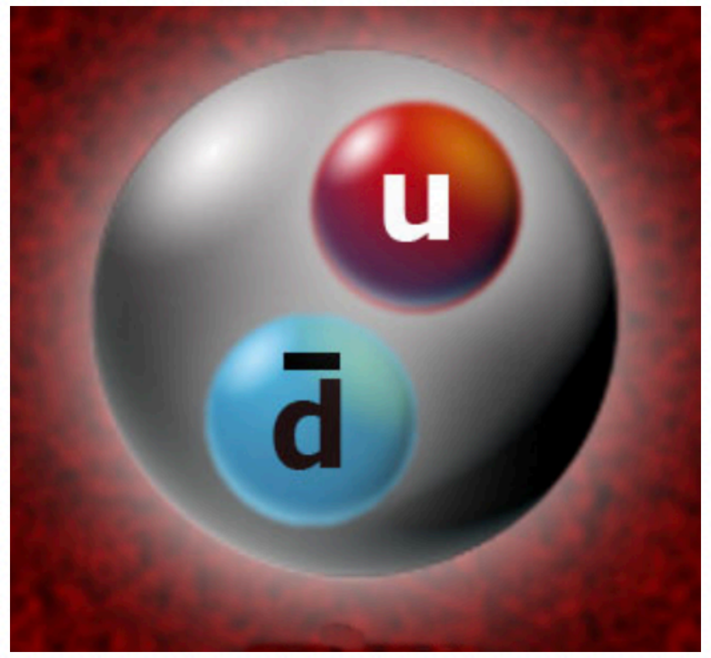
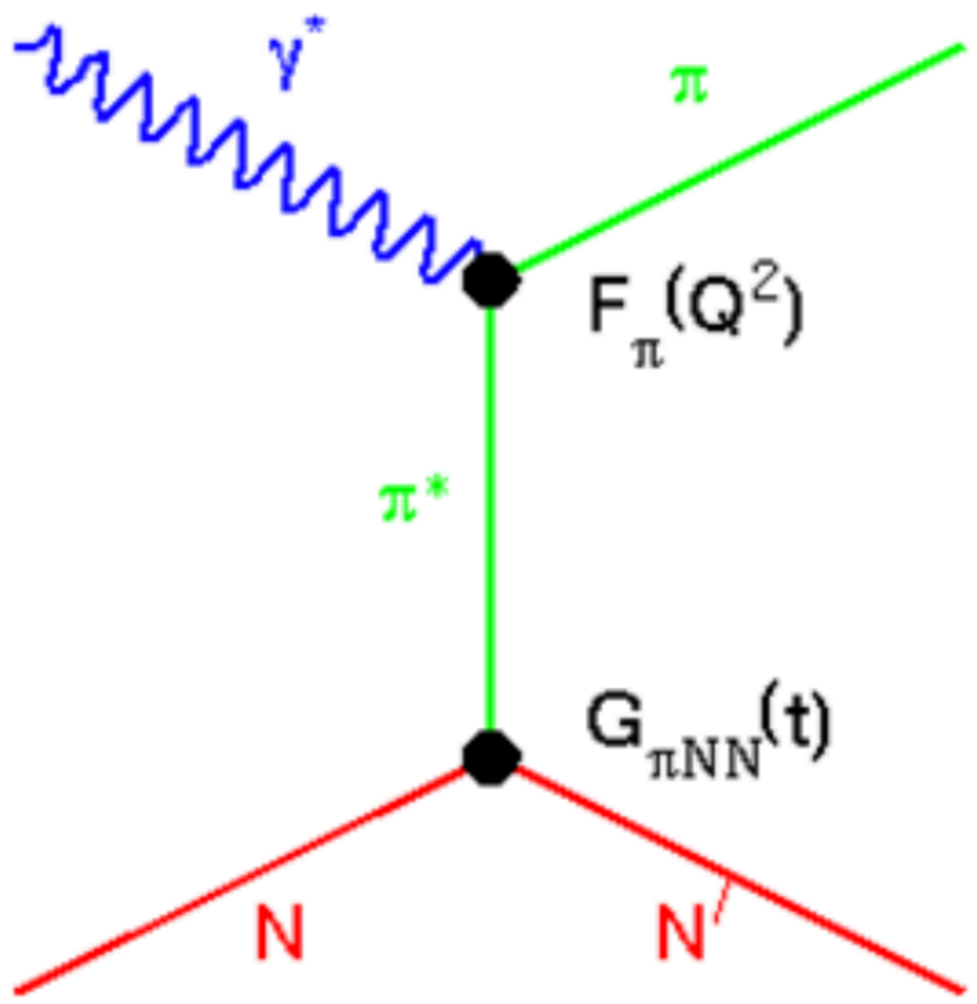
Argonne NATIONAL LABORATORY

