

$K \rightarrow \pi\pi$ and ϵ'

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$K \rightarrow \pi\pi$ & direct CPV

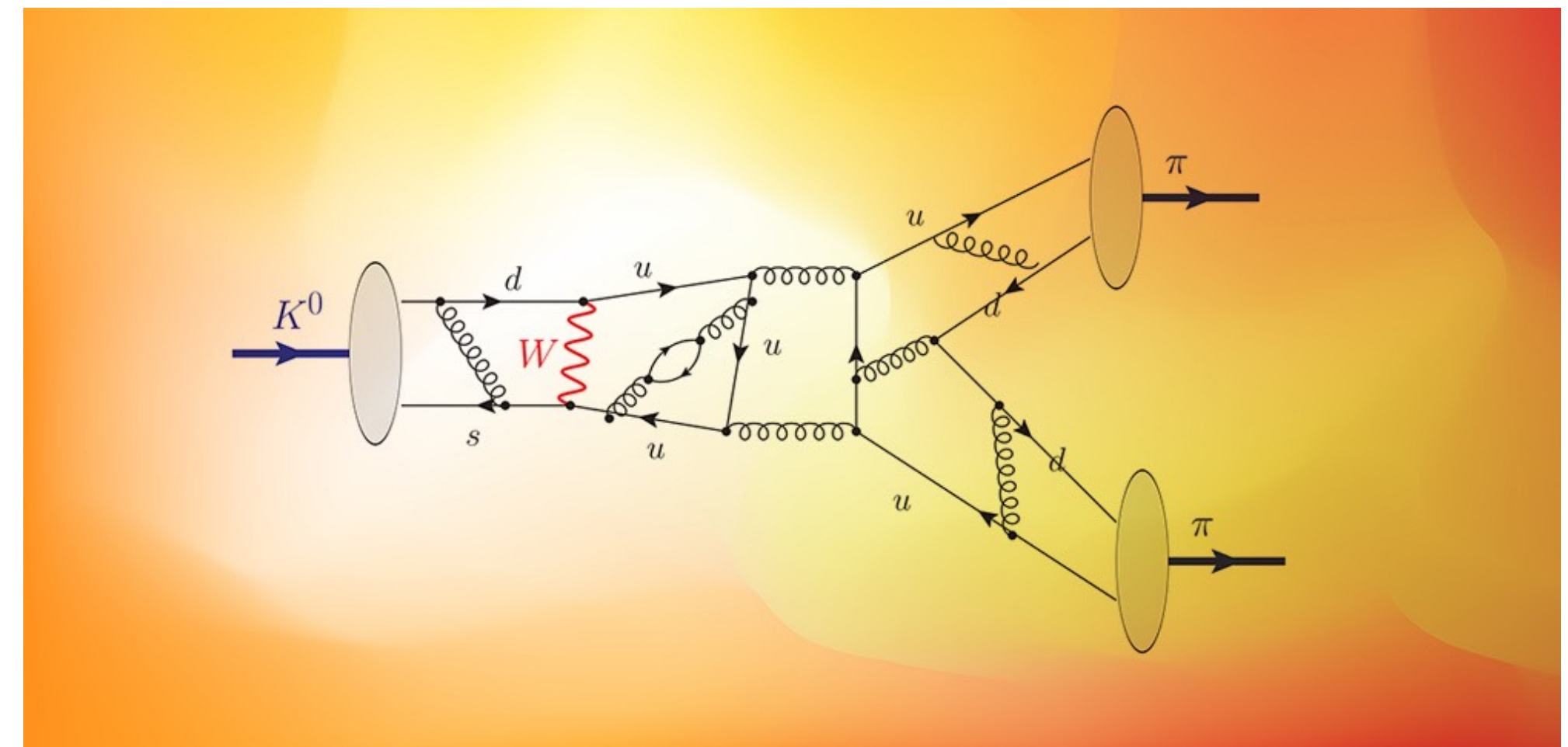
$$|K_L\rangle = |K_2\rangle + \varepsilon |K_1\rangle$$

CP odd CP even
direct CPV indirect CPV

$|K_2\rangle$ (CP odd) $\xrightarrow{\varepsilon'}$ $|\pi\pi\rangle$ (CP even)
 $|K_1\rangle$ (CP even) $\xrightarrow{\varepsilon}$ $|\pi\pi\rangle$ (CP even)

- ε' vs ε

- $\text{Re} (\varepsilon'/\varepsilon)_{\text{exp}} = 16.6(2.3) \times 10^{-4}$ (KTeV, NA48)
- Explained by SM?



