From the past to the future: α_s in e^+e^-

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2nd U.S. Future Circular Collider Workshop

March 26, 2024

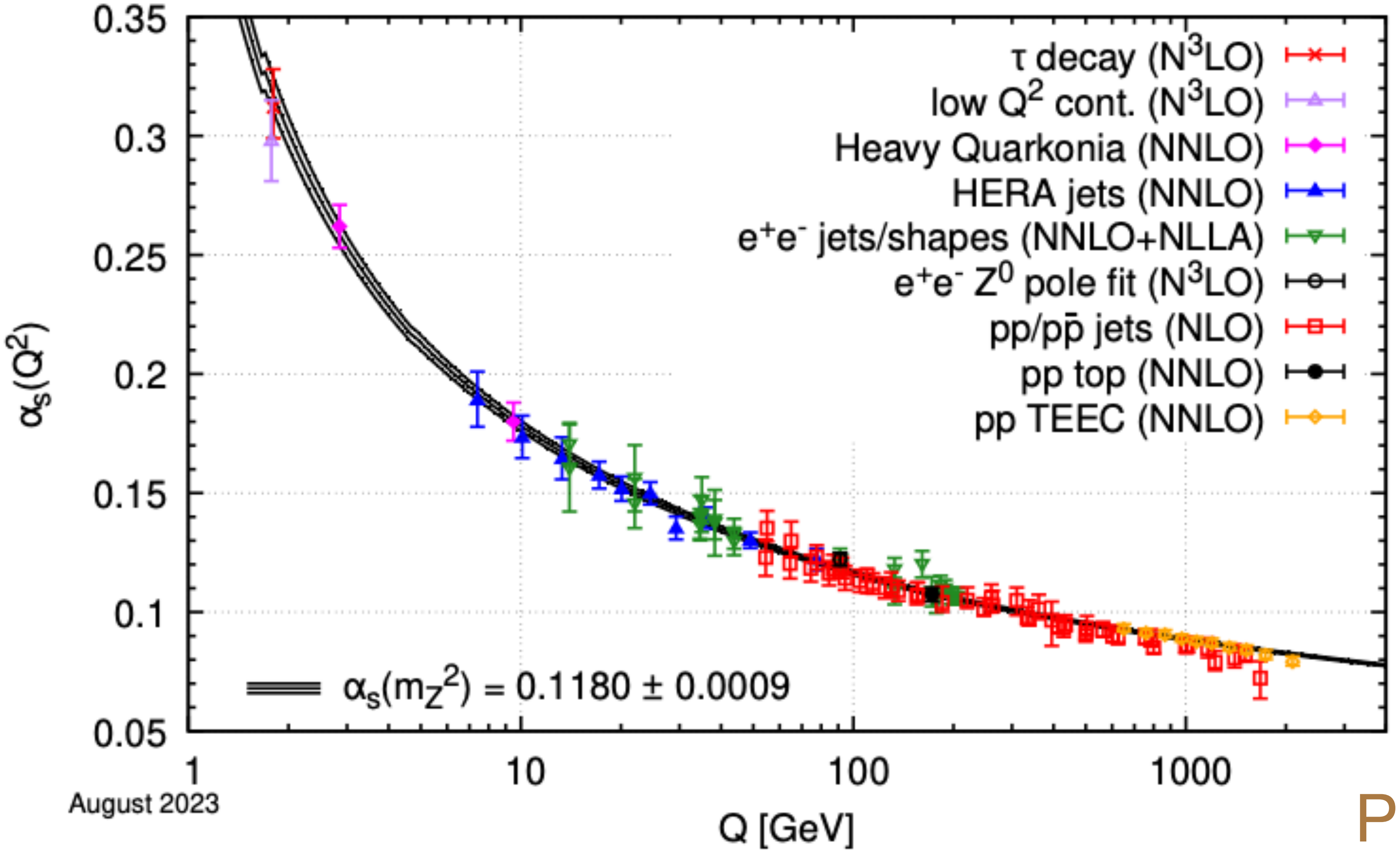
Massachusetts Institute of Technology

Outline

- α_s determinations: from LEP → today
- Post LEP theory advances for $e^+e^- \rightarrow$ jets
- A puzzle in using precision theory to analyze LEP
- Future prospects (opportunities at FCC-ee)

Importance of α_s

- Key parameter for collider physics (MC, FO, resum., ...)
- Important for determining other SM parameters (e.weak, m_t, ...)
- Plays a role in search for new physics ($b \to s\gamma$, coupling unif., ...)



PDG 2023

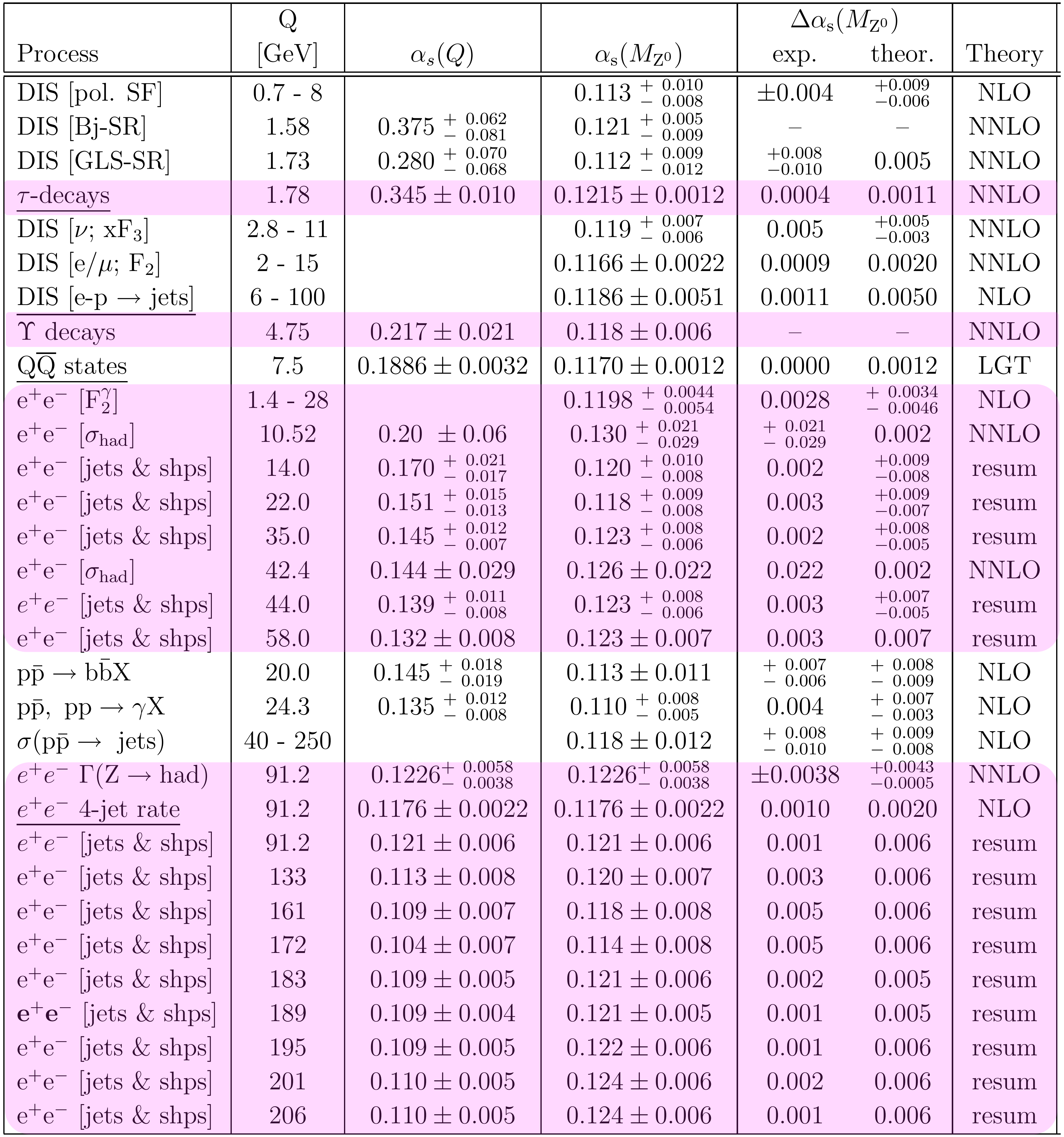
End of LEP era
Inputs for $\alpha_s(m_Z)$

$$\boxed{\alpha_s(m_Z) = 0.1170 \pm 0.0012}$$

PDG 2005

e^+e^- collider results

theory uncertainties dominated



Process	Q			$\Delta\alpha_s(M_{Z^0})$		
Process	[GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	exp.	theor.	Theory
DIS [pol. SF]	0.7 - 8		$0.113^{+0.010}_{-0.008}$	± 0.004	$^{+0.009}_{-0.006}$	NLO
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO
DIS [GLS-SR]	1.73	$0.280^{+0.070}_{-0.068}$	$0.112^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO
τ-decays	1.78	0.345 ± 0.010	0.1215 ± 0.0012	0.0004	0.0011	NNLO
DIS [ν; xF_3]	2.8 - 11		$0.119^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO
DIS [e/μ; F_2]	2 - 15		0.1166 ± 0.0022	0.0009	0.0020	NNLO
DIS [e-p → jets]	6 - 100		0.1186 ± 0.0051	0.0011	0.0050	NLO
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO
$Q\overline{Q}$ states	7.5	0.1886 ± 0.0032	0.1170 ± 0.0012	0.0000	0.0012	LGT
e^+e^- [F_2^γ]	1.4 - 28		$0.1198^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO
e^+e^- [jets & shps]	14.0	$0.170^{+0.021}_{-0.017}$	$0.120^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum
e^+e^- [jets & shps]	22.0	$0.151^{+0.015}_{-0.013}$	$0.118^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum
e^+e^- [jets & shps]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO
e^+e^- [jets & shps]	44.0	$0.139^{+0.011}_{-0.008}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum
e^+e^- [jets & shps]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
$p\bar{p} \to b\bar{b}X$	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO
$p\bar{p}$, $pp \to \gamma X$	24.3	$0.135^{+0.012}_{-0.008}$	$0.110^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO
$\sigma(p\bar{p} \to \text{jets})$	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO
e^+e^- $\Gamma(Z \to \text{had})$	91.2	$0.1226^{+0.0058}_{-0.0038}$	$0.1226^{+0.0058}_{-0.0038}$	± 0.0038	$^{+0.0043}_{-0.0005}$	NNLO
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1176 ± 0.0022	0.0010	0.0020	NLO
e^+e^- [jets & shps]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shps]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum
e^+e^- [jets & shps]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shps]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shps]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum
e^+e^- [jets & shps]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum
e^+e^- [jets & shps]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shps]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum
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DIS [e/μ; F$_2$]	2 -					
DIS [e-p $\rightarrow$ jets]	6 - 1					
Υ decays	4.7					
$\mathrm{Q\overline{Q}}$ states	7.					
e^+e^- [F$_2^\gamma$]	1.4 -					
e^+e^- [$\sigma_{\rm had}$]	10.					
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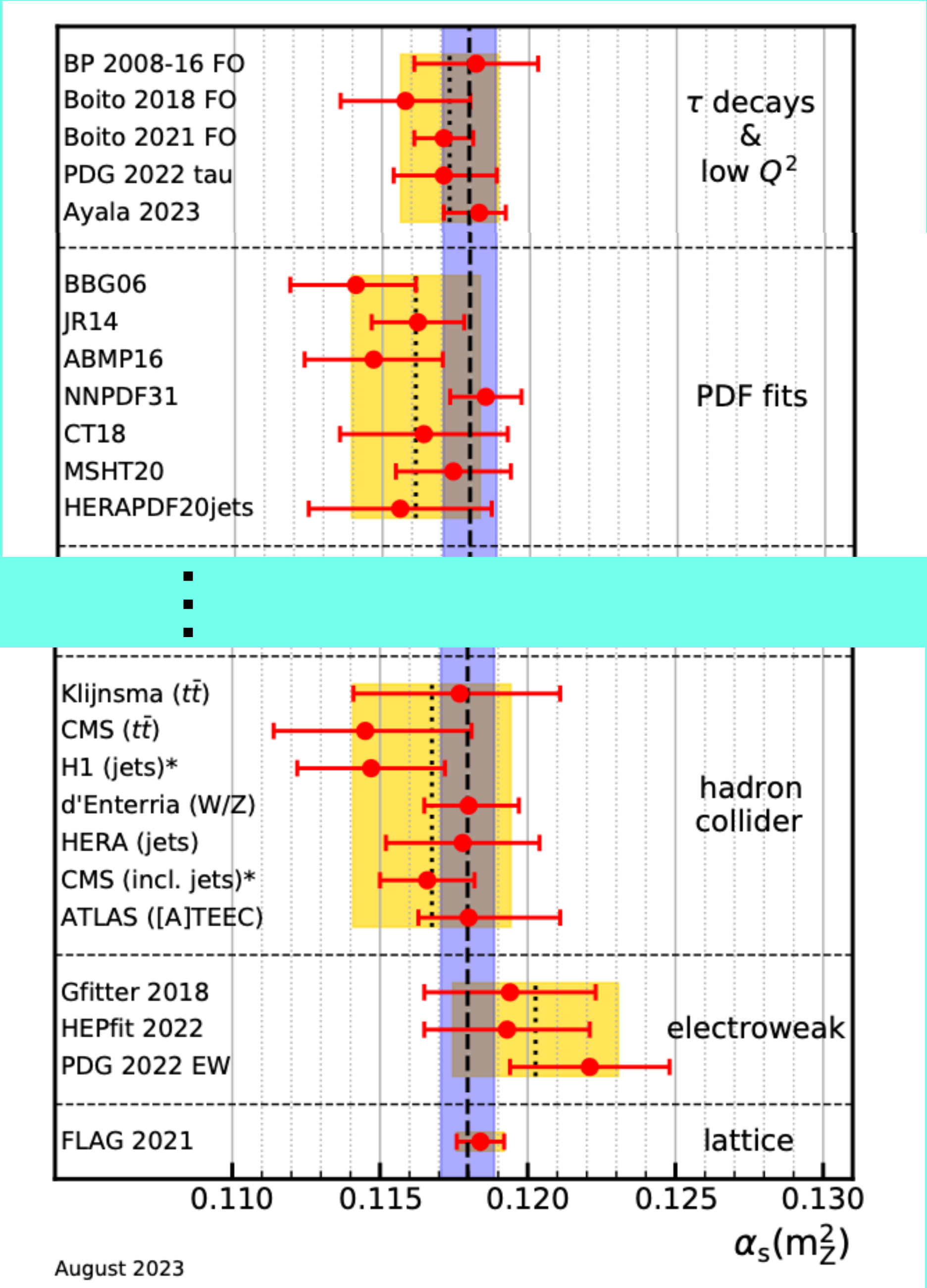
PDG 2023

