Resonances in QCD and their Couplings from Anisotropic Clover Lattices

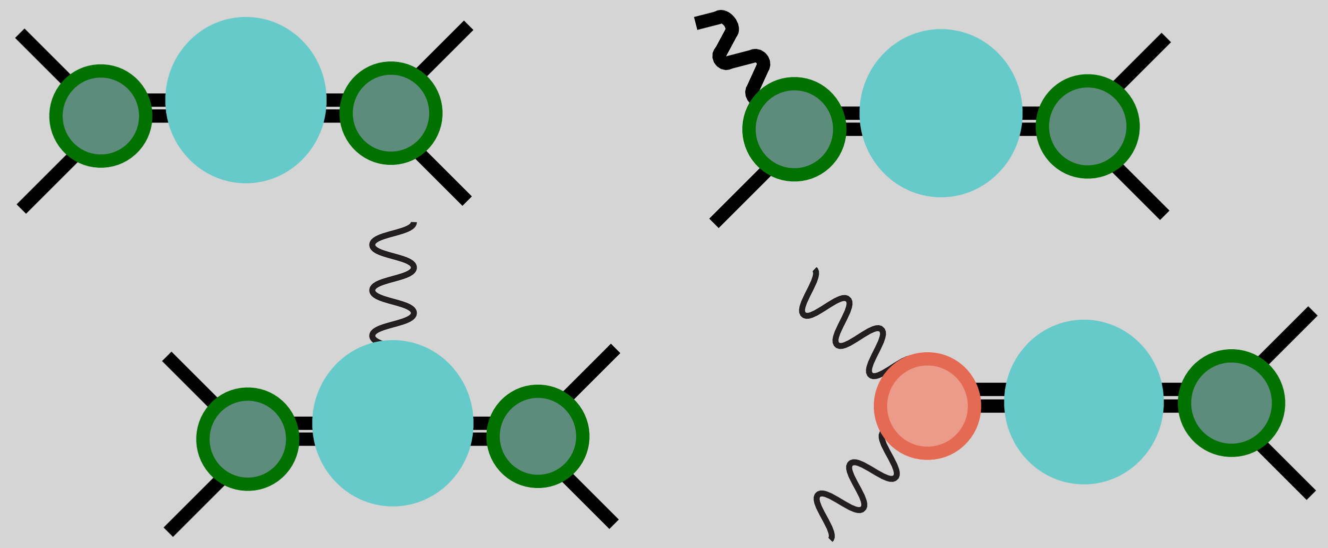
USQCD 2023

Arkaitz Rodas

Jefferson Lab
Thomas Jefferson National Accelerator Facility

hadspec

Goals



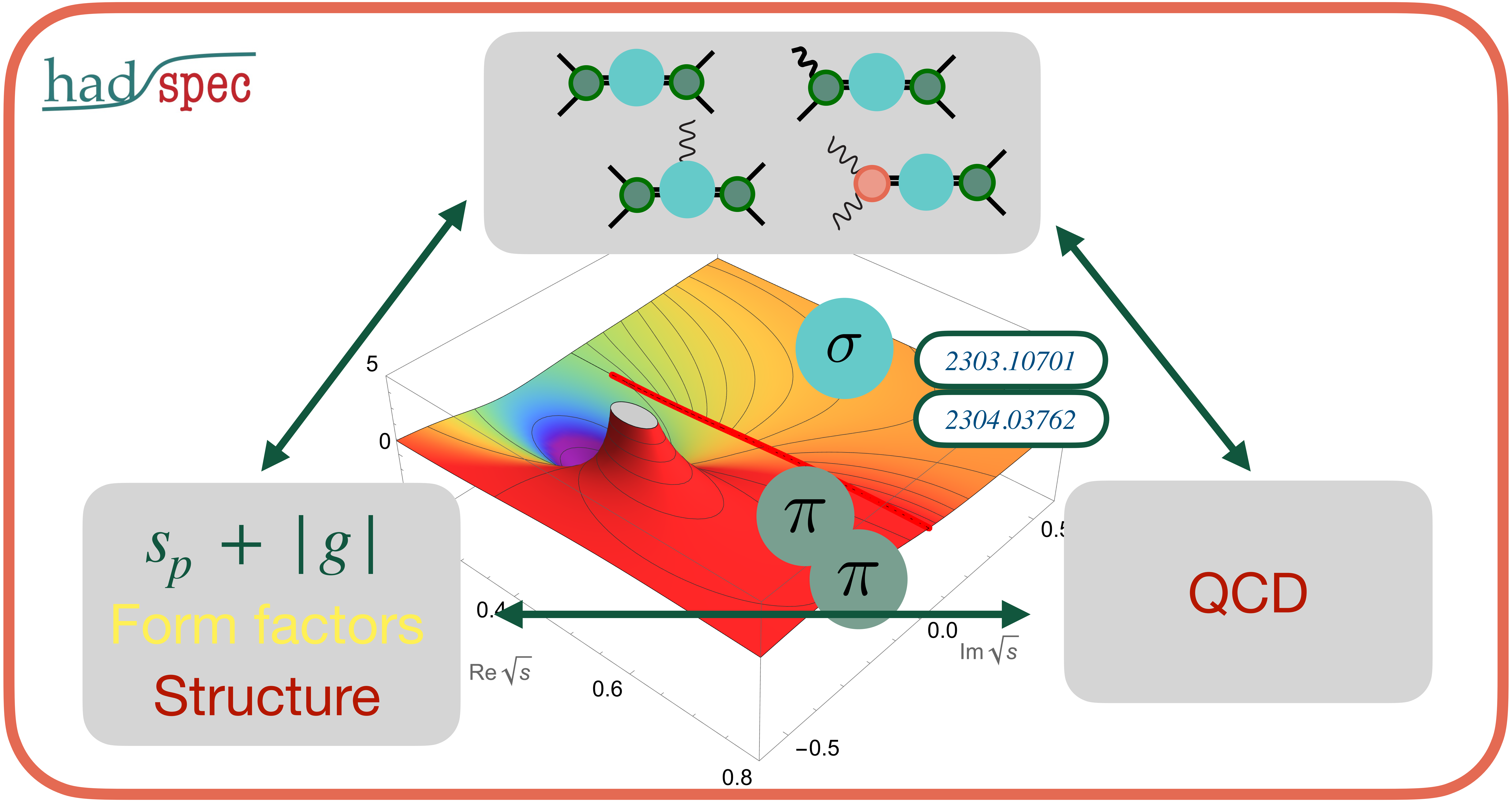
$s_p + |g|$

Form factors

Structure

QCD

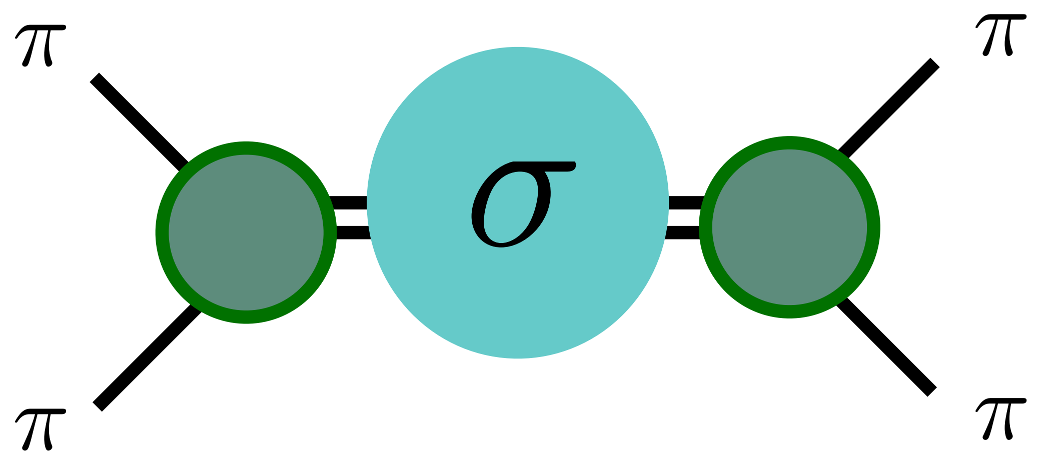
Goals

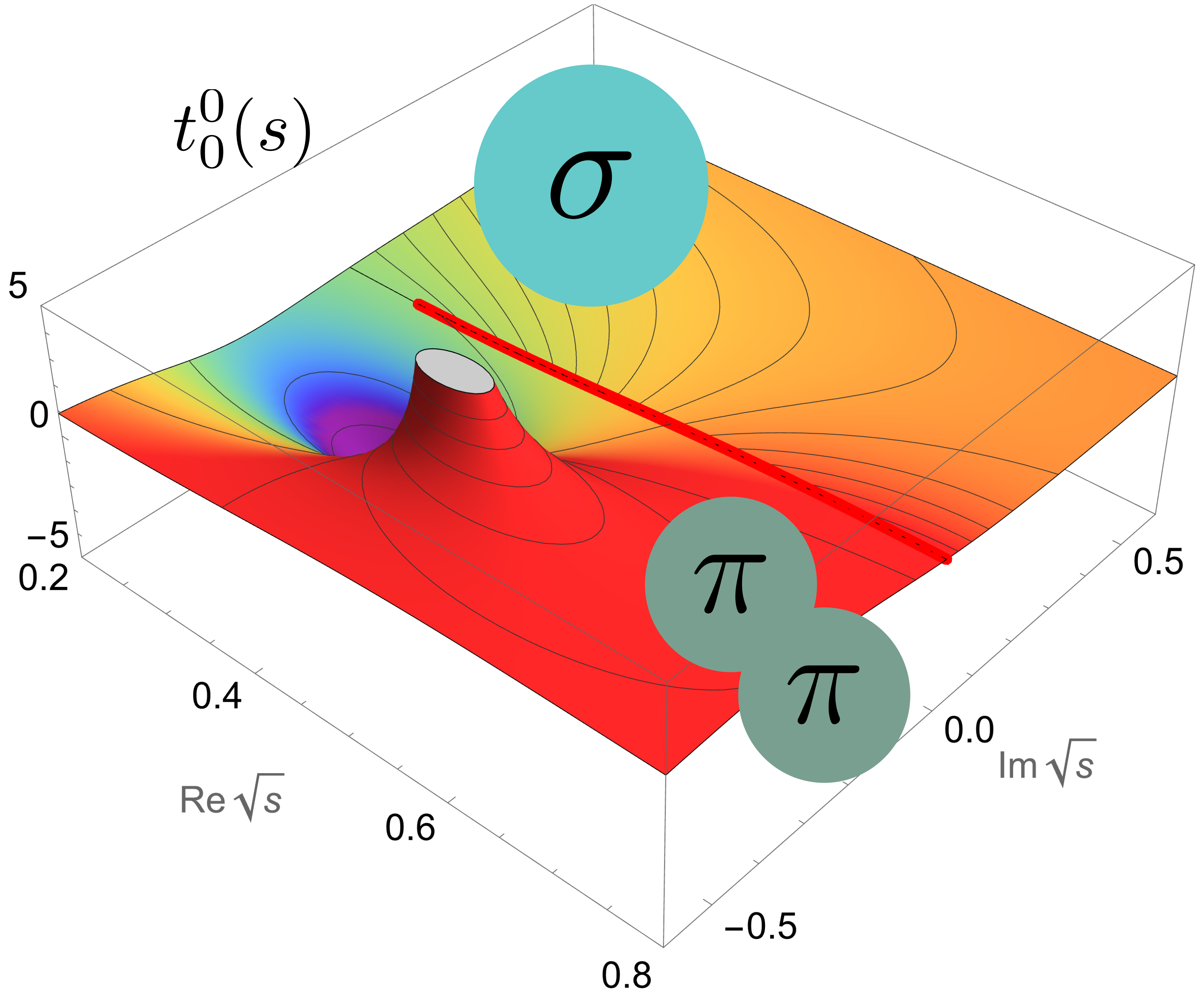


σ
π
π
2303.10701
2304.03762
5
0
0.4
0.6
0.8
Re √s
0.5
0.0
−0.5
Im √s
$s_p + |g|$
Form factors
Structure
QCD

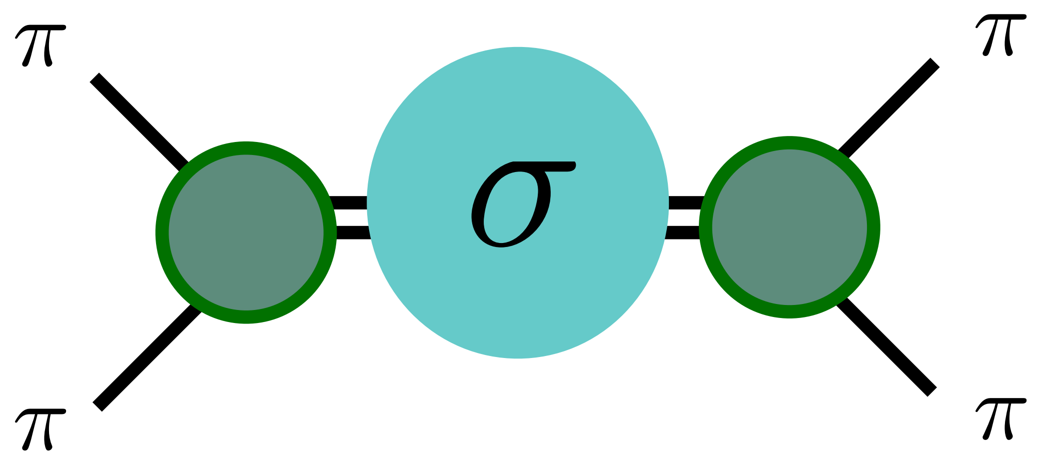
Light Scalars: the σ

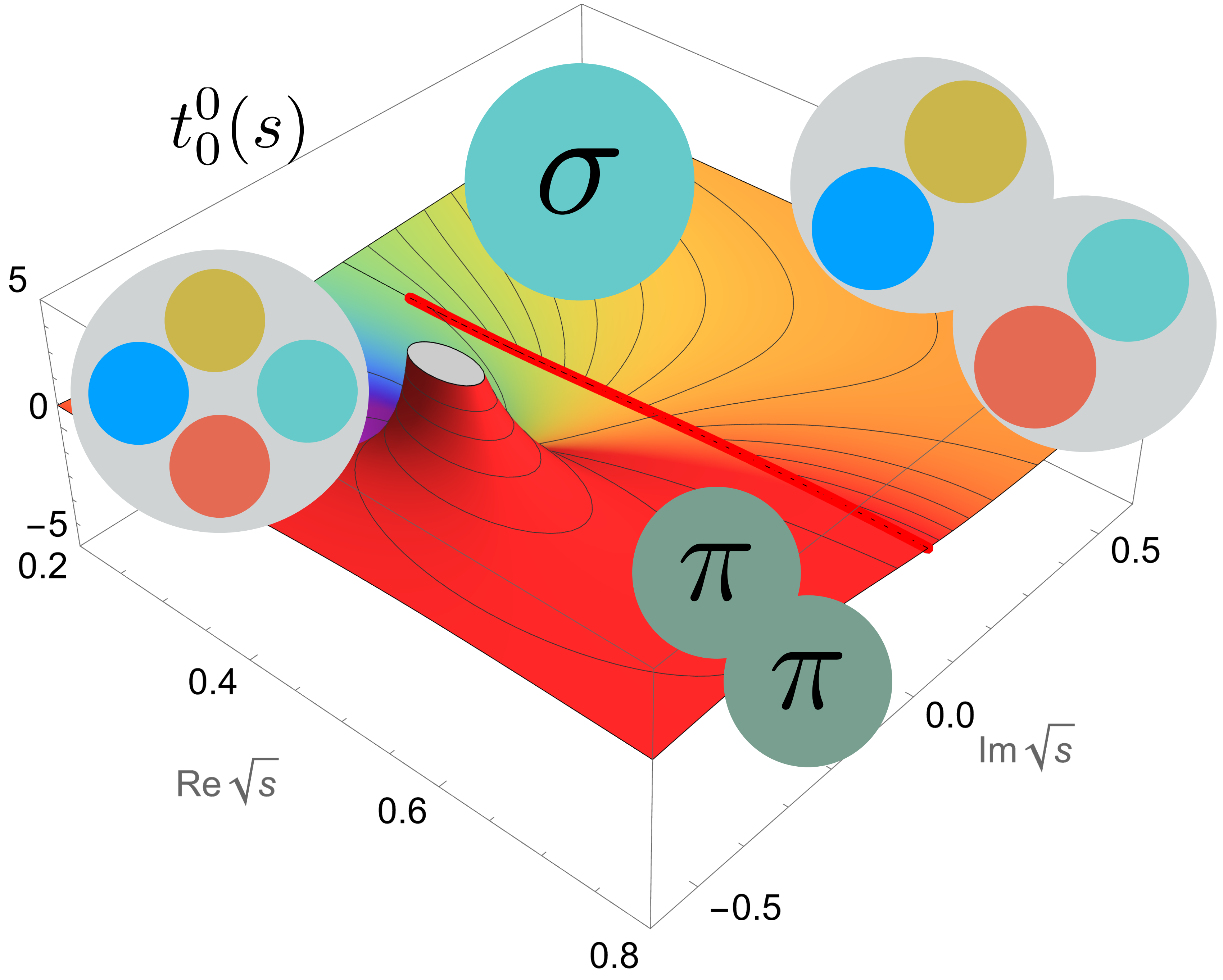
Lightest resonance in QCD





Light Scalars: the σ





Lightest resonance in QCD

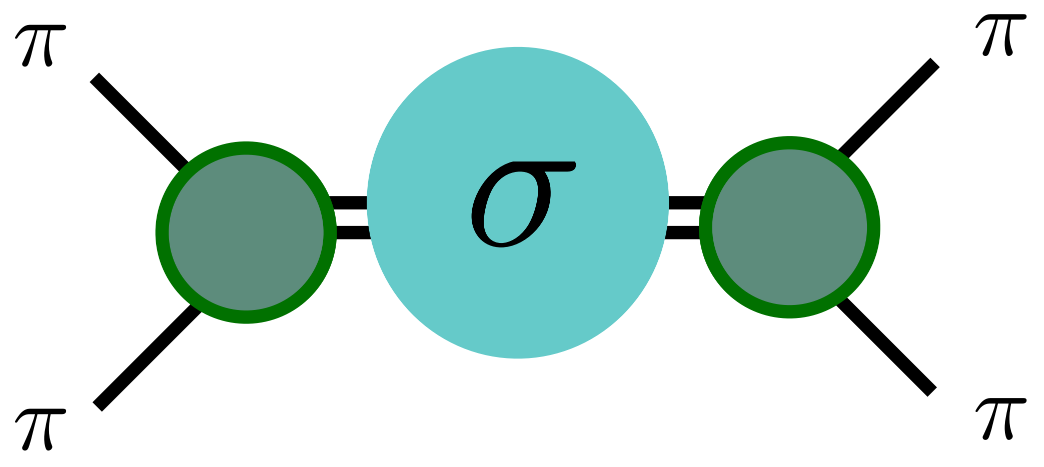
Extremely broad → extremely short-lived

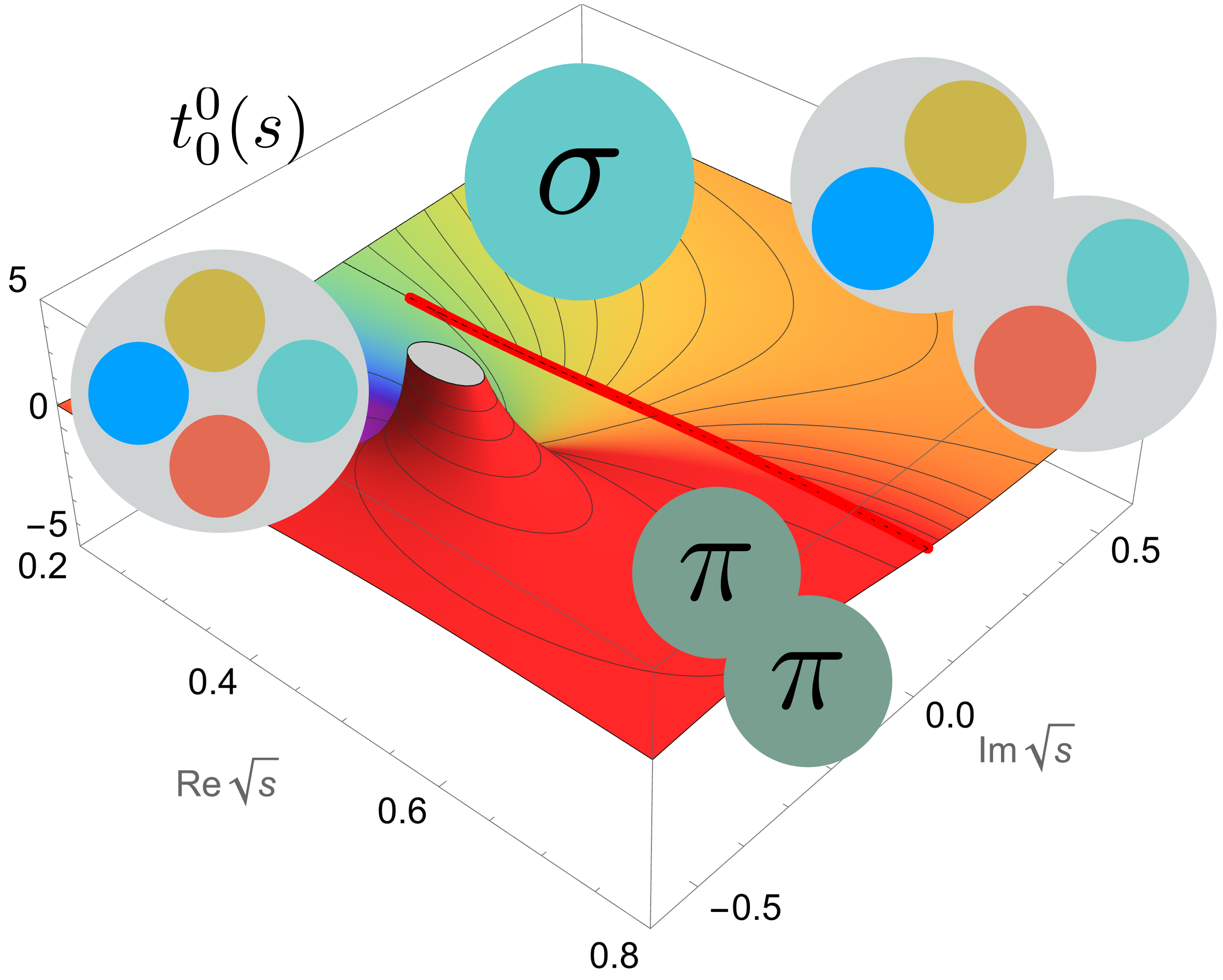
Correlated with chiral symmetry-breaking phenomena

Not well-understood → new observables ??

Input to hadron physics observables

Light Scalars: the σ





Lightest resonance in QCD

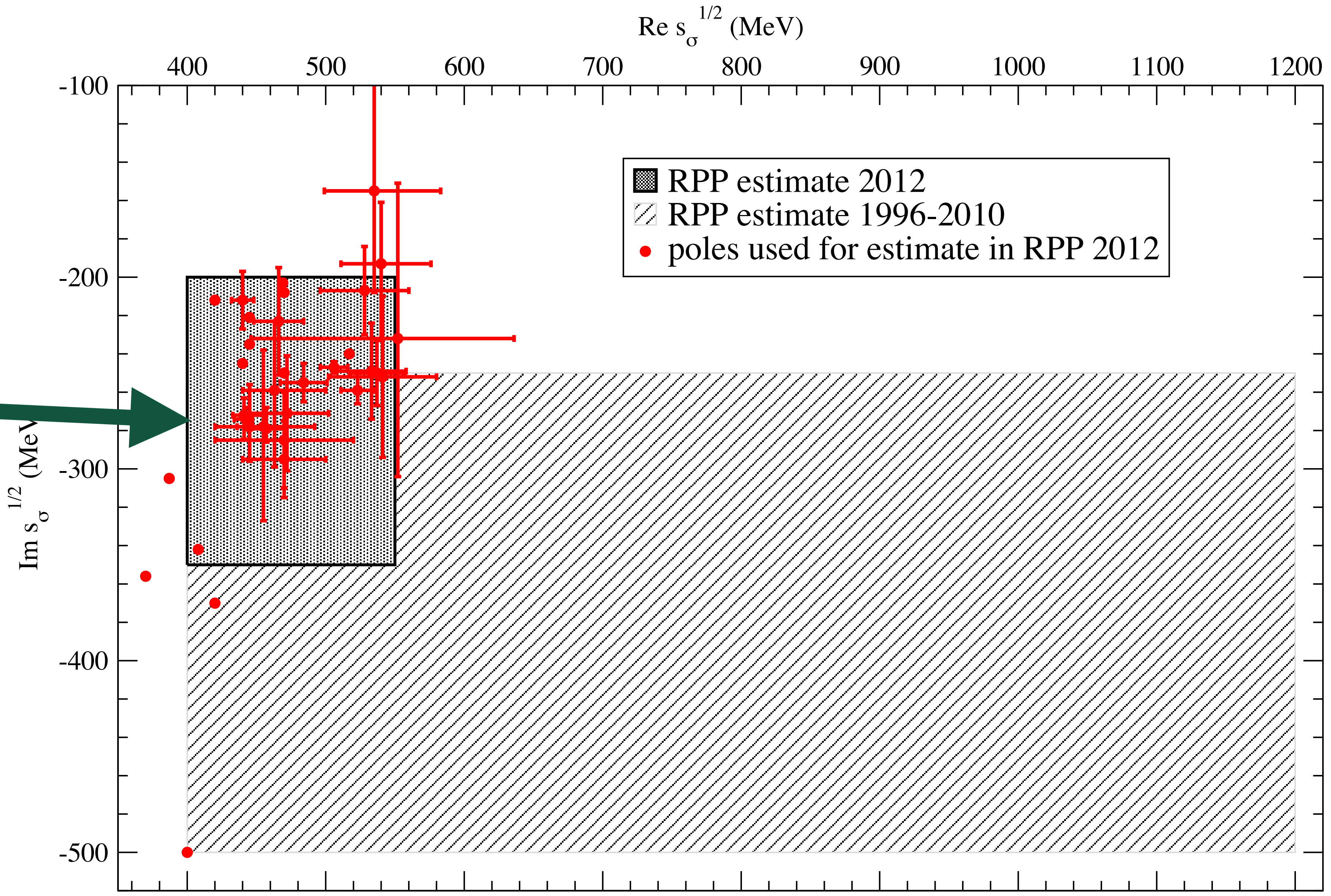
Extremely broad → extremely short-lived

Correlated with chiral symmetry-breaking phenomena

Not well-understood → new observables ??

Input to hadron physics observables

Very challenging experimental extraction

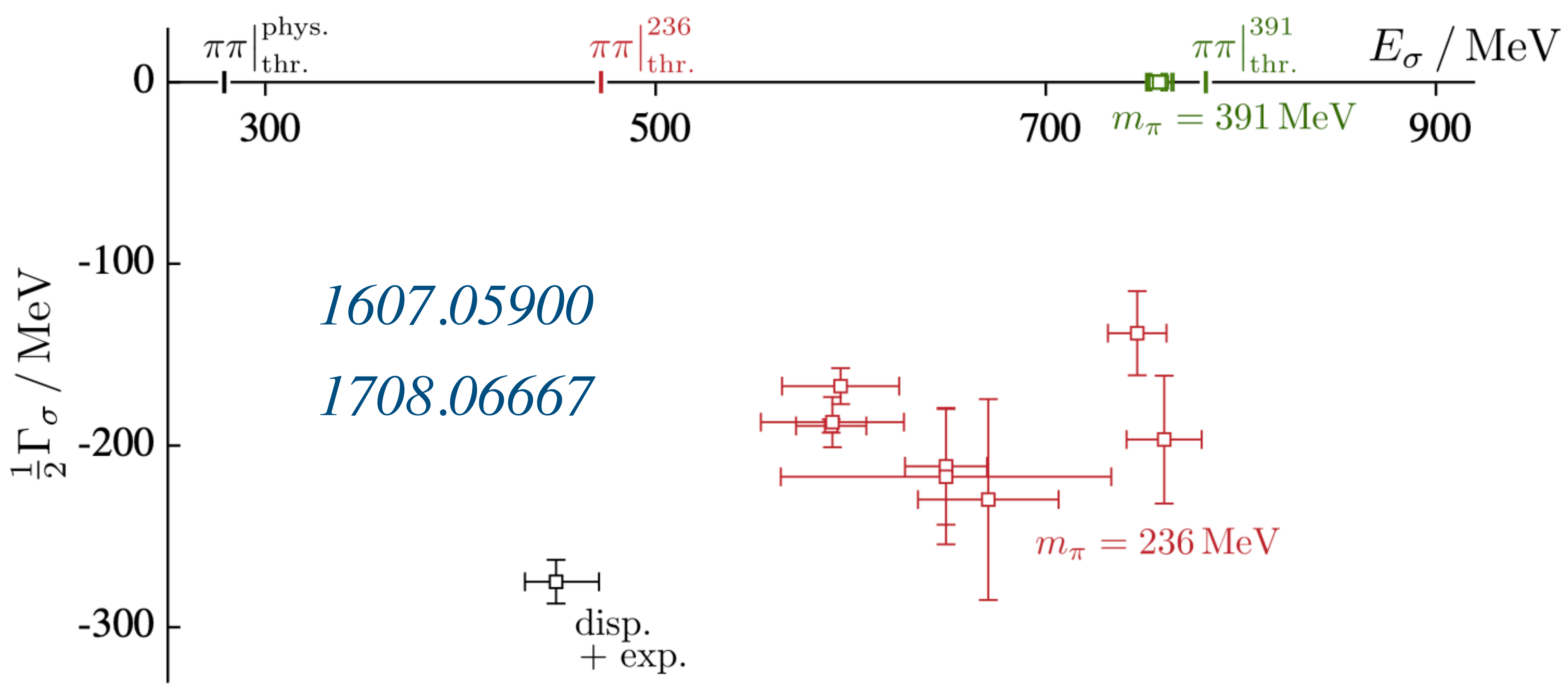


What happens for Lattice QCD ??

Light Scalars: the σ

Other works

16010.10070 1803.02897



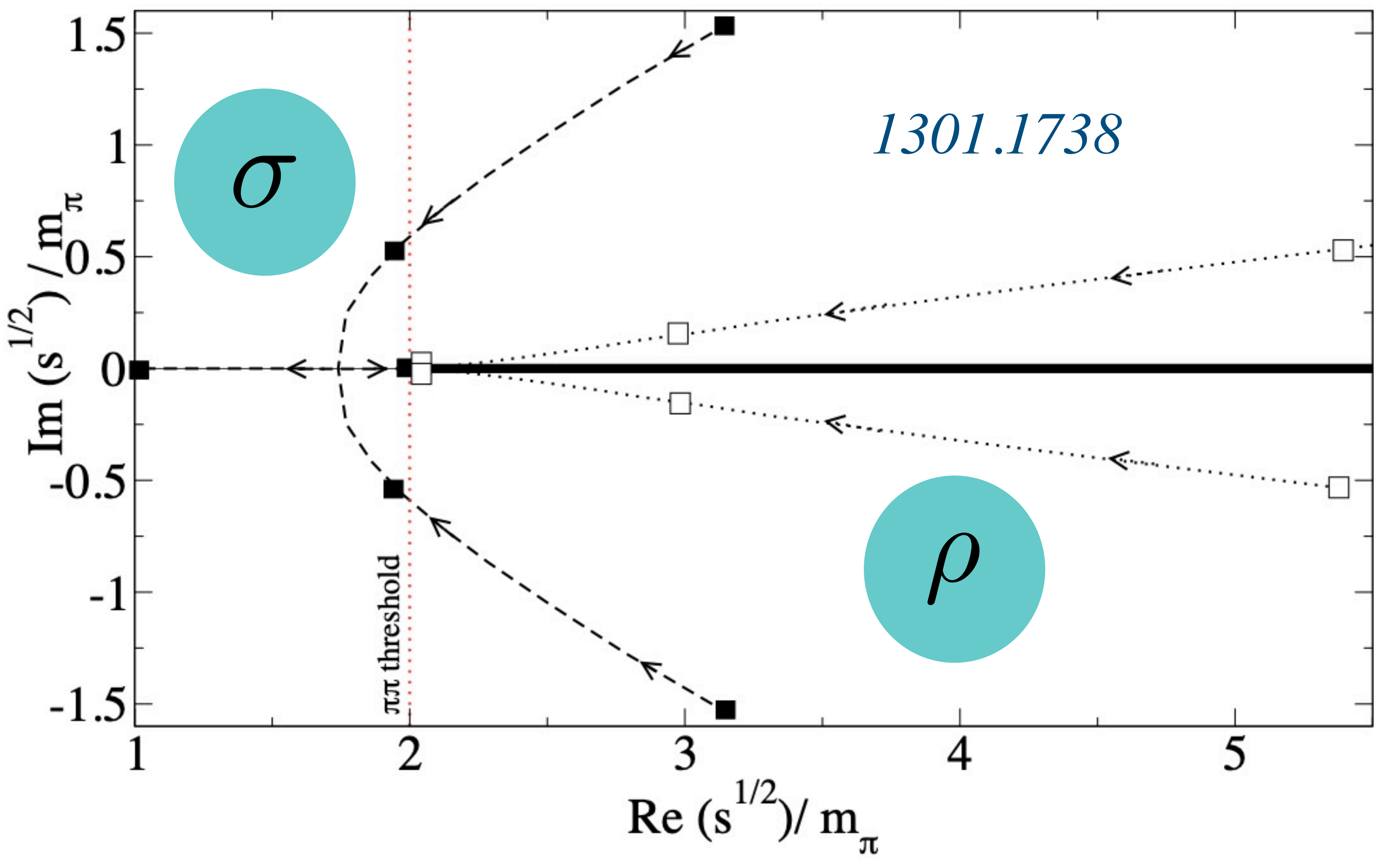
Past

- $m_\pi \sim 391\ MeV \rightarrow$ **Stable**
- $m_\pi \sim 239\ MeV \rightarrow$ **Broad resonance**

This year

2303.10701

- $m_\pi \sim 330\ MeV \rightarrow$??
- $m_\pi \sim 283\ MeV \rightarrow$??



Light Scalars: the σ

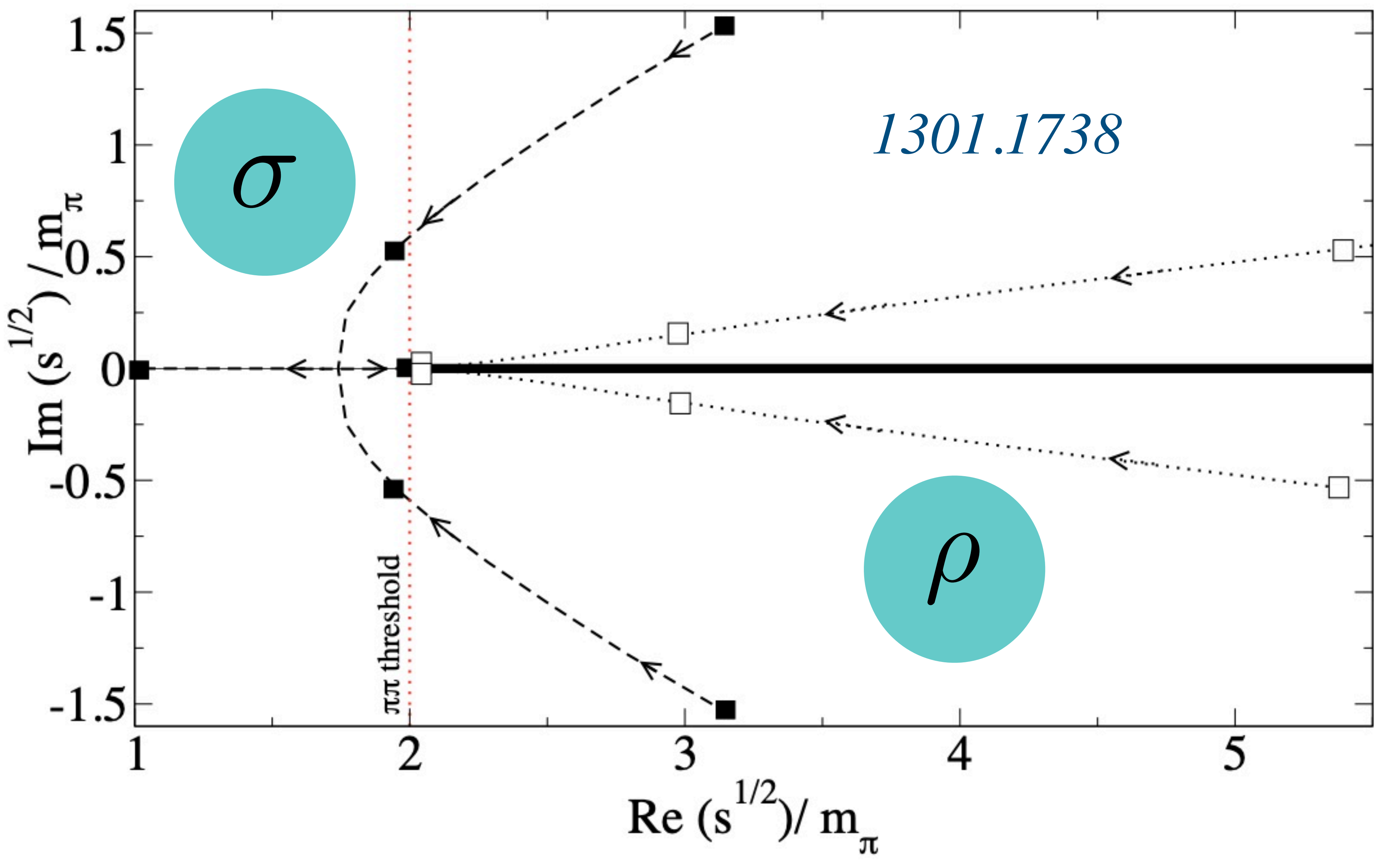
Other works

16010.10070 1803.02897

$-a_t m_\ell$	$(L/a_s)^3 \times T/a_t$	N_{cfgs}	N_{vecs}	$N_{t_{\mathrm{src}}}$	$a_t m_\pi$	$a_t m_K$	$a_t m_\eta$	$a_t m_\Omega$	ξ	m_π/MeV
0.0850	$24^3 \times 256$	473	160	4–16	0.05635(14)	0.09027(15)	0.09790(100)	0.2857(8)	3.467(8)	330
0.0856	$24^3, 32^3 \times 256$	392, 475	160, 256	4–8	0.04720(11)	0.08659(14)	0.09602(70)	0.2793(8)	3.457(6)	283

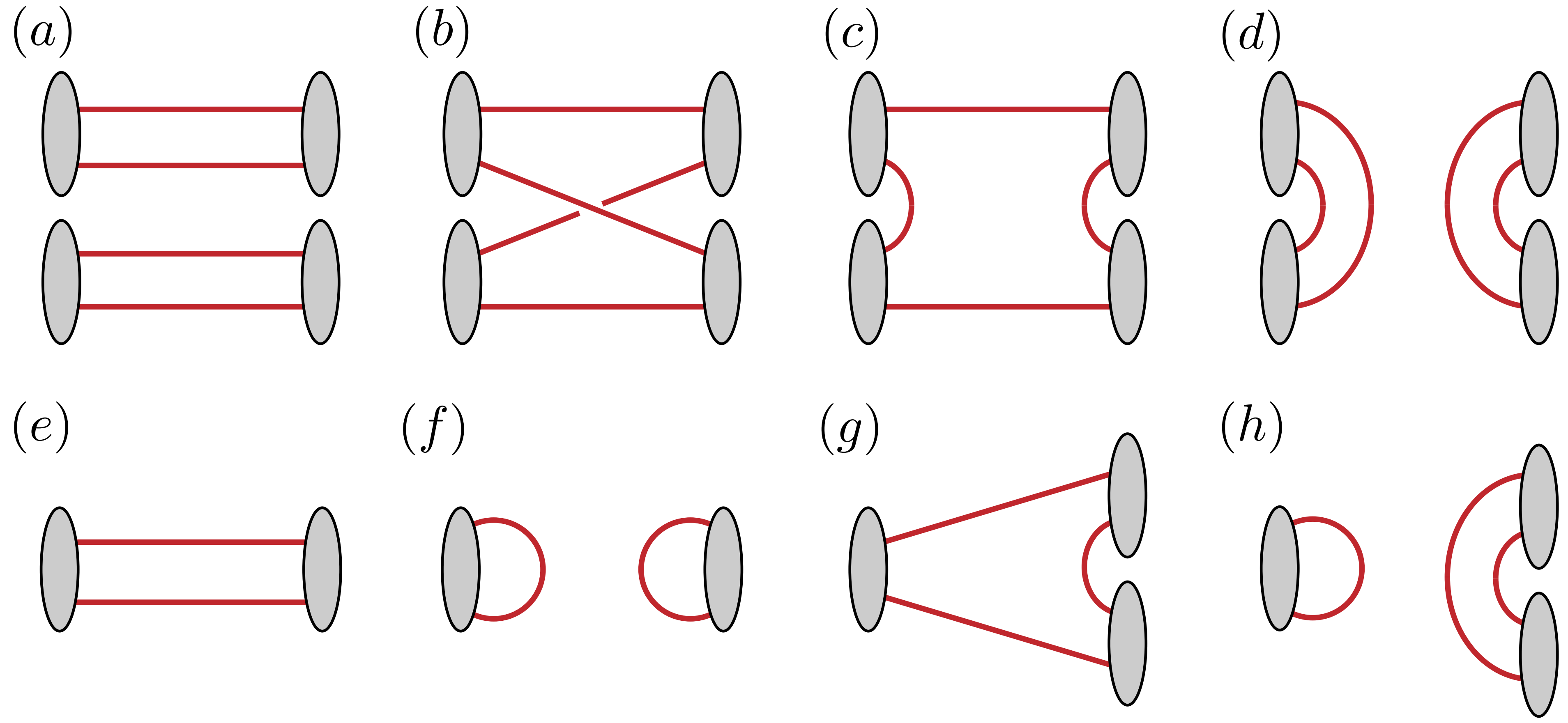
This year 2303.10701

- $m_\pi \sim 330\, MeV \to$??
- $m_\pi \sim 283\, MeV \to$??