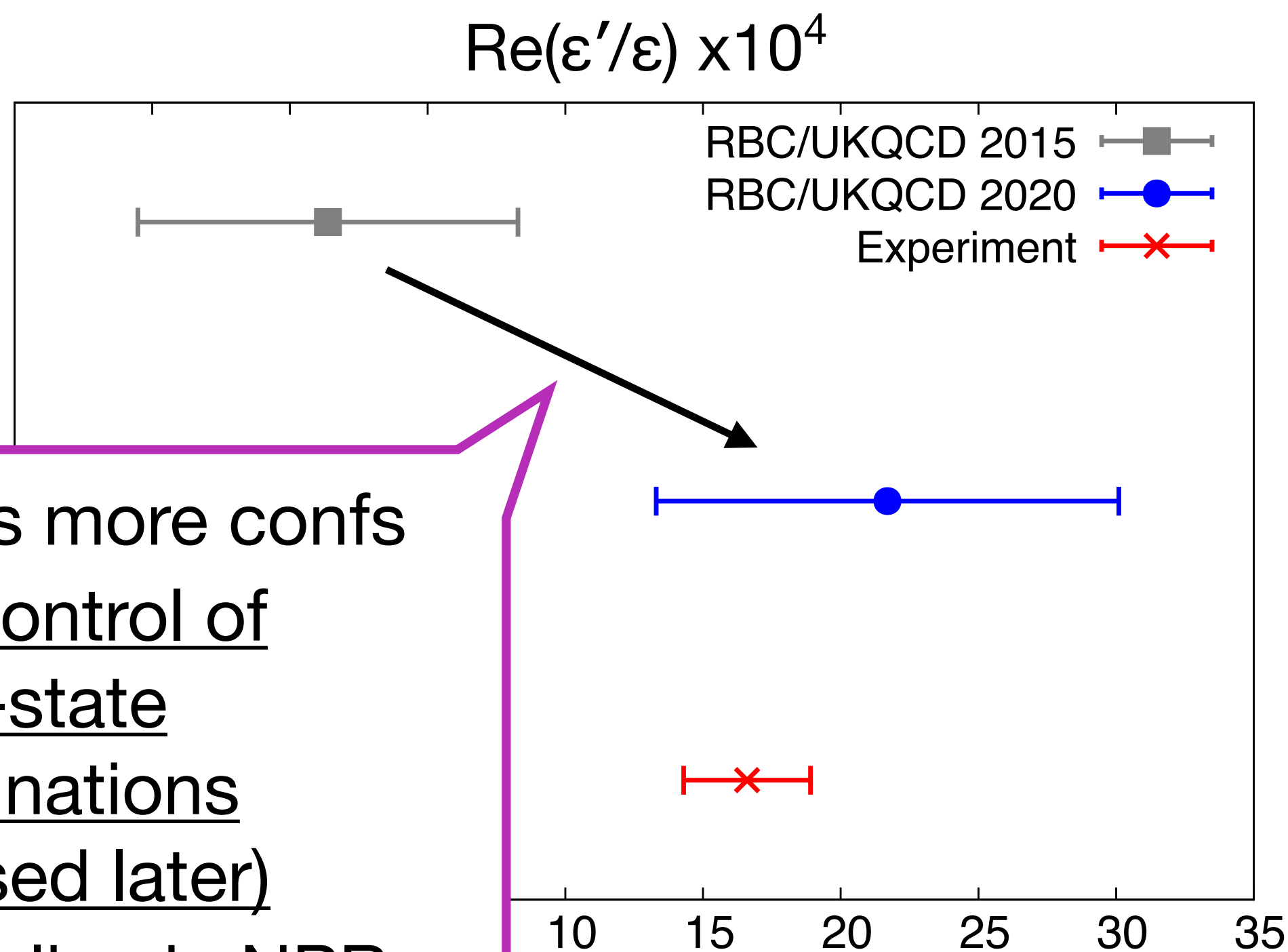
Calculation with G-parity BC

Ground $\pi\pi$ final state can express on-shell kinematics

$$\text{Re} \left(\frac{\epsilon'}{\epsilon} \right) \approx \frac{\omega}{\sqrt{2}|\epsilon|} \left[\frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right]$$

(Amplitudes: $A_l = \langle (\pi\pi)_l | H_W | K \rangle$)

($\omega = \text{Re} A_2 / \text{Re} A_0$)



- 3+ times more confs
- Better control of excited-state contaminations (discussed later)
- Step scaling in NPR

- $\text{Re} \left(\frac{\epsilon'}{\epsilon} \right)_{\text{SM}} = 21.7(2.6)_{\text{stat}}(6.2)_{\text{sys}}(5.0)_{\text{EM/IB}} \times 10^{-4}$

• PRD 102,054509 (2020)

🤝

$$\text{Re}(\epsilon'/\epsilon)_{\text{exp}} = 16.6(2.3) \times 10^{-4}$$

- Independent calculations desired
 - ◆ Phenomenological importance of ϵ'
 - ◆ Relatively large uncertainty compared to exp
- Major sources of systematic errors
 - ◆ QED & IB corrections
 - ◆ Finite lattice spacing effects
 - ◆ Wilson coefficients

Why periodic BC?

- Already have lattice ensembles with physical pion mass
 - $a^{-1} = 1 \text{ GeV}$, $24^3 \times 64$ & $a^{-1} = 1.4 \text{ GeV}$, $32^3 \times 64$ & ...
 - Continuum limit without new ensemble generation
- Hope to introduce QED/IB effects near future
 - Complicated with G-parity BC (violation of charge conservation)
 - Maybe possible with periodic BC
- Presence of $E_{\pi\pi} = 2m_{\pi}$ state challenging
 - interesting to see feasibility of extracting signal of excited states

