Nucleon Matrix Elements using Clover-on-Clover Fermions

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Outline

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  - Physics from **nucleon form factors and charges**
  - Methodology for calculation of **nucleon matrix elements** using **lattice QCD**

- Excited-state effect on nucleon matrix elements
  - Effect from $N\pi$ / $N\pi\pi$ multihadron excited states

- Results
  - Isovector axial, electric and magnetic form factors
  - Flavor diagonal axial, scalar, and tensor charges

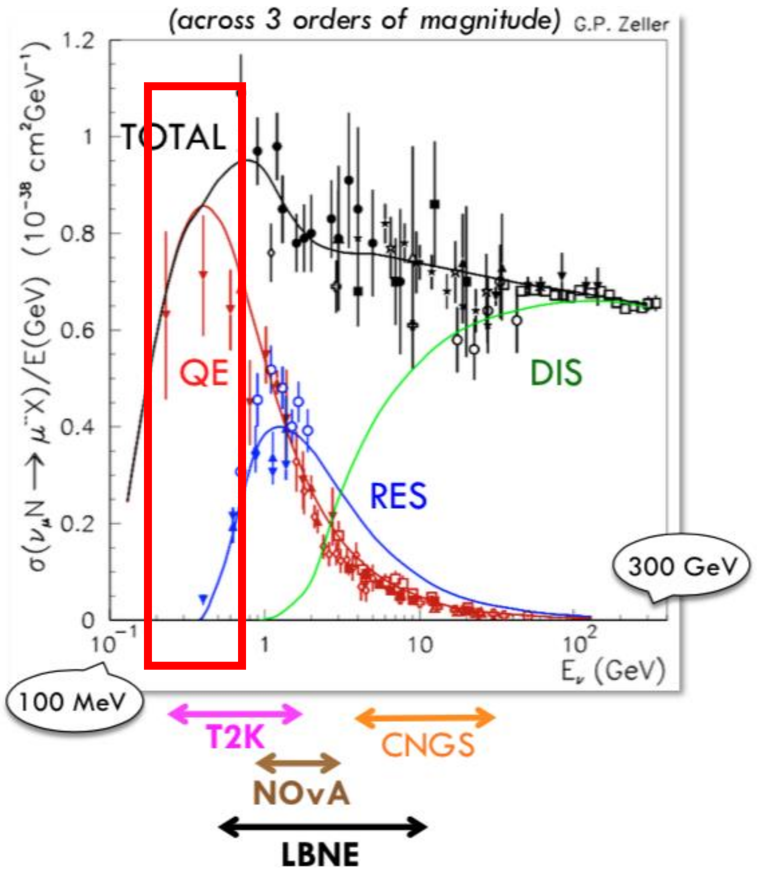
Introduction

Physics from **nucleon form factors and charges**

Methodology for calculation of **nucleon matrix elements** using **lattice QCD**

Lepton-nucleon scattering

- Nucleon charges and form factors give the strength of the interaction of external probes (electrons, neutrinos, $\cdots$) with nucleons and are critical inputs in experimental searches of physics beyond the standard model.
- High precision results for axial, electric and magnetic form factors versus Q^2 needed for determining **(quasi-) elastic cross-section** of (ν, e, μ) scattering off nuclei



$$\frac{d\sigma}{dQ^2}\begin{pmatrix}\nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n\end{pmatrix} = \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

F_A = axial form factor
$\tilde{F}_P$ = induced pseudoscalar
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
$M = M_n = M_p \approx 939$ MeV
$m = M_\pi$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\ &\quad - \frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A + 2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\Big], \\ B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

Physics from flavor diagonal nucleon charges

- $g_A^q = \Delta q$: Quark contributions to the nucleon spin

$$\frac{1}{2} = \sum_{u,d,s,\cdots}\left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

X. Ji (1997),
L_q: orbital angular momentum of the quark
J_g: total angular momentum of the gluons

- g_T^q : Quark EDM contributions to the neutron EDM d_n

C. Baker et al. (2006)

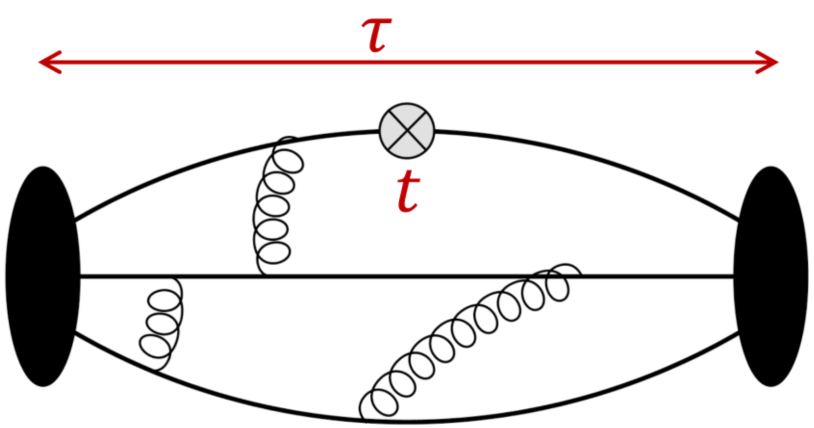
$$|d_n| = |d_u^\gamma g_T^u + d_d^\gamma g_T^d + d_s^\gamma g_T^s + \cdots| \le 2.9 \times 10^{-26}\ e\ \mathrm{cm}$$

- $g_S^q = \frac{\partial M_N}{\partial m_q}$: Slope of the nucleon mass with respect to the quark mass

$\sigma_{\pi N} = m_l g_S^{u+d}$: Quark contributions to the nucleon mass
$\sigma_s = m_s g_S^s$

Connected and disconnected diagrams

- Charges / Form factors are obtained from the nucleon ME $\langle N|\bar{q}\,\Gamma\,q|N\rangle$
- Require high precision measurements of quark bilinear operators within the nucleon state for both “connected” and “disconnected” 3-point correlation functions,