Light Scalars: the σ

1607.05900



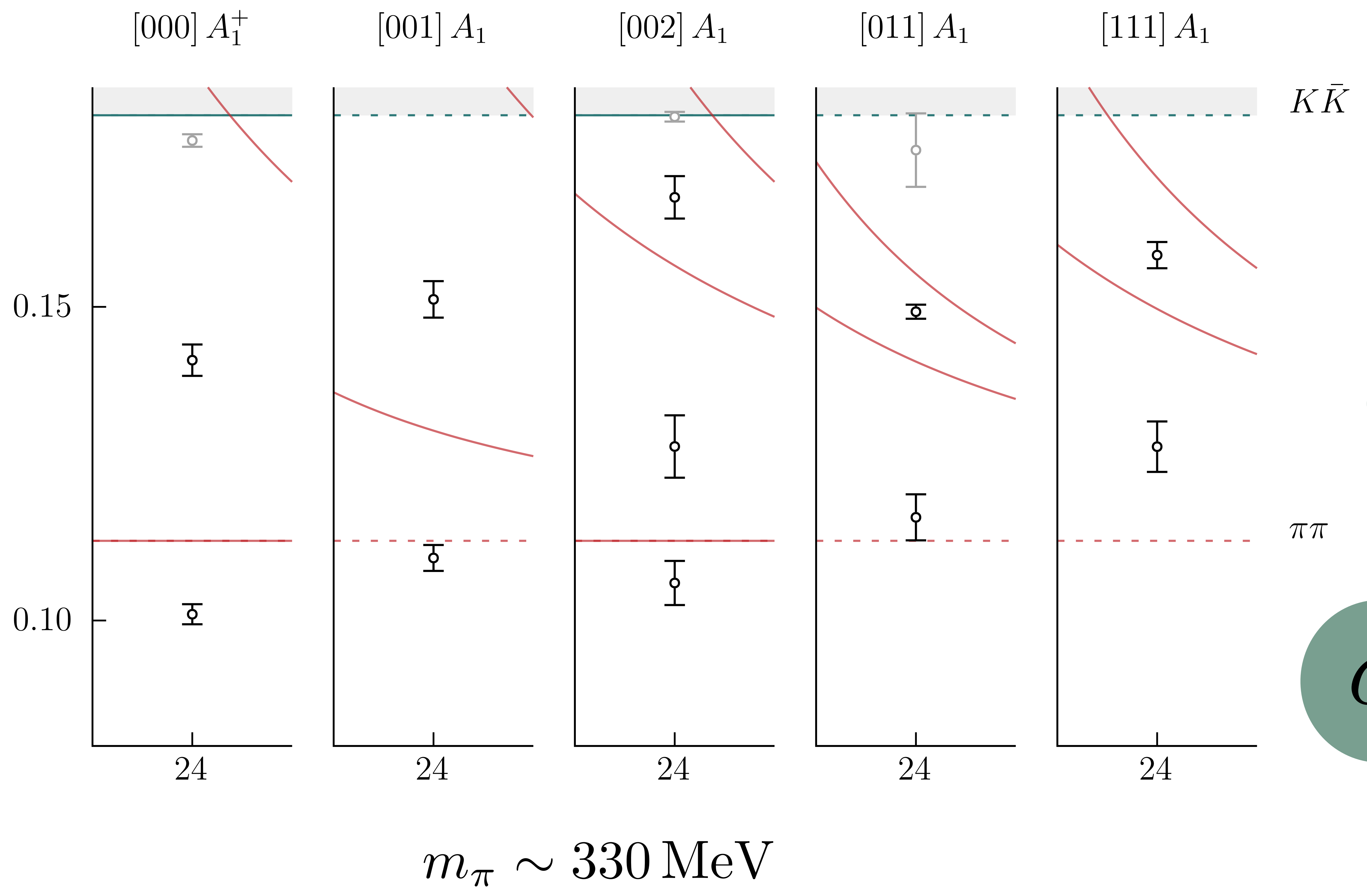
Lattice QCD challenge:

- ☑ **Disconnected diagrams → distillation**
- ☑ **Particles that decay → Lüscher**

Amplitude analysis challenge:

- ☑ **Resonances are poles → unitarity**
- ☐ **Resonances require extrapolations**

$I = 0\ \pi\pi$



Similar spectrum to previous masses

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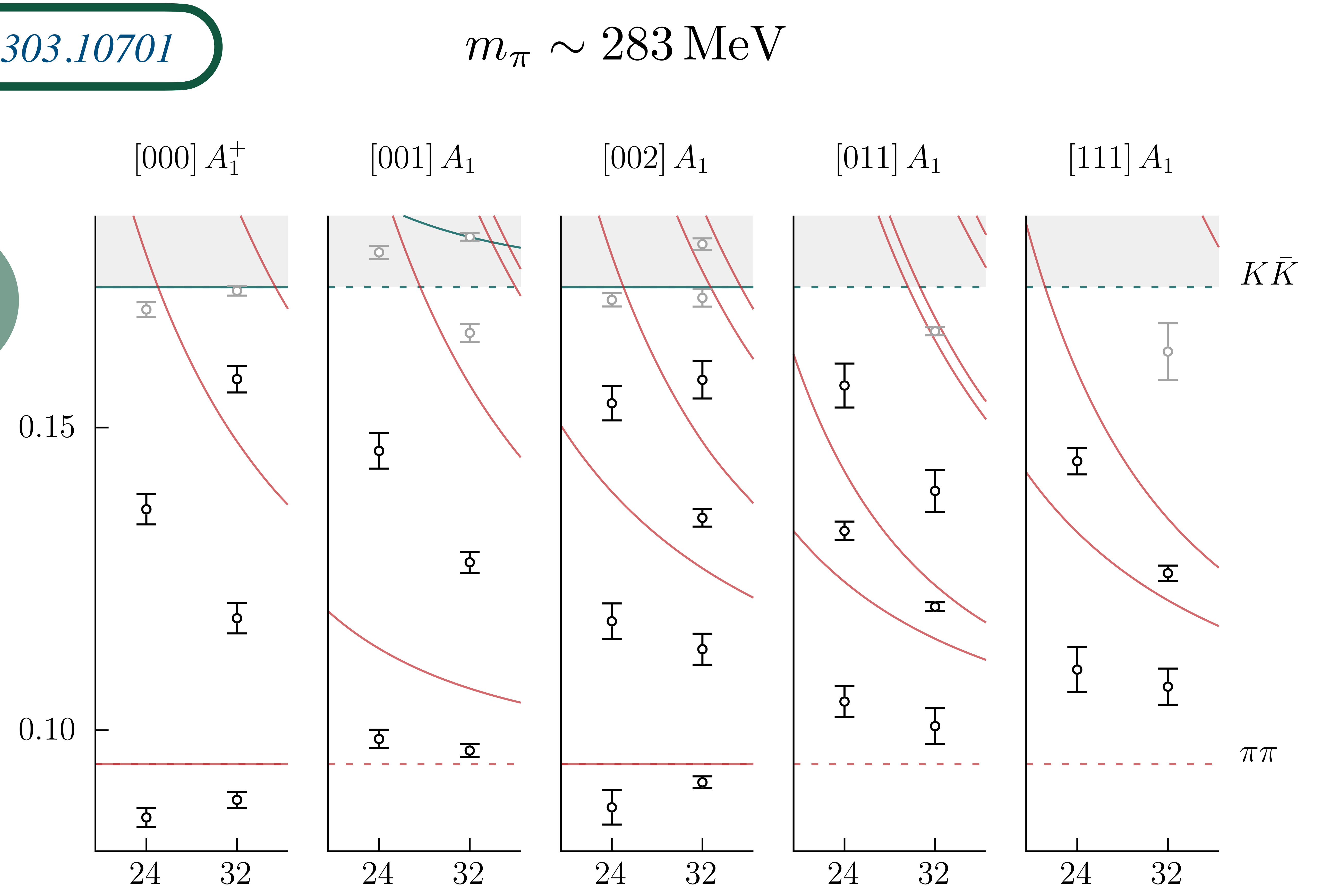
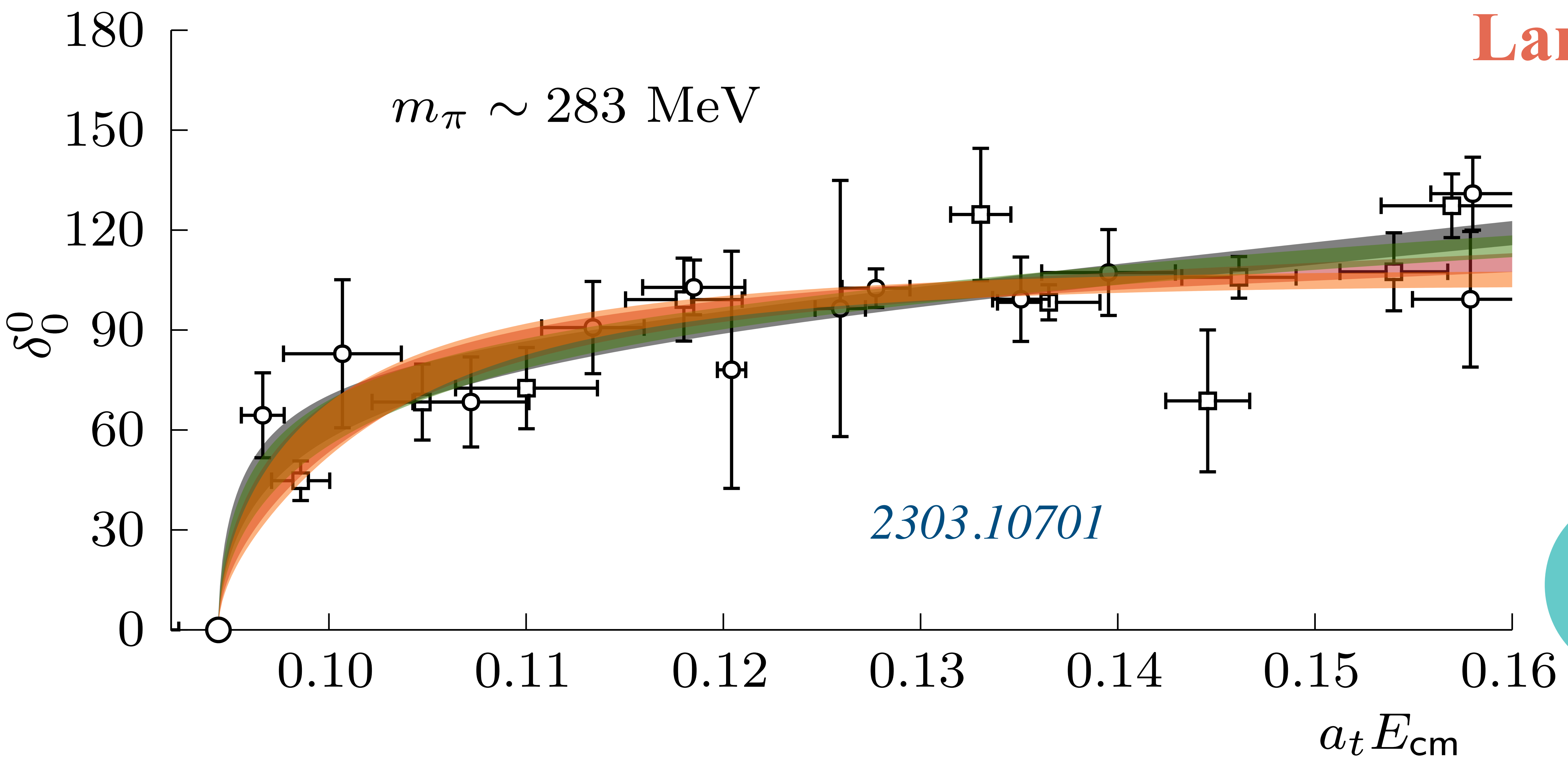
σ



Over 60 "elastic" levels for I=0

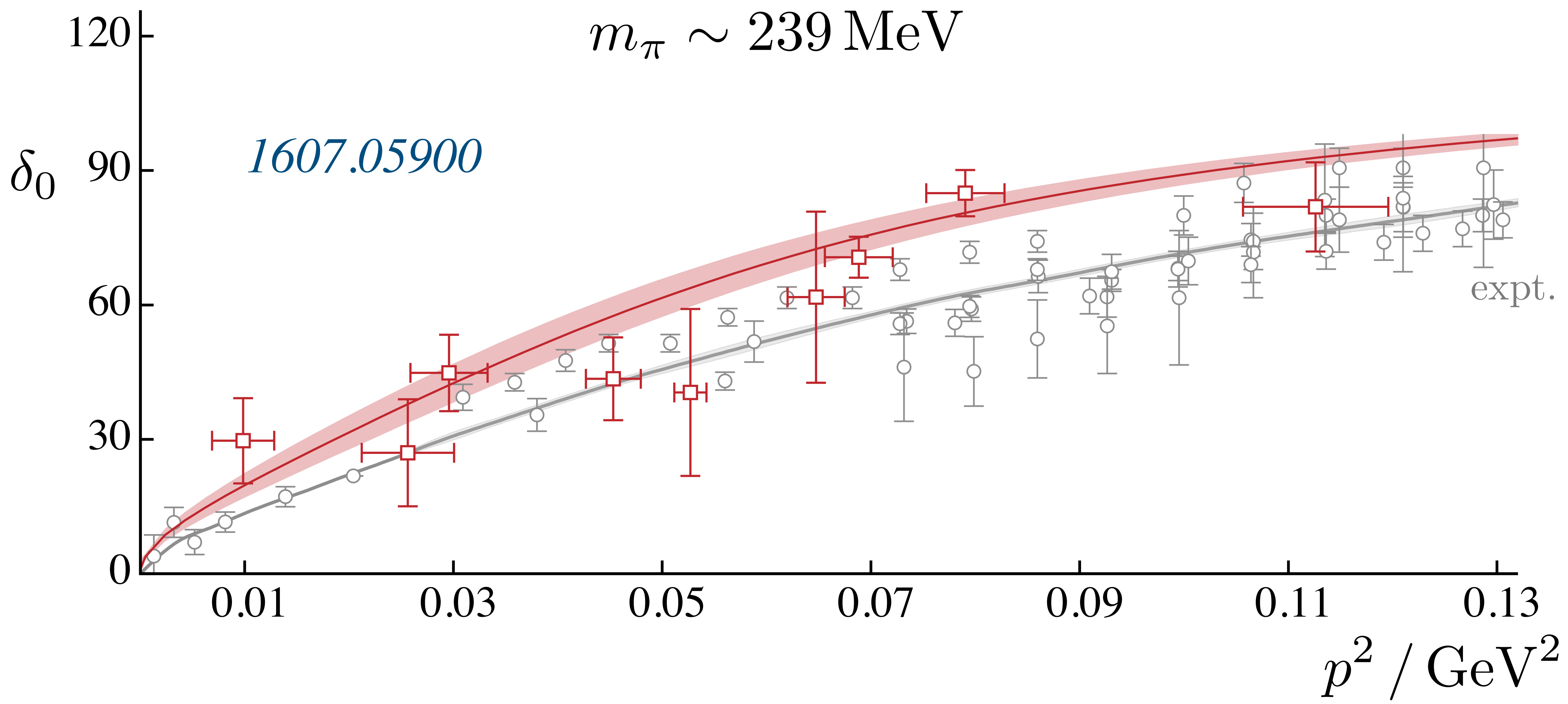
$I = 0\ \pi\pi$



Large derivative at threshold

$\delta(s) = \pi/2$ is far from threshold

σ



Over 20 parameterizations

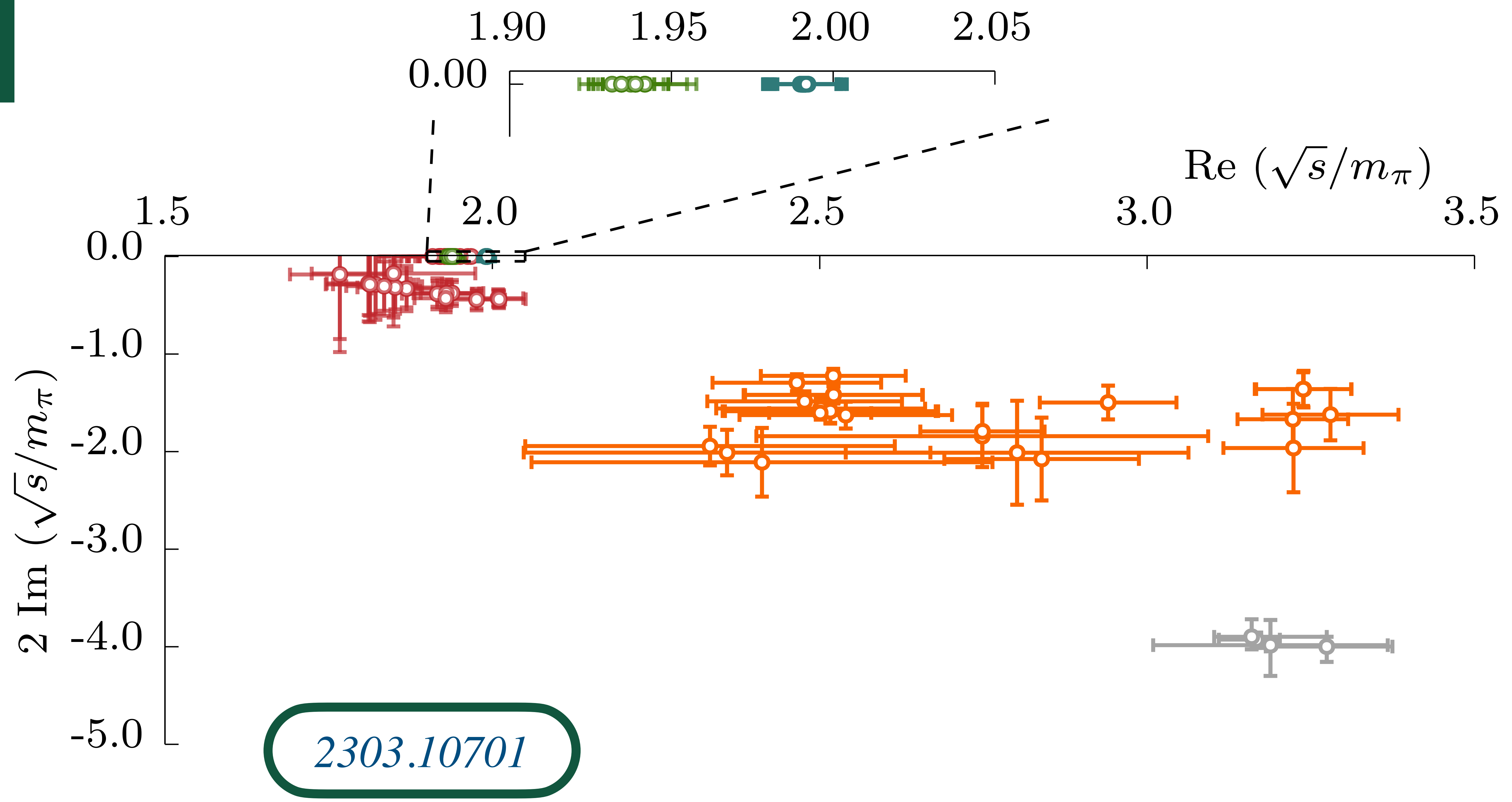
Smaller derivative at threshold

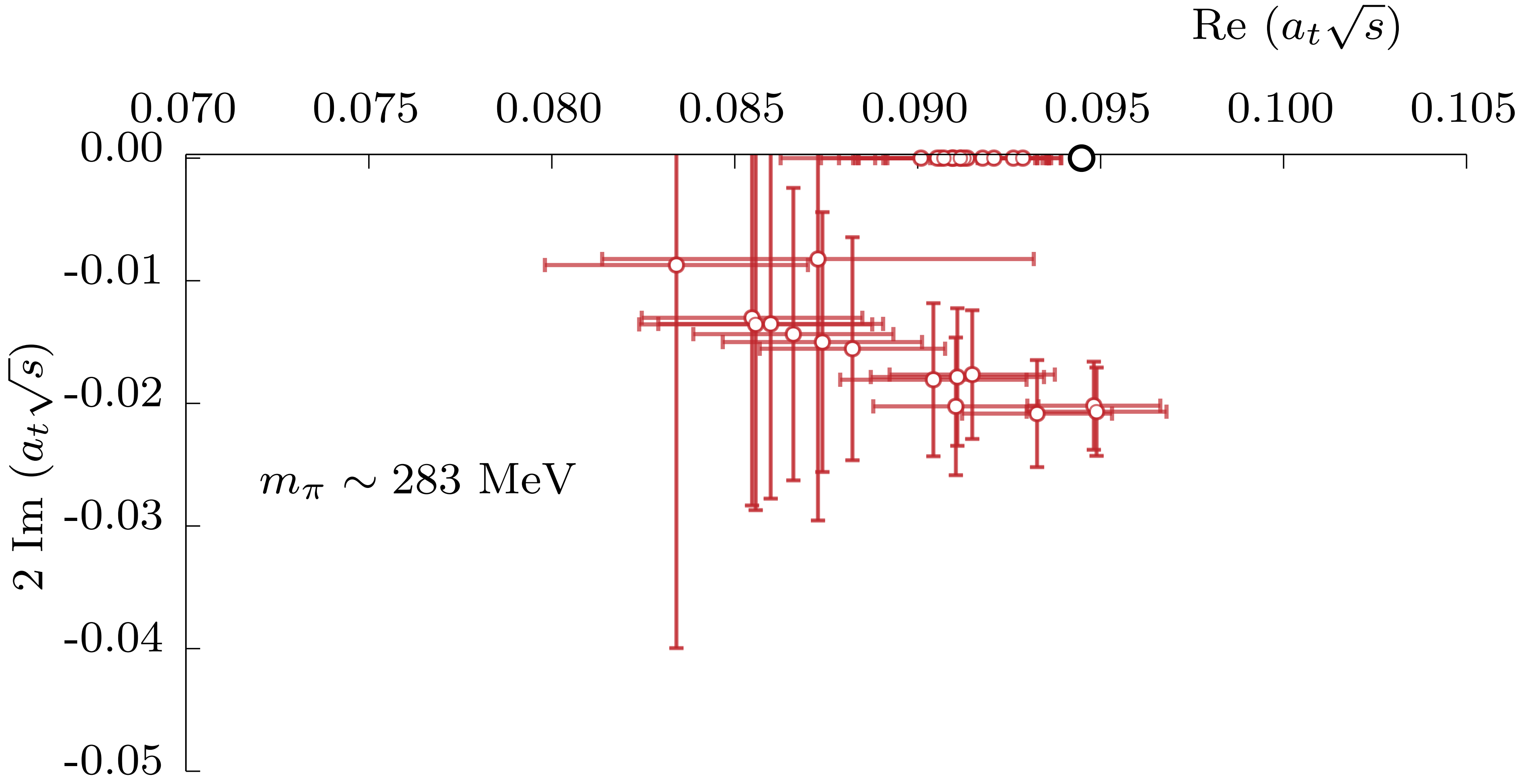
Resonances require extrapolations

Over 20 parameterizations

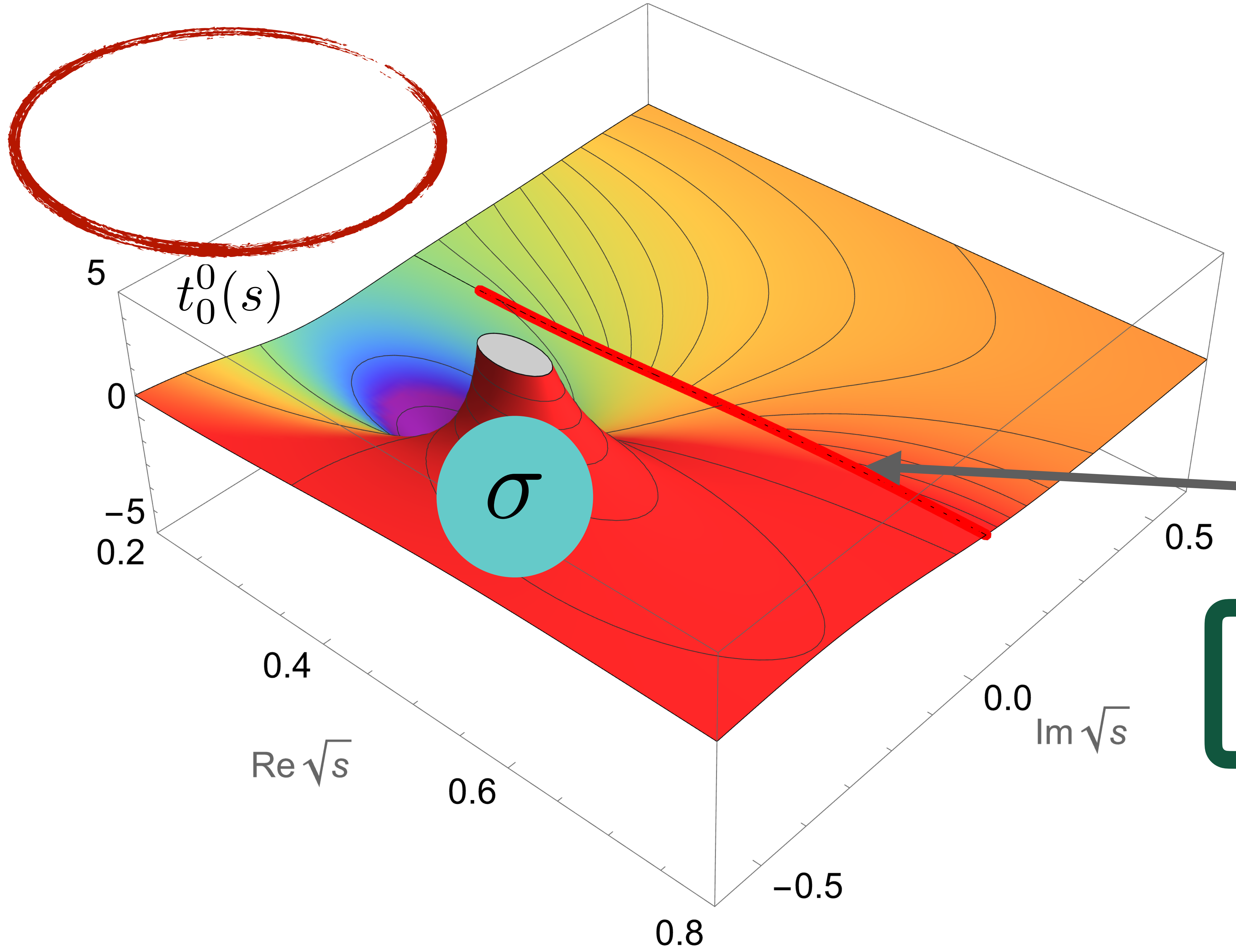
$m_\pi \sim 239\, MeV \rightarrow$ **Broad resonance**

$m_\pi \sim 283\, MeV \rightarrow$ **??**

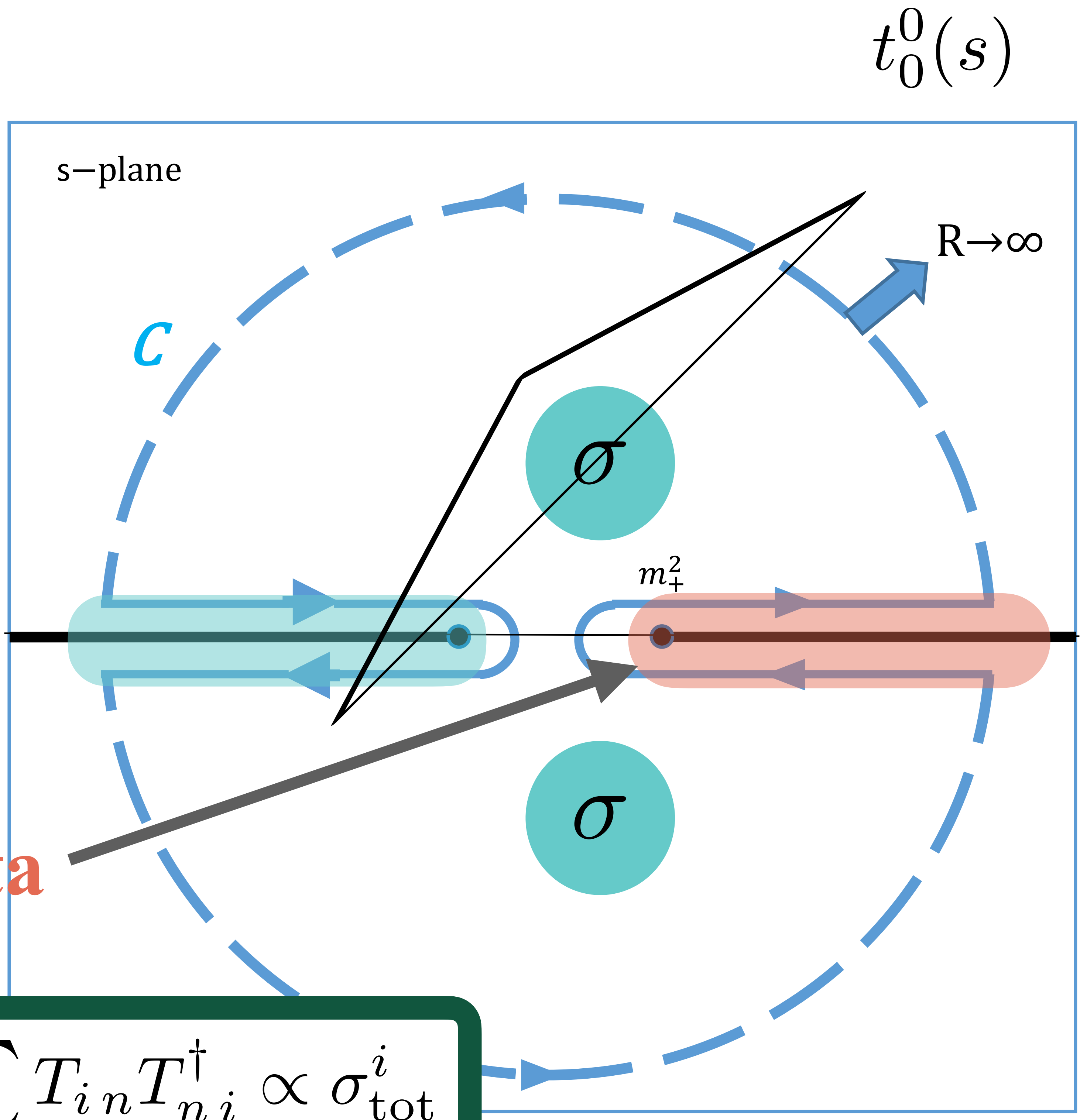




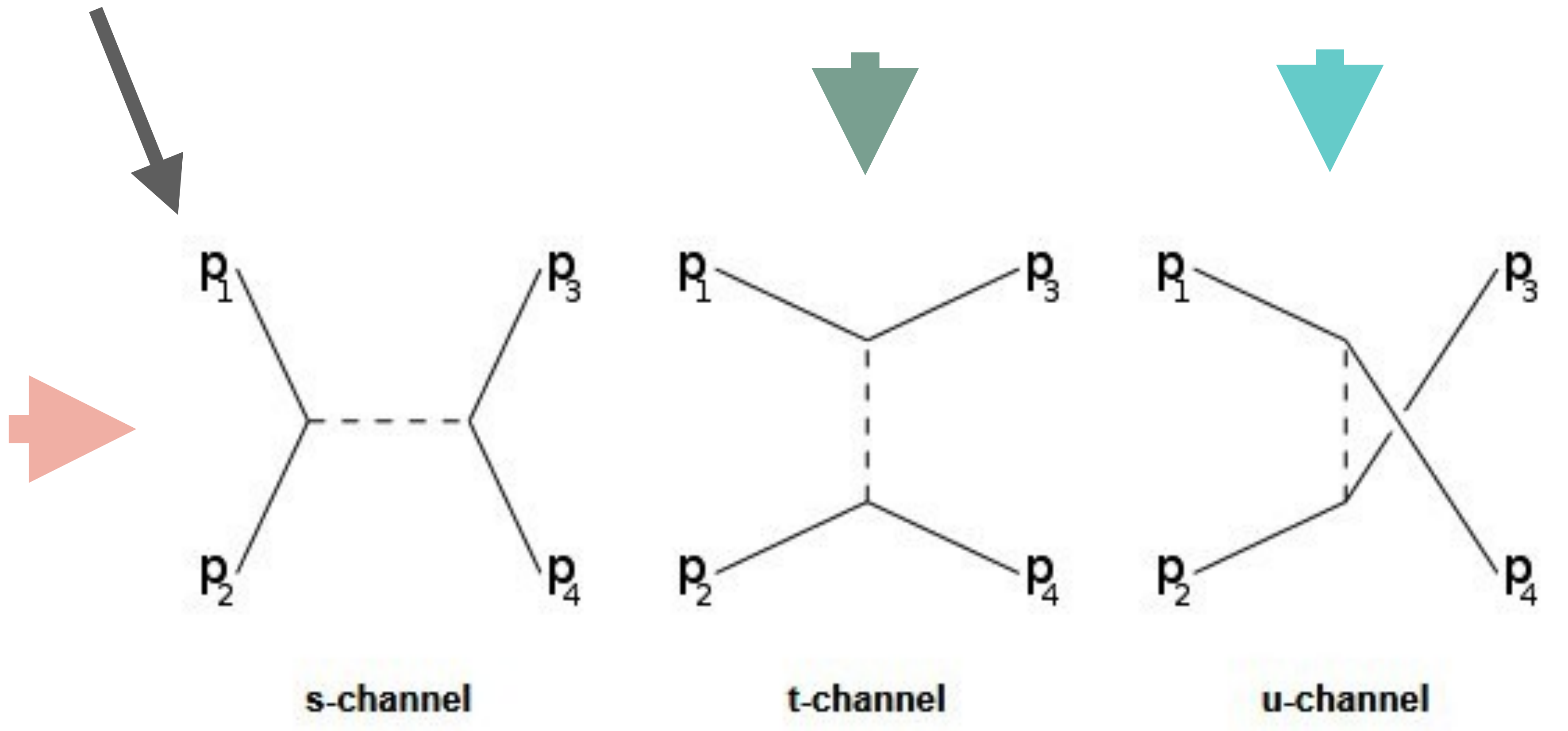
Crossing



Data



$$\mathrm{Im}\, T_{i\,i}(s, t=0) \propto \sum_n T_{i\,n} T^\dagger_{n\,i} \propto \sigma^i_{\mathrm{tot}}$$