Setup

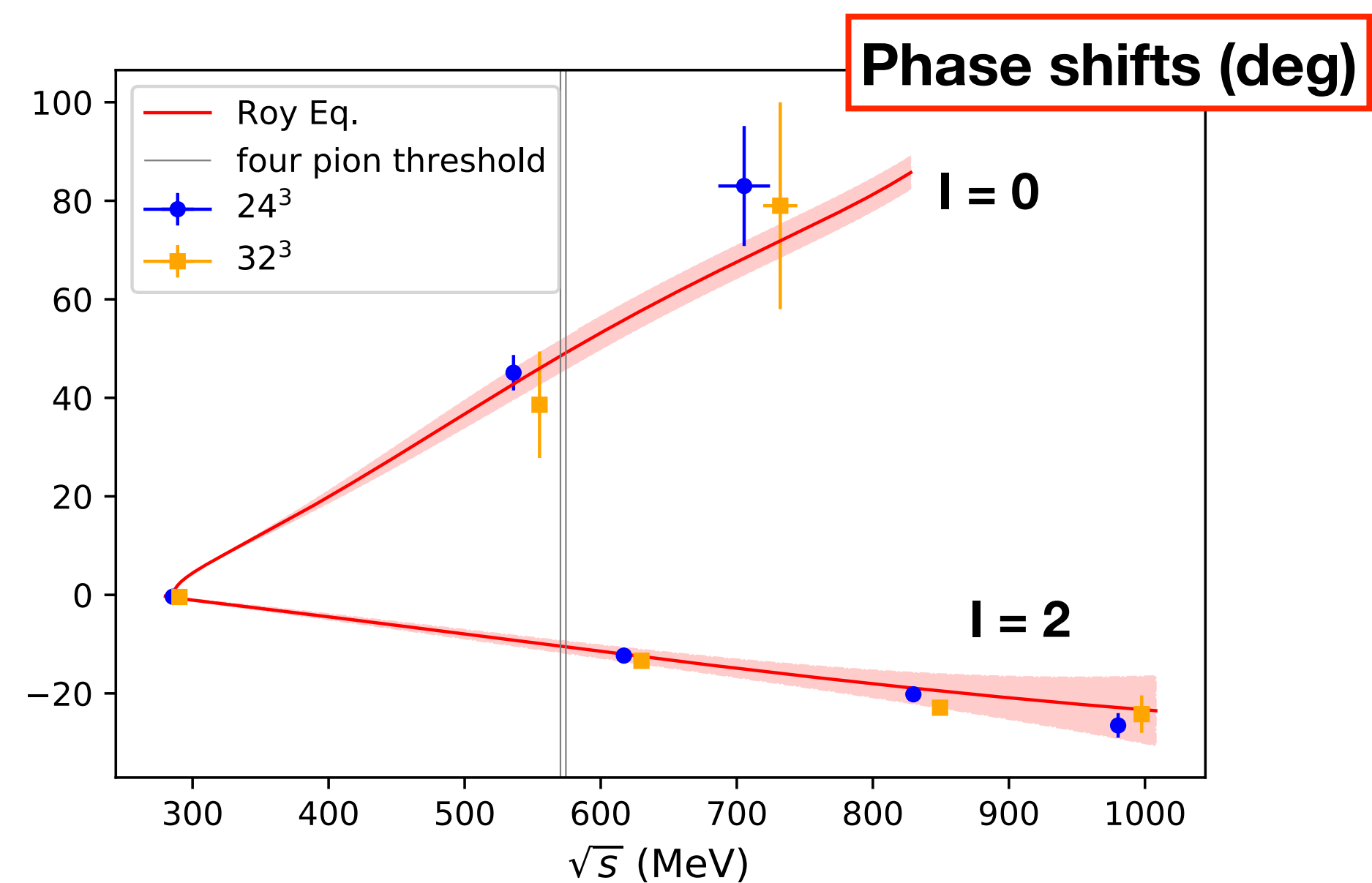
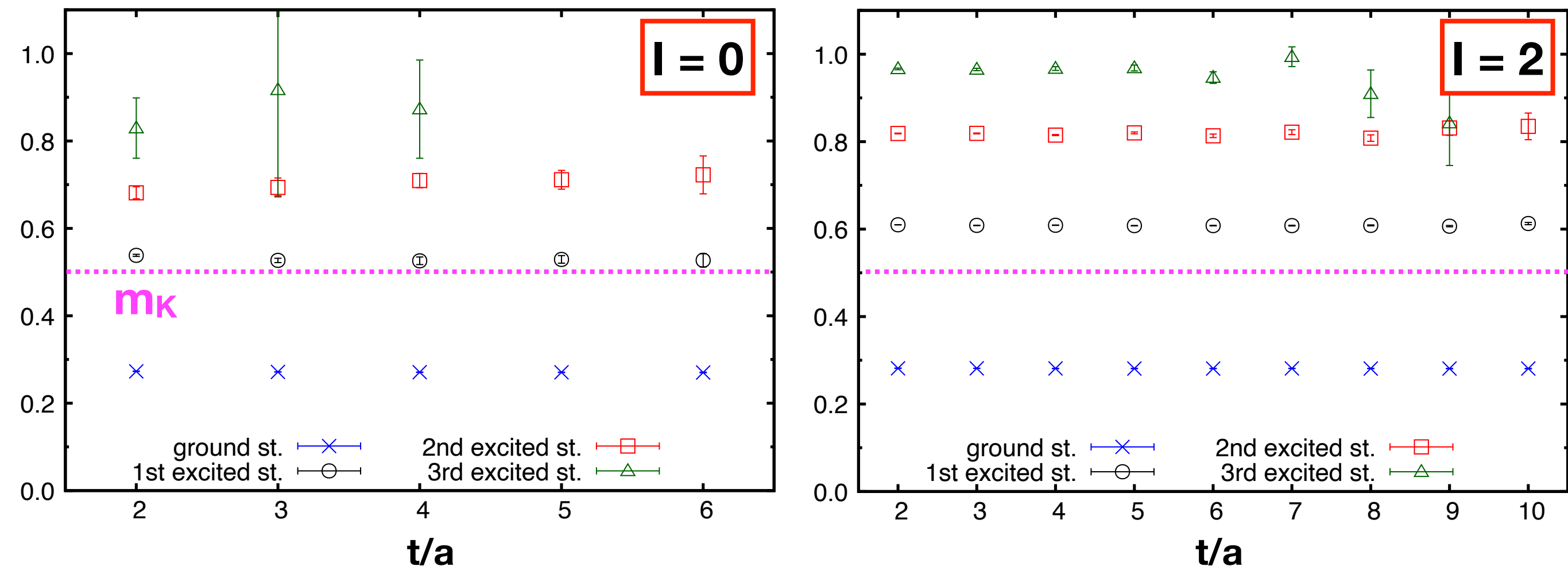
- 2+1 MDWF ensembles with physical m_π (RBC/UKQCD)
 - $a^{-1} = 1.02$ GeV, $24^3 \times 64$
 - $a^{-1} = 1.38$ GeV, $32^3 \times 64$
- GEVP to extract ME with excited $\pi\pi$ states
 - $O_\pi(\vec{p})O_\pi(-\vec{p})$ w various \vec{p}
 - O_σ : iso-singlet scalar operator for $l = 0$
 - also control higher-state contamination
- All-to-all propagator method
- AMA correction (exact calculation w fewer confs)