Pion/kaon are pseudo-Goldstone bosons of QCD



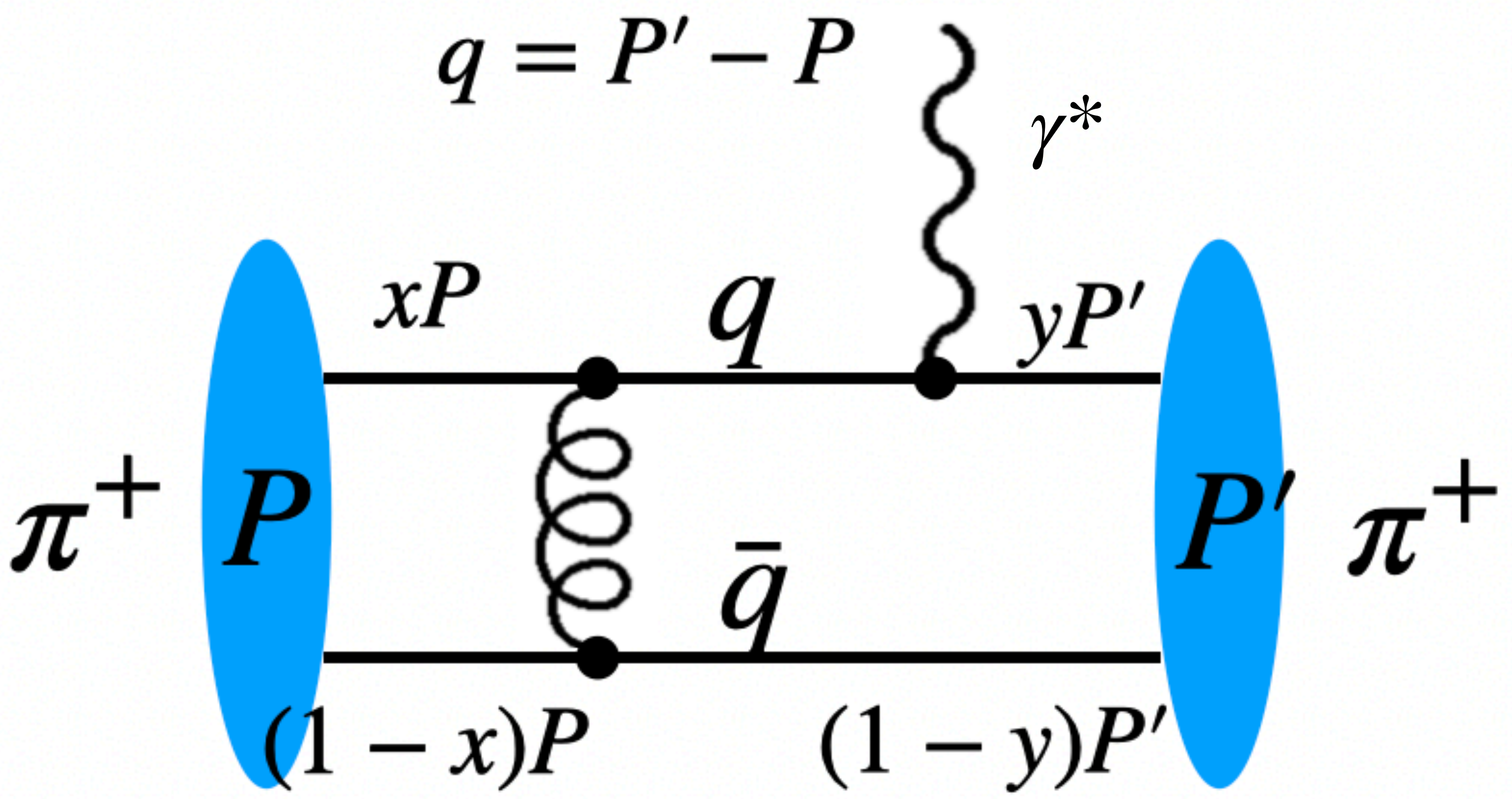


- ➡ Critical ingredient for understanding the dynamical chiral symmetry breaking in QCD.
- ➡ Improve our knowledge of the emergent mass generation and its competition with the Higgs mechanism.
- ➡ Experimental knowledge of pion/kaon internal structure are Limited.



Form factors in the partonic regime

➡ The form factors of mesons provide a clearest opportunity to study the transition from non-perturbative to perturbative QCD.



- Electromagnetic form factor

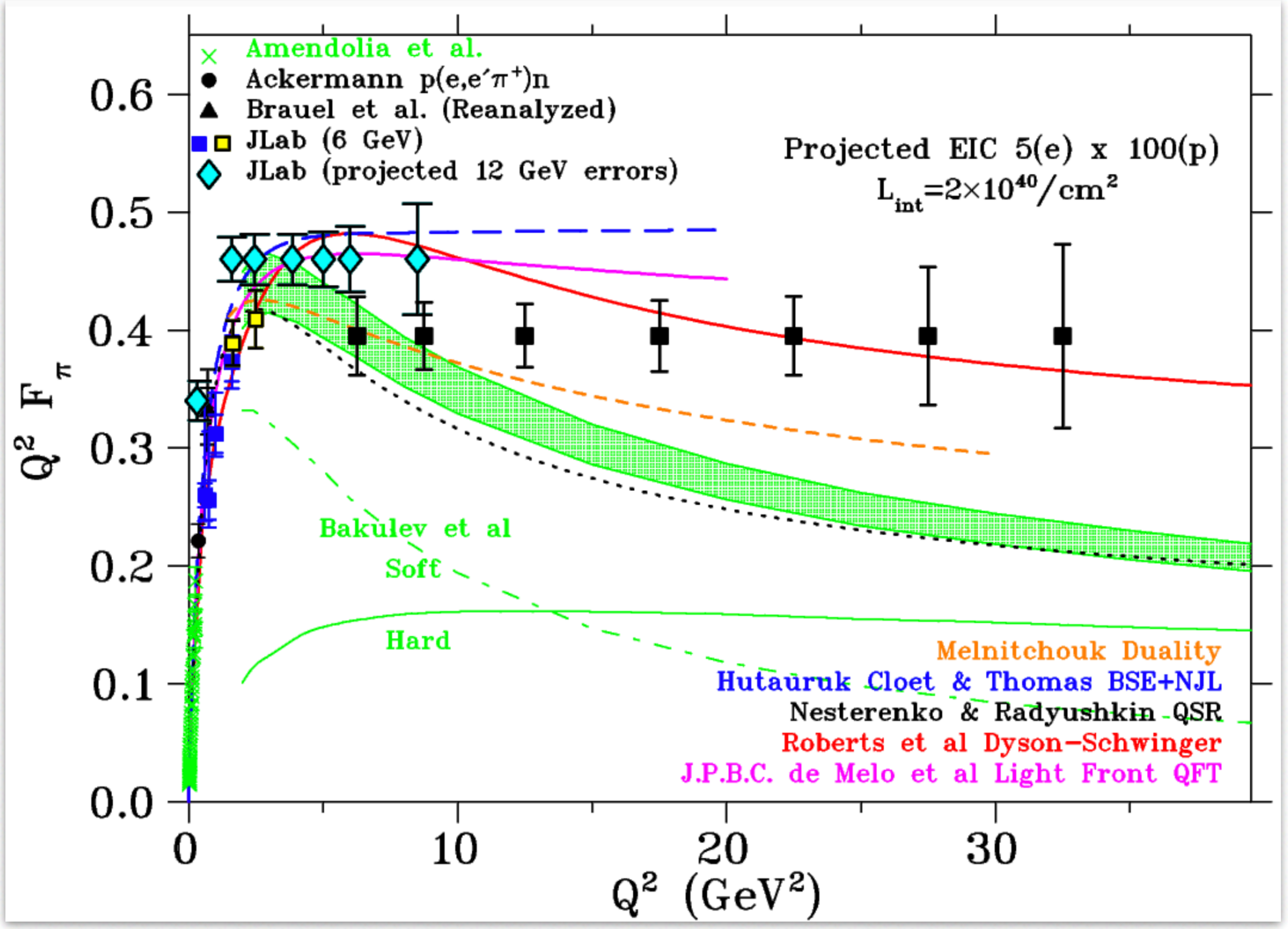
$\gamma^* \pi^+ \to \pi^+$

$$F_\pi(Q^2) \overset{Q^2 \to \infty}{=} \mathcal{N} \int_0^1 \int_0^1 dx dy\, \phi^*(v, \mu_F^2) \times T_F^V(u, v, Q^2, \mu_R^2, \mu_F^2) \phi(u, \mu_F^2) + \mathcal{O}(\frac{1}{Q^2})$$

