Training spiking neural networks via adjoint sensitivity analysis

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subMIT IAP Workshop 2024

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Big idea: The brain is extraordinarily effective on a modest energy budget

Luccioni et al. 2022

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$1e^{14}$ synapses

$4e^{3}$ MJ/year (human)

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Big idea: The brain is extraordinarily effective on a modest energy budget

GPT-3

$1e^{14}$ synapses

$2e^{11}$ parameters

$4e^{3}$ MJ/year (human)

$4e^{6}$ MJ (training)

Luccioni et al. 2022

How are biological neurons different from artificial neurons?

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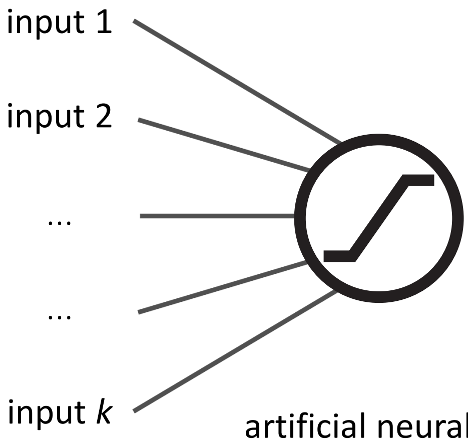
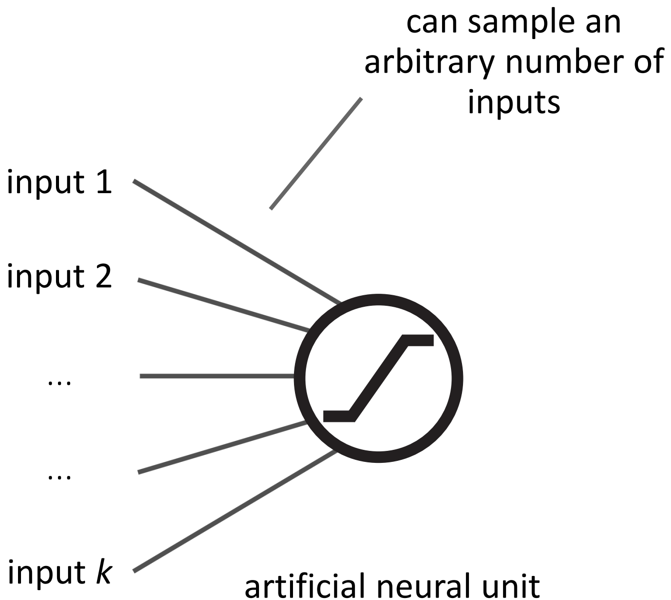


artificial neural unit

aka

“point” neuron

How are biological neurons different from artificial neurons?



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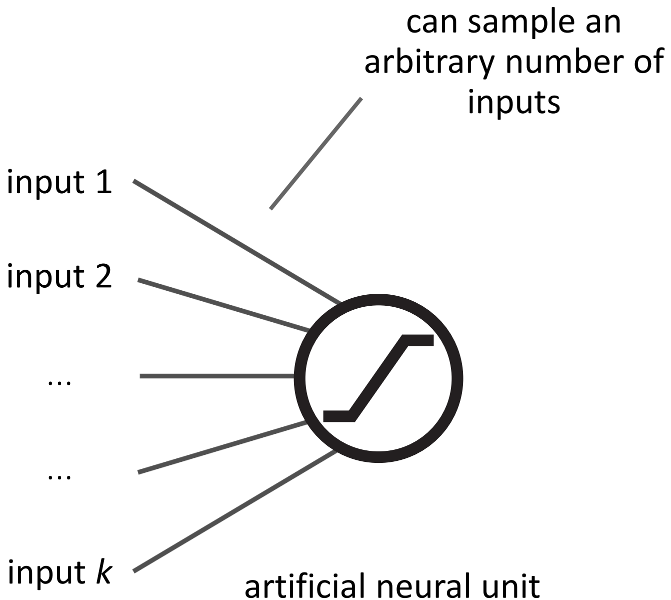

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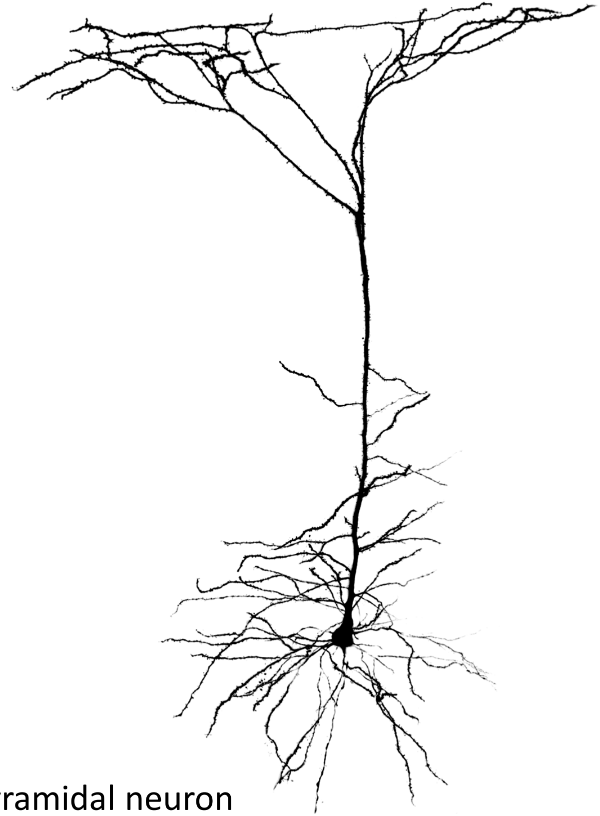

layer V pyramidal neuron

Mark Harnett

Vardalaki et al. 2022

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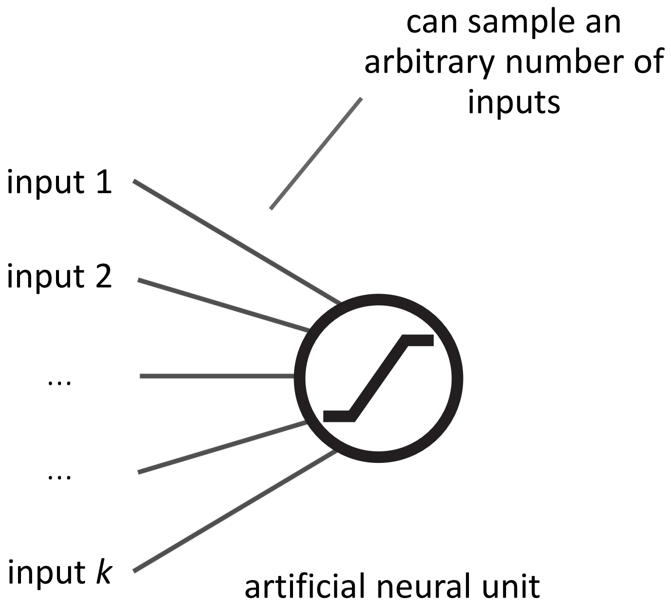