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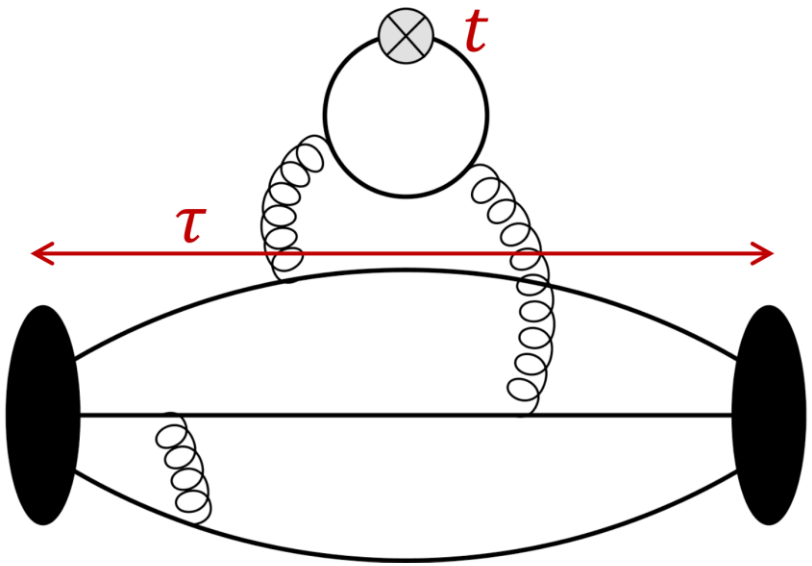
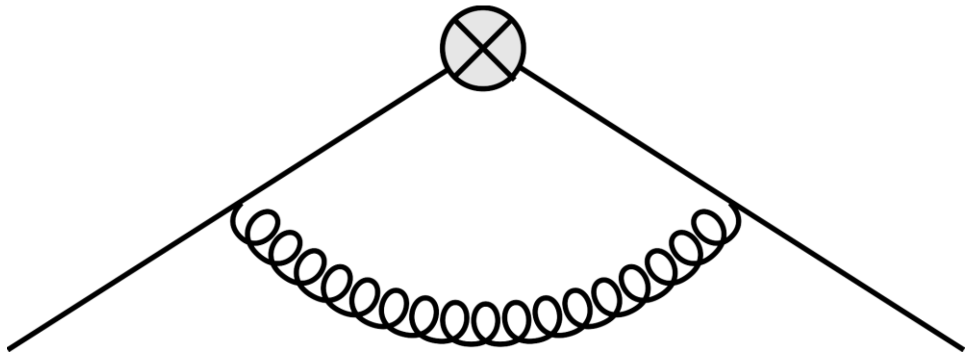


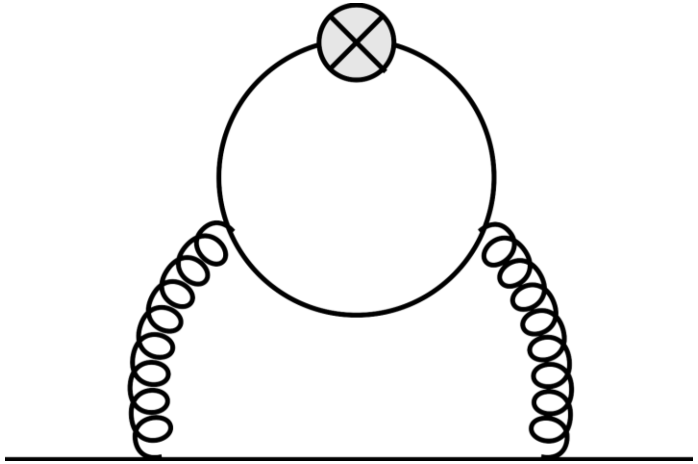
Calculated with covariant Gaussian source smearing, multiple source-sink separation $0.9 \lesssim \tau \lesssim 1.4$, accelerated with coherent sequential inversions and the truncated solver method with bias correction. PNDME (2018)

All-to-all quark propagator estimated by stochastic method using Z_4 random sources, accelerated with the truncated solver method with bias correction and hoping parameter expansion. PNDME (2015)

Nonperturbative renormalization

- We explicitly evaluated the 3×3 flavor mixing matrices in RI-sMOM scheme and convert into $\overline{\mathrm{MS}}$ scheme value 2 GeV.
- Results on the corrections from the flavor mixing
  - Small and negligible for $g_{A,T}^{u,d,s}$ and $g_S^{u,d}$
  - g_S^s gets a correction about ~20% at $a \approx 0.15\mathrm{fm}$, and ~6% at $a \approx 0.06\mathrm{fm}$ from the off-diagonal $Z_S^{s,u+d}$.

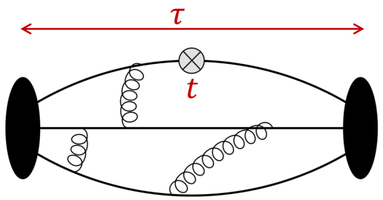




Clover fermions on 2+1-flavor Clover Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	$N_{\rm conf}$	N_{HP}	N_{LP}
a127m285	0.127	285	5.87	2002	8008	256256
a094m270	0.094	269	4.09	2469	7407	237024
a094m270L	0.094	269	6.15	4510	18040	577280
a093m220	0.093	216	4.95	2000	8000	256000
a093m220X	0.093	214	4.81	2005	8020	256640
a091m170	0.091	169	3.35	4012	16048	513536
a091m170L	0.091	170	5.01	3500	17500	560000
a073m270	0.073	272	4.81	4720	18800	604160
a072m220	0.072	223	5.10	2000	12000	192000
a071m170	0.071	166	4.28	3120	18720	299520
a070m130	0.070	127	4.37	2500	15000	240000
a056m280	0.056	281	5.10	3250	19500	312000
a056m220	0.056	215	4.38	2550	15300	244800

Analyzed for the calculation of isovector charges and form factors



Summit (GPU) at OLCF

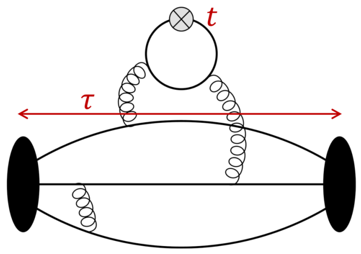
- **13 gauge ensembles with $N_f = 2+1$** generated by the JLab/W&M/LANL/MIT collaborations
- $O(2-6\times 10^5)$ measurements done,

$$C^{\rm imp} = \sum_{i=1}^{N_{\rm LP}} \frac{C_{\rm LP}(\mathbf{x}_i^{\rm LP})}{N_{\rm LP}} + \sum_{i=1}^{N_{\rm HP}} \left[\frac{C_{\rm HP}(\mathbf{x}_i^{\rm HP}) - C_{\rm LP}(\mathbf{x}_i^{\rm HP})}{N_{\rm HP}}\right]$$