Process dependent hard kernel from perturbation theory

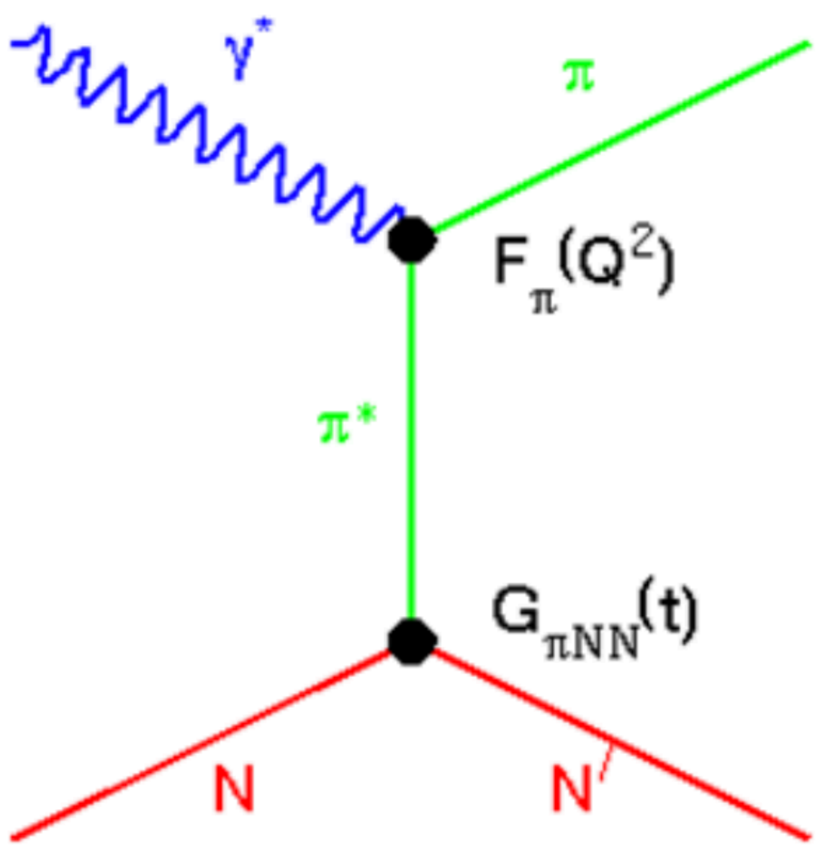
Process independent non-perturbative DAs

Form factors in the partonic regime



- Current EMFF of pion has existed for decades and only for Q^2 smaller than 3 GeV^2.
- At this stage, the pQCD prediction using asymptotic DA is challenged by existing EMFF data.
- Measuring the EMFF of kaon is even harder from experiment.

➡ Studies of the EMFF of pion and kaon with Q^2 up to $10\ \mathrm{GeV}^2$ and $6\ \mathrm{GeV}^2$ are underway at the ongoing JLAB 12 GeV experiment, and will be extended up to $30\ \mathrm{GeV}^2$ at the future Electron Ion Collider (EIC) facility.

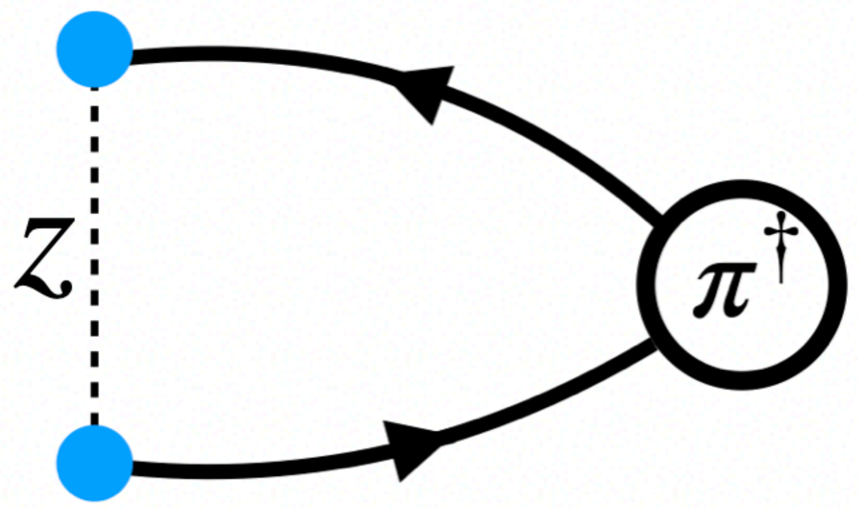


Lattice Setup

- Mixed fermion action
  - Sea quark action: N_f = 2+1 HISQ with physical quark masses, $L_s^3 \times L_t = 64^3 \times 64$, a = 0.076 fm
  - Valence quark action: Wilson-Clover with physical quark masses, 1-HYP smeared gauge links
- Use momentum smearing for quarks to achieve better overlap with boosted hadrons in brit frame
- Previous USQCD allocation related to this work: pion form factors up to 10 GeV^2, pion distribution amplitude and kaon distribution amplitude.
- This proposal: Kaon form factor
  - $P_z = 3,5,6\ [\frac{2\pi}{L_s a}]$ and $t_{sep} = 6,8,10,12$, providing Q^2 up to 10 GeV^2.