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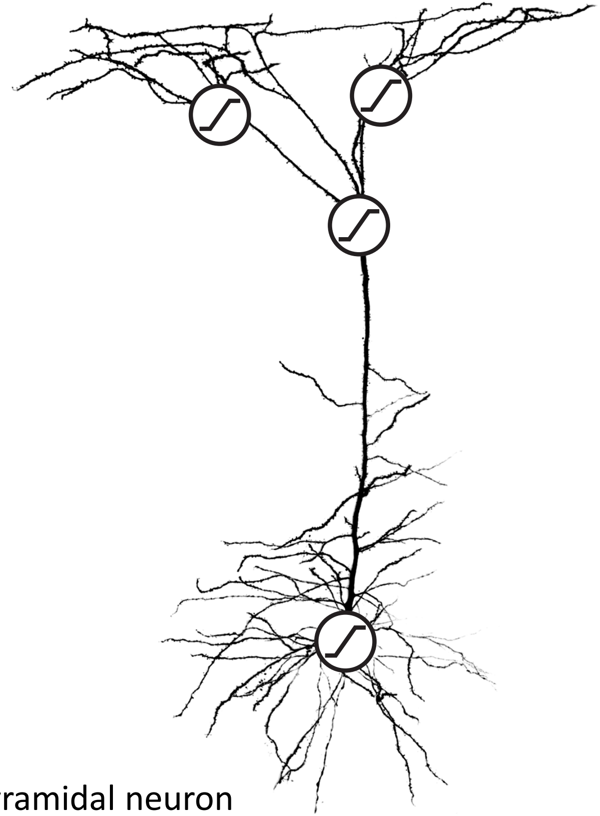

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Why do we have dendrites?

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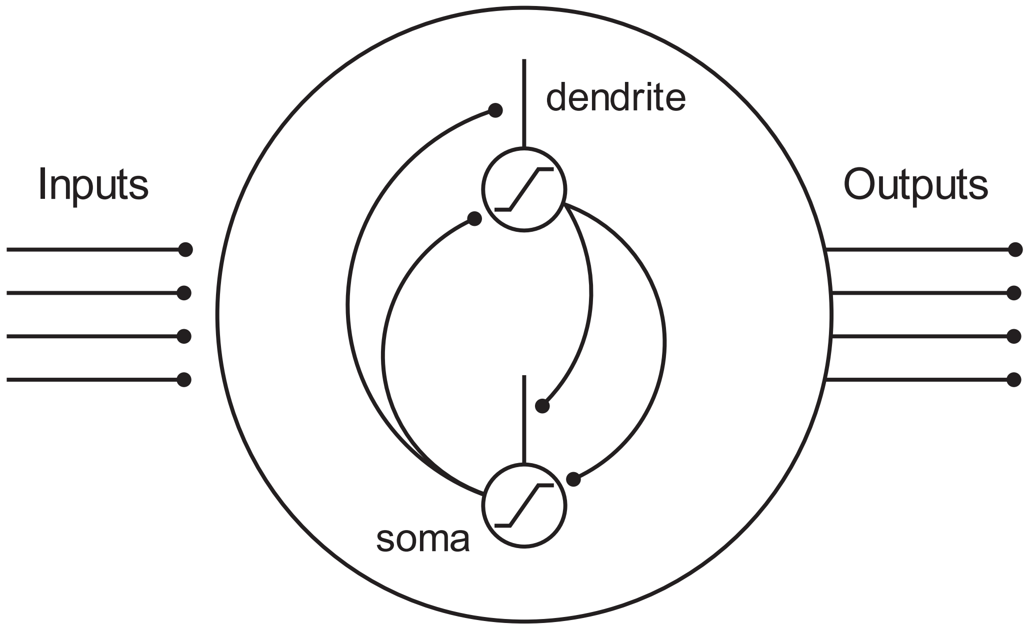
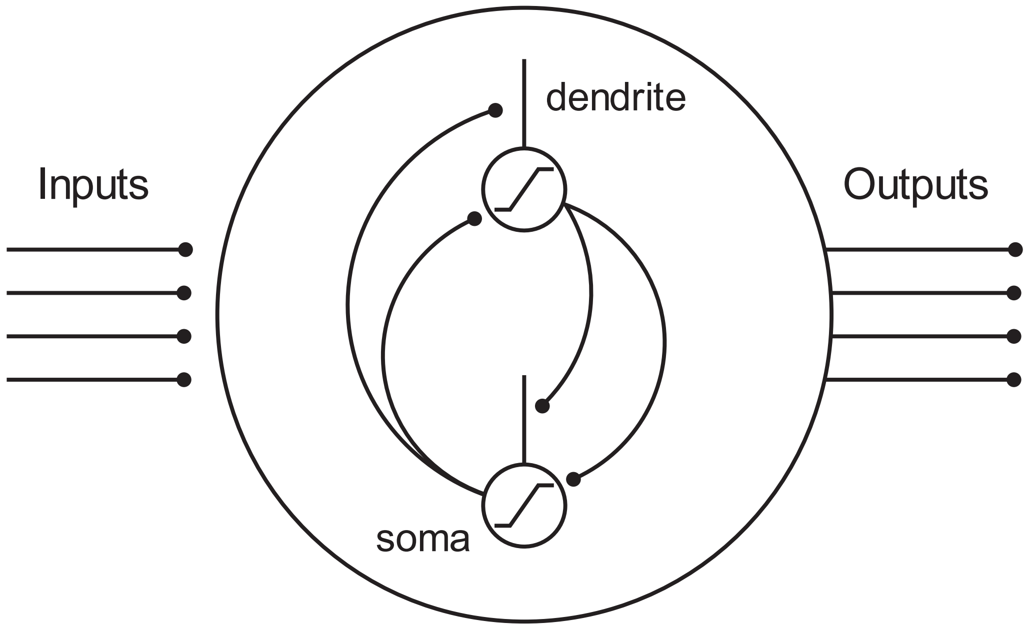
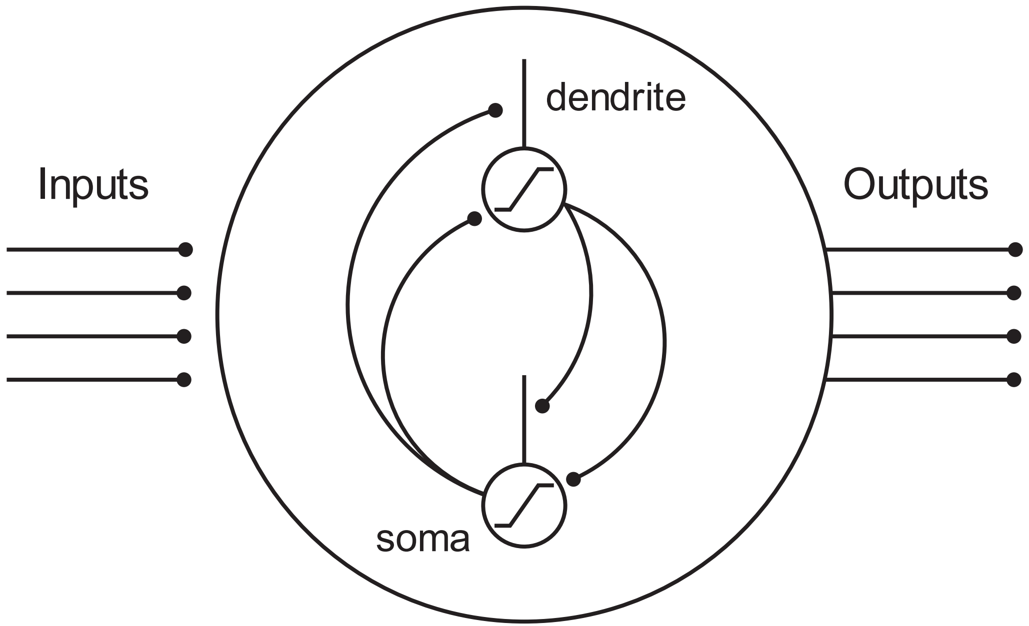
↓

