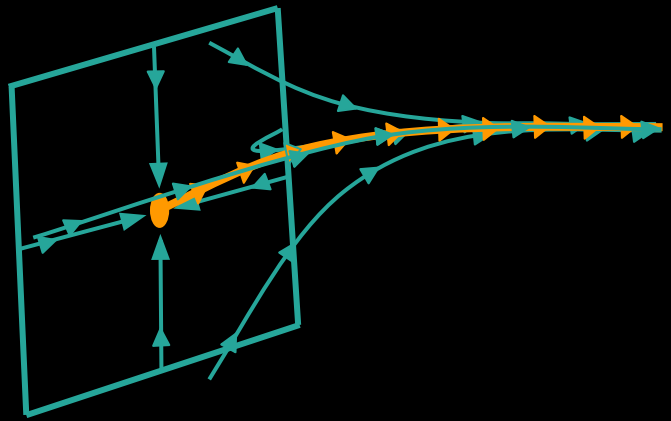
US Lattice Quantum Chromodynamics
All Hands Meeting 2023

Non-perturbative RG β-function of 8-flavor SU(3) gauge theory



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in collaboration with Anna Hasenfratz
[†]graduate student (CU Boulder)

Motivation: massless $N_f = 8$ & symmetric mass generation (SMG)

[Hasenfratz, A., Schaich, D., Veernala, A. *JHEP* **06** (2015) 143]
*[LatKMI *PRD* **96**, 014508 (2017)]
*[LSD Collaboration *PRD* **99**, 014509 (2019)]
*[Appelquist, T., Ingoldby, J., Piai, M. *PRD* **126**,191804 (2021)]

- SU(3) gauge-fermion system with 8 flavors popular BSM model
  - e.g., investigations by LatKMI*, LSD* and Appelquist et al.*

But that's not all…

- Evidence for chirally symmetric & confining phase (SMG)*
  - SU(3) gauge fields + 2 massless staggered fermions†
    - Even number of species needed to cancel $\mathbb{Z}_4$'t Hooft anomaly*
    - $\Longrightarrow$ Transition to SMG phase triggered by 4-fermion interaction?
  - May lead to chiral fermions

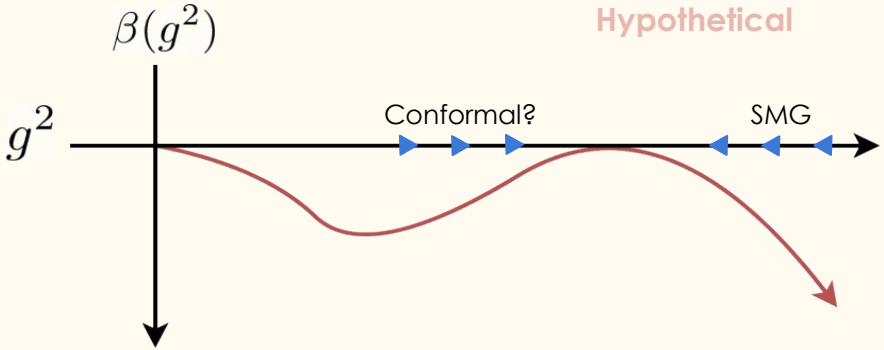
†2 Kähler-Dirac (8 continuum Dirac)

[Wang, .J., Wen, X.-G. *PRD* **99**, 111501 (2018)]
[Wang, J., You, Y.-Z. *Symmetry* **2022**, 14, 1475]
*[Catterall, S. *PRD* **107**, 014501 (2023)]
*[Hasenfratz, A. *PRD* **106**, 014513 (2022)]

Background: Pauli-Villars (PV) improvement

[Hasenfratz, A., Shamir, Y., Svetitsky, B. *PRD* **104**, 074509 (2021)]
*[Hasenfratz, A. *PRD* **106**, 014513 (2022)]

- Previous studies did not reach strong coupling
  - Bulk 1st-order phase transition at finite $\beta_b = 6/g_0^2$
  - Triggered by strong UV fluctuations
- Pauli-Villars (PV) bosons suppress UV fluctuations
  - Push phase transition to larger g_0^2
  - Transition becomes continuous for stag. $N_f = 8$
- Finite-size scaling* suggests renormalization group (RG) β-function just touches zero