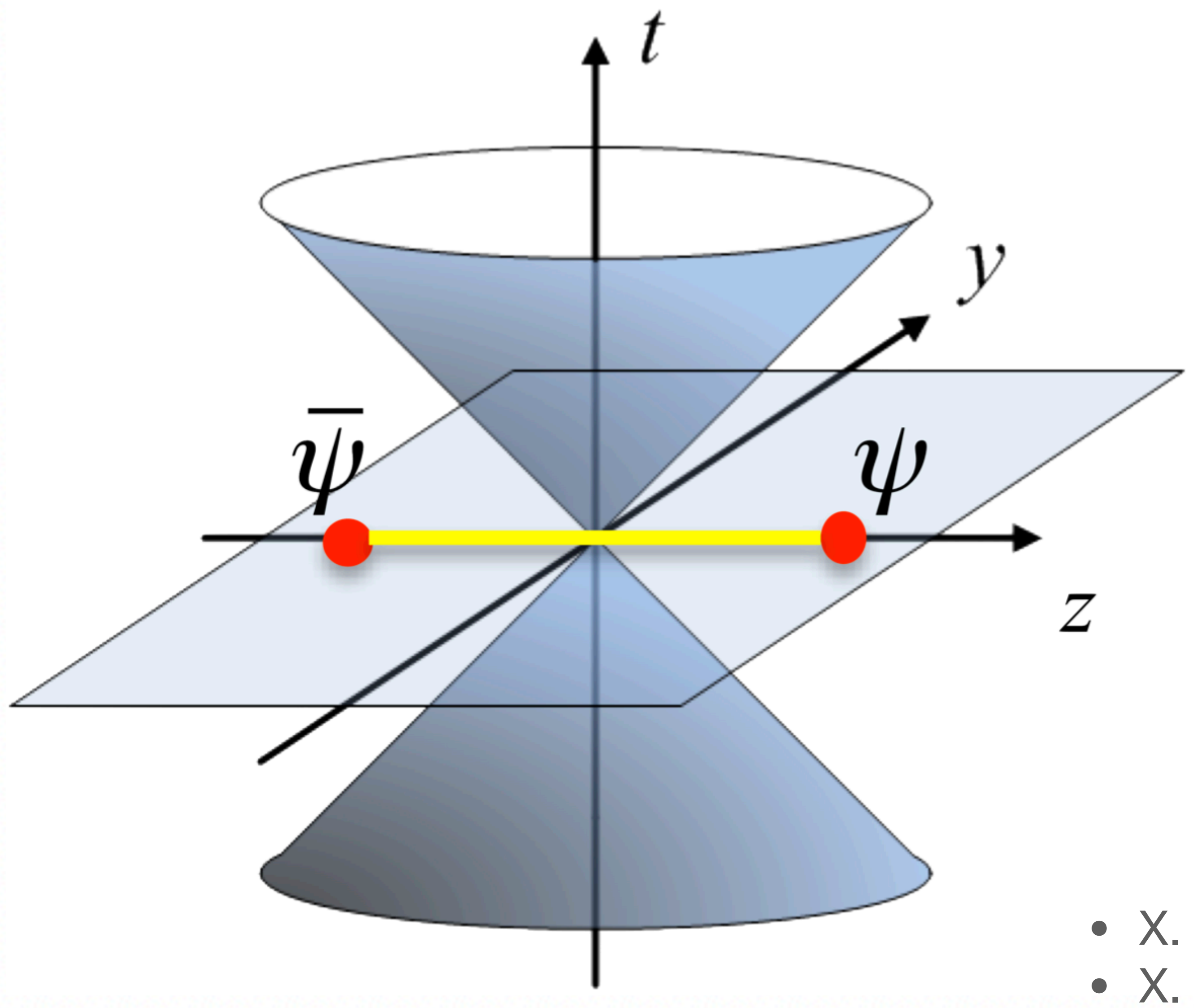
Pion distribution amplitude from lattice

The quasi-DA matrix elements,



$$iP_z h(z, P_z) = \langle 0 | \bar{d}(-z_3/2)\gamma_z\gamma_5 W_{z_3} u(z_3/2) | \pi^+; P\rangle$$

$t = 0, \quad z \neq 0$



- X. Ji, PRL 2013
- X. Ji, et al, RevModPhys 2021

F.T.

The short distance factorization:

$$h^R(z, P_z, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m\rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

Mellin moments

$\lambda = zP_z$

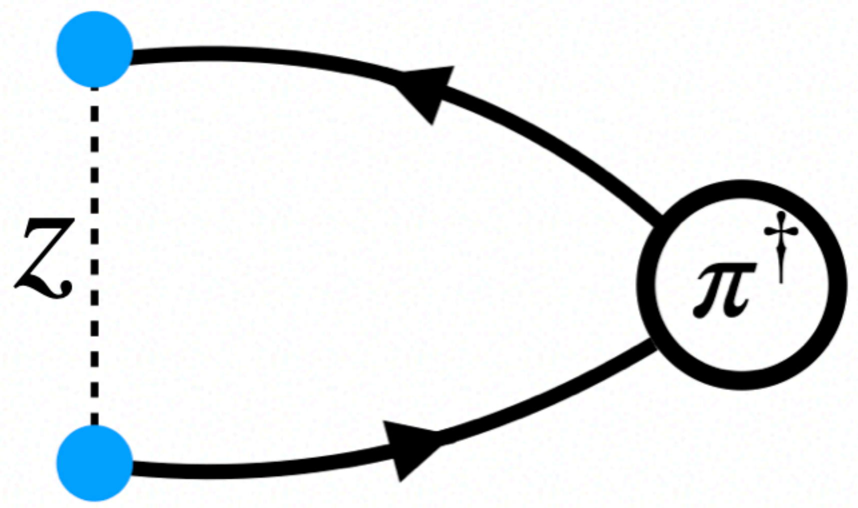
Large-momentum effective theory:

$$\tilde{\phi}(x, P_z, \mu) = \int \frac{dy}{|y|} C(x, y, P_z, \mu)\phi(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(1-x)^2P_z^2}, \frac{\Lambda^2_{QCD}}{x^2P_z^2}\right)$$

- A. Radyushkin, PRD 100 (2019)
- A. Radyushkin, Int.J.Mod.Phys.A 2020

Pion distribution amplitude from lattice

The quasi-DA matrix elements,



$$iP_z h(z, P_z) = \langle 0 | \bar{d}(-z_3/2)\gamma_z\gamma_5 W_{z_3} u(z_3/2) | \pi^+; P\rangle$$

