nEDM from the clover-on-clover formulation

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Introduction

Introduction
BSM Operators

Standard model CP violation in the weak sector.

Strong CP violation from dimension 4 operators anomalously small.

- Dimension 4:
  - CP violating mass $m\bar{\psi}\gamma_5\psi$.
  - Topological charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.

- Suppressed by $v_{\mathrm{EW}}/M_{\mathrm{BSM}}^2$:
  - Quark Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Quark Chromo-electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.

- Suppressed by $1/M_{\mathrm{BSM}}^2$:
  - Gluon Chromo-electric Dipole moment: $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
States

Asymptotic states are 'free' particles: always have a P symmetry.

If interaction does not have these symmetries, the symmetry generator will be different on different asymptotic states.
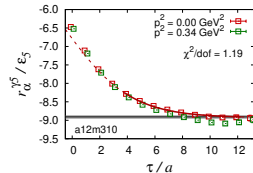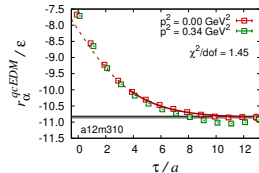Change the nucleon interpolating operator

$$\hat{N} \rightarrow e^{-i\alpha_N \gamma_5} \hat{N}$$

to get standard parity for nucleon.
α_N can be chosen real if interactions have PT symmetry.
α_N can be calculated in chiral perturbation theory and as

$$\lim_{\tau \rightarrow \infty} [r_\alpha(\tau) \equiv \frac{\Im \, \mathrm{Tr}\, \gamma_5 (1 + \gamma_4)\langle N(\tau)\bar{N}(0)\rangle}{\Re \, \mathrm{Tr}(1 + \gamma_4)\langle N(\tau)\bar{N}(0)\rangle}]$$

arXiv:2304.09929 [hep-lat]

Introduction
Form Factors

Vector form-factors in standard representation for C, P, and T:

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\langle N | V_\mu(q) | N \rangle = \bar{u}_N \left[\gamma_\mu \, F_1(q^2) + i \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \, \frac{F_2(q^2)}{2m_N} \right.$$

$$+ \left(2i \, m_N \gamma_5 q_\mu - \gamma_\mu \gamma_5 q^2 \right) \frac{F_A(q^2)}{m_N^2}$$

$$\left. + \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \gamma_5 \, \frac{F_3(q^2)}{2m_N} \right] u_N$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
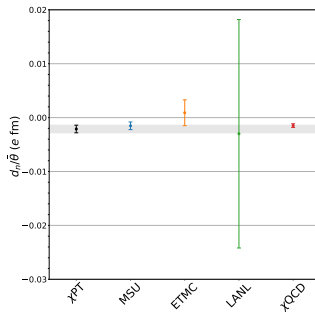- F_A violates PT; F_3 violates CP.

Lattice Calculation

Lattice Calculation
Theta term

Calculation is essentially a reweighting of the 3-pt function by the topological charge.
Difficult to obtain a signal at physical quark masses.
Systematics are being studied.
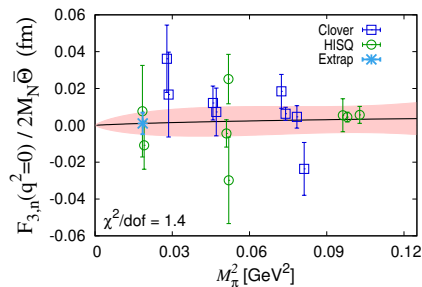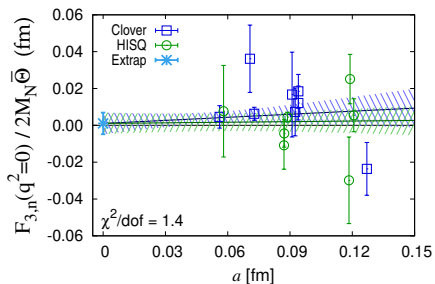