What computational role do dendrites serve?


Inputs
dendrite
Outputs
soma



What do dendrites get us?



What do dendrites get us?

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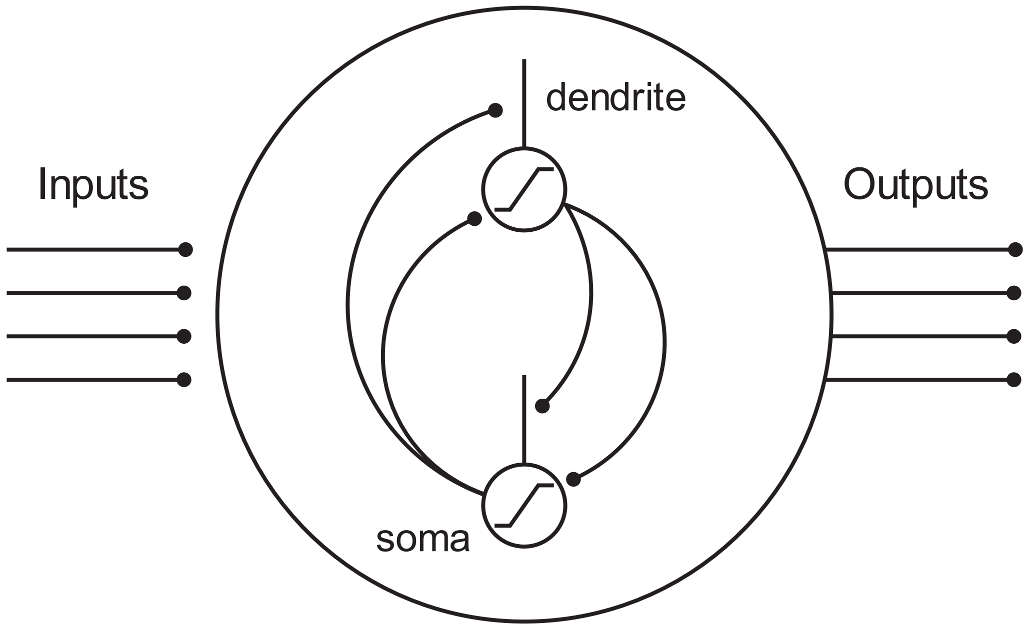

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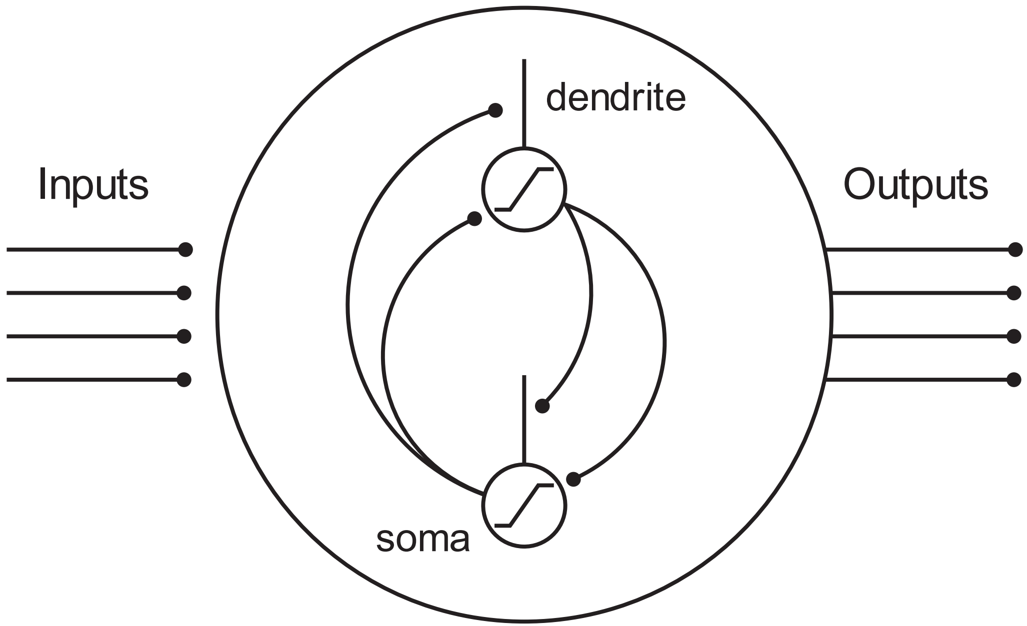