Disconnected on 2+1-flavor Clover Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	$N_{\rm conf}^{\rm disc}$ light/strange	Random srcs light/strange
a127m285	0.127	285	5.87	1002 / 1002	7200 / 7200
a094m270	0.094	269	4.09	1197 / 1197	7200 / 7200
a094m270L	0.094	269	6.15	1000 / 1000	7200 / 7200
a093m220	0.093	216	4.95	985 / 1368	7200 / 7200
a093m220X	0.093	214	4.81		
a091m170	0.091	169	3.35	1155 / 1155	7200 / 7200
a091m170L	0.091	170	5.01		
a073m270	0.073	272	4.81	1132 / 1378	7200 / 7200
a072m220	0.072	223	5.10	1000 / 1000	7200 / 7200
a071m170	0.071	166	4.28		
a070m130	0.070	127	4.37		
a056m280	0.056	281	5.10		
a056m220	0.056	215	4.38		

Analyzed and proposed for the disconnected diagrams



[PNDME, PRL 127 (2021) 242002]

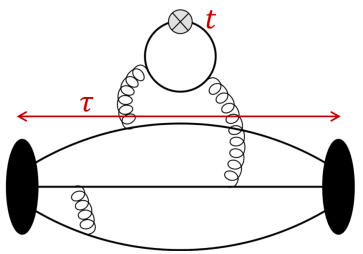
Nucleon sigma term $\sigma_{\pi N}$ calculation using $N_f = 2+1+1$ HISQ ensemble gives an interesting result, and we want to verify the result using $N_f = 2+1$ clover ensembles.

Disconnected on 2+1+1-flavor HISQ Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	N_{conf}^{conn}	N_{conf}^{disc} light/strange
a15m310	~0.15	320	3.93	1917	1917 / 1917
a12m310	~0.12	310	4.55	1013	1013 / 1013
a12m220	~0.12	228	4.38	744	958 / 870
a09m310	~0.09	313	4.51	2263	1017 / 1024
a09m220	~0.09	226	4.79	964	712 / 847
a09m130	~0.09	138	3.90	1290	1270 / 994
a06m310	~0.06	320	4.52	500	808 / 976
a06m220	~0.06	235	4.41	649	1001 / 1002

Flavor diagonal charge study [PNDME]

Analyzed for the disconnected diagrams



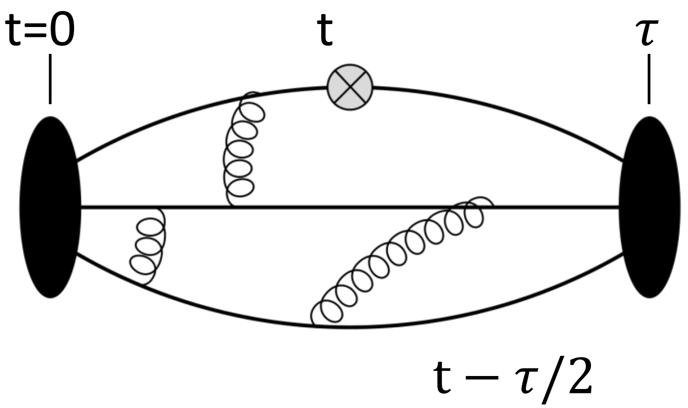
- Ensembles generated by MILC Collaboration
- 8 ensembles including one physical M_π^{phys} ensemble
- HYP smeared $N_f = 2+1+1$ MILC HISQ lattices, Clover fermion with a tree-level tadpole improved c_{SW}

Excited-state effect

Effect from $N\pi$ / $N\pi\pi$ multihadron excited states

Excited state contamination (ESC)

- Nucleon signal/noise decays $\propto e^{-(E-1.5M_\pi)\tau}$ with Euclidean time τ.
- Nucleon operator creates ground state nucleons (N) plus all excited states (ES) with the same quantum number, including $N\pi$, $N\pi\pi$, $N\rho$, $N(1440)$, $N(1710)$, $\cdots$.
  - Excited states that give significant contribution to a particular correlation function are not known a priori. $\rightarrow$ χPT is a very useful guide to understand ESC
  - Physical mass ensemble (a070m130) is crucial as the mass gap of $N\pi$ state ($\approx$ 1200 MeV) becomes significantly smaller than the lowest radial excitation $N(1440)$



$$C^{3pt} = \langle 0|\mathcal{O}|0\rangle |A_0|^2 e^{-M_0\tau} \times \left[1 + \frac{\langle 1|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_1|^2}{|A_0|^2} e^{-\Delta M_1 \tau} + \frac{\langle 2|\mathcal{O}|2\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_2|^2}{|A_0|^2} e^{-(\Delta M_2+\Delta M_1)\tau} + \frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_1|}{|A_0|} e^{-\Delta M_1 \frac{\tau}{2}} \times 2\cosh\left(\Delta M_1 (t-\frac{\tau}{2})\right) + (\cdots)\right]$$

ESC!

Ground-state matrix element

→Nucleon charge or Form Factor

G_P^{u-d}: Excited state effect

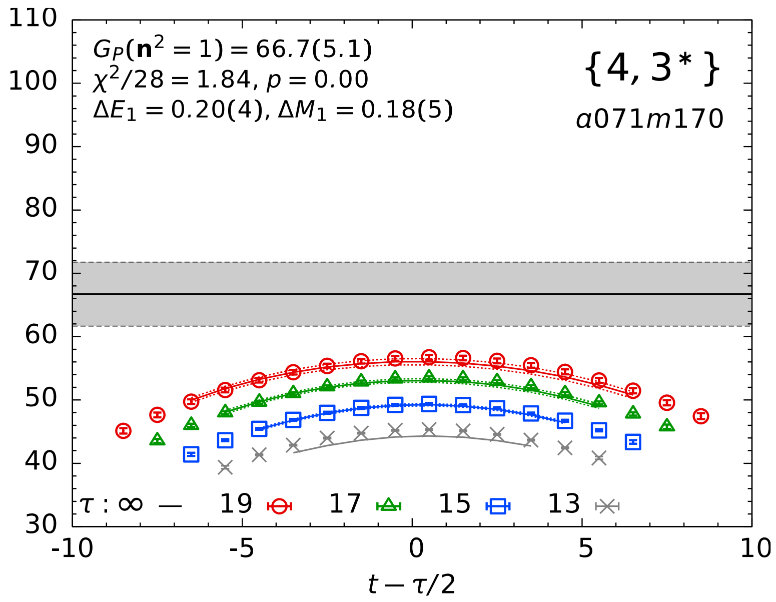
[NME (2021), PRD 105 054505]
$a \approx 0.071$ fm, $M_\pi \approx 170$ MeV
At $\vec{q} = \frac{2\pi}{L}(1,0,0)$