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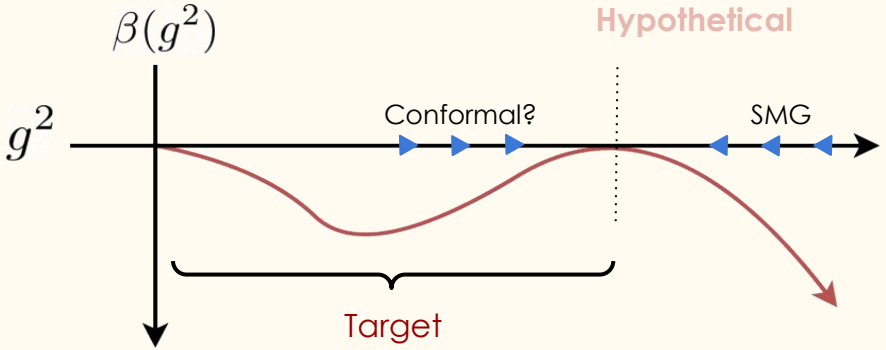
*[Hasenfratz, A. *PRD* **106**, 014513 (2022)]

- Previous studies limited by reach in strong coupling
  - ➢ Bulk 1st-order phase transition at finite $\beta_b = 6/g_0^2$
  - ➢
- P
  - ➢ $= 8$
- Fi (RG) β-function just touches zero

Goal

Calculate the RG β-function from "weak coupling" side of phase transition

Aided by PV improvement



Simulation details

*[Osborn, J., Jin, X.Y. *PoS* 256 (2016)]
[Hasenfratz, A., Shamir, Y., Svetitsky, B. *PRD* **104**, 074509 (2021)]
*[Hasenfratz, A., Neil, E., Shamir., Y, Svetitsky, B., Witzel, O. *In Preparation* (2023)]

- nHYP-smeared staggered fermions & adjoint-plaquette gauge action
  - Pauli-Villars (PV) improvement
  - Use MILC* & Quantum EXpressions* (QEX)
  - Symmetric volumes ($L/a = 32, 36, 40$)
    - (Anti-“”)periodic BC’s for fermion(gauge)
  - $8.8 \leq \beta_b \equiv 6/g_0^2 \leq 9.9$ (7 total)
- Gauge flows (GF) run with MILC & QEX
  - Run Wilson flow & modified rectangle flow*
  - Measure Wilson & clover operator

Pauli-Villars action

4 degenerate PV/fermion with
$am_{\mathrm{PV}} = 0.75$

Flow action

$\mathcal{S}_{\mathrm{rect.}} = (1 - 8c_1)\mathcal{S}_{\mathrm{Wils.}} + c_1\mathcal{S}_{\mathrm{clov.}}$
e.g., $c_1 = -1/12 \rightarrow$ “Symanzik flow”
$c_1 = +1/12 \rightarrow$ “C13 flow”

*[David Schaich’s modified MILC code: github.com/daschaich/KS_nHYP_FA]
*[QEX main branch: github.com/jcosborn/qex]
*[Curtis Peterson’s fork of QEX: github.com/ctpeterson/qex]

Continuous β-function method (CBFM)

[Fodor, Z., Holland, K., Kuti, J., Mondal, S., Nogradi, D., *JHEP* (2014) 018]
*[Kuti, J., Fodor, Z., Holland, K., Wong, K. H. *PoS*, LATTICE2021 (**2021**) 321]
*[Hasenfratz, A., Peterson, C.T., Witzel, O., van Sickle, J., arXiv:2303.00704, *sub. to PRD*]

- Calculate GF β-function using the CBFM*
  1. $L/a \to \infty$ extrapolation of $g^2_{\rm GF}(t; g_0^2, L)$ and $\beta_{\rm GF}(t; g_0^2, L)$ at fixed t/a^2 & β_b
  2. $a^2/t \to 0$ extrapolation of $\beta_{\rm GF}(t; g_0^2)$ at fixed $g^2_{\rm GF}$