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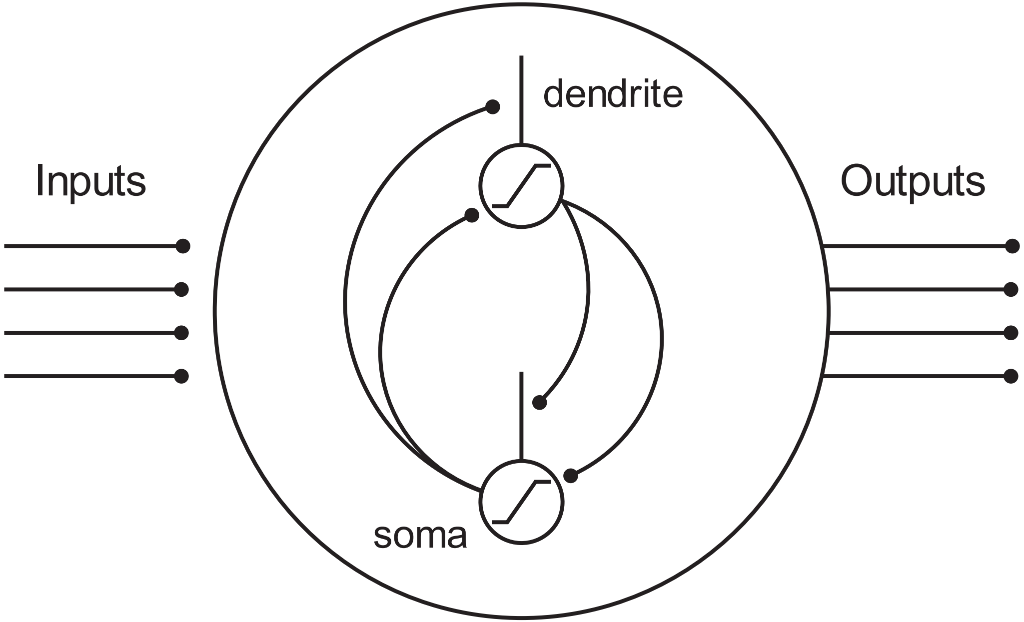

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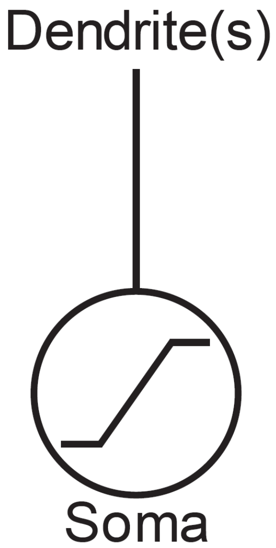
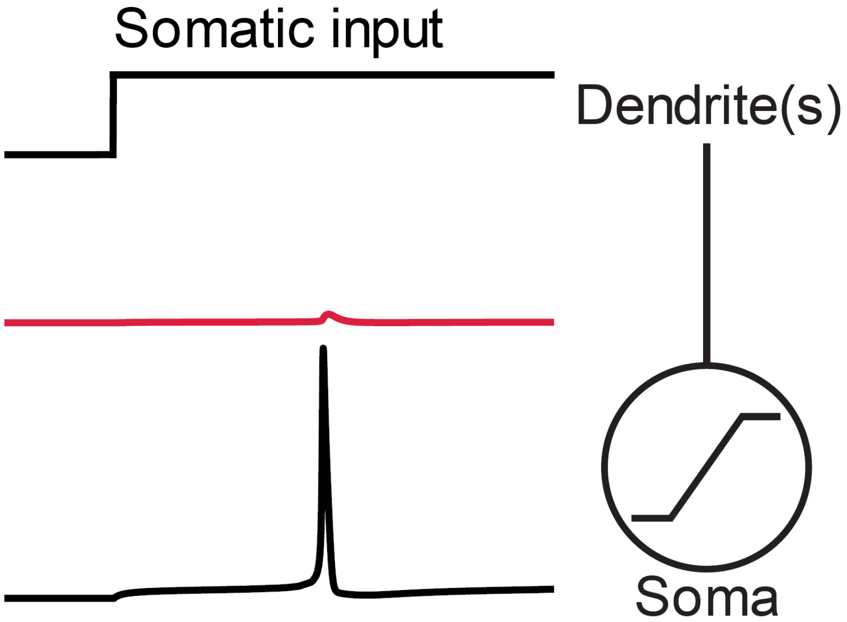

Dendrite(s)
Soma


Somatic input
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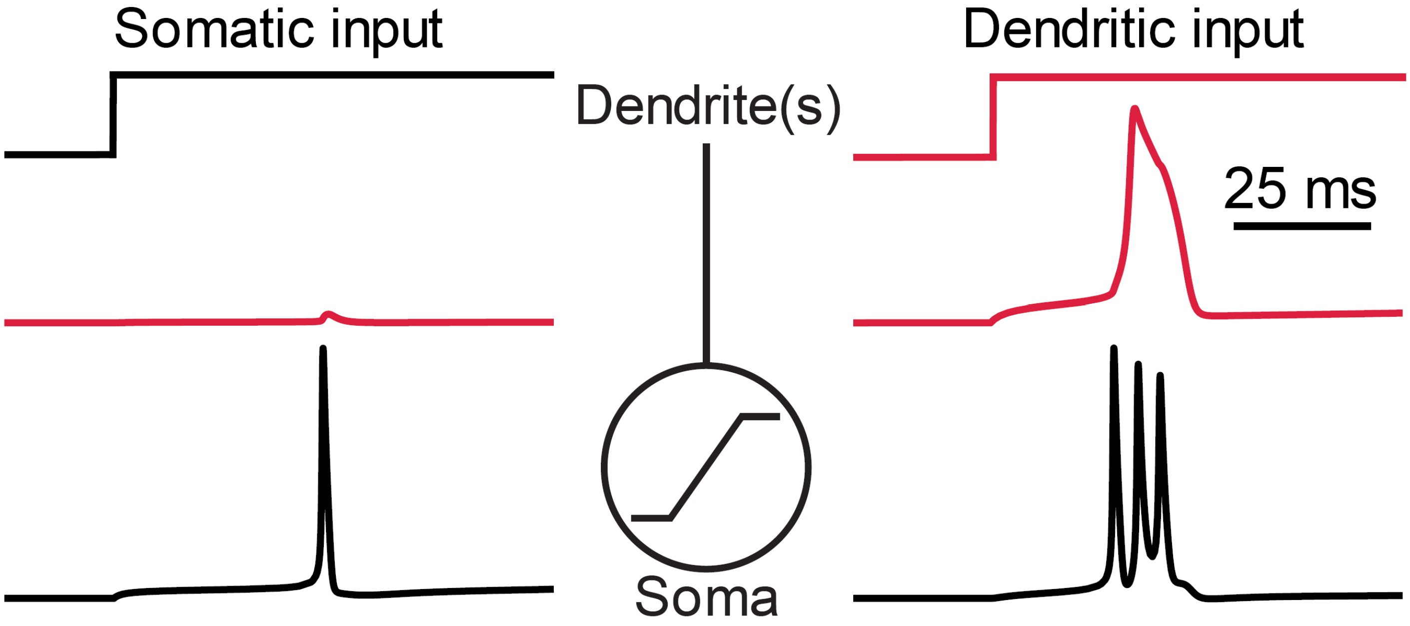
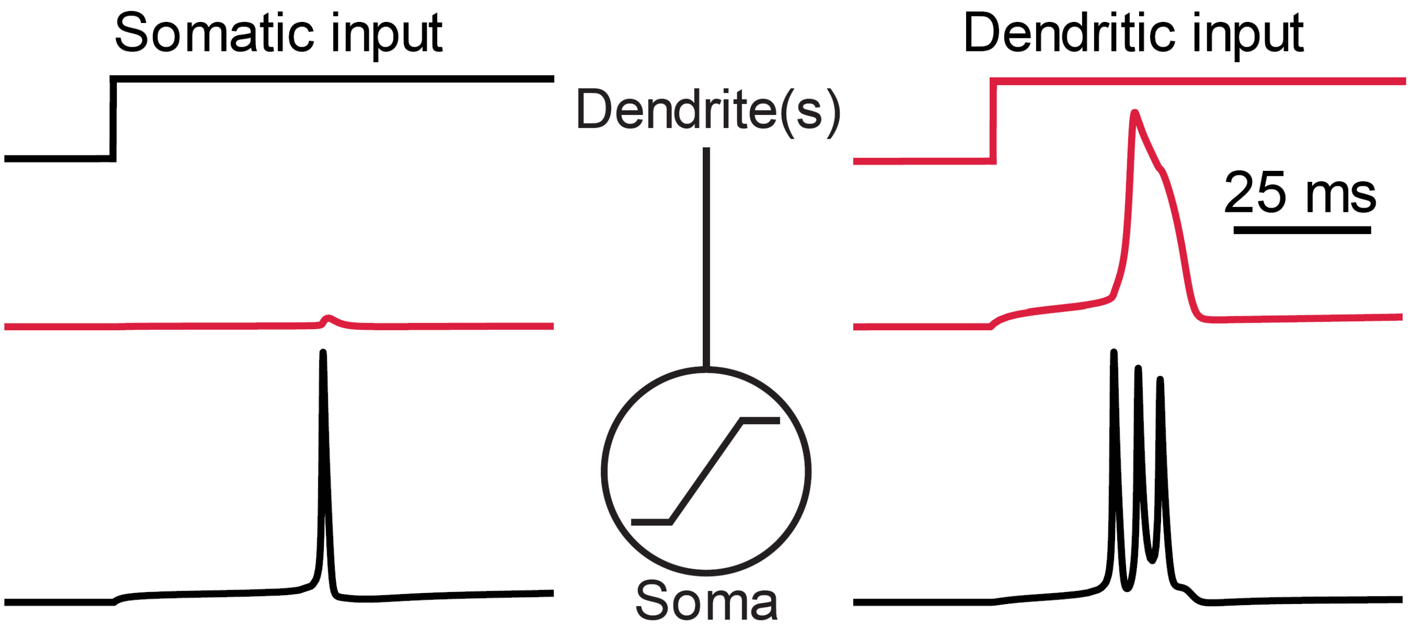