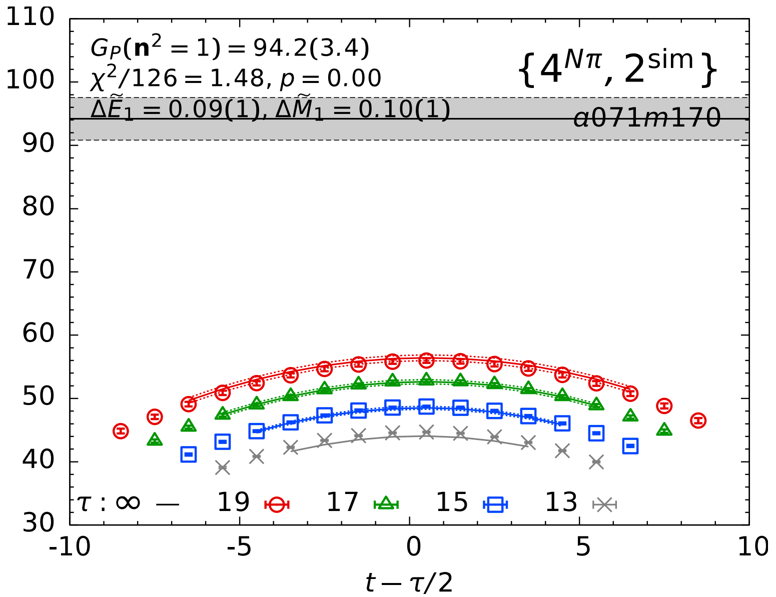
- Data displayed: 3-point/2-point ratio of correlation functions showing dependence on t, τ due to ES
- Gray band: $G_P^{u-d}(\vec{q})$ determined from the ES fit.

Standard 3-state fit to $\langle P \rangle$

$\Delta E_1 \sim \Delta M_1 \sim 0.5 GeV$

from 2pt analysis





Simultaneous 2-state to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators

$E_1 \sim N(\vec{0})\pi(\vec{q})$
$M_1 \sim N(\vec{q})\pi(-\vec{q})$

$\Delta E_1 \sim \Delta M_1 \sim 0.25 GeV$

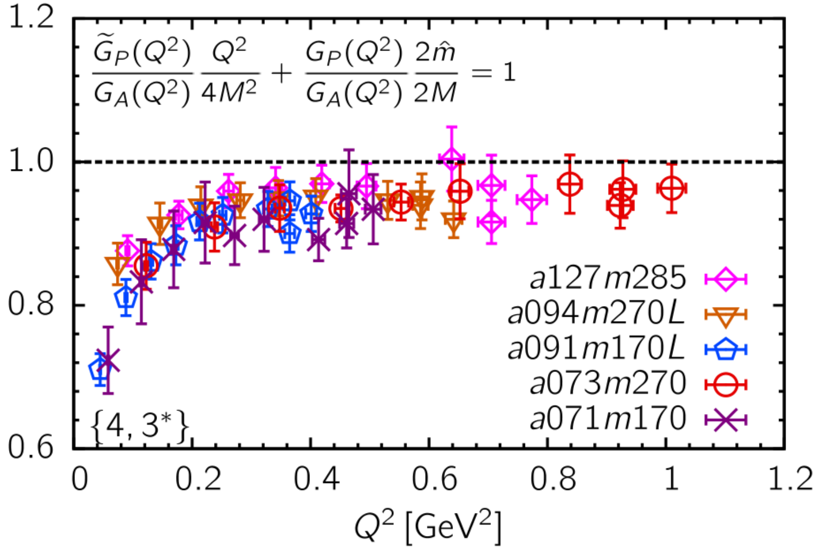
$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

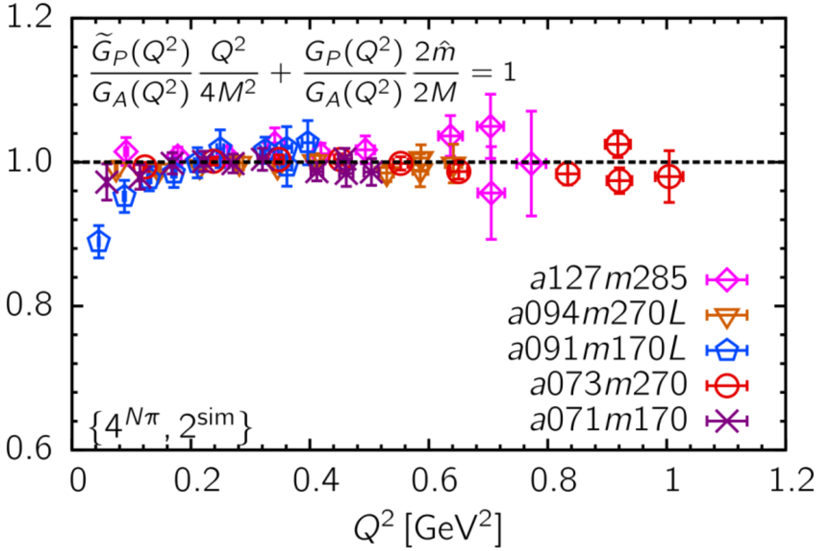
- χPT: $N\pi$ state coupling large in the axial current
- Output of a simultaneous fit *increases* the axial form factors by $G_A \sim 5\,\%$, $\tilde{G}_P \sim 35\,\%$, $G_P \sim 35\,\%$
- Satisfies PCAC relation!