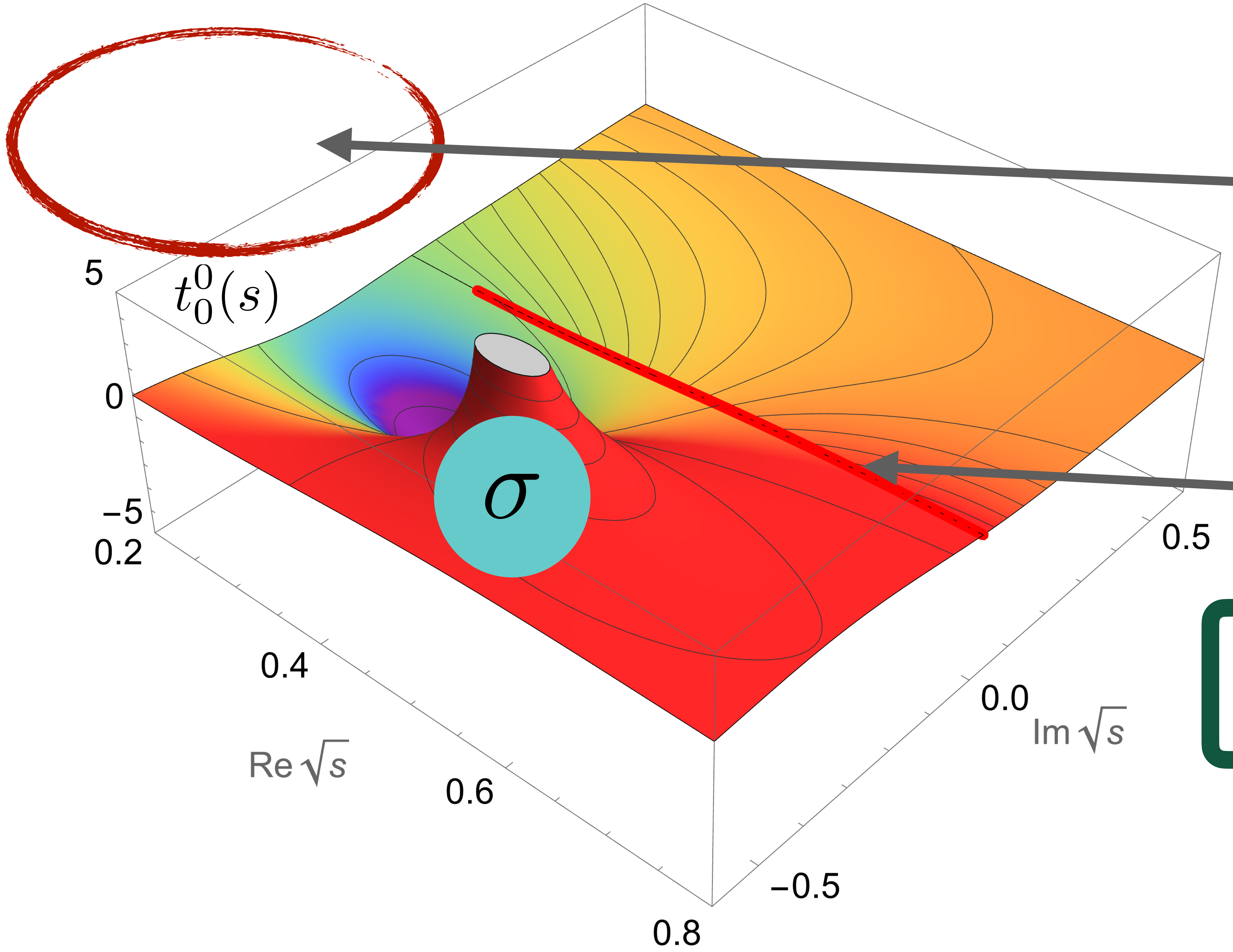
Particles and anti-particles are related

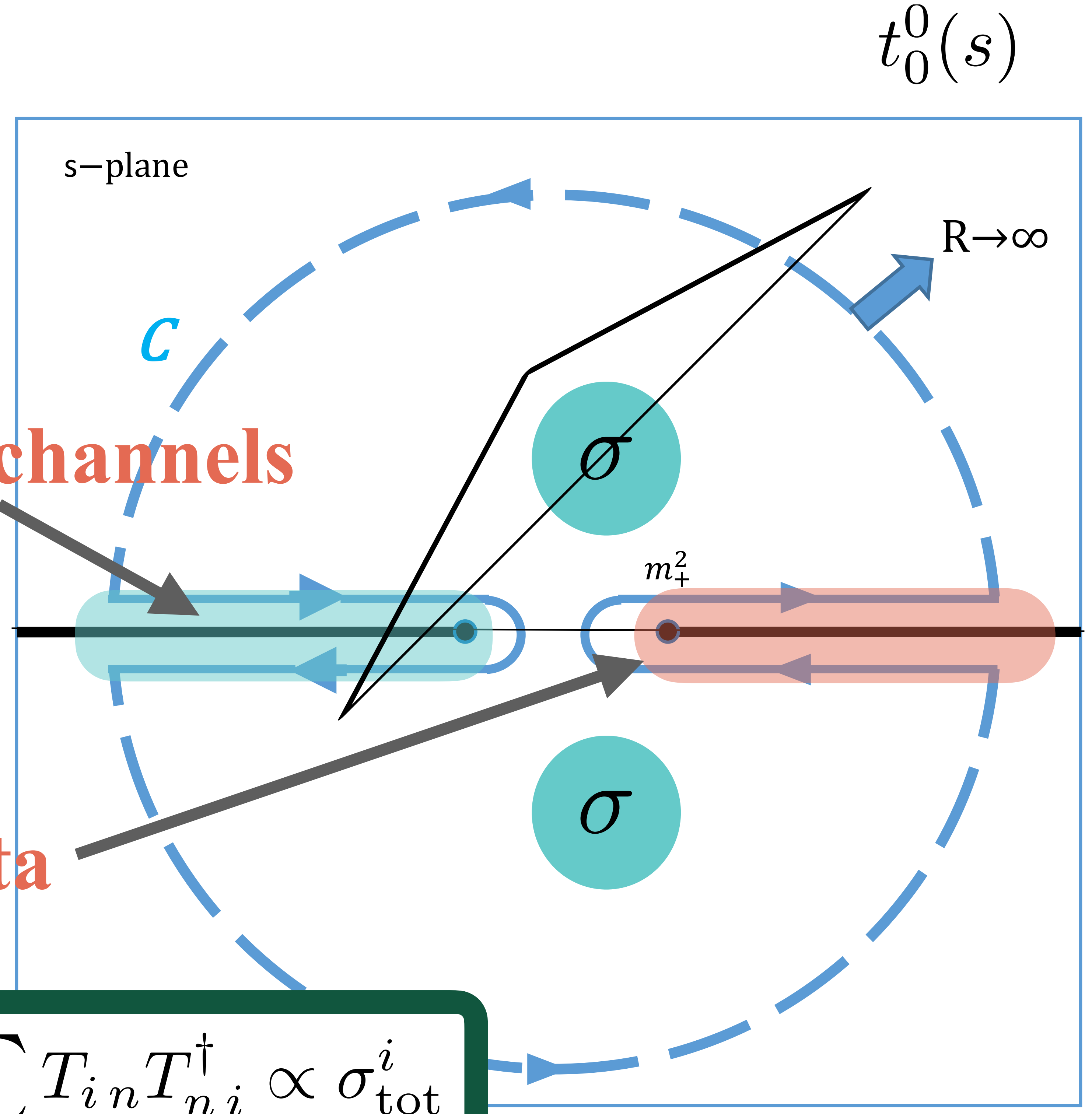
s-channel

t-channel

u-channel

Crossing





Cross-channels

Data

$$\text{Im}\, T_{i\,i}(s,t=0) \propto \sum_n T_{i\,n} T^\dagger_{n\,i} \propto \sigma^i_{\text{tot}}$$

Particles and anti-particles are related

s-channel

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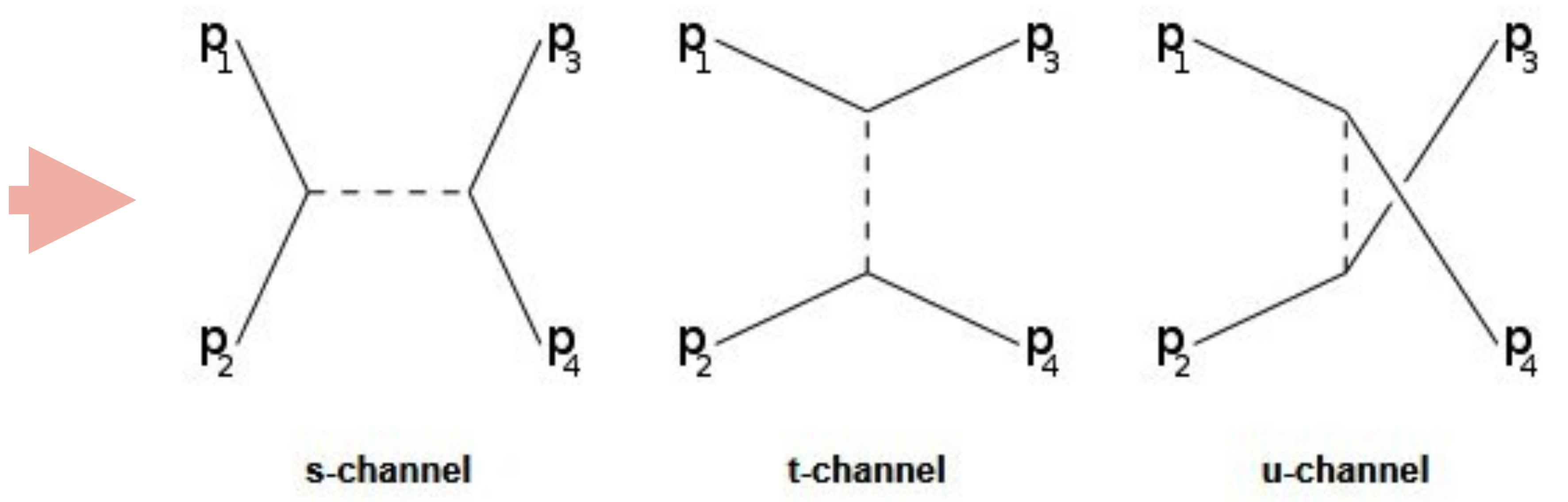

Dispersion relations

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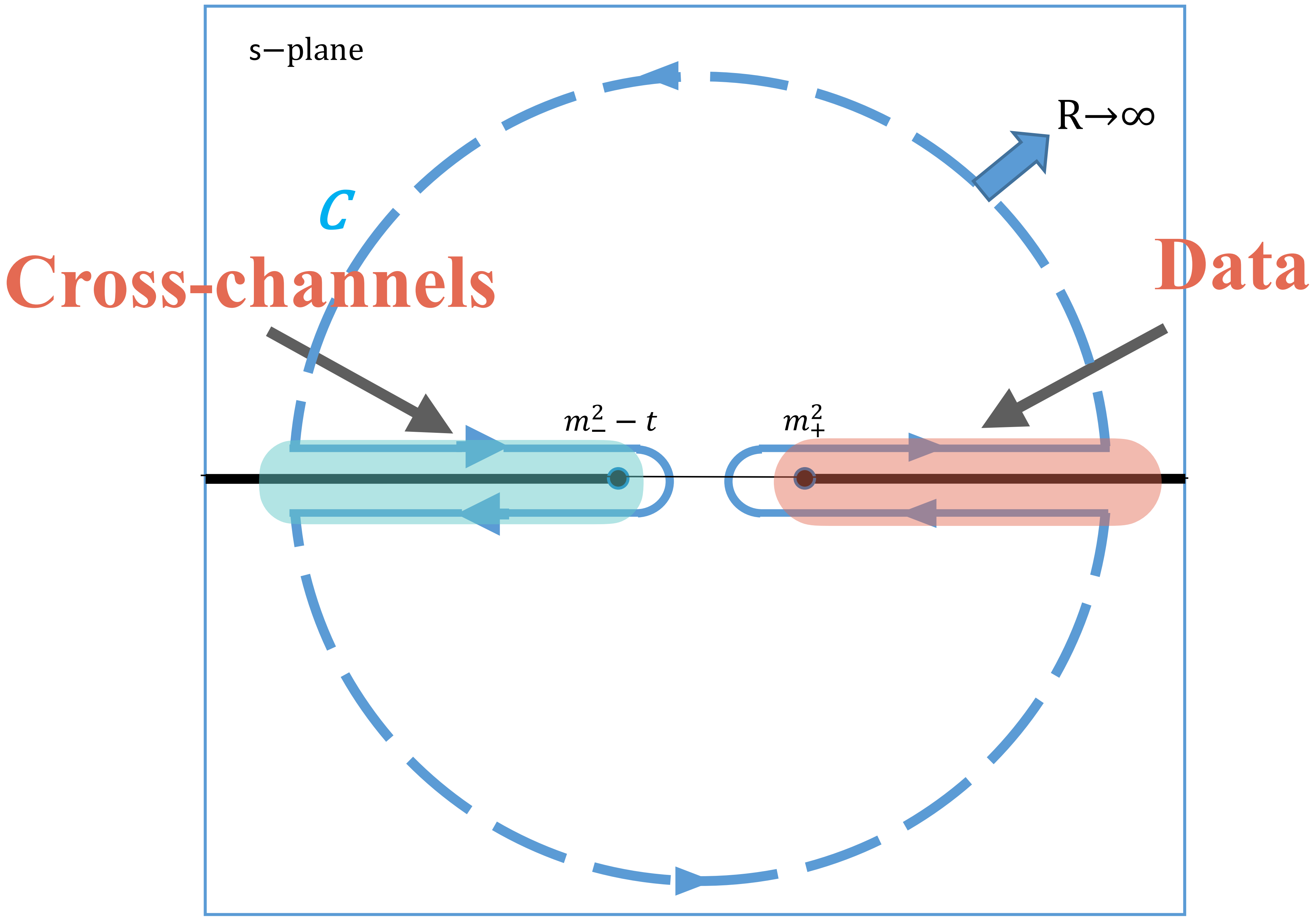
Causality ↔ Analiticity

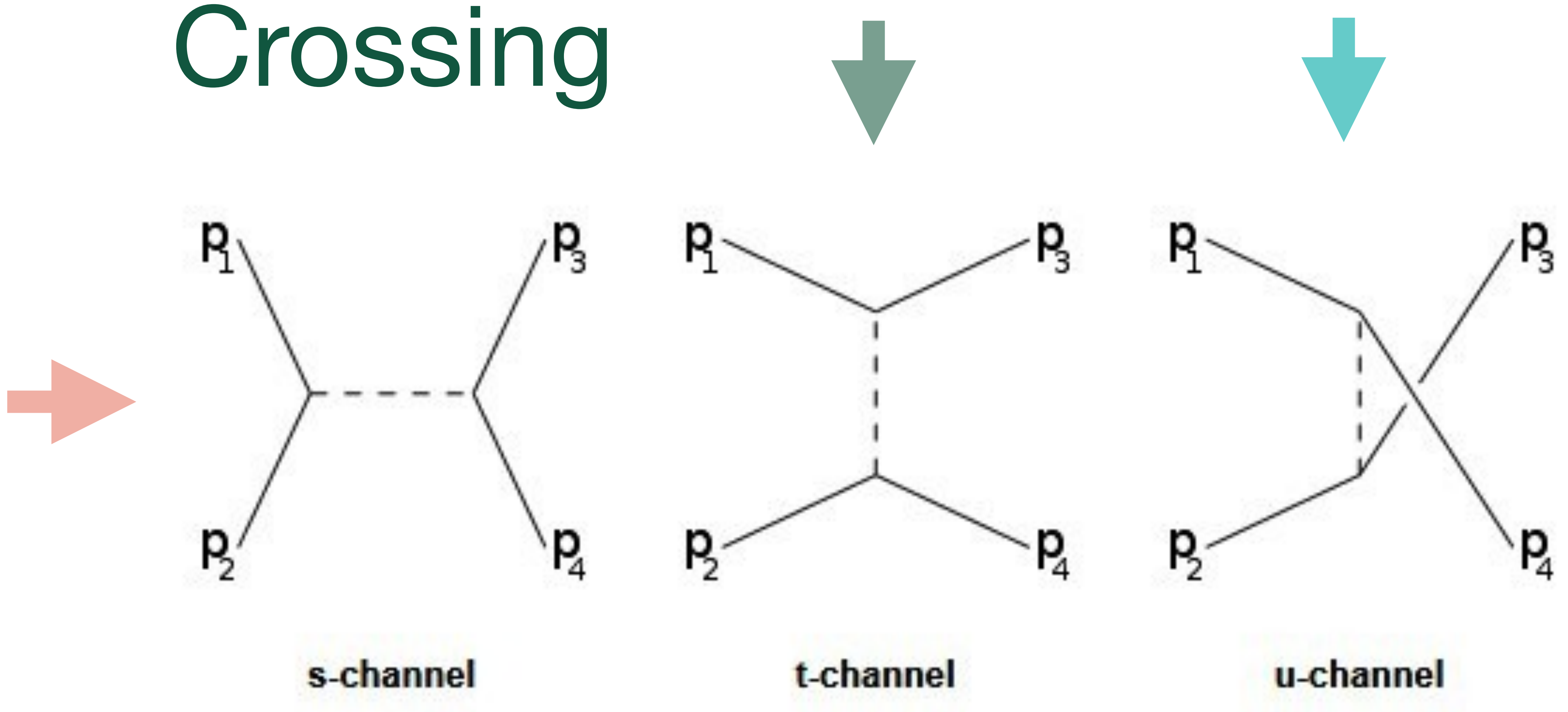
Built using Cauchy's theorem

$$t(z) = \oint_C \frac{t(z')}{z' - z} dz'$$



They can implement both analyticity AND crossing

Crossing



Fit → In

DR → Out

$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)(s - 4m_\pi^2) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s',s)\,\mathrm{Im}\, t_{\ell'}^{I'}(s')$$

“Model-independent”

- ☑ **Obtain your DRs** ⟷ **Crossing+analyticity**
- ☑ **Use all PWs available** ⟷ **Necessary Input**
- ☐ **Make** *Fit → In* *DR → Out* **compatible** ⟷ **Unitarity**

Make *Fit → In* *DR → Out* compatible

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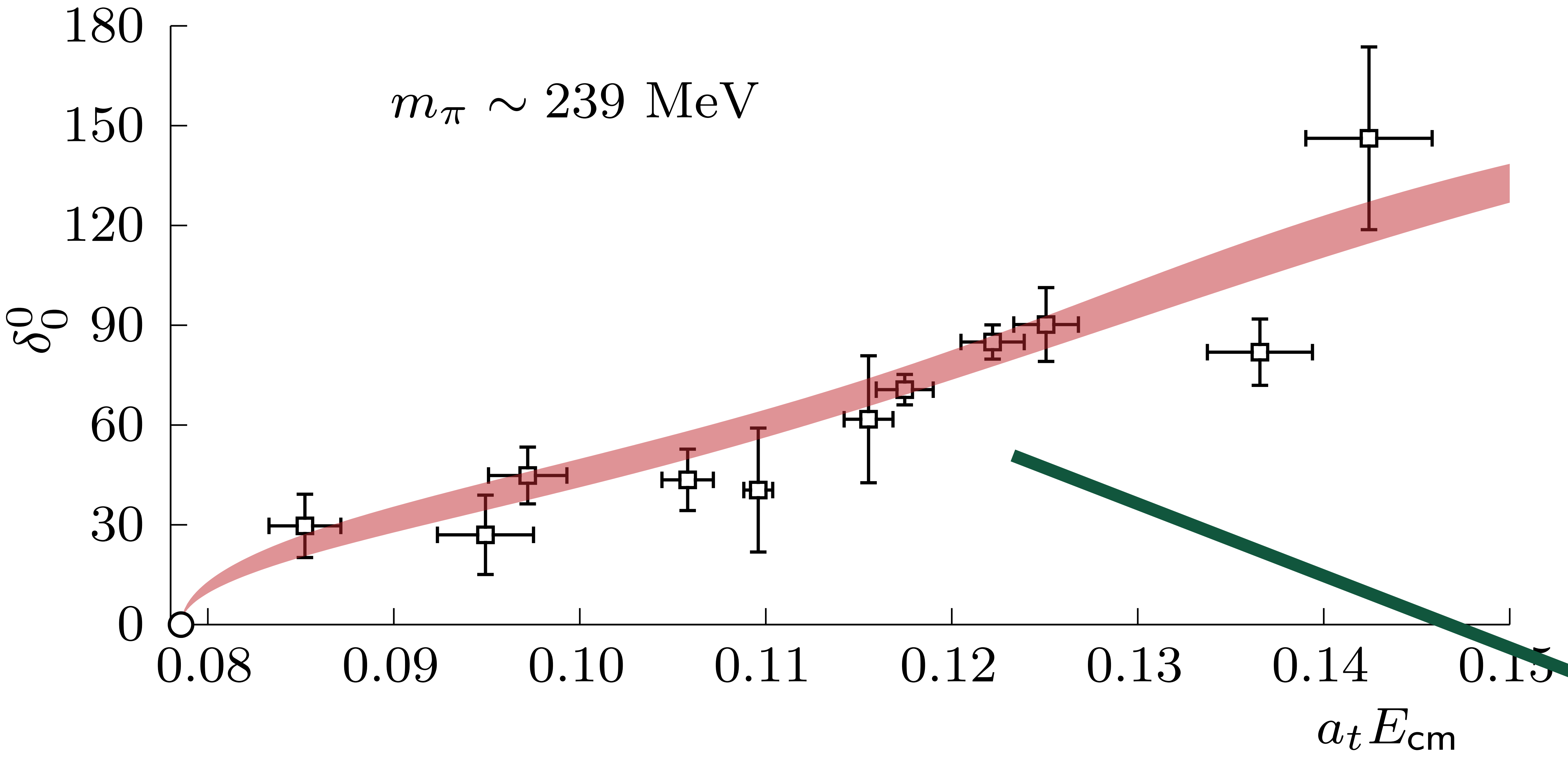
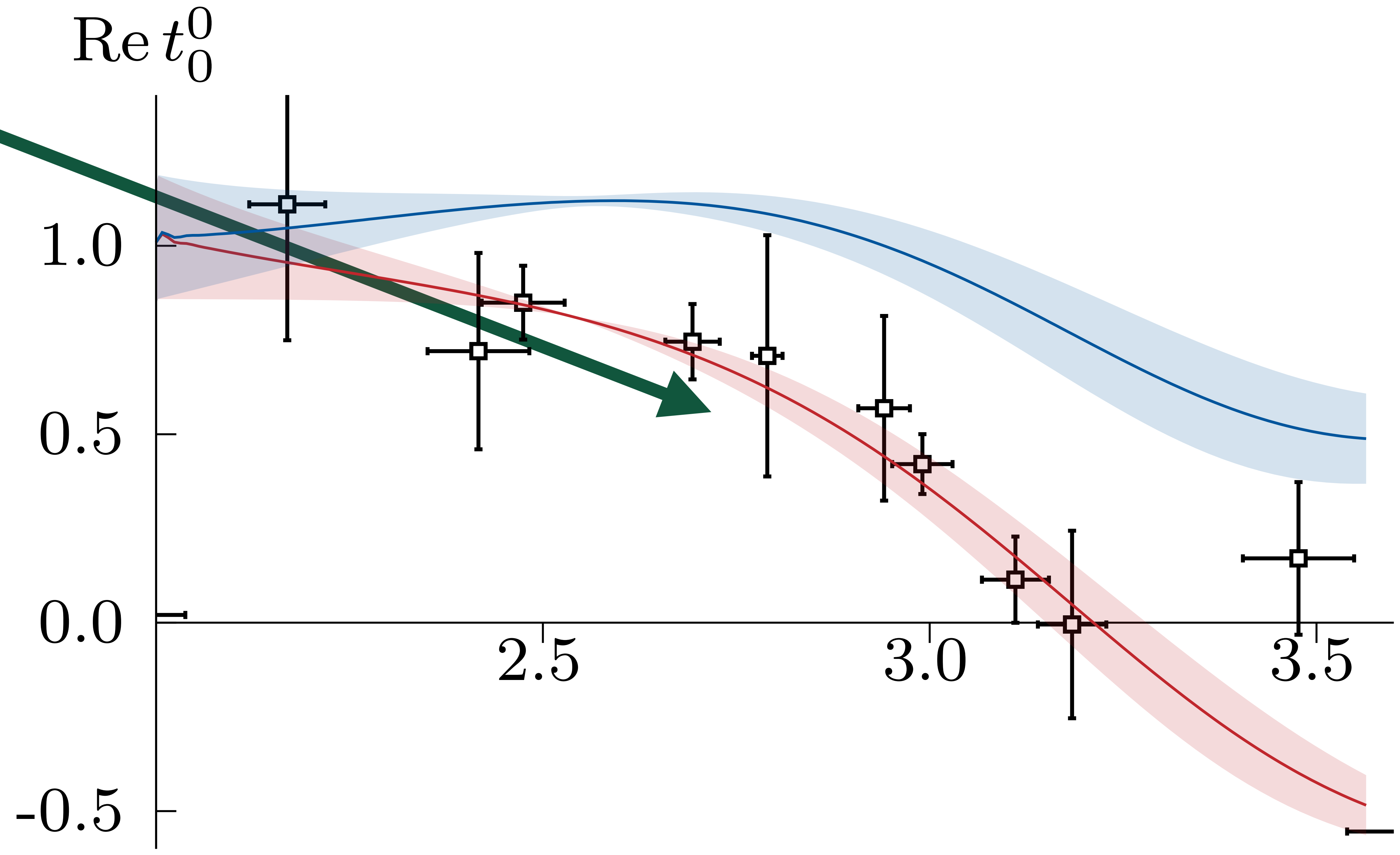


Model 1



Make ***Fit → In*** ***DR → Out*** **compatible**

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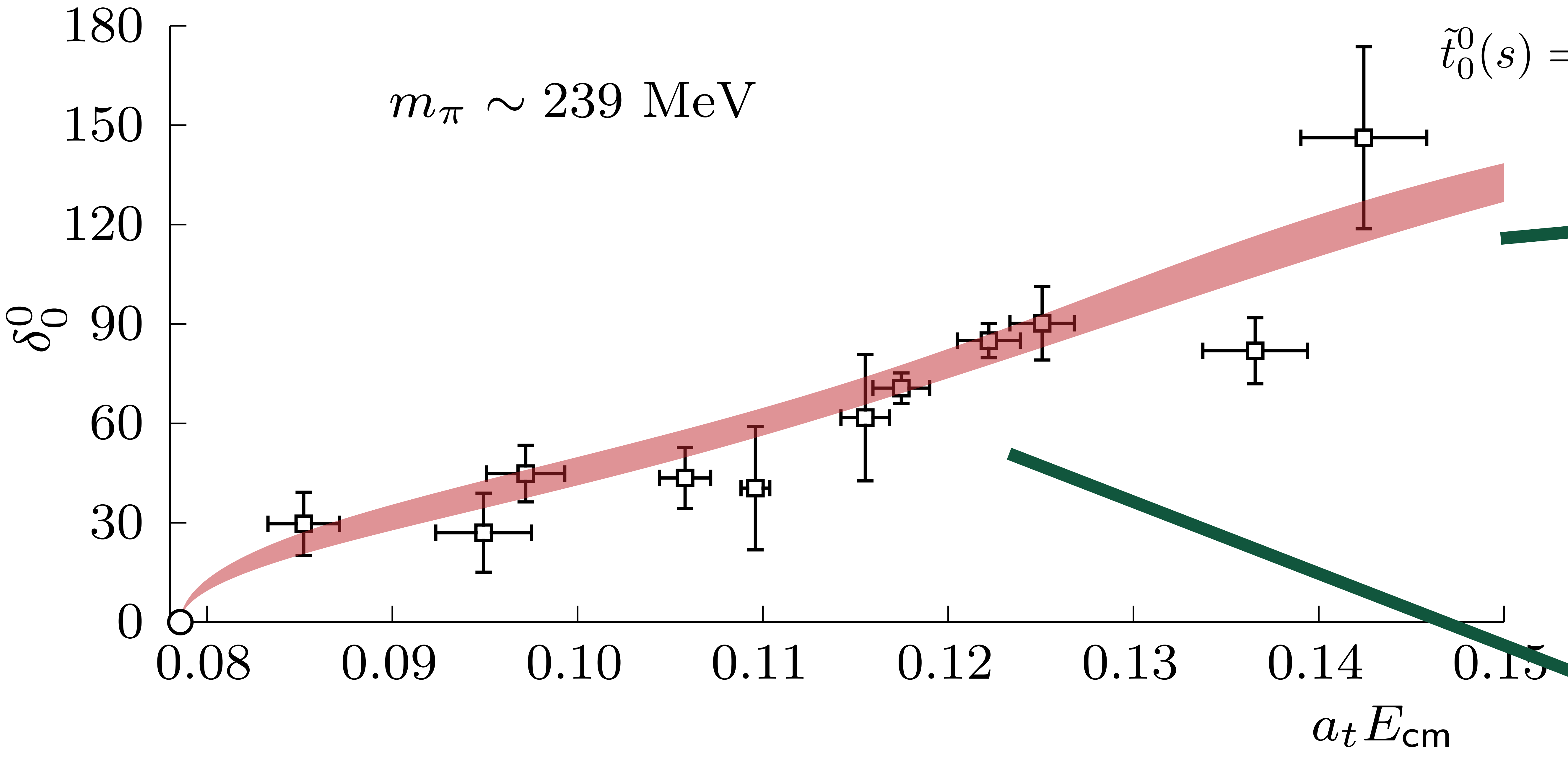
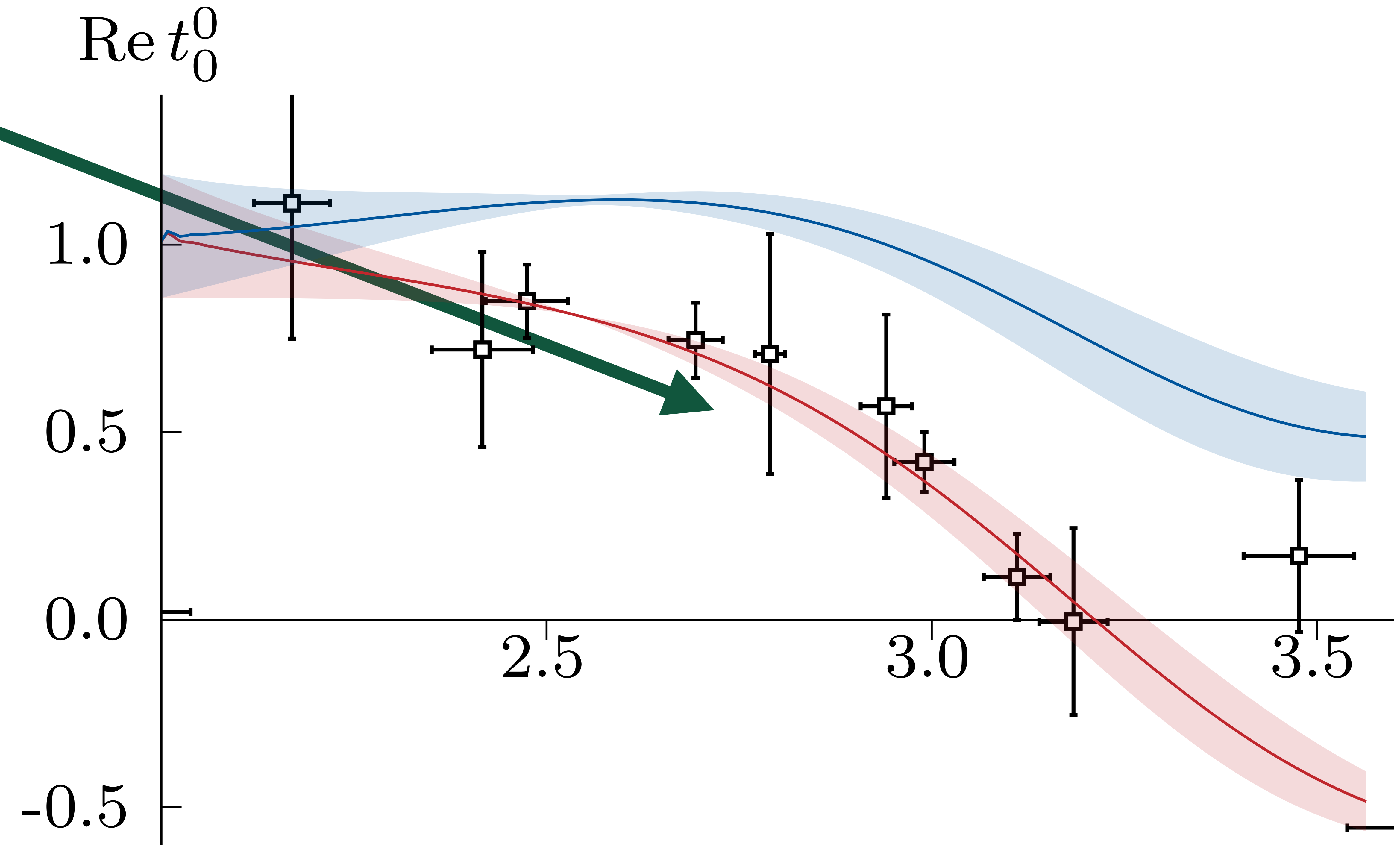
$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s',s)\,\mathrm{Im}\,t_{\ell'}^{I'}(s')$$





Model 1

Make *Fit → In* *DR → Out* **compatible**

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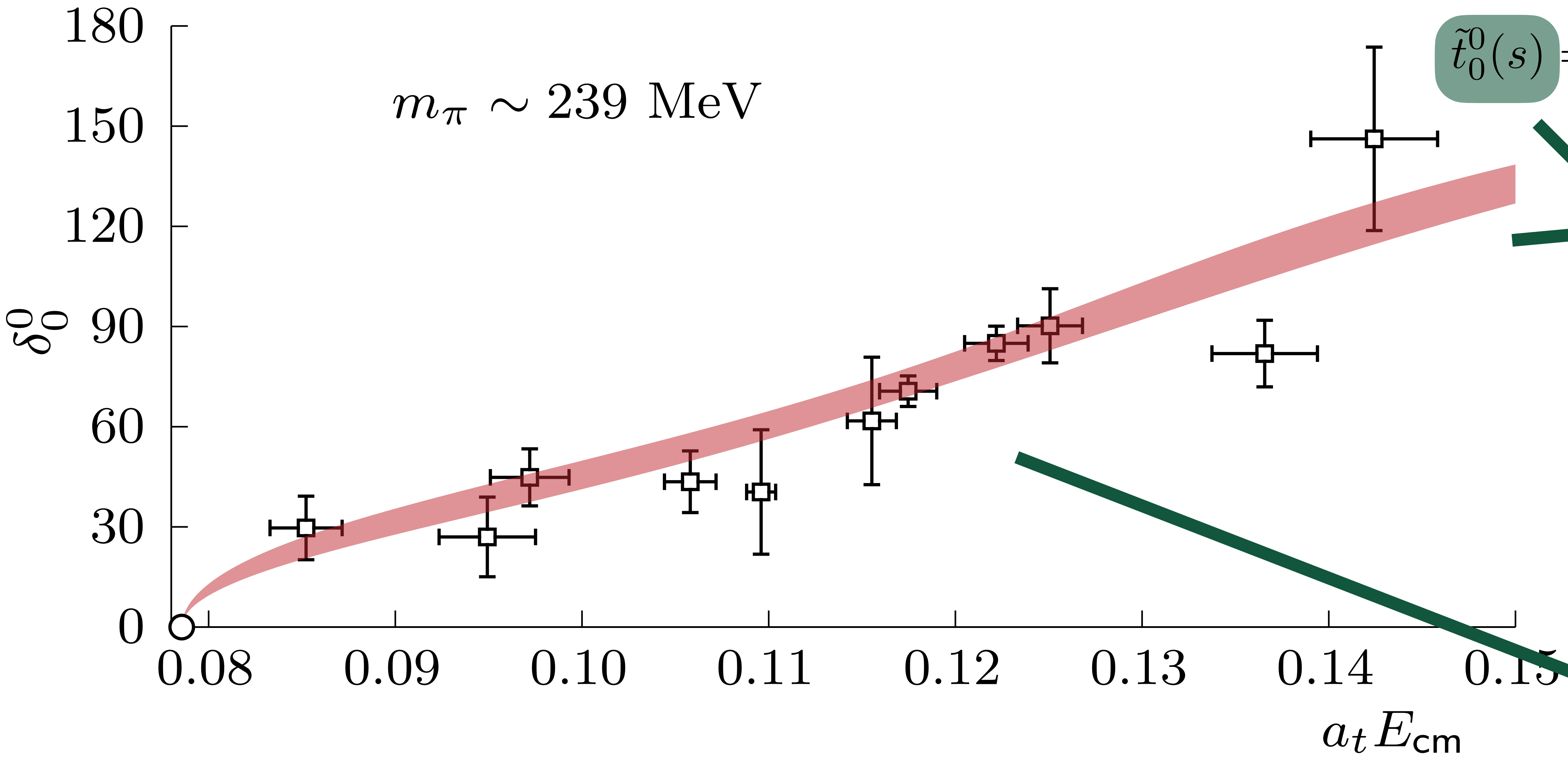
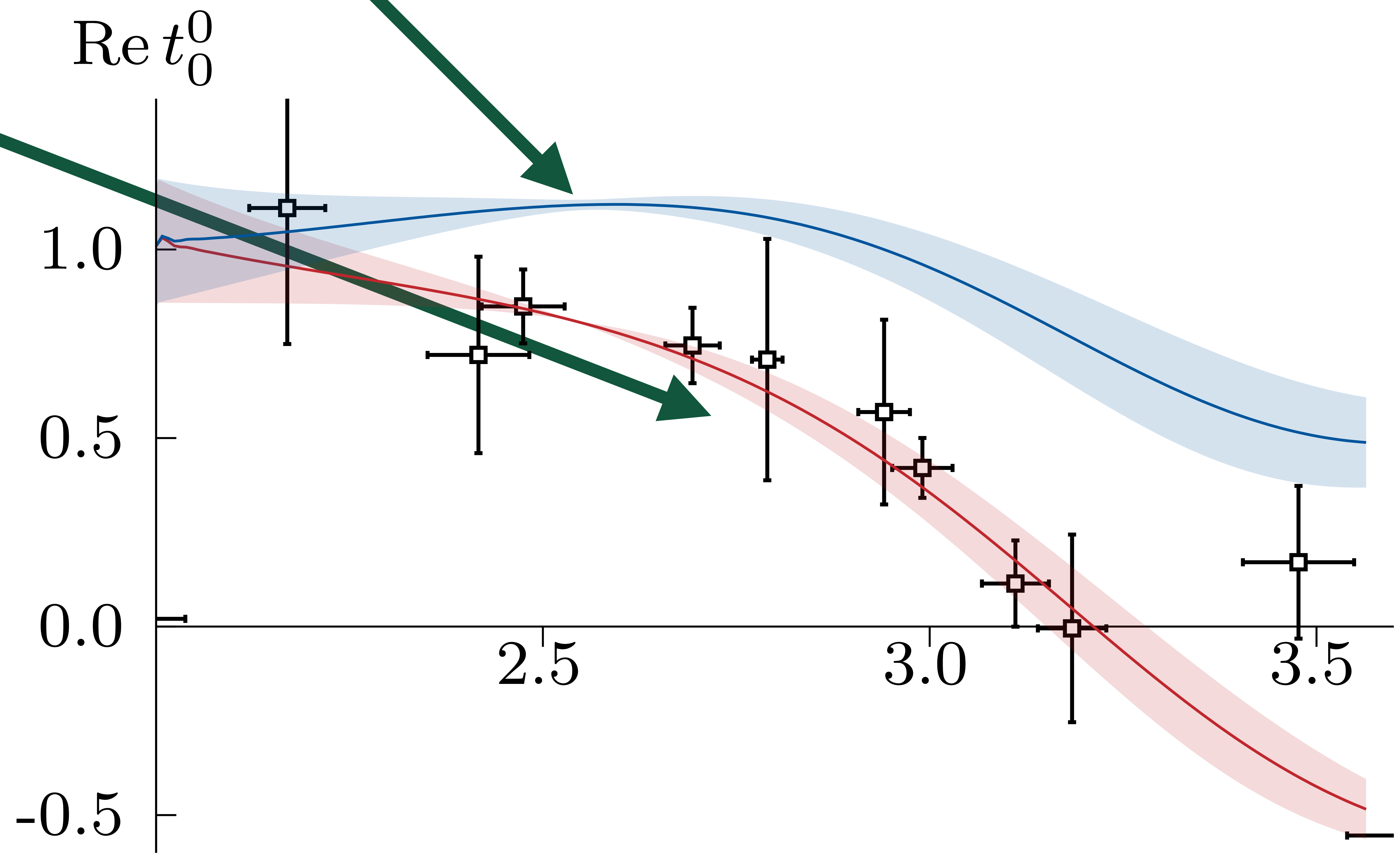
$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s',s)\,\mathrm{Im}\, t_{\ell'}^{I'}(s')$$