GF coupling and β-function in finite volume

$$g^2_{\rm GF}(t; L, g_0^2) \sim \langle t^2 E(t) \rangle^*$$

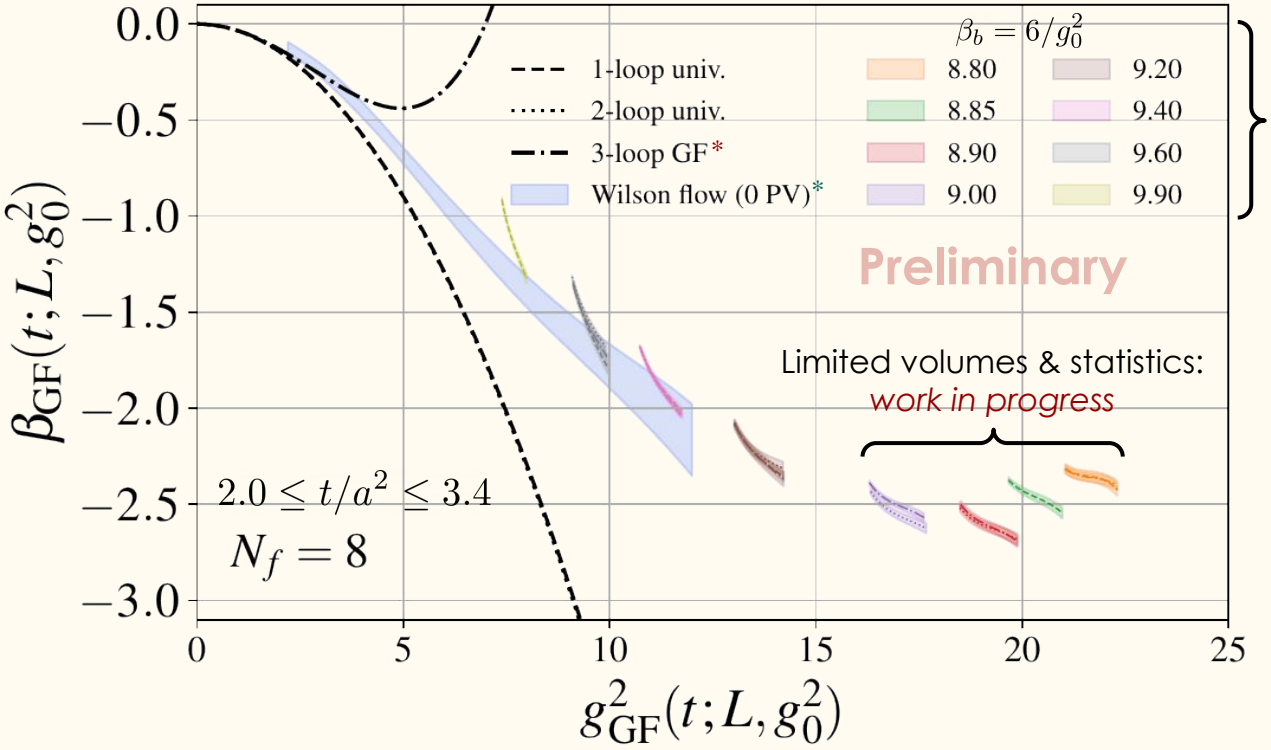
$$\beta_{\rm GF}(t; g_0^2, L) \equiv -t \frac{\rm d}{{\rm d}t} g^2_{\rm GF}(t; g_0^2, L)$$

*$E(t)$ is the Yang-Mills energy density; we consider Wilson & clover "operators"

details & results

Where we are

*[Hasenfratz, A., Schaich, D., Veernala, A. *JHEP* **06** (2015) 143]
*[Artz, Harlander, Lange, Neumann, Prausa *JHEP* **06** (2019) 121]



Raw data (no extrapolations) at each β_b (different colors) on $L/a = 32, 36, 40^\dagger$