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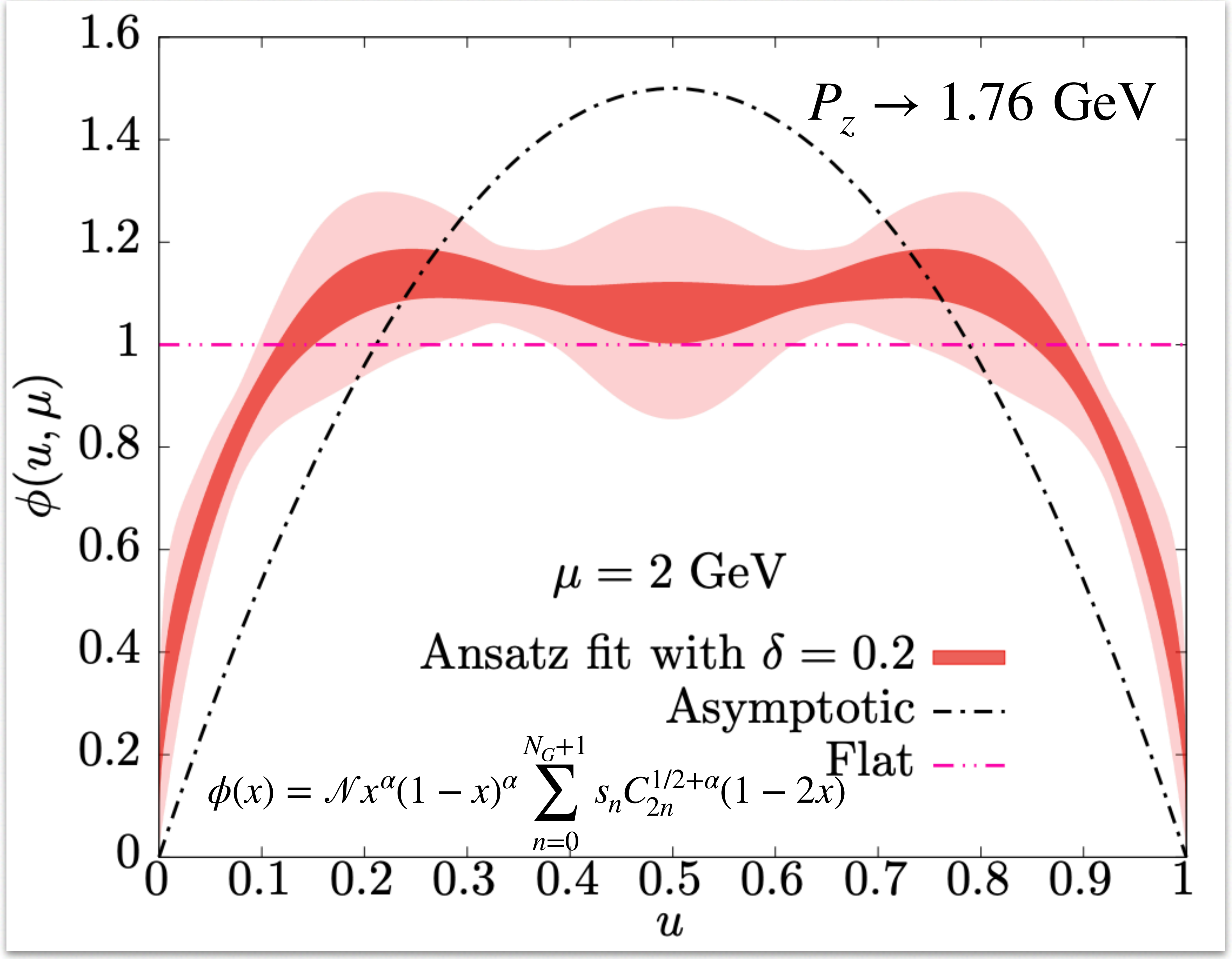
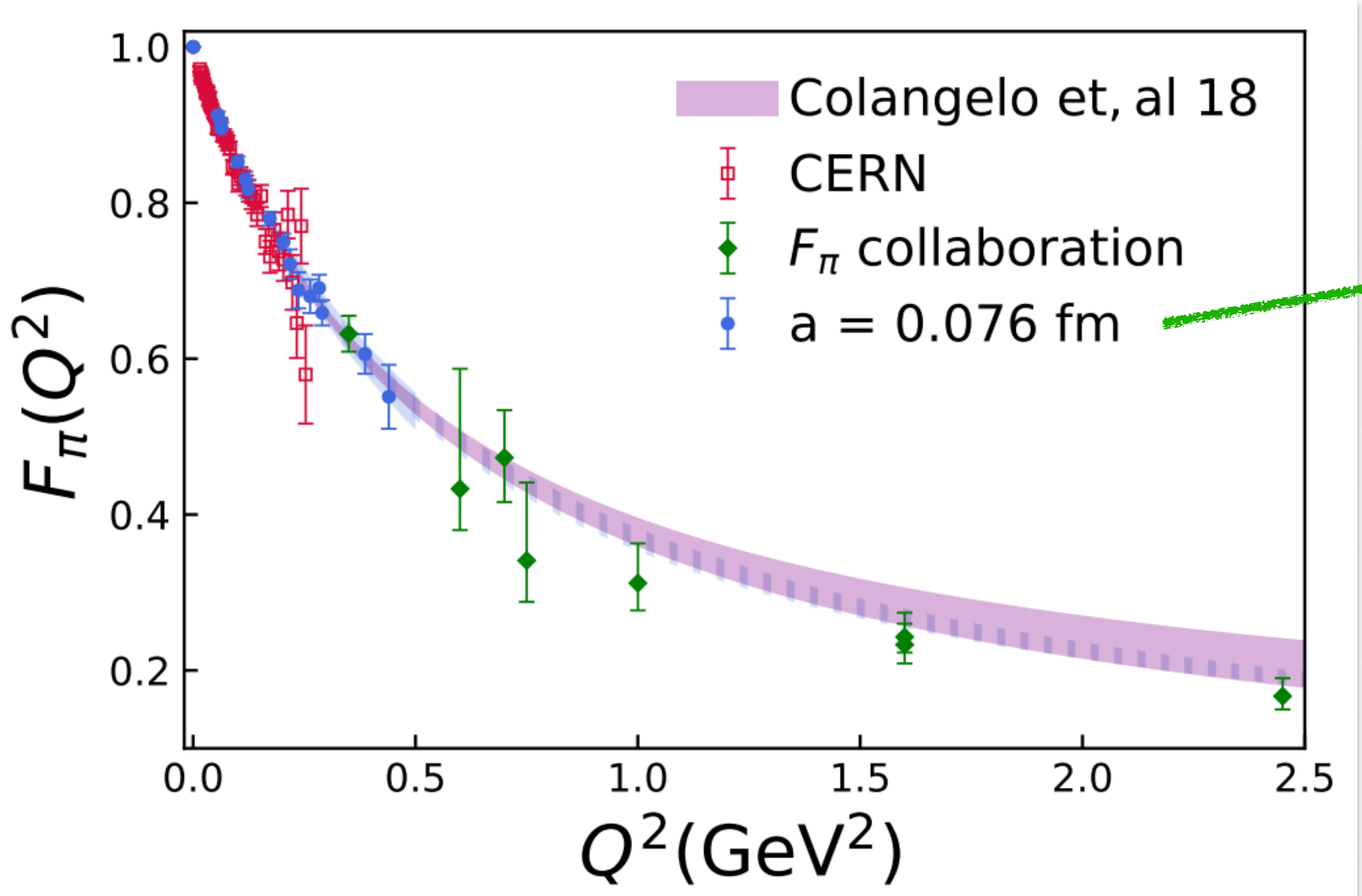
Mellin moments