Model 1



Make *Fit → In* *DR → Out* compatible

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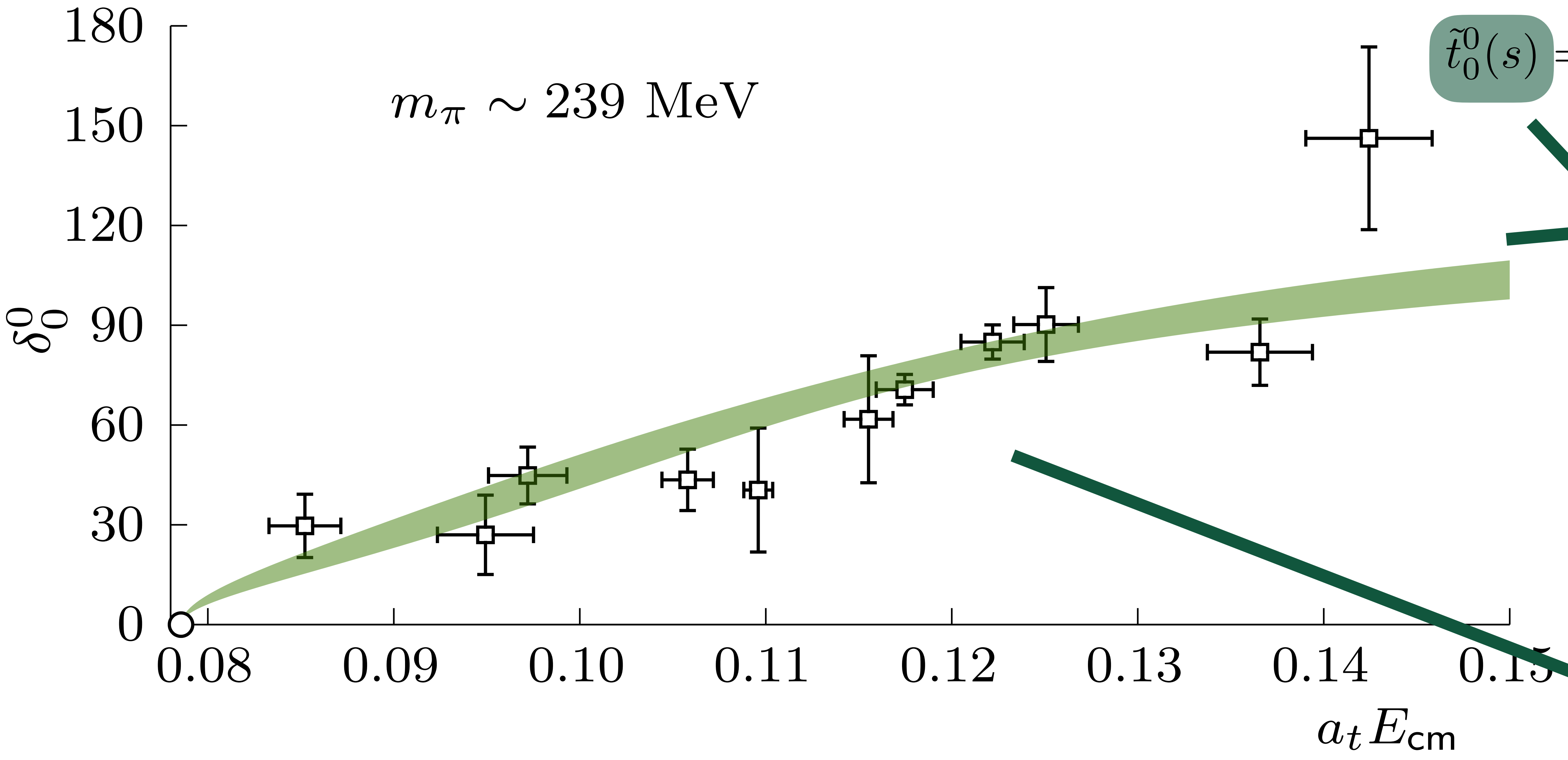
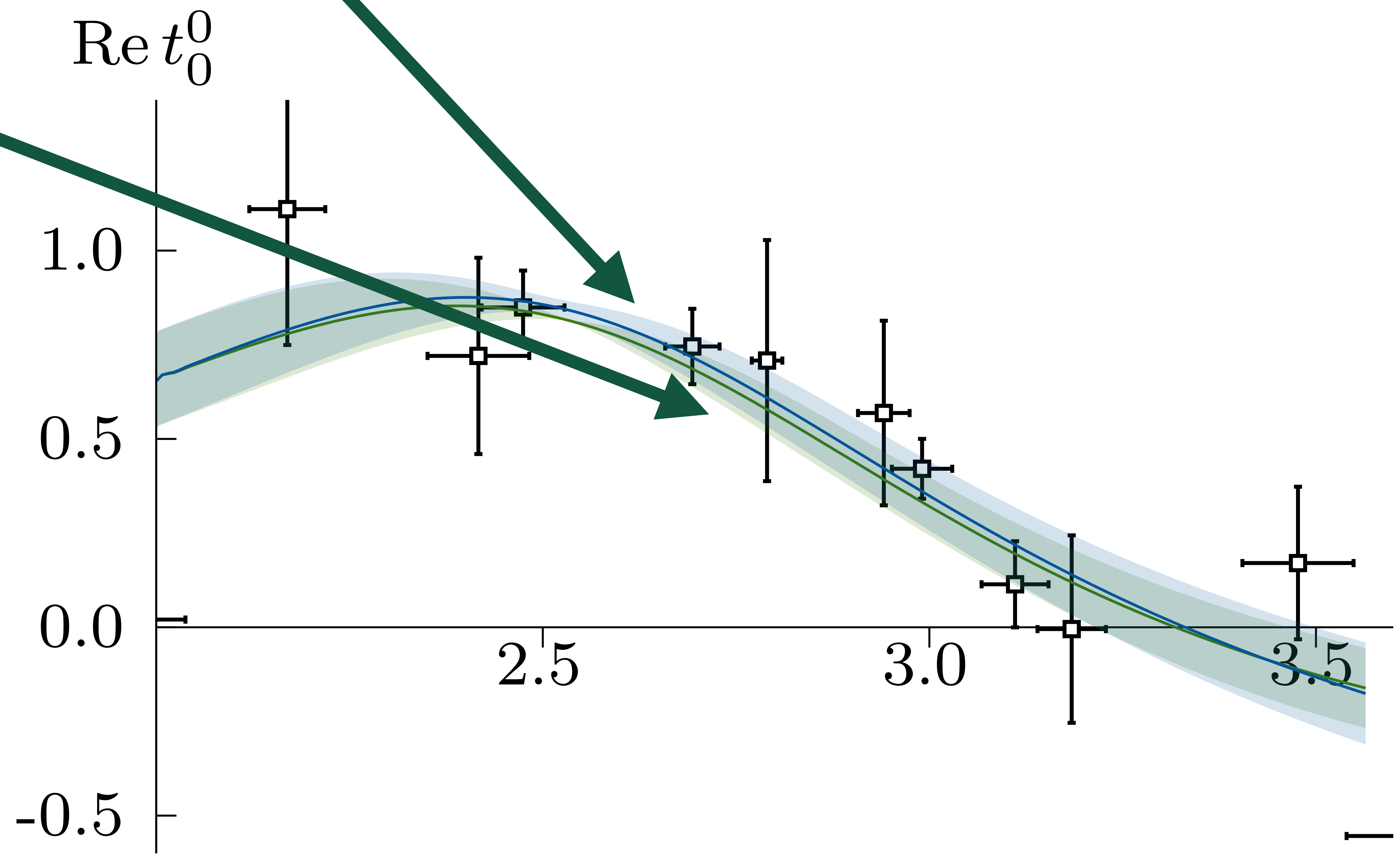
$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s', s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$



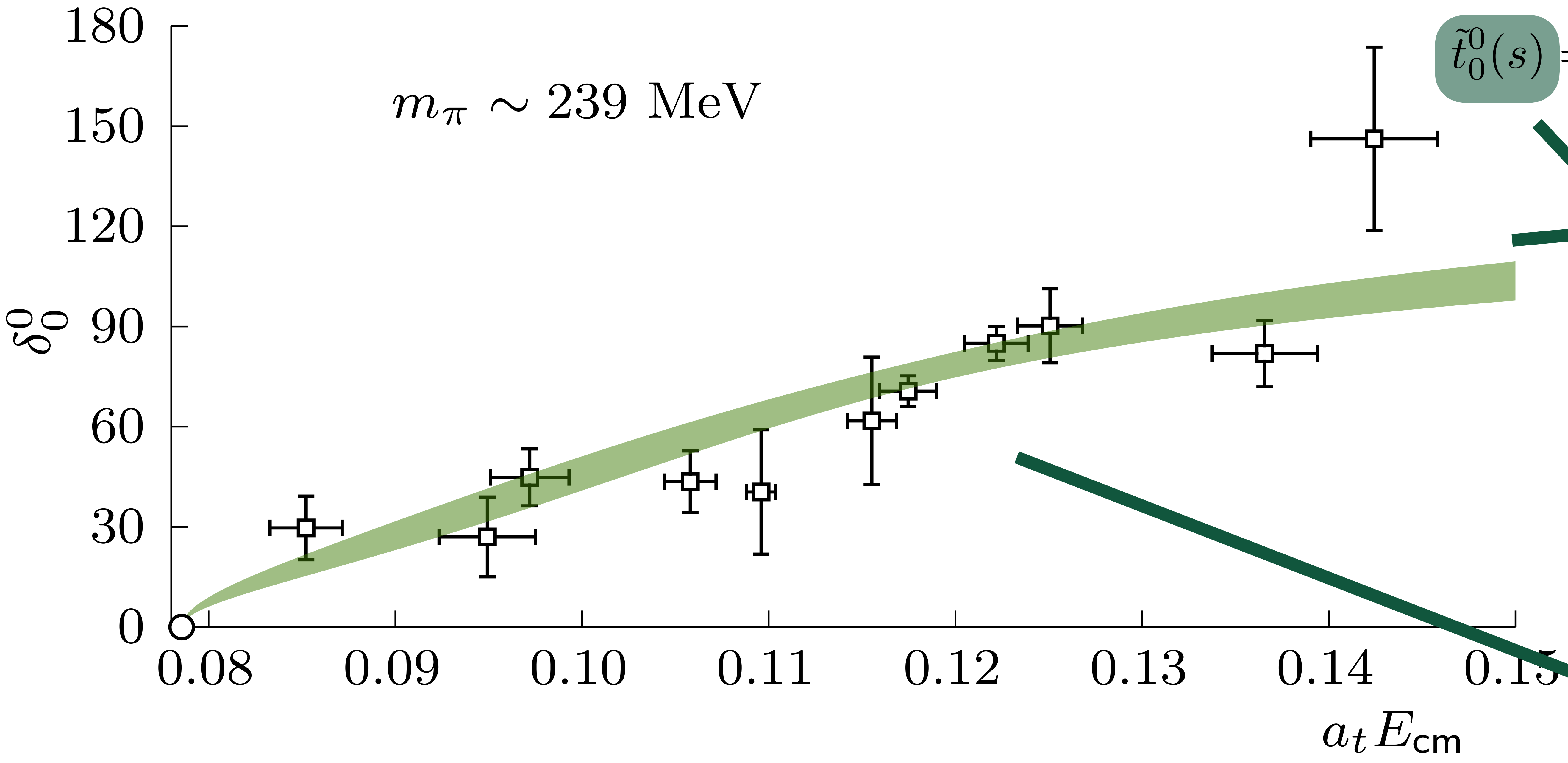
Model 2



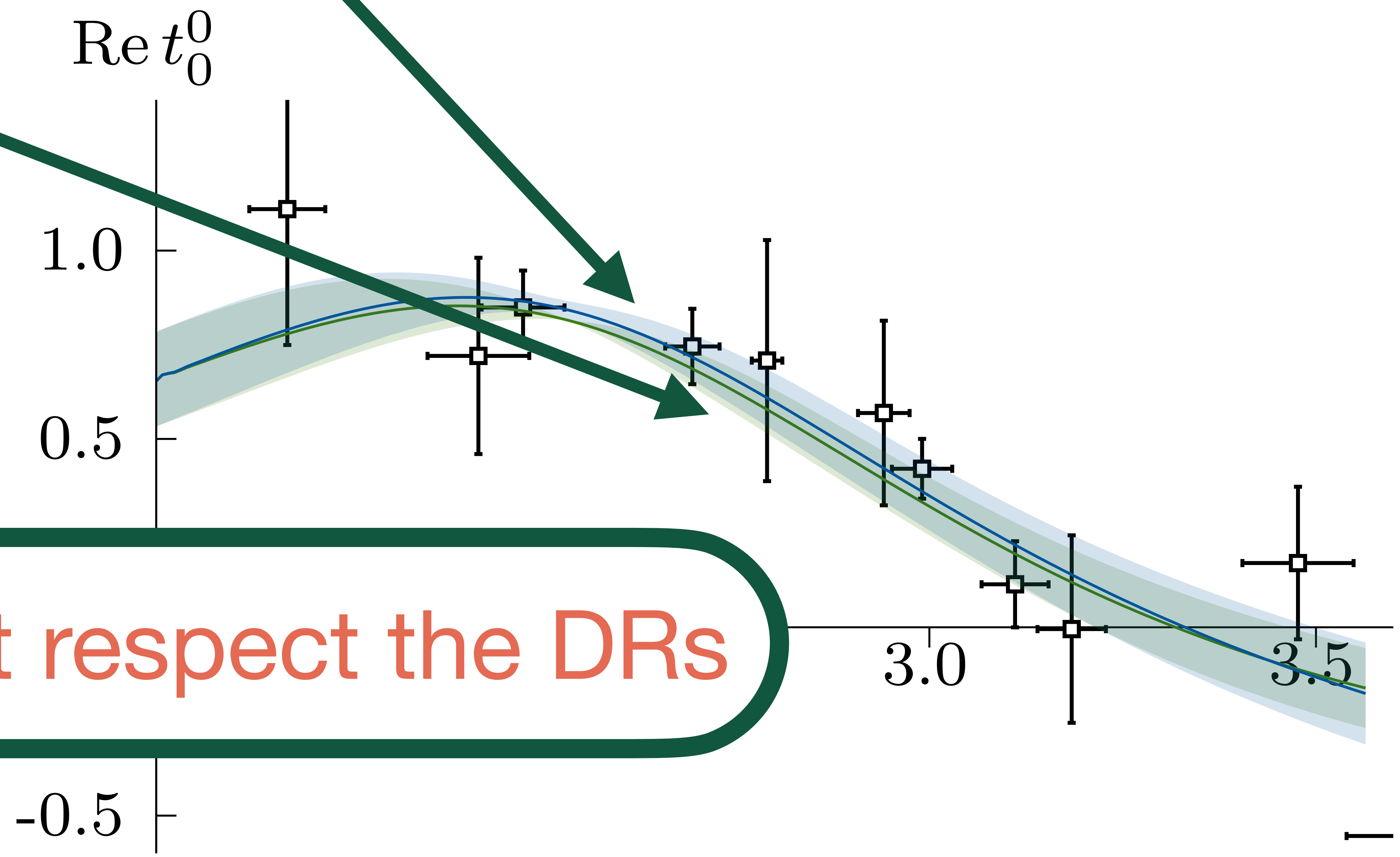
Make *Fit → In* *DR → Out* compatible

2304.03762

$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s',s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$



Model 2



We select those models that respect the DRs

Outside the physical region

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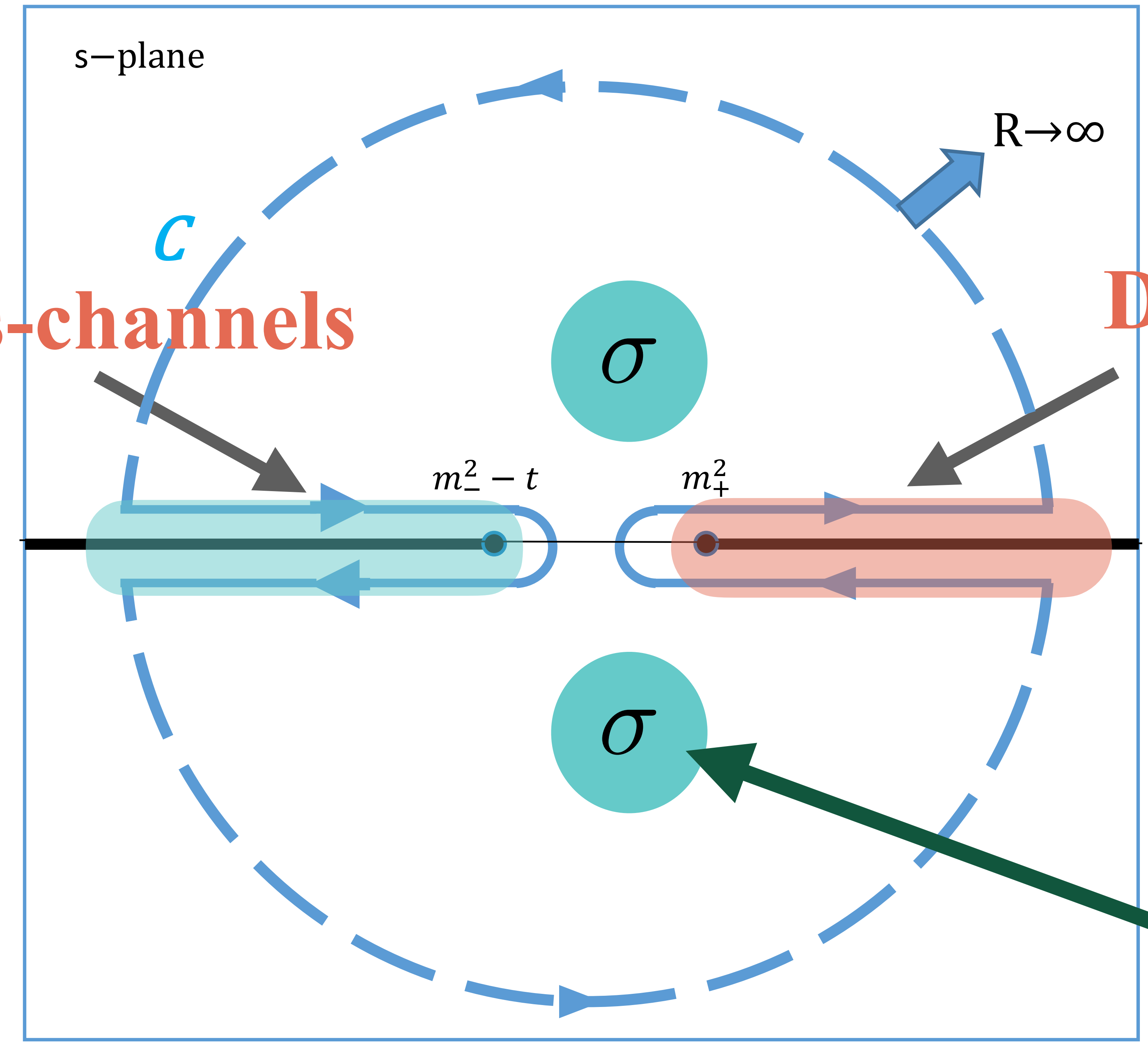
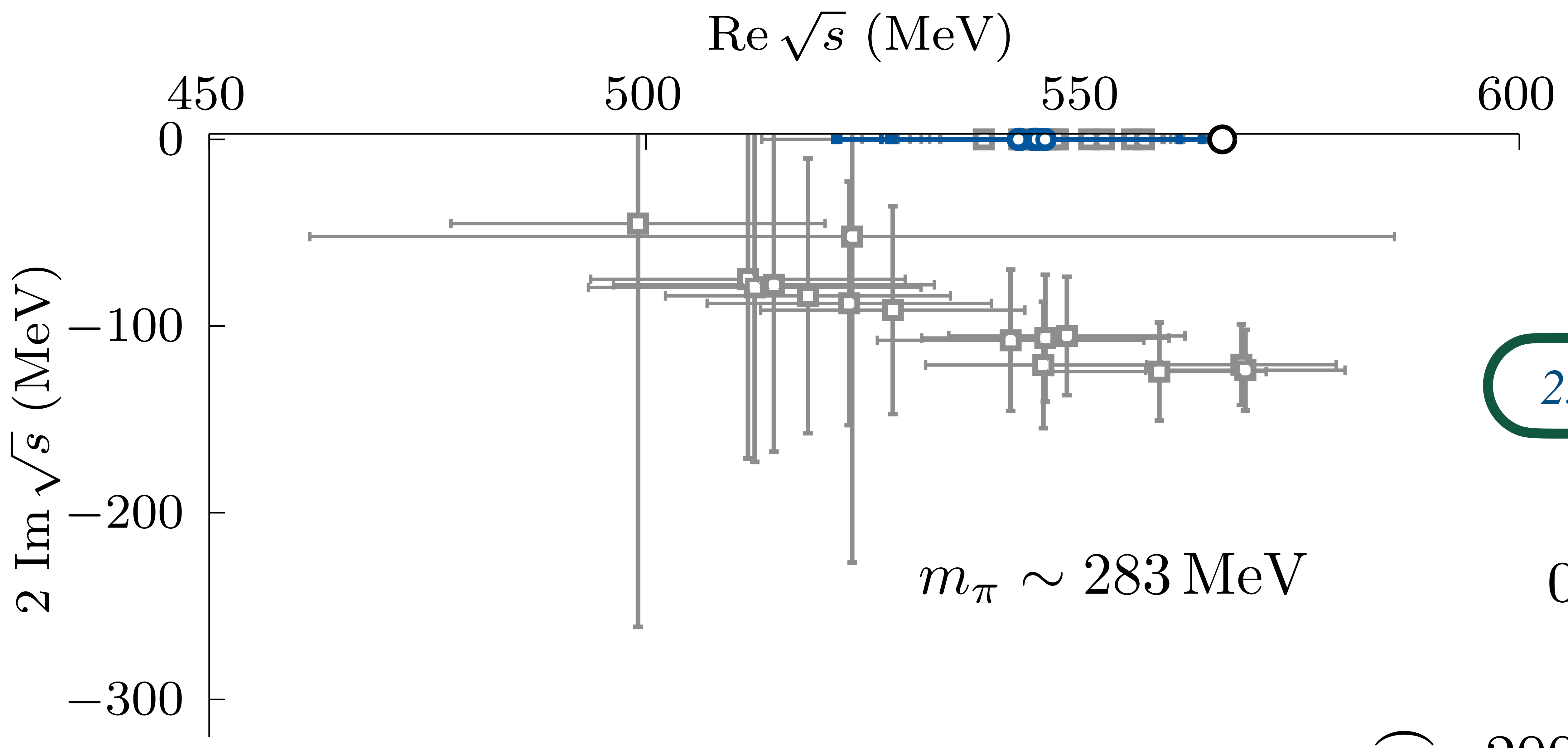


Both sides are good now

What happens everywhere else??

What happens here??

Dispersive σ



No tension anymore

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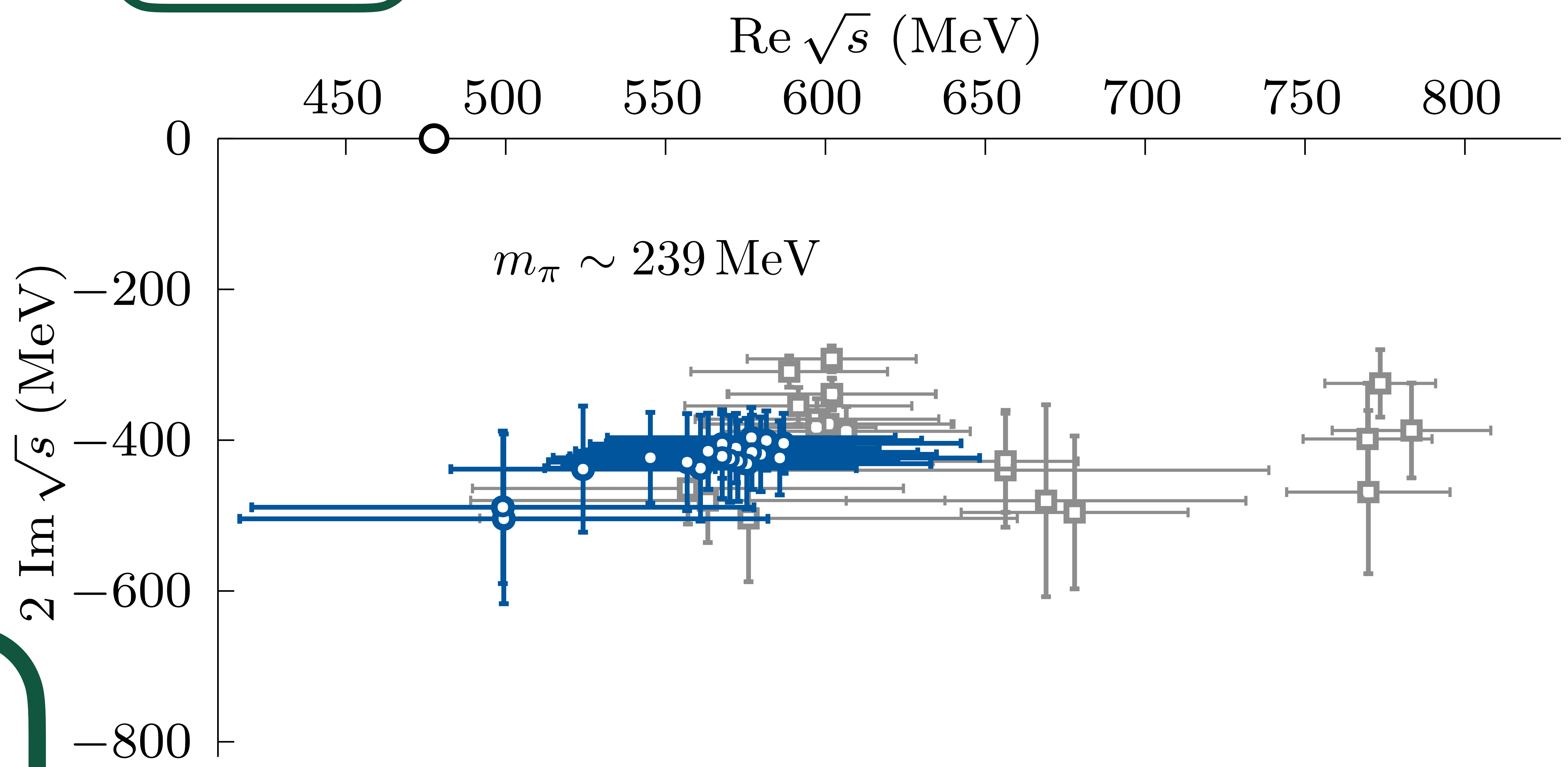
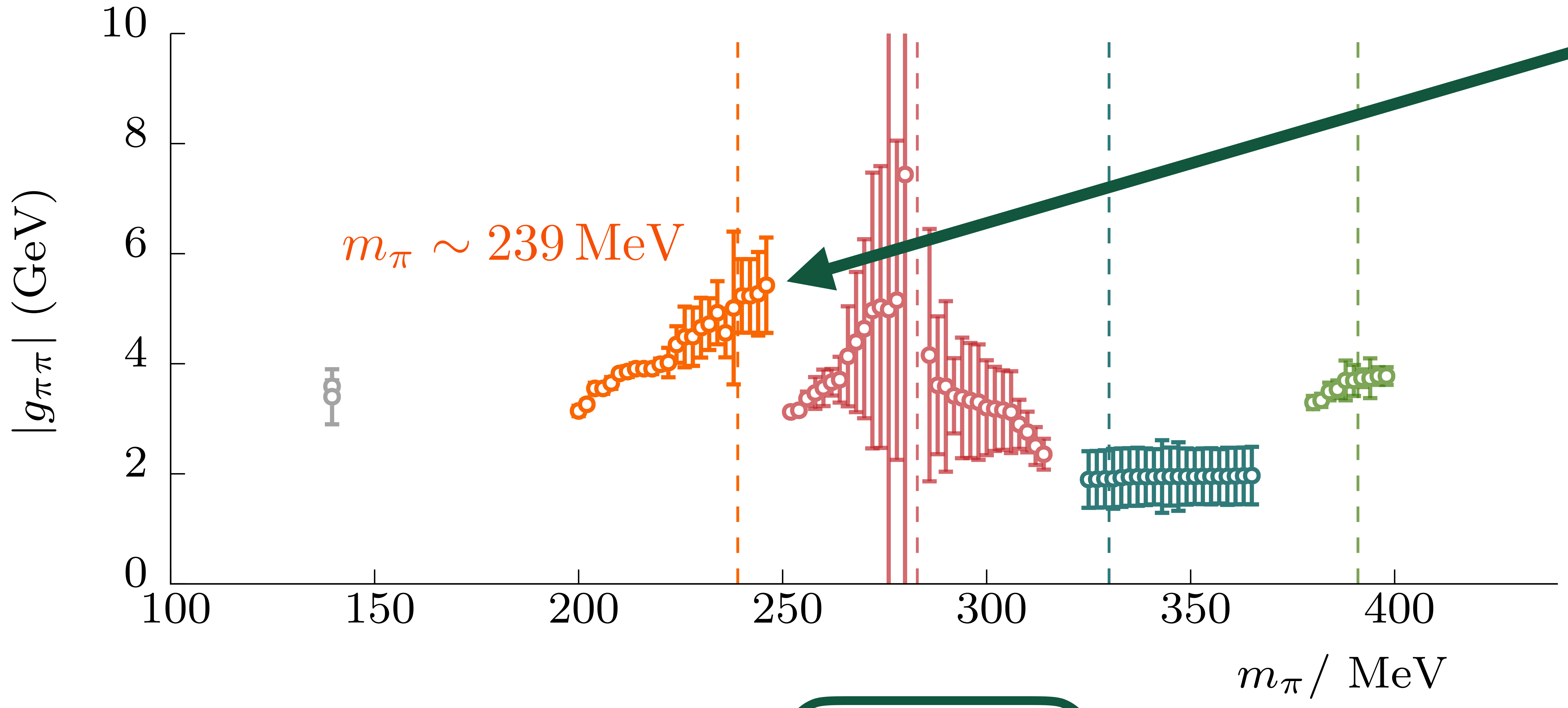


We traded statistical uncertainty increase by large systematic reduction

Couplings

Models: Large systematic dependence



DRs: No systematics

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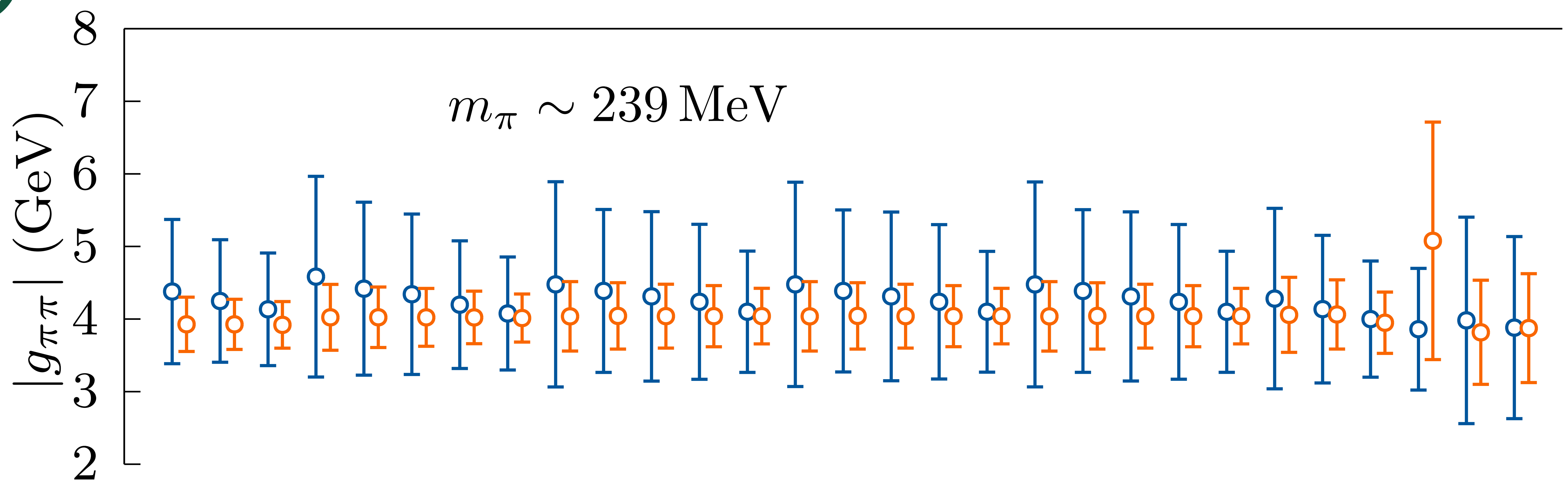
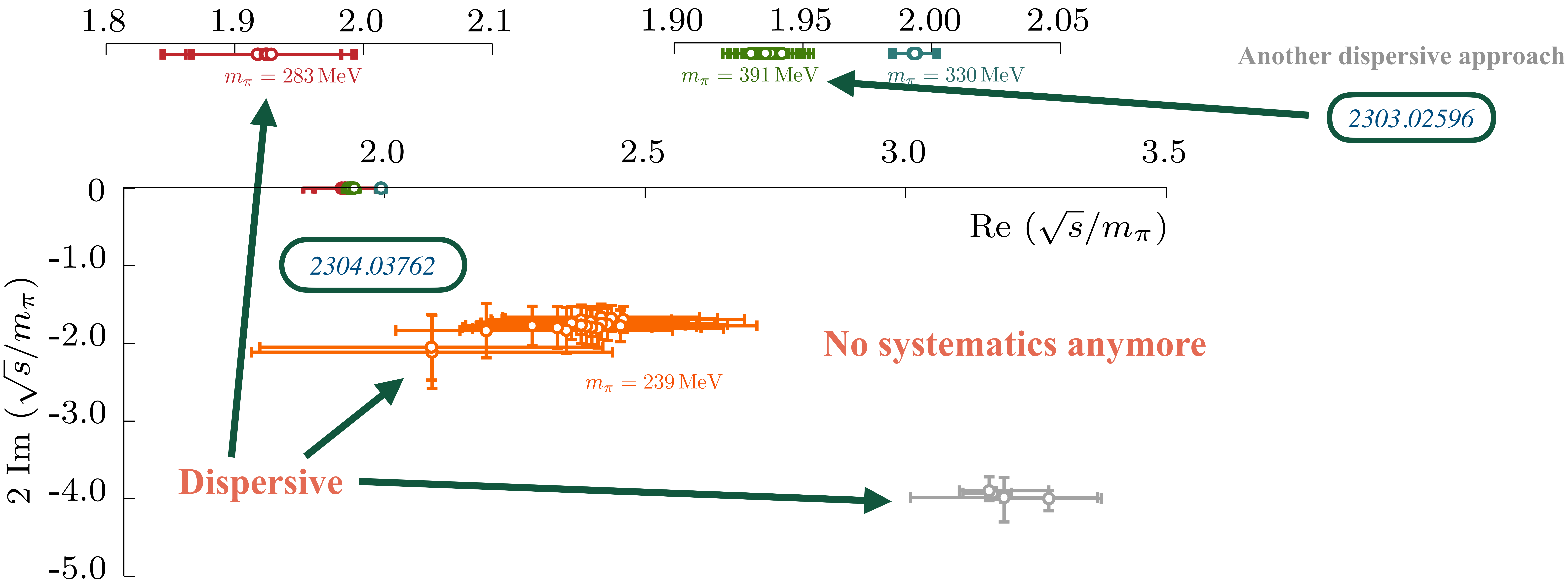


Dispersive σ



We traded statistical uncertainty increase by large systematic reduction

Summary and outlook

First-principles extraction of a broad resonance directly from QCD

The lighter the π, the more relevant this approach is

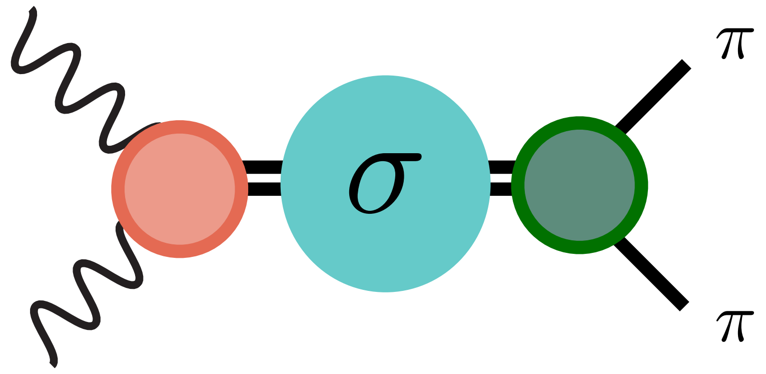
Better constraints over scattering lengths

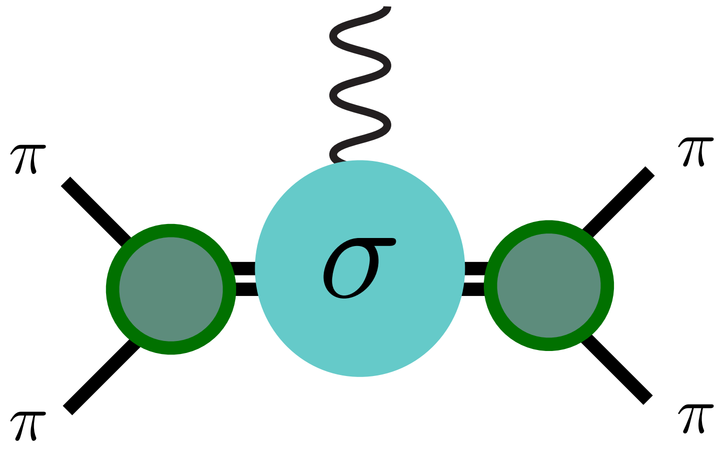
Future

Include second, larger volume for the lighter pion mass

Extract the $f_0(980)$??

