

Setting the scale of HISQ ensembles



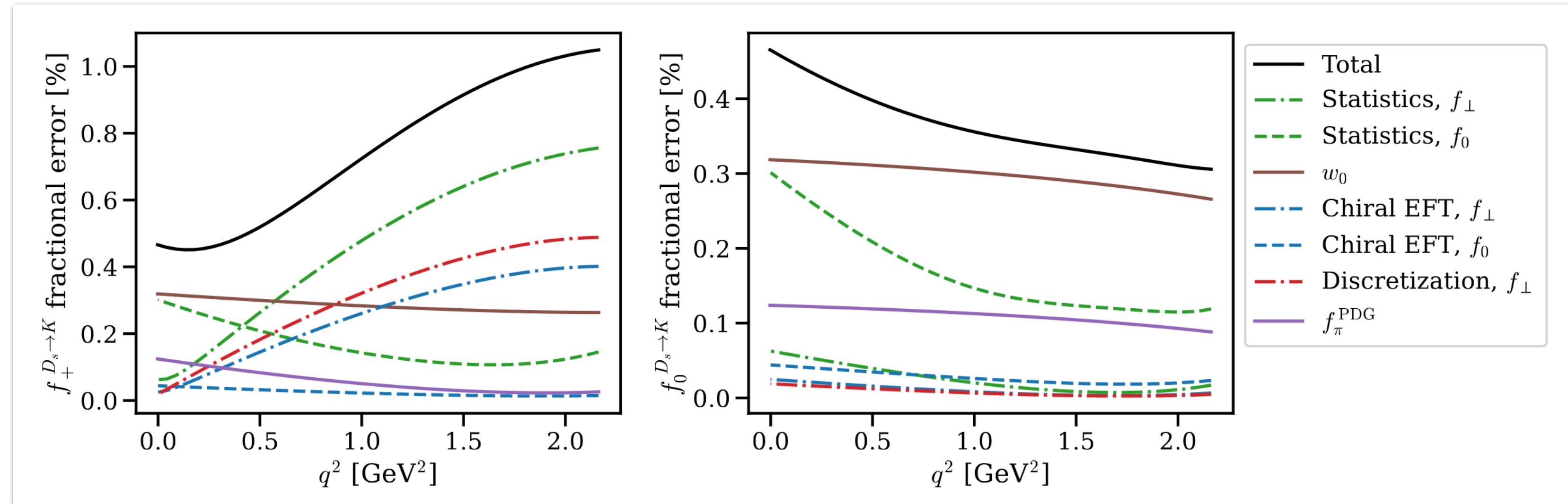
Yin Lin 林胤
yin01@mit.edu

On behalf of
the Fermilab lattice and MILC
working group



Scale setting for precision physics

For semileptonic D decay processes, the scale setting error is one of the dominant sources of uncertainties.



Error (%)	m_b/m_c	m_c/m_s	m_b/m_s	$m_{s,\overline{MS}}(2 \text{ GeV})$	\overline{m}_c	\overline{m}_b	$\overline{\Lambda}_{\text{MRS}}$
Statistics and EFT fit	0.10	0.09	0.11	0.43	0.31	0.29	4.6
Two-point correlator fits	0.07	0.01	0.08	0.07	0.05	0.00	1.3
Scale setting and tuning	0.02	0.14	0.16	0.18	0.03	0.02	0.2
Finite-volume corrections	0.00	0.02	0.01	0.01	0.01	0.00	0.0
Topological charge distribution	0.01	0.00	0.00	0.01	0.01	0.01	0.2
Electromagnetic corrections	0.12	0.11	0.01	0.01	0.08	0.00	0.0
α_s	0.01	0.00	0.01	0.56	0.75	0.18	2.9
$f_{\pi,\text{PDG}}$	0.03	0.07	0.10	0.12	0.04	0.02	0.2

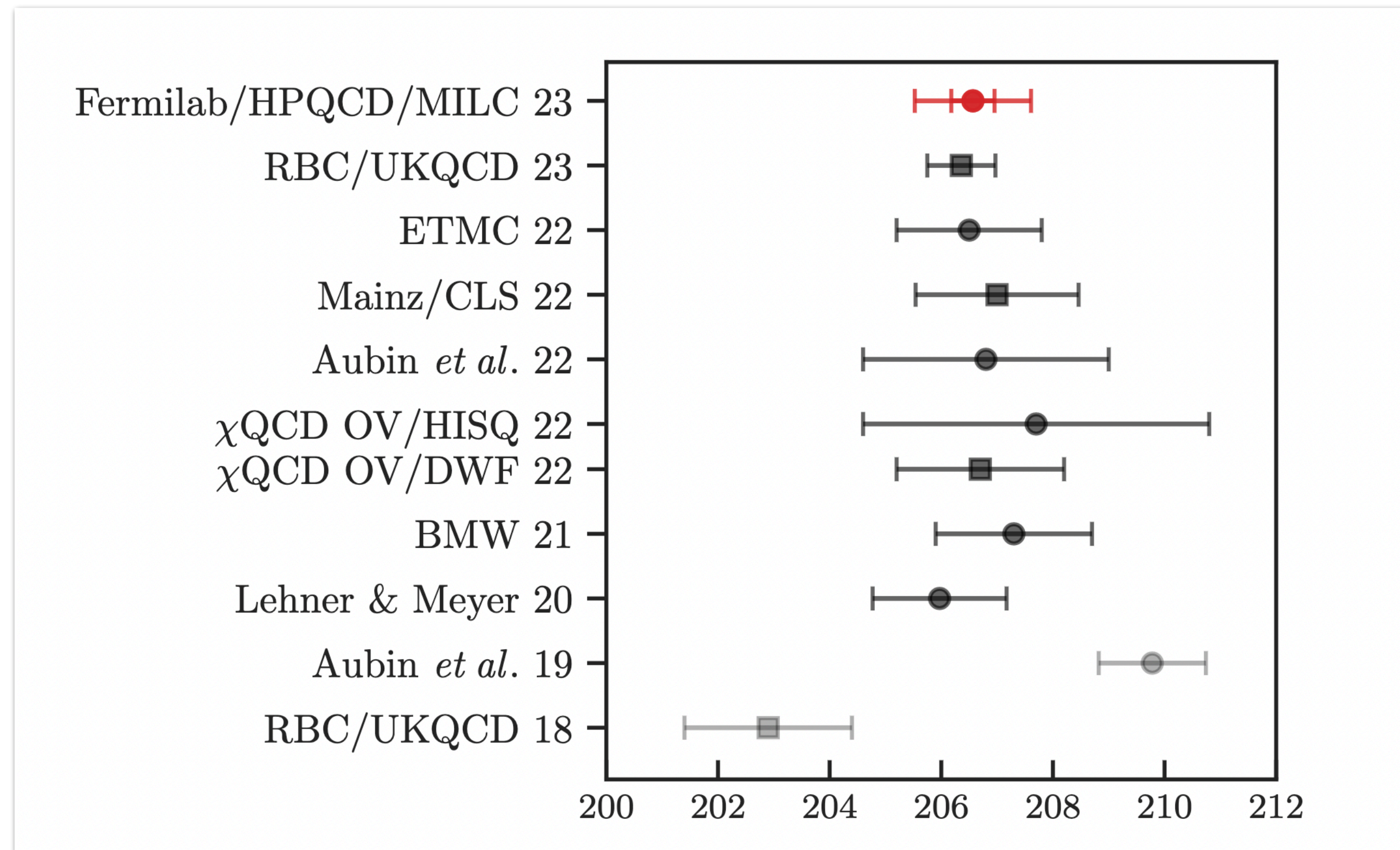
Fermilab Lattice and MILC. [arXiv:2212.12648](https://arxiv.org/abs/2212.12648)

For quark masses, the scale setting error is also a dominant source of error.

Fermilab Lattice, MILC, TUMQCD. [arXiv:1802.04248](https://arxiv.org/abs/1802.04248)

Scale setting for muon g-2

Precision scale setting on HISQ to constrain the leading order HVP to muon g-2



$$\frac{\Delta a_{hvp}}{a_{hvp}} \approx 2 \left(\frac{\Delta \Lambda}{\Lambda} \right)$$

M. Della Morte et al. [arXiv:1705.01775](https://arxiv.org/abs/1705.01775)

A sub-percent scale determination, accounting for all systematics, is needed.