PCAC: Need to Remove Excited State Contributions

Standard 3-state fit to A_μ, P correlators

$\Delta E_1 \sim \Delta M_1 \sim 0.5 GeV$





Simultaneous 2-state to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators

$E_1 \sim N(\vec{0})\pi(\vec{q})$
$M_1 \sim N(\vec{q})\pi(-\vec{q})$

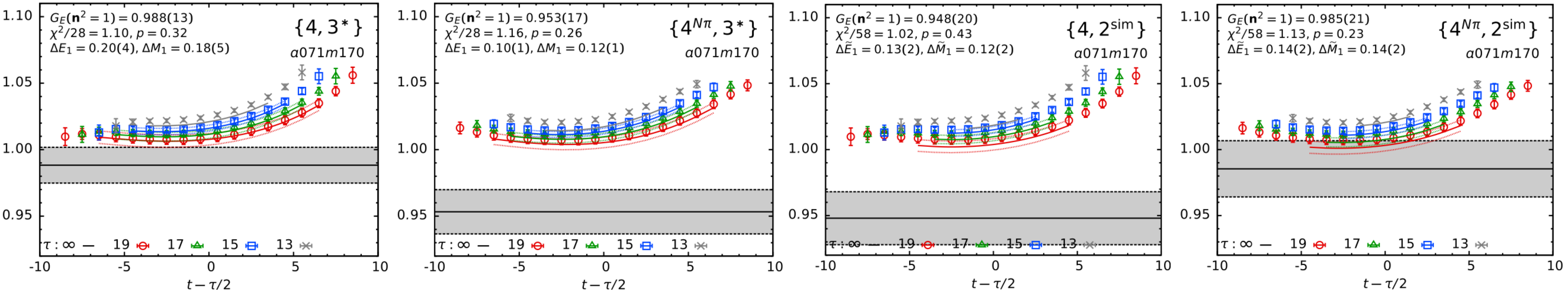
[NME (2021), PRD 105 054505]

$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

G_E^{u-d}: Excited state effect

[NME (2021), PRD 105 054505]
$a \approx 0.071$ fm, $M_\pi \approx 170$ MeV
At $\vec{q} = \frac{2\pi}{L}(1,0,0)$

- Over 4 different strategies to control the ES effect, $G_E^{u-d}(\vec{q})$ has $\approx 4\%$ variation
- At larger momentum transfer $\vec{q}$, the data and fit become less sensitive to ES



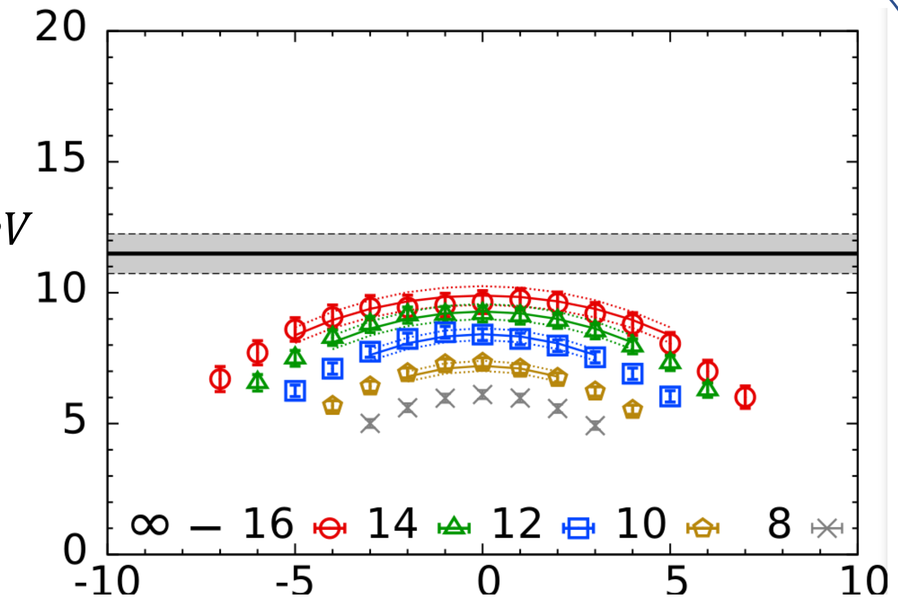
- Data displayed: 3-point/2-point ratio of correlation functions showing dependence on t, τ due to ES
- Gray band: $G_E^{u-d}(\vec{q})$ determined from the ES fit.

g_S^{u+d}: Excited state effect

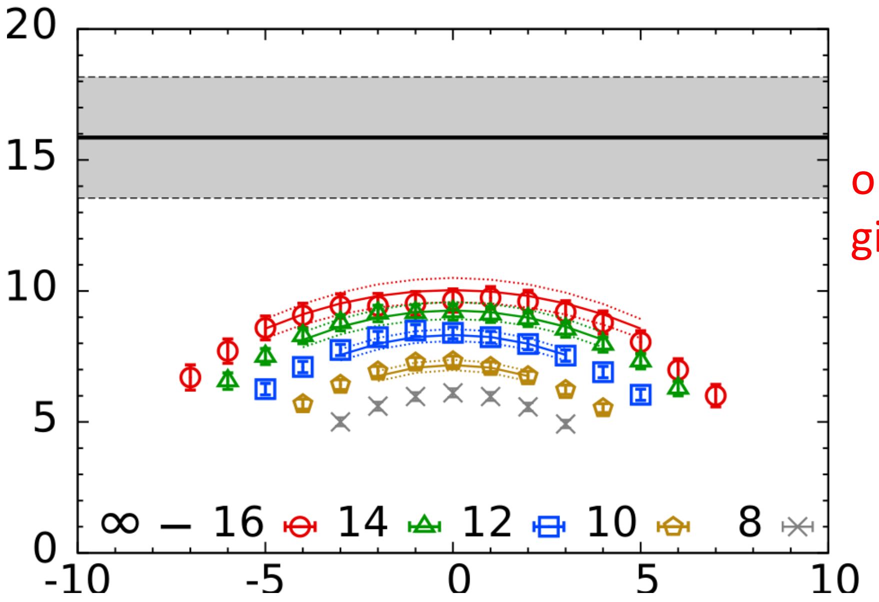
$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

PNDME, PRL 127 (2021) 242002

$M_1 \approx 1.6\ GeV$
without $N\pi$



$\sigma_{\pi N} = m_l g_S^{u+d}$ ~ 40 MeV, $\frac{\chi^2}{dof} = 1.1$



$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $M_1 \approx 1.2\ GeV$
with $N\pi$

$\sigma_{\pi N} = m_l g_S^{u+d}$ ~ 60 MeV, $\frac{\chi^2}{dof} = 1.2$

- Scalar is sensitive to $N\pi$ state
- Output is close to the phenomenological determination