Achievements

- $\pi\pi$ scattering
 - ▶ GEVP successfully worked
 - ▶ m_K close to 1st excited state
 - ▶ Phase shifts from Lüscher's method consistent with Roy equation

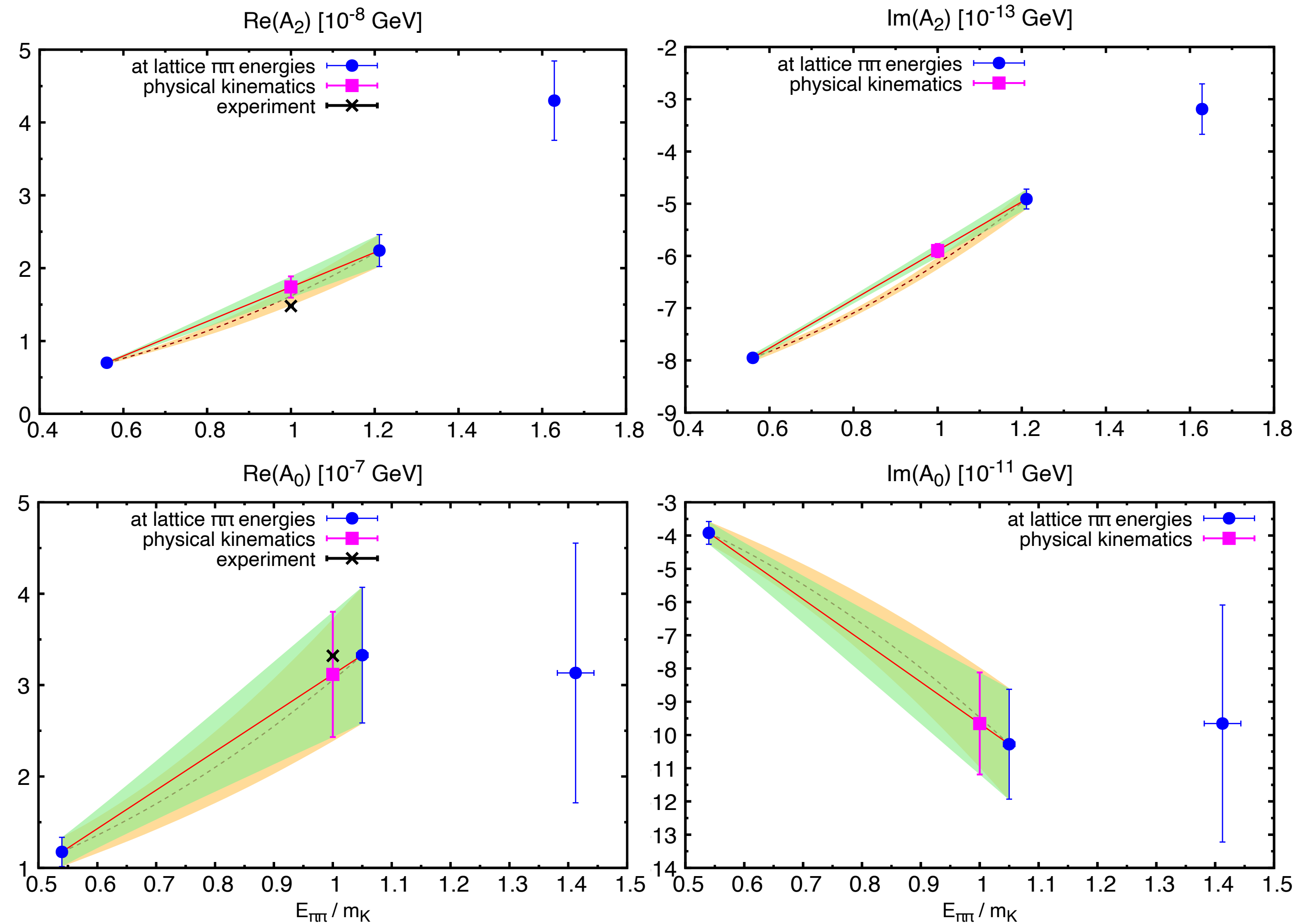
- arXiv: 2301.09286 [hep-lat]

Effective energies (lattice units)



Achievements

- $K \rightarrow \pi\pi$ amplitudes
- Interpolation to physical kinematics
- Paper to be on arXiv soon