Connected light quark contribution to the intermediate window observable

Fermilab Lattice, HPQCD, MILC. [arXiv: 2301.08274](https://arxiv.org/abs/2301.08274)

Gradient flow scales on HISQ



Gradient flow scales

- ▶ Smoothing procedure, Lüscher, [1006.4518](#):

$$\frac{dV_{x,\mu}}{dt} = - \left\{ \partial_{x,\mu} S^f(t) \right\} V_{x,\mu}, \quad V_{x,\mu}(t=0) = U_{x,\mu},$$

where the flow action $S^f = S_{Wilson}$ or $S^f = S_{Symanzik}$

- ▶ Scale setting:

$$t^2 \langle S^o(t) \rangle \Big|_{t=t_0} = Const \quad \text{or} \quad \left[t \frac{d}{dt} t^2 \langle S^o(t) \rangle \right]_{t=w_0^2} = Const,$$

where the observable $S^o = S_{clover}$ or $S^o = S_{Wilson}$ or $S^o = S_{Symanzik}$

Improving the lattice gradient flow

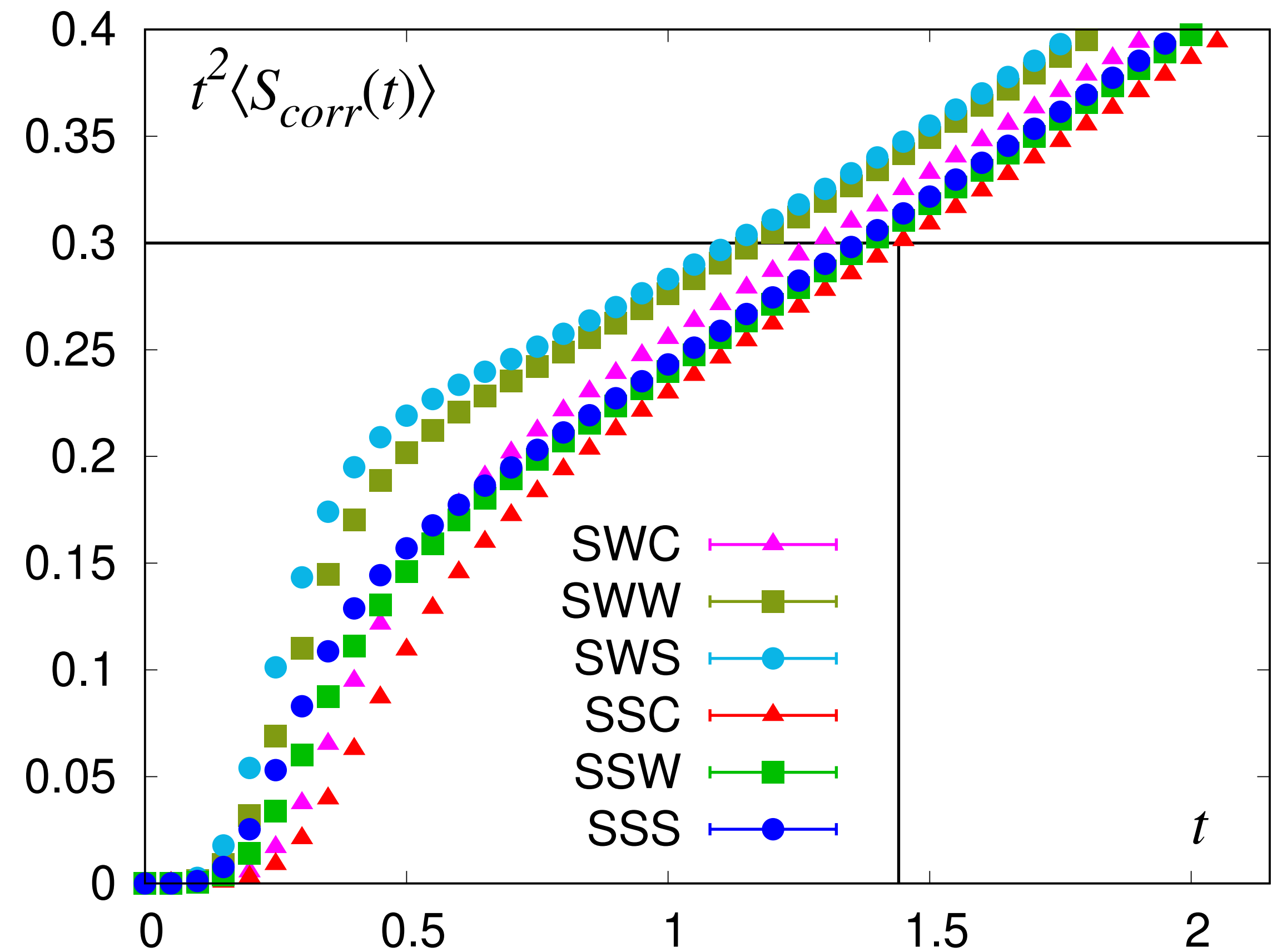
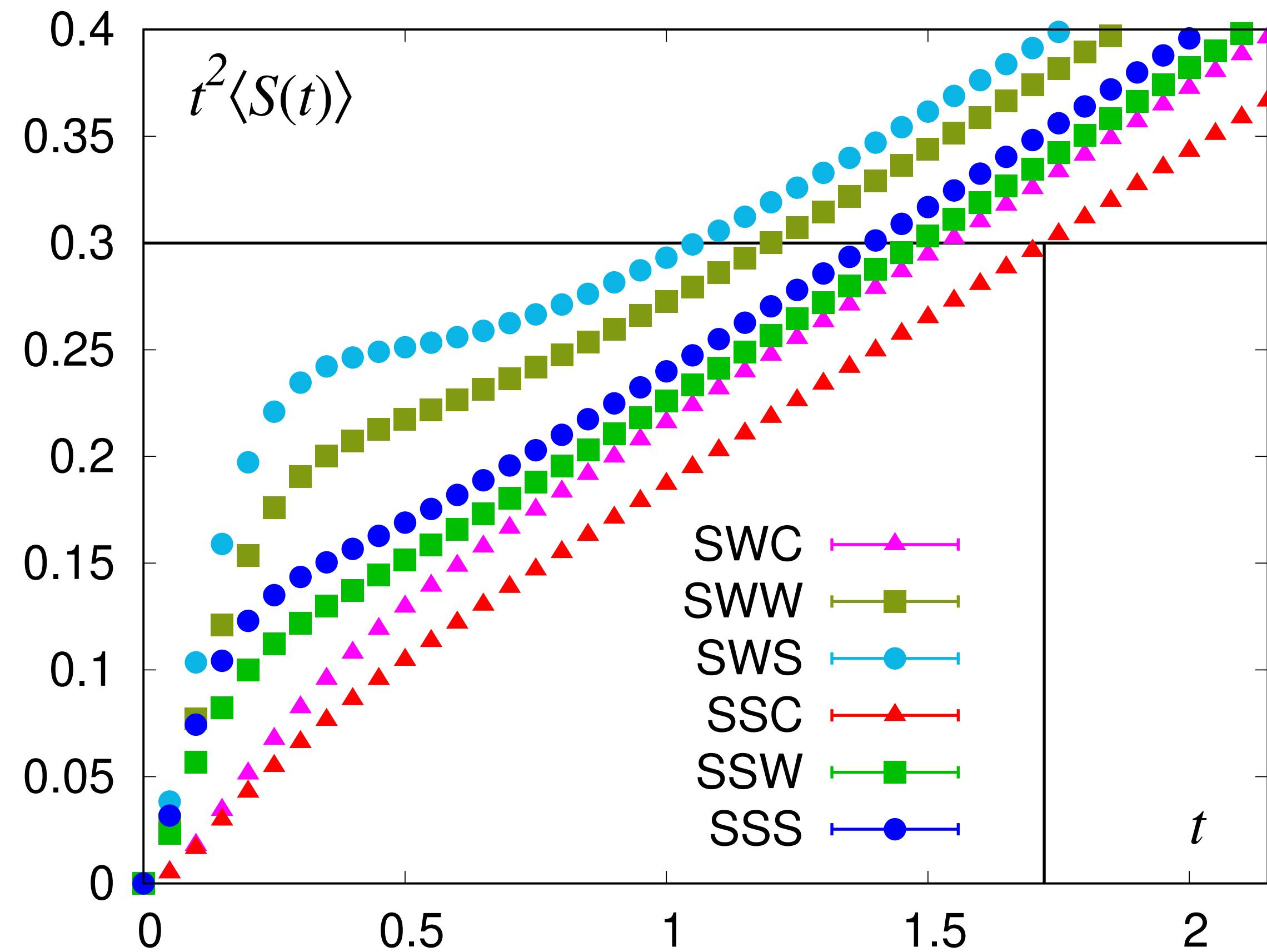
For a given combination of the dynamical action, flow action and the observable the leading discretization effects can be canceled at tree level, [Fodor et al, 1406.0827](#):

$$t^2 S(t) \rightarrow t^2 S_{\text{corr}}(t) = \frac{t^2 S(t)}{1 + \sum_{m=1}^4 C_m (a^{2m} / t^m)}$$