Results

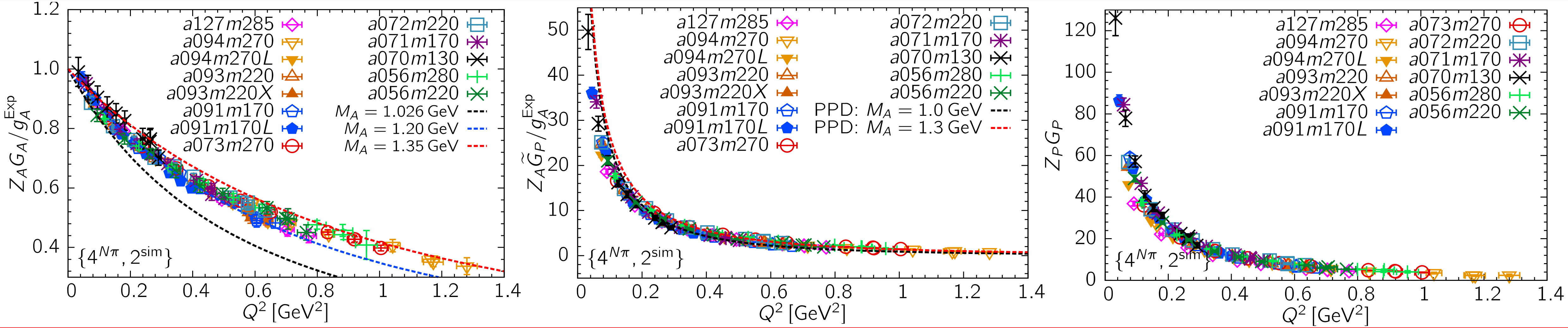
Isovector axial, electric and magnetic form factors

Flavor diagonal axial, scalar, and tensor charges

Nucleon Isovector Form Factors

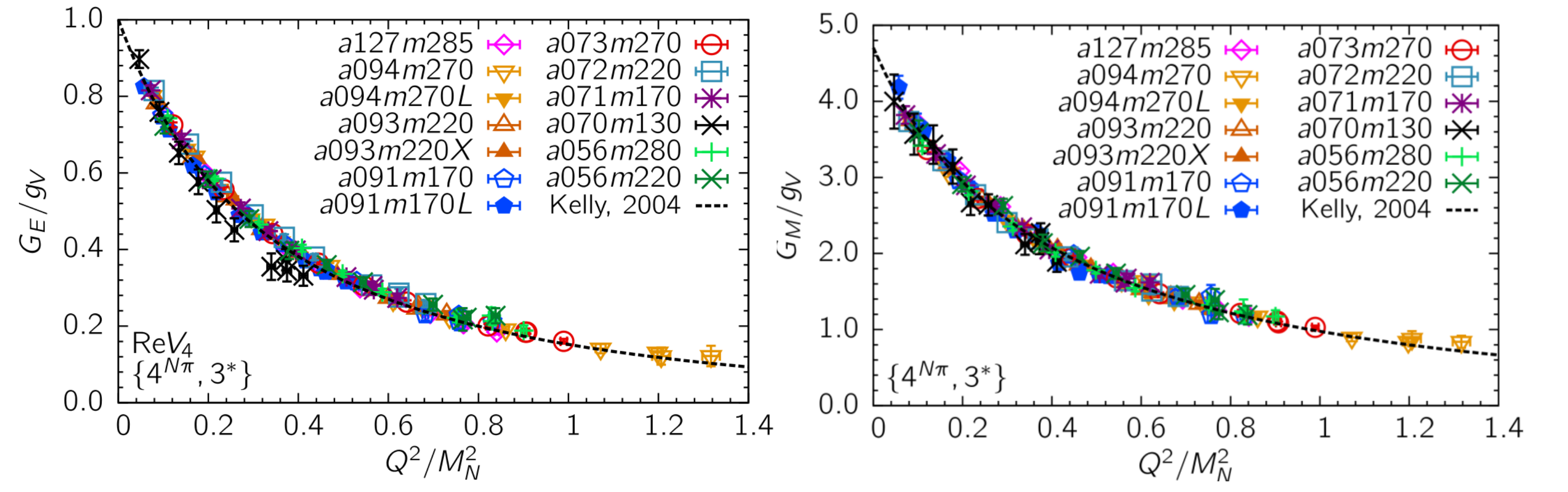
[NME, Lattice 2022 preliminary, arXiv:2301.07885]