Somatic input
Dendrite(s)
Dendritic input
25 ms
Soma

A minimal but biologically-realistic model of neural spiking



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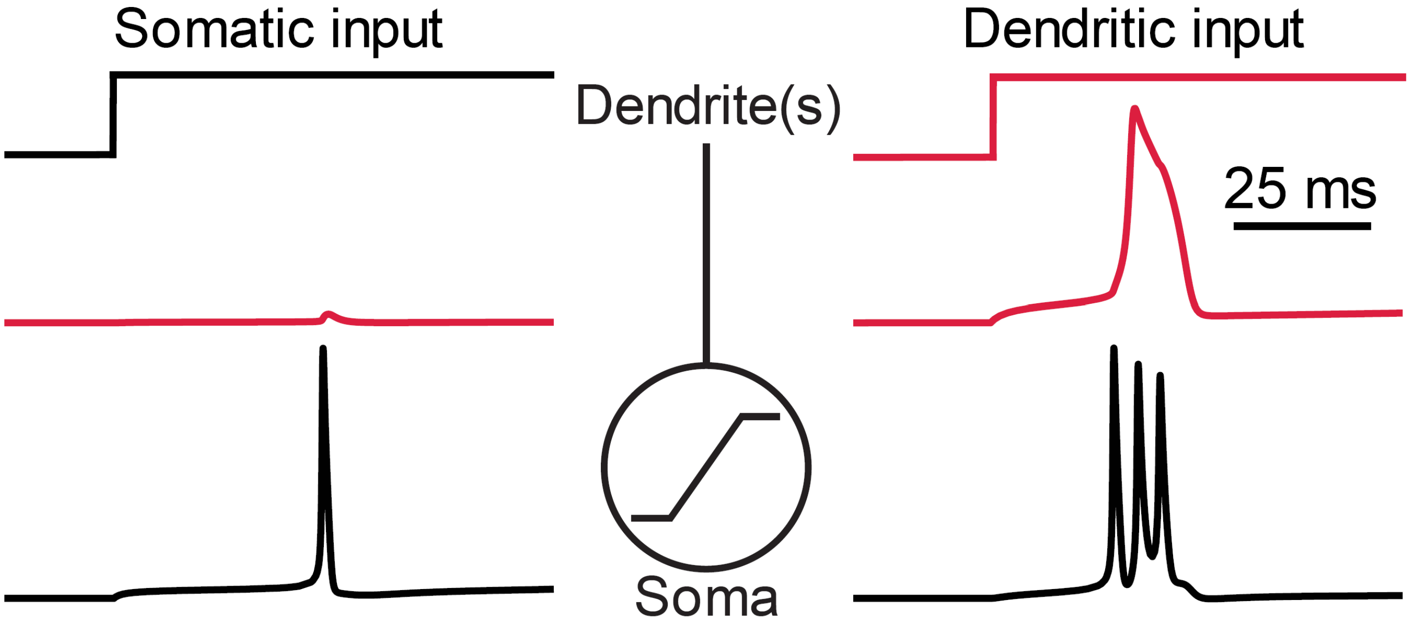

Implemented in Julia!

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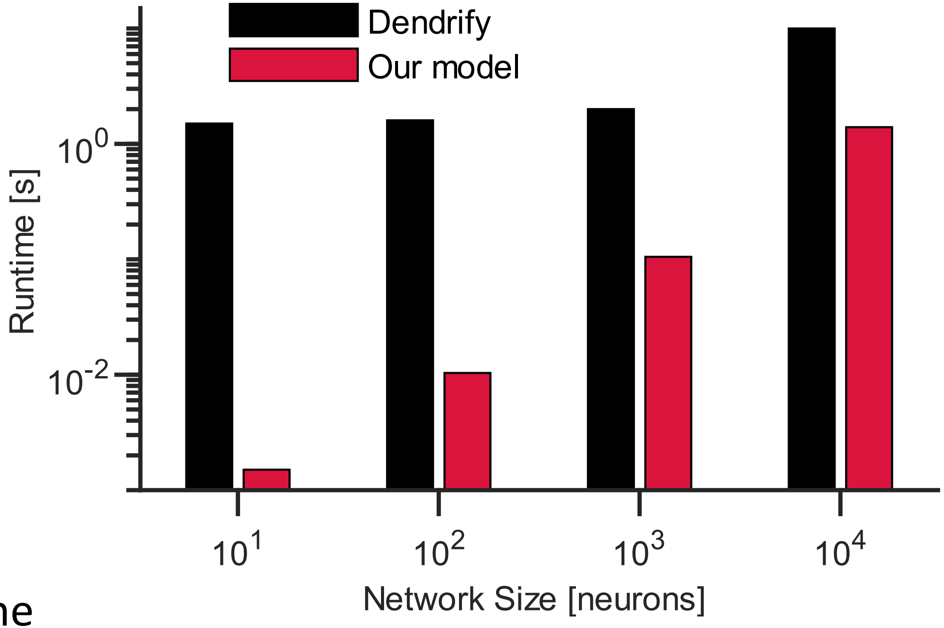