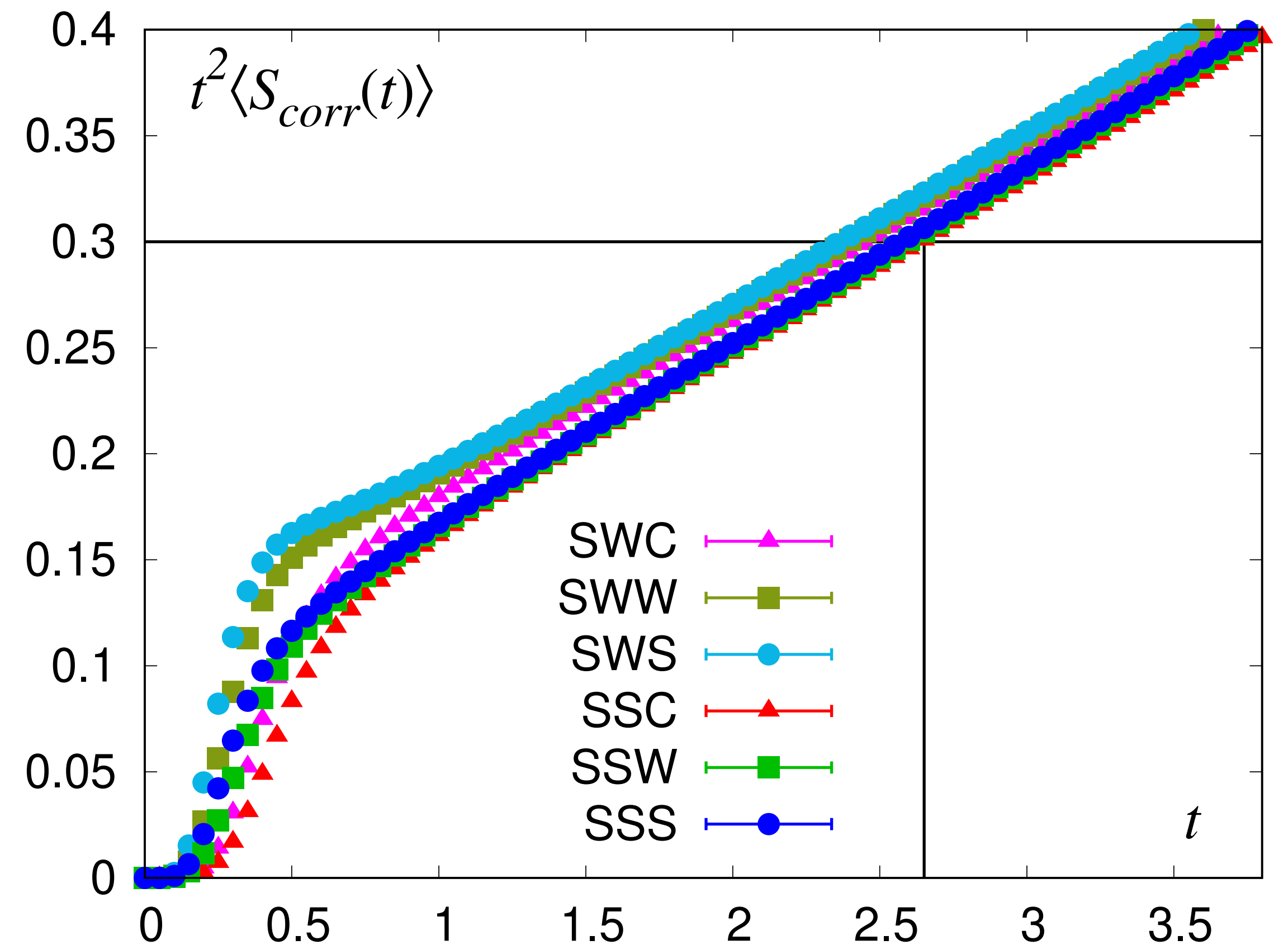
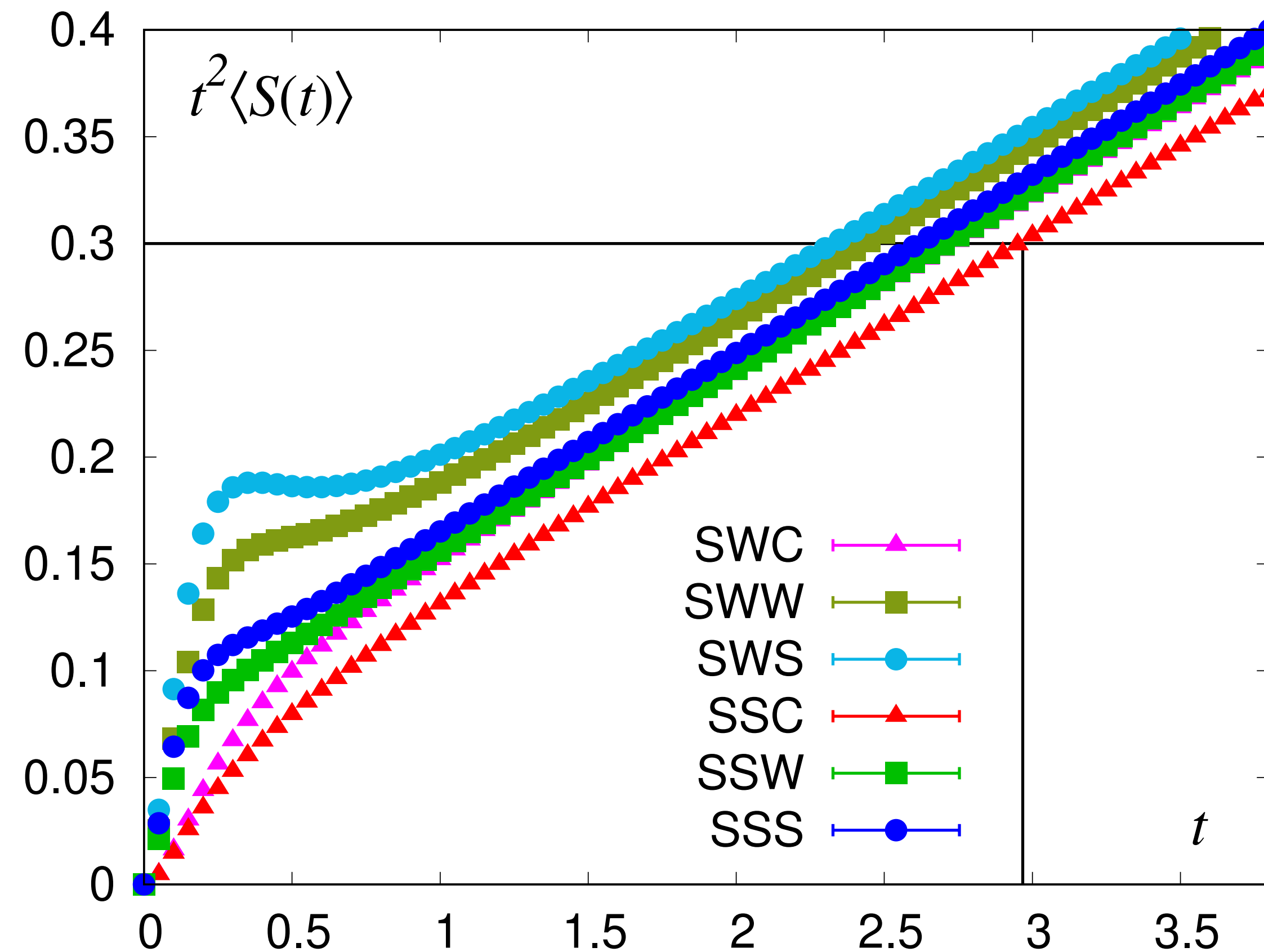
Parameters of the calculation