$$\alpha_s(m_Z) = 0.1180 \pm 0.0009$$



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How has e^+e^- theory progressed?

τ

electroweak

jets*

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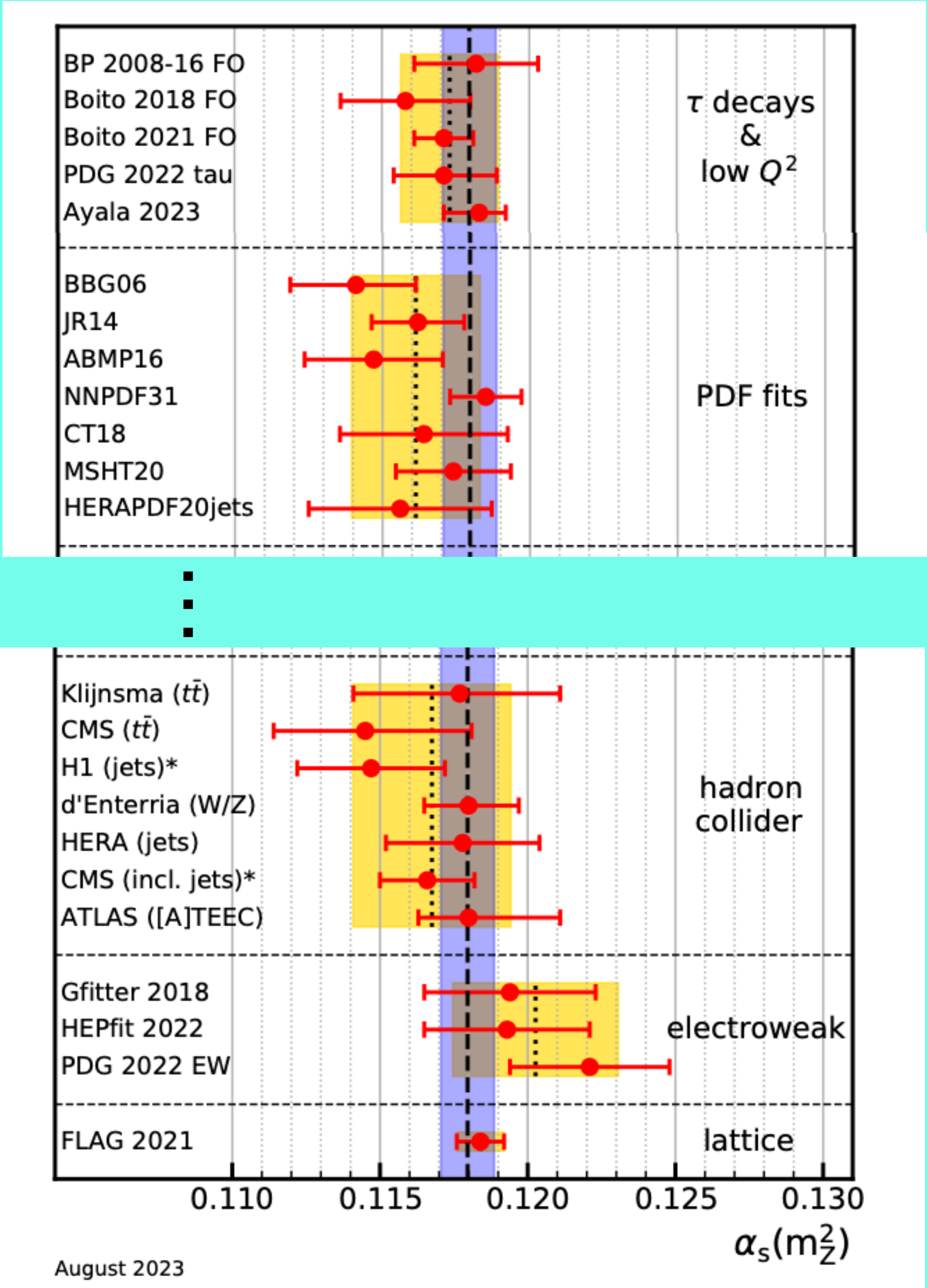

$(e^+e^- \to q\bar{q}g, \ldots)$

Jets are directly sensitive to α_s

$$e^+e^- \xrightarrow{Q} \text{jets}$$

Probed by event shapes (thrust, heavy-jet mass, C, ...)

$$\tau = \sum_i \tau_i$$

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In the period 2005-2015 the theoretical description of event shapes $d\sigma/d\tau$, got boosted

- Fixed-order predictions @ α_s^3 `07-`09 (Gehrmann-De Ridder et.al.; Weinzierl)
- Resummation at N^{3}LL (SCET) `08-15 (Becher, Schwartz, Chien, Hoang, Mateu, IS, ...)
- Dijet power corrections with operators `07 (Lee, Sterman)
- Renormalon free power correction scheme `08 (Hoang, IS)

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$e^+e^- \xrightarrow{Q}$ jets

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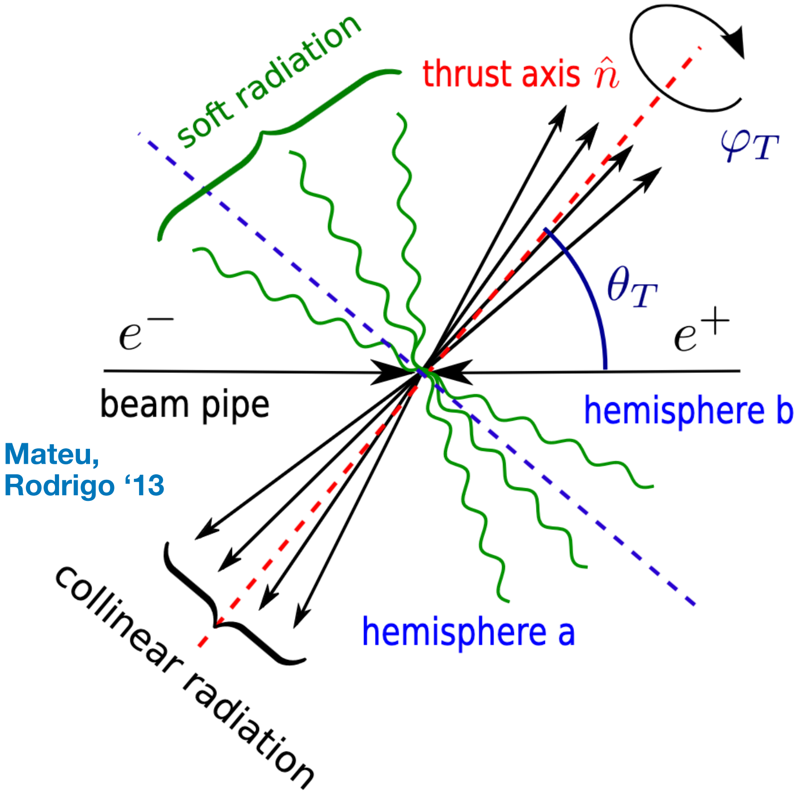
This motivated carrying out Global Fits to e^+e^- data at various c.m.(Q) to obtain $\alpha_s(m_Z)$

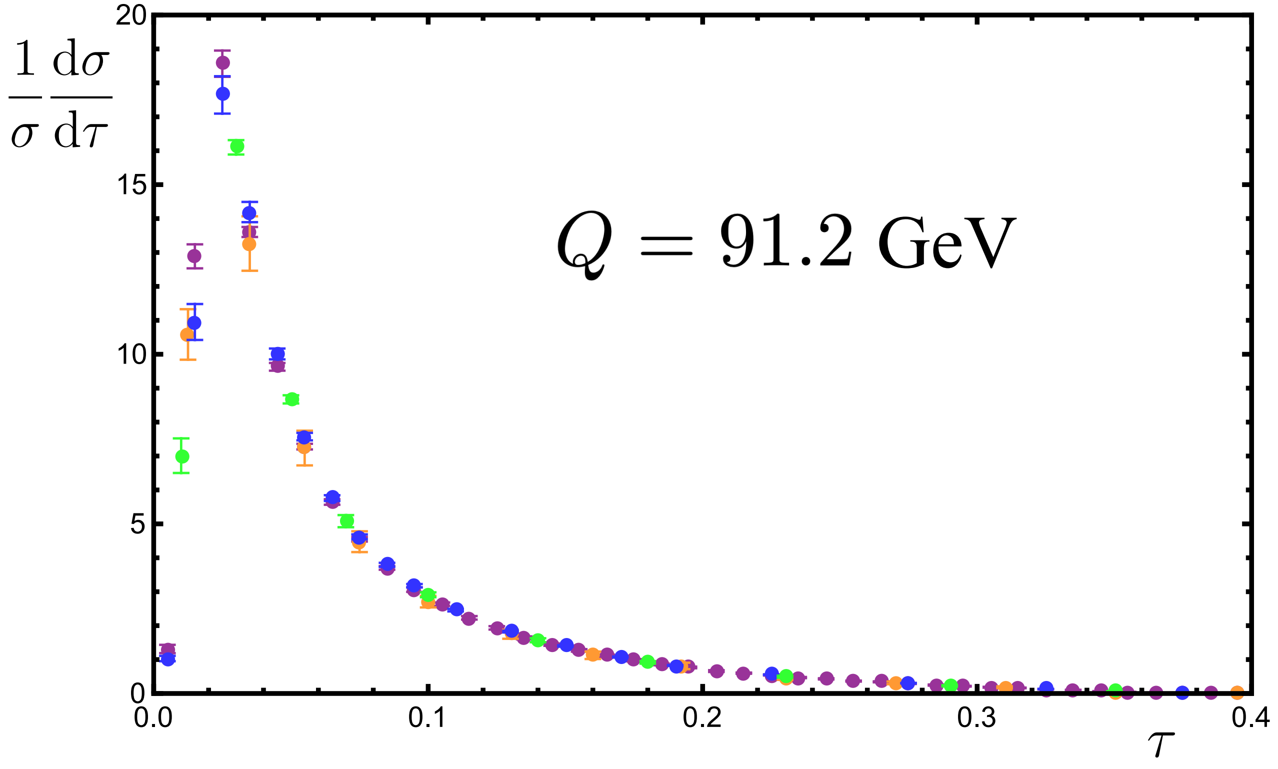
- Thrust (Abbate,Fickinger, Hoang, Mateu, IS 2011; Gehrmann, Luisoni, Monni 2013)
- Thrust moments (Abbate et.al. 2012)
- C-parameter (Hoang, Kolodrubetz, Mateu, IS 2015)
- Moments to many event-shapes (Gehrmann, Jaquier, Luisoni 2010)

Experimental data (eg. thrust fit)



Mateu, Rodrigo '13

	Experiment	Values of the center of mass energy Q
LEP	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
	DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
	OPAL	{91.0, 133.0, 177.0, 197.0}
	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLAC	SLD	{91.2}
DESY	TASSO	{14.0, 22.0, 35.0, 44.0}
	JADE	{35.0, 44.0}
KEK	AMY	{55.2}