Study new observables ??

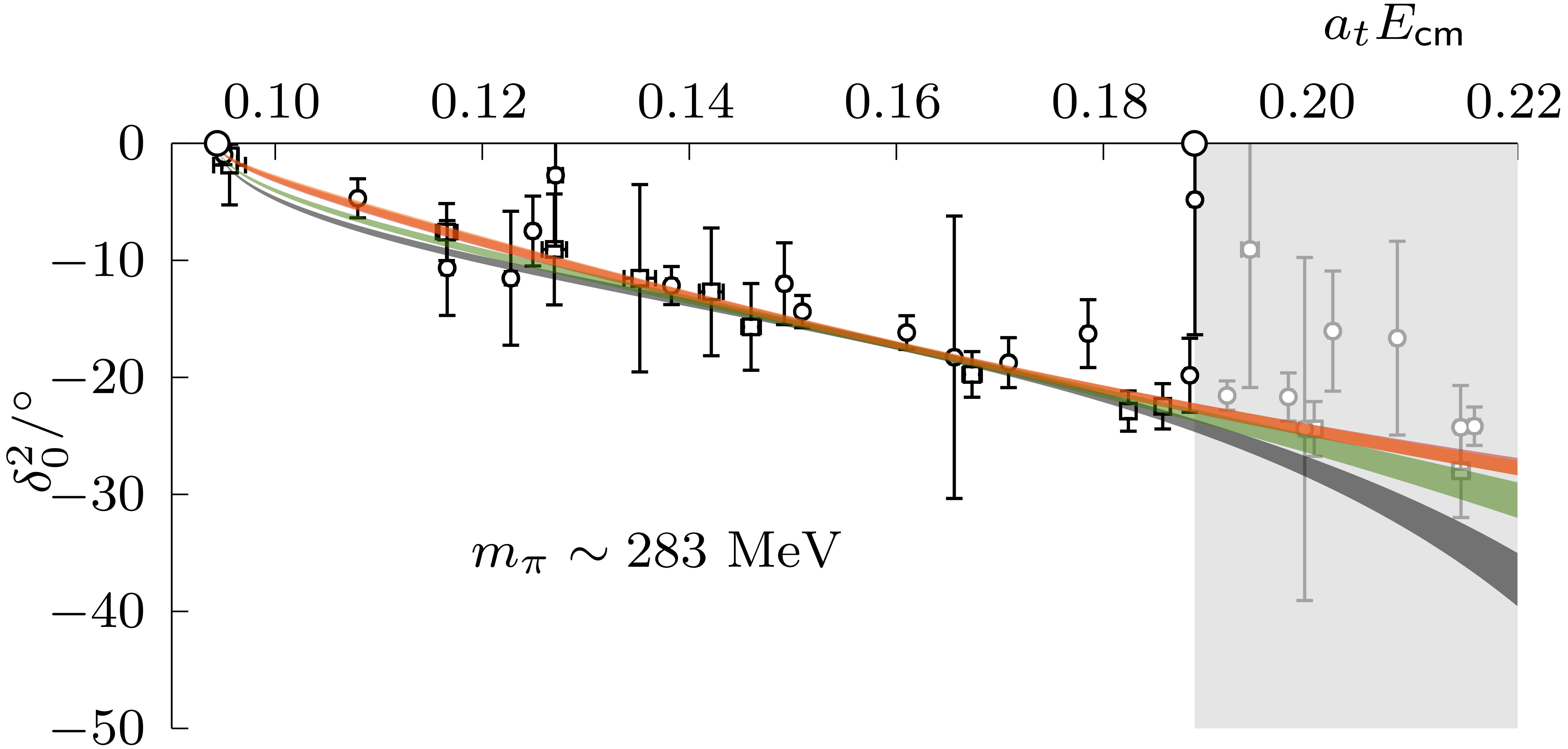




Thank you!!

$I = 2\ \pi\pi$

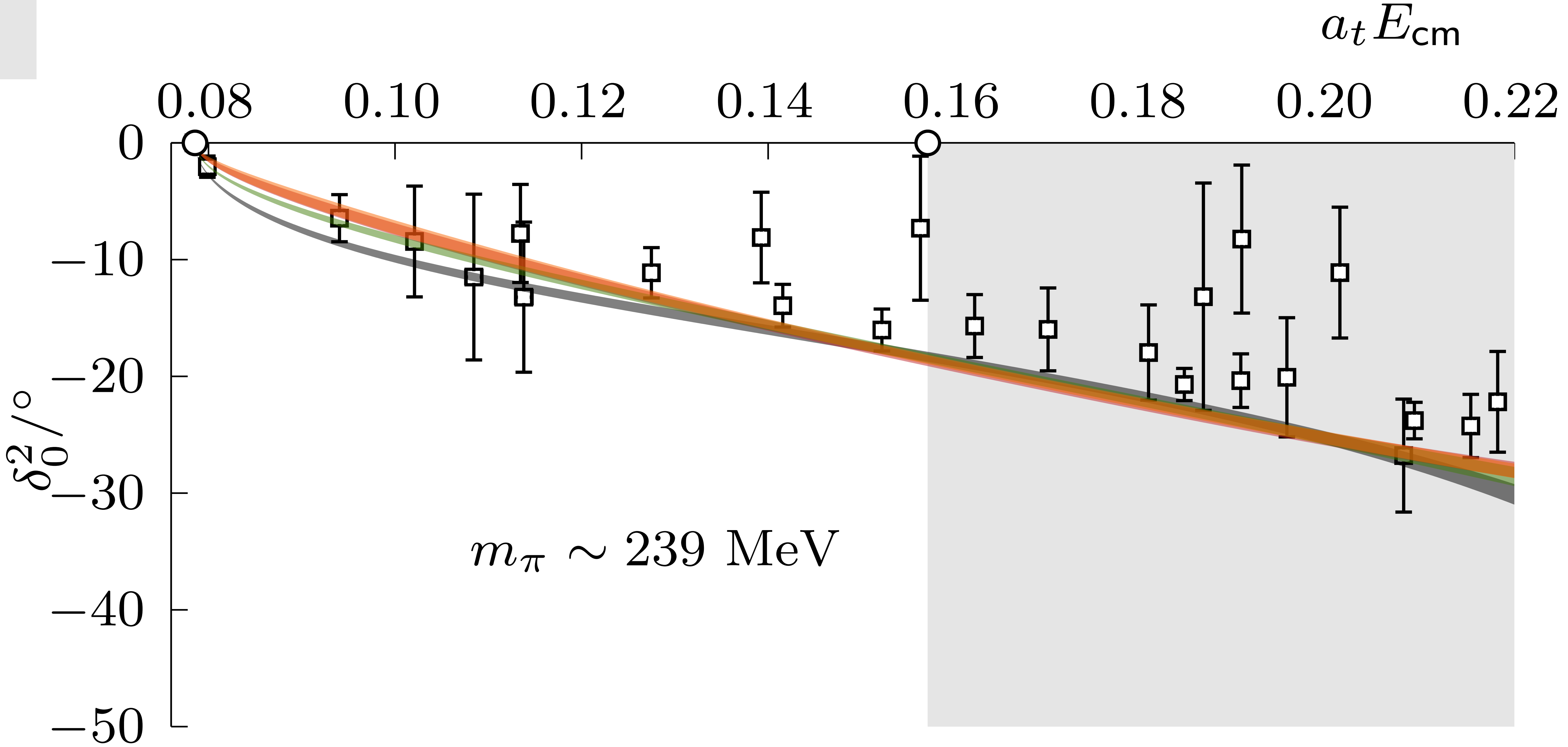
2303.10701



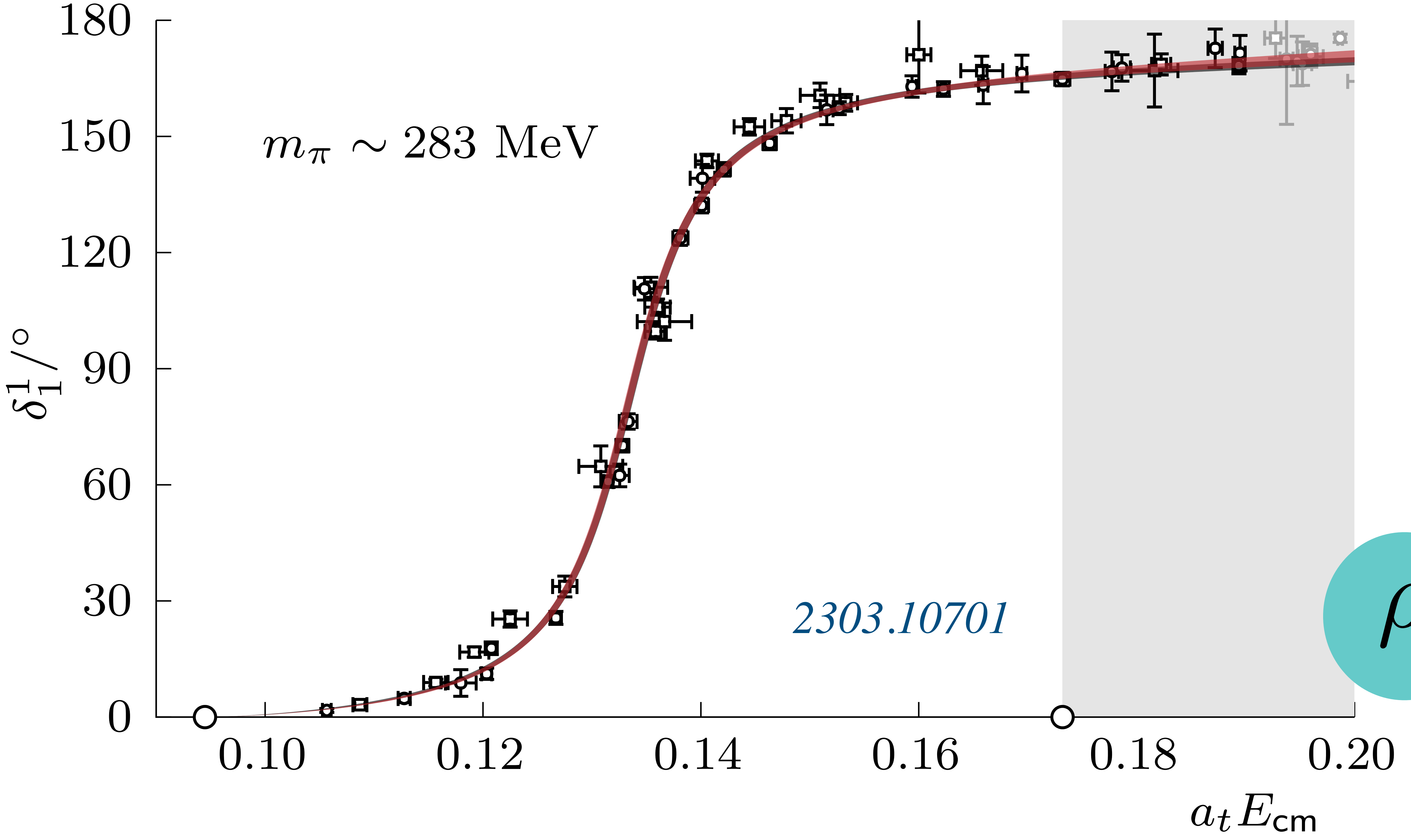
Percent error for $\delta(s)$

10+ parameterizations

Systematic spread at threshold

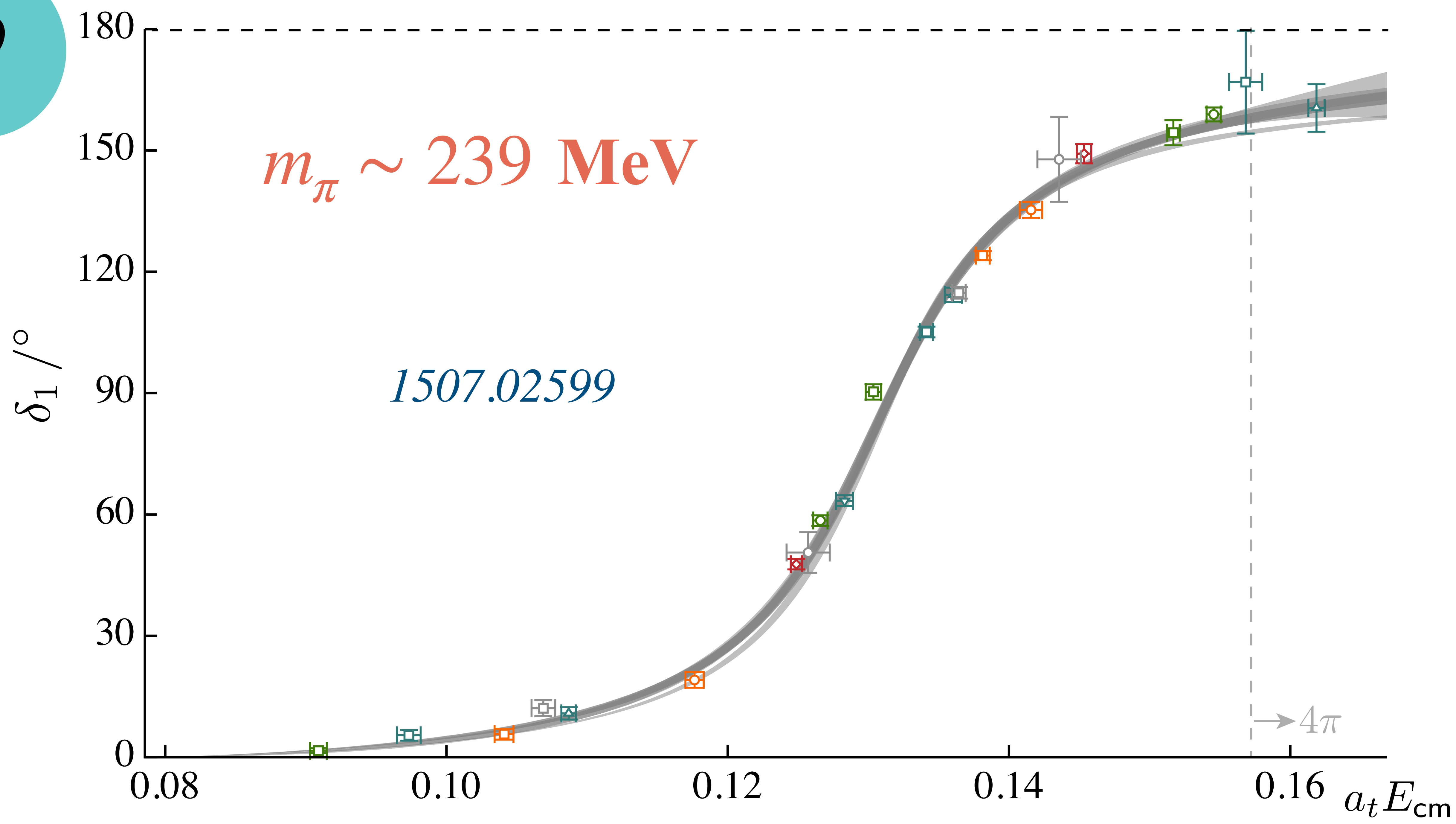


$I = 1\ \pi\pi$



Percent error for $\delta(s)$

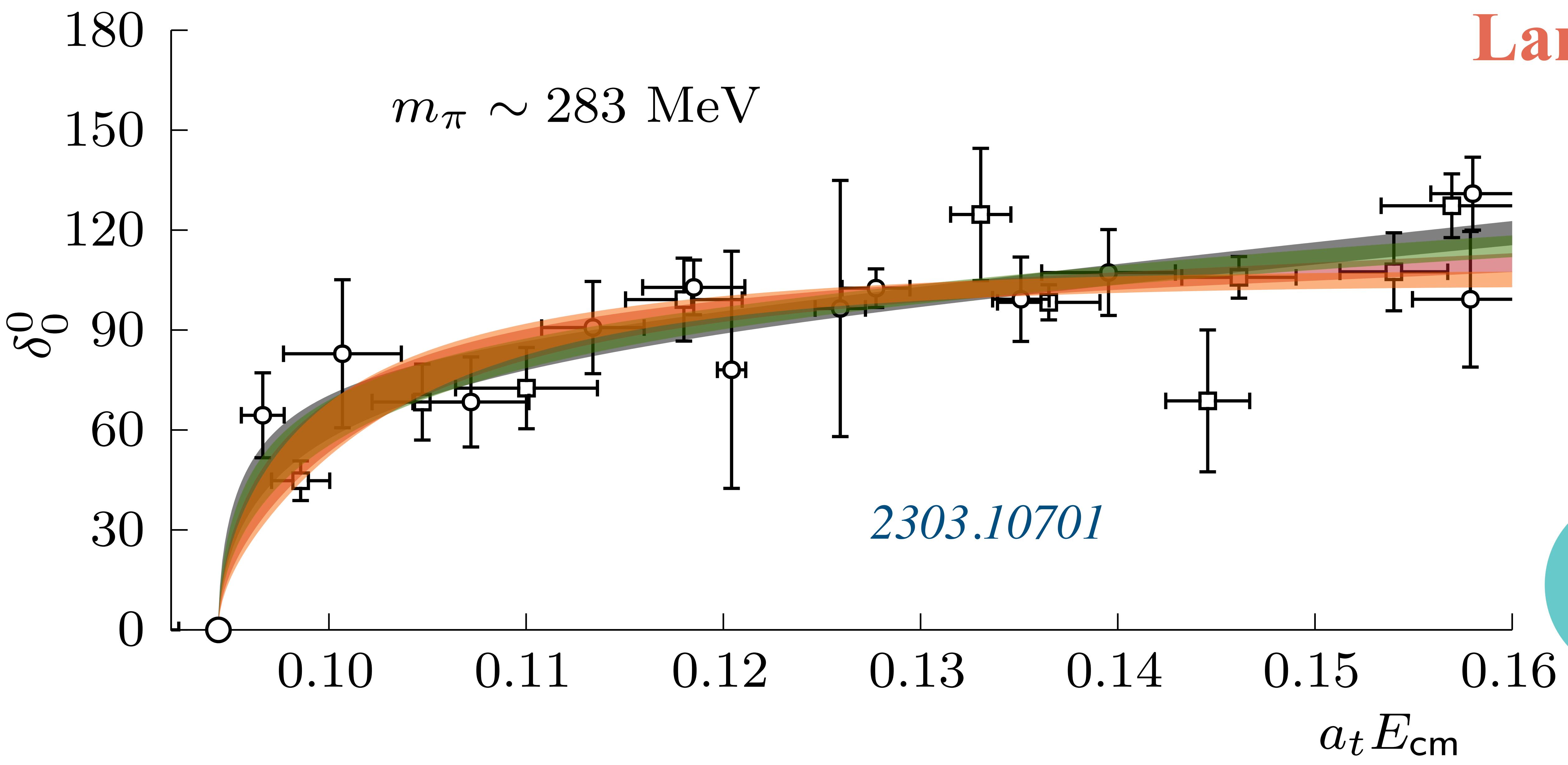
ρ



Around 10 parameterizations

Very consistent amplitude fits

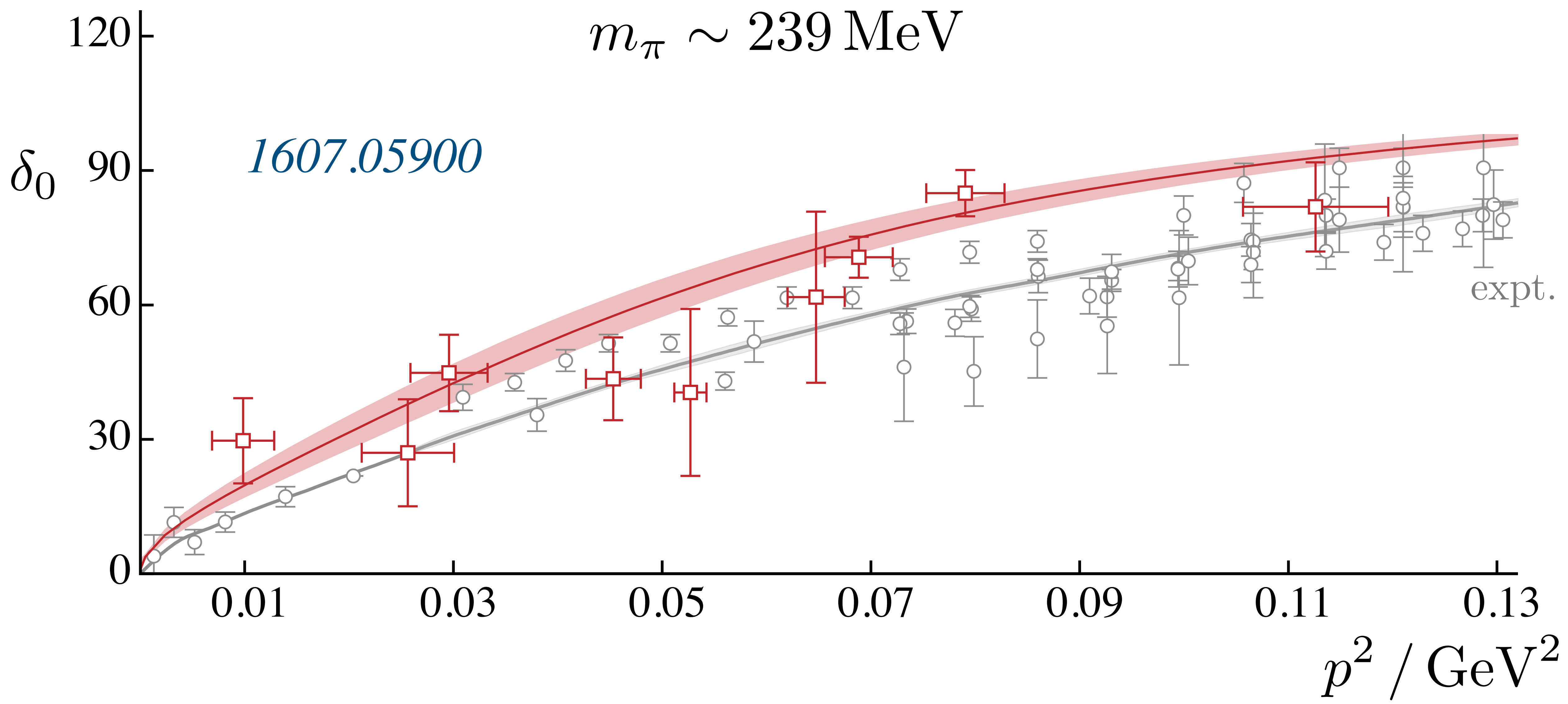
$I = 0\ \pi\pi$



Large derivative at threshold

$\delta(s) = \pi/2$ is far from threshold

σ



Over 20 parameterizations

Smaller derivative at threshold

Use all PWs available

Scalar $\ell = 0$ waves dominate the DRs

But we extracted/fitted several waves

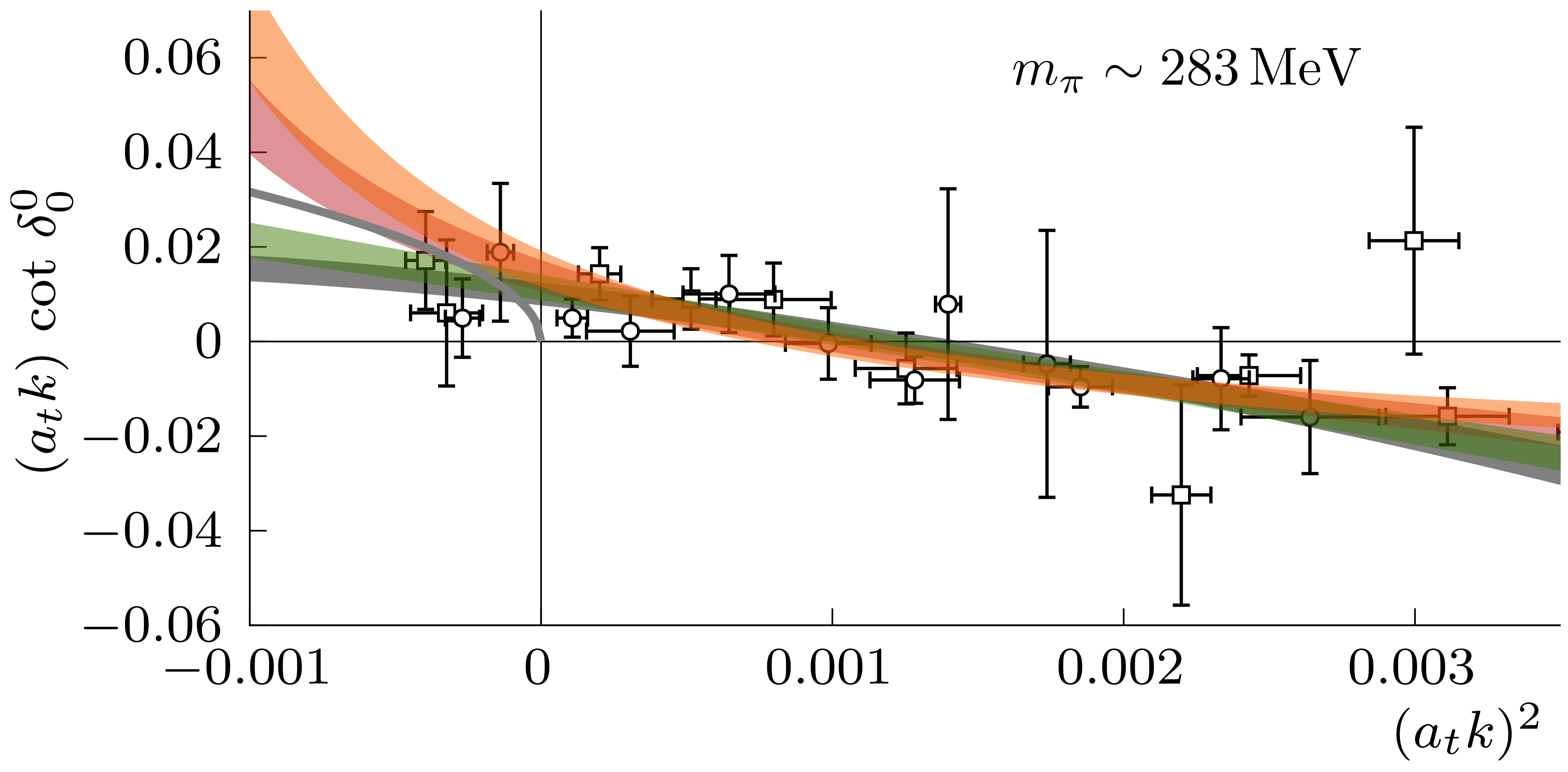
Every band is a different model fit

$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{1}{\cot \delta_\ell^I(s) - i}$$

Large SL spreads at threshold

$$k \cot \delta_0^I(s) \sim 1/a_0^I$$

$\ell = 0,\ I = 0\ \pi\pi$



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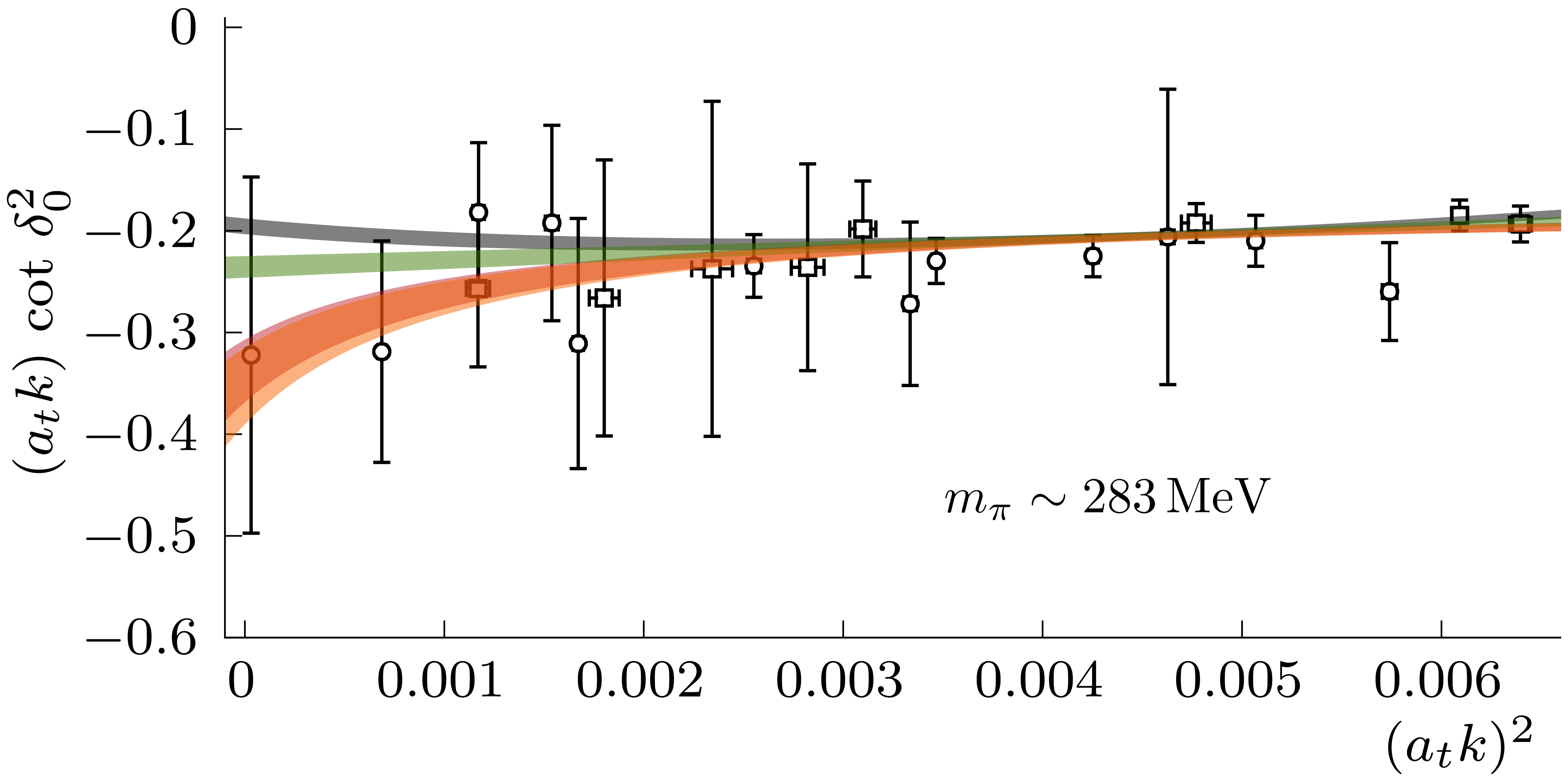
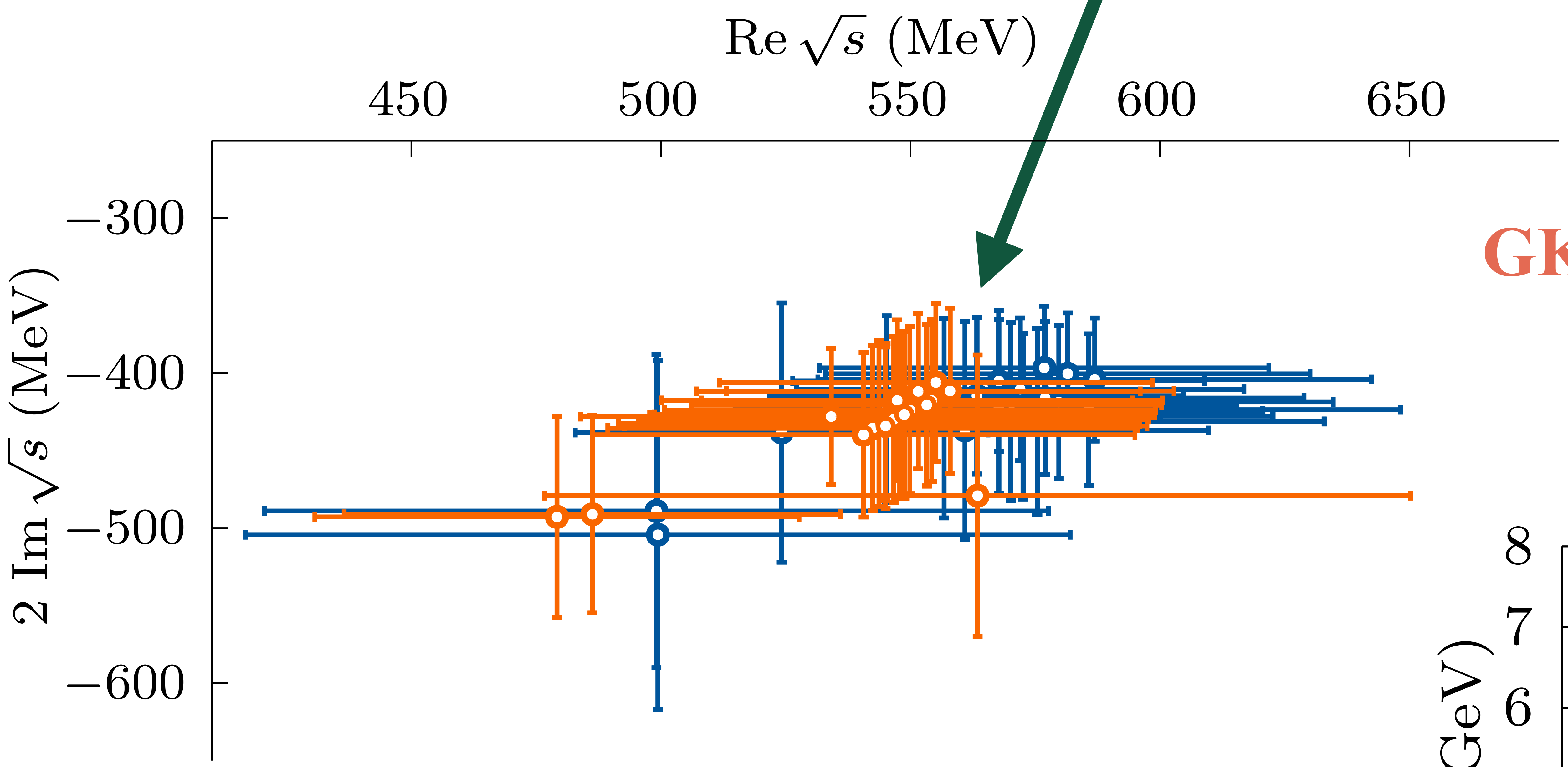
$\ell = 0,\ I = 2\ \pi\pi$

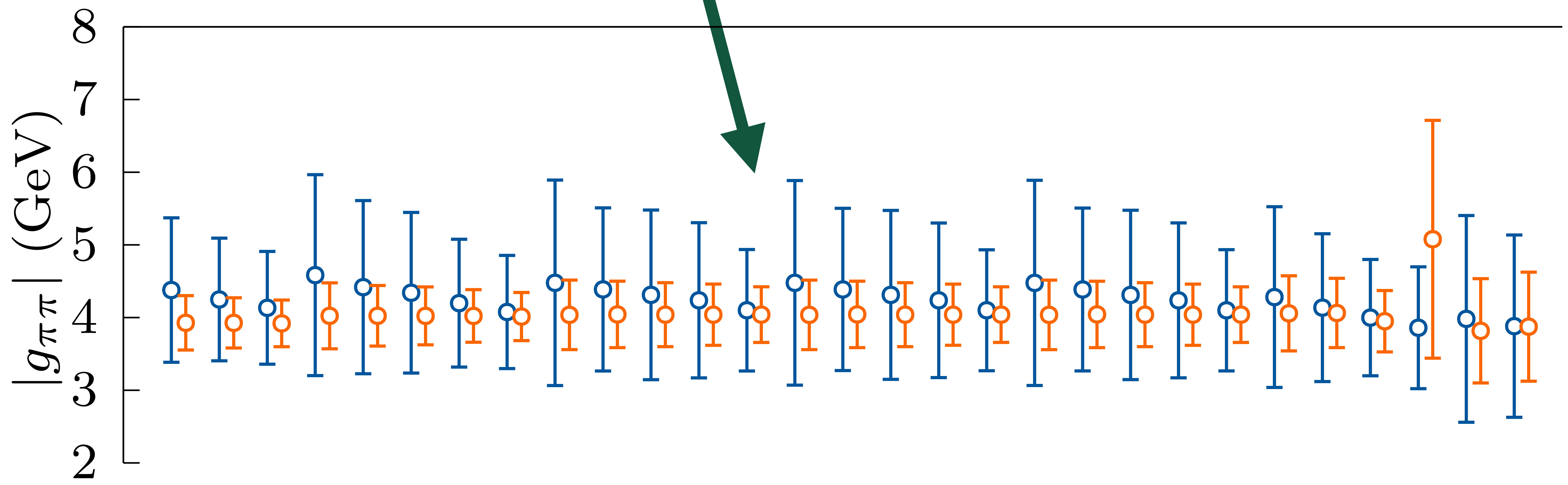


Couplings

However, pole extraction is more accurate in most cases, particularly for the coupling