- 1.0 s simulation time
- Euler with 0.1 ms time step
- Neglecting recurrence for fair comparison
- Dendrify timings include “build time”

Pagkalos et al., 2023

Training via adjoint sensitivity analysis

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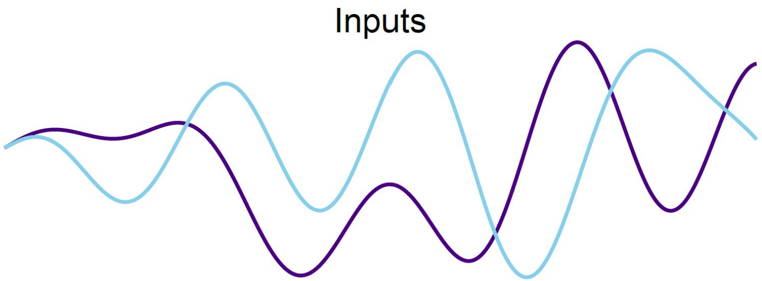

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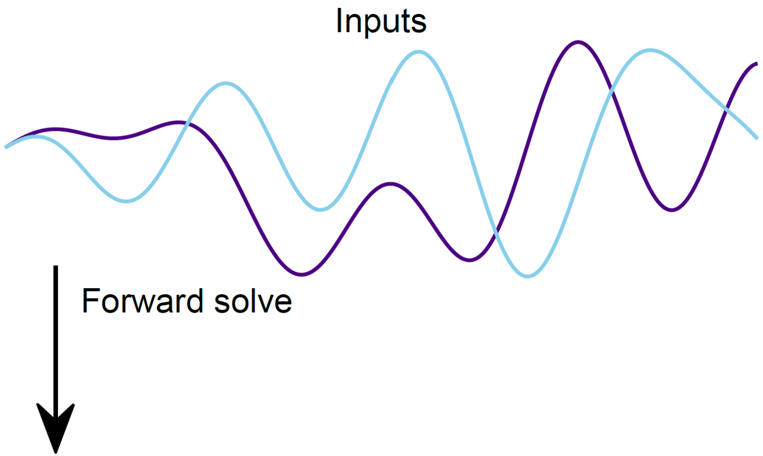

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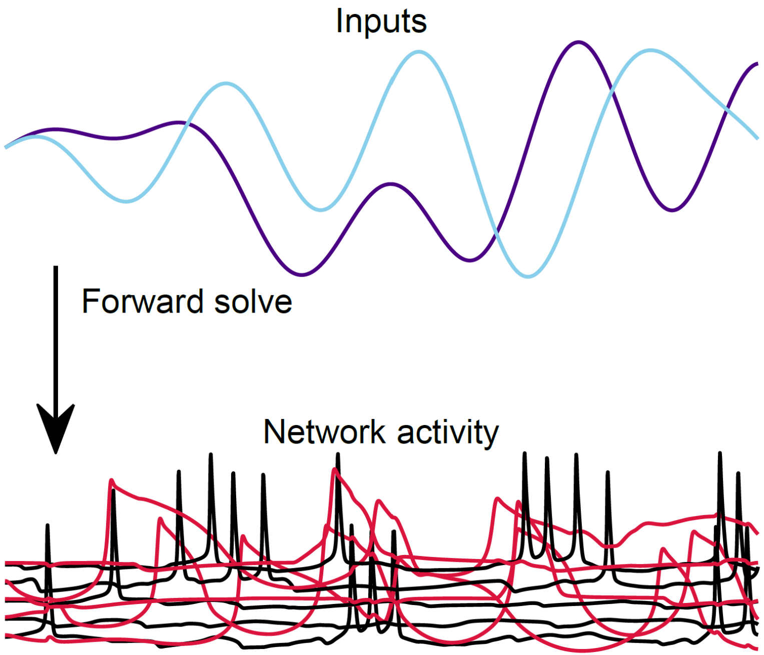

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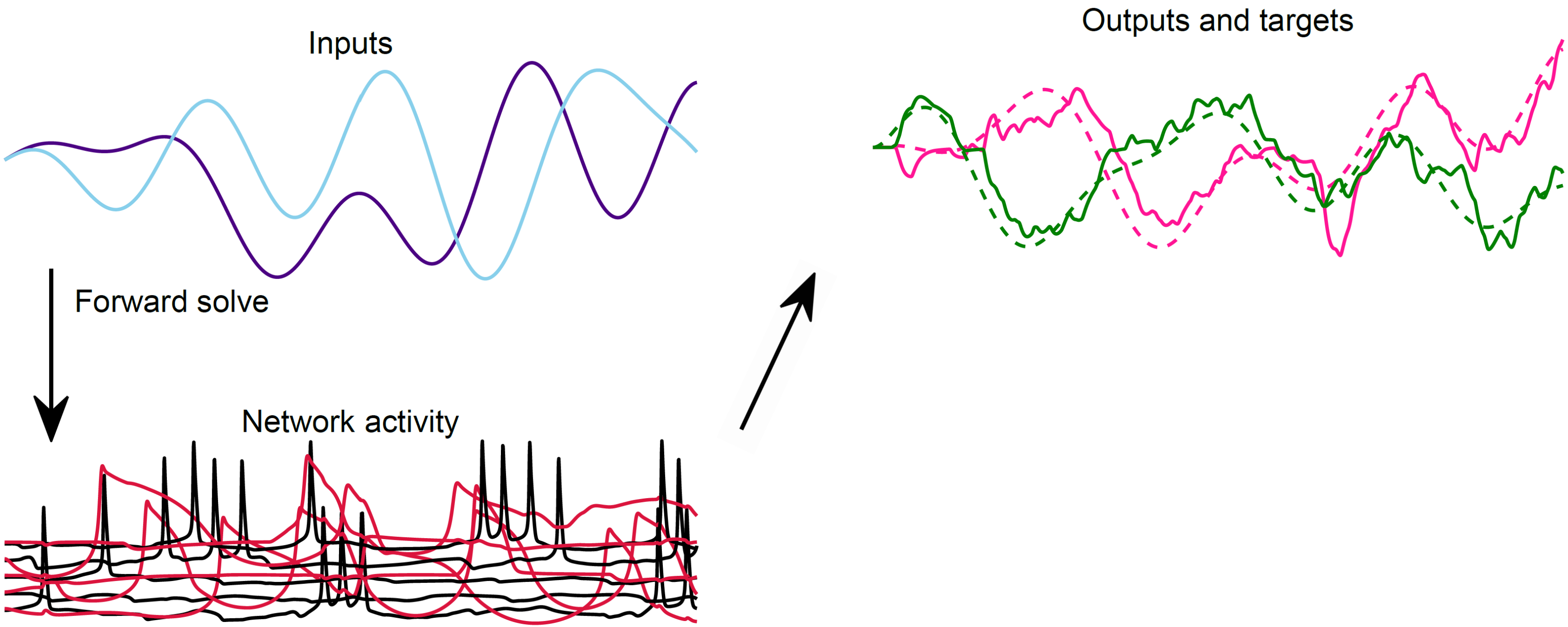