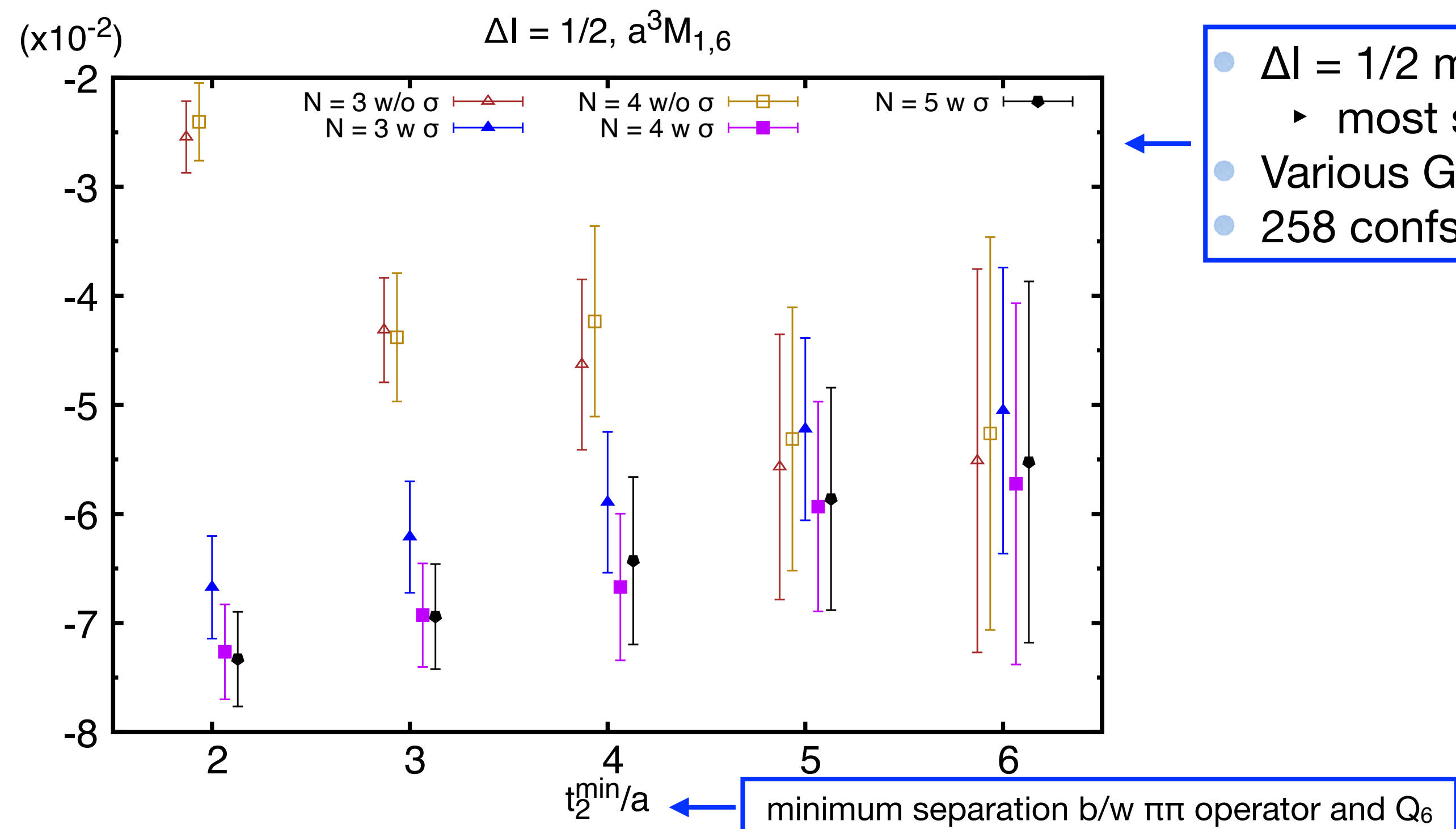
Goal vs where we are

- Goal (for next allocation year): continuum limit of A_0 with $< 10\%$ stat error
 - then only a few sources of systematic errors to be taken into account
- Measurements so far
 - 450 configs on 24^3 done
 - 250 configs on 32^3 almost done → 500 in the 23–24 allocation year
- Precision performance and expect

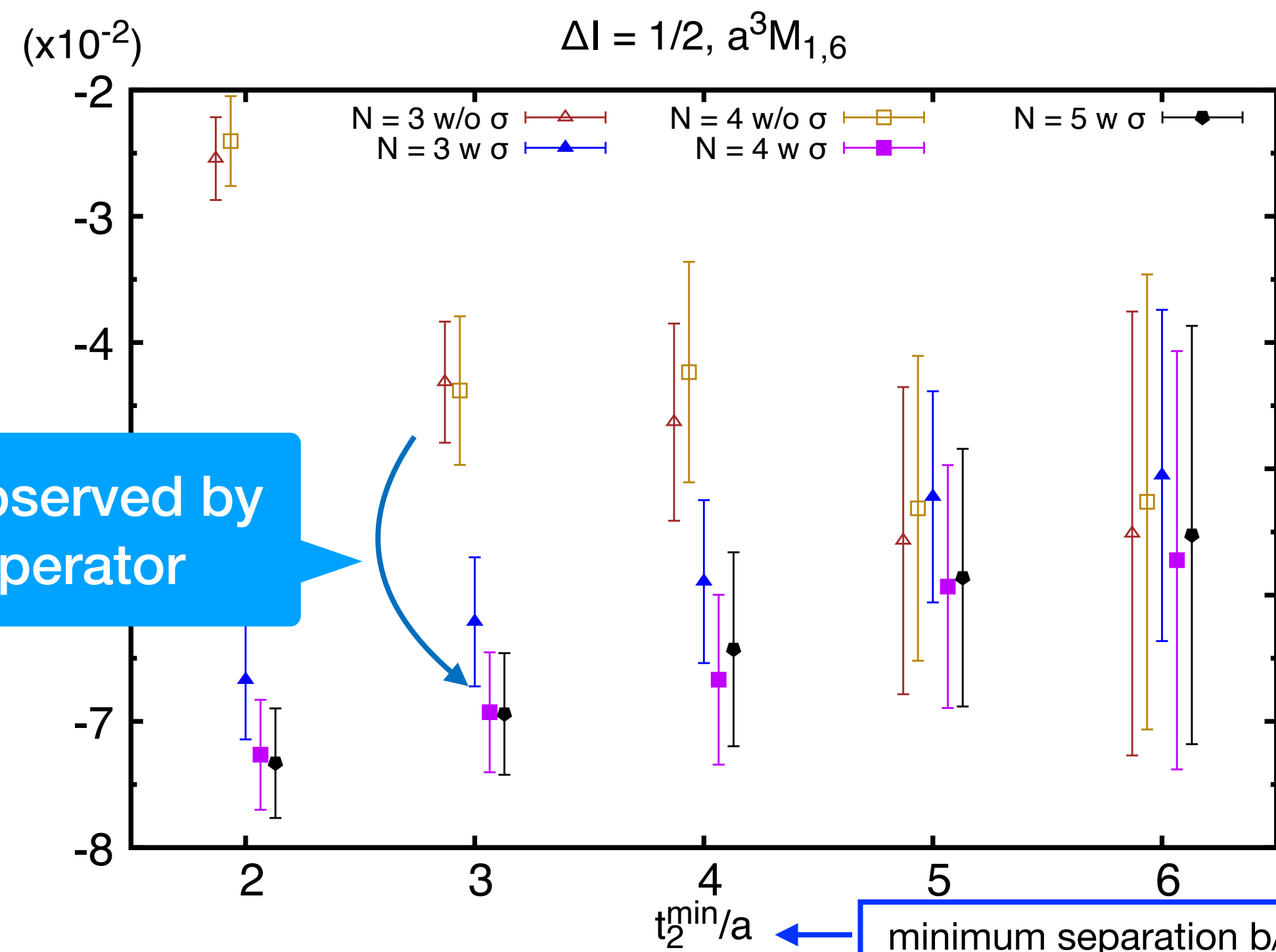
	$24^3 \times 64$	$32^3 \times 64$		expectation	
	$24^3 \times 64$	$32^3 \times 64$		$24^3 \times 64$	$32^3 \times 64$
N_{conf}	258	107	→	450	500
Stat error on $\text{Re}A_0$	22%	16%	→	17%	7.4%
Stat error on $\text{Im}A_0$	15%	16%	→	11%	7.4%