GKPY produces $\sim 40\,\%$ uncertainty in most cases



Permutations

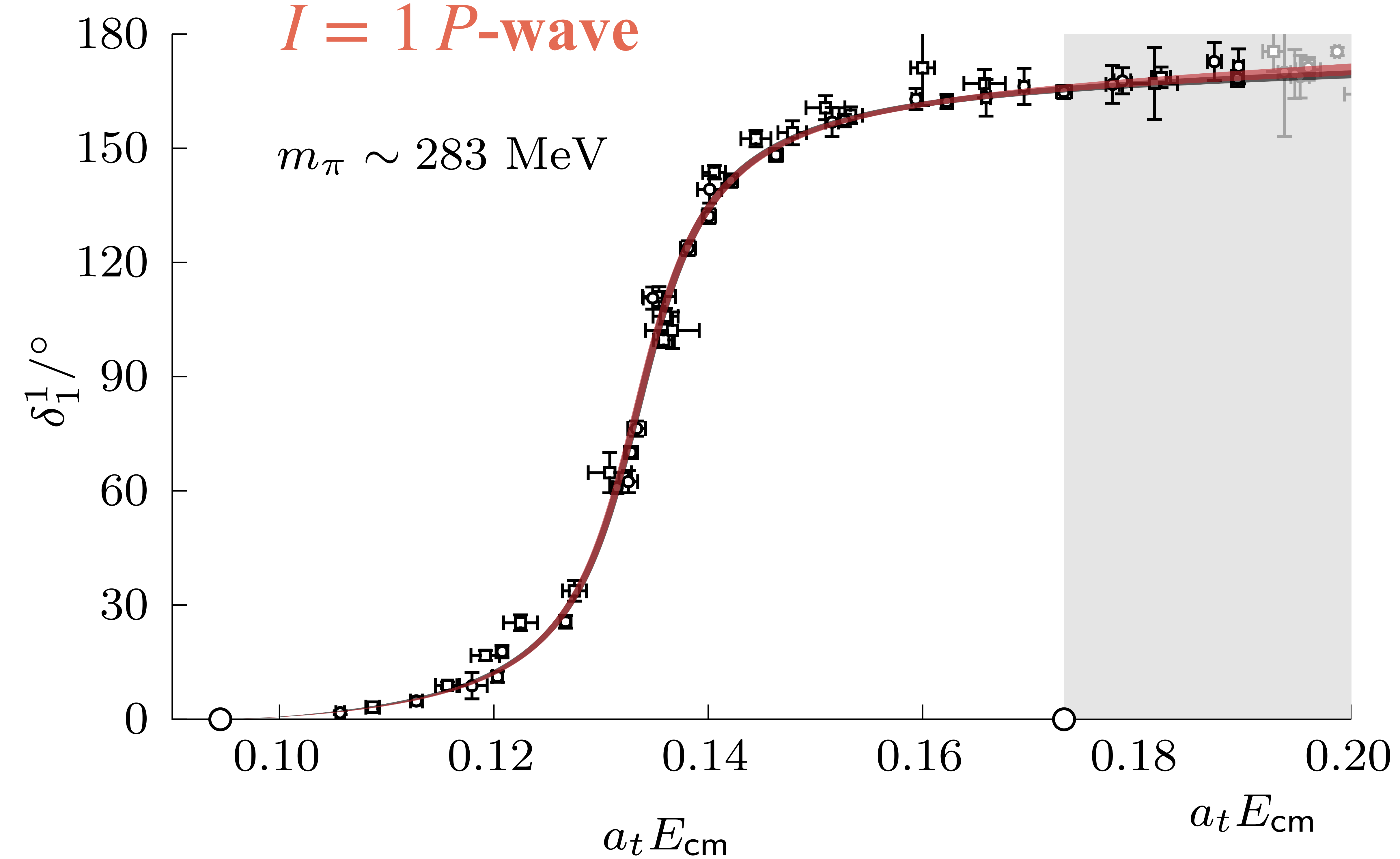
$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s',s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

For ℓ_{max} partial waves

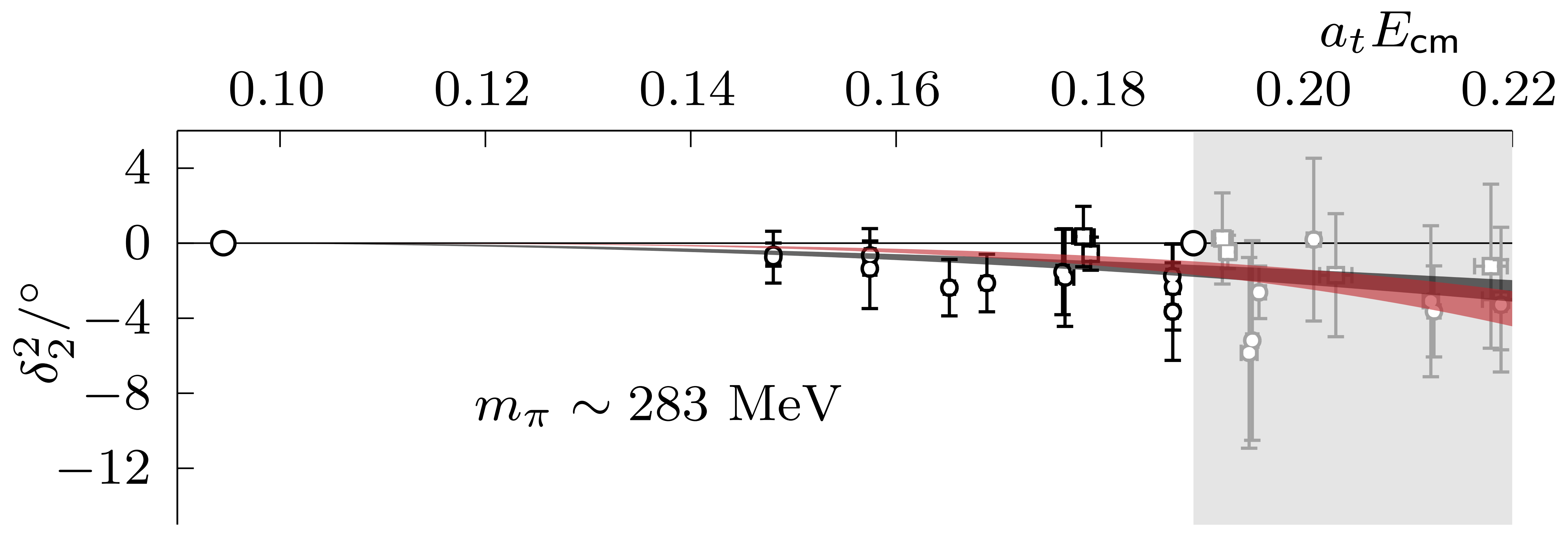
$$N_I \ell_{max} N_{params} \sim 10^5$$

We can fix most

$I = 1$ P-wave



$I = 2$ D-wave



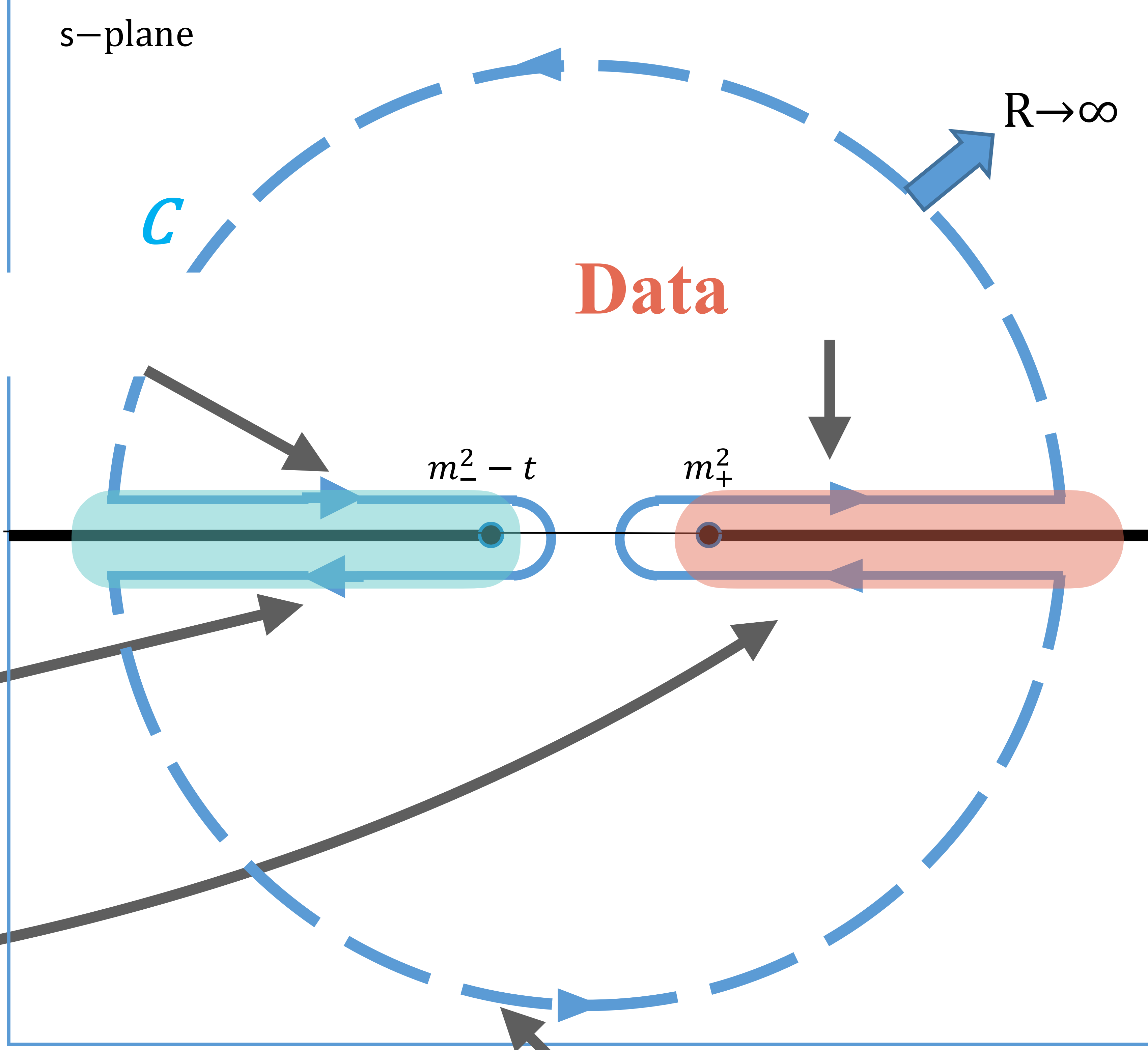
Dispersion relations

Cauchy

$$t(z) = \oint_C \frac{t(z')}{z'-z} dz'$$

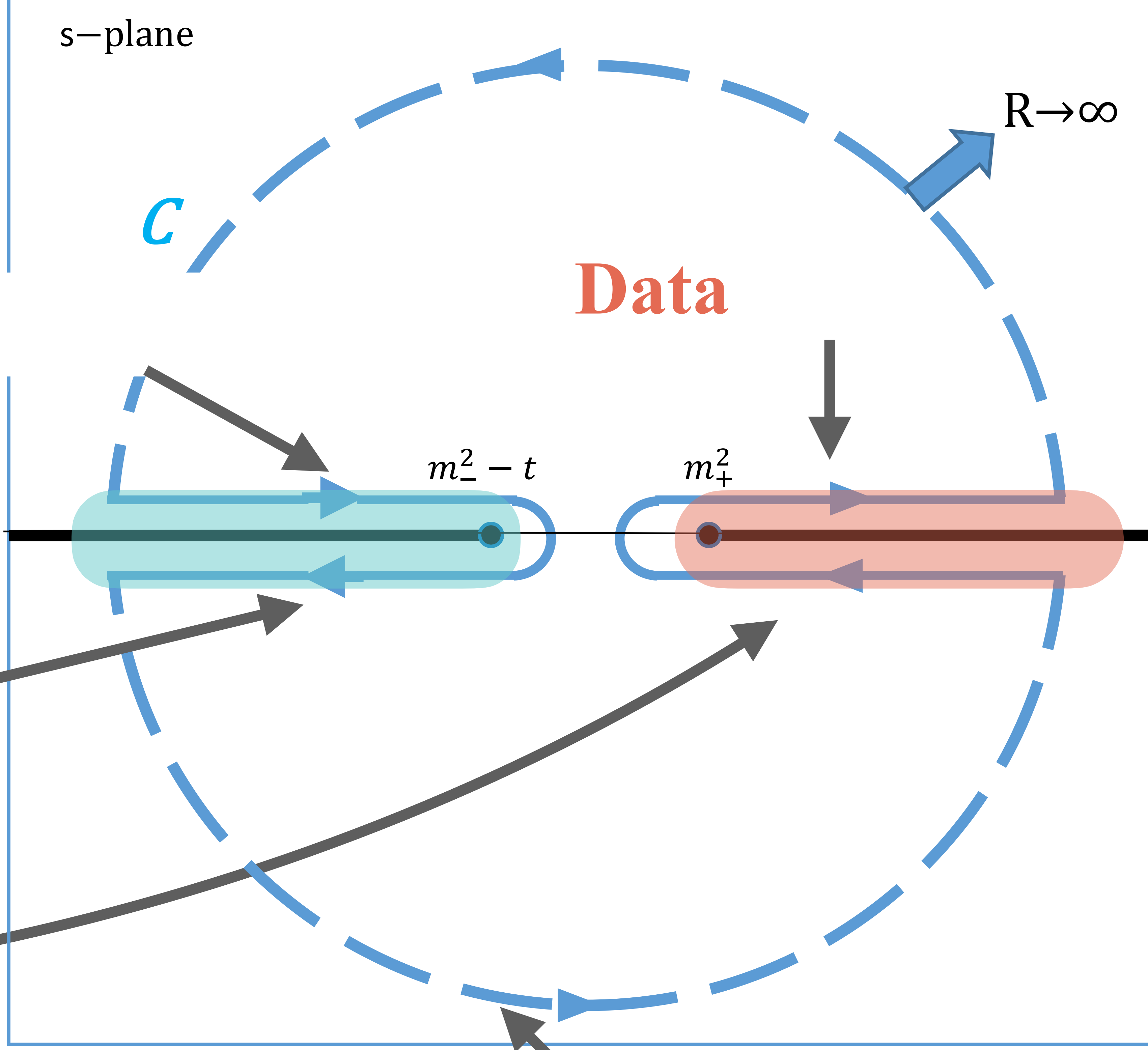
Crossed Data

$$T^I(s,t) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}\, T^I(s',t)}{s'-s} + LHC$$



0 contribution

Dispersion relations



Cauchy

Crossed Data

$$t(z) = \oint_C \frac{t(z')}{z' - z} dz'$$

$$T^I(s,t) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}\, T^I(s',t)}{s' - s} + LHC$$

Crossing

$$T^I(s,t) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \left(\frac{\mathrm{Im}\, T^I(s',t)}{(s' - s)} + \frac{\sum C_{su}^{I\,I'}\, \mathrm{Im}\, T^{I'}(s',t)}{(s' - u)} \right)$$

Partial wave dispersion relations

2304.03762

Amplitudes are decomposed in partial waves

$$T^I(s,t) = 32\pi \sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s)$$

Fit to the data

Fit → In

DR → Out

Dispersive's result

$$\tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

The most well-known are the ROY eqs., for example, for $I = \ell = 0$ they look

Roy PLB (1971)

$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Fit → In

DR → Out

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds'\, K^{II'}_{\ell\ell'}(s',s)\, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

"Model-independent"

Algebraic functions

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds'\, K^{II'}_{\ell\ell'}(s',s)\, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

All Isospins and partial waves

Make *Fit → In* *DR → Out* **compatible** ⟷ **Unitarity**

$$\left[d^2\right]_\ell^I \equiv \sum_{i=1}^{N_{\rm smpl}} \left(\frac{\mathrm{Re}\,\tilde{t}_\ell^I(s_i) - \mathrm{Re}\,t_\ell^I(s_i)}{\Delta\left[\mathrm{Re}\,\tilde{t}_\ell^I(s_i) - \mathrm{Re}\,t_\ell^I(s_i)\right]} \right)^2$$

Make *DR → Out* **and data compatible** ⟷ **Lattice QCD data description**

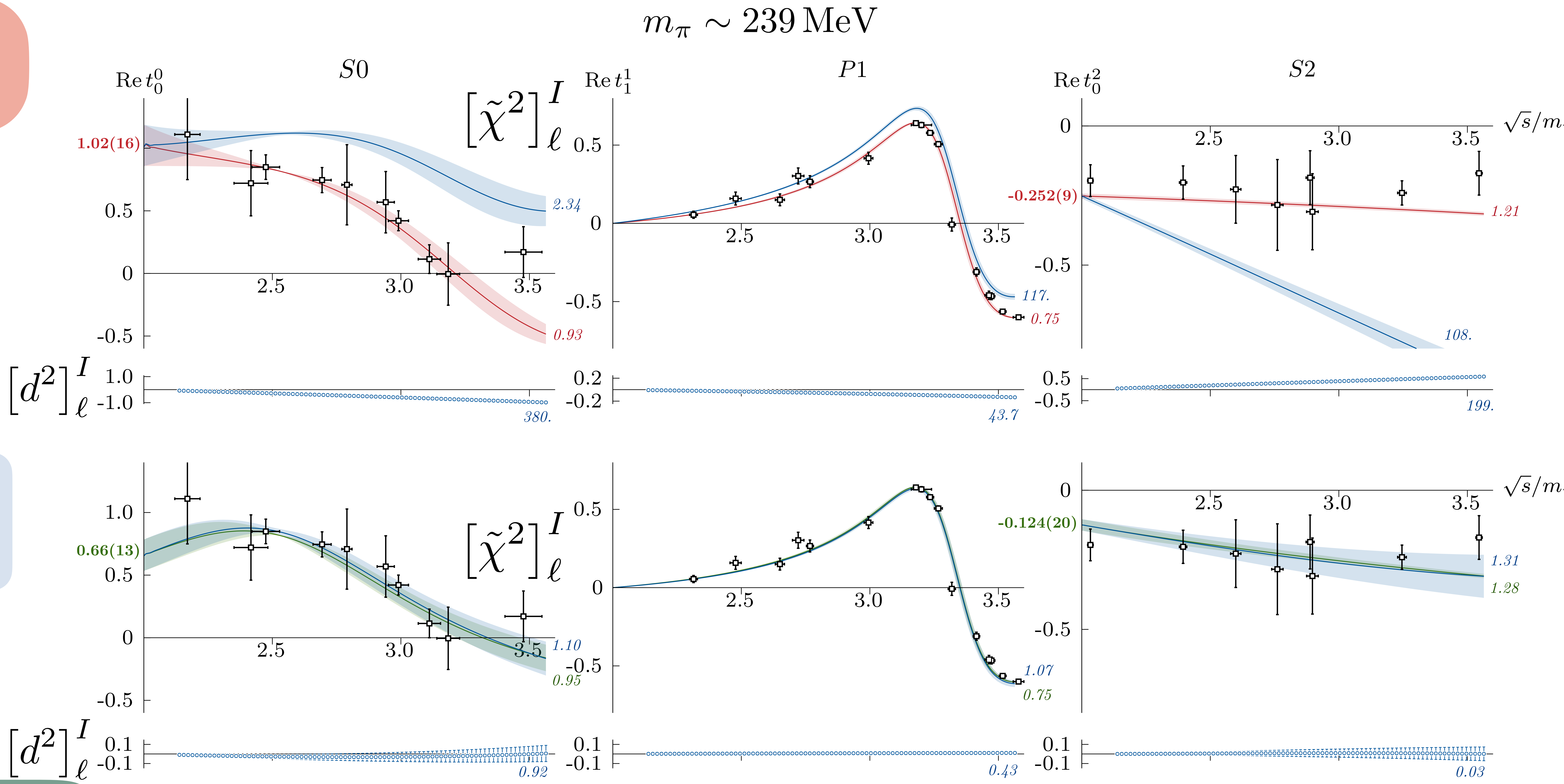
$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}_\ell^I(s_i)}{\Delta_i} \right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right) \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}_\ell^I(s_j)}{\Delta_j} \right)$$

Tests: good vs bad

Bad fit combination

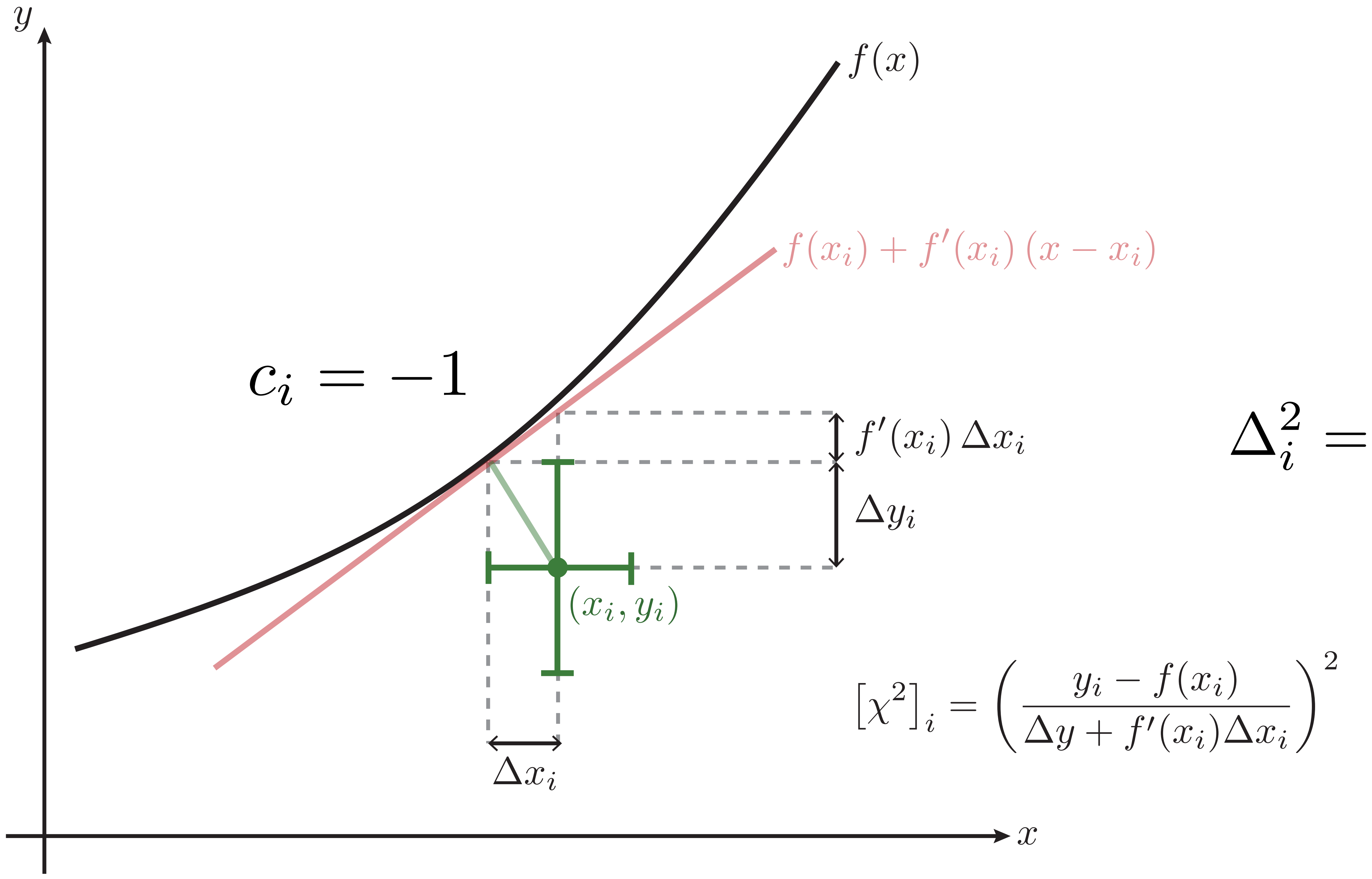
Dispersive output

Good fit combination



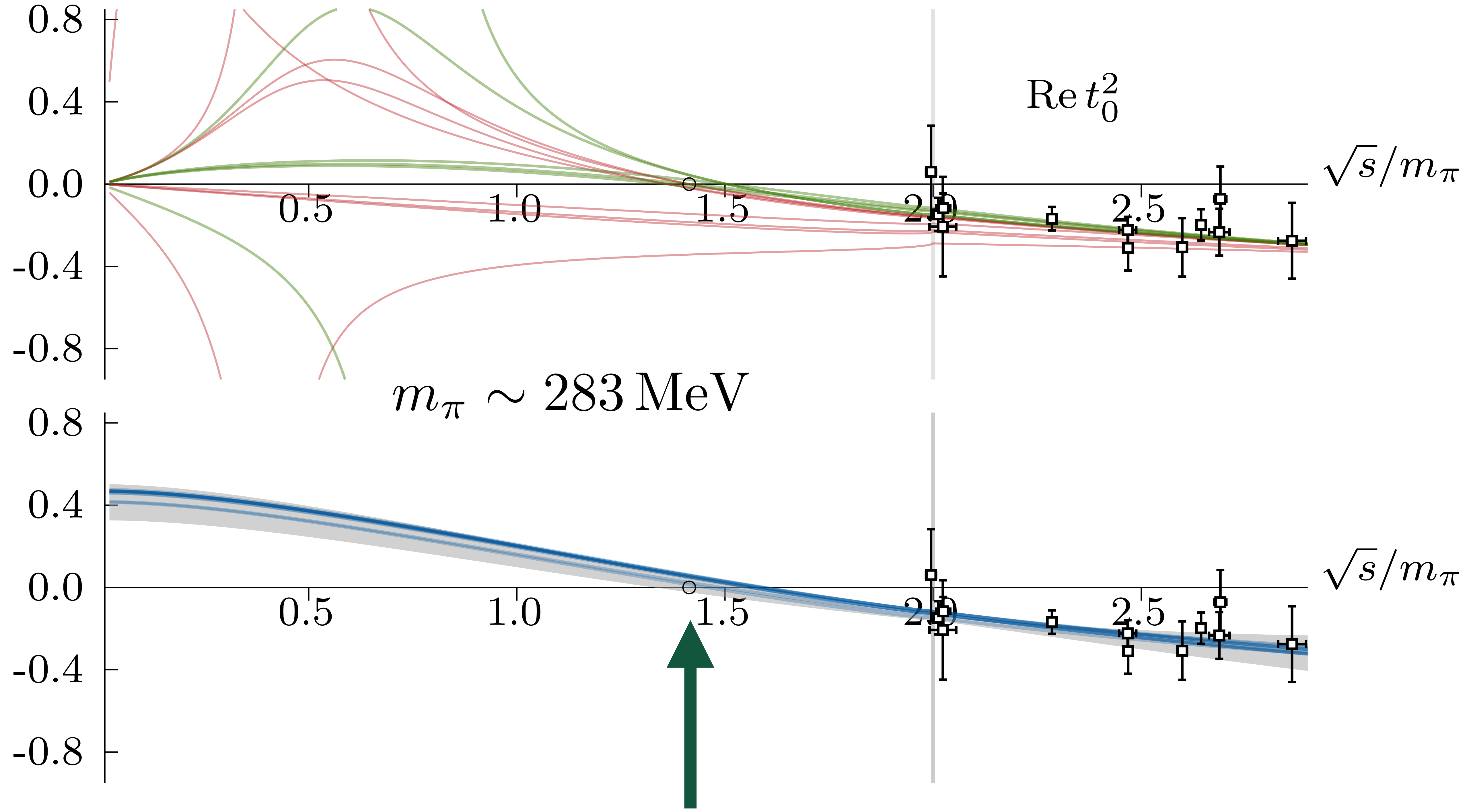
Make $DR \rightarrow Out$ **and data compatible** ⟷ **Lattice QCD data description**

$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - {\rm Re}\,\tilde{t}_\ell^I(s_i)}{\Delta_i}\right) {\rm corr}\,(\mathfrak{f}_i,\mathfrak{f}_j) \left(\frac{\mathfrak{f}_j - {\rm Re}\,\tilde{t}_\ell^I(s_j)}{\Delta_j}\right)$$



$$\Delta_i^2 = \begin{pmatrix} \Delta\mathfrak{f}_i & \frac{d\tilde{f}_\ell^I(s_i)}{dE_i}\Delta E_i \end{pmatrix} \begin{pmatrix} 1 & -c_i \\ -c_i & 1 \end{pmatrix} \begin{pmatrix} \Delta\tilde{f}_i \\ \frac{d\tilde{f}_\ell^I(s_i)}{dE_i}\Delta E_i \end{pmatrix}$$

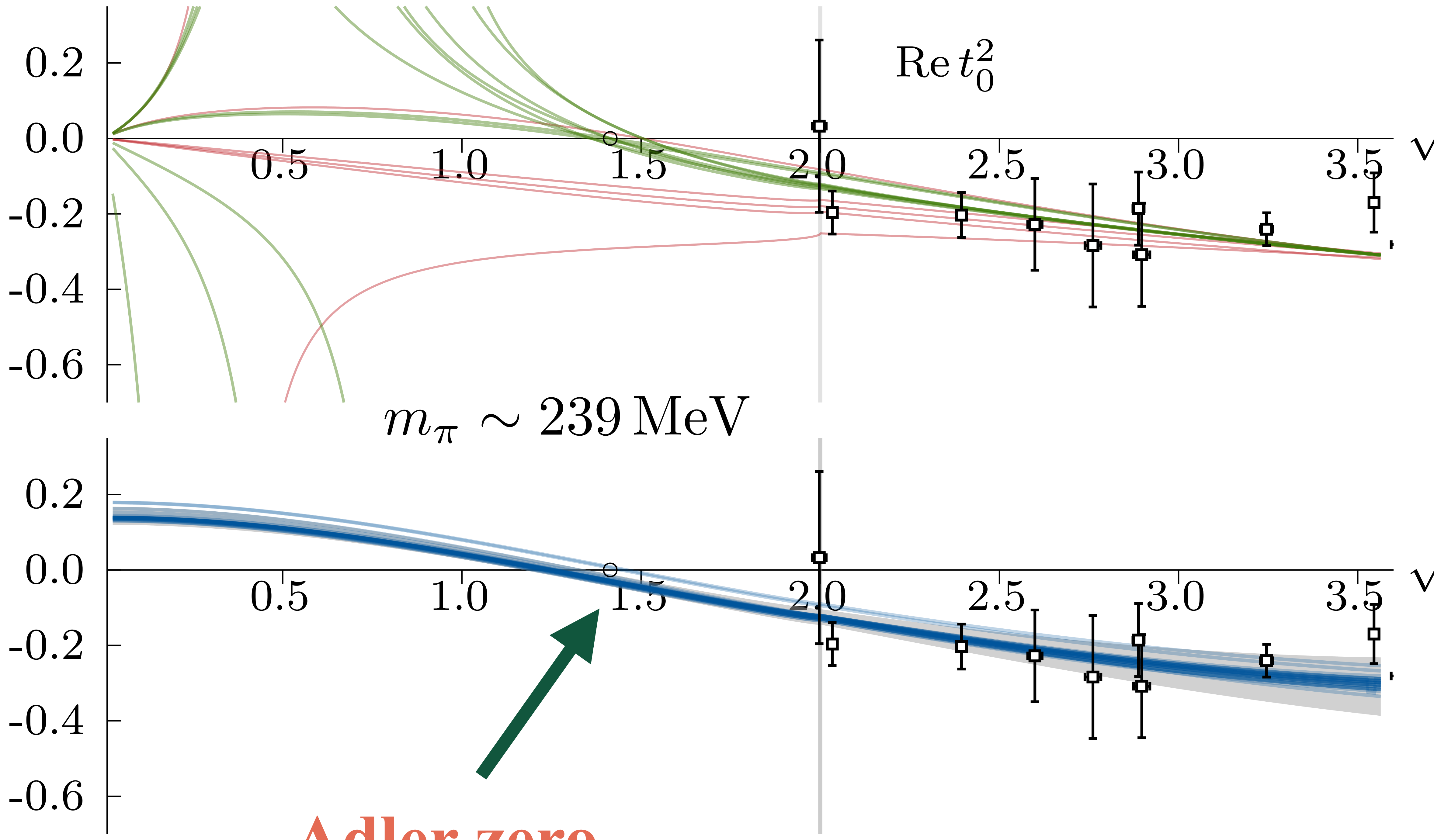
Sub-threshold



Adler zero

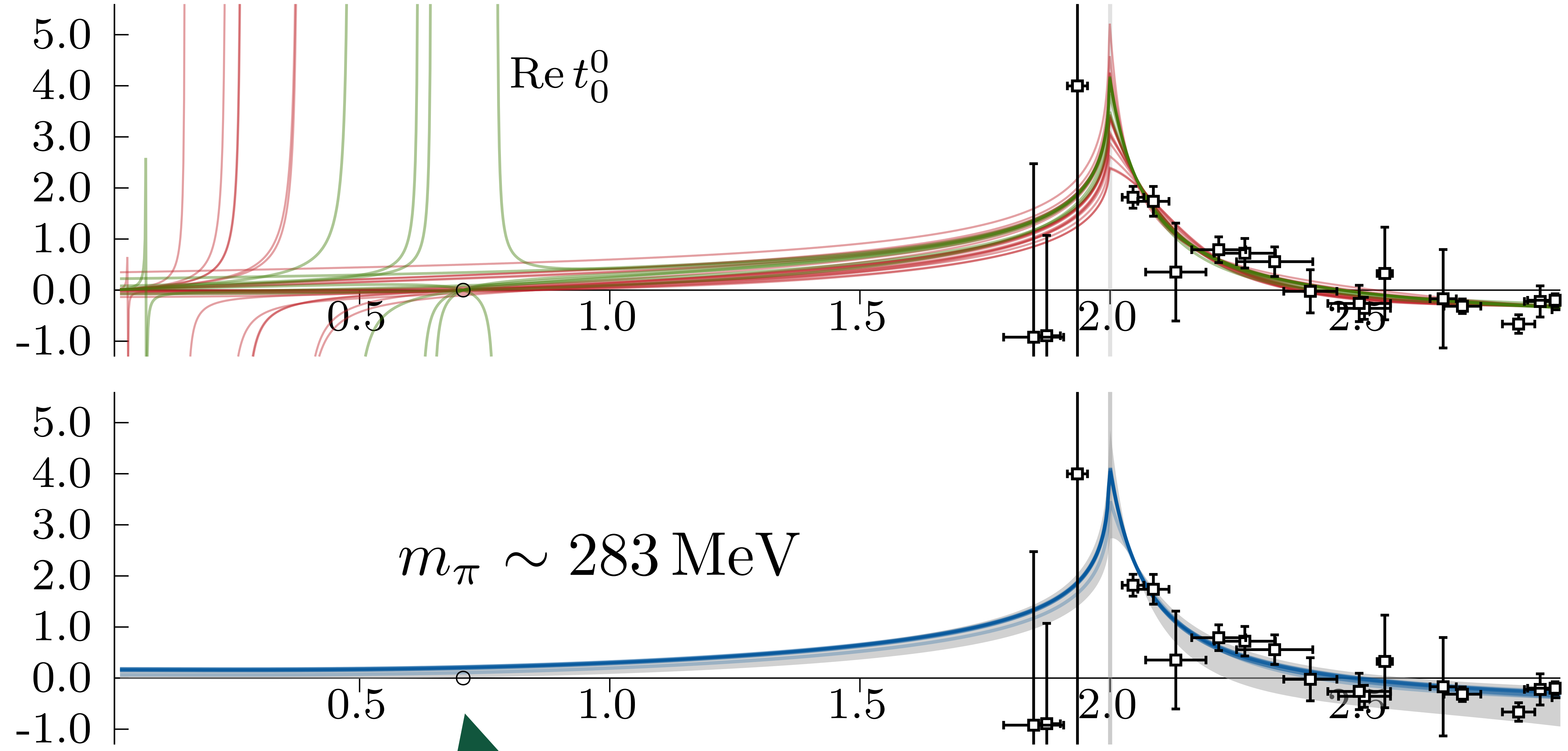
Even “bad” DRs produce Adler zeroes for I=2