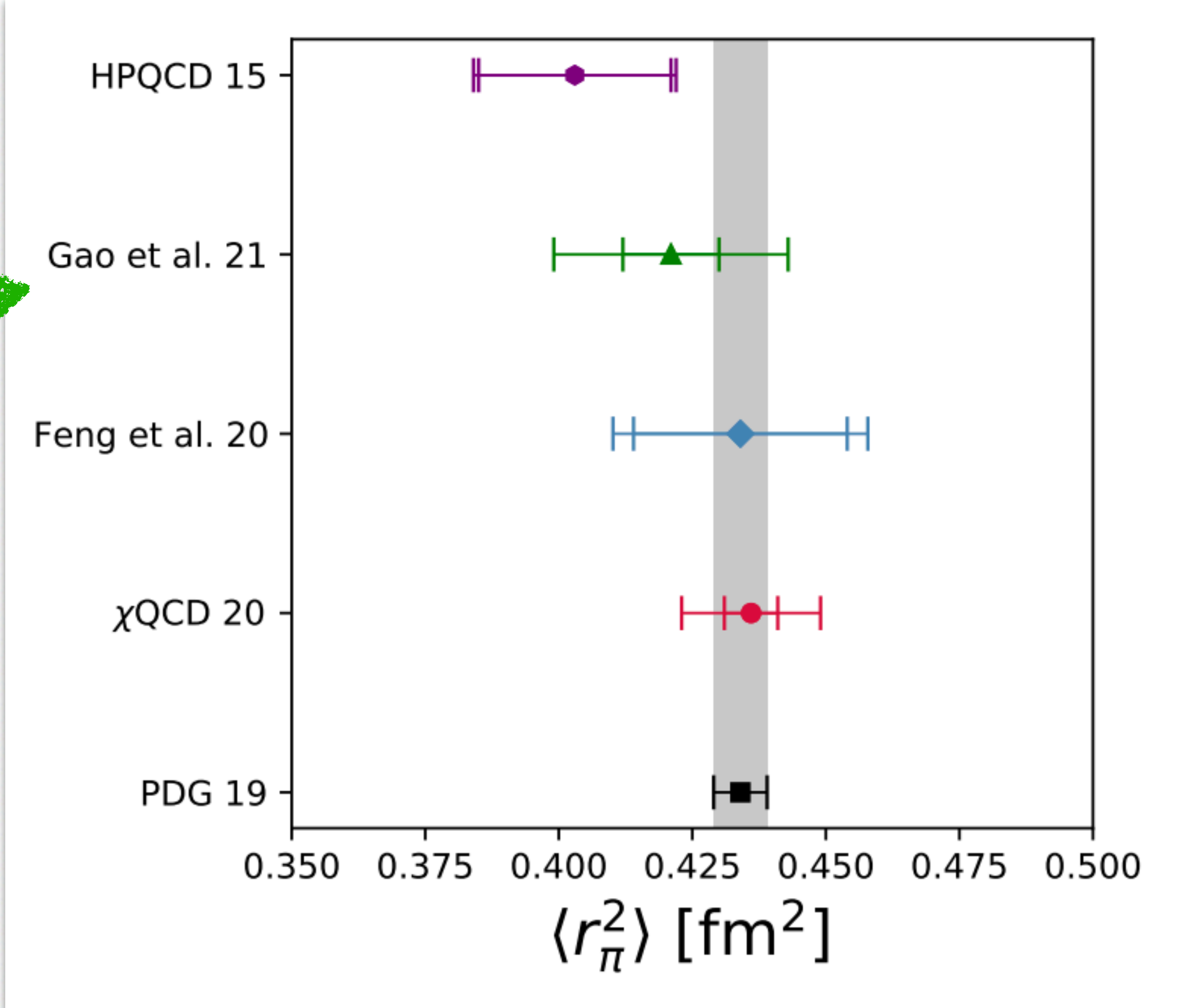
$$\lambda = zP_z$$



Phys.Rev.D 106 (2022) 7, 074505

Pion form factors and charge radius



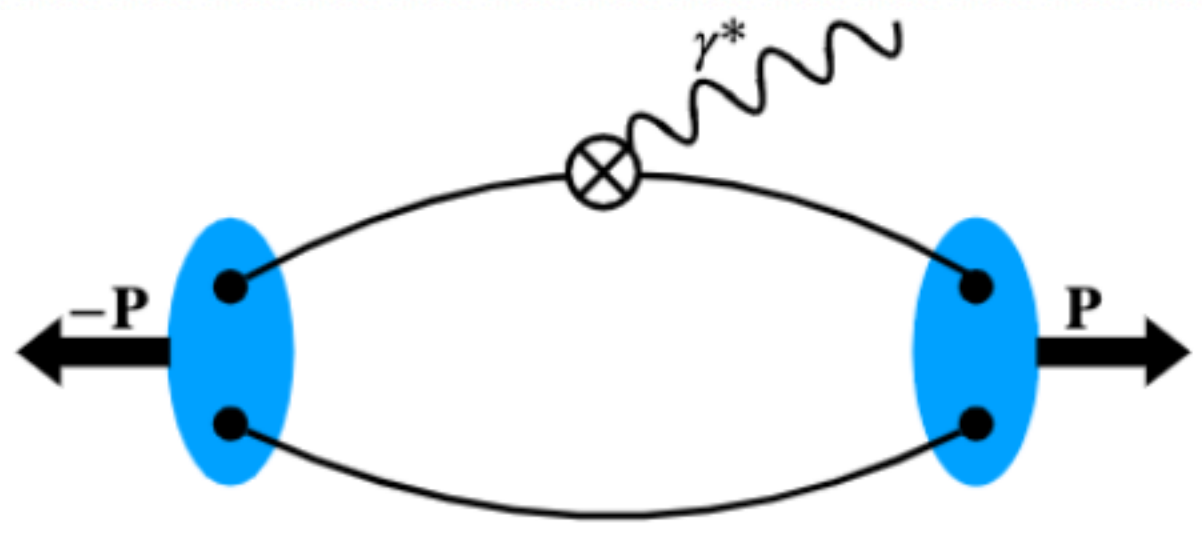


Phys.Rev.D 104 (2021) 11, 11

$$(P_1 + P_2)_\mu F_\pi(Q^2) = \langle \pi^+; P_1 | \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d | \pi^+; P_2 \rangle$$

Pion form factors at large Q^2