- might need some improvement to achieve 10% after continuum limit

Dominant source of stat error on $\text{Im}A_0$



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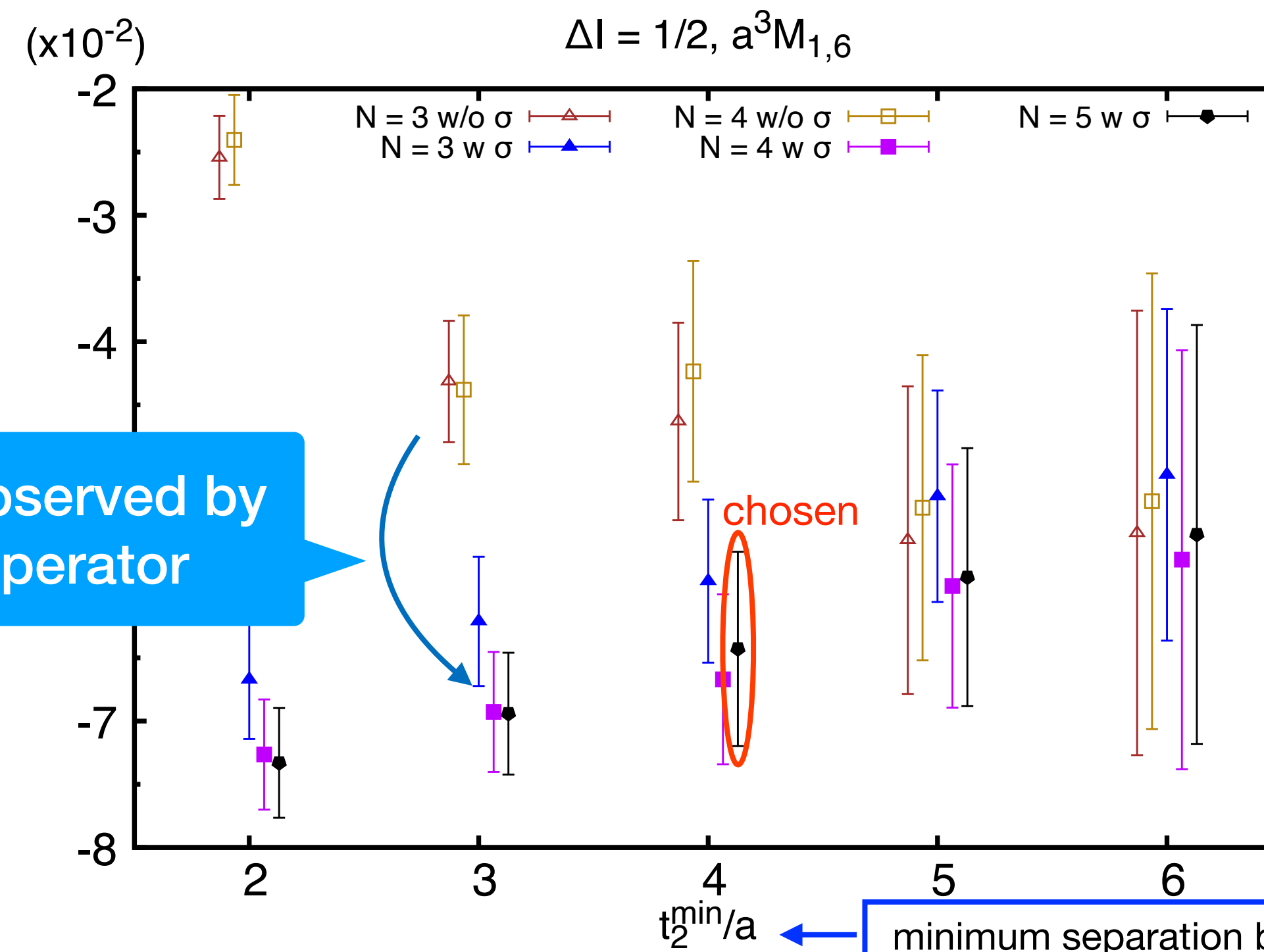


Significant shift observed by introducing σ operator

- $\Delta I = 1/2$ matrix element of Q_6
 - most significant for $\text{Im}A_0$
- Various GEVP bases w N operators
- 258 confs on $24^3 \times 64$