Very “stable” for $I = 2\ \pi\pi$

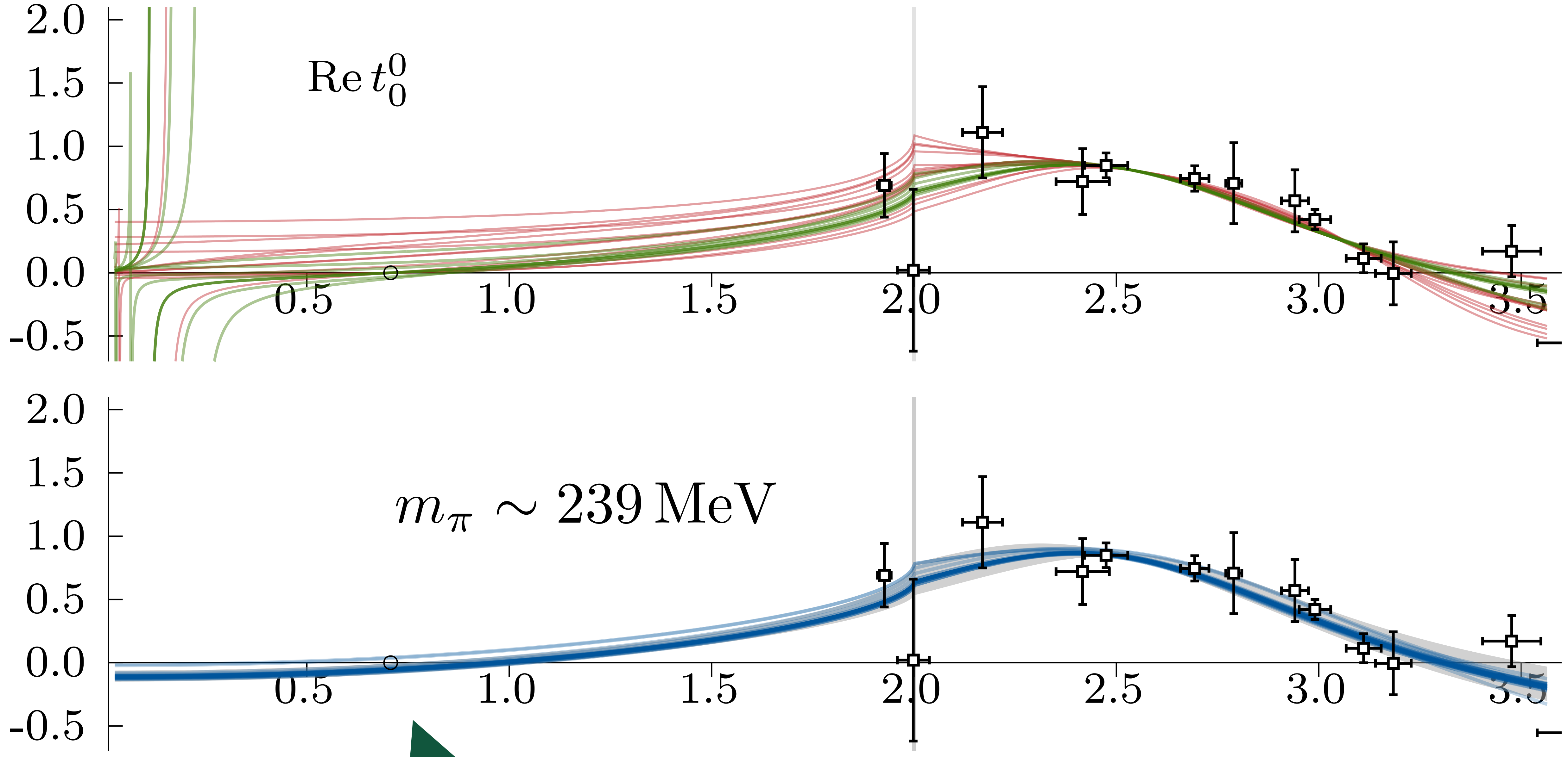


Adler zero

Sub-threshold



No good DR produces an $I=0$ $\pi\pi$ Adler zero for the heavier mass



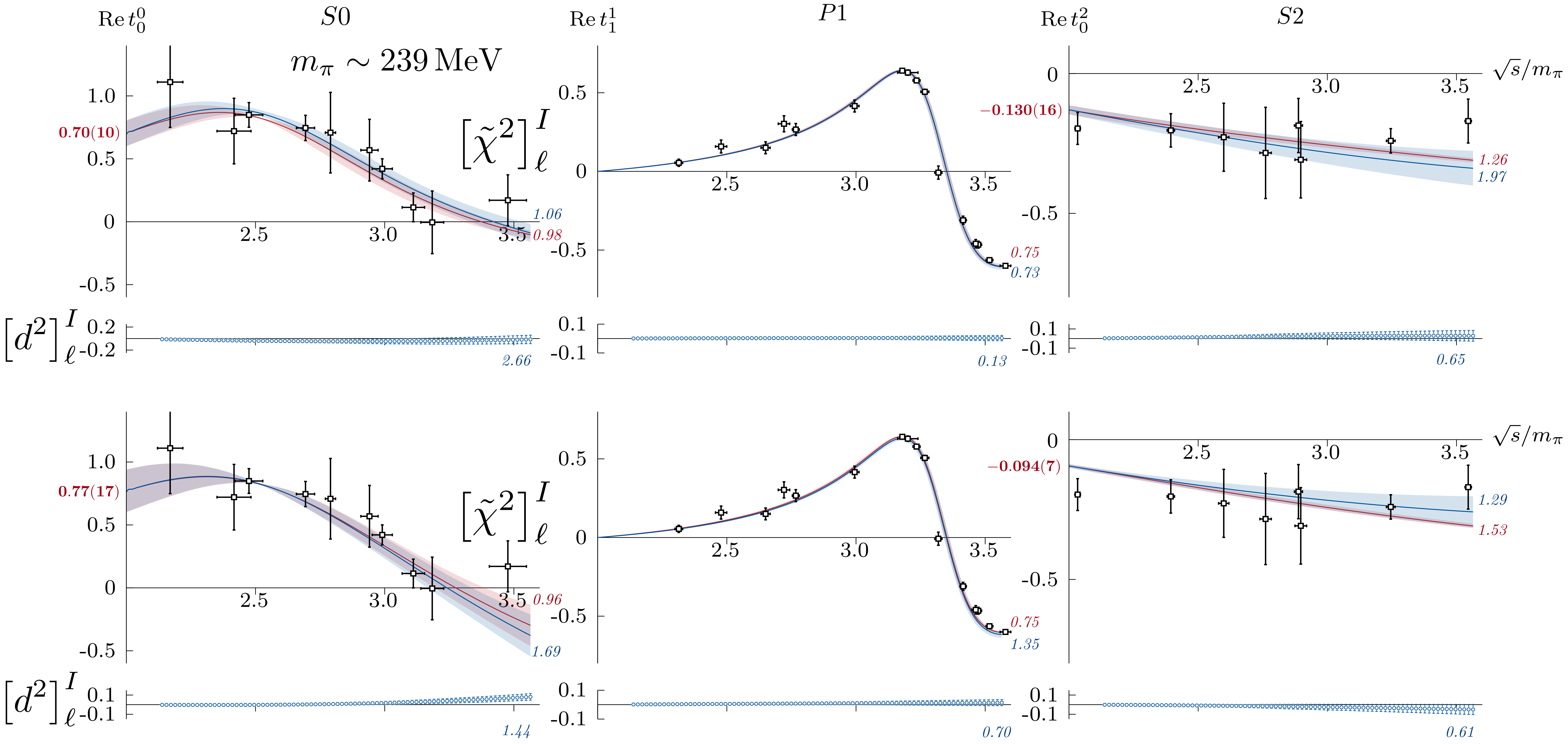
NO Adler zero

All good DRs produce an $I=0$ $\pi\pi$ Adler zero for the lighter mass

Adler zero

Ok but not great

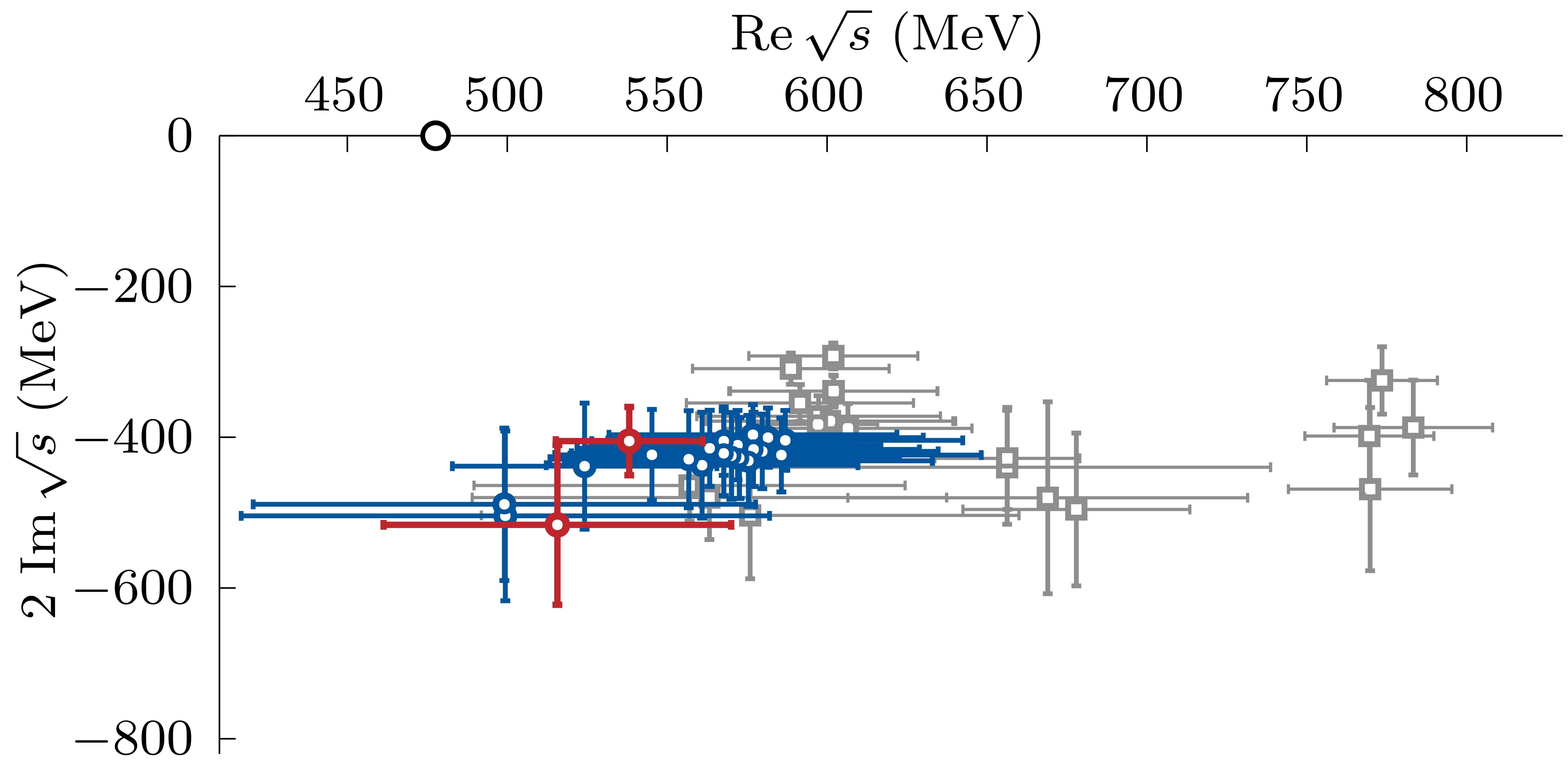
Visually, they describe the data and fit, but they are not perfect



Ok but not great

Visually, they describe the data and fit, but they are not perfect

$m_\pi \sim 239\,\mathrm{MeV}$

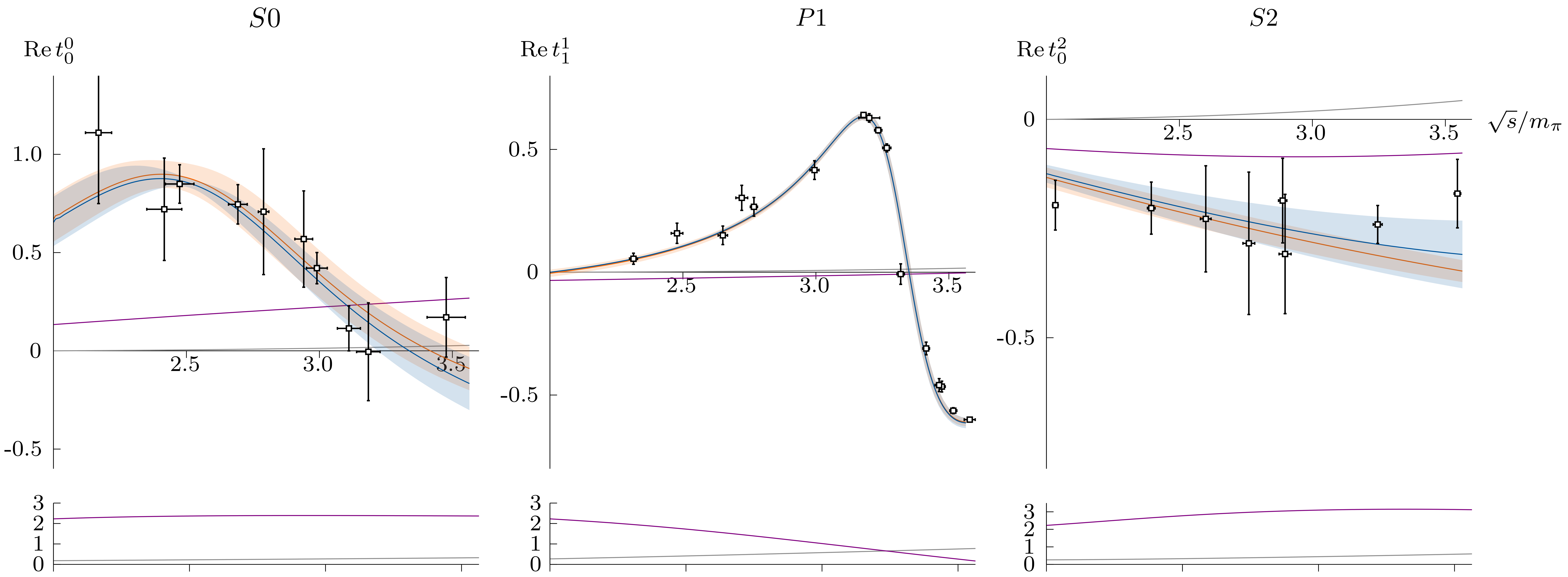


GKPY vs ROY

GKPY: Minimally subtracted → one less subtraction than ROY

For our analysis, Regge contribution too large for d^2

$m_\pi \sim 239\,\mathrm{MeV}$



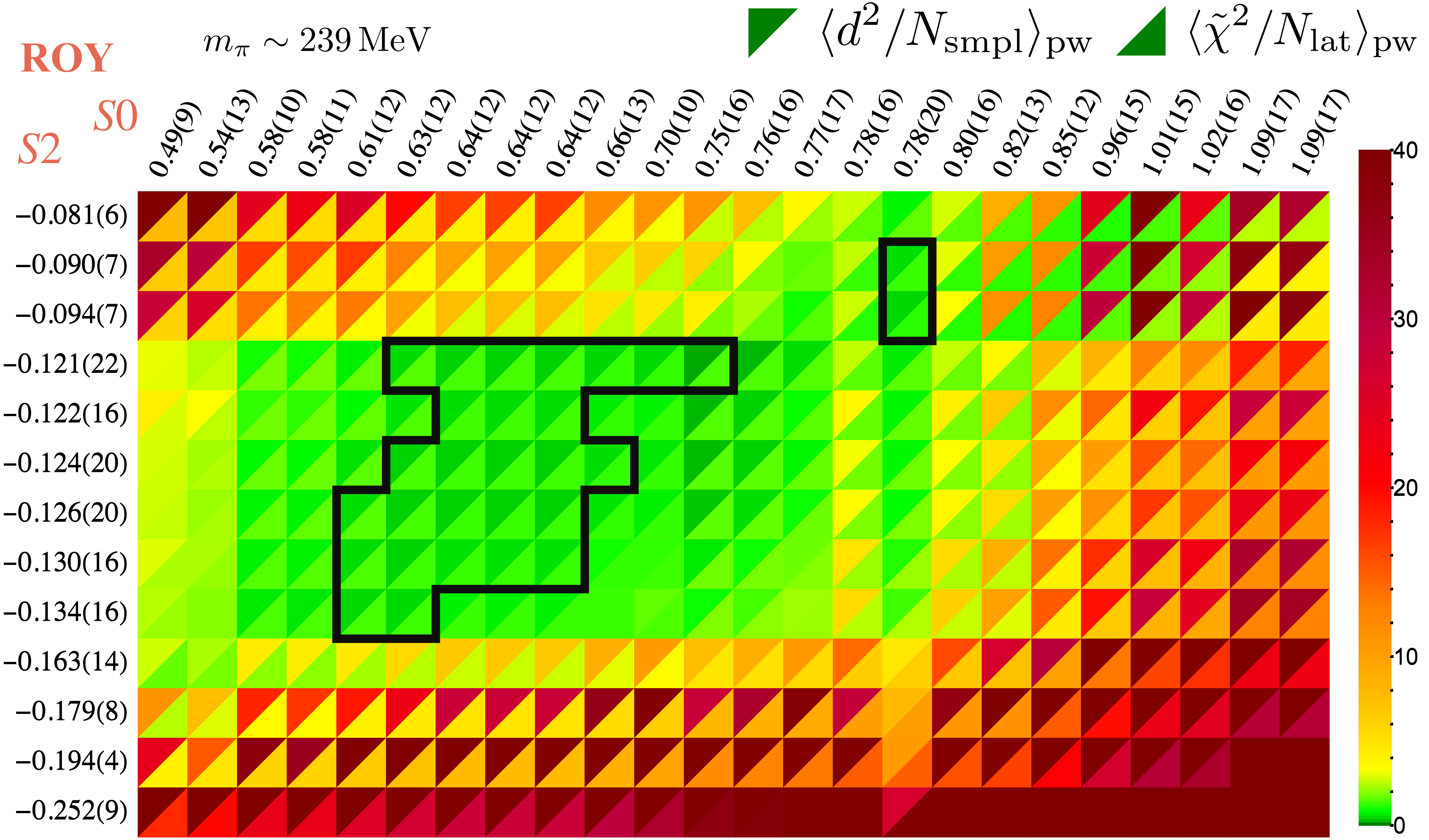
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black $d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

ROY



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

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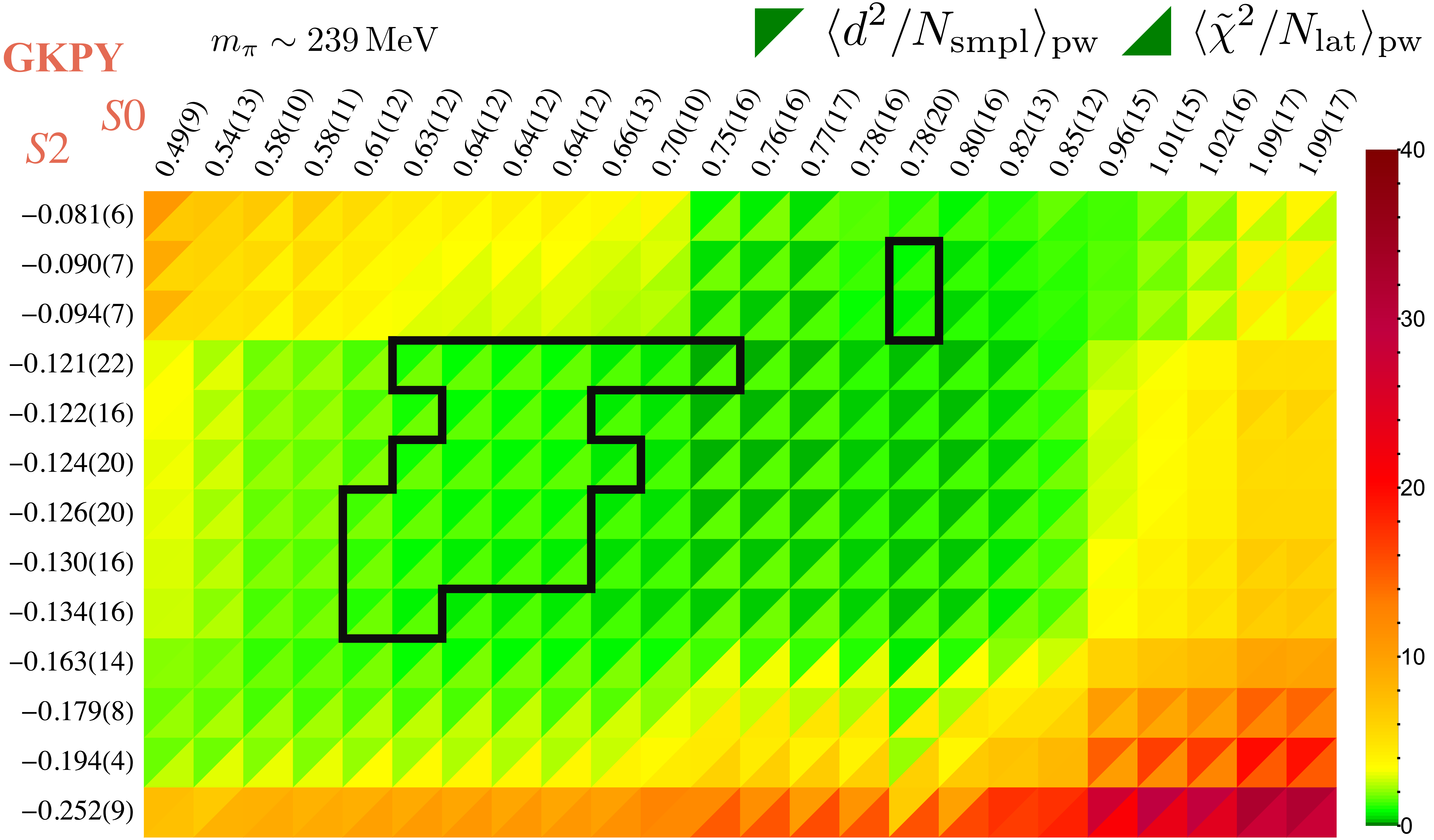
$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



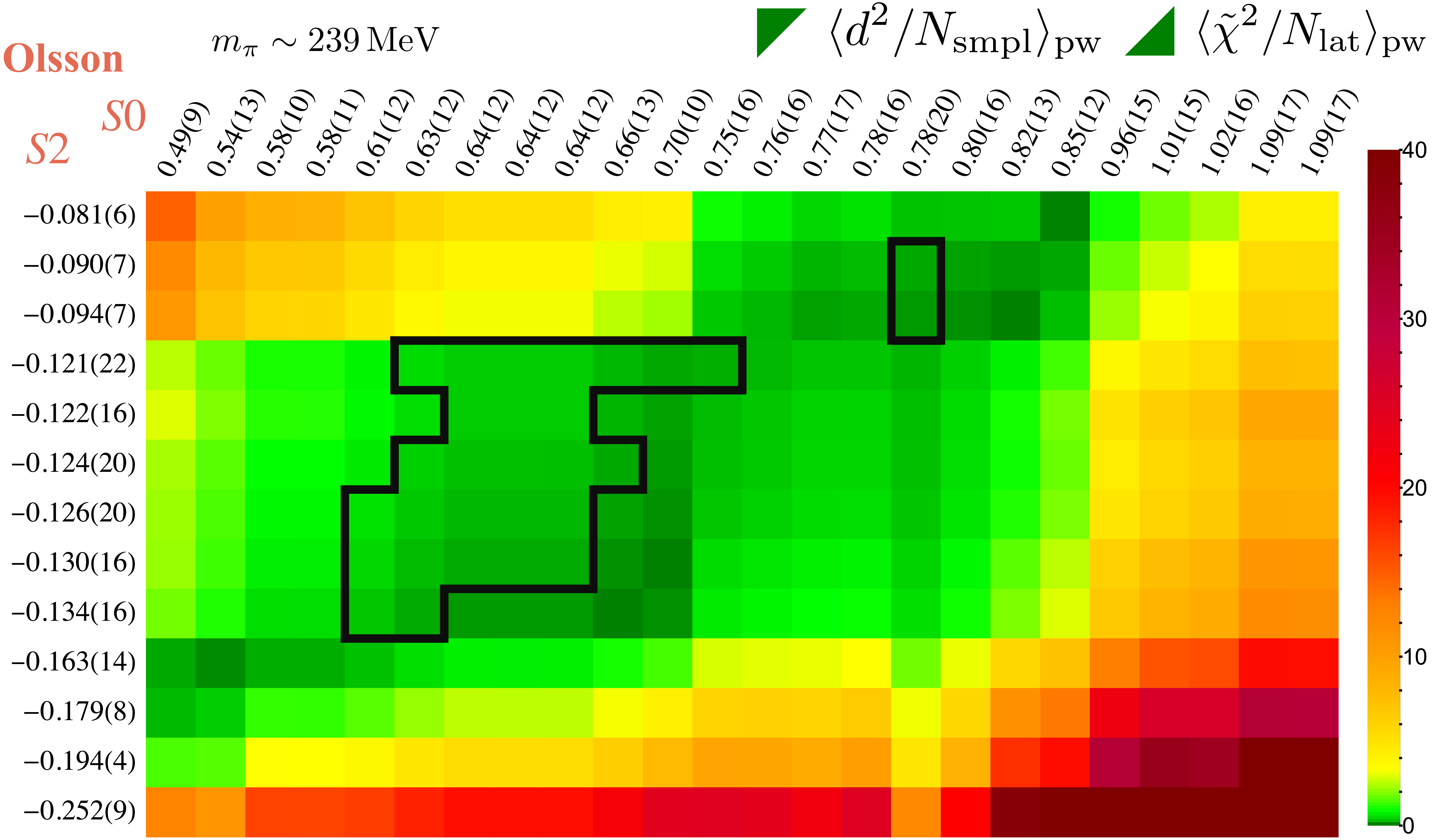
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black $d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

Olsson



Scattering plane

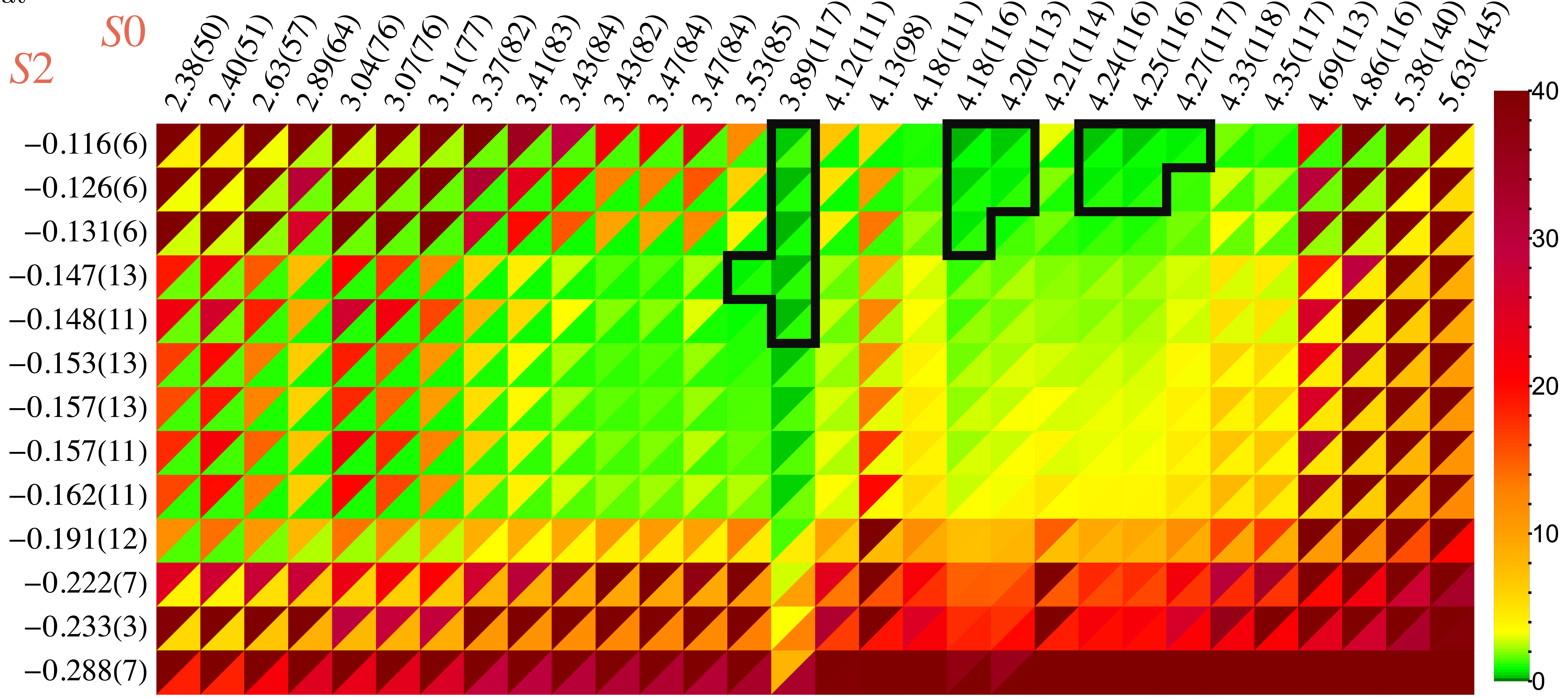
$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

$\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

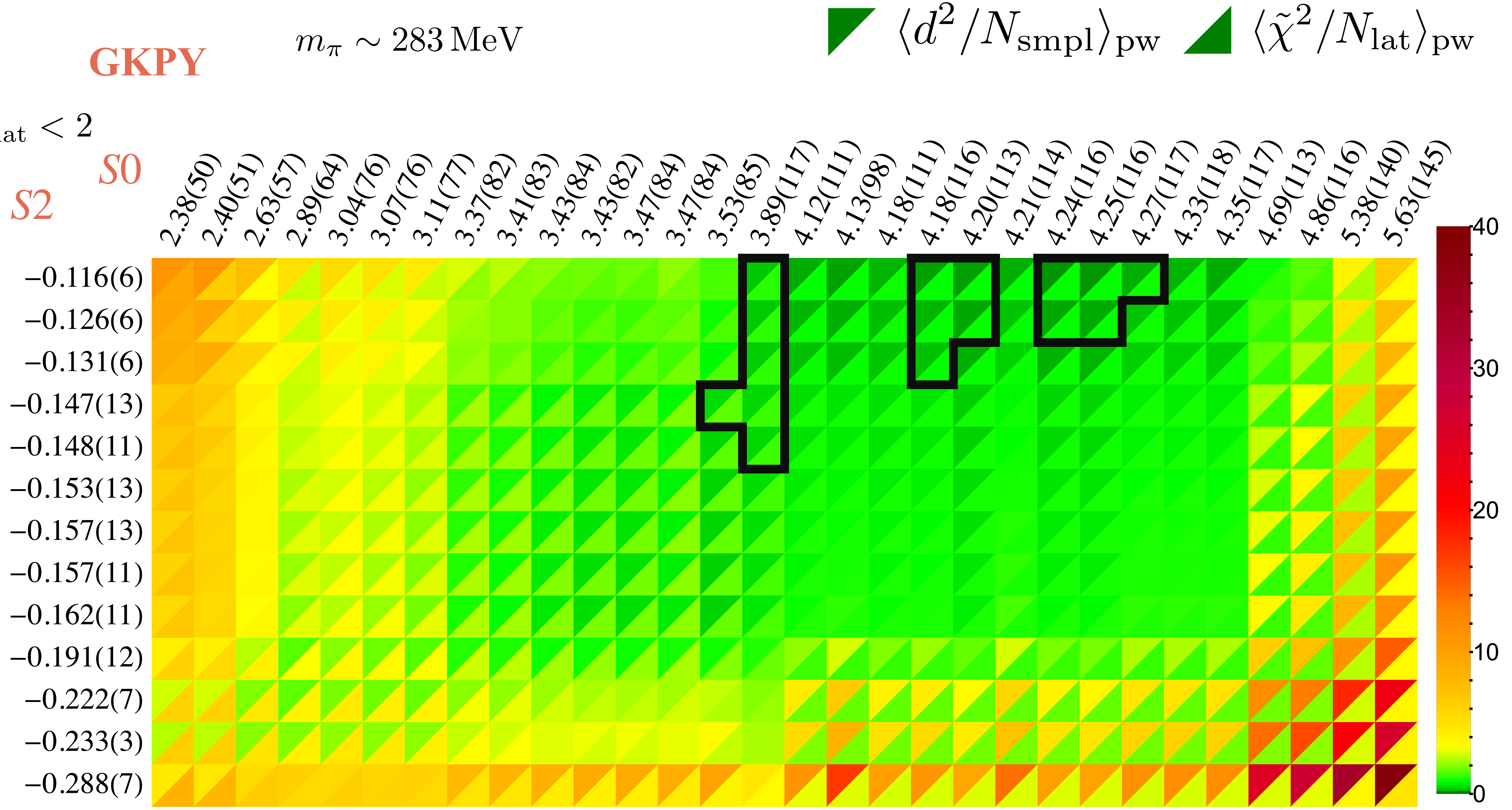
$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

Black

GKPY

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

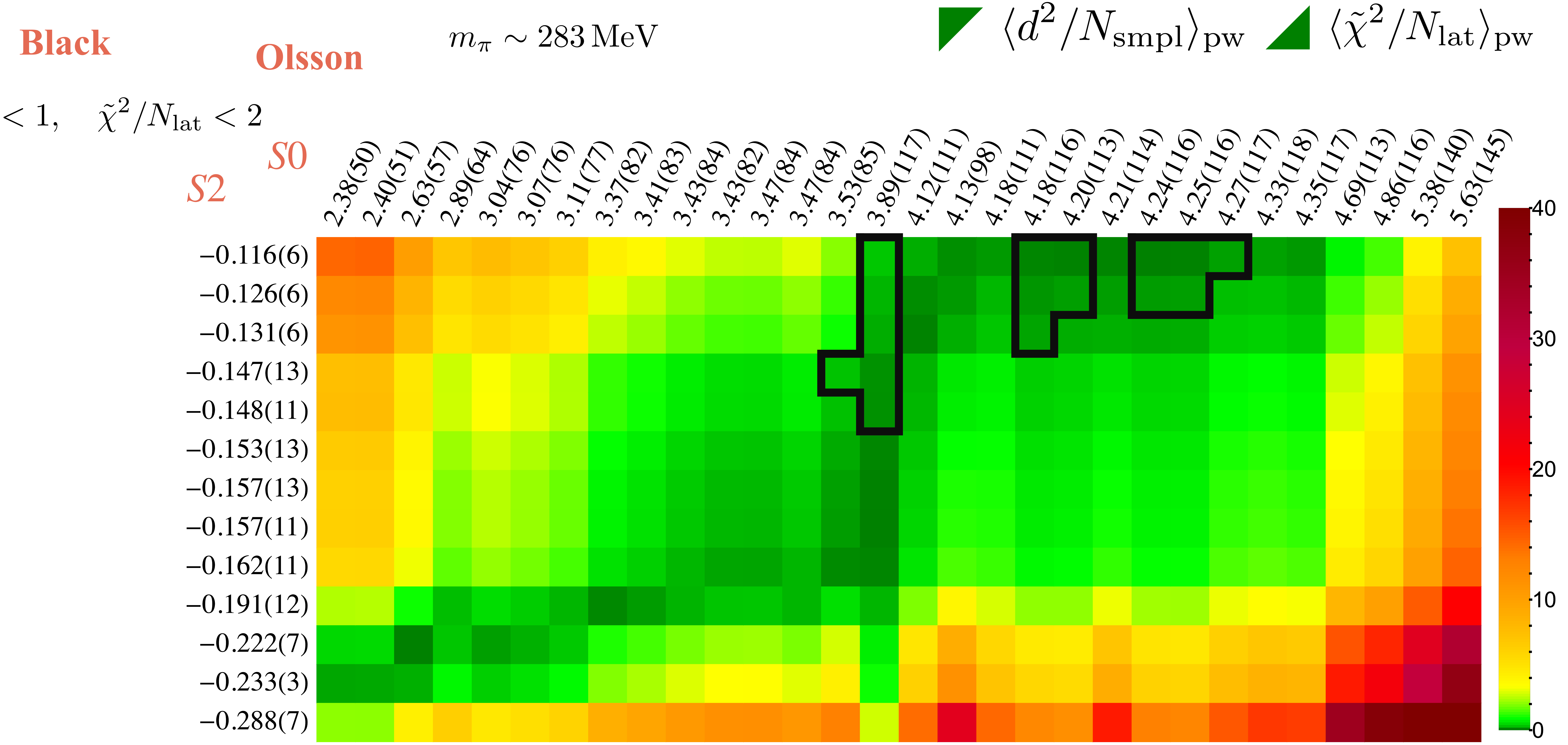


Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



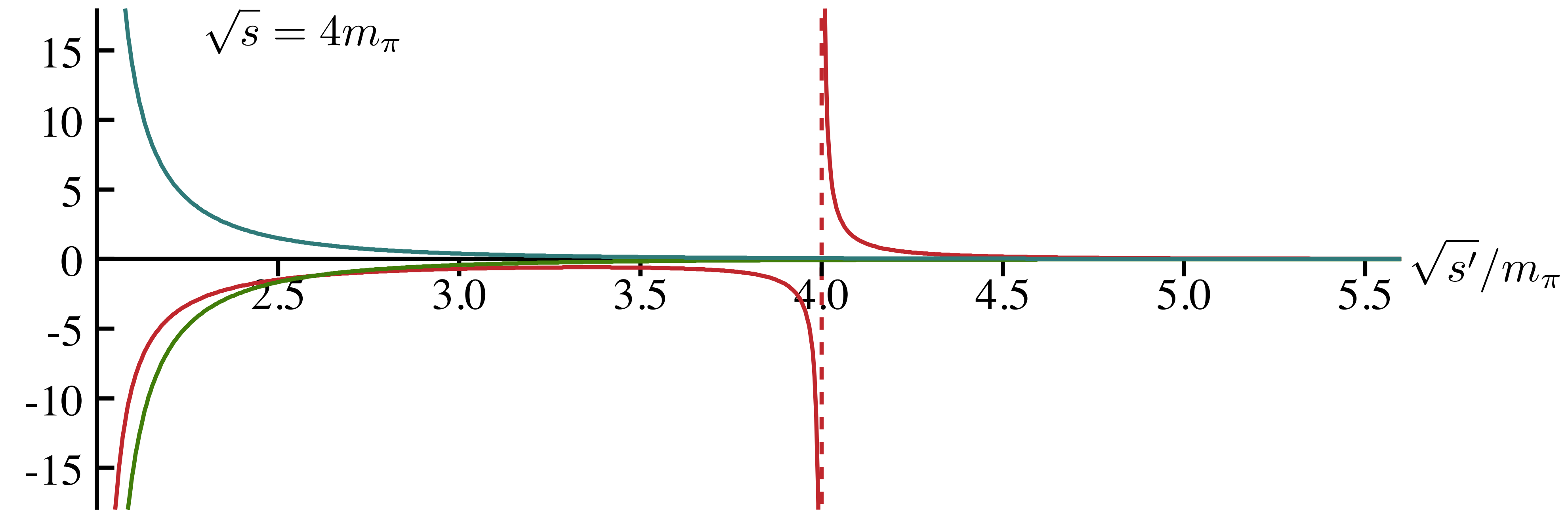
The good

Fit → In

DR → Out

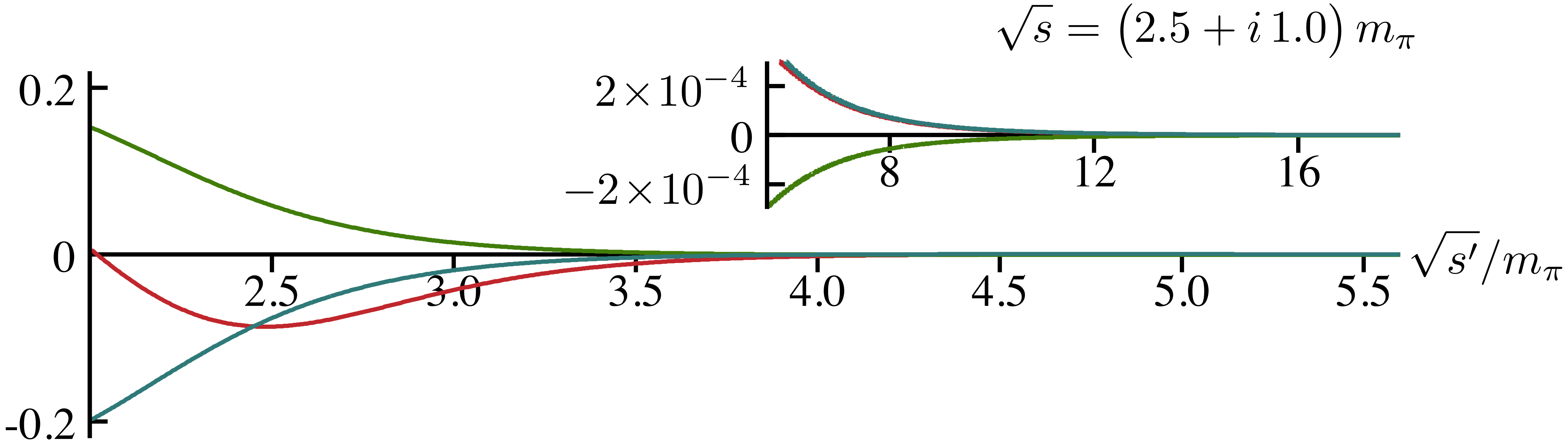
$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s)\, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