- ▶ Two flows: Wilson, Symanzik
- ▶ Observable: Clover, Wilson, Symanzik
- ▶ Tree-level corrections
- ▶ Fourth-order commutator-free Lie group integrator, [2007.04225](#), [2101.05320](#)
- ▶ Integrate the flow at two step sizes 0.05, 0.025
- ▶ The flow on all MILC HISQ ensembles at $a \approx 0.09$ fm, 0.12 fm and 0.15 fm is completed, $a \leq 0.06$ fm is ongoing on USQCD and non-USQCD resources
- ▶ Computing time in 2023-2024 is requested for the retuned CalLat physical pion mass $a \approx 0.09$ fm ensemble the incorporated in several MILC projects

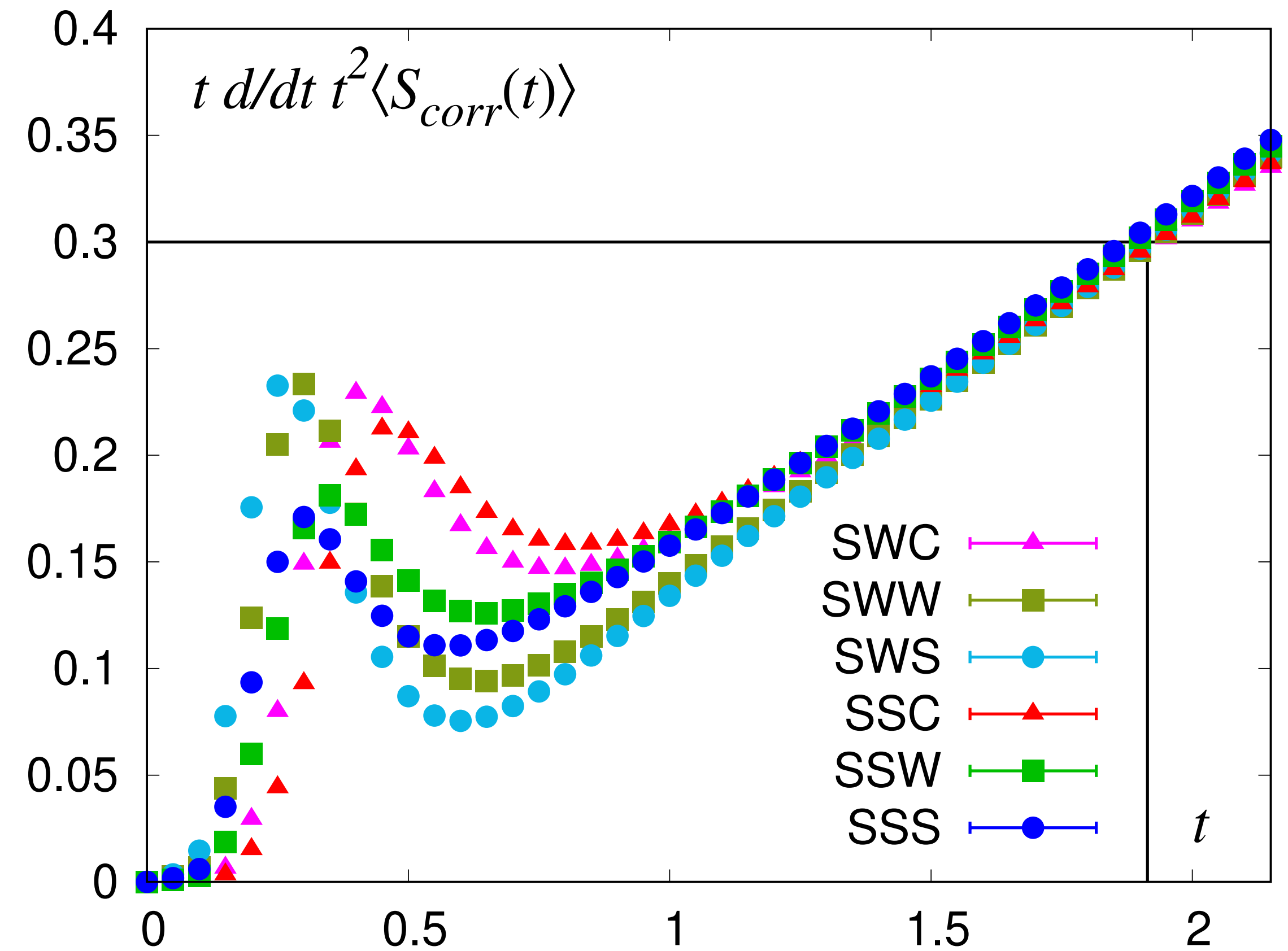
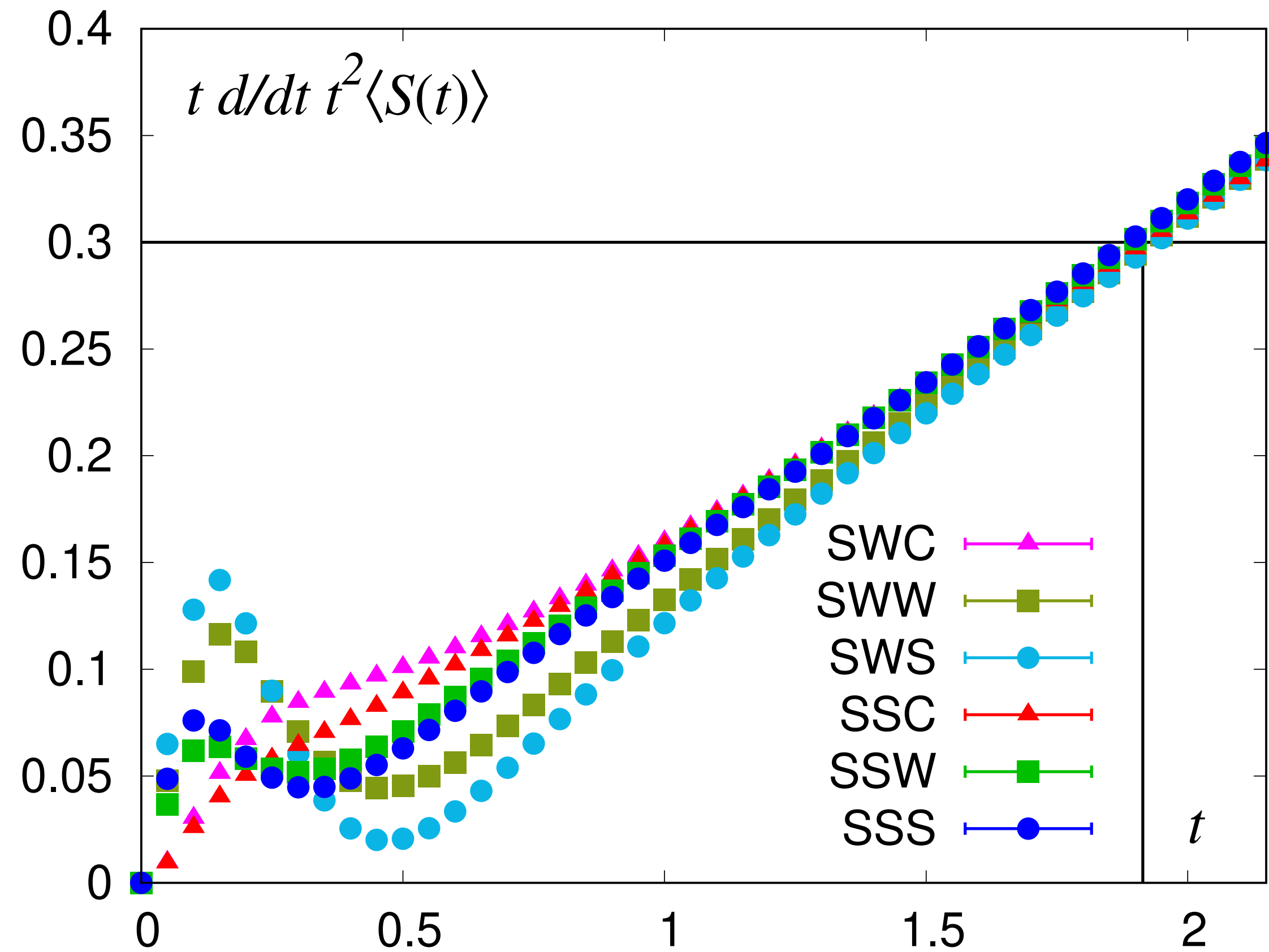
Action density vs flow time, $a \approx 0.12$ fm