The pion form factors in breit frame,



$$Q^2 = (2P_z)^2$$

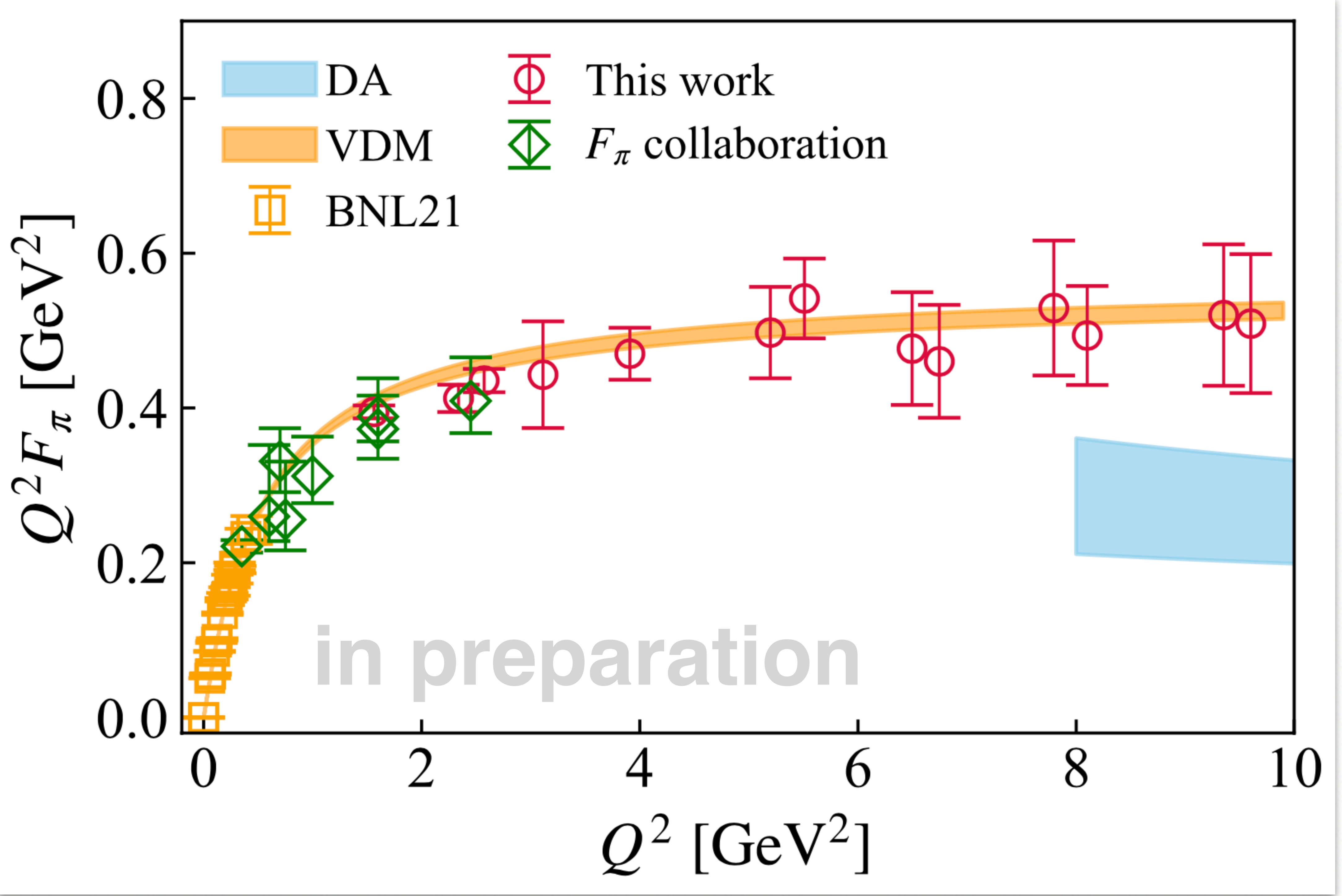
$$P_z \to 1.53\ \text{GeV}$$

$$2P_0 F_\pi(Q^2)$$

$$= \langle \pi^+; -P | \frac{2}{3}\bar{u}\gamma_0 u - \frac{1}{3}\bar{d}\gamma_0 d | \pi^+; P \rangle$$

➡ We keep the quark boost along with the hadron momentum to optimize the boosted smearing