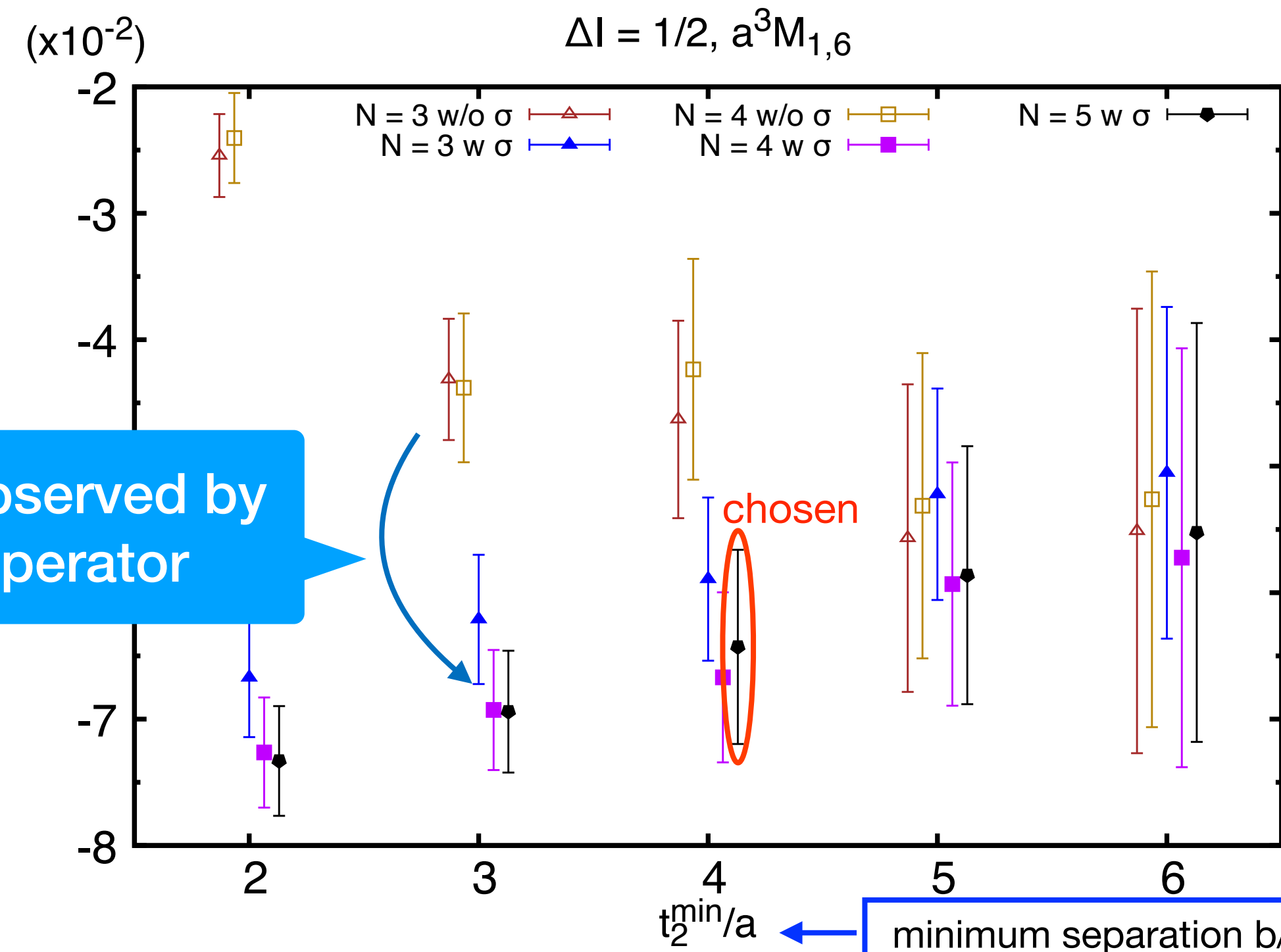
minimum separation b/w $\pi\pi$ operator and Q_6