Blue band is continuum prediction from *old dataset**

Black lines are prediction from perturbation theory*

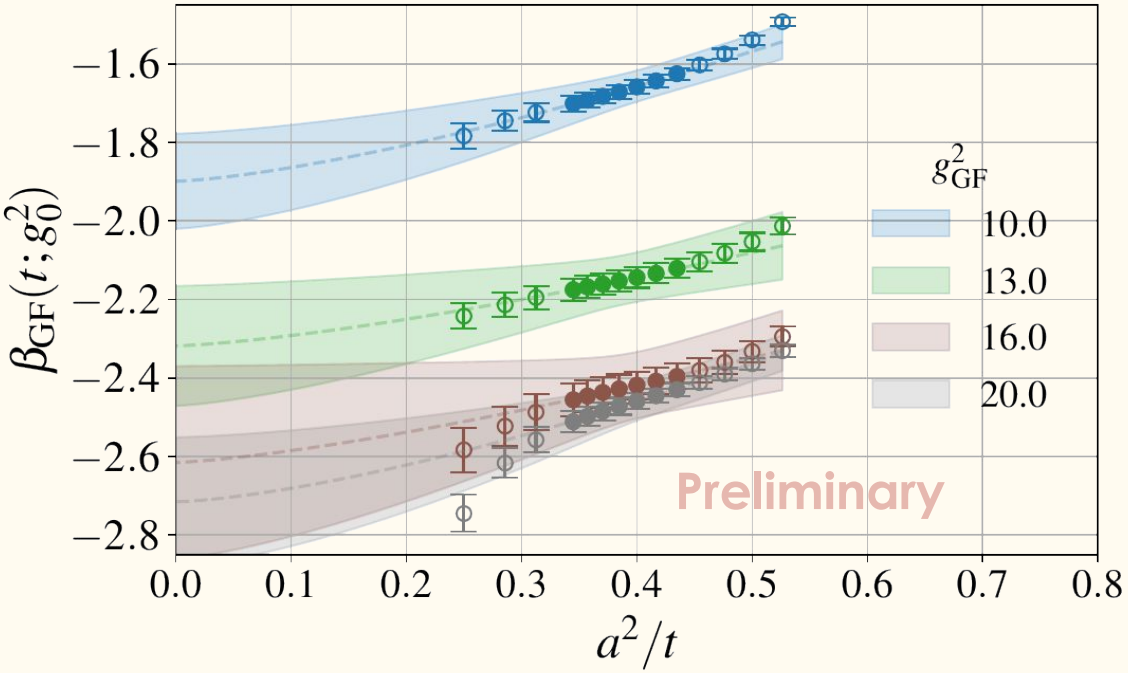
†(colored) dash = 32, dot = 36, dash-dot = 40

details & results

Continuum extrapolation $a^2/t \to 0$

Leading discretization effects*:
$\beta_{\mathrm{GF},\mathcal{O}}(t; g_0^2) \approx \beta_{\mathrm{GF}}(t) + C_{\mathcal{O}}(a^2/t)^{\zeta}$,
where $\zeta(g^2_{\mathrm{GF}}) \to 1$ as $g^2_{\mathrm{GF}} \to 0$