E. Mereghetti, personal communication

Lattice Calculation
Theta term: Clover-on-clover

New calculation: simultaneous fit to Clover-on-HISQ and Clover-on-Clover results
PRELIMINARY (errors only statistical):

$$d_N = c_1 M_\pi^2 + c_2 M_\pi^2 \log\left(\frac{M_\pi^2}{M_N^2}\right) + c_3^{\text{HISQ}} a + c_3^{\text{Clover}} a = 0.0010(59)$$
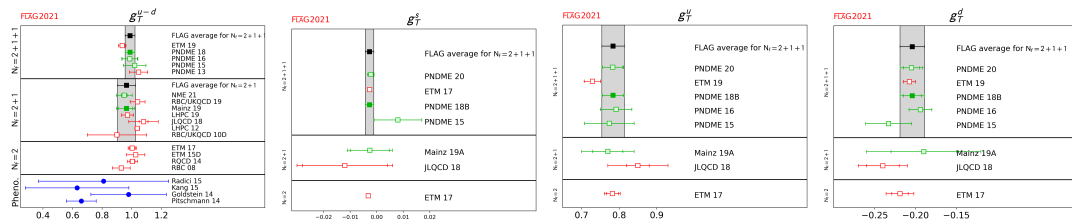
Lattice Calculation
Quark EDM

nEDM due to the quark EDM operator proportional to the tensor charge.
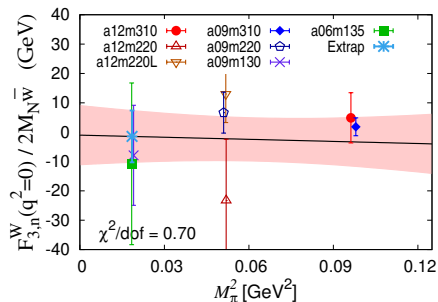
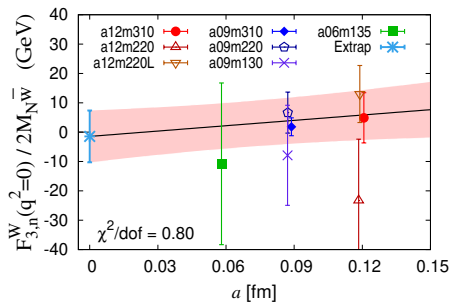Lattice calculations dominate determination

New clover-on-clover analysis in Phys. Rev. D105 (2022) 054505.

Lattice Calculation
Weinberg term

PRELIMINARY (errors only statistical):

Mixing with the Theta term to be removed.

Data collected with both clover-on-HISQ and clover-on-clover.

Analysis in progress.

Lattice Calculation
Technique: quark chromo-EDM

The quark chromo-EDM operator can be handled by the Schwinger source method

$$P \equiv \left[\slashed{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \right]^{-1} \longrightarrow \left[\slashed{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu}) \right]^{-1}$$

The fermion determinant

$$\exp \mathrm{Tr} \ln \left[1 + i\epsilon \, \Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\slashed{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1} \right]$$

vanishes for the isovector case
Schwinger-source does not work with gradient-flow! Instead, we can use

$$\left[\slashed{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu}) \right]^{-1} \approx P - i\epsilon P\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}P$$

Lattice Calculation
Three-point function

ϵ small to avoid multiple insertions.

Lattice Calculation
quark chromo-EDM status

- HISQ analysis of isovector chromo-EDM available. (arXiv:)
- Developed techniques for subtraction of power-divergence.
- Large lattice corrections because of $O(a^2)/am$ corrections.
- Available at arXiv:2304.09929 [hep-lat]
- Isoscalar analysis in progress.

Quark Chromoelectric Dipole Moment

Quark Chromoelectric Dipole Moment
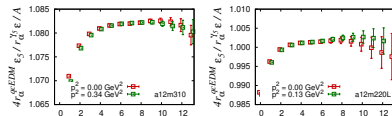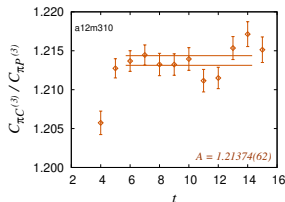## Power divergence

Quark chromo-EDM operator is power-divergent:

$$\tilde{C} = i\bar{\psi}\Sigma^{\mu\nu}\gamma_5 G_{\mu\nu}T^a\psi - i\frac{A}{a^2}\bar{\psi}\gamma_5 T^a\psi$$

Demanding $\langle\Omega|\,\tilde{C}\,|\pi\rangle = 0$ fixes A:

Leading order χPT:

$$\alpha_N(\tilde{C}) = 0 \implies \frac{1}{A}\frac{\alpha_N(C)}{\alpha_N(\gamma_5)} = 1$$