- Clover fermion on $N_f = 2 + 1$ clover ensembles



Axial form factors

- $N\pi$ excited state needed to satisfy PCAC relation. Impact on FF is large

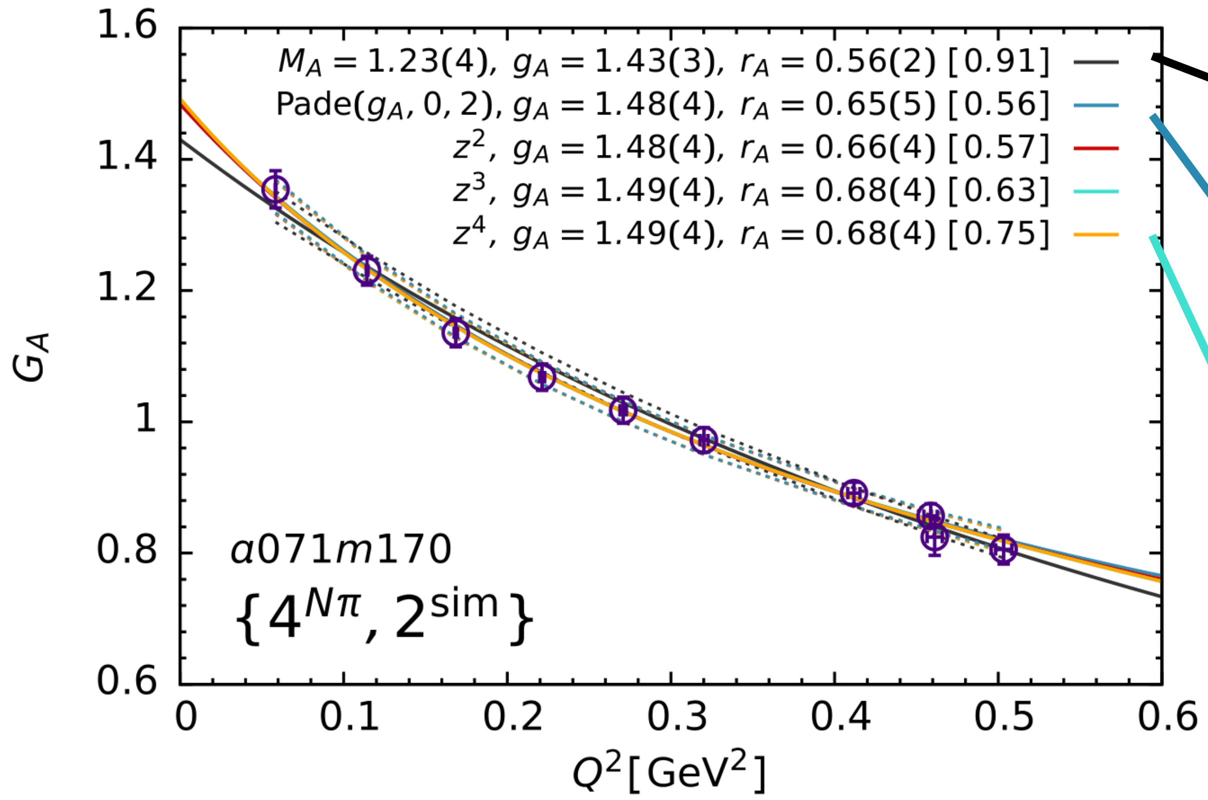


Electric & Magnetic form factors

- Less sensitive to the details of the excited states
- Good agreement with the Kelly curve

[J.J.Kelly, PRC 70, 068202 (2004)]

G_A^{u-d} : Examined Dipole, Pade and z-expansion fits

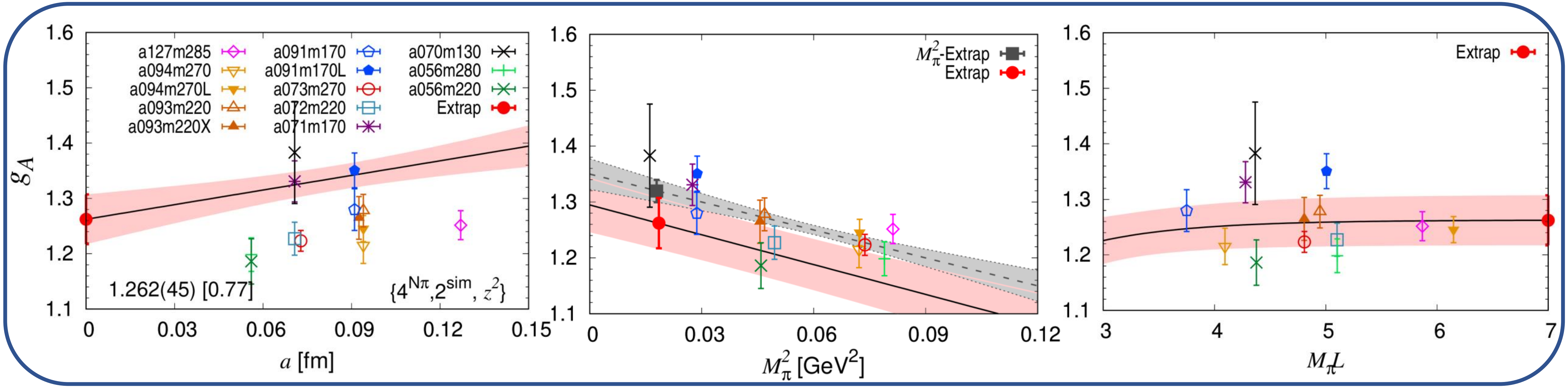


- Dipole: $\frac{G_A(0)}{(1+Q^2/M_E^2)}$
- Pade: $\frac{g}{1+b_1 Q^2+b_2 Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

[NME (2021), PRD 105 054505]

g_A^{u-d}: chiral continuum extrapolation

[NME (2022), preliminary]



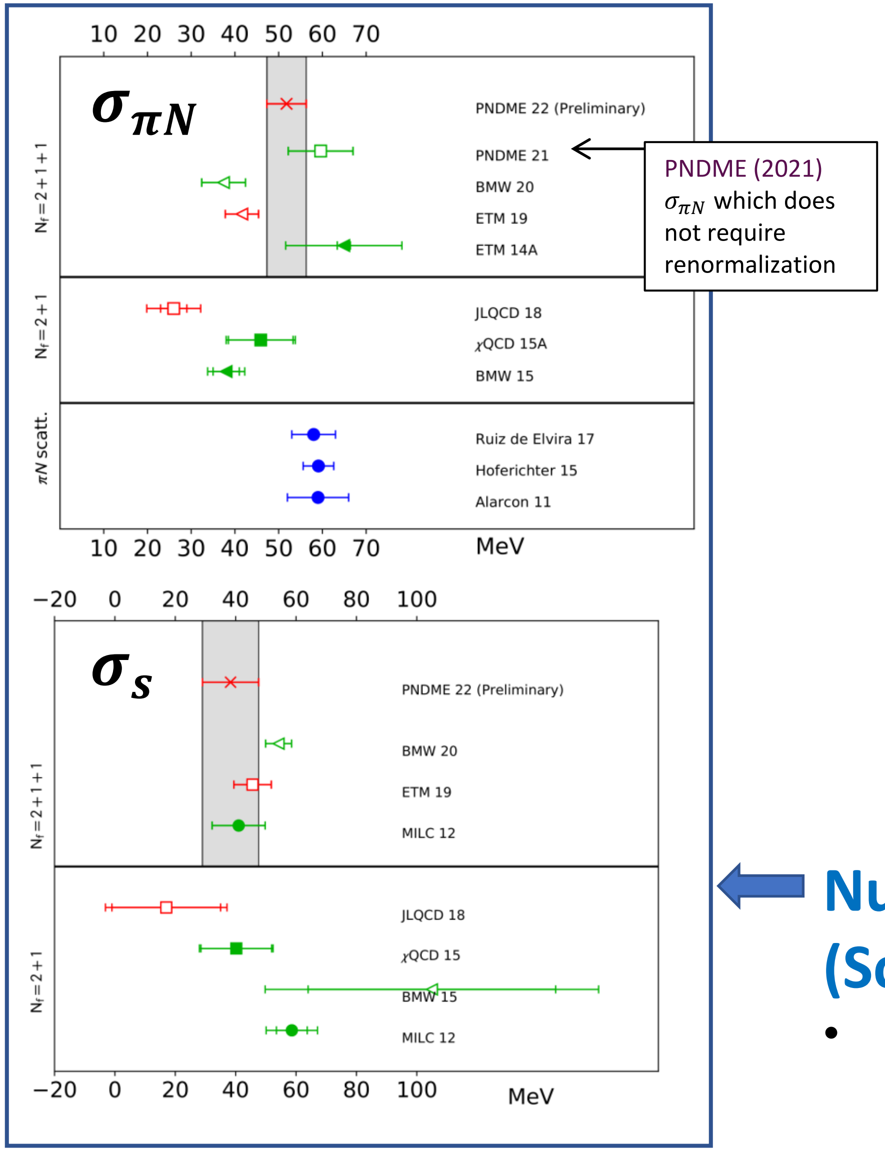
- Axial charges obtained from the $Q^2 \to 0$ extrapolation to $G_A(Q^2)$