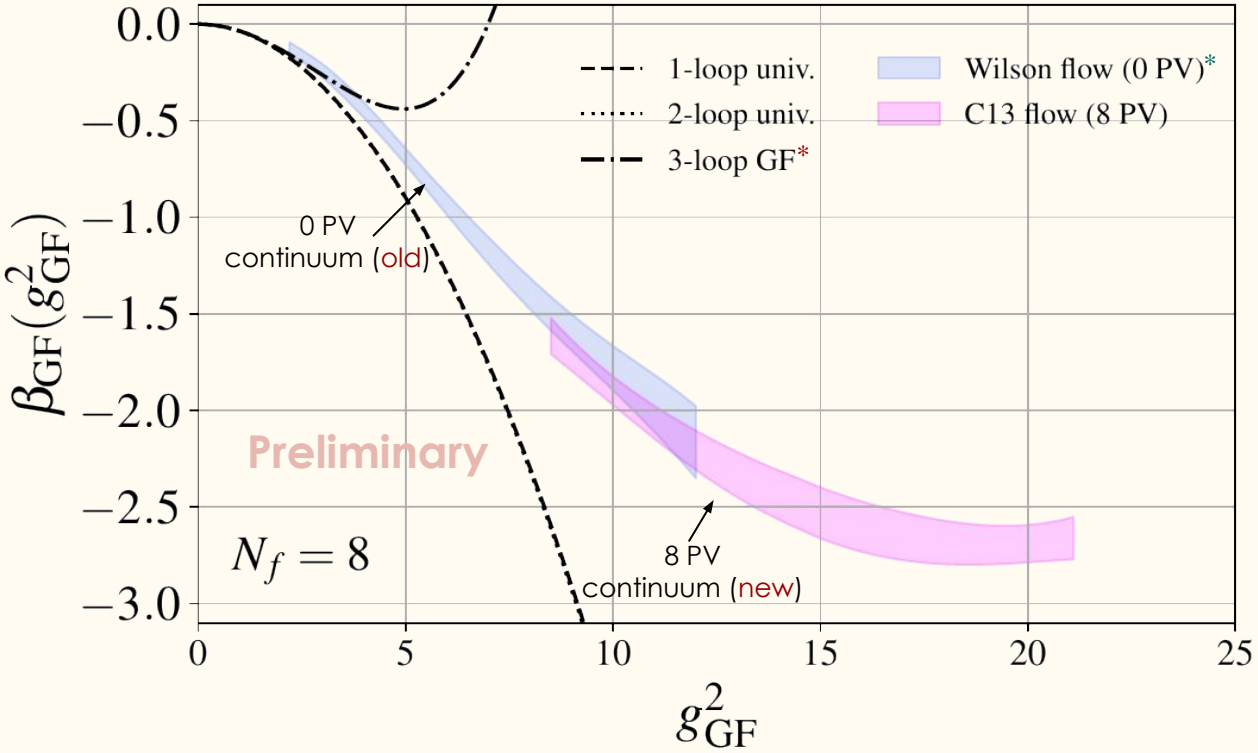
Determine $\beta_{\mathrm{GF}}(t)$ and ζ from joint fit to combinations of operators



*ζ = leading irrelevant exponent

Preliminary $N_f = 8$ continuum β-function

*[Hasenfratz, A., Schaich, D., Veernala, A. *JHEP* **06** (2015) 143]
*[Artz, Harlander, Lange, Neumann, Prausa *JHEP* **06** (2019) 121]



Weak coupling beginning to converge on old dataset.

Preliminary strong coupling showing signs of upward curvature in $g^2_{\rm GF}$

Summary & role of USQCD resources

*2 Kähler-Dirac (8 continuum Dirac)
*May consider small volume simulations in SMG phase

Motivation

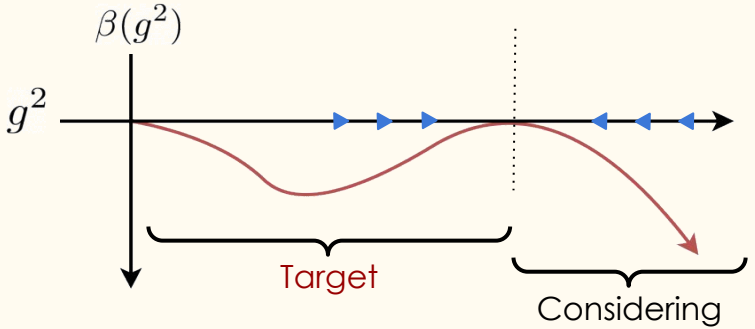
- 8-flavor SU(3) gauge-fermion system popular BSM model
- Evidence of SMG phase (potentially with continuum limit)
- Want to calculate RG β-function from "weak coupling"

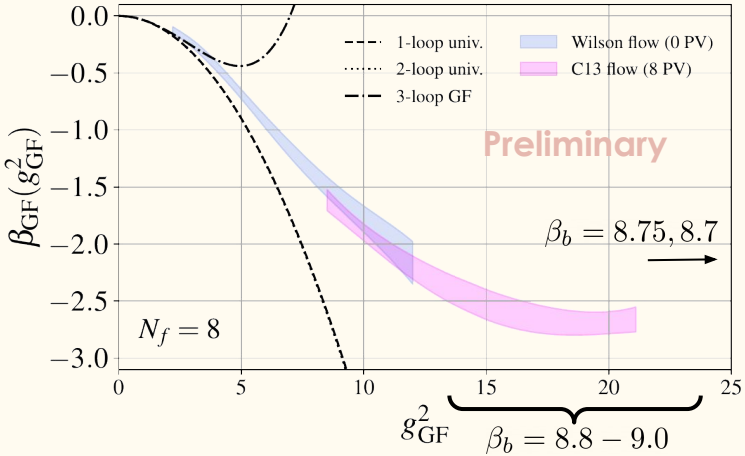
Preliminary results

- Signs of upward curvature in strong coupling β-function
  - ➢ Pushing to larger $g^2_{\rm GF}$ *may* reveal an IRFP