Ensemble	c_{SW}	a (fm)	t-range	A
a12m310	1.05094	0.1207(11)	6–14	1.21374(62)
a12m220L	1.05091	0.1189(09)	7–14	1.21800(33)
a09m310	1.04243	0.0888(08)	8–22	0.99621(30)
a06m310	1.03493	0.0582(04)	14–30	0.77917(24)

Quark Chromoelectric Dipole Moment
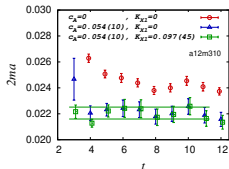## Multiplicative renormalization

Isovector pseudoscalar can be rotated away up to $O(a)$ effects!
We can determine the $O(a)$ effects non-perturbatively:

$$\frac{\langle \pi \left[a\partial_\mu A^\mu - \bar{c}_A a^2 \partial P + \bar{K}(a^2 C - AP) \right] \rangle}{\langle \pi P \rangle} = 2\bar{m}a(1 + O(a^2))$$

So, on-shell zero-momentum

$$\text{M.E. of } P = \text{M.E. of } \frac{x \equiv a^2 \bar{K}}{y \equiv 2\bar{m}a + A\bar{K}} C .$$


| Ensemble | fit-range | | χ^2/d.o.f | | c_A | $\bar{K}_{X1}$ | $2\bar{m}a$ | $\dfrac{2ma}{\bar{K}_{X1}}$ | $\dfrac{2ma}{2\bar{m}a + A\bar{K}_{X1}}$ |
	c_A	$\bar{K}_{X1}$	c_A	$\bar{K}_{X1}$					
a12m310	4–11	3–11	0.66	0.88	0.054(10)	0.097(45)	0.02205(46)	0.23(10)	0.158(58)
a12m220L	4–11	3–11	2.08	3.09	0.0342(77)	0.183(35)	0.01152(21)	0.063(12)	0.0491(86)
a09m310	5–15	4–15	0.99	1.09	0.0277(40)	0.047(15)	0.01684(15)	0.35(11)	0.263(61)
a06m310	6–20	5–20	0.29	1.53	0.0093(17)	0.0272(60)	0.010460(37)	0.385(87)	0.331(50)
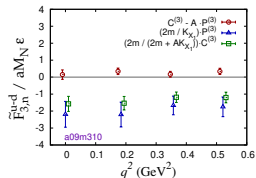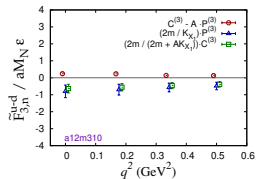
Quark Chromoelectric Dipole Moment
Results
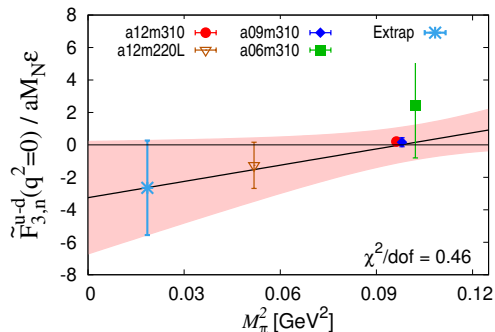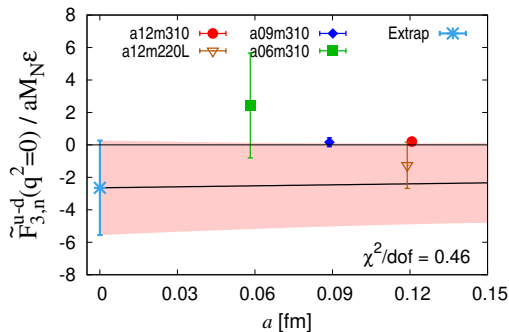
Preliminary data for power-divergent-subtracted tree-level matched operator $\tilde{C}$ in $\overline{\text{MS}}$ scheme 1-loop run to 2 GeV.

Quark Chromoelectric Dipole Moment
Request

Move away from mixed actions.

- $a = 0.127(2)$ fm, $M_\pi = 285(3)$ MeV, $32^3 \times 96$, 2.3 M Sky-core-hrs/500 confs
- $a = 0.094(1)$ fm, $M_\pi = 269(3)$ MeV, $48^3 \times 128$, 10.4 M Sky-core-hrs/500 confs
- $a = 0.0728(8)$ fm, $M_\pi = 272(3)$ MeV, $48^3 \times 128$, 10.4 M Sky-core-hrs/500 confs

Using Chroma codebase.

Code works best on KNL; parts of the code not yet optimized on GPUs.