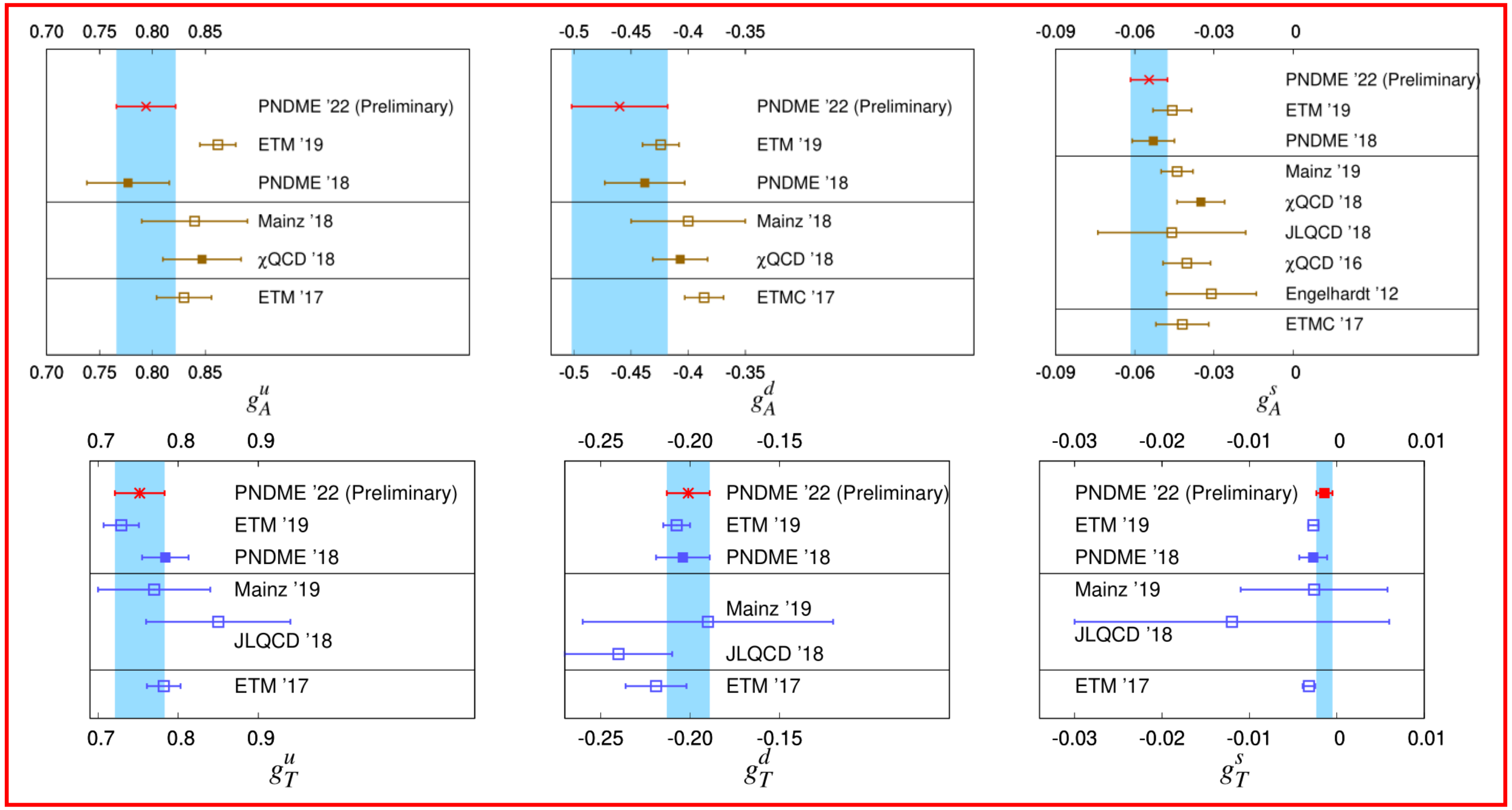
$$g(a, M_\pi, M_\pi L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 \frac{M_\pi^2 e^{-M_\pi L}}{\sqrt{M_\pi L}}$$

Nucleon Flavor Diagonal Charges
: Comparison with FLAG 2021 results

[PNDME, Lattice 2022 preliminary, arXiv:2301.07890]

- Clover fermion on $N_f = 2 + 1 + 1$ HISQ ensembles
- Flavor mixing calculated nonperturbatively
- Chiral-Continuum extrapolation including a data at M_π^{Phys}





Nucleon sigma terms (Scalar charges)

- $\sigma_{N\pi}$: Excited-state effects are large and results very sensitive to $N\pi$ / $N\pi\pi$ states

Axial and Tensor charges

- Less sensitive to the details of the excited states

Summary

- We are calculating nucleon isovector form factors and flavor diagonal charges as part of a comprehensive analysis of nucleon structure
- Form factors presented as a function of Q^2 over $0.04 < Q^2 < 1\ \mathrm{GeV}^2$.
- We are investigating excited state effects
  - Contributions from $N\pi$ / $N\pi\pi$ multihadron excited states
    - Evidence of large ES for $\tilde{G}_P, G_P$, and $g_S^{u,d}$ ($\sigma^{\pi N}$).
  - Need higher statistics to resolve the ES at M_π^{Phys} and on finer lattices (smaller a)
  - Higher order ES fits are under investigation
  - Study with multihadron $N\pi$ operators is in progress

Acknowledgements

- The calculations used the CHROMA software suite.
- We thank DOE for computer time allocations at NERSC and OLCF.
- We thank the USQCD collaboration for computer time
- Institutional Computing at LANL for computer time