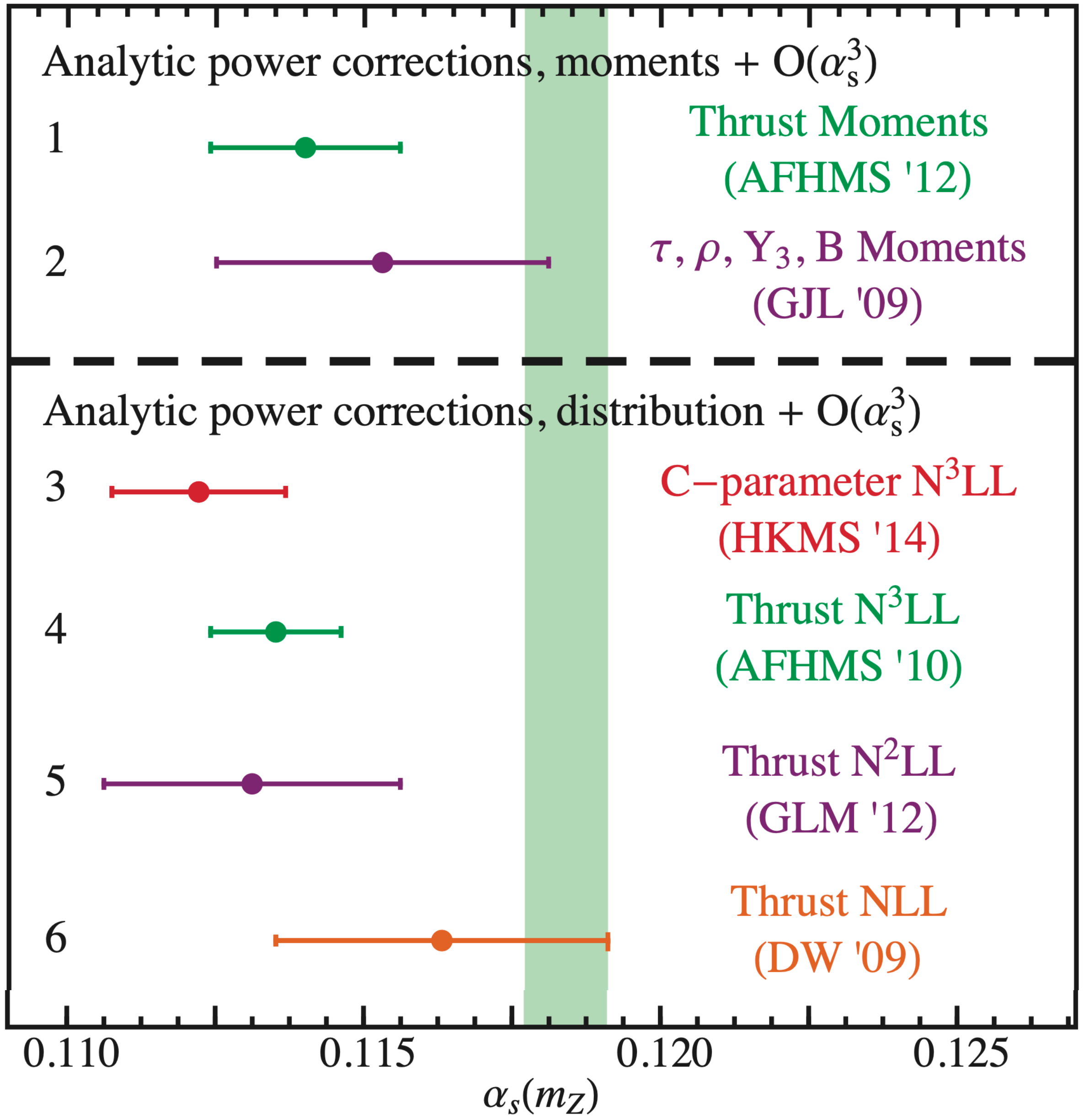
Standard dataset: $35\text{ GeV} \leq Q$

$$\frac{6\text{ GeV}}{Q} \leq \tau \leq 0.33$$

487 bins

Correlations treated in minimal overlap model

Precision e^+e^- Jet Puzzle



Although α_s was obtained with high accuracy, the central values are uncomfortably small (compared to world average)

Low values confirmed by 2023 analysis (Bell, Lee, Makris, Talbert, Yan)

Jets in e^+e^- with QFT

Systematically improvable!

Factorization theorems, resum logs, jet hadronization, combining regions, ...

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

$$\boxed{\frac{\Delta\alpha_s}{\alpha_s} \sim 0.5\%}$$

b mass corrections

nonperturbative soft function , $\left(\frac{\Omega_1}{Q\tau} \right)$

$$\frac{d\hat{\sigma}_s}{d\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l \tau}{\tau} \right]_+$$
$$= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S)$$

Singular partonic for massless quarks QCD+QED final states

$$\frac{d\hat{\sigma}_{ns}}{d\tau} = \sum_{n,l} \alpha_s^n \ln^l \tau + \sum_n \alpha_s^n f_n(\tau)$$

Nonsingular partonic

Resummation for singular partonic

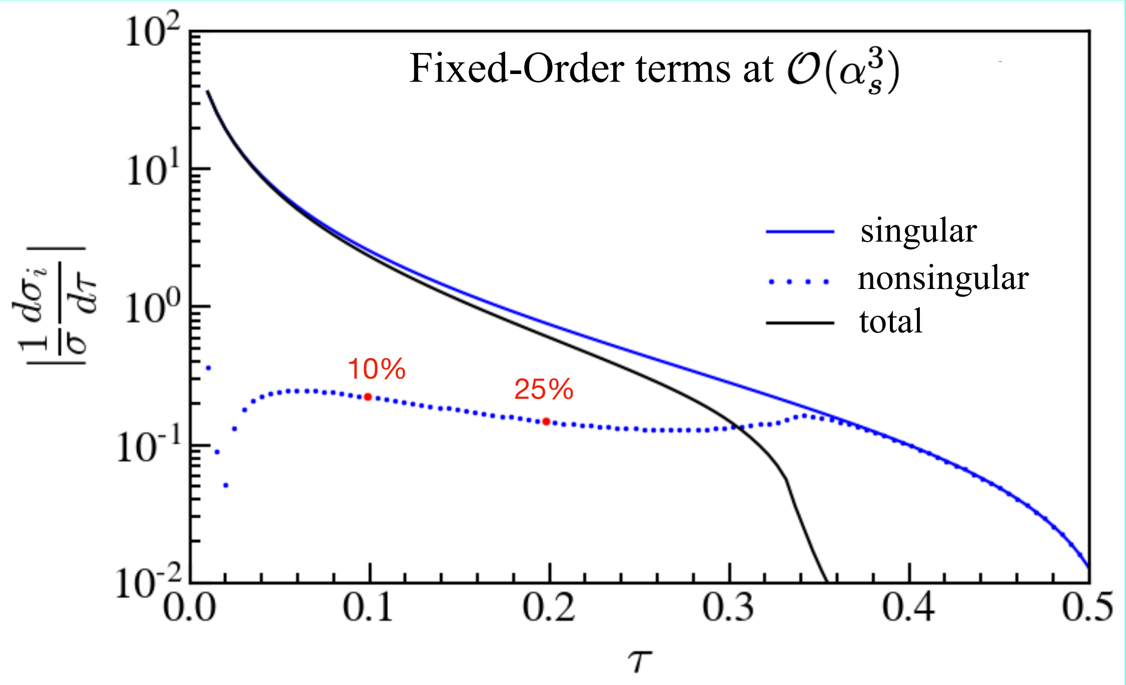
Single hadronization parameter Ω_1 when $\Lambda_{\rm QCD} \ll Q\tau$

$$S(\tau) = S_{\rm pert}(\tau) - S'_{\rm pert}(\tau) \frac{2\Omega_1}{Q} \approx S_{\rm pert}\left(\tau - \frac{2\Omega_1}{Q} \right) \qquad \Omega_1 \sim \Lambda_{QCD}$$

$$\bar{\Omega}_1 \equiv \frac{1}{2N_C} \langle 0 | {\rm tr} \bar{Y}_{\bar{n}}(0) Y_n(0)\, i\partial_\tau Y_n^\dagger(0) \bar{Y}_{\bar{n}}^\dagger(0) | 0 \rangle \qquad \overline{\rm MS}$$

Jets in e^+e^- with QFT

Singular terms dominate



Factorization theorems, res ons, ...

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat\sigma_{\mathrm{s}}}{d\tau} + \frac{d\hat\sigma_{\mathrm{ns}}}{d\tau} + \right.$$

$$\frac{d\hat\sigma_s}{d\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l \tau}{\tau}\right]_+$$

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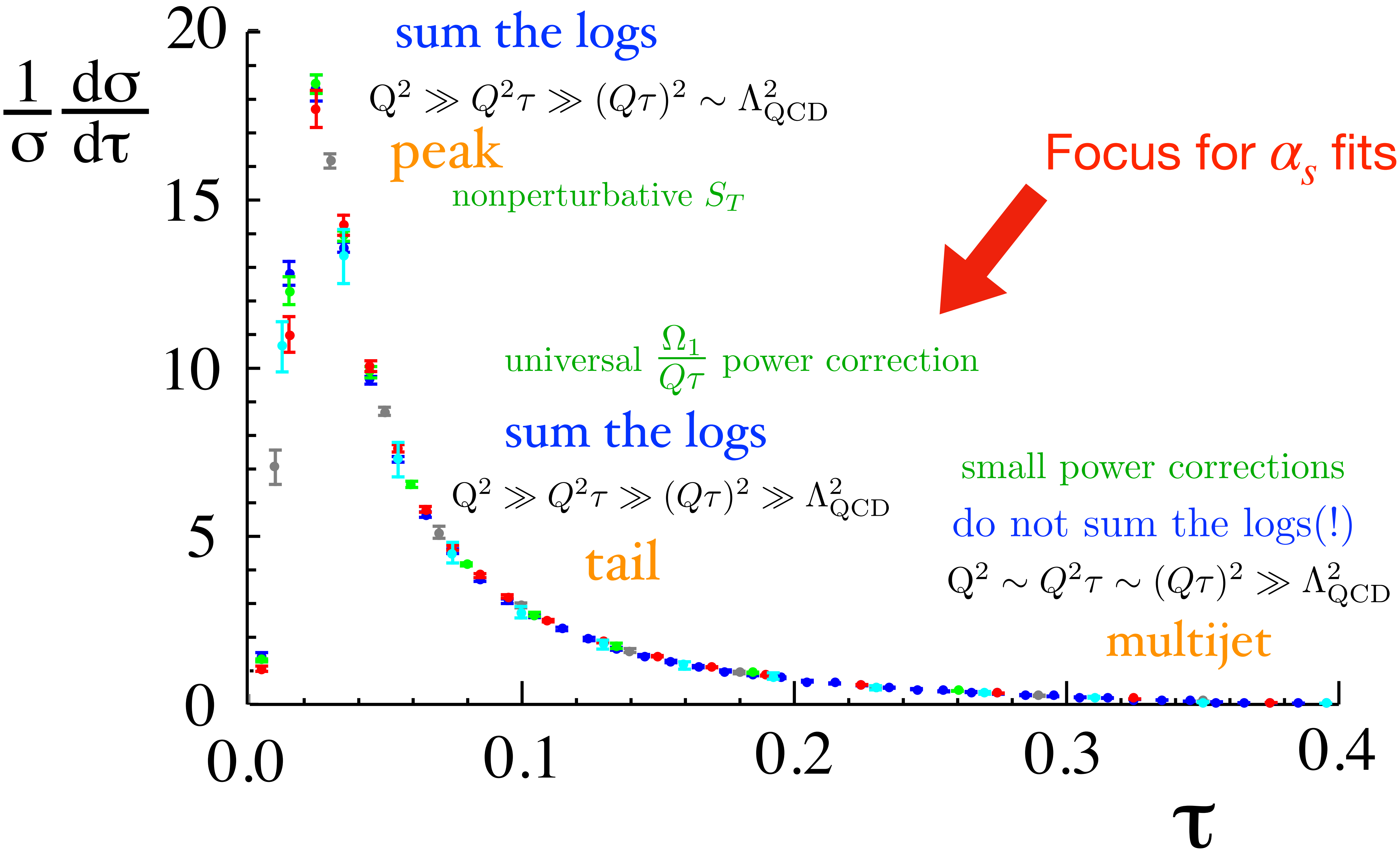
$$S(\tau) = S_{\mathrm{pert}}(\tau) - S'_{\mathrm{pert}}(\tau)\frac{2\Omega_1}{Q} \approx S_{\mathrm{pert}}\left(\tau - \frac{2\Omega_1}{Q}\right) \qquad \Omega_1 \sim \Lambda_{QCD}$$

$$\bar\Omega_1 \equiv \frac{1}{2N_C}\left\langle 0\left|\mathrm{tr}\,\overline{\mathrm{Y}}_{\bar n}(0)\mathrm{Y}_n(0)\, i\partial_\tau \mathrm{Y}_n^\dagger(0)\overline{\mathrm{Y}}_{\bar n}^\dagger(0)\right|0\right\rangle \qquad \overline{\mathrm{MS}}$$