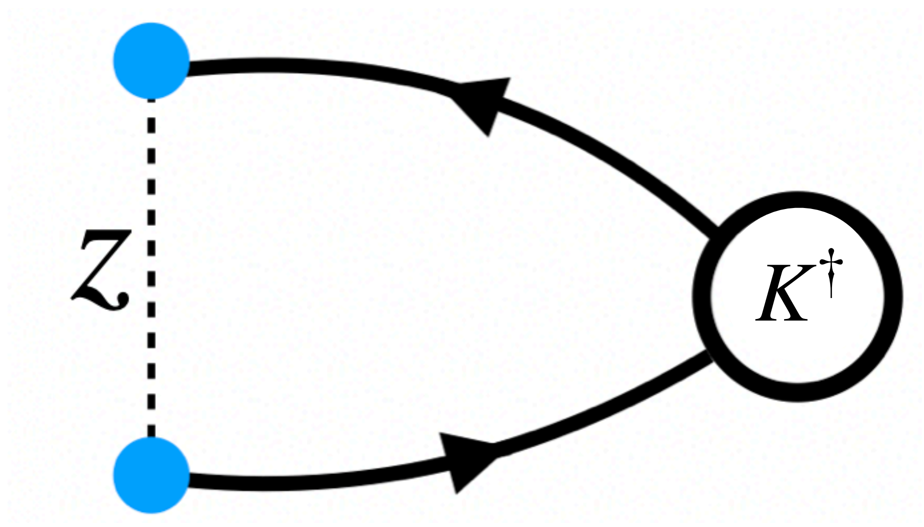
$$\xi = \frac{-k}{-P} = \frac{k}{P}$$



- Overlap with existing experimental extraction and extend the range of Q^2 up to 10 GeV^2.
- Can be described by the vector dominance model using the pion radius computed from the same lattice.
- Higher than the pQCD prediction using the DA (blue band) from the same lattice.

Kaon DA and form factors

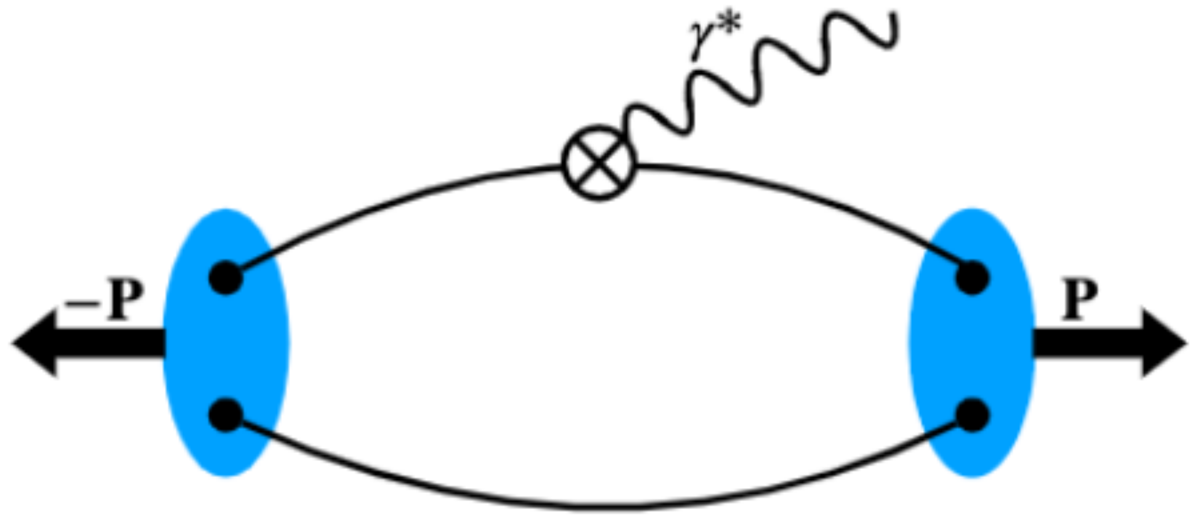
The quasi-DA matrix elements of kaon,



$$iP_z h(z, P_z) = \langle 0 | \bar{s}(-z_3/2)\gamma_z\gamma_5 W_{z_3} u(z_3/2) | K^+; P\rangle$$

- ➡ $m_\pi = 140$ MeV, $m_K = 495$ MeV
- ➡ Under analysis with momentum P_z up to 1.76 GeV.

The kaon form factors in breit frame,



$$(P_1 + P_2)_\mu F_K(Q^2) = \langle K^+; -P | \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{s}\gamma_\mu s | K^+; P\rangle$$

- ➡ $Q^2 = (2P_z)^2$ with $P_z \to 1.53$ GeV
- ➡ This proposal.

- The impact of isospin symmetry breaking.
- Competition between the emergent mass generation and Higgs mechanism.

Proposal request

- **Software:**
  - Uses the Qlua package
  - Inversions done with multi-grid solvers
  - Interfaces with QUDA
- **Statistics:**
  - 350 configs
  - 1 exact and 32 sloppy solves for $P_z = 3\frac{2\pi}{L_s a}$
  - 2 exact and 64 sloppy solves for $P_z = 5\frac{2\pi}{L_s a}$ & $6\frac{2\pi}{L_s a}$
- **Requested resources:**
  - 204k K80 hours
  - 25 TB disk space
  - 50 TB archival storage

Timings using 16 nodes of the BNL IC cluster (32 K80 cards)

extended source/sink creation	t_{ext}	10 sec.
MG u/d-quark inversion sloppy	t^u_{inv}	26 sec.
MG u/d-quark inversion exact	t^u_{inv}	50 sec.
MG s-quark inversion sloppy	t^s_{inv}	18 sec.
MG s-quark inversion exact	t^s_{inv}	27 sec.
contraction for kaon 2pt function	t^K_{2pt}	10 sec.
contraction for 3pt function for FF	t^{FF}_{3pt}	4 sec.

$$t_{src} = t^u_{inv} + t^s_{inv} + 2t^K_{2pt} + 4t_{ext}$$
$$+(t^u_{inv} + t^s_{inv} + 2t_{ext} + 2t^{FF}_{3pt}) \times N_{sep}$$

Summary

- Propose to calculate large-Q^2 kaon form factors as a continuation of pion form factors and DA calculations from last year.
- Our collaboration has made significant progress toward the understanding of the pion structure, including pion charge radius, form factors, distribution amplitude as well as valence quark PDF.
- We will apply the techniques described here to produce and analyze the data for the new allocation.

Thanks for your attention