What we need*

- Request: **6.7 M Sky-core-hours** at FNAL
  - ➢ Continue $\beta_b = 8.8 - 9.0$ on $L/a = 32, 36, 40$
  - ➢ Thermalize/run $\beta_b = 8.75, 8.7$ on $L/a = 32, 36, 40$
- + **10 TB (13 k Sky-core-hours)** of tape storage





Acknowledgements

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US Lattice Quantum Chromodynamics

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curtistaylorpeterson.com

GF β-function

[Carosso, A., Hasenfratz, A., Neil, E. *PRL* **121**, 201601 (2018)]
[Lüscher, M., *JHEP* **08** (2010) 71]
[Makino, H., Morikawa, O., Suzuki, H. *PTEP* **05**, 099201 (2021)]

- GF describes a real-space RG transformation in infinite volume when combined with appropriately-defined coarse graining step

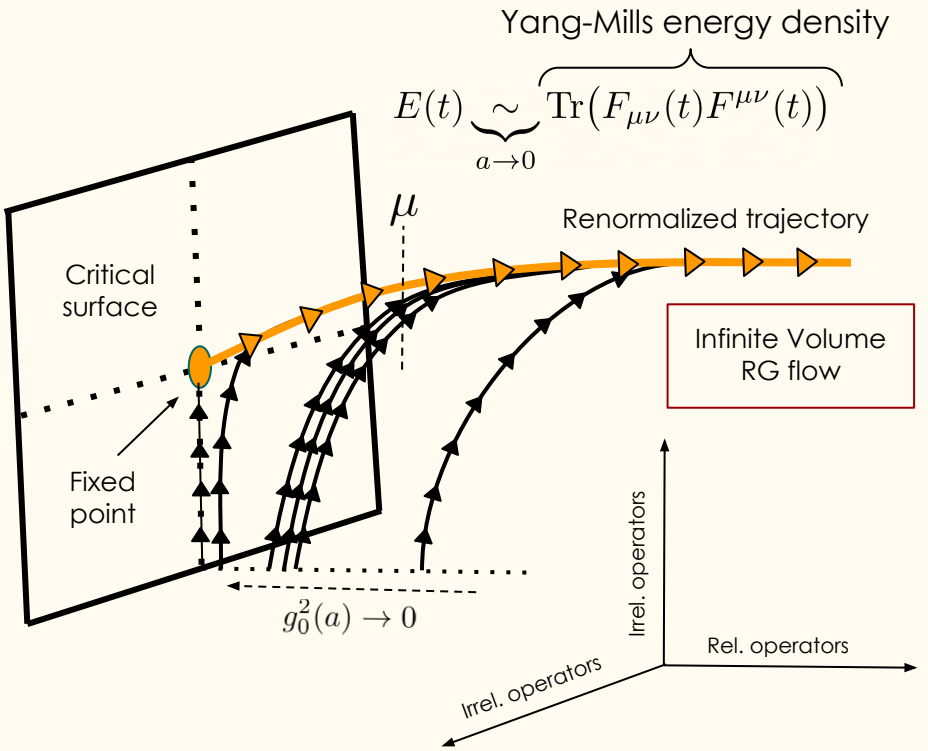
- Define a renormalized running coupling ($\mu^2 \propto 1/8t$)
  - Common choice in LGT studies is to use the flowed Yang-Mills energy density, since it does not renormalize*

$$g^2_{\rm GF}(t; g_0^2) \equiv \mathcal{N}\langle t^2 E(t)\rangle$$

  - Describes flow along renormalized trajectory with corresponding β-function

$$\beta_{\rm GF}(t; g_0^2) = -t\frac{\rm d}{{\rm d}t} g^2_{\rm GF}(t; g_0^2)$$

*$\mathcal{N} = 128\pi^2/3(N^2-1)$ chosen such that the GF coupling matches $\overline{\rm MS}$ at tree level



supplement