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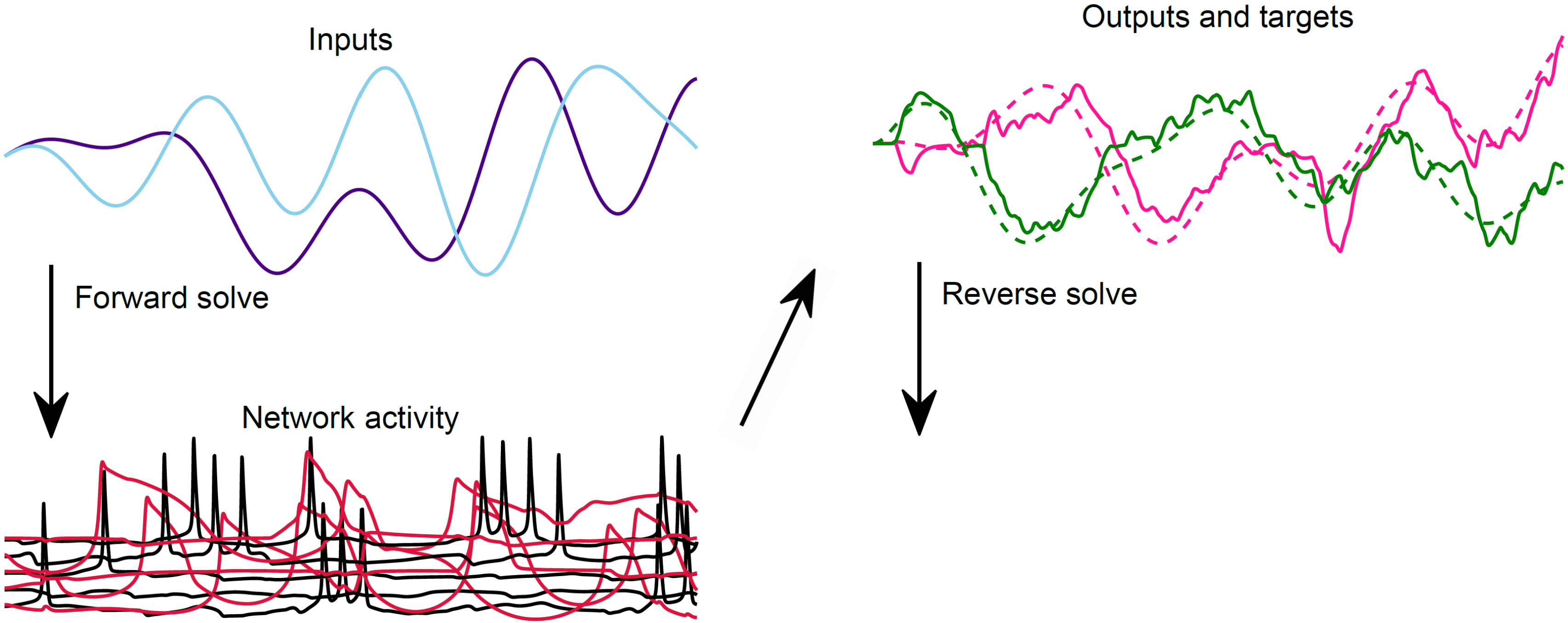

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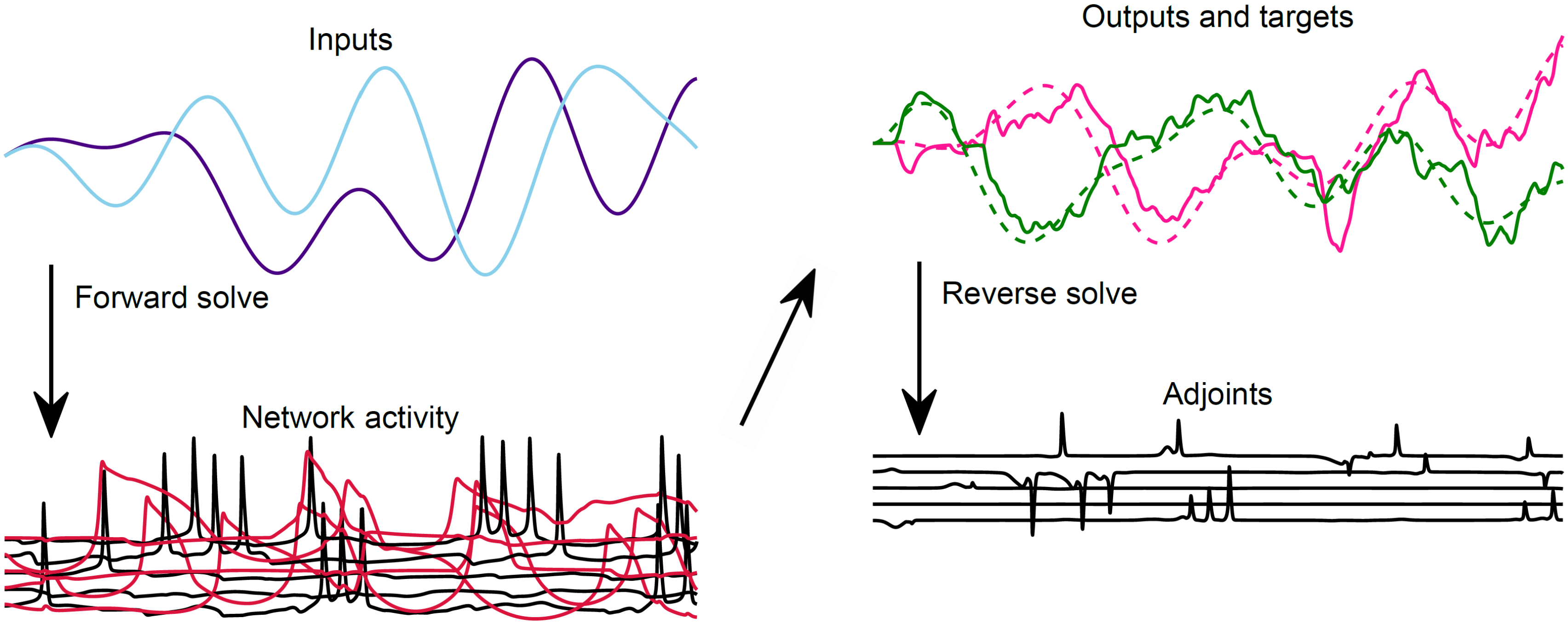