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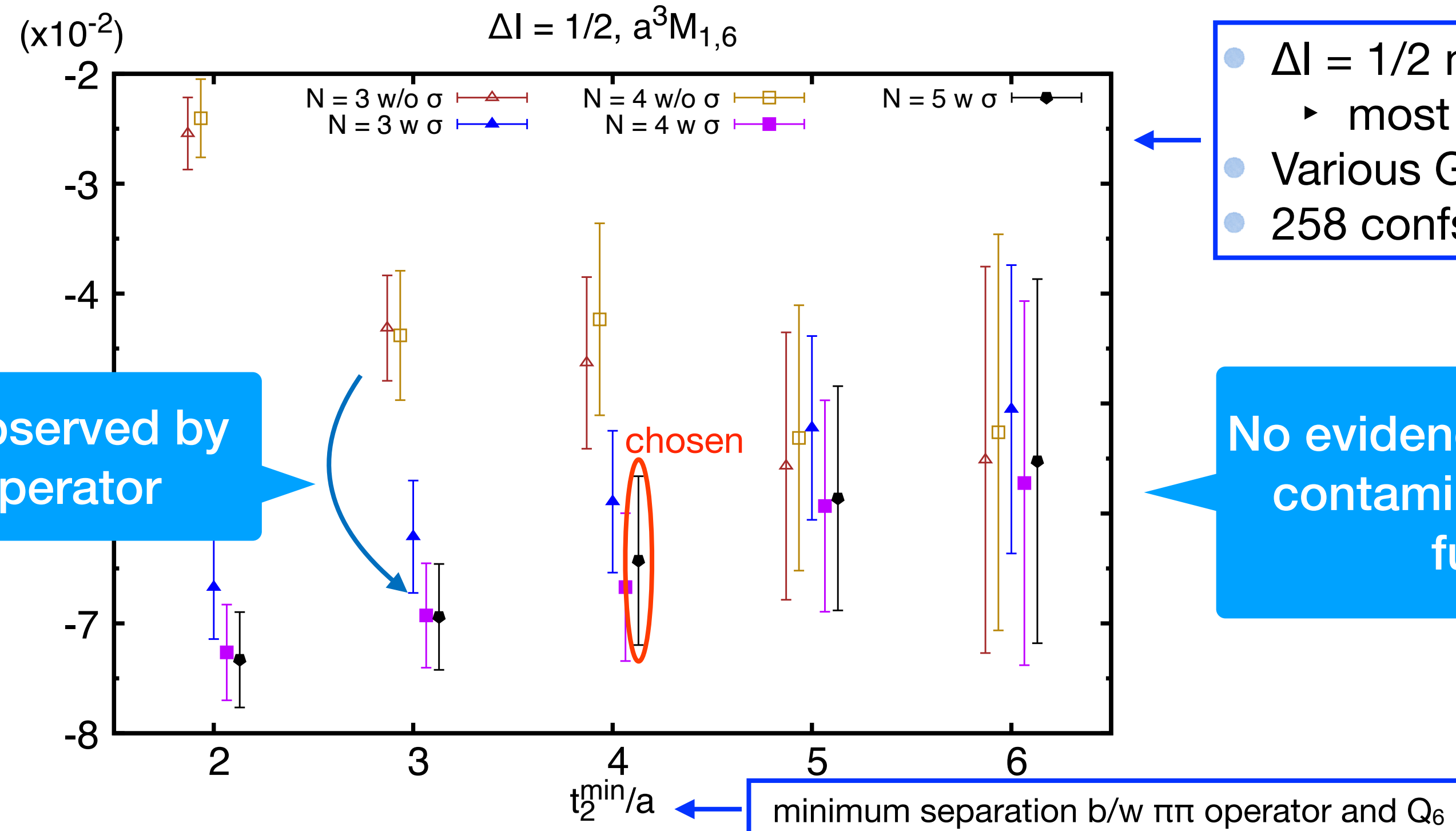
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No evidence of significant excited-state contamination w σ but any room for further improvement?

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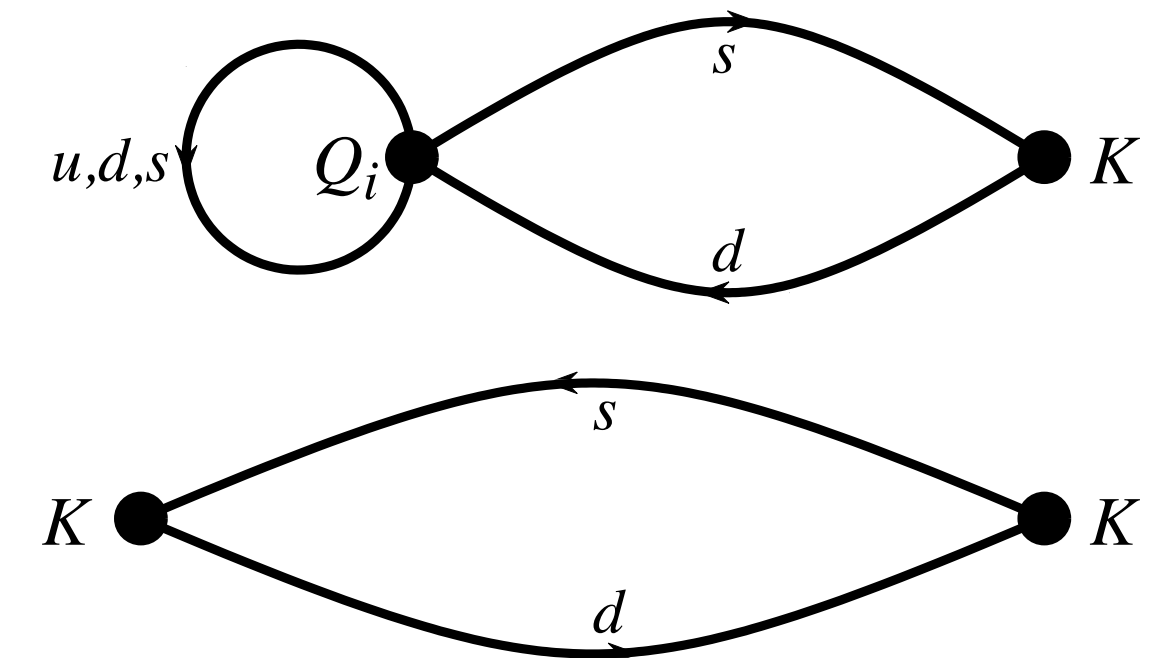


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Introducing a KK sink operator in 23–24 alloc year

- very cheap to implement (we have saved needed meson fields)
- can take care of potential contamination from $\langle KK | Q_i | K \rangle$
- give a few new types of diagrams



NPR

- Significant dependence on intermediate renormalization scheme & scale found
 - RI-SMOM(ϑ, ϑ) vs RI-SMOM(γ_μ, γ_μ) schemes
 - $\mu \approx 1.48a^{-1} \rightarrow$ big difference up to 15%
 - $\mu \approx 1.28a^{-1} \rightarrow$ 3–4%, ok but
 - $(1.48/1.28)^2 \neq 15\% / 4\%$ is concerning
- Plan to calculate with a few more intermediate scales to better understand in 23–24 alloc year
- Code is ready

Summary

- Goal: continuum limit with $\leq 10\%$ stat error on A_0
 - More statistics: additional measurements on 32^3 lattice to 500 confs
 - Better control of systematic errors – NPR
 - Improvement ideas (KK sink operator, ...)
- Request at JLab
 - 37 M KNL core hours
 - Carry forward current 200 TB disk
 - Additional 530 TB on short-term tape