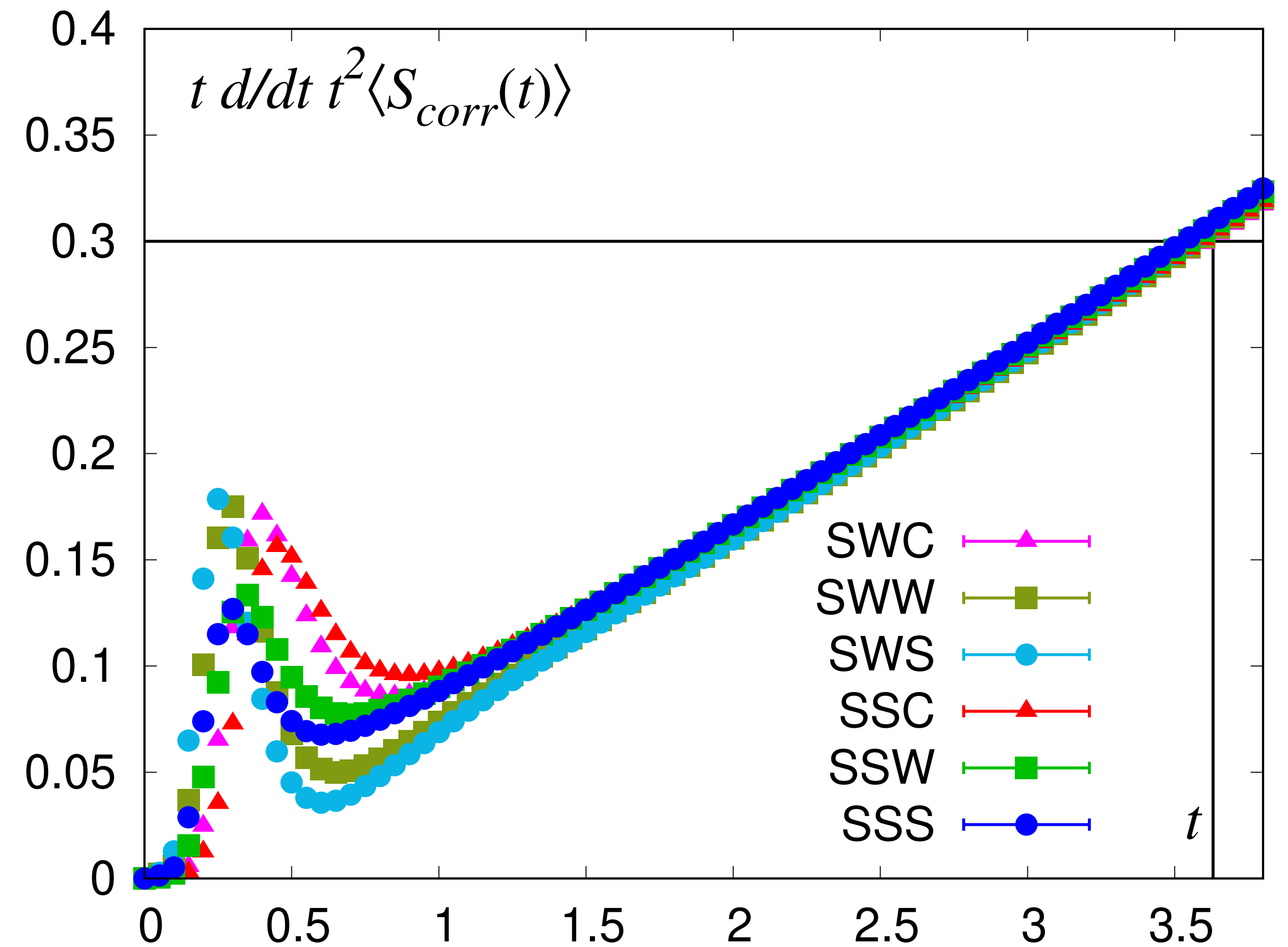
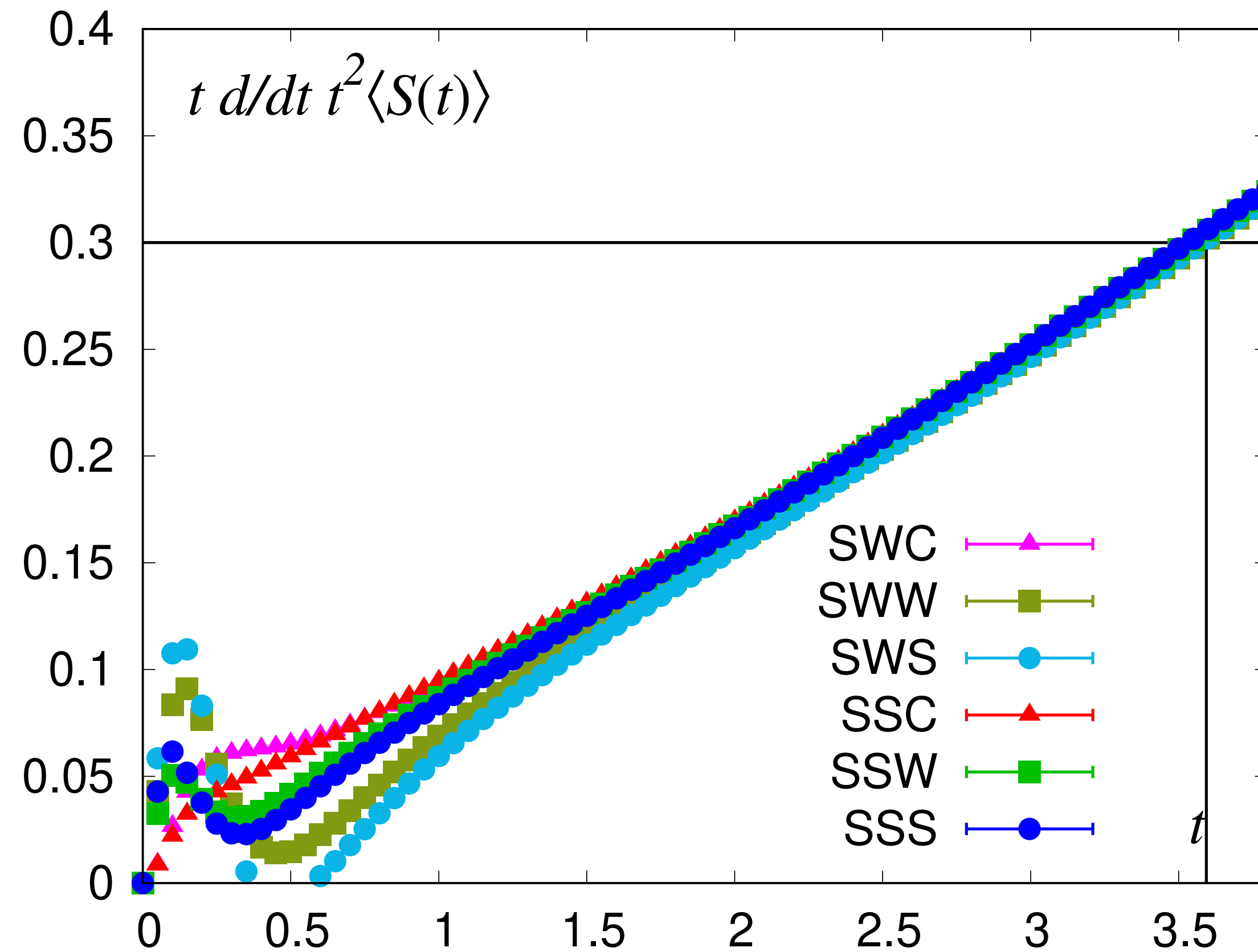
Action density vs flow time, $a \approx 0.09$ fm