Three Regions:



peak

sum the logs

$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \sim \Lambda^2_{\mathrm{QCD}}$

nonperturbative S_T

Focus for α_s fits

universal $\frac{\Omega_1}{Q\tau}$ power correction

tail

sum the logs

$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \gg \Lambda^2_{\mathrm{QCD}}$

multijet

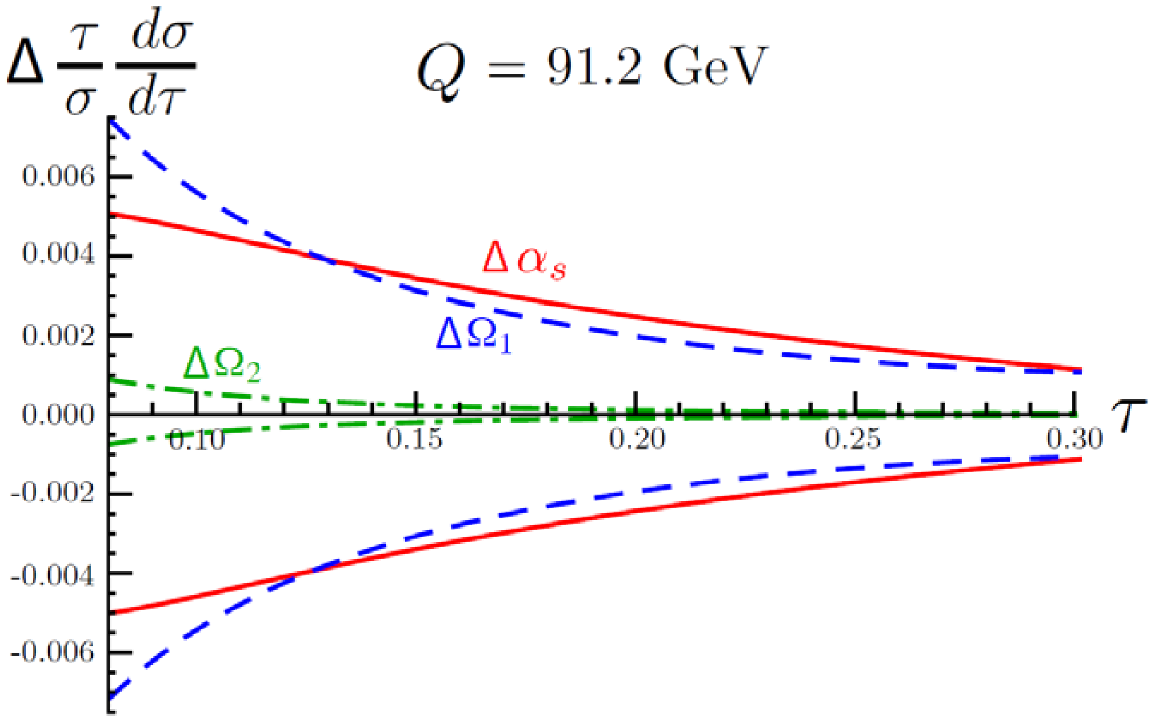
small power corrections

do not sum the logs(!)

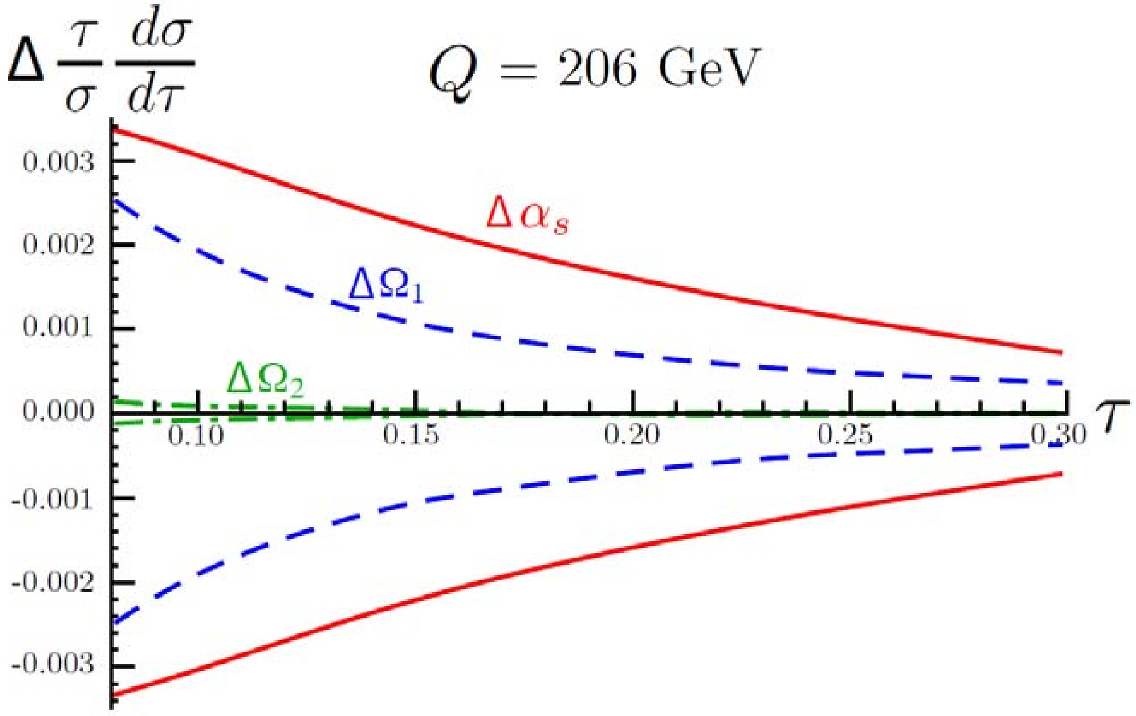
$Q^2 \sim Q^2\tau \sim (Q\tau)^2 \gg \Lambda^2_{\mathrm{QCD}}$

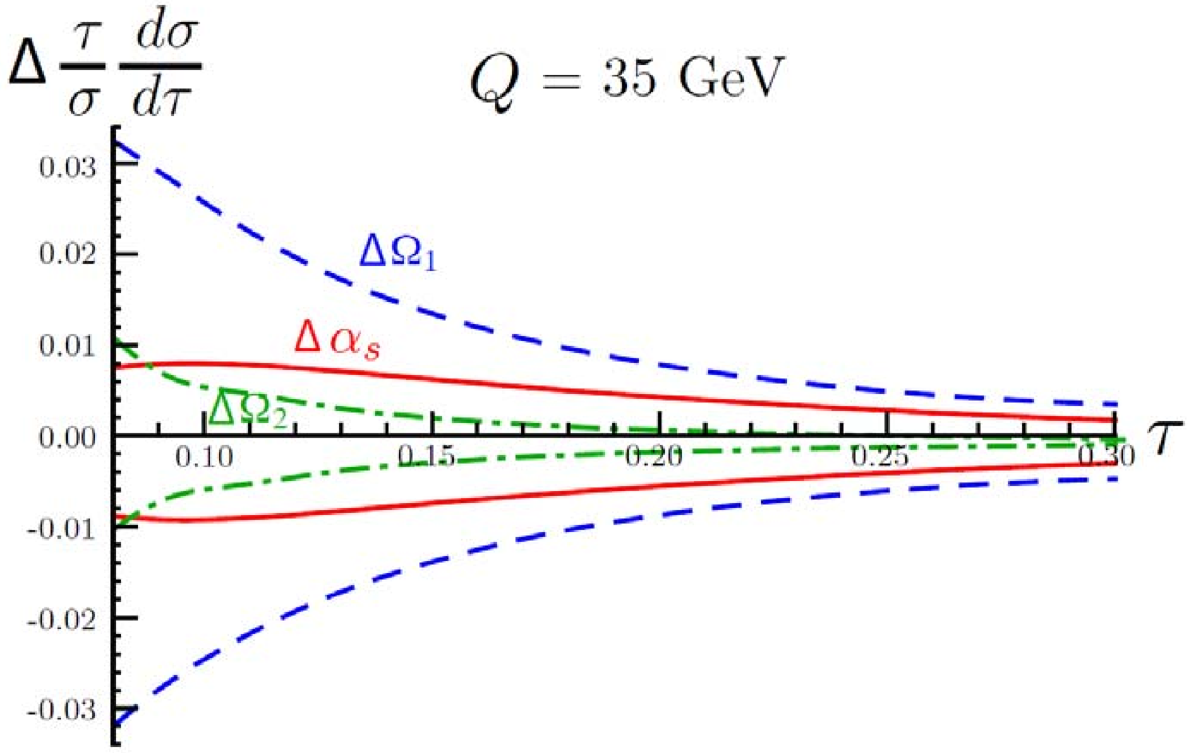
Two fit parameters $\{\alpha_s, \Omega_1\}$

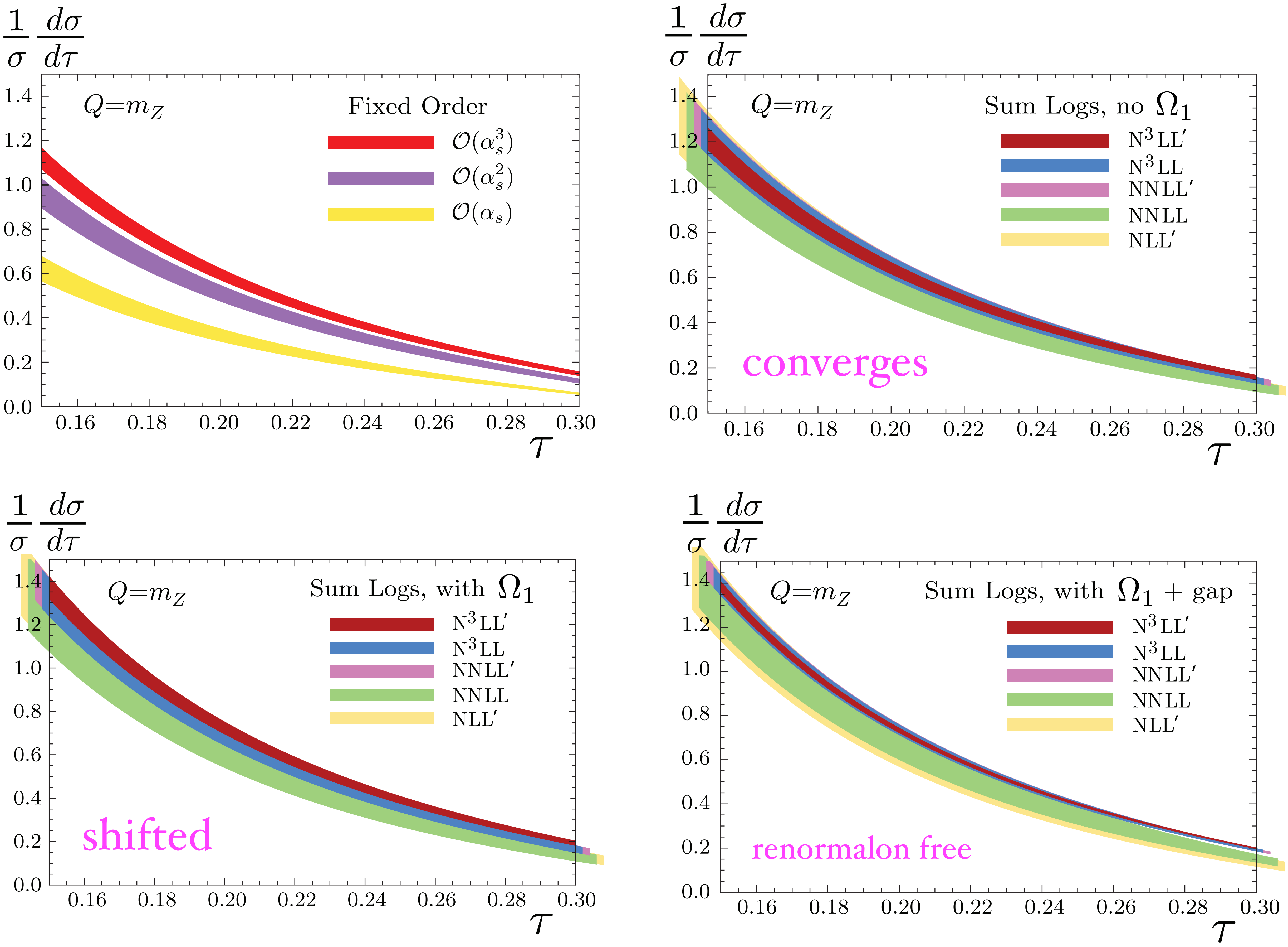
We fit for Ω_1 & $\alpha_s(m_Z)$ simultaneously. Strong degeneracy lifted by many Q's.