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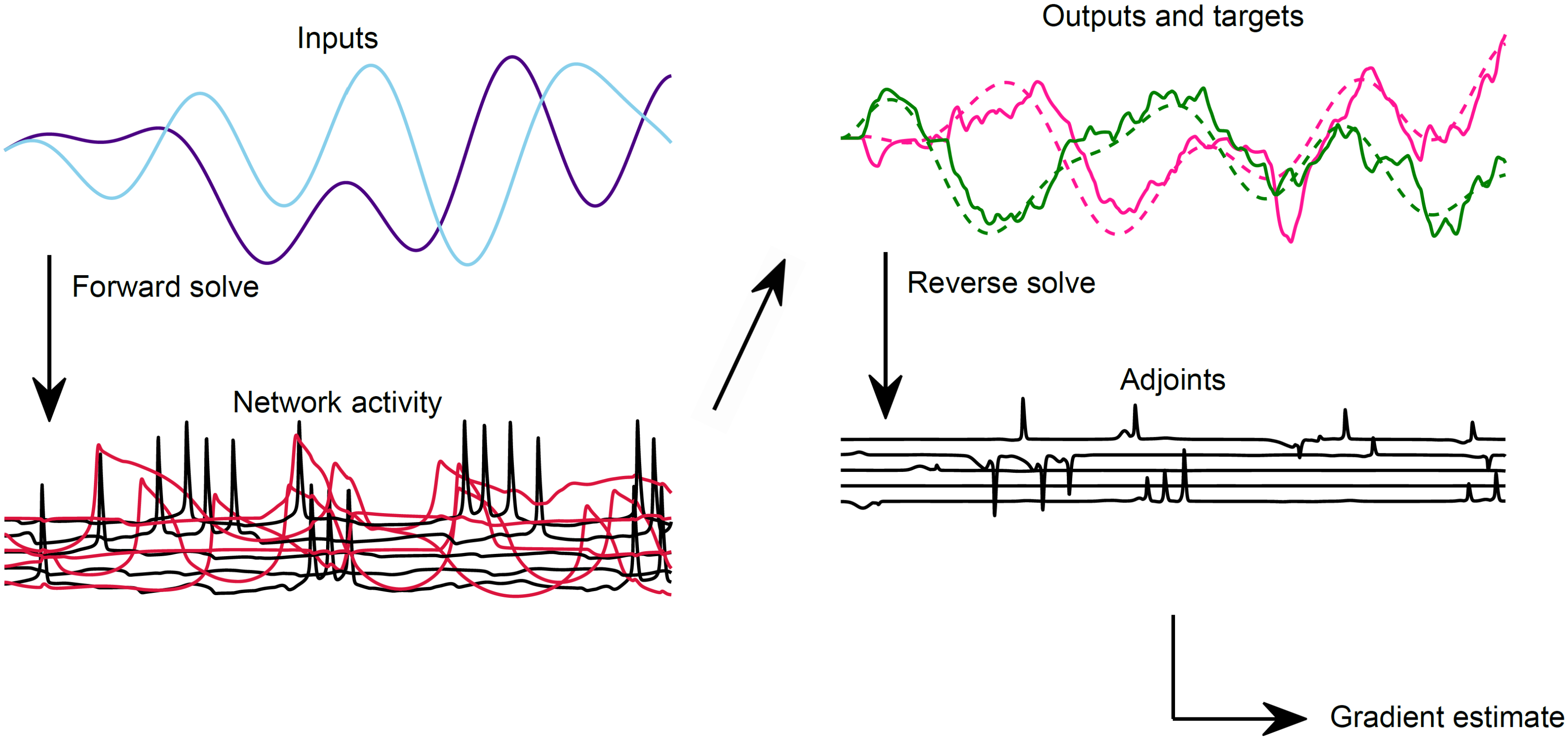

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6. Repeat >10,000 times

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- Approach: train many networks simultaneously

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- Will typically submit 100 – 300 batch requests at a time
- 1 – 3 cores per job
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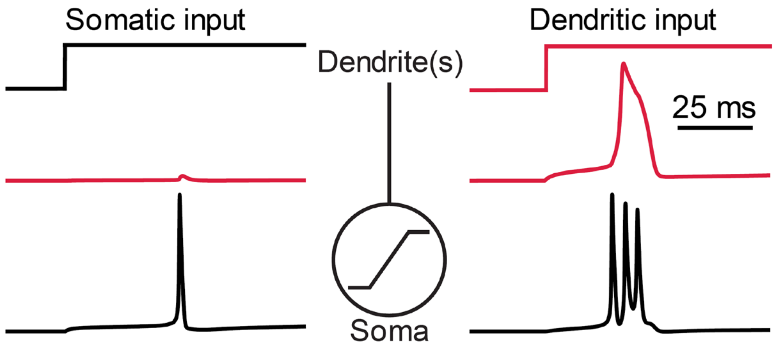
- subMIT’s cores are relatively fast, so I use it on my most expensive networks

Questions? Feedback?

A minimal but biologically-realistic model of neural spiking



$$\dot{v}_i = (a_i(v_i - b_i)^2 + c_i)(v_i - 1) - j_i u_i(v_i - l_i) - k_i w_i(v_i - m_i) + \sigma_{ji}(v_j - \mu_j) + J_i$$

$$\tau_{u_i}\dot{u}_i = d_i(v_i - e_i)^4 + f_i - u_i$$

$$\tau_{w_i}\dot{w}_i = g_i(v_i - h_i)^2 + i_i - w_i$$

$$[i, j \neq i] \in [\text{soma}, \text{dendrite}]$$

Training via adjoint sensitivity analysis

Given an instantaneous loss *L*, we seek to minimize a cost function *C* such that

$$\delta C = \delta \int_{t_1}^{t_2} dt\, L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{\theta}) = 0$$

We can perform a Legendre transformation on *L* to obtain a “Hamiltonian” that introduces new adjoint variables ***p***

$$H(t, \boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\theta}) = \dot{\boldsymbol{x}} \cdot \boldsymbol{p} + L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{\theta})$$

where ***p*** are defined by

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{x}}$$

The parameter gradients of the initial cost function can then be computed as

$$\frac{\partial C}{\partial \boldsymbol{\theta}} = \int_{t_1}^{t_2} dt\, \frac{\partial H}{\partial \boldsymbol{\theta}}$$