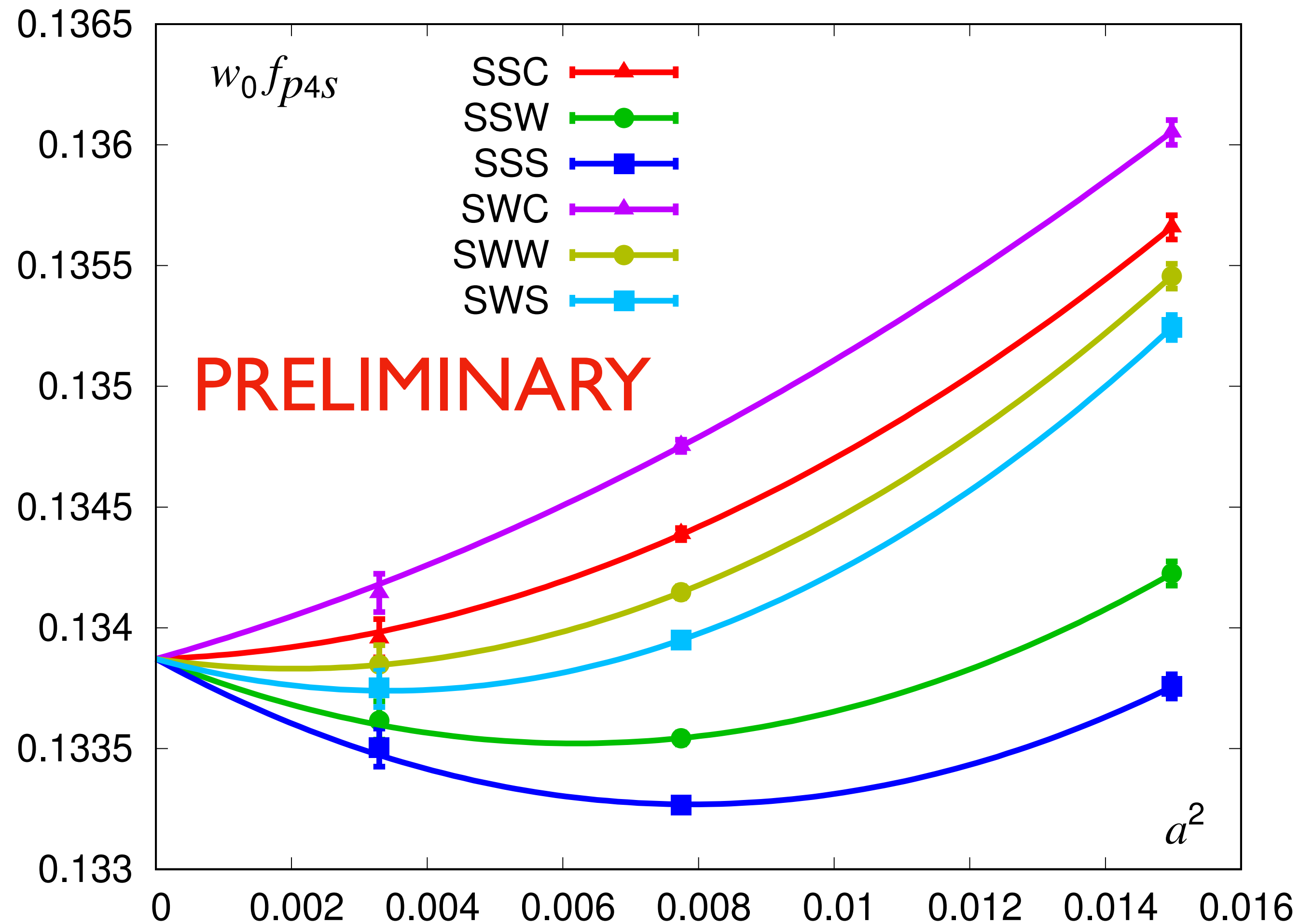
Action density vs flow time, $a \approx 0.12$ fm



Action density vs flow time, $a \approx 0.09$ fm



Simple continuum extrapolation



- ▶ Simultaneous fit for all six available flow/observable combinations on the physical pion $a \approx 0.06, 0.09$ and 0.12 fm ensembles
- ▶ Quadratic in a^2 , 18 data points, 13 parameters with common intercept

Ω Baryons on HISQ

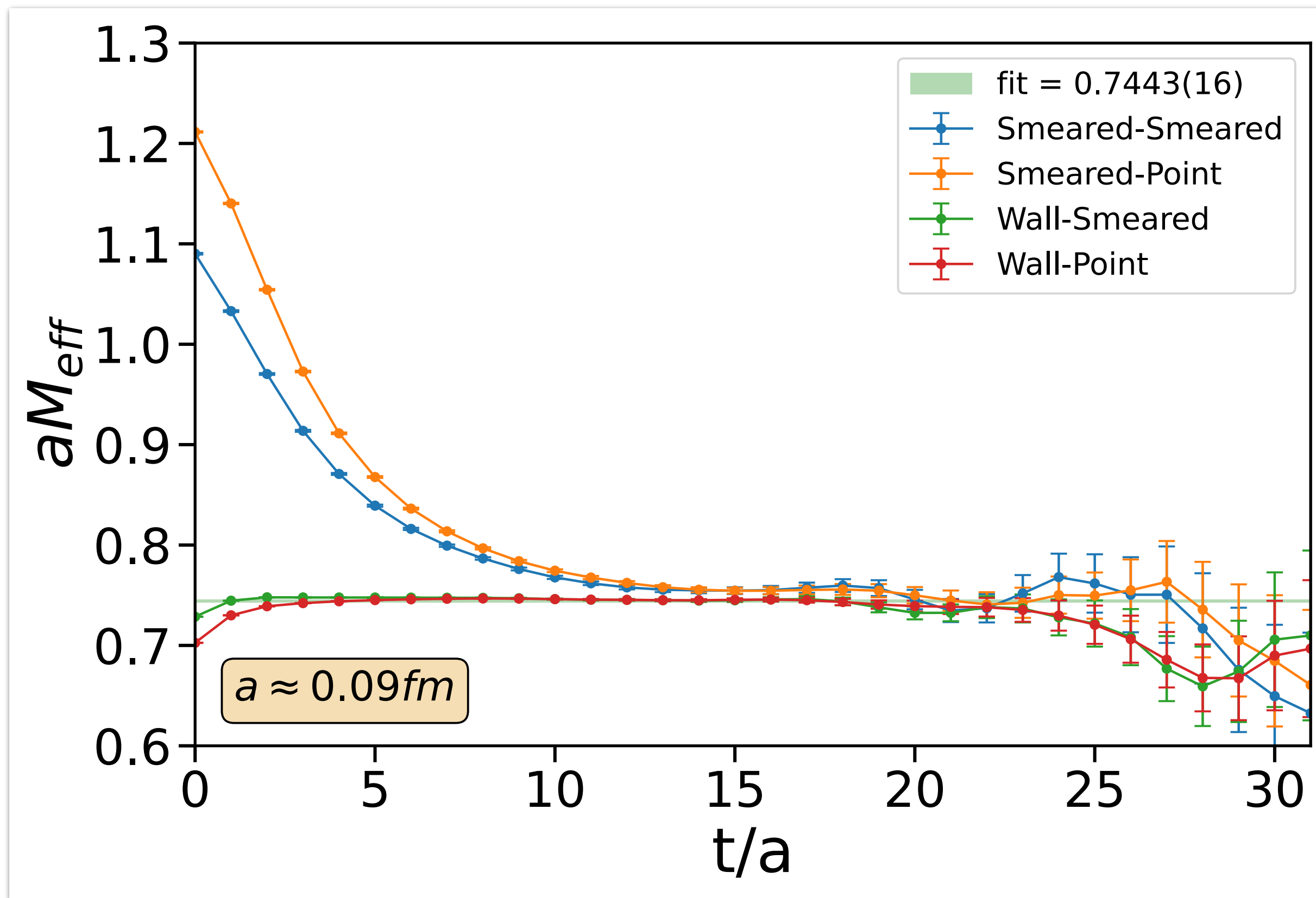
Details of aM_Ω on HISQ

- ▶ So far only computed on **physical-mass ensembles** $a \approx 0.15, 0.12, 0.09,$ **and 0.06 fm**. Cheap to compute on most ensembles
- ▶ Three Ω baryon tastes, three interpolators. [J. Bailey. arXiv:hep-lat/0611023](#), [C. Hughes, Y. Lin, A. Meyer. arXiv:1912.00028](#)
- ▶ Two types of sources, **wall and Gaussian smeared sources**, to better constrain the excited-state contamination

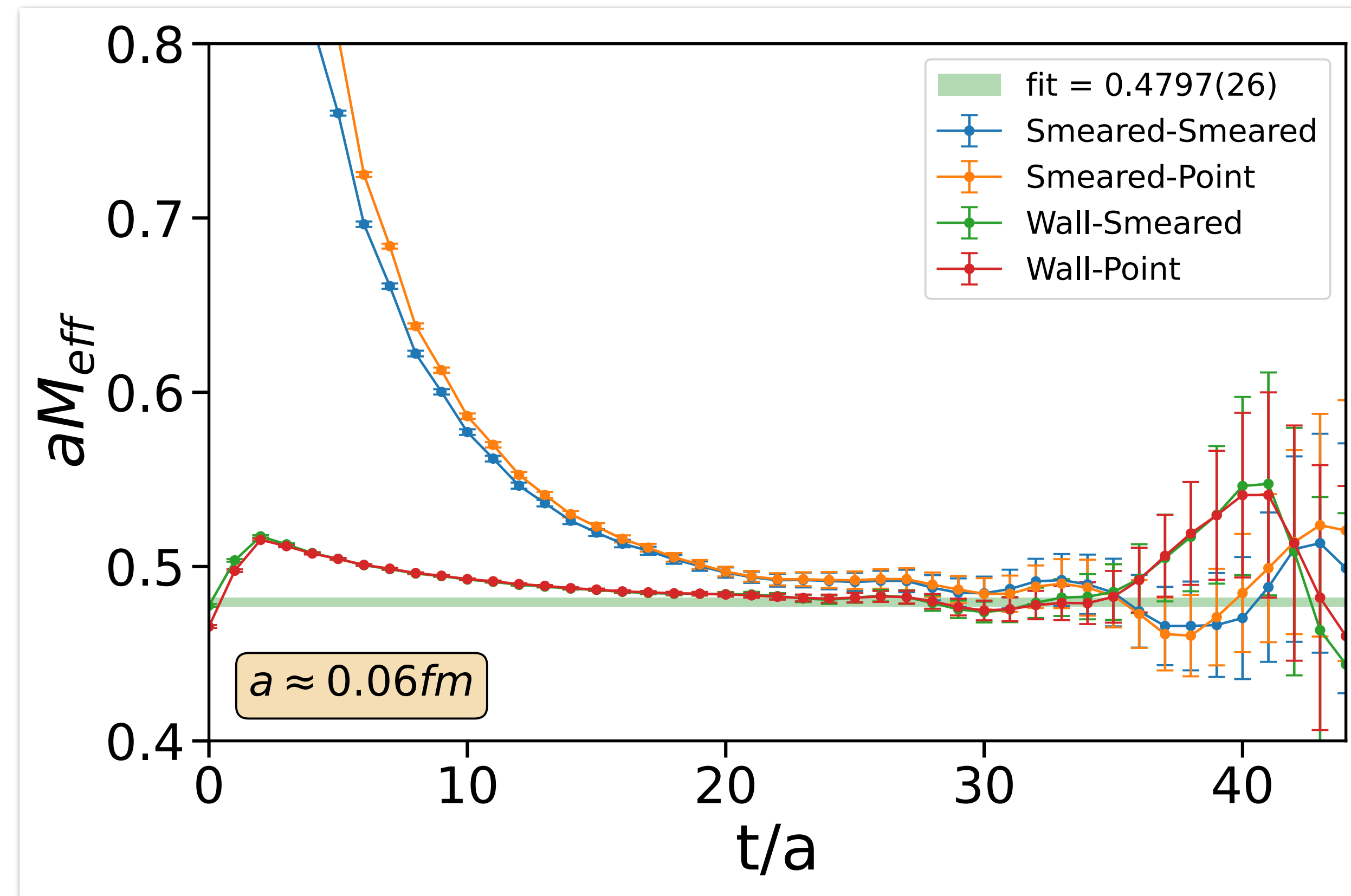
$$O_{\Omega,1} = \text{[Diagram of a cube with three quarks (red, blue, green) at the bottom-left-front corner]}$$

$$O_{\Omega,2} = \text{[Diagram of a cube with quarks at (red, blue, green) at bottom-left-front]} - \text{[Diagram of a cube with quarks at (red, blue, green) at bottom-left-back]} + \text{[Diagram of a cube with quarks at (red, blue, green) at top-left-back]}$$

aM_{Ω} , $a \approx 0.09, 0.06$ fm physical masses

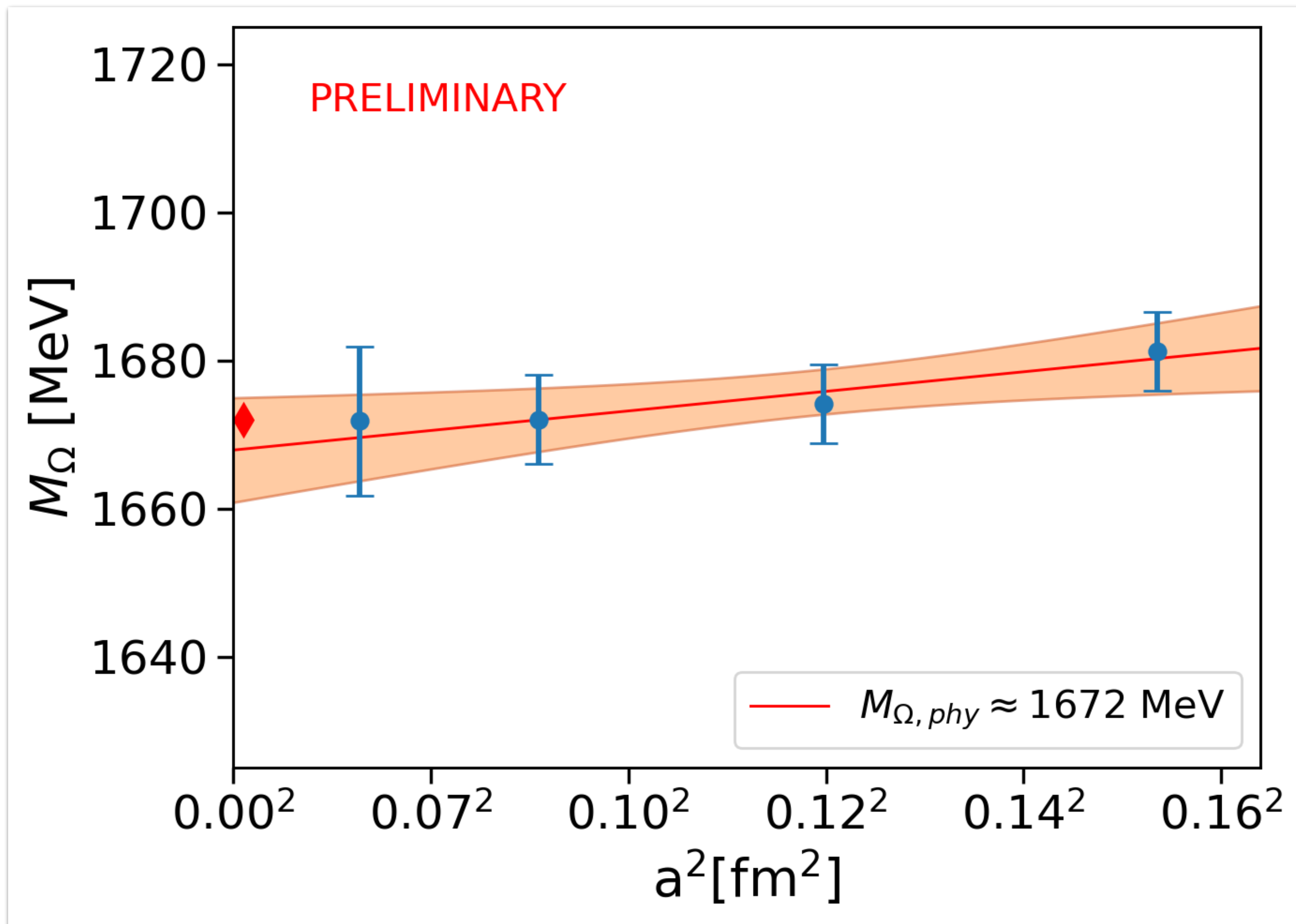


error $\approx 0.2\%$



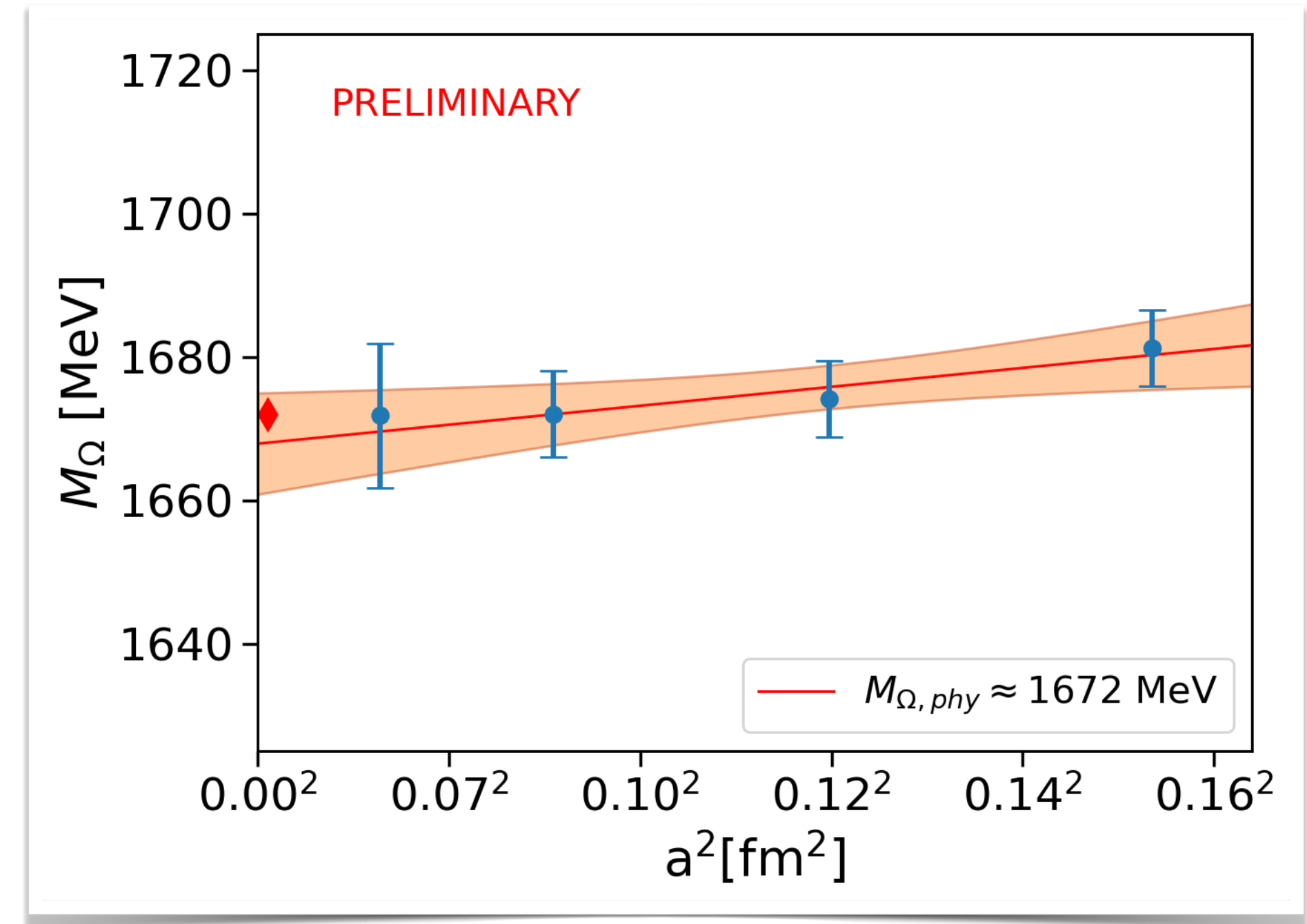
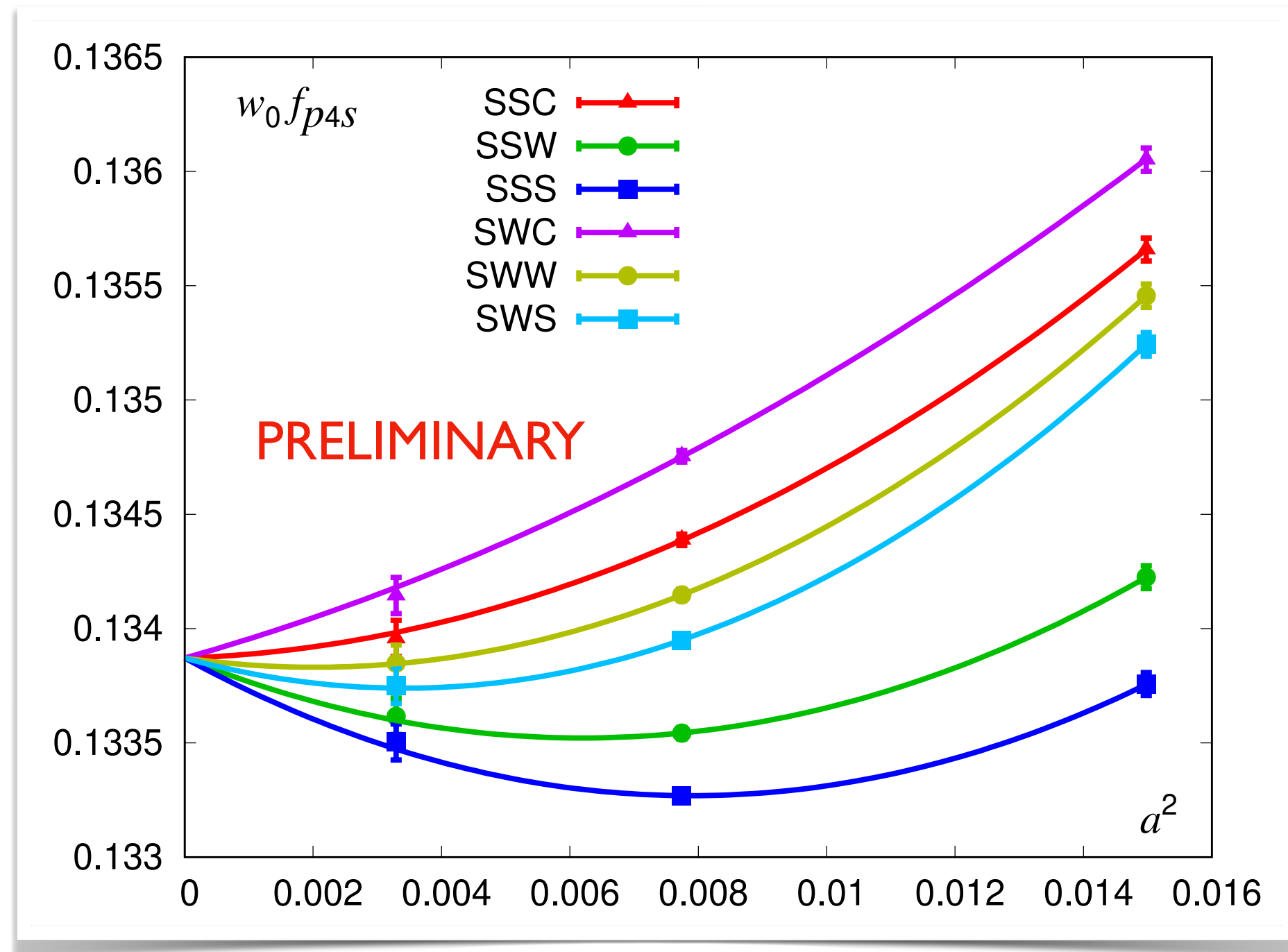
error $\approx 0.5\%$

Simple continuum extrapolation of M_Ω



- ▶ Preliminary fit result:
 $M_\Omega = 1668(7)$ MeV
- ▶ **error $\approx 0.4\%$** — on par/
better with the current best
lattice determinations
- ▶ More statistics are being
added
- ▶ **Systematic errors** are
important

Summary and outlook



- ▶ Perform measurements of the retuned $a \approx 0.09$ fm from CalLat
- ▶ Continue accumulating statistics on $a \leq 0.06$ fm ensembles with external resources

- ▶ Constraining the isospin breaking effects on $a \approx 0.12, 0.09$ fm ensembles
- ▶ Continuing accumulating statistics