Power correction needed with 20% accuracy to get α_s at the 1% level



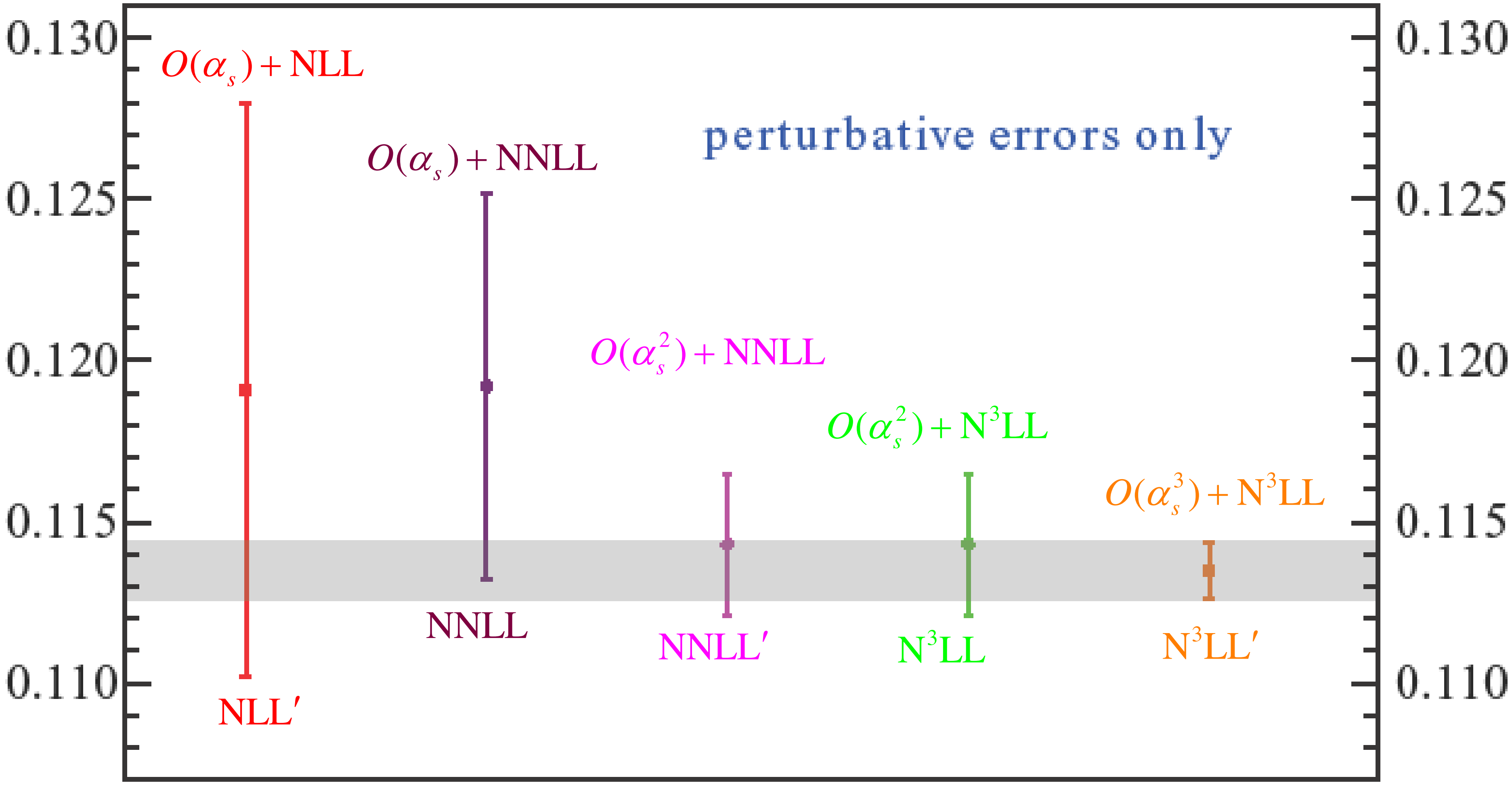


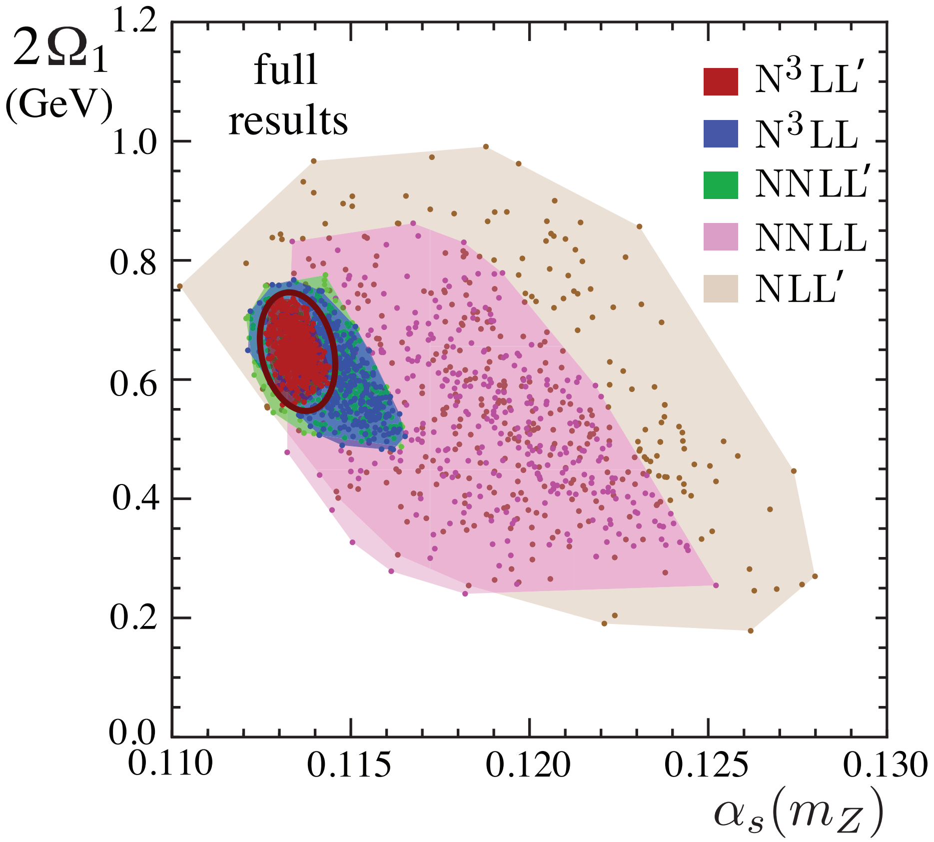


converges

shifted

renormalon free


0.130
0.125
0.120
0.115
0.110
$O(\alpha_s)$ + NLL
$O(\alpha_s)$ + NNLL
$O(\alpha_s^2)$ + NNLL
$O(\alpha_s^2)$ + N^{3}LL
$O(\alpha_s^3)$ + N^{3}LL
perturbative errors only
NLL′
NNLL
NNLL′
N^{3}LL
N^{3}LL′


$2\Omega_1$ (GeV)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
full results
N^{3}LL′
N^{3}LL
NNLL′
NNLL
NLL′
0.110
0.115
0.120
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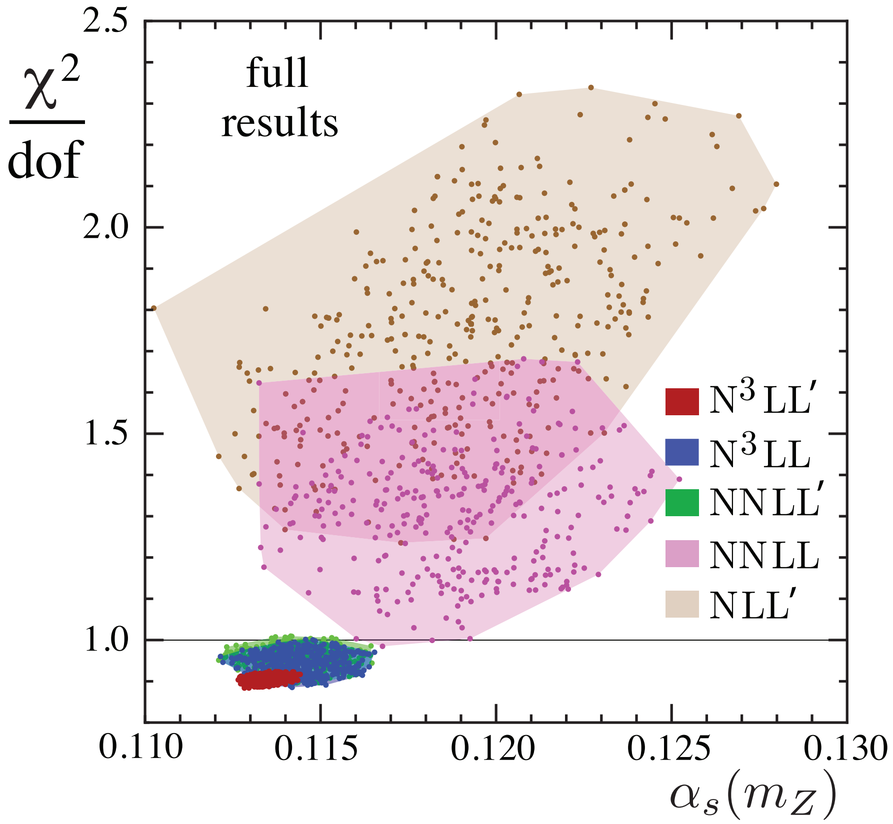
0.130
$\alpha_s(m_Z)$