Real s



Smeared over a large energy region

Complex s



An ϵ on the real axis → ϵ' in the complex plane

The bad

Not happening

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Partial waves

Extrapolated

$$\int_{4m_\pi^2}^{\infty} = \int_{4m_\pi^2}^{s_{max}} + \int_{s_{max}}^{\infty} = \int_{4m_\pi^2}^{s_{fit}} + \int_{s_{fit}}^{s_{max}} + \int_{s_{max}}^{\infty}$$

Regge

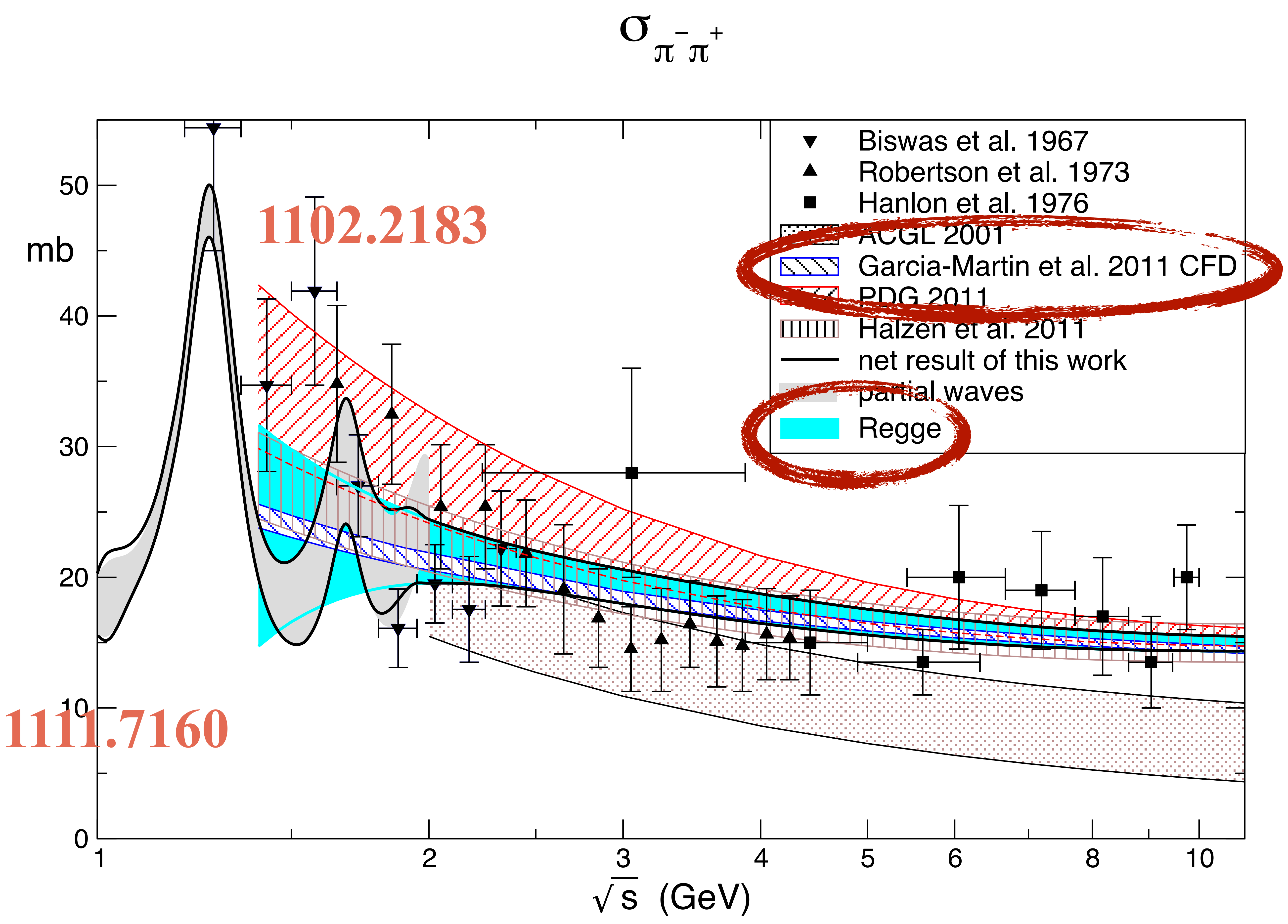
- Regge must be extrapolated from phys. m_π
- Regge is wrong below $a_t m_\pi \sim 0.22 - 0.25$

The Regge

Regge must be extrapolated from phys. m_π

$\mathbb{P} \rightarrow$ gluon exchanges $\rightarrow$ constant over m_q

$\rho, f_2 \rightarrow$ resonances, not constant $\rightarrow \lambda \sim \Gamma/M$

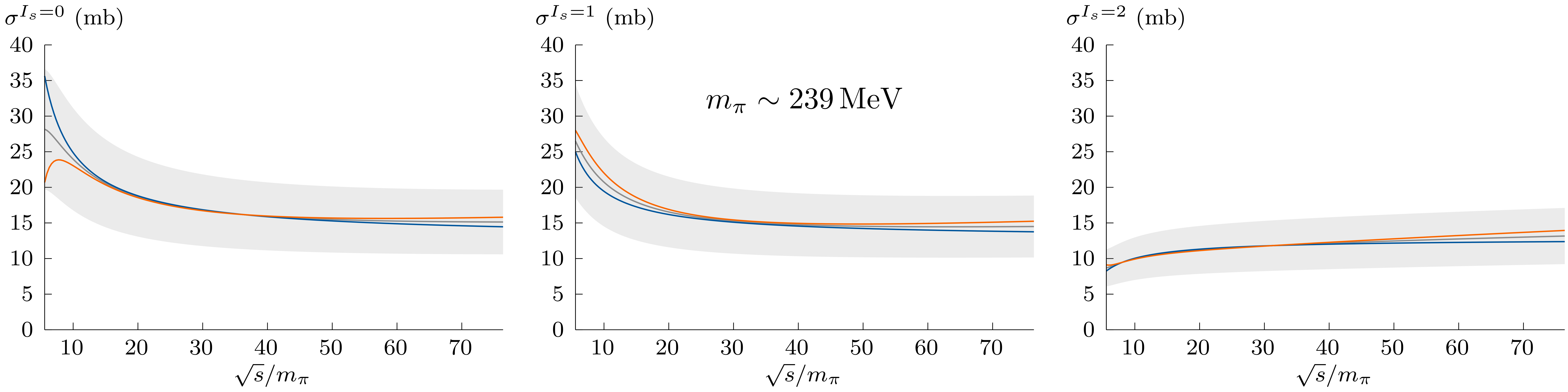


Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

Regge

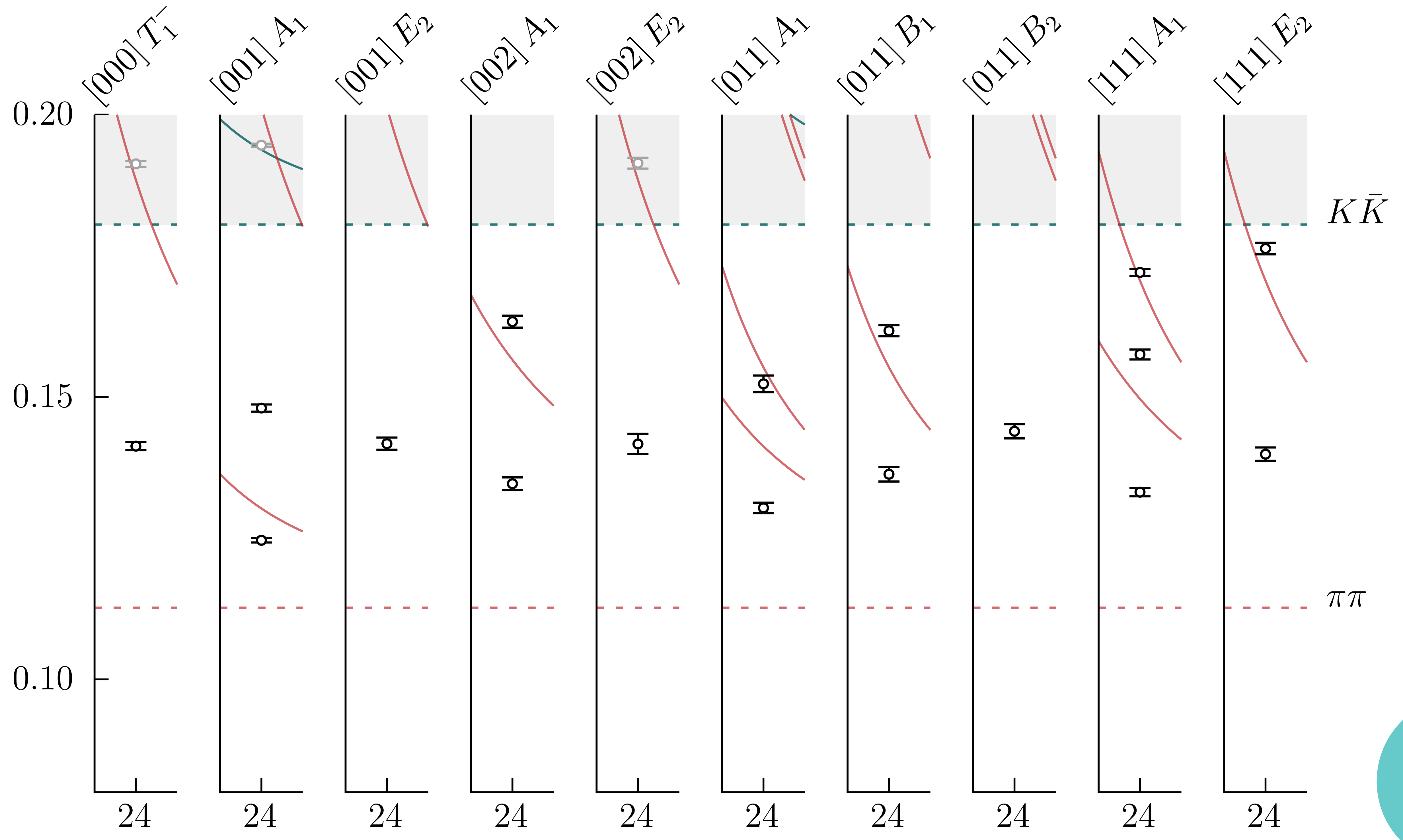
Regge must be extrapolated from phys. m_π



$$\textbf{Our } F_{Regge} = \frac{F_{Regge1} + F_{Regge2}}{2}$$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

$I = 1\ \pi\pi$

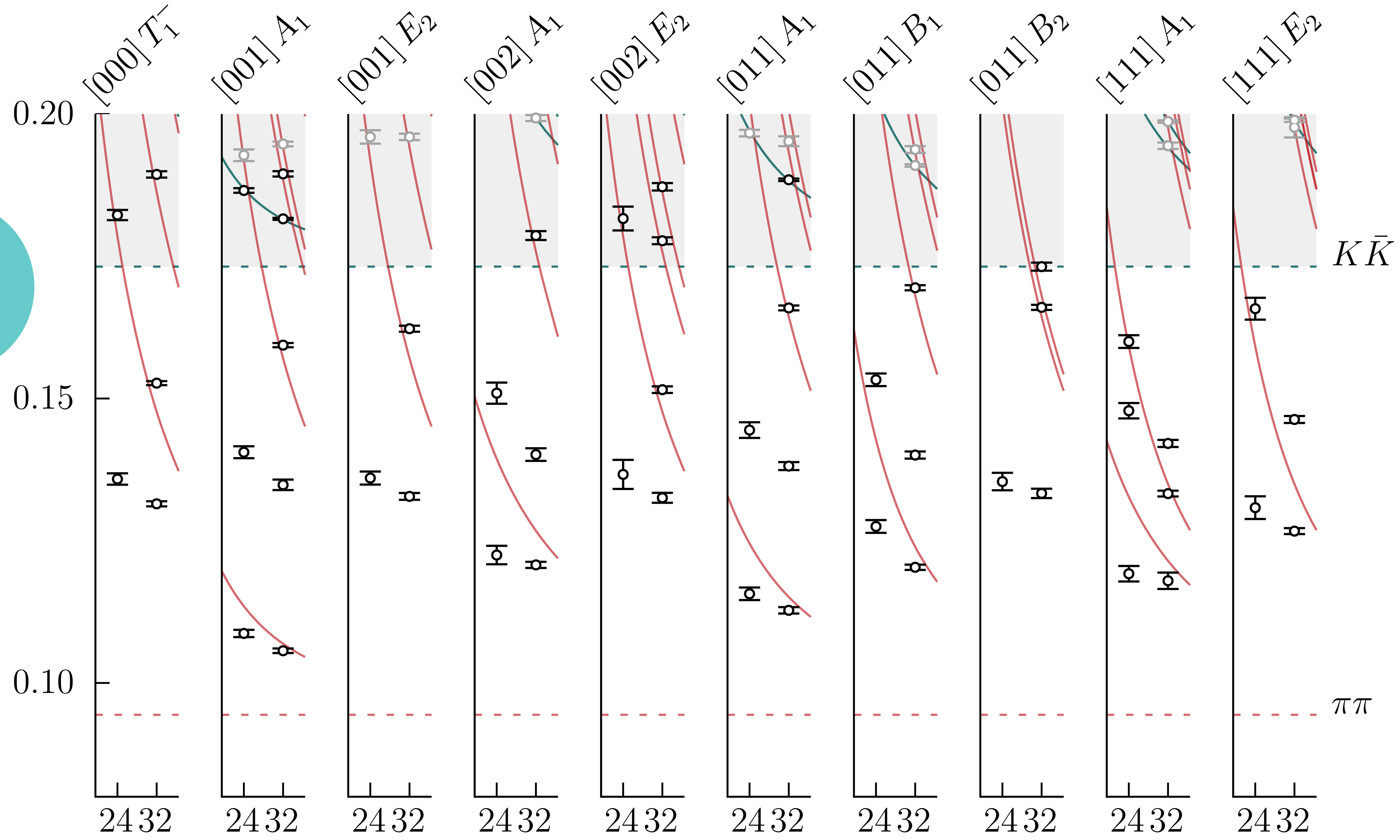


$m_\pi \sim 330\,\text{MeV}$

Similar spectrum to previous masses

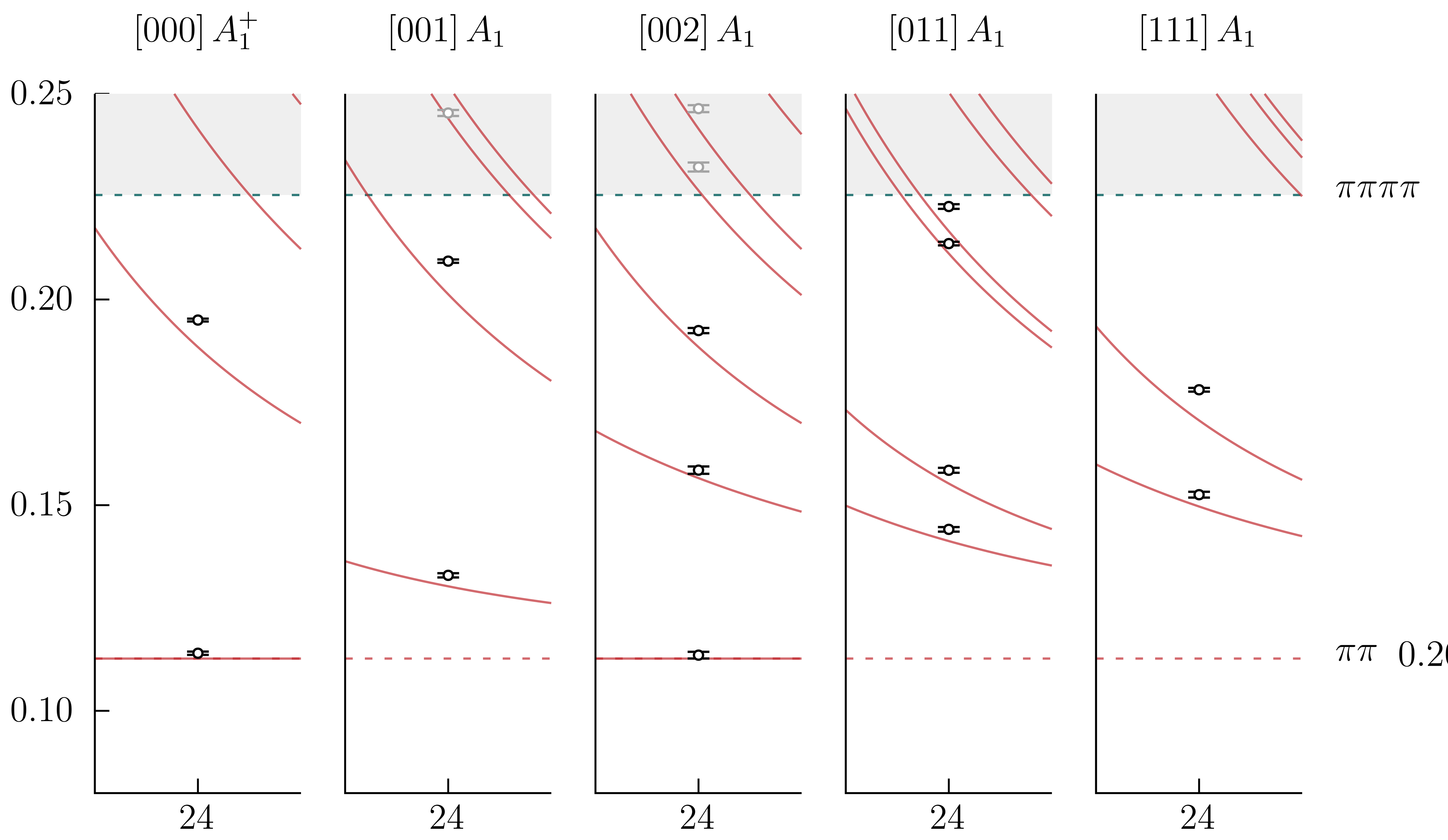
ρ

$m_\pi \sim 283\,\text{MeV}$



Allows us to study the ρ resonance m_q dependence

$I = 2\ \pi\pi$



$m_\pi \sim 330\,\text{MeV}$

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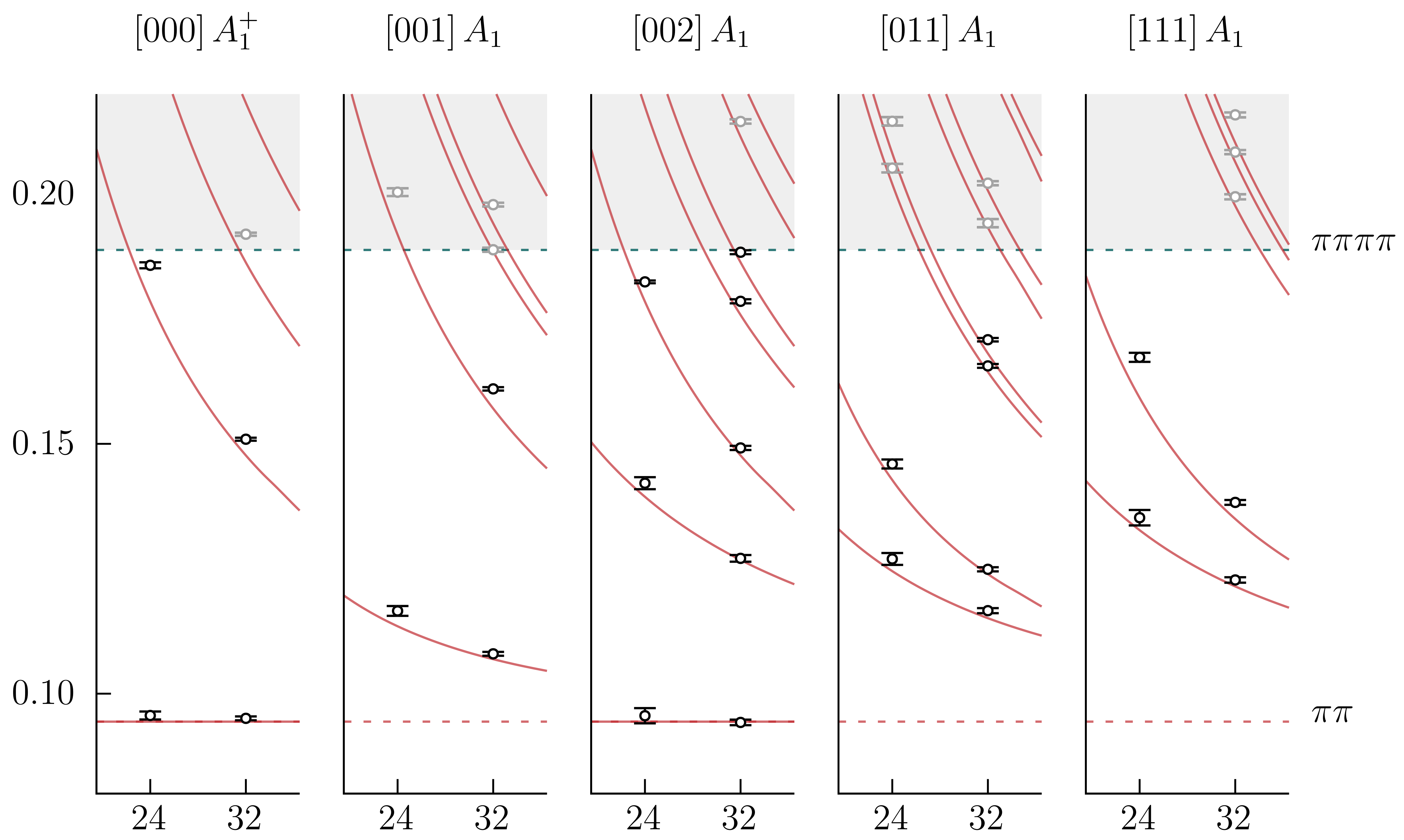
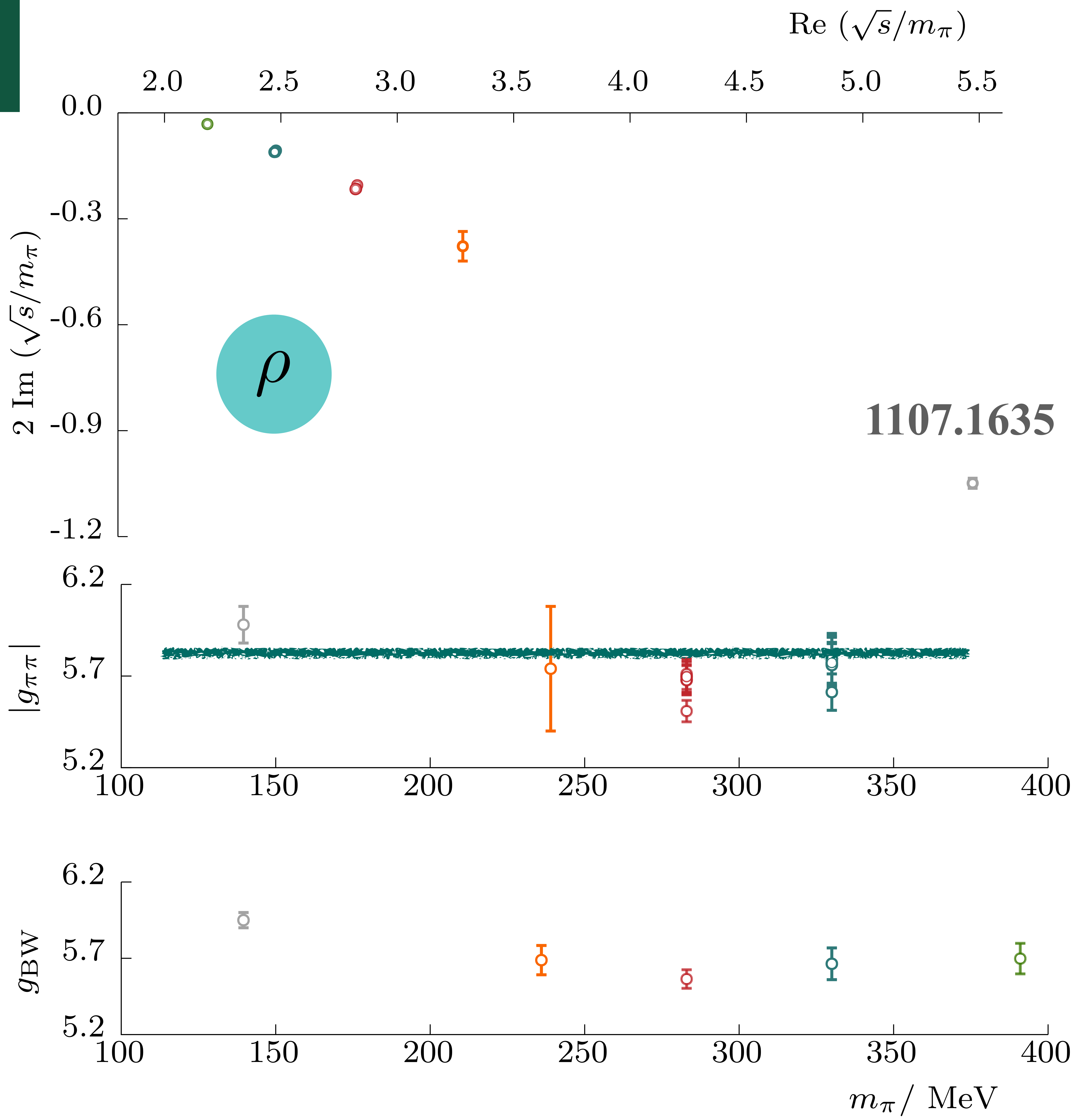


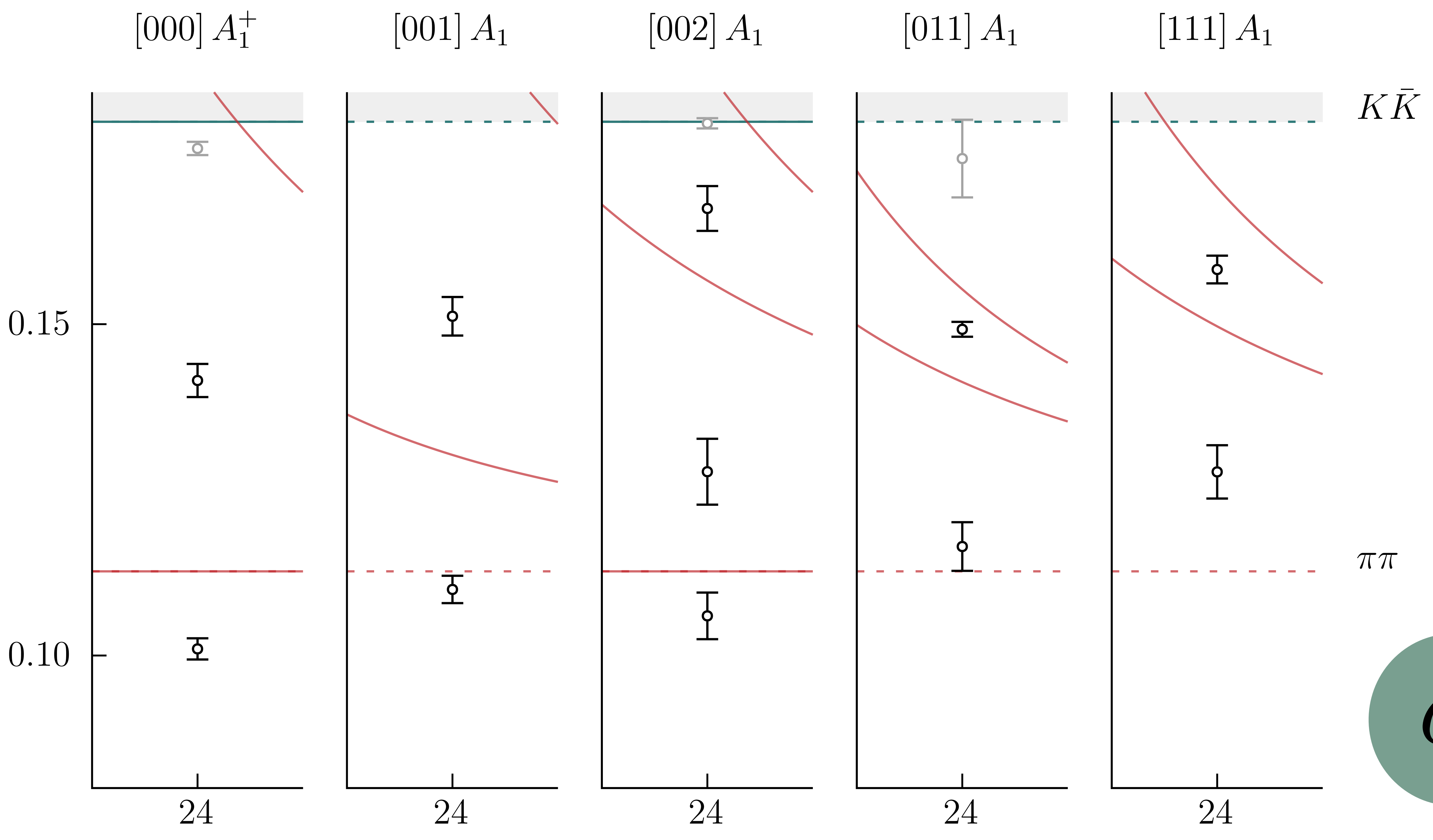
$I = 1 \ \pi\pi$

Ordinary m_q dependence



g **constant**

$I = 0 \ \pi\pi$



$m_\pi \sim 330\,\text{MeV}$

Similar spectrum to previous masses

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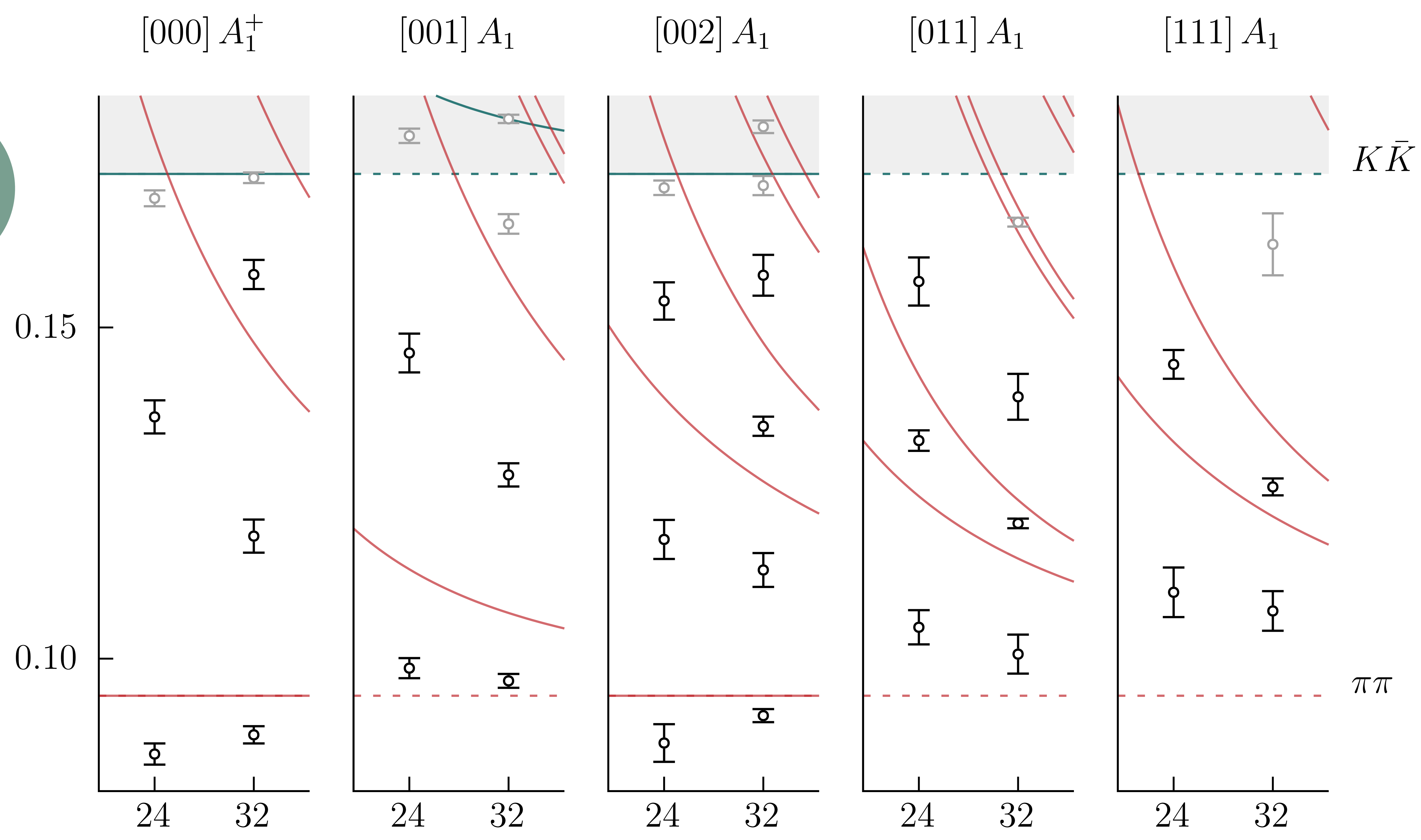


σ

Over 60 "elastic" levels for I=0

$I = 0\ \pi\pi$

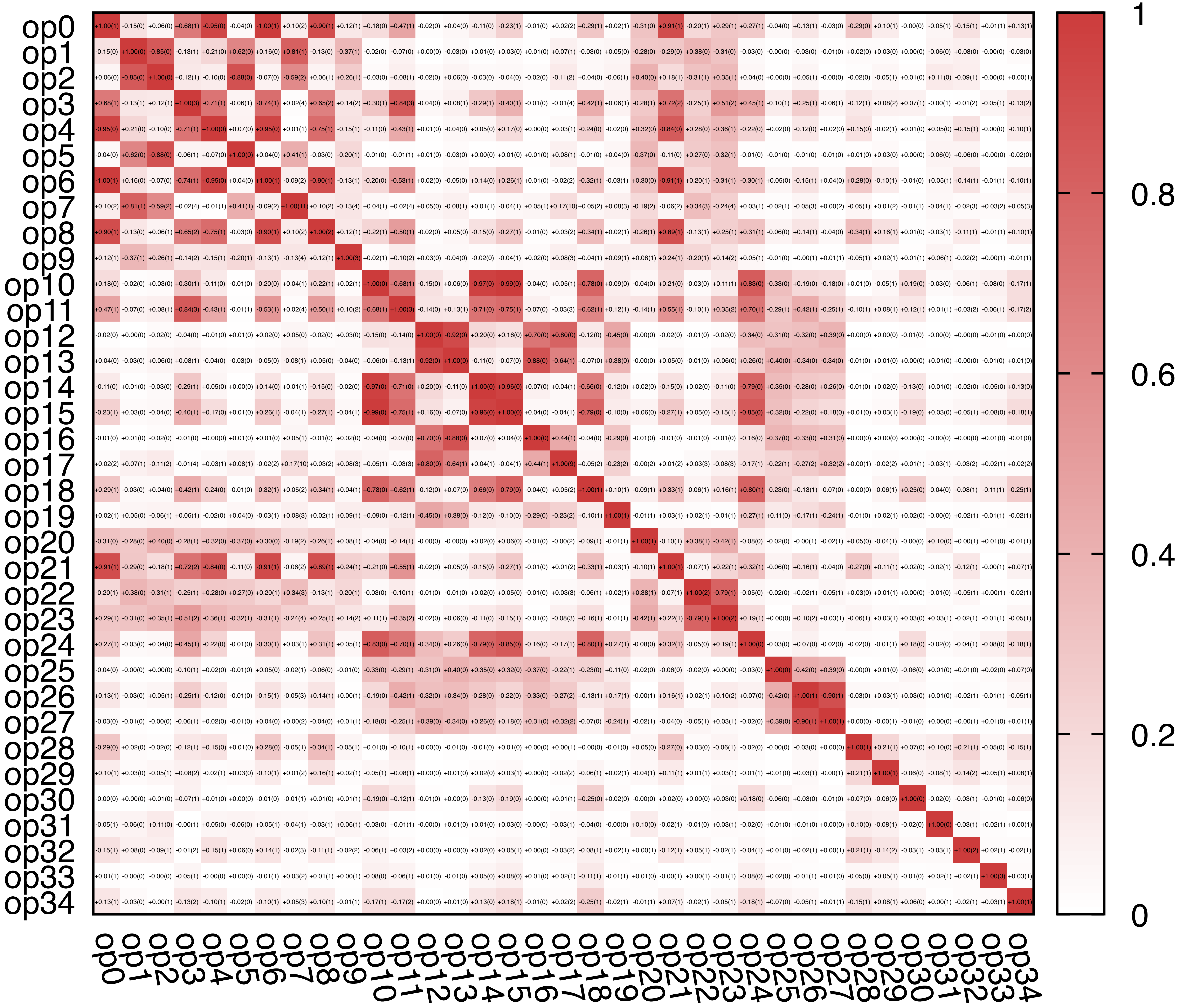
Many ops. for a good GEVP

Distillation → 0905.2160

Time src avg. correlations

Some highly correlated

More than a few relevant ops.



$I = 0\ \pi\pi$

Many fits for a good E_n

Many fits for diff

t_0

t_{min}, t_{max}

N_{exp}**(1-2)**

Model averaging technique

2008.01069

2208.13755