$\frac{\chi^2}{\text{dof}}$
2.5
2.0
1.5
1.0
full results
N^{3}LL′
N^{3}LL
NNLL′
NNLL
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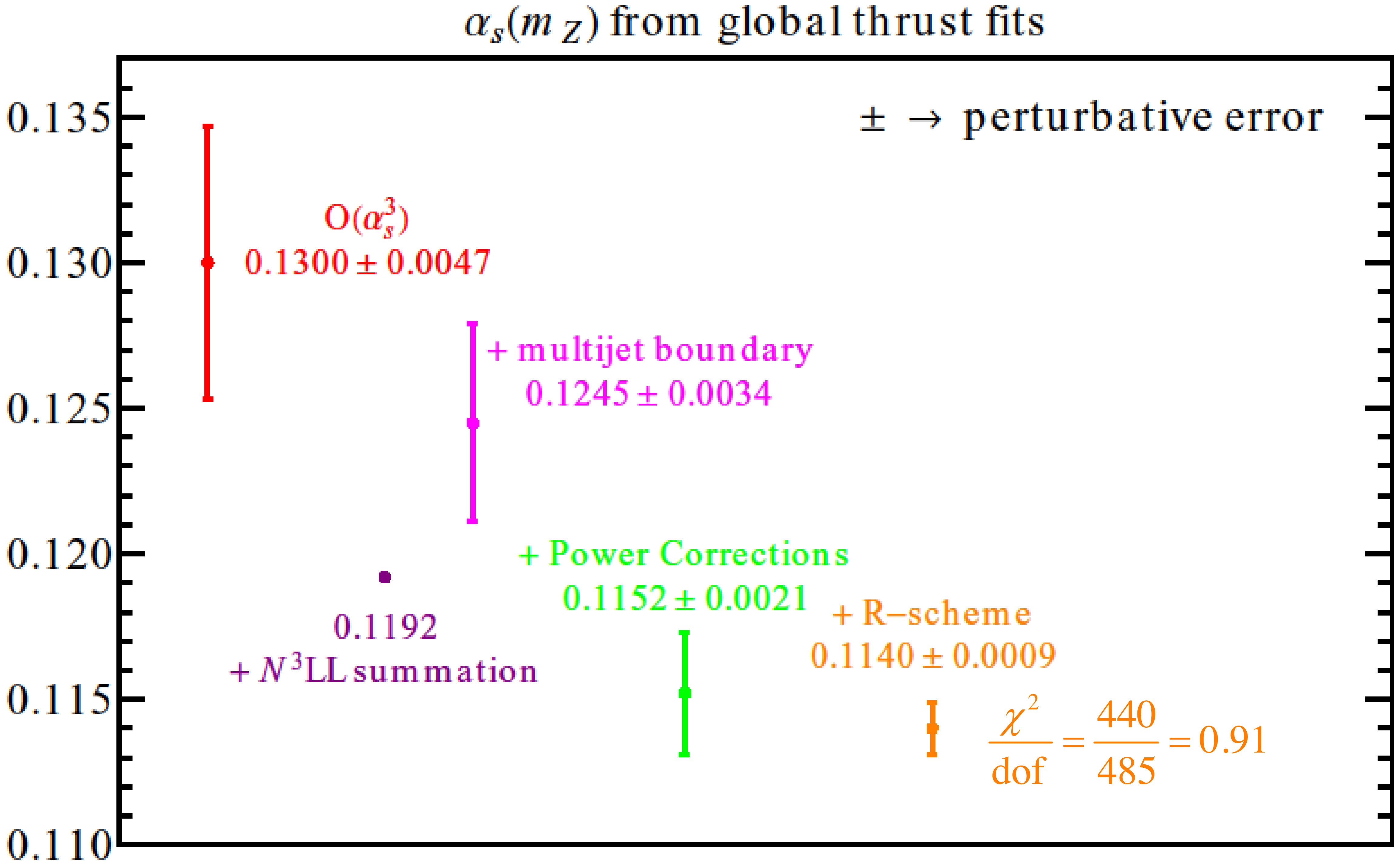
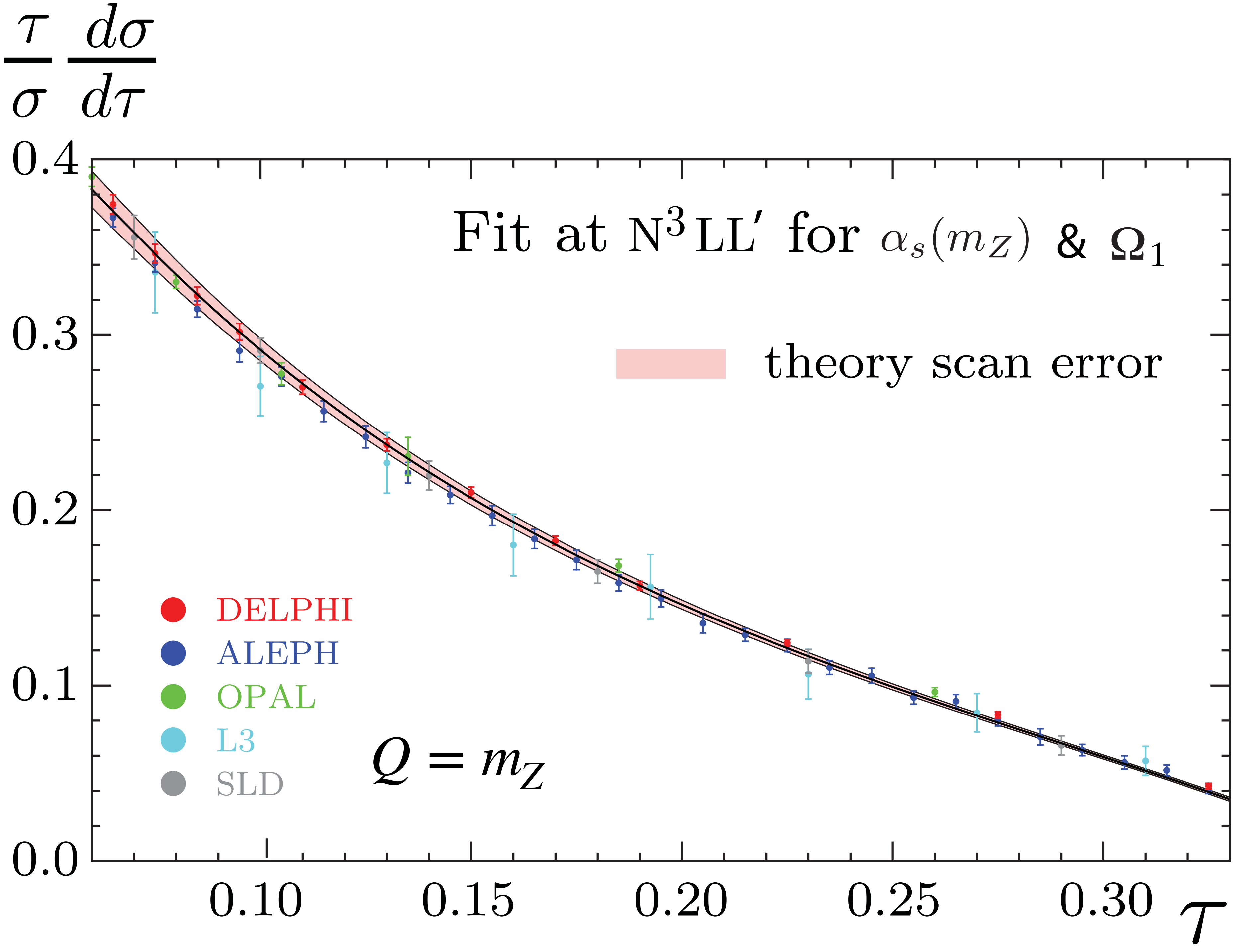


Power correction has expected impact: $\frac{\delta\alpha_s}{\alpha_s} \approx -(9 \pm 3)\ \%$



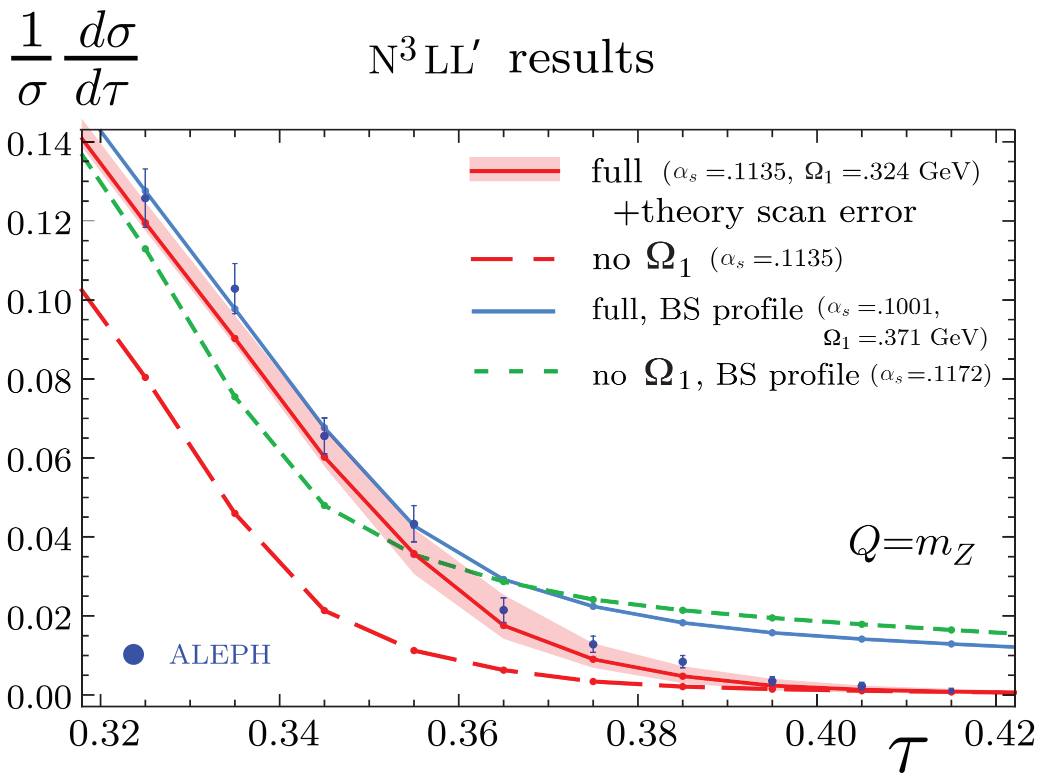
\+ agreement for many other Q's

Consistency checks

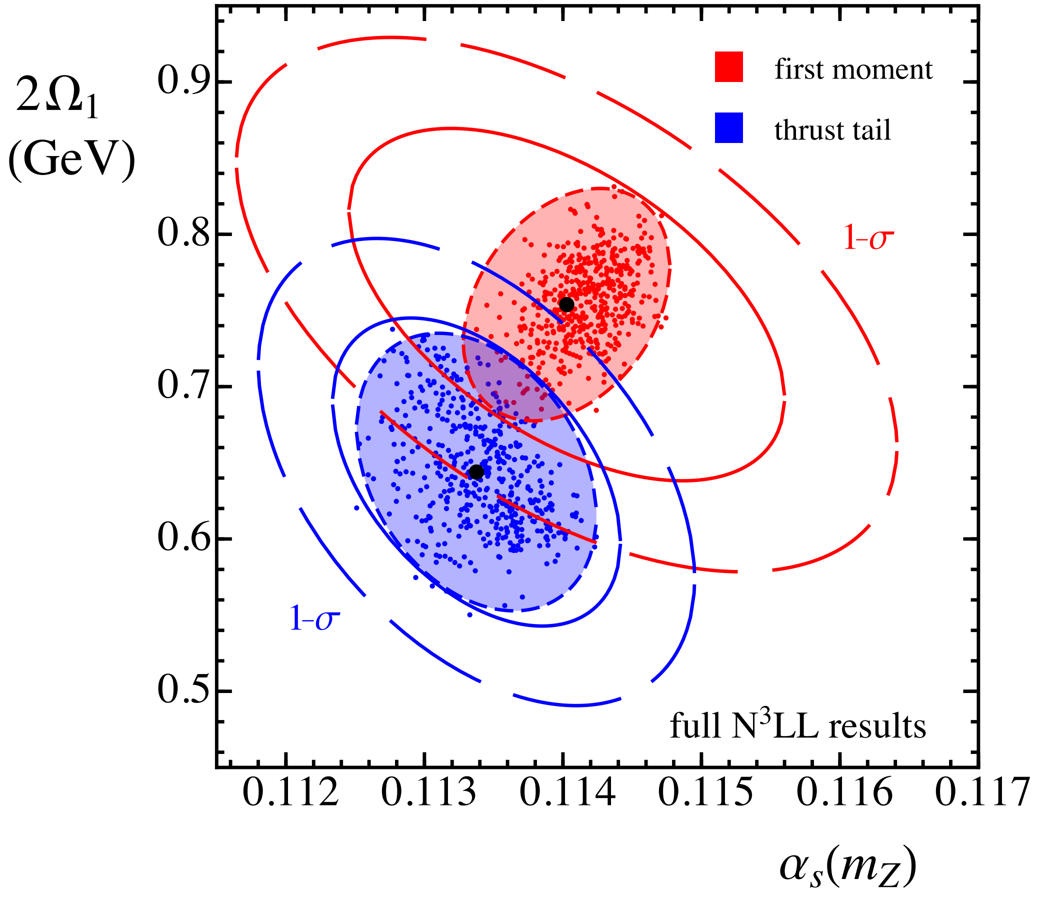
QED & b-mass effects small

$$\Delta\alpha_s(m_Z) = -0.0005$$

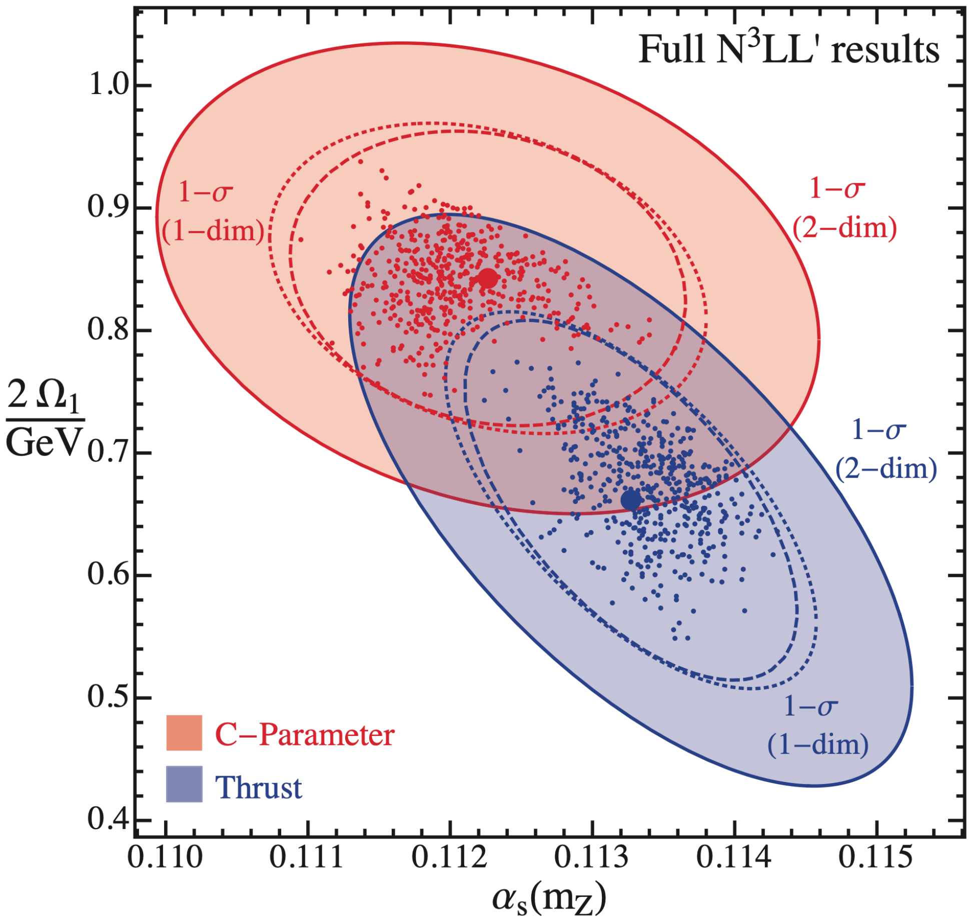
Agreement beyond the fit region



Thrust vs. thrust moments



Thrust vs. C-parameter

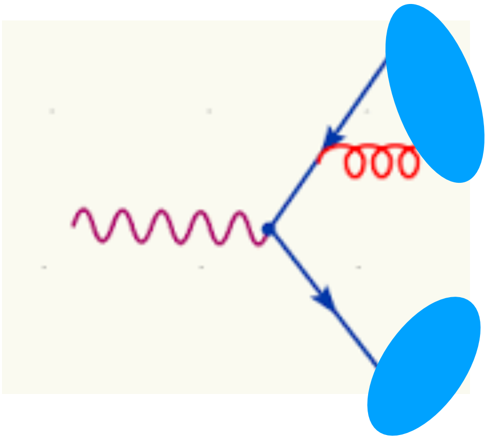


What could be going on?

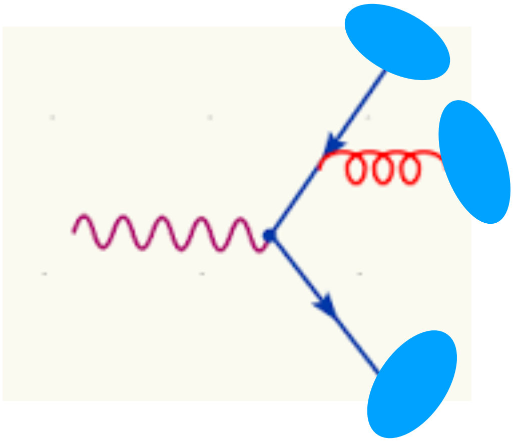
Issue 1: Power corrections for 2-jets (Ω_1) versus 3-jets ($\neq \Omega_1$)

- perturbative dijet cross section dominates

- fit used dijet power correction for all τ



3-jet power corrections in fit region?



- Luisoni, Monni, Salam (2021)

Use dispersive model to “compute” power correction at C = 3/4

Find within that model that: $\Omega_1^{2-\mathrm{jet}} \simeq 2\Omega_1^{3-\mathrm{jet}}$ (model has one NP param.)

Note: In QCD 3-jet configuration is described by operators with 6 Wilson lines (generalizing the 4 lines for dijets), and hence ratio is non-perturbative too.

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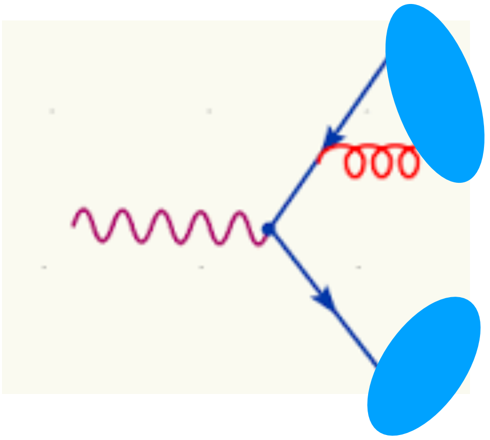

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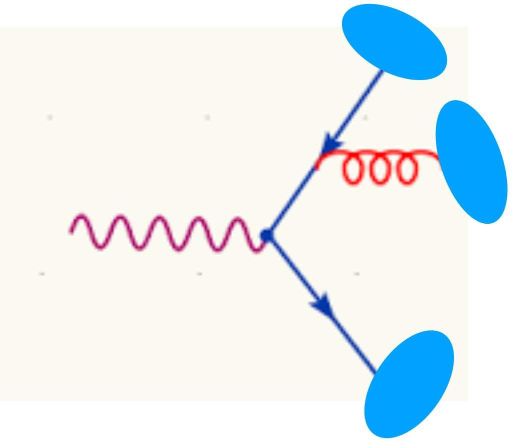


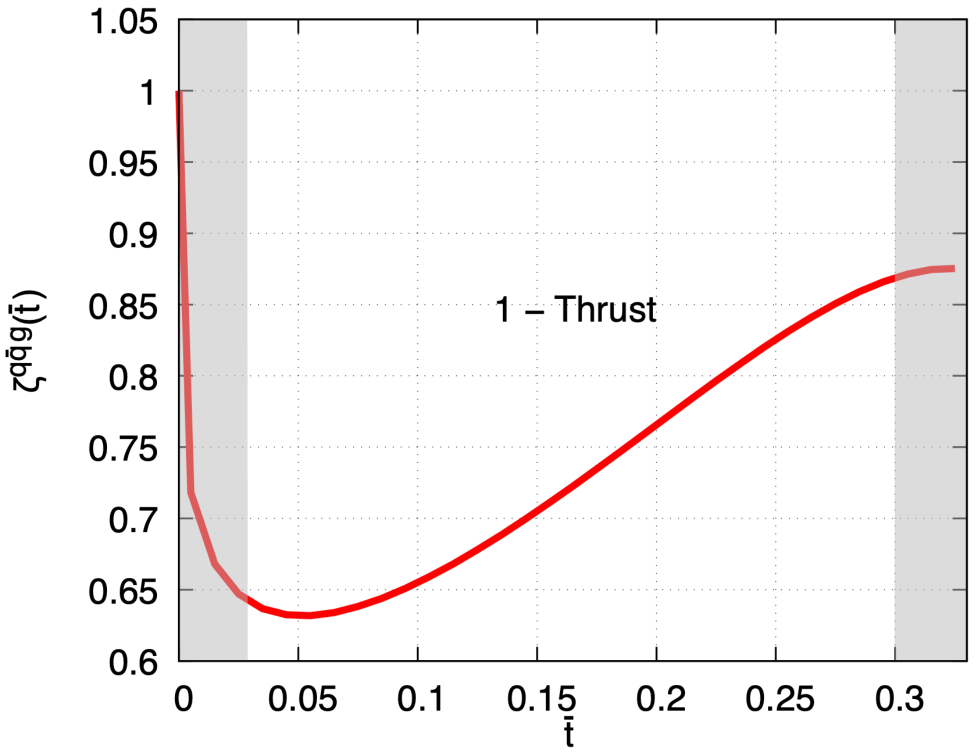
- Caola et.al. (2022)

Large β_0 limit, probe power corrections with small gluon mass

$$\frac{d\sigma}{d\tau} = \frac{d\hat{\sigma}}{d\tau}\left(\tau - \frac{\zeta_\tau(\tau)2\Omega_1}{Q}\right)$$

Rapid change at small τ.
Increases by 30% in fit region.

Note: method implicitly assumes 3-jets can always be resolved



Issue 1: Power corrections for 2-jets (Ω_1) versus 3-jets ($\neq \Omega_1$)

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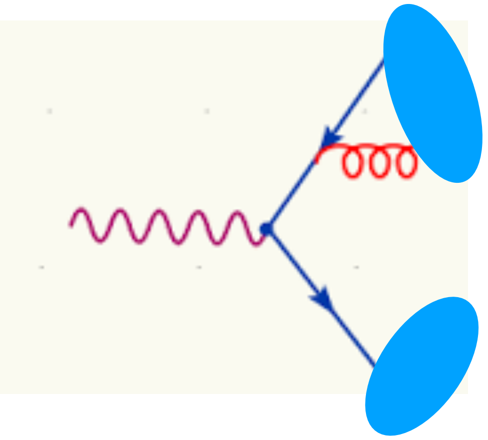
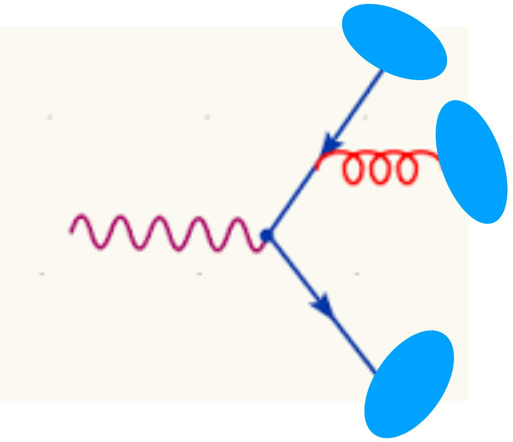


3-jet power corrections in fit region?

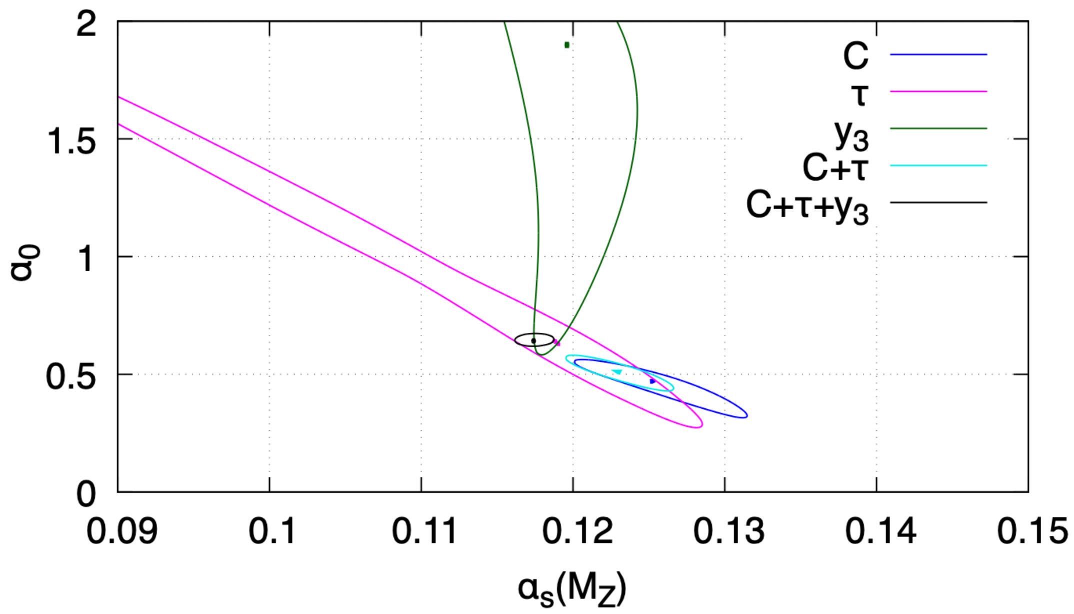


- **Nason, Zanderighi (2023)**

  Analysis using ALEPH at $Q = m_Z$, various event shapes

  Implements 3-jet $\zeta(\tau)$ model

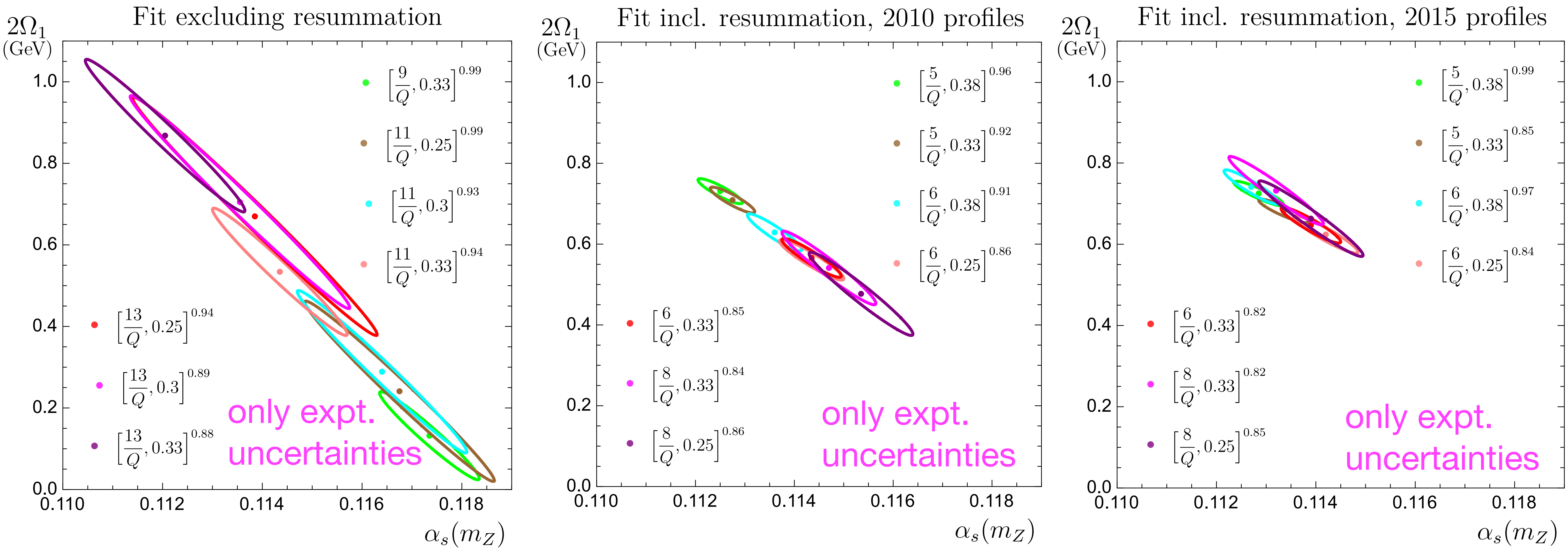
  Conclude that 3-jet power correction can lead to larger α_s



Note: Neglects resummation
Does not correlate theory across data bins

Issue 2: Fit range

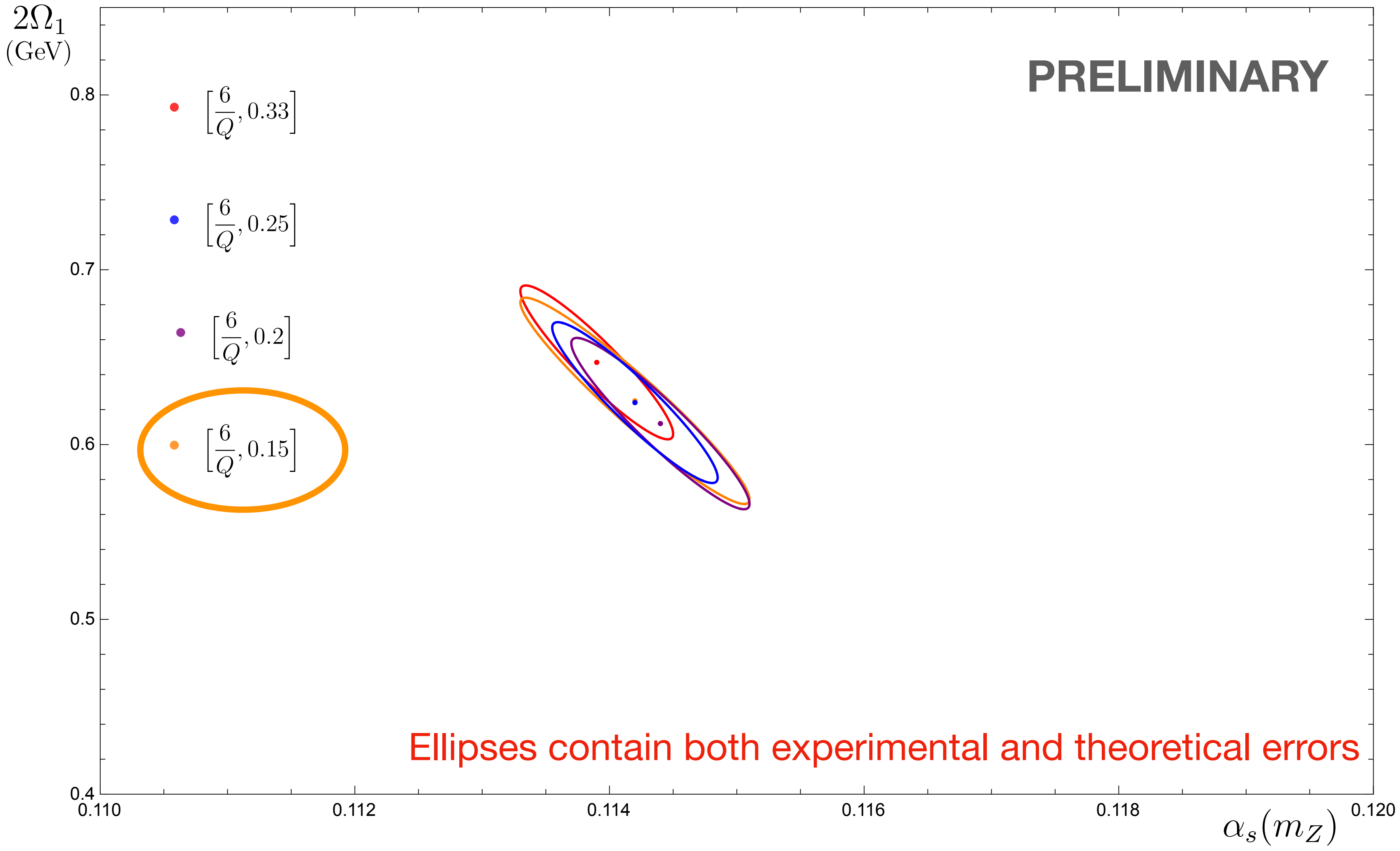
Benitez-Rathgeb, Hoang, Mateu, IS, Vita
(work in progress)



$[\tau_{\min}, \tau_{\max}]^{\chi^2/dof}$ use of improved 2015 theory leads to stability

Opens up the possibility of a fit that focuses on region that is clearly dijet: $\tau \in [6/Q, .15]$

2024 Analysis — 3-Loop NS from Colorful + new Profile functions



- Increased experimental uncertainty due to smaller fit region [6/*Q*, .15]:

$$\alpha_s = 0.1142 \pm 0.0006_{\text{pert}} \pm 0.0009_{\text{exp}+\Omega_1} \pm 0.0004_{\,\text{3jet}} = 0.1142 \pm 0.0012_{\text{tot}}$$

$\langle \chi^2 \rangle/\text{dof} = 0.86$

Compare to 2010 thrust fit ([6/*Q*, .33], without QED corrections):

$$\alpha_s = 0.1140 \pm 0.0008_{\text{pert}} \pm 0.0005_{\text{exp}+\Omega_1} \pm 0.0001_{\text{had}} = 0.1140 \pm 0.0010_{\text{tot}}$$

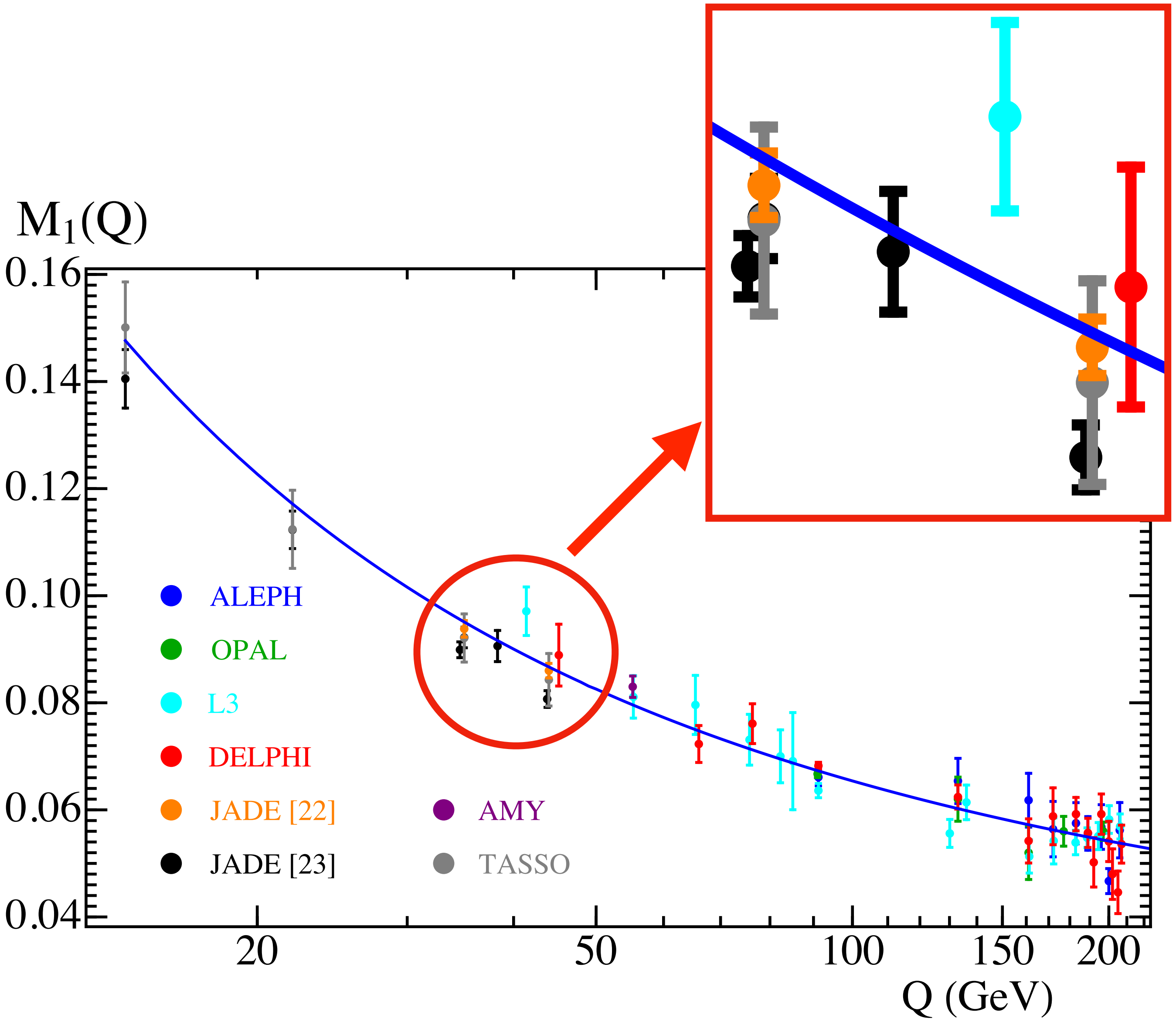
- Estimated size for 3-jet power corrections here ≃ 0.0004

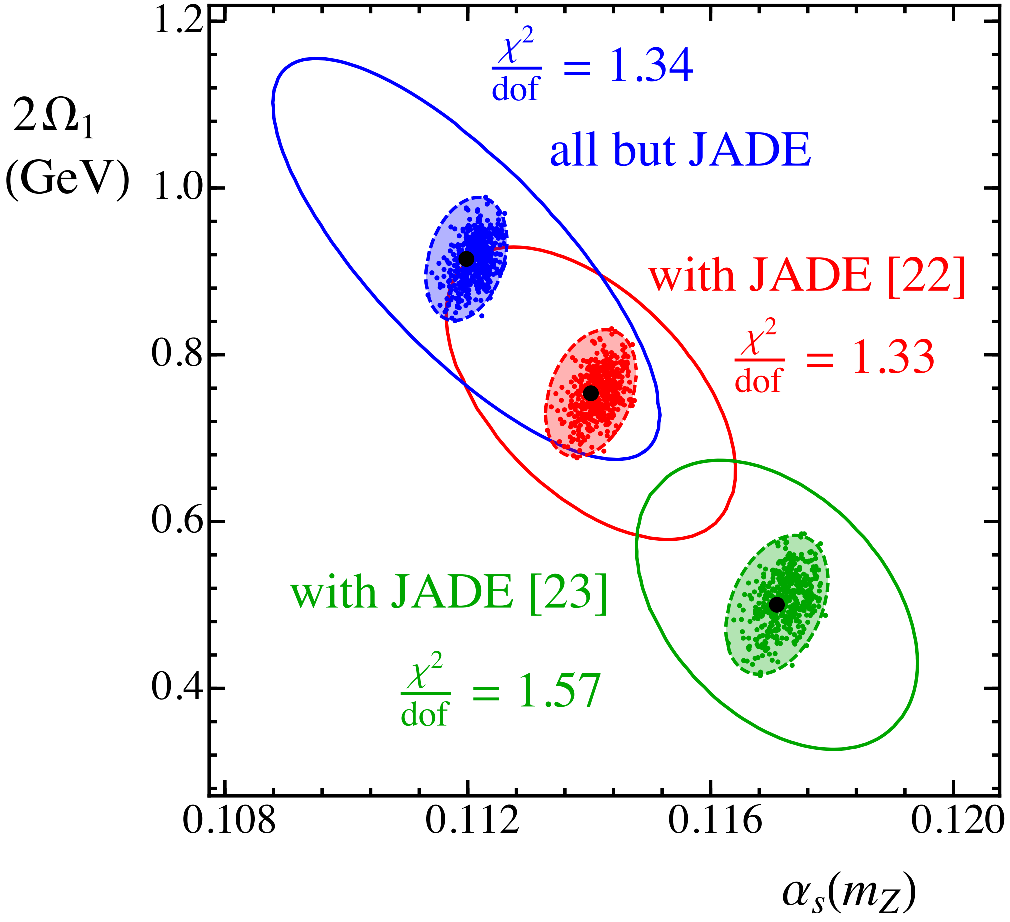
Puzzle remains

Issue 3: Worth reconsidering systematics.

At the time of LEP, MC generators (used for calibration, determining acceptances, resolution ...) were much less sophisticated than those we have now.

An analysis that updated this was done by JADE for thrust moments in 2009 [23] (updating original 1998 [22])



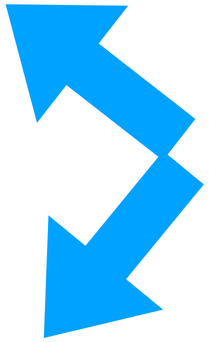


Summary & Outlook

$\alpha_s(m_Z)$ from e^+e^- jets, looking forward to the next e^+e^- collider

Theory wish list

- More rigorous treatment of 3-jet power corrections is crucial (including transition to 2-jet)
- Analyses with more observables (heavy-jet mass, EECs, ...), and combined observables while including all theory correlations
- Subleading power resummation, extension to N^{4}LL, ...



Synergy!

Experimental wish list

- More kinematic info: multi-differential distributions, jet substructure, ...
- Full correlation matrices
- Impact of using modern Monte Carlo generators